Analysing the motion of an object in a uniform gravitational field

REVISED

The equations of uniformly accelerated motion can be used to analyse the motion of an object moving vertically under the influence of gravity. In this type of example, it is important to call one direction positive and the other negative and to be consistent throughout your calculation. The following worked example demonstrates this.

WORKED EXAMPLE

A boy throws a stone vertically up into the air with a velocity of 6.0 m s^{-1}. The stone reaches a maximum height and falls into the sea, which is 12 m below the point of release (Figure 2.6). Calculate the velocity at which the stone hits the water surface.
(acceleration due to gravity = 9.81 m s^{-2})

Answer

$u = 6.0\,\mathrm{m\,s^{-1}}$

$a = -9.81\,\mathrm{m\,s^{-2}}$

$s = -12\,\mathrm{m}$

$v = ?$

Required equation:

$$v^2 = u^2 + 2as$$

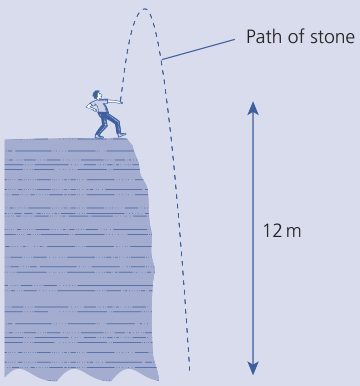
$$v^2 = 6.0^2 + [2 \times (-9.81) \times (-12)] = 36 + 235 = 271$$

$$v = \pm 16.47\,\mathrm{m\,s^{-1}}$$

In this example, upwards has been chosen as the positive direction; hence, *u* is +6.0 m s^{-1}. Consequently, the distance of the sea below the point of release (12 m) and the acceleration due to gravity (9.81 m s^{-2}) are considered negative because they are both in the downward direction.

The final velocity of the stone is also in the downward direction. Therefore, it should be recorded as −16.47 m s^{-1} and rounded to −16 m s^{-1}.

It is also worth noting that air resistance on a stone moving at these speeds is negligible and can be ignored.



▲ Figure 2.6

PRACTICAL SKILL

When timing moving objects, readings must always be repeated and then averaged. One method of finding the uncertainty in timing is to halve the difference between the maximum and minimum readings. For further information on determining uncertainties, see 'Evaluation' on pp. 105–107.

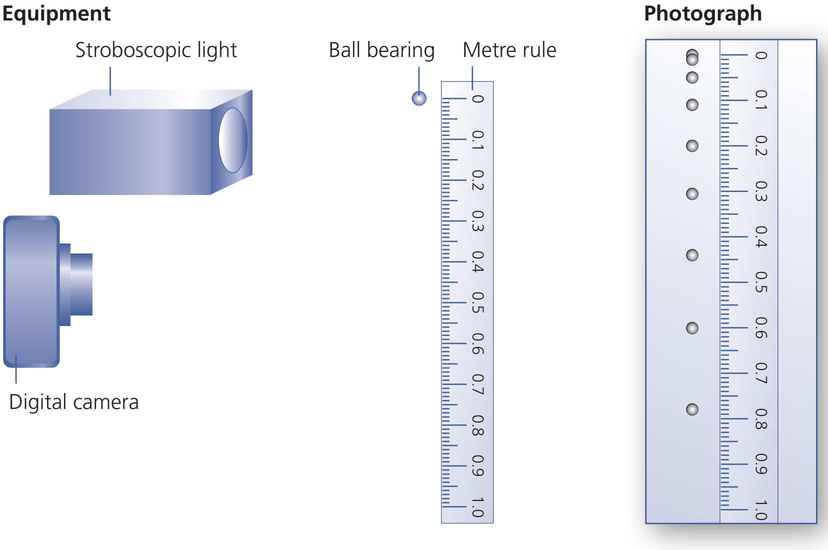
Acceleration of free fall

REVISED

In the absence of air resistance, all objects near the Earth fall with the same acceleration. This is known as the acceleration of free fall. Similarly, objects near any other planet will fall with equal accelerations. However, these accelerations will be different from those near the Earth. This is explored further in the section on dynamics.

