TeeJay Publishers

SQA - Higher Maths

Higher Course Planner Using TeeJay's

Higher Maths Textbook

This **Course Planner** for **Higher**, is based on **TeeJay's New Higher Book**, and it comes in two parts :-

Part A - Each Outcome is listed in order, directly from the SQA site, with a reference as to how our Higher Maths Book covers the entire contents as listed in the official documents, including the new topics

Part B - The Book Chapters are listed **in order** from our Higher Maths Book i, with references to the official SQA list of Outcomes. (**A practical course planner**)

TeeJay Publishers

SQA - Higher

Higher Course Planner - Following Outcome Order

This Course Planner for Higher, is based on TeeJay's Higher Maths Book.

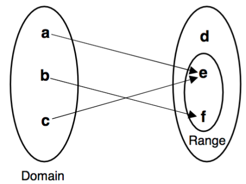
Part A

In this section, the Skills, as presented in the SQA Document "Higher Course Specifications" May 2018, are reproduced to provide teachers and lecturers with all the mandatory information needed to deliver the Higher Course to students.

See Appendix 2 pages 19-31 of the SQA's Higher Mathematics Course Support Notes, by visiting their web page :-

https://www.sqa.org.uk/files_ccc/HigherCourseSpecMathematics.pdf.

Skill	Explanation	TJ Higher Book	Comments/Methodology/Suggestions	√
Applying algebraic and trigonometric skills				
Manipulating Algebraic Expressions	• Factorising a cubic or quartic polynomial expression • Simplifying numerical expressions using the laws of logarithms and exponentials	Chapter 12 Polynomials. Chapter 17 Logs/Exponentials.	Teachers or lecturers could: ♦ demonstrate strategies for factorising polynomials, that is synthetic division, inspection, algebraic long division (From previous learning, candidates should be able to factorise quadratic expressions. They can link these solution(s) to the graph of a function. Factorising polynomials beyond degree two allows extension of this concept.) ♦ link the logarithmic scale to science applications, for example decibel scale for sound, Richter scale of earthquake magnitude, astronomical scale of stellar brightness, acidity and pH in chemistry and biology (Note link between scientific notation and logs to base 10.)	☐ ☐
Manipulating Trigonometric Expressions	• Applying the addition and/or double angle formulae • Applying trigonometric identities • Converting $a\cos x + b\sin x$ to $k\cos(x \pm \alpha)$ or $k\sin(x \pm \alpha)$	Chapter 13 Trigonometry Chapter 13 Trigonometry Chapter 16 The Wave Function	Teachers or lecturers could: ♦ show candidates how formulae for $\cos(\alpha+\beta)$ and $\sin(\alpha+\beta)$ can be used to prove formulae for $\sin 2\alpha$, $\cos 2\alpha$ and $\tan(\alpha+\beta)$ ♦ emphasise the distinction between $\sin x°$ and $\sin x$ (degrees and radians) ♦ give candidates practice in applying the standard formulae, for example expand $\sin 3x$ or $\cos 4x$ ♦ set candidates geometric problems which require the use of addition or double angle formulae	☐ ☐ ☐
Identifying and sketching related functions.	• Identifying a function from a graph, or sketching a function after a transformation of the form $kf(x),\ f(kx),\ f(x)+k,\ f(x+k)$ or a combination of these. • Sketch $y = f'(x)$ given the graph of $y = f(x)$. • Sketch the inverse of a logarithmic or an exponential function. • Completing the square in a quadratic expression where the coefficient of x^2 is non-unitary.	Chapter 5 Transformation/graphs Chapter 8 Calculus 2 Chapter 5 Transformation/graphs Chapter 7 Quadratic Theory	Candidates could use graphic calculators to explore various transformations of functions. Candidates should be able to: ♦ recognise a function from its graph ♦ interpret formulae or equations for maximum and minimum values and identify when they occur	☐ ☐ ☐ ☐

