Problem Solving in Mathematics

3–4

Sample Pages

Hyacinth Dorleon
Karen Morrison
Rodney Julien

HODDER EDUCATION

1 Problem solving and why it is important

Today's students are growing up in an increasingly technological and information-rich world. This has led to some changes in education, notably a move away from rote learning to more enquiry- and skills-based approaches. Students, even at Grade 3 and 4 levels, are expected to develop a range of skills, including problem-solving skills.

Creative thinking and problem-solving skills are valued at school, but they are also very relevant in the real world. Students still need to learn basic mathematical skills, but they also have to be able to apply these in unfamiliar contexts and to communicate their thinking and explain how they got to their solutions.

This book aims to provide challenging but supportive contexts for teaching and learning how to approach problem solving in mathematics. The course is underpinned by the 'Five Es' approach to learning.

The course will help students to:

- **Engage** with problems meaningfully
- **Explore** different options for approaching and solving problems
- **Explain** how they are thinking
- **Elaborate** on their working and processes and see connections between different areas of mathematics
- **Evaluate** what worked and what didn't, learning from mistakes as well as successes.

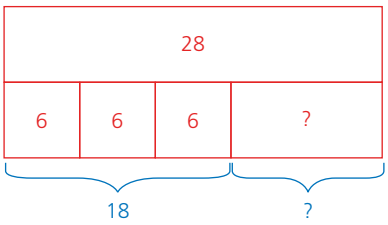
Mathematical problems

In primary mathematics, problem solving is often associated with 'word problems' or 'story sums'. For example, Natalie picked 28 ripe mangoes in the morning. She gave six mangoes to each of her three neighbours. How many mangoes did she have left?

For many students, the main 'problem' in these types of questions is that they do not know what is being asked of them. Once they work out what they are being asked to find, they can often do the calculations easily. One method of approaching these types of questions is to teach students to represent them visually. This is a key feature of successful mathematics teaching in Singapore and elsewhere around the world. For example, the problem above could be shown like this:

This is called a bar model and it is one of the strategies we will explore in the course.

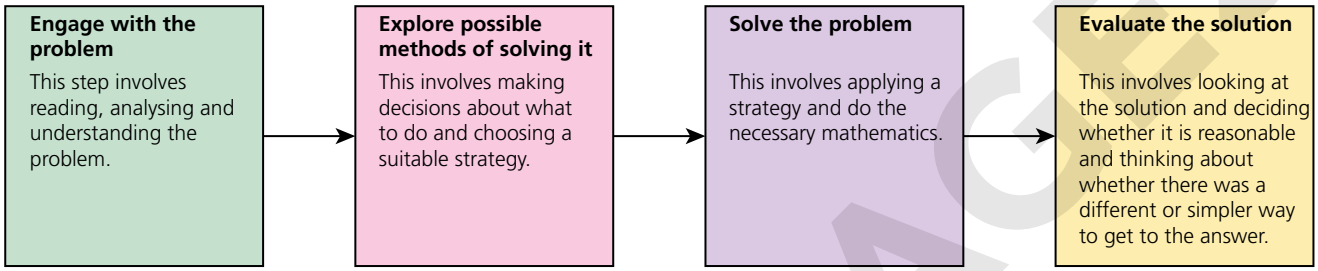
The model above allows the students to see that they need to subtract 18 from 28 to find out how many are left: 28 – 18 = 10 mangoes left.



In this book we include word problems but we also provide problems involving patterns, measurement, shape and space and data. In fact, any question where the solution and method of working is not immediately clear is considered to be a problem.

The process of problem solving

Problem solving refers to the process or steps that you take to find a solution (what we traditionally call the answer). There are four main steps in the process:



Problem-solving strategies

The strategies that we introduce and teach in this book are designed to help students approach different types of problems. Some strategies are more suitable for particular types of problems and it is useful to give students experience of using these with carefully selected problems. Research suggests that this is an efficient method of developing problem-solving skills.

The key strategies for Grades 3–4 are:

* Modelling using real objects
* Drawing diagrams
* Making organised lists
* Using tables
* Looking for patterns and making connections
* Guess, check and improve
* Eliminating possibilities.