Determination of the acceleration of free fall

Figure 2.7 shows apparatus that can be used to determine the acceleration of free fall.



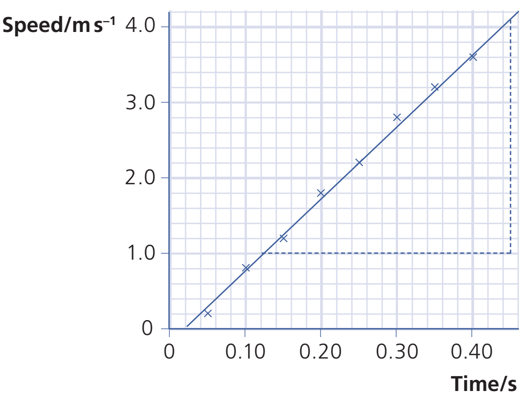
▲ **Figure 2.7 Apparatus to determine the acceleration of free fall**

The stroboscopic light flashes at a fixed frequency and the shutter of the camera is held open. This results in a photograph that shows the position of the ball in successive time intervals, as in Figure 2.7. In this example, the stroboscopic light was set to flash at 20 Hz. In Table 2.1, the third column shows the distance the ball travels in each time interval and the fourth column shows the average speed during each interval.

▼ Table 2.1

Time/s	Position/m	Distance/m	Speed/m s^{-1}
0.00	0.00	0.00	0.0
0.05	0.01	0.01	0.2
0.10	0.05	0.04	0.8
0.15	0.11	0.06	1.2
0.20	0.20	0.09	1.8
0.25	0.30	0.11	2.2
0.30	0.44	0.14	2.8
0.35	0.60	0.16	3.2
0.40	0.78	0.18	3.6

A graph of the displacement against time is plotted (Figure 2.8). Acceleration is equal to the gradient of this graph.



▲ Figure 2.8

Readings from Figure 2.8: (0.45, 4.1) and (0.125, 1.0).

$$a = \frac{4.1 - 1.0}{0.45 - 0.125} = 9.5\,\text{m s}^{-2}$$

NOW TEST YOURSELF

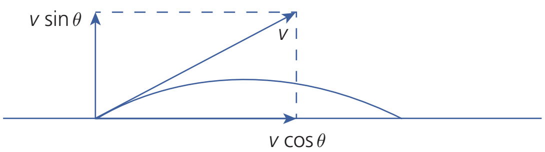
TESTED

7 An astronaut standing on the Moon drops a hammer from a height of 1.2 m. The hammer strikes the ground 1.2 s after being released. Calculate the acceleration of free fall on the Moon.

Motion in two dimensions

REVISED

Consider an object thrown from near the Earth's surface with an initial velocity v at an angle θ to the horizontal. The velocity v can be resolved into two components, one horizontal and one vertical, as can be seen in Figure 2.9.



▲ **Figure 2.9 Motion of an object in a gravitational field can be resolved into components in the vertical and horizontal directions**

The two components of the motion can be analysed separately.

If air resistance is negligible, then there is zero force in the horizontal direction and this component of the velocity is constant.

The vertical component can be treated as a one-dimensional problem of an object in a gravitational field.

The path that the object follows is called a parabola.

WORKED EXAMPLE

A golf ball is hit so that it leaves the club at a velocity of 45 m s^{-1} at an angle of 40° to the horizontal. Calculate:

a the horizontal component of the initial velocity

b the vertical component of the initial velocity

c the time taken for the ball to reach its maximum height

d the horizontal distance travelled when the ball is at its maximum height

(Ignore the effects of air resistance and spin on the ball.)

