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KEY STAGE 3
Mastering
Mathematics

Extend

Practice Book

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Contents

CHAPTER 1

Sequences

1.1 Arithmetic sequences

The position of a term in a sequence is the value of n for that term.

1 The position-to-term formula for a sequence is $3n - 5$.
   a 106 is in the sequence.
   What number term is it?
   b Explain how to use the position-to-term formula to show that 999 is not in the sequence.

2 Muhammad says, '$3n + 4$ and $3n + 5$ are the same sequence because they both go up in 3s'.
   Without writing out either sequence, explain why Muhammad is wrong.

3 Gina says, '$5n + 4$ and $5n + 9$ are the same sequence because they both go up in 5s and they both contain 14'.
   Without writing out either sequence, explain why Gina is wrong.

4 Matilda says that the first three terms of a sequence are in the ratio 2:5:7.
   Explain why Matilda's sequence is not an arithmetic sequence.

An expression is something involving algebra. Here it has n somewhere and some numbers.

5 An arithmetic sequence has 4th term 30 and 7th term 51.
   a Find an expression for the nth term.
   b 744 is in the sequence.
   What number term is it?
   c Is 88 888 in the sequence?
   Justify your answer.

6 An arithmetic sequence has 6th term 79 and 10th term 65.
   a Find an expression for the nth term.
   b 2 is in the sequence.
   What number term is it?
   c Is 55.5 in the sequence?
   Justify your answer.

7 Scarlett runs a taxi service.
   She charges £3.50 for a 1 km journey and £5 for a 2 km journey.
   What formula does she use to work out the cost of a journey of d km?

8 Is it true that the position-to-term formula $2n + 1$ generates the odd numbers?
   Justify your answer.

In the following diagrams, 'matchsticks' are used for the sides of the squares.

9 **a** Write down the position-to-term formula for the number of squares in the terms of this sequence.
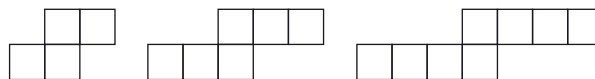
   b Write down the position-to-term formula for the number of 'matchsticks' used to make each term of the sequence in part **a**.
   c Write down the position-to-term formula for the number of squares in each term of this sequence.

   d Write down the position-to-term formula for the number of 'matchsticks' used to make each term of the sequence in **c**.
   e Explain why the answers to parts **a** and **c** are the same.
   f Explain the difference between the answers to parts **b** and **d**.

10 Find the sequence.
   The ratio of the first term to the third term is 1:8.
   The ratio of the second term to the fifth term is 3:10.
   The fourth term is 23.

11 Find the sequence.
   The ratio of the first term to the third term is 4:3.
   The ratio of the second term to the fourth term is 7 : 5.
   The fifth term is 10.

12 These four numbers generate arithmetic sequences that fill the rows and columns that form the rest of the table.

1	3		
4	6		

   a Copy and complete the table.

   b Write down the position-to-term formula for the numbers in each column.

   c What do you notice about your answers to part **b**?

   d Where is the sequence with position-to-term formula $5n - 4$?

13 The numbers in the table are generated by the four numbers in the top left-hand corner.

		6	3
		7.5	4.5
15	12	9	6
16.5	13.5	10.5	7.5

   a What are those numbers?

   b Write down the position-to-term formula for the numbers in each row.

   c Describe the numbers in the first and third rows.

   d The numbers in the first and third rows are the same type of number.

      Explain why the position-to-term formulas are different.

14 Write down the position-to-term formula for the sequence:

$a, a + d, a + 2d, a + 3d, \dots$

15 $n + 2, 3n - 1, 5n - 4, 4n + 2 \dots$

is an arithmetic sequence for a particular value of n.

Find n.

1.2 Other sequences

Remember
A position-to-term formula is an expression involving n.

1 **a** Write down the next term in the sequence 2, 5, 10, 17, …

   b Write down the position-to-term formula for the sequence.

   c Is 100 in the sequence?

      Justify your answer.

2 Write down the two missing terms in the sequence:

$8, 4, 2, \boxed{}, \boxed{}, \dfrac{1}{4}, \dfrac{1}{8}.$

3 In a sequence, the ratio of the first term to the second term is the same as the ratio of the second term to the third term.

What kind of sequence is it?

4 Write down the next two terms in the sequence: $a, b, a + b, a + 2b, \dots$

5 The sequence with position-to-term formula $n^2 + 3$ and the sequence with position-to-term formula $n^2 + 7$ are added.

