

Essential Maths Skills

for AS/A-level

Computer Science

Gavin Craddock and Victoria Ellis



Series Editor Heather Davis
Educational Consultant with Cornwall Learning

Contents

The listed content is assessed by the awarding bodies AQA, OCR, Pearson Edexcel and WJEC at AS and A-level.

1 Number systems and sets

Number systems

Sets (AQA A-level only)

2 Number bases

Converting a Base 2 or Base 16 Number to Base 10

Converting a Base 10 to a Base 2 or Base 16 Number

3 Units (AQA only)

Binary prefixes

Decimal prefixes

Converting Between Values

4 Arithmetic operations in a programming language

Mod

DIV

Power

Rounding and Truncating

Logical Operations

5 Binary numbers

Signed vs Unsigned Binary

Key values for n bits

Binary Addition

Binary Multiplication

Representing Negative Numbers

Two's Complement

Fixed Point

Logical Shifts (A-level only)
Bitwise Operators (A-level only)

6 Floating Point Data Representation

Representation of real numbers in binary
Floating point binary
Normalisation
Range and precision
Absolute and relative errors

7 Representing images, sound and other data (AQA only)

Bitmap images
Sound samples

8 Boolean algebra

Logic gates
Boolean algebra
Boolean identities and De Morgan's laws

9 Vectors (AQA A-level only)

Vector Representation
Vector Addition
Scalar Vector Multiplication
Dot product of two vectors
Convex combination of two vectors

10 Big-O notation and Complexity of Algorithms

Functions
Complexity of algorithms (A-level only)

Exam-style questions

Appendix: Cross referencing essential maths skills with exam board topics

2 Number bases

Decimal, binary and hexadecimal number base

‘Number base’ means the number of different symbols that are used. So if you have base 10 (known as decimal, or denary), there are 10 different symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. These symbols are then combined to represent any number, e.g. 12, 99, 386.

Binary is base 2 and there are two different symbols: 0 and 1.

Hexadecimal is base 16 and there are 16 different symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. Letters are used in hexadecimal instead of two digit numbers.

The base of a number can be identified by its subscript:

- If the subscript is 10, e.g. 9_{10} , this means it is a base 10 number (decimal).
- If the subscript is 2, e.g. 1011_2 , this means it is a base 2 number (binary).
- If the subscript is 16, e.g. 281_{16} , this means it is a base 16 number (hexadecimal).

A = 10
B = 11
C = 12
D = 13
E = 14
F = 15

Converting a base 2 or base 16 number to base 10

Each digit in a number can be converted into its decimal (base 10) value using four steps. Table 2.1 shows how the digits are divided. In computer science we start counting at 0. The first digit (on the right) is Digit 0 (D0). This is multiplied by its base to the power of 0 (base⁰). Digit 1 (D1) is multiplied by its base to the power of 1 (base¹). Digit 2 (D2) is multiplied by its base to the power of 2 (base²) and so on.

Table 2.1

D4	D3	D2	D1	D0
$\times \text{base}^4$	$\times \text{base}^3$	$\times \text{base}^2$	$\times \text{base}^1$	$\times \text{base}^0$

Multiplying to the power of 2 means the number multiplied by itself, i.e. $4^2 = 4 \times 4$.

Multiplying to the power of 3 means the number multiplied by itself, multiplied by itself, i.e. $4^3 = 4 \times 4 \times 4$.

A Worked examples

a Convert the binary number 0101 to decimal.

Step 1: put the binary number into the table format as shown in Table 2.1.

Table 2.2

0	1	0	1
$\times \text{base}^3$	$\times \text{base}^2$	$\times \text{base}^1$	$\times \text{base}^0$

Step 2: binary is base 2, so replace the word 'base' with 2.

Table 2.3

0	1	0	1
$\times 2^3$	$\times 2^2$	$\times 2^1$	$\times 2^0$

Step 3: calculate each column.

Table 2.4

$0 \times 2^3 = 0$	$1 \times 2^2 = 4$	$0 \times 2^1 = 0$	$1 \times 2^0 = 1$
--------------------	--------------------	--------------------	--------------------

Step 4: Add the columns together.

$$0 + 4 + 0 + 1 = 5$$

b Convert the hexadecimal number 1D9 to decimal.

Step 1: put the hexadecimal number into the table format as shown in Table 2.1.

Table 2.5

0	D	9
$\times \text{base}^2$	$\times \text{base}^1$	$\times \text{base}^0$

Step 2: hexadecimal is base 16, so replace the word 'base' with 16.

Table 2.6

0	D	9
$\times 16^2$	$\times 16^1$	$\times 16^0$

Step 3: Calculate each column, replacing any letters with their numerical equivalent. (D = 13; see box on the previous page)

Table 2.7

$1 \times 16^2 = 256$	$13 \times 16^1 = 208$	$9 \times 16^0 = 9$
-----------------------	------------------------	---------------------

Step 4: add the columns together.

$$256 + 208 + 9 = 473$$

B Guided questions

Copy out the workings and complete the answers on a separate piece of paper.

1 Convert the binary number 11100 to denary. Show your working.

Step 1: fill in the number base for binary in each column. The first one has been done for you.

Table 2.8

1	1	1	0	0
$\times 2^4$	$\times \underline{\quad}^3$	$\times \underline{\quad}^2$	$\times \underline{\quad}^1$	$\times \underline{\quad}^0$

Step 2: complete the calculation for each column. Again, the first one has been done for you.

$$16 + \underline{\quad\quad\quad} + \underline{\quad\quad\quad} + \underline{\quad\quad\quad} + \underline{\quad\quad\quad}$$

The answer is $\underline{\quad\quad\quad}$.

2 Convert the hexadecimal number 3F to denary. Show your working.

Step 1: fill in the second hexadecimal number on the top row and the bases on the bottom row.

Table 2.9

3	$\underline{\quad}$
$\times \underline{\quad}^1$	$\times \underline{\quad}^0$

Step 2: complete the calculation for each column.

$$\underline{\quad\quad\quad} + \underline{\quad\quad\quad}$$

The answer is $\underline{\quad\quad\quad}$.

C Practice questions

- 3 Convert the binary number 1011 to decimal.
- 4 Convert the hexadecimal number 29 to decimal.
- 5 Convert the binary number 10111 to decimal.
- 6 Convert the hexadecimal number AB to decimal.
- 7 Convert the binary number 10110110 to decimal.
- 8 Convert the hexadecimal number 32F1 to decimal.