Problem Solving in Mathematics

5–6

Sample Pages

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HODDER EDUCATION

1 Problem solving and why it is important

Today's students are growing up in an increasingly technological and information-rich world. This has led to some changes in education, notably a move away from rote learning to more enquiry- and skills-based approaches. Students are expected to develop a range of skills, including diverse and adaptable problem-solving skills.

Creative thinking and problem-solving skills are valued at school, but they are also very relevant in the real world. Students still need to learn basic mathematical skills, but they also have to be able to apply these in unfamiliar contexts and to communicate their thinking and explain how they got to their solutions.

This book aims to provide challenging but supportive contexts for teaching and learning how to approach problem solving in mathematics. The course is underpinned by the 'Five Es' approach to learning.

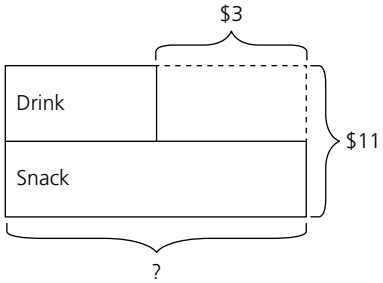
The course will help student to:

* **Engage** with problems meaningfully
* **Explore** different options for approaching and solving problems
* **Explain** how they are thinking
* **Elaborate** on their working and processes and see connections between different areas of mathematics, as well as connections to real-world contexts
* **Evaluate** what worked and what did not, learning from mistakes as well as successes

Mathematical problems

In primary mathematics, problem solving is often associated with 'word problems' or 'story sums'. For example, Sherilyn bought a drink and a snack for $11. The drink cost $3 less than the snack. What did the snack cost?

For many students, the main 'problem' in these types of questions is that they do not know what is being asked of them. Once they work out what they are being asked to find, they can often do the calculations easily. One method of approaching these types of questions is to teach students to represent them visually. This is a key feature of successful mathematics teaching in Singapore and elsewhere around the world. For example, the problem above could be shown like this:



This type of diagram is called a bar model and it is one of the strategies that students have met in previous grades. At this level, we will develop this further to show its application in a range of different types of problems.

The bar model helps students to visualise, and then solve the problem.

They know that drink + snack = $11

So, drink + $3 + snack = $11 + $3

The diagram makes it clear that if you add $3 to the price, you need to add it to the total.

Now they have two items of the same price that together cost $14.

$14 ÷ 2 = $7

So, the snack cost $7.

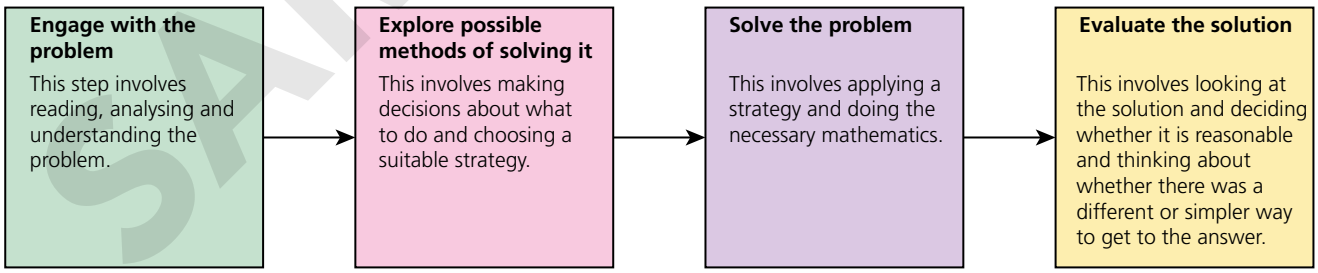
You can check the answer by going back to the original problem. If the snack cost $7, then the drink must cost $4.

$7 + $4 = $11

In this book we include word problems but we also provide problems involving patterns, measurement, shape and space and data. In fact, any question where the solution and method of working is not immediately clear is considered to be a problem.

The process of problem solving

Problem solving refers to the process or steps that you take to find a solution (what we traditionally call the answer). There are four main steps in the process:



Problem-solving strategies

The strategies that we introduce and teach in this book are designed to help students approach different types of problems. Some strategies are more suitable for particular types of problems and it is useful to give students experience of using these with carefully selected problems. Research suggests that this is an efficient method of developing problem-solving skills.

The key strategies for Grades 5–6 are:

* Drawing diagrams
* Using tables and looking for patterns
* Organised lists
* Guess, check and improve
* Eliminating possibilities
* Working backwards
* Using equations
* Simplifying the problem.

Chapters 2–9 each deal with a single strategy. The strategy is introduced and a number of step-by-step worked examples are provided to show the strategy in use. Once the strategy has been taught, students can then work through a set of graded problems to try it out and gain experience in both using the strategy and recognising the types of problems that can be solved by applying the strategy.