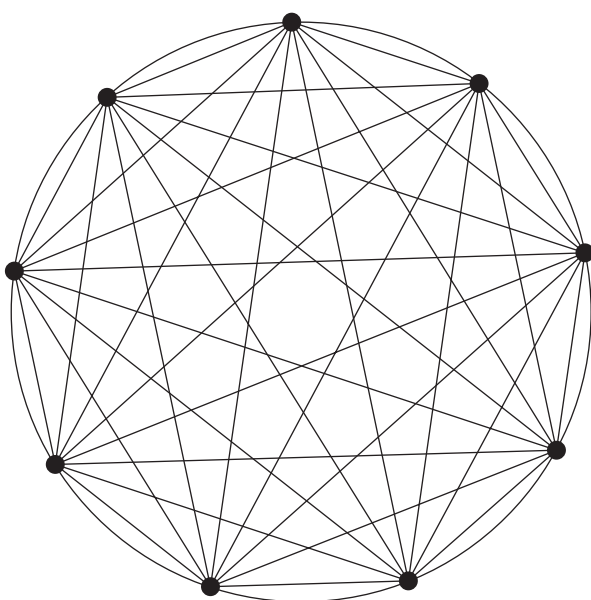What is the position-to-term formula of the new sequence?

Remember
Each term in a Fibonacci sequence is created by adding the two previous terms.

6 The Fibonacci sequence is 1, 1, 2, 3, 5, 8, …

   a Work out the differences between the terms.

   b What do you notice?

   c Explain the pattern you noticed in part **b**.

7 The first term of a Fibonacci sequence is 4 and the fifth term is 29.

Find the second, third and fourth terms.

8 The first term of a Fibonacci sequence is -1 and the fifth term is 4.

Find the second, third and fourth terms.

9 Find the sequence in the following example.

The product of the first and fifth terms is the same as the product of the second and fourth terms.

The fourth term is three times the third term, which is the square root of the fifth term.

The first term is 1.

10 Find the sequence in the following example.

The terms increase by an extra 2 each time.

The ratio of the first two terms is 2:3.

The ratio of the third and fourth terms is also 2:3.

The fifth term is 5 times the first term.

11 A frog can jump up sets of steps 1 step or 2 steps at a time.

a Show that the frog has two ways to jump up a set of 2 steps.

b Show that the frog has three ways to jump up a set of 3 steps.

c How many ways can the frog jump up:

i 4 steps

ii 5 steps

iii 6 steps?

d Write down the number of ways that the frog can jump up a set of 7 steps.

12 The diagram shows a mystic rose with nine points.


a How many straight lines are in the diagram?

b Fill in the table to show the number of straight lines in mystic roses with other numbers of points.

Points	2	3	4	5	6	7	8	9
Lines	1			10				

c Write down the position-to-term formula for the sequence of the number of straight lines.

d Use the answer to part **c** to find the formula for the number of straight lines in a mystic rose with N points.

13 **a** There are three people in a room.
Each pair shakes hands once.
How many handshakes are there?

b **i** How many handshakes are there with four people in the room?

ii How many handshakes are there with five people in the room?

iii How many handshakes are there with two people in the room?

c Write down a formula for the number of handshakes when there are N people in the room.

14 **a** Draw the next two shapes in the sequence.
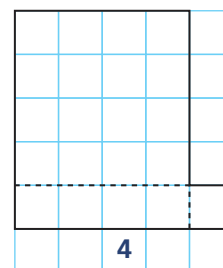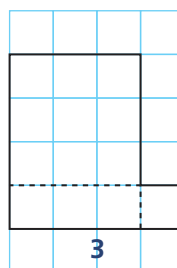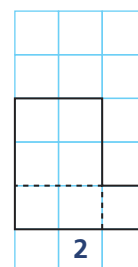
b Write down the sequence formed by the areas of the shapes.

c Write down the calculations shown by the dotted lines on the shapes, starting with $1 + 1 + 1$.

d Find a position-to-term formula for the areas of the shapes.

