ESSENTIAL

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Practice Questions & Exam Papers

QUESTIONS & PAPERS

- Practise **60+ questions** covering every topic
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Practice Questions

Topic	Practice Questions
Algebraic and trigonometric skills	
▸ Functions and graphs	1–9
▸ Quadratics	10–13
▸ Polynomials	14–16
▸ Logarithms and exponentials	17–22
▸ Trigonometric formulae	23–27
▸ The wave function	28–29
Geometric skills	
▸ Vectors	30–35
Calculus skills	
▸ Differentiation	36–39
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▸ Applications of integration	48–51
Algebraic and geometric skills	
▸ Recurrence relations	52–55
▸ The straight line	56–58
▸ The circle	59–62

Practice Papers

Topic	Practice Paper A Paper 1	Practice Paper A Paper 2	Practice Paper B Paper 1	Practice Paper B Paper 2
Algebraic and trigonometric skills				
▸ Functions and graphs	3, 8, 10, 14		2, 5, 9	2
▸ Quadratics	6	2		1, 11
▸ Polynomials		5(a)	15	
▸ Logarithms and exponentials	13, 15	8	8	10, 13
▸ Trigonometric formulae	18	11(a)	11, 13	15
▸ The wave function		9	16	
Geometric skills				
▸ Vectors	7, 17	6	4, 12	3, 5
Calculus skills				
▸ Differentiation	5		6	7
▸ Applications of differentiation	11, 12, 19	3	1	12, 14(a), (b)
▸ Integration	16	11(b)	7	
▸ Applications of integration		5(b), 10	14	9, 14(c)
Algebraic and geometric skills				
▸ Recurrence relations	2			8
▸ The straight line	4, 9	1	3	6(a), (b)
▸ The circle	1	4, 7	10	4, 6(c)

MARKS

Algebraic and trigonometric skills

1 A function f is given by $f(x) = \frac{1}{5-x}$.

a) What value of x cannot be in the domain of f? **1**

b) Find $f^{-1}(x)$. **3**

> **Hint!**
> $f(x)$ is undefined when the denominator = 0.

2 A function h is given by $h(x) = \sqrt{4-x^2}$.
Write down a suitable domain of h. **2**

3 Functions $f(x) = 4x$ and $g(x) = 3\sin x$ are defined on suitable domains.

a) Evaluate $f\left(g\left(\frac{\pi}{6}\right)\right)$. **1**

b) Find an expression for $g(f(x))$. **2**

4 Functions $f(x) = \frac{1}{2}x - \frac{3}{4}$ and $g(x) = 2x + \frac{3}{2}$ are defined on suitable domains.

> **Hint!**
> $f(f^{-1}(x)) = x$

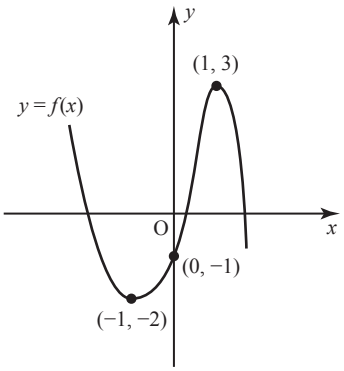
a) Find an expression for $f(g(x))$. **1**

b) What is the connection between the functions f and g? **2**

> **Hint!**
> This table shows the transformations associated with different functions.

Function	Transformation
$-f(x)$	Reflect in the x-axis
$f(-x)$	Reflect in the y-axis
$f(x) + a$	Vertical translation a units up
$f(x + a)$	Horizontal translation a units left
$kf(x)$	Vertical stretch by factor k
$f(kx)$	Horizontal compression by factor k
$f^{-1}(x)$	Reflect in $y = x$
$f'(x)$	Stationary points of $f(x)$ are roots of $f'(x)$

5 The diagram shows the graph of a function $y = f(x)$.
On separate diagrams sketch the graphs of:



a) $y = 3 - f(x)$; **2**

b) $y = 2f(x) - 1$; **2**

c) $y = f'(x)$. **3**

Paper 1 (non-calculator)