Skill	Explanation	TJ Higher Book	Comments/Methodology/Suggestions	√
Applying algebraic and trigonometric skills to functions				
Determining composite and inverse function	• Know and use terms domain and range. • Determining a composite function given $f(x)$ and $g(x)$, where $f(x)$ and $g(x)$ can be trigonometric, logarithmic, exponential or algebraic functions • determining $f^{-1}(x)$ of functions	Chapter 3 Functions & graphs Chapter 3 Functions & graphs Chapter 3 Functions & graphs	The use of balloon or arrow diagrams and Cartesian graphs can help reinforce the definition of function, domain, and range. Arrow diagrams can also be used to demonstrate that the composite function $f(g(x))$ may not always exist. Diagrams or graphs can also be used to establish whether or not a given function has an inverse. Candidates should be aware that $f(g(x))=x$ implies $f(x)$ and $g(x)$ are inverses.  	☐ ☐ ☐
Solving algebraic equations	• Solving a cubic or quartic polynomial equation • Use the discriminant to find an unknown, given the nature of the roots of an equation, • Solve quadratic inequalities, $ax^2 + bx + c \geq 0$ (or ≤ 0). • Solving logarithmic and exponential equations: • Using the laws of logarithms and exponents • Solving equations of the following forms for a and b, given two pairs of corresponding values of x and y :- $\log y = b\log x + \log a,\ y = ax^b$ $\log y = x\log b + \log a,\ y = ab^x$ • Using a straight line graph to confirm relationships of the form $y = ax^b$ and $y = ab^x$ • Mathematically modelling situations involving the logarithmic or exponential functions. • Finding the coordinates of the point(s) of intersection of a straight line and a curve or of two curves.	Chapter 12 Polynomials Chapter 7 Quadratic Theory Chapter 7 Quadratic Theory Chapter 17 Logs/Exponentials Chapter 17 Logs/Exponentials Chapter 17 Logs/Exponentials Chapter 17 Logs/Exponentials Chapter 17 Logs/Exponentials Chapter 1 Systems of Equations	Teachers or lecturers could: ♦ demonstrate when expressions are not polynomial (negative or fractional powers) ♦ explain that a repeated root is also a stationary point ♦ emphasise the meaning of solving $f(x)=g(x)$ ♦ introduce the Remainder Theorem by: — demonstrating how, for a polynomial equation, this leads to the fact that $f(x)=0$, if $(x-h)$ is a factor of $f(x)$ and h is a root of the equation and vice versa — explaining that communication should include a statement such as 'since $f(h)=0$' or 'since remainder is 0' — using divisors and/or factors of the form $(ax-b)$ ♦ link the solutions of algebraic equations to a graph of function(s), where possible, and encourage candidates to make this connection — candidates could use graphic calculators or refer to diagrams in the question or sketch diagrams to check their solutions ♦ use real-life contexts involving logarithmic and exponential characteristics, for example rate of growth of bacteria, calculations of money earned at various interest rates over time, decay rates of radioactive materials	☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐
Solving trigonometric equations	• Solving trigonometric equations in degrees or radians, including those involving the wave function or trigonometric formulae or identities, in a given interval.	Chapter 6 Trigonometry 1 Chapter 13 Trigonometry 2 Chapter 16 The Wave Function	♦ use real-life contexts, for example: — A possible application is the refraction of a thin light beam passing from air into glass. Its direction of travel is bent towards the line normal to the surface, according to Snell's law. ♦ demonstrate how trigonometric equations can be solved graphically ♦ explain that in the absence of a degree symbol, candidates should use radians in solutions, for example $0 \leq x \leq \pi$	☐

Skill	Explanation	TJ Higher Book	Comments/Methodology/Suggestions	√
Applying geometric skills				
Determining vector connections	• Determining the resultant of vector pathways in three dimensions • Working with collinearity. • Determining the coordinates of an internal division point of a line.	Chapter 14 Vectors Chapter 14 Vectors Chapter 14 Vectors	Candidates should: ♦ work with vectors in both two and three dimensions ♦ mention 'parallel vectors' and 'common point' in their solutions to show collinearity ♦ be able to distinguish between coordinate and component forms	☐ ☐ ☐
Working with vectors	• Evaluate a scalar product given suitable information and determining the angle between two vectors. • Apply properties of the scalar product. • Using and finding unit vectors including $\mathbf{i}, \mathbf{j}, \mathbf{k}$ as a basis.	Chapter 14 Vectors Chapter 14 Vectors Chapter 14 Vectors	Teachers and lecturers could: ♦ introduce candidates to the zero vector, for example through its broader application: — They could sketch a vector diagram of the three forces on a kite, when stationary: its weight, force from the wind (assume normal to centre of kite inclined facing the breeze) and its tethering string. These must sum to zero. ♦ explain the perpendicular and distributive properties of vectors, for example if $\lvert\mathbf{a}\rvert, \lvert\mathbf{b}\rvert \neq 0$ then $\mathbf{a}\cdot\mathbf{b}=0$ if and only if the directions of $\mathbf{a}$ and $\mathbf{b}$ are at right angles	☐ ☐ ☐