Answer

a $v_h = v\cos\theta = 45\cos 40 = 34.5\,\text{m s}^{-1}$

b $v_v = v\sin\theta = 45\sin 40 = 28.9\,\text{m s}^{-1}$

c In the vertical direction, the motion can be considered to be that of a ball thrown vertically upwards, decelerating under the effect of gravity. At the top of the flight, the vertical velocity will be, momentarily, zero. Use the equation:

$$v = u + at$$

$$0 = 28.9 + (-9.81)t$$

$$t = 2.94\,\text{m s}^{-1}$$

d The horizontal component of the velocity remains constant throughout the flight.

$$s = ut = 34.5 \times 2.94 = 101 = 100\,\text{m}$$

If the effects of air resistance and spin are ignored, the flight path would be symmetrical. This means that if the ball were hit on a level field, it would travel a total horizontal distance of 200 m before bouncing.

STUDY TIP

In the worked example, v_h, v_v and t are interim values used to find the value of s. Interim values should be calculated to one more significant figure than the original data, then the final answer for s is given to 2 significant figures.

Experimental skills and investigations

The questions

Almost one-quarter of the marks for the AS Level examination are for experimental skills and investigations. These are assessed on Paper 3, which is a practical examination.

A total of 40 marks are available on Paper 3, divided equally between two questions. Although the questions are different on each Paper 3, the number of marks assigned to each skill is always similar.

The paper is designed to assess three skill areas:

- manipulation, measurement and observation
- presentation of data and observations
- analysis, conclusions and evaluation

Manipulation, measurement and observation

Collection of data

REVISED

You must be familiar with common laboratory apparatus, including rulers with millimetre scales, calipers, micrometer screw gauges, protractors, top-pan balances, newton meters, electrical meters (both analogue and digital), measuring cylinders, thermometers and stopwatches.

You will be expected to set up the apparatus supplied according to the instructions and diagram given on the question paper. The instructions detail the method to be followed and which measurements to take.

Variables

REVISED

You should be familiar with the terms **independent variable** and **dependent variable** and be able to recognise them in different experiments. The following table shows some examples.

Investigation		Independent variable	Dependent variable
1	Investigating the height of a bouncing ball	Height from which the ball is dropped	Height to which the ball bounces
2	Investigating the period of vibration of masses suspended by a spring	Mass on the end of the spring	Periodic time
3	Investigating the melting of ice in water	Temperature of water	Time taken to melt
4	Investigating the current through resistors	Resistance of resistor	Current
5	Investigating e.m.f. using a potentiometer	e.m.f.	Balance length

We will refer to these examples later in the text, so you might like to put a marker on this page so that you can easily flip back as you read.

KEY TERMS

The **independent variable** is the variable that you control or change in an experiment.

The **dependent variable** is the variable that changes as a result of the changing of the independent variable.

Range of readings

REVISED

- When you plan your experiment, you should use as wide a range of values for the independent variable as the equipment allows.
- In general, the readings should be evenly distributed between the extremes of the range.
- If you consider Investigation 3 in the table above – the melting ice experiment – the range of temperatures of the water in the beaker should be from nearly 100°C to about 10°C. You will probably be told how many readings to take, but it is likely to be a minimum of six sets. The values chosen for the independent variable should be taken at roughly equal intervals. A sensible spread might be 95°C, 80°C, 60°C, 45°C, 30°C and 15°C.
- It sometimes makes sense to take several readings near a particular value – for example, if the peak value of a curved graph is being investigated. Practice in carrying out experiments will give you experience in deciding if this type of approach is necessary.

REVISION ACTIVITY

Accuracy, precision and uncertainty, including combining uncertainties, are discussed in detail in the 'Errors and uncertainties' section on pp. 10–13. You should refer back to this to refresh your memory.

Presentation of data and observations

Table of results

REVISED

The table must be drawn before collecting results. It must have a sufficient number of columns to record the independent and dependent variables, as well as any calculated data. Each column must have a heading that includes the quantity and unit.

Raw data

REVISED

The degree of precision of raw data in a column should be consistent. It will be determined by the measuring instrument used or the precision to which you can measure. This means that the number of significant figures may not be consistent. An example might be when measuring across different resistors using a potentiometer, where the balance points might vary from 9.3 cm to 54.5 cm.