15 Find the position-to-term formula for the sequence formed by the areas of these shapes.
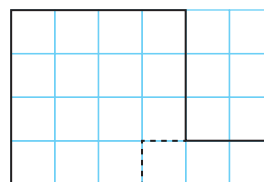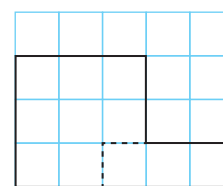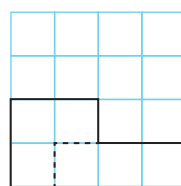
Graphs

● Fluency	● Reasoning	● Problem solving

2.1 Straight line graphs

1 a Write down the coordinates of three points with the y coordinate 3 more than the x coordinate.

b Write down the equation of the straight line through the points with the y coordinate 3 more than the x coordinate.

2 Is the point $(2, -3)$ on the line $y = 3x - 4$?

Justify your answer.

> An appropriate scale goes up in steps that are easy to work with (2s, 5s, 10s) and goes up to a sensible amount.
> 20 would work with the questions here.

3 The exchange rate between British pounds and euros is 1.08 € to £1.

a Explain why you can use the equation $y = 1.08x$ to work out how many euros you get for your money in pounds.

b Plot the line $y = 1.08x$ using values of x from 0 to 20. Choose an appropriate scale.

c What does the x coordinate of a point on the line represent?

d What does the y coordinate of a point on the line represent?

e How many euros do you get for £15?

f How many pounds do you get for 15 €?

4 a Write down the coordinates of three points where the sum of the coordinates is -1.

b Write down the equation of the straight line through the points where the sum of the coordinates is -1.

5 Is the point $(3, -4)$ on, above or below the line $y = 3 - 4x$?
Justify your answer.

6 5 miles is equivalent to 8 km.

a Explain why you can use the equation $y = \frac{5}{8}x$ to convert a distance in km to a distance in miles.

b Plot the line $y = \frac{5}{8}x$ using values of x from 0 to 40.
Choose an appropriate scale.

c What does the x coordinate of a point on the line represent?

d What does the y coordinate of a point on the line represent?

e How many miles is 25 km?

f How many kilometres is 25 miles?

7 Is the point $(-1, 5)$ above or below the line $x + y = 3$?
Justify your answer.

8 a Write down the coordinates of three points where double the x coordinate is 3 more than the y coordinate.

b Write down the equation of the straight line through the points where double the x coordinate is 3 more than the y coordinate.

9 Pencils cost 40p each.

a Explain why you can use the equation $y = 0.4x$ to work out the cost of several pencils.

b Plot the line $y = 0.4x$ using values of x from 0 to 50. Choose an appropriate scale.

c What does the x coordinate of a point on the line represent?

d What does the y coordinate of a point on the line represent?

e How much do 45 pencils cost?

f How many pencils can you get for £10?

The x and y axes divide a graph into quadrants. The first quadrant is where both coordinates are positive. The quadrants are counted anti-clockwise.

10 Find the equation of the straight line.

It passes through the point $(1, 3)$.

It does not pass through the fourth quadrant.

The ratio of the change in x to the change in y is 1:2.

11 Find the equation of the straight line.

It passes through the point $(-5, 3)$.

It does not pass through the first quadrant.

The ratio of the change in x to the change in y is 1:1.

12 The cost of renting a steam cleaner is £40 plus £7 per day.

a Explain why you can use the equation $y = 7x + 40$ to work out the rental cost.

b Plot the line $y = 7x + 40$ using values of x from 0 to 10. Choose an appropriate scale.

c What does the x coordinate of a point on the line represent?

d What does the y coordinate of a point on the line represent?

e How much does it cost to rent the cleaner for 7 days?

f How many whole days can you get for £65?

13 Write down the equations of the lines:

a where the y coordinate is 2 less than the x coordinate

b where the y coordinate is 5 times the x coordinate

c where the y coordinate is half the x coordinate

d where the x coordinate is 3 less than the y coordinate

e where the x coordinate is double the y coordinate.

14 Find the equation of the straight line.

It passes through the point $(6, 0)$.

It does not pass through the second quadrant.

It makes a triangle with area 9 square units with the x and y axes.

15 Find the equation of the straight line.

It passes through the point $(6, -2)$.

It does not pass through the third quadrant.

It makes a triangle with area 3 square units with the x and y axes.

2.2 Real life graphs

A graph should include:

(i) clear scales on both axes

(ii) labels on the axes

(iii) a title indicating what it shows.

Note: The graphs in the text here do not require these features as they are for illustrative purposes relating to questions.