Chapters 2–8 each deal with a single strategy. The strategy is introduced and a number of step-by-step worked examples are provided to show the strategy in use. Once the strategy has been taught, students can then work through a set of graded problems to try it out and gain experience in both using the strategy and recognising the types of problems that can be solved by applying the strategy.

In exercises, tests and examinations, students will not be told what strategy to use to solve problems. Chapter 9 provides a set of mixed problems to allow students to apply what they've learned and to choose their own strategies for solving the problems.

The answers to all problems are given on pages 59–61. A set of worked solutions can be found online at:
www.hoddereducation.com/Problem-Solving-Answers

2 Model problem situations using real objects

Explain

Some problems can be solved by modelling the problem using real objects (manipulatives). This strategy allows students to work at the concrete level and it is linked to, and prepares them for, visual representation (drawings) of problems as they progress and learn more.

Counters, cubes, bottle tops, buttons, beans and any other small objects can be used to represent numbers and items in the problems. It is useful to give students a collection of their own to use and a small tray (a plastic lid or a paper plate works well) that they can use to place the items to model the situation.

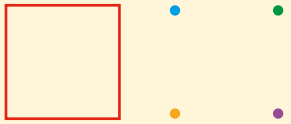
Using models is a good strategy when the problem gives you information that can easily be shown using objects.

For example: Four friends are going to eat lunch. They will all sit at a square table with one seat at each side. How many different ways are there for them to arrange themselves?

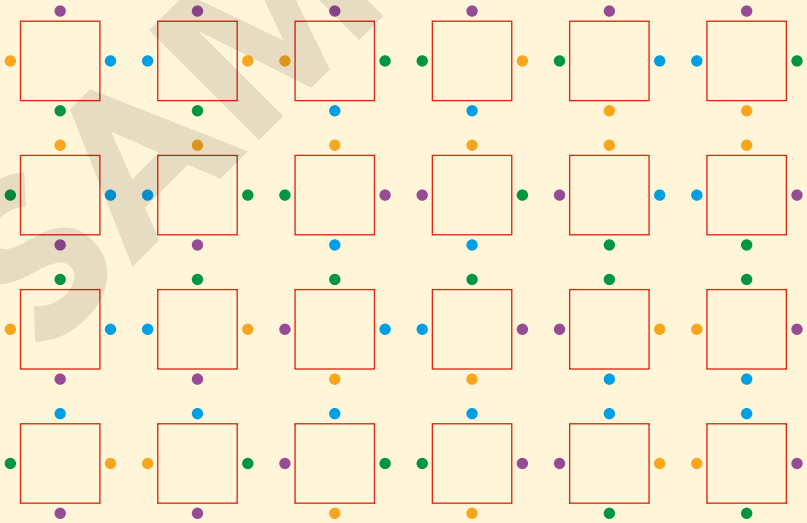
Students can model this using a drawn square and any four different objects to represent the four friends.

Find a square object or draw a square to show the table.

Choose four objects to represent the friends. We have used four different coloured counters.



Arrange the counters around the square. It is useful to record (draw or list) the arrangements as you work to make sure you don't repeat any.

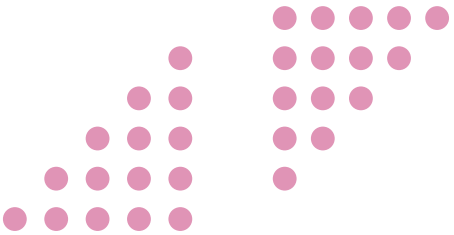


Answer: There are 24 different ways to do this.

As students begin to work in a higher and higher number range, this strategy is less useful because it is not convenient or efficient to use objects when the numbers are large.