Skill	Explanation	TJ Higher Book	Comments/Methodology/Suggestions	√
Applying calculus skills				
Differentiating functions .	• Differentiating an algebraic function which is, or can be simplified to, an expression in powers of x • Differentiating $k\sin x$, $k\cos x$ • Differentiating a composite function using the chain rule.	Chapter 4 Calculus 1 Chapter 15 Calculus 4 Chapter 15 Calculus 4	Teachers and lecturers could use examples from science and terms associated with rates of change, for example acceleration, velocity.	☐ ☐ ☐
Using differentiation to investigate the nature and properties of functions	• Determining the equation of a tangent to a curve at a given point by differentiation. • Determining where a function is strictly increasing/decreasing. • Sketching the graph of an algebraic function by determining stationary points and their nature as well as intersections with the axes and behaviour of $f(x)$ for large positive and negative values of x.	Chapter 4 Calculus 1 Chapter 8 Calculus 2 Chapter 8 Calculus 2	Candidates should know: ♦ that the gradient of a curve at a point is defined to be the gradient of the tangent to the curve at that point ♦ when a function is either strictly increasing, decreasing or has a stationary value, and the conditions for these Candidates can use the second derivative or a detailed nature table. Stationary points should include horizontal points of inflexion.	☐ ☐ ☐
Integrating functions.	• Integrating an algebraic function which is, or can be, simplified to an expression of powers of x. • Integrating functions of the form $f(x) = (x + q)^n$, $n \neq -1$. • Integrating functions of the form $f(x) = p\cos x$ and $f(x) = p\sin x$ • Integrating functions of the form $f(x) = (px + q)^n$, $n \neq -1$. • Integrating functions of the form $f(x) = p\cos(qx + r)$ and $f(x) = p\sin(qx + r)$. • Solving differential equations of the form $\frac{dy}{dx} = f(x)$	Chapter 11 Calculus 3 Chapter 15 Calculus 4 Chapter 15 Calculus 4 Chapter 15 Calculus 4 Chapter 15 Calculus 4 Chapter 11 Calculus 3	Candidates should know: ♦ the meaning of the terms integral, integrate, constant of integration, definite integral, limits of integration, indefinite integral, area under a curve ♦ that if $f(x) = F'(x)$ then $\int_a^b f(x)dx = F(b) - F(a)$ and $\int f(x)dx = F(x) + C$ where C is the constant of integration Teachers and lecturers could: ♦ introduce integration as the process of finding anti-derivatives ♦ demonstrate how to integrate $\cos^2 x$ and $\sin^2 x$ using $\cos^2 x = \frac{1}{2}(1+\cos 2x)$ $\sin^2 x = \frac{1}{2}(1-\cos 2x)$	☐ ☐ ☐ ☐ ☐ ☐
Using integrating to calculate definite integrals.	• Calculating definite integrals of polynomial functions with integer limits. • Calculating definite integrals of functions with limits which are integers, radians, surds or fractions.	Chapter 11 Calculus 3 Chapter 15 Calculus 4		☐ ☐