**1** Which one of the following situations can be represented by the graph?
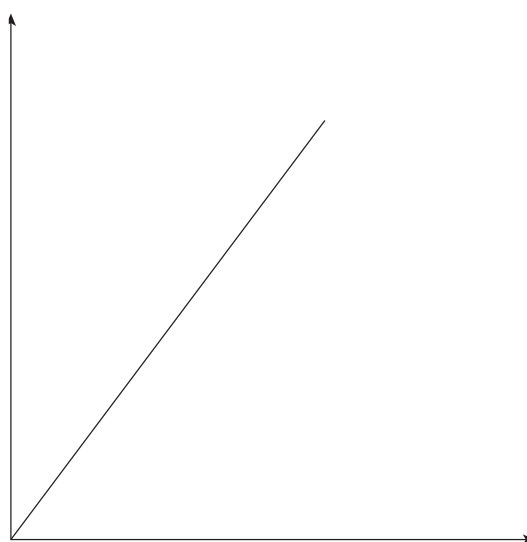
A Rise in temperature during a summer day.

B Converting Australian dollars to Malaysian ringgits.

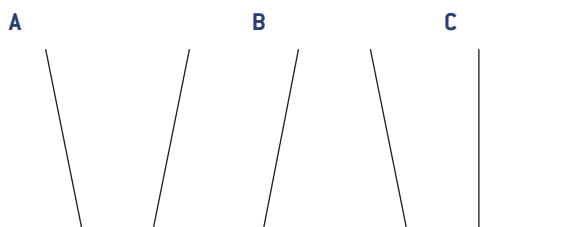C Rise in heart rate during an hour of exercise.

Justify your choice.

Explain why the other two situations are not accurately represented by the graph.

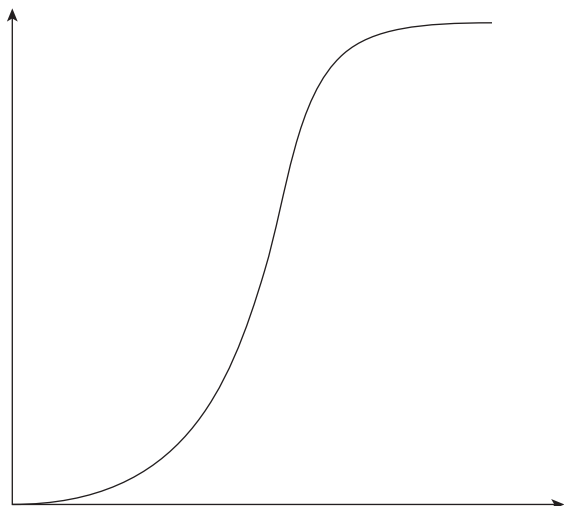**2** The graph represents the height of water in a container as it fills at a constant rate.
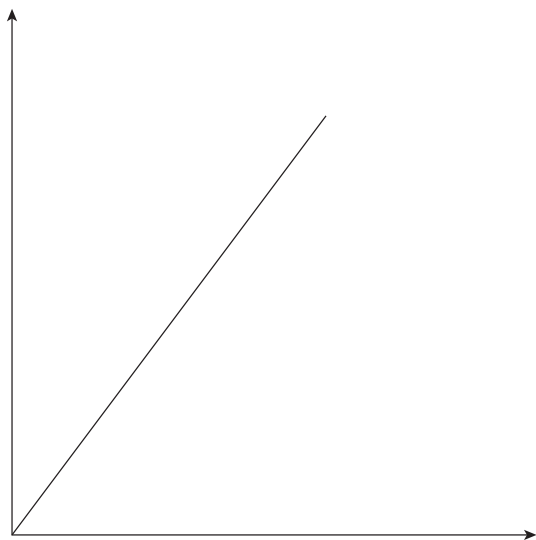
Which one of the following containers would produce the graph?

A   B   C


Justify your choice of container.

Explain why the other two containers would not produce the graph.

3 The horizontal axis is time in minutes.
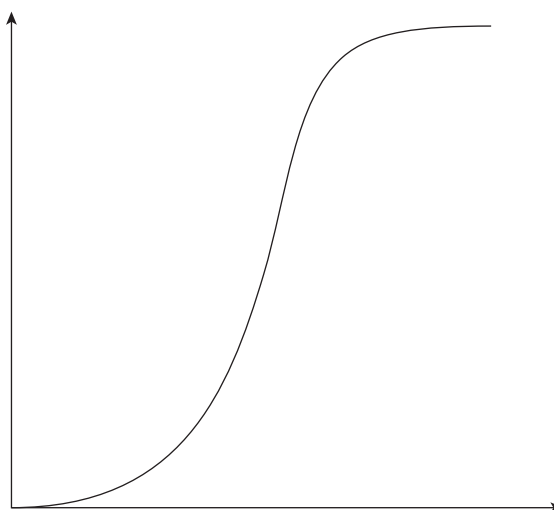The vertical axis is volume of water.

a Explain what the graph shows.
b Describe a real-life situation represented by this graph.