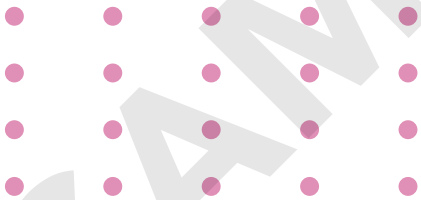
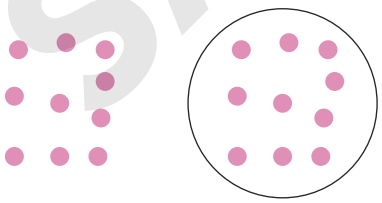
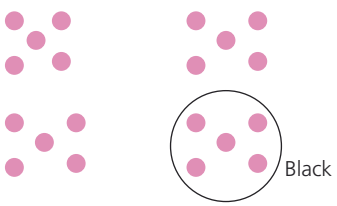
Worked example 1

Jamila and Susan have 5 comic books between them. How many ways could these be split between the girls?

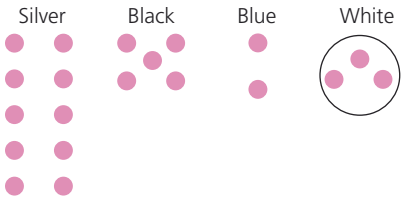
Model	Thinking
	Find 5 items to model the comics. Or use real comics. Explore ways to divide the items into 2 groups. Remember to keep a record of your work.
JAMILA SUSAN 	Answer: There are 6 ways of splitting the comics.

Worked example 2

There are 20 vehicles in a parking lot. Half are silver, a quarter of them are black, one-tenth are blue and the rest are white. How many vehicles of each colour are there?

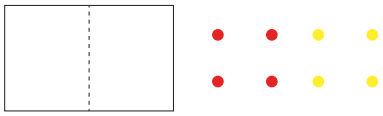
Model	Thinking
	Count out 20 counters.
	Make 2 equal groups to find half. Half = 10 cars
	Make 4 equal groups to find a quarter. One quarter = 5 cars

Worked example 2 Continued...

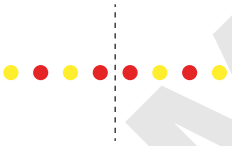
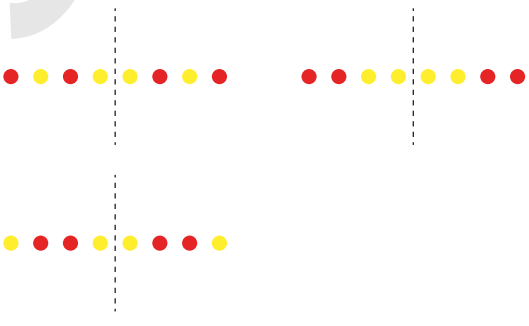
	Make 10 equal groups to find one-tenth. $\frac{1}{10}$ = 2 cars This means there are 10 silver, 5 black and 2 blue cars. 10 + 5 + 2 = 17
	There are 3 left over, so there are 3 white cars. Answer: 10 cars are silver, 5 are black, 2 are blue and 3 are white.

Worked example 3

Micah has 4 red counters and 4 yellow counters. He has to arrange them in a row to make a symmetrical pattern. The dashed line is the line of symmetry.



How many different symmetrical patterns can he make?

Model	Thinking
	Use 4 red and 4 four yellow counters. **Hint:** Remember that the colours in a symmetrical pattern have to be symmetrical as well as the shapes.
	For a symmetrical pattern you will need 2 two red and 2 yellow on each side of the line. Odd numbers won't work.
	Arrange them so that the counters on each side of the line are a mirror image of each other. Answer: He can make 6 symmetrical patterns.

Now you try …

Solve these problems by modelling them using real objects. Write the answers only in this book.

1 If you shared $45 equally among 5 people, how much money would each receive?

Each person will receive _____ .

2 I left school at 2:45 p.m. and arrived home 45 minutes later. What time did I arrive home?

I arrived home at _____ .

3 There were 32 parents at a PTA meeting for the junior department. Fruit juice was served after the meeting. One jug holds 4 cups of juice. How many jugs of juice would the PTA need to give each person 1 cup?

They will need _____ jugs.

4 Kimaya has 30 beads. Her teacher gives her 6 boxes and asks her to put an equal amount of beads in each box. How many beads should there be in each box?

There should be _____ beads in each box.

5 Shuryelle has 15 metres of ribbon that she uses to weave handmade gifts. Each gift needs 3 metres of ribbon. How many gifts can she make?