In exercises, tests and examinations, students will not be told what strategy to use to solve problems. Chapter 10 provides a set of mixed problems to allow students to apply what they've learned and to choose their own strategies for solving the problems.

The answers to all problems are given on pages 75–77. A set of worked solutions can be found online at:
www.hoddereducation.com/Problem-Solving-Answers

2 Draw diagrams to help you solve problems

Explain

Drawing simple pictures or diagrams is a useful strategy for solving problems. The pictures help students to visualise the problem and work out what they need to do to solve it.

Pictures or diagrams can be any visual representation of a problem: for example, diagrams of shapes or areas, one-to-one matching diagrams, number lines, tree diagrams (inverted V-diagrams) and bar model diagrams. When you teach students to use diagrams, it is important to stress that they should use simple representations. For example, if they are using a diagram to show animals, they could just draw a simple circle to represent each one.

Note

The bar model does not give the solution; rather it helps the student work out what to do. It is the visual representation of the problem that is of value in this method.

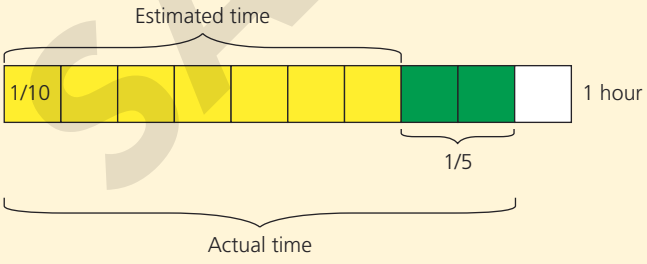
Bar models are simple rectangles that are drawn and labelled to represent word problems. The bar model allows students to show what is known and to see quite clearly what they need to work out (what is unknown). Bar models also help students clarify concepts and deepen their mathematical understanding.

Most number and algebra problems, including those involving fractions, can be represented using bar models.

Consider this problem: Tamika estimated that it would take her $\frac{7}{10}$ of an hour to do her homework. In reality, it took her $\frac{1}{5}$ of an hour more. How long did she spend doing homework? Give your answer as a fraction of an hour.

Students can draw a bar to represent this situation like this:

This bar allows them to use equivalent fractions to solve the problem.



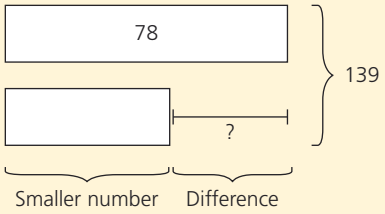
It took her $\frac{9}{10}$ of an hour to complete her homework.

Explain Continued...

Here is another example:

The sum of two numbers is 139. The greater number is 78. What is the difference between the two numbers?

This can be represented using two bars like this:



The model shows the problem clearly and students can see that they can solve the problem in two steps.

First, they have to find the smaller number: 139 – 78 = 61.

Once they have worked that out, they can find the difference: 78 – 61 = 17.

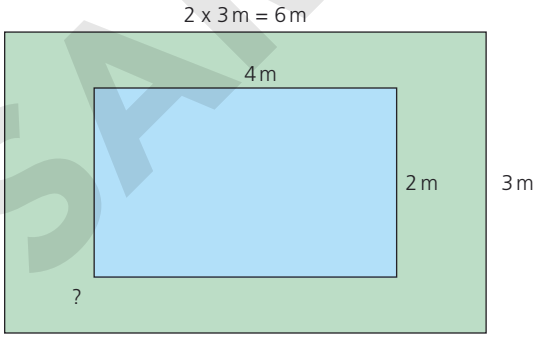
The difference between the numbers is 17.

International research has shown that students who are consistently taught to use this type of model perform better and struggle less with worded problems. The bar models are easy to draw and they can be used to represent large numbers, proportions and even fractions efficiently.

Worked example 1

A rectangular room is 3m wide. It is twice as long as it is wide. If a carpet that is 4m long and 2m wide is placed on the floor of the room, how much of the floor is not covered?

Draw a diagram of the room and label it with the information you are given in the problem.



The diagram shows that the area of the room less the area of the carpet is how much floor is not covered.

$3m \times 6m = 18m^2$

The area of the carpet is $4m \times 2m = 8m^2$

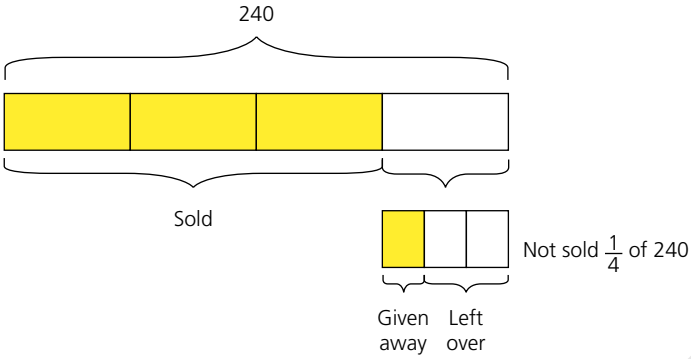
The area of the floor that is not covered is $18 - 8 = 10m^2$

Answer: 10 m^2 of floor is not covered by carpet.