4 The horizontal axis is time in years.
The vertical axis is life expectancy.
Describe the real-life situation represented by this graph.
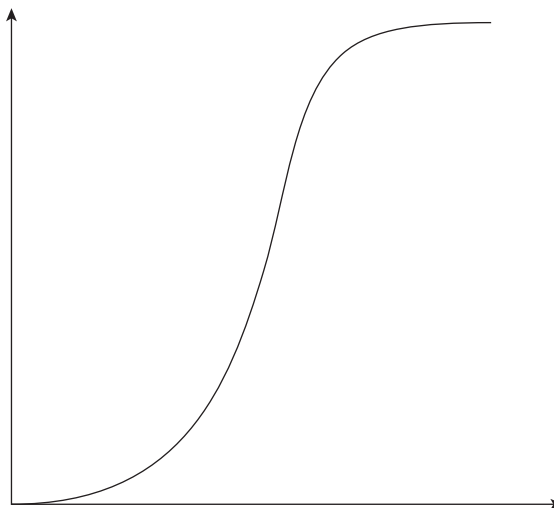
5 The horizontal axis is time in minutes.
The vertical axis is height of water.

a Explain what the graph shows.
b Draw a container that will produce this graph when filled at a constant rate.

Many graphs show change over time. As time is the independent variable in these cases, it is always on the x-axis.

6 The graph shows the number (N) in a population of rabbits over a period of 10 months.
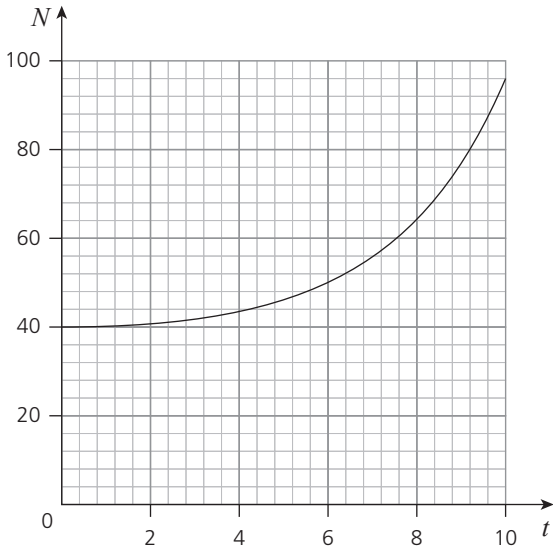
a How many rabbits were there at the start of the 10-month period?

b How long did it take for the population to increase by 30?

c How long did it take for the population to increase by 25%?

d What percentage of the final population is the original population?

**7** The graph shows the number of sandwiches sold per day by a small business over a period of 10 weeks.
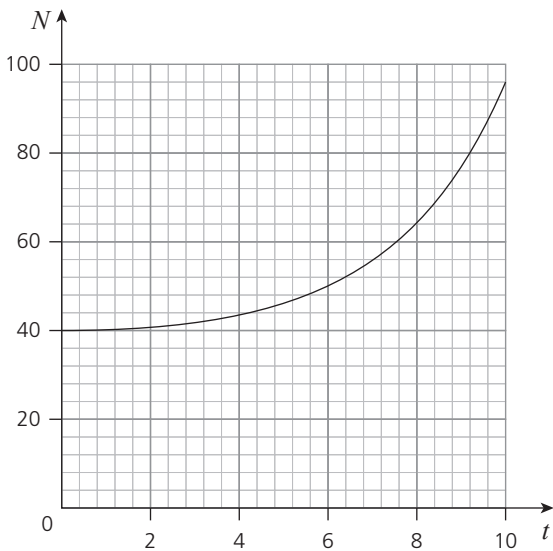
a How many sandwiches are sold per day at the end of the 10-week period?

b When have the sales increased by 50%?

c Describe the situation represented by the graph.

The graph for questions 8 and 9 is a different type of graph showing change over time.