She can make _____ gifts.

6 A principal drives to the supermarket on the way home from work. The table shows some information about the journey.

	Time
Leaves work	4:30 p.m.
Gets to supermarket	4:50 p.m.
Leaves supermarket	5:15 p.m.

How many minutes did the principal spend in the supermarket?

The principal spent _____ minutes in the supermarket.

7 On the playground there are 13 boys and 5 more girls than boys. How many children are on the playground?

There are _____ children on the playground.

8 Jeavon has an elastic band that is $8\frac{1}{2}$ cm long. He stretches it until it is 20 cm long. What is the difference between the length of the elastic band before Jeavon stretched it and the length after it is stretched?

There is a difference of ☐.

9 Some oranges are put into boxes. One box holds 8 oranges. How many boxes are needed to hold 50 oranges?

☐ boxes are needed.

10 Mrs Celeste uses half a kilogram of cheese to make 3 pies. How much cheese is needed for her to make 15 pies?

☐ cheese is needed for 15 pies.

11 Fruits are gathered in buckets to make fruit bowls. There are 2 water melons and 3 mangoes in every bucket. There are 45 fruits altogether. How many mangoes are there?

There are ☐ mangoes.

12 On a map, 6 cm represents 1 km on the ground. A road on the map is 27 cm long. What is the length of the road in kilometres?

The road is ☐ kilometres long.

13 A calculator costs 3 times as much as a pack of copy paper. If the copy paper costs $14, what is the total cost of the calculator and the pack of copy paper?

The total cost is ☐.

14 Nadia has a round, a square and a rectangular sticker. She wants to arrange them in a triangular shape in the centre of a page. How many ways are there for her to do this?

There are ☐ ways to do this.

15 Javon wants to make eggnog. He found a recipe that calls for 4 eggs and it serves 6 people. If he wants to make eggnog for 15 people, how many eggs does he need?

He needs ☐ eggs.

We live in an increasingly technological and information-rich world. Education is focusing more on enquiry and skills-based approaches. Students, even at the early levels of schooling, are expected to develop a range of skills, including problem-solving skills.

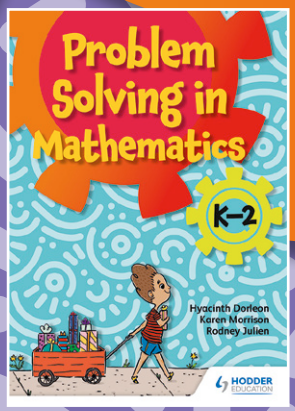
Creative thinking, using information appropriately and problem solving are important for students in the school curriculum and in daily life. This book will help students learn how to apply mathematical skills in different contexts and explain how they got to their solutions.

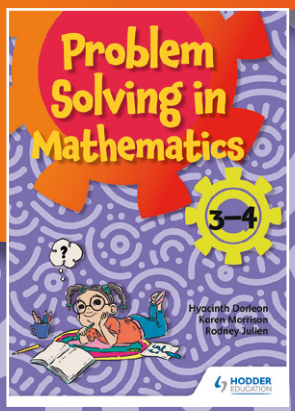
This book is easy for parents and teachers to use, and teaches multiple strategies for solving problems, using stimulating and supportive contexts. The approach is underpinned by the Five Es approach to learning.

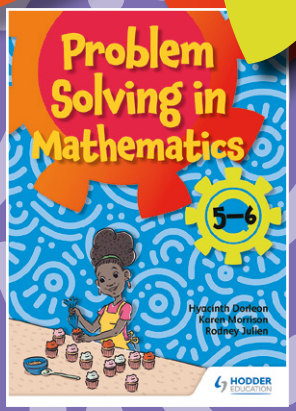
The course will help students to:

- Engage with problems meaningfully
- Explore different options for approaching and solving problems
- Explain their thought processes
- Elaborate on their working and processes and see connections between different areas of mathematics
- Evaluate what worked and what didn't, learning from mistakes as well as successes.

Sample Pages







HODDER EDUCATION
e: education@hachette.co.uk
w: hoddereducation.com

ISBN 978-1-510-48076-6
9 781510 480766
FSC