Worked example 2

Mr Morgan picked 240 avocados. He sold $\frac{3}{4}$ of them at the market. On the way home from market, he gave $\frac{1}{3}$ of the remaining avocados to people he met. How many did he have left?

Draw bars to represent the problem situation.



Now you can do the mathematics to solve the problem.

$240 \div 4 = 60$, so $\frac{1}{4} = 60$ That is the number of avocados he did not sell.

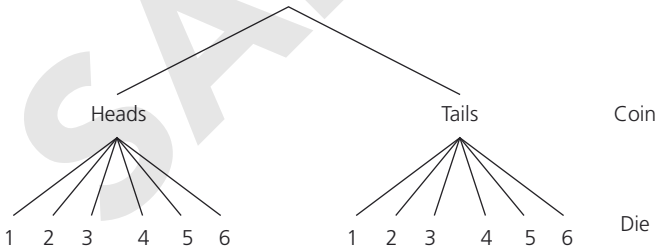
$60 \div 3 = 20$ That is the number of avocados he gave away.

Answer: He was left with 60 – 20 = 40 avocados.

Worked example 3

Treyvon tosses a coin and rolls a normal die at the same time. How many possible results could he get?

Draw a tree diagram to show the results.



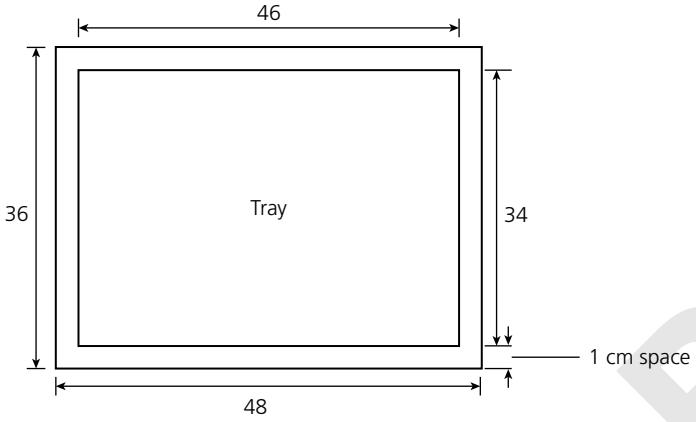
Answer: He could get 12 possible results.

Hint: A result could be heads and 4, or tails and 2.

Worked example 4

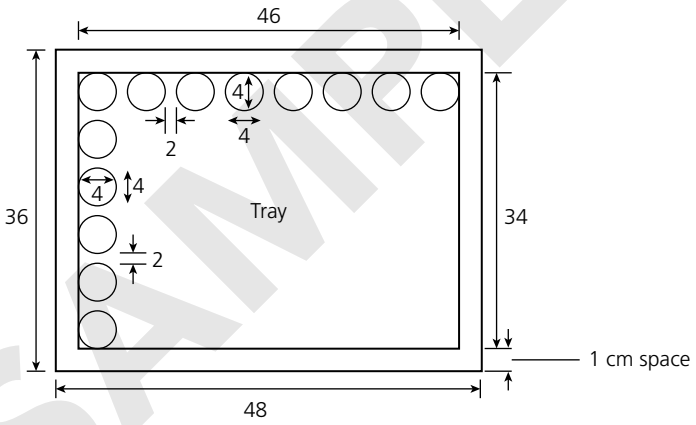
Zara is placing chocolate brownies on a tray to bake them. The tray is a rectangle 36cm wide and 48cm long. The brownies have to be 2cm apart from each other and 1cm away from the edge of the tray. If each brownie is a circle with diameter 4cm, how many can she fit onto the tray?

Start by drawing the tray.



Remember, you have to have 1cm open all around the edge.

Use your diagram to work out how many brownies can fit along each length. Remember: they must be 2cm away from each other, but only 1cm from the edge.



The diagram shows that you make can six rows with eight brownies in each row.

$6 \times 8 = 48$

Answer: She can fit 48 brownies onto the tray.

Now you try …

Draw a diagram to represent each of these problems.

Show how you work out the answers.

1 The path from Zach's front door to the gate is 11 tiles long. Zach plays a game where he jumps onto every third tile and then jumps back a tile. How many jumps will it take him to get through the gate?