Vertical line graphs and bar graphs are common in statistics to show how things change over time.

**8** The graph shows the number of days in each month where the wind was stronger than 30 miles per hour.
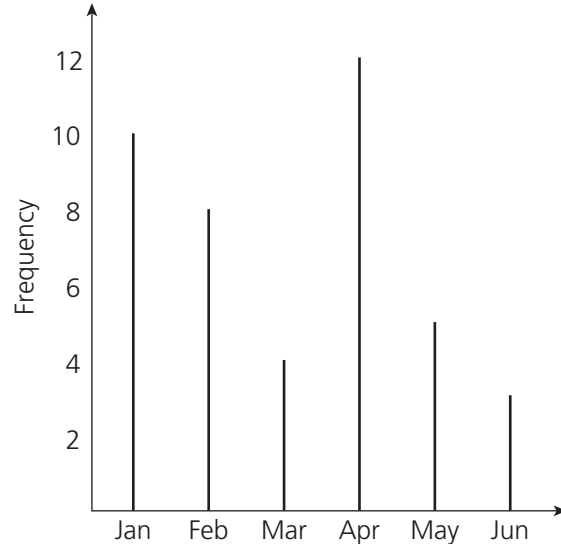
a Which month has half the number of windy days compared with the previous month?

b Two months have a number of windy days in the ratio 1:3.
Which ones?

c There is a 20% decrease in the number of windy days between two successive months.
Which ones?

9 The graph in question 8 shows the number of books Daisy read each month in 2020.

Suggest possible reasons for at least three features of the graph.

10 Draw a graph that shows the height of water in the container as it fills at a rate of 50 cm³ per second. The container is made up of six cubes.
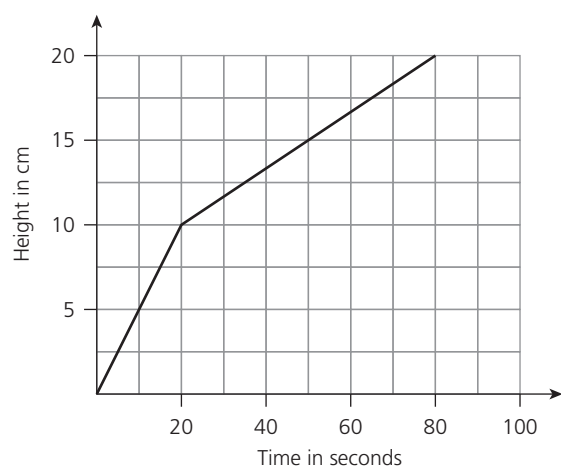

30 cm
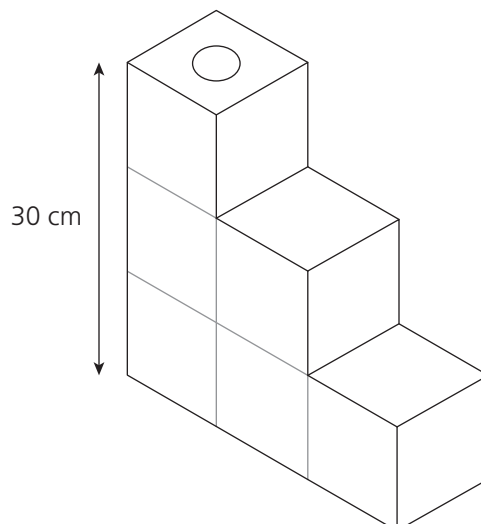
11 Draw a graph that shows the height of water in the container as it fills at a rate of 50 cm³ per second. The container is made up of 6 cubes.