Calculated data

REVISED

With data calculated from raw measurements, the number of significant figures must be consistent with the raw measurements. This usually means that, except where they are produced by addition or subtraction, calculated quantities should be given to the same number of significant figures as (or one more than) the measured quantity of least precision. If a time is measured as 4.1 s, squaring this gives 16.81 s^2. However, you would record the value as 16.8 s^2 (or perhaps 17 s^2). As with the raw data, this means that the number of significant figures in the column is not necessarily consistent.

The following table shows some readings from a resistance experiment and demonstrates how readings should be set out with:

- the column headings, with quantity and unit
- the raw data to the same precision
- the calculated data to the relevant number of significant figures

p.d./V	Current_1/A	Current_2/A	Average current/A	Resistance/Ω
1.21	0.301	0.304	0.303	3.99
1.46	0.351	0.349	0.350*	4.17
1.73	0.358	0.364	0.361	4.79

* Do not forget to include the zero, to show that the current has been measured to the nearest milliamp.

Exam-style questions

This section contains structured questions similar to those you will meet in Paper 2.

In the actual examination, you have 1 hour and 15 minutes to do the paper. There are 60 marks on the paper, so you can spend just over 1 minute per mark. If you find you are spending too long on one question, then move on to another that you can answer more quickly. If you have time at the end, then come back to the difficult one. When you do the exam-style questions in the book, try to stick to this time schedule – it will give you a guide to the rate you need to work at in the examination.

Some questions require you to recall information that you have learned. Be guided by the number of marks awarded to suggest how much detail you should give in your answer. The more marks there are, the more information you need to give.

Some questions require you to use your knowledge and understanding in new situations. Do not be surprised to find something completely new in a question – something you have not seen before. Just think carefully about it, and find something that you do know that will help you to answer it.

Think carefully before you begin to write. The best answers are short and relevant – if you target your answer well, you can get a lot of marks for a small amount of writing. As a general rule, there will be twice as many answer lines as marks. So you should try to answer a 3 mark question in no more than 6 lines of writing. If you are writing much more than that, you almost certainly have not focused your answer tightly enough.

Look carefully at exactly what each question wants you to do. For example, if it asks you to 'Explain', then you need to say *how* or *why* something happens, not just *describe* what happens. Many students lose large numbers of marks by not reading the question carefully.

Where there are calculations to be done, write out the formula and show the examiner what you are doing. There are often several marks for the calculation and even if a simple error, such as an arithmetic error, occurs early in the calculation, the examiner can still give you marks for the rest of the calculation, as long as they can understand what you are doing.

Go to **www.hoddereducation.com/cambridgeextras** for sample answers and commentaries to the questions.

For each question in this practice paper, there is an answer that might get a C or D grade, followed by expert comments (shown by the icon e). Then there is an answer that might get an A or B grade, again followed by expert comments. Try to answer the questions yourself first, before looking at the answers and comments online.

Chapter 1

1 The frequency f of a stationary wave on a string is given by the formula:

$$f = \frac{\sqrt{T/\mu}}{2L}$$

where L is the length of the string, T is the tension in the string and μ is a constant for the string.

a State which of the quantities f, L and T are base quantities. [1]

b **i** State the SI base units of T. [1]

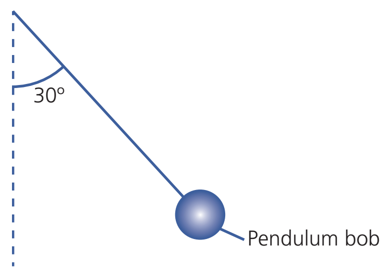
ii Determine the SI base units of μ. [1]

[Total: 3]

2 **a** Describe the difference between a scalar quantity and a vector quantity. [2]

b A simple pendulum has a bob of mass 50 g. Determine the weight of the bob. [2]

c The bob is pulled to one side by a horizontal force so that the pendulum string makes an angle of 30° with the vertical.



i Add arrows to the diagram to show the direction of:

1 the weight of the pendulum bob (label this *W*)

2 the horizontal force (label this *F*)

3 the tension (label this *T*) [3]