2 A school bought 15 cases of drinks for the school fair. One-third of the drinks were sorrel and two-thirds were ginger. How many cases of ginger drinks did the school buy?

3 A recipe calls for 2.5 kilograms of seafood to make 4 kilograms of jambalaya. If the chef only has 1.8 kilograms of seafood, how much jambalaya can she make?

4 Ms Dorleon gave $\frac{2}{5}$ of the money in her purse to charity and spent half of the remainder on food. If she had $30 left in her purse, how much did she start with?

5 The protocol for the reopening of a school after a viral outbreak required three out of every five children in the sixth grade classroom to move to the fifth grade classroom to ensure physical distancing. If the school started with 40 children in the sixth grade, how many had to move to the fifth grade room?

6 In a group of students, 27 are members of the Red Cross Society and 25 are members of the choir. Of these, nine students are members of both groups. How many students are there in the two groups?

7 Zuriel and his brother Kenaz wanted to buy a gift for their mother. Together, they had $120. Zuriel contributed $20 more than Kenaz. How much money did they each contribute?

8 Eucharia has to cut a length of bamboo into seven pieces to make a kite. If it takes her 90 seconds to make one cut, how long will it take her to cut the bamboo into seven pieces?

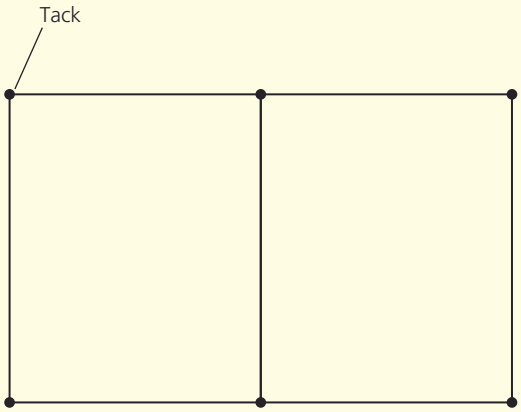
9 Ken had $75. He spent $\frac{2}{5}$ of his money and gave the remainder to his brother. How much money did he give his brother?

10 Kenaz bought six pizzas for a party. If he served each person who attended $\frac{2}{3}$ of a pizza, how many people were at the party?

11 The sum of two numbers is 7.26. If the smaller number is 3.4, what is the larger number?

12 Bronte and her three friends had to buy snacks for the annual grade 6 hike. They bought cheese twists for $5.40, groundnut sugar cakes for $2.55 and chocolate bars for $2.65. If the four friends split the cost of the snacks equally, how much should each friend pay?

13 Five photographs the same shape and size are to be tacked onto a board. Each photograph has to be tacked on all four corners, but the corners can overlap, like this:



Here six tacks have been used to display two photos.

What is the least number of tacks you can use to display five photos?

We live in an increasingly technological and information-rich world. Education is focusing more on enquiry and skills-based approaches. Students, even at the early levels of schooling, are expected to develop a range of skills, including problem-solving skills.

Creative thinking, using information appropriately and problem solving are important for students in the school curriculum and in daily life. This book will help students learn how to apply mathematical skills in different contexts and explain how they got to their solutions.

This book is easy for parents and teachers to use, and teaches multiple strategies for solving problems, using stimulating and supportive contexts. The approach is underpinned by the Five Es approach to learning.

The course will help students to:

- Engage with problems meaningfully
- Explore different options for approaching and solving problems
- Explain their thought processes
- Elaborate on their working and processes and see connections between different areas of mathematics
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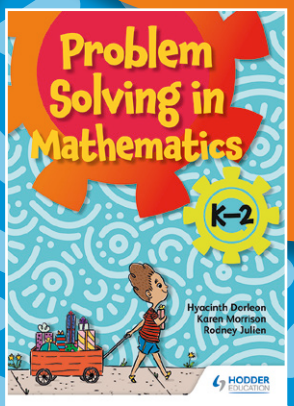

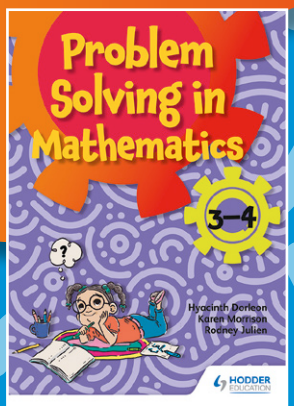

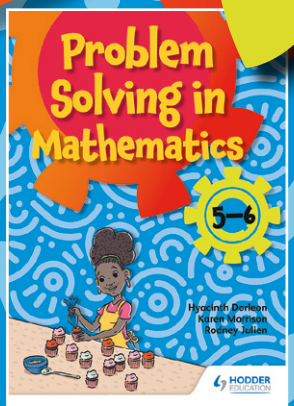

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