Skill	Explanation	TJ Higher Book	Comments/Methodology/Suggestions	√
Applying calculus skills				
Applying differential calculus.	• Determine the optimal solution to a given problem. • Determine the greatest/least values of a function on a closed interval. • Solving problems using rate of change.	Chapter 8 Calculus 2 Chapter 8 Calculus 2 Chapter 8 Calculus 2	Teachers and lecturers could: ♦ apply maximum and/or minimum problems in real-life contexts, for example minimum amount of card for creating a box, maximum output from machines ♦ link rate of change to science contexts, for example optimisation in science: An aeroplane cruising at speed v at a steady height has to use power to push air downwards to counter the force of gravity and to overcome air resistance to sustain its speed. The energy cost per km of travel is given approximately by: $E = Av^2 + Bv^2$. (A and B depend on the size and weight of the plane.) At the optimum speed $\frac{dE}{dv} = 0$, thus get an expression for v_{opt} in terms of A and B.	☐ ☐ ☐
Applying integral calculus	• Finding the area between a curve and the x-axis. • Finding the area between a straight line and a curve or two curves. • Determine and use a function from a given rate of change and initial conditions.	Chapter 11 Calculus 3 Chapter 11 Calculus 3 Chapter 11 Calculus 3	Teachers and lecturers could demonstrate how to: ♦ use graphical calculators as part of an investigative approach ♦ calculate the area between curves by subtracting individual areas, using diagrams or graphing packages ♦ reduce the area to be determined to smaller components to estimate a segment of area between the curve and x-axis and then use the area formulae (triangle or rectangle) A practical application of the integral of $\frac{1}{x^2}$ is to calculate the energy required to lift an object from the earth's surface into space. The work energy required is $E = \int F dr$, where F is the force due to the earth's gravity and r is the distance from the centre of the earth. For a 1 kg object $E = -\int \left(\frac{GM}{r^2}\right) dr$, where M is the mass of the earth and G is the universal gravitational constant. $GM = 4 \cdot 0 \times 10^{14}\ \mathrm{m^3\,s^{-2}}$ The integration extends from $r = 6 \cdot 4 \times 10^6$ m (the radius of the earth) to infinity.	☐ ☐ ☐

Skill	Explanation	TJ Higher Book	Comments/Methodology/Suggestions	√
Applying algebraic and geometric skills				
Applying algebraic skills to rectilinear shapes.	• Finding the equation of a line parallel to and a line perpendicular to a given line. • Using $m = tan\theta$ to calculate a gradient or angle. • Using properties of medians, altitudes and perpendicular bisectors in problems involving the equation of a line and intersection of lines. • Determine whether or not two lines are perpendicular.	Chapter 2 Equation of a Line Chapter 2 Equation of a Line Chapter 2 Equation of a Line Chapter 2 Equation of a Line	Teachers and lecturers could: ♦ emphasise the 'gradient properties' of $m_1 = m_2$ and $m_1m_2 = -1$ ♦ use practical contexts for triangle work, where possible ♦ emphasise differences in median, altitude, etc ♦ investigate properties and intersections Candidates should: ♦ avoid approximating gradients to decimals ♦ have knowledge of the basic properties of triangles and quadrilaterals ♦ include the phrases 'parallel' and 'common point' in their answers to show collinearity, for example since $m_{AB} = m_{BC} \Rightarrow$ AB and BC are parallel, and B is a common point ♦ understand terms such as orthocentre, circumcentre and concurrency	☐ ☐ ☐ ☐
Applying algebraic skills to circles and graphs	• Determining and using the equation of a circle. • Using properties of tangency in the solution of a problem. • Determining the intersection of circles or a line and a circle.	Chapter 9 The Circle Chapter 9 The Circle Chapter 9 The Circle	Teachers and lecturers could: ♦ develop the equation of a circle (centre the origin) from Pythagoras, and extend this to a circle with centre (a,b) or relate to transformations ♦ link the properties of tangency with the application of the discriminant ♦ make candidates aware of different ways in which more than one circle can be positioned, for example intersecting at one, two, or no points, sharing the same centre (concentric), one circle inside another ♦ give candidates practice in applying knowledge of geometric properties of circles to find related points (for example the stepping-out method) — solutions should not be obtained from scale drawings	☐ ☐ ☐
Modelling situations using sequences.	• Determining a recurrence relation from given information and using it to calculate a required term • Finding and interpreting the limit of a sequence, where it exists	Chapter 10 Recurrence Relations Chapter 10 Recurrence Relations	Teachers and lecturers could use examples from a real-life context, for example a situation where the concentration of chemicals or medicines is important.	☐ ☐

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Part B

In this section, we list the Chapters covering the entire content of the SQA's Higher Mathematics Course. The electronic version of this part is available for download from teejaymaths.com and may be used as a course planner, the final column providing a means of adding supplementary materials and comments.

See Page Appendix 2 pages 19-31 of the SQA's Higher Mathematics Course Support Notes, by visiting their web page :-

https://www.sqa.org.uk/files_ccc/HigherCourseSpecMathematics.pdf.

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