30 cm
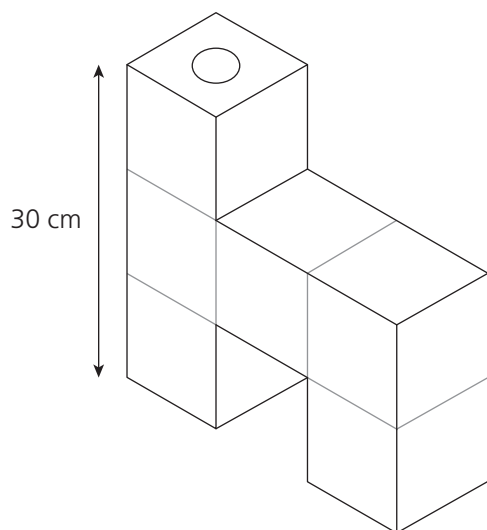
12 Draw a graph that shows the height of water in the container as it fills at a rate of 50 cm³ per second. The container is made up of 7 cubes.
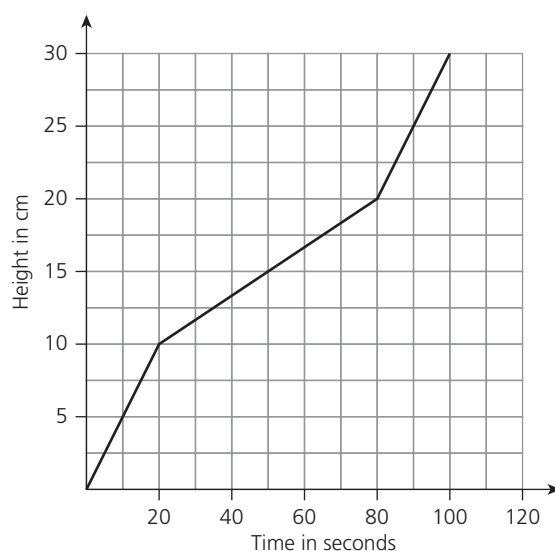
30 cm
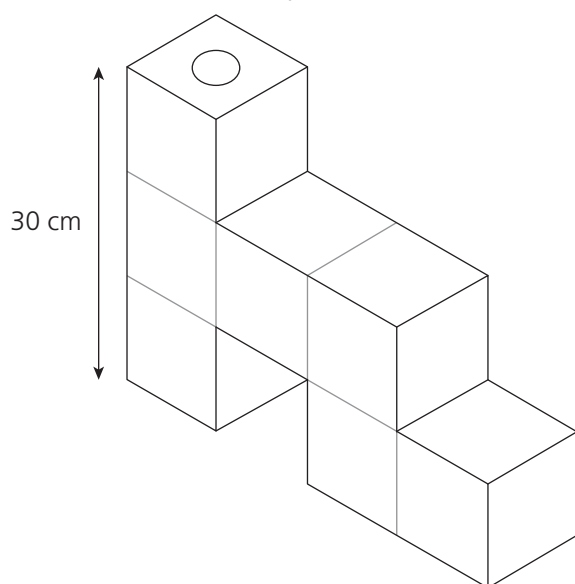
13 The graph shows the height of water in a container as it fills at a rate of 50 cm³ per second. The container is made up of 4 cubes, each of side length 10 cm.

Height in cm

Time in seconds

Draw a possible container.

14 The graph shows the height of water in a container as it fills at a rate of 50 cm³ per second. The container is made up of 5 cubes, each of side length 10 cm.
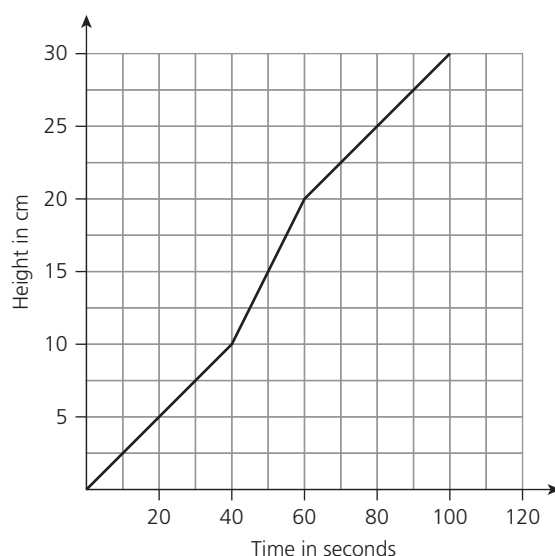
Height in cm

Time in seconds

Draw a possible container.

15 The graph shows the height of water in a container as it fills at a rate of 50 cm³ per second. The container is made up of 5 cubes, each of side length 10 cm.

Height in cm

Time in seconds

Draw a possible container.

BOOK
2

KEY STAGE 3
Mastering
Mathematics

Extend
Practice
Book

Strengthen problem-solving skills and challenge yourself with extra practice. This book is packed with questions that are perfect for extension activities and learning beyond the curriculum.

- Ensure sustained progress with challenging practice questions designed to improve fluency, reasoning and problem-solving skills
- Follow a Mastery approach to learning with questions that get progressively more difficult through small steps
- Apply and reflect on knowledge with questions that promote conceptual understanding
- Easily learn at home with answers available online

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