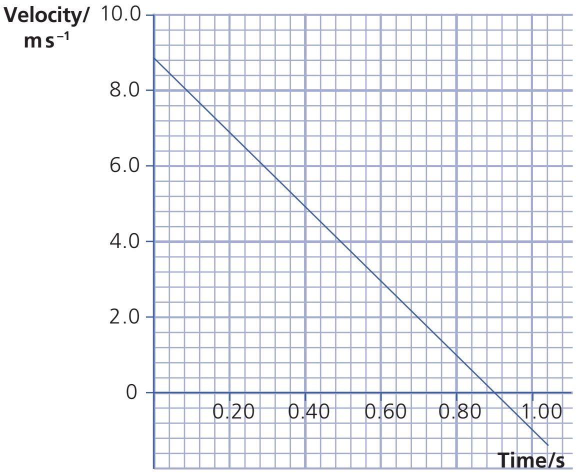
ii Calculate the magnitude of the tension in the string. [2]

iii Calculate the magnitude of the horizontal force. [2]

[Total: 11]

Chapter 2

1 A builder throws a brick up to a second builder on a scaffold, who catches it. The graph shows the velocity of the brick from when it leaves the hand of the first builder to when the second builder catches it.



a Show that the acceleration of the brick is $9.8\,m\,s^{-2}$. [2]

b The gradient of the velocity–time graph is negative.
Explain what this shows. [1]

c The second builder catches the brick 1.04 s after the first builder released it.
Calculate the height the second builder is above the first builder. [2]

d The second builder drops a brick for the builder on the ground to catch.
Suggest why it is much more difficult to catch this brick than the one in the previous case. [1]

[Total: 6]

2 A remote controlled vehicle on the planet Mars fires an object at a velocity of $25\,m\,s^{-1}$. The initial direction of the velocity is at 60° to the surface of Mars. You may assume that the air resistance on Mars is negligible.
(acceleration of free fall on Mars = $3.7\,m\,s^{-2}$)

a Sketch the flight path of the object. [1]

b Calculate:

i the vertical component of the object's initial velocity [1]

ii the horizontal component of the object's velocity [1]

c Show that the time of flight of the object before it hits the ground is 12 s. [3]

d Calculate the horizontal distance that the object travels before landing on the ground. [2]

e Explain why the object would travel further when fired on Mars compared with firing it with the same velocity on Earth. [2]

[Total: 10]

Chapter 3

1 a Explain the difference between mass and weight. [2]

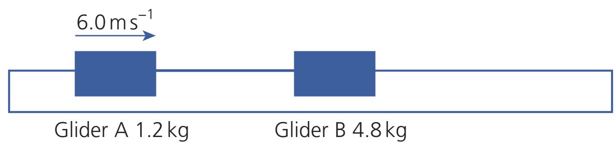
b A golf ball of mass 46 g travelling horizontally collides with a vertical brick wall at 90°. The speed of the ball just before it collides with the wall is $25\,m\,s^{-1}$. It rebounds horizontally from the wall at a speed of $23\,m\,s^{-1}$.
Calculate the change in momentum of the ball. [3]

c The ball is in contact with the wall for 4.0×10^{-2} s.
Calculate the average force acting on the ball during the collision. [2]

d Sketch the path the ball will take after it hits the wall. [1]

[Total: 8]

2 Glider A on an air track has a mass of 1.2 kg. It moves at $6.0\,m\,s^{-1}$ towards glider B which is stationary and has a of mass 4.8 kg (Figure 3.8).



The two gliders collide and glider A rebounds with a speed of $3.6\,m\,s^{-1}$.

a State what is meant by an elastic collision. [1]

b Show that the speed of the second glider after the collision is $2.4\,m\,s^{-1}$. [2]

c Show that the collision is elastic. [3]

The gliders are in contact for 30 ms during the collision.

d i Calculate the average force on the stationary glider during the collision. [2]

ii Compare the forces on the two gliders during the collision. [2]

[Total: 10]

Chapter 4

1 a State the conditions for an object to be in equilibrium. [2]

b A non-uniform trap door of mass 6.8 kg is held open by a force of magnitude 24 N applied perpendicular to the door. The trap door is 1.8 m long.
Calculate the distance from the hinge to the centre of gravity. [3]