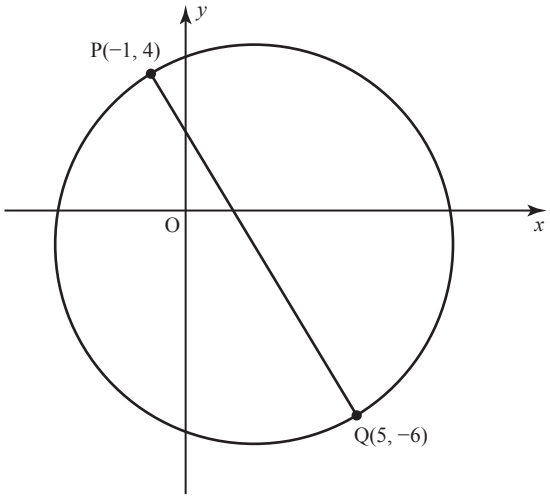
Duration: 1 hour 30 minutes
Total marks: 70
Attempt ALL questions.
You may NOT use a calculator.
Full credit will be given only to solutions which contain appropriate working.
State the units for your answer where appropriate.
Answers obtained by readings from scale drawings will not receive any credit.

MARKS

1 P(−1, 4) and Q(5, −6) are points on the circumference of a circle, as shown in the diagram. **3**

PQ is a diameter.

Find the equation of the circle.



2 A sequence is generated by the recurrence relation $u_{n+1} = au_n + b$, where $-1 < a < 1$ and $u_0 = 40$.

a) Given $u_1 = 28$ and $u_2 = 19$, find the values of a and b. **4**

b) Find the limit of this sequence as $n \to \infty$. **2**

3 Functions $f(x) = \dfrac{1}{x+2}$ and $g(x) = 3x - 4$ are defined on suitable domains.

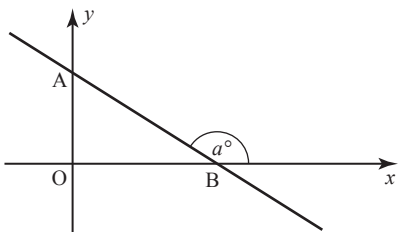
a) Find an expression for $h(x)$ where $h(x) = f(g(x))$. **2**

b) What value of x cannot be in the domain of h? **1**

4 Line AB has equation $\sqrt{3}x + y = 3$ as shown in the diagram. **2**

The angle between AB and the positive direction of the x-axis is $a°$.

Calculate the value of a.



Paper 2

Duration: 1 hour 45 minutes
Total marks: 80
Attempt ALL questions.
You may use a calculator.
Full credit will be given only to solutions which contain appropriate working.
State the units for your answer where appropriate.
Answers obtained by readings from scale drawings will not receive any credit.

MARKS

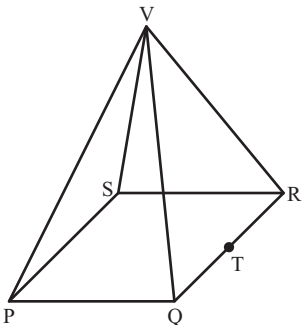
1 Solve $10 - 3x - x^2 < 0$, where x is a real number. **2**

2 Functions $f(x) = \dfrac{1}{x+3}$ and $g(x) = \dfrac{1}{x} - 3$ are defined on suitable domains.

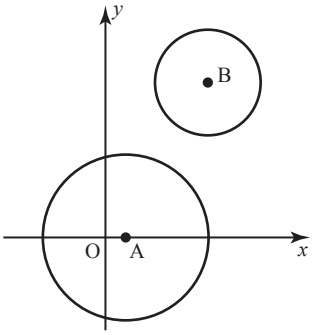
a) Find an expression for $f(g(x))$. **3**
Give your answer in its simplest form.

b) What is the connection between the functions f and g? **1**

3 VPQRS is a pyramid with rectangular base PQRS. **3**
T is the midpoint of QR.
Relative to some appropriate axes,
$\overrightarrow{PQ}$ represents $5\mathbf{i}+5\mathbf{j}-5\mathbf{k}$
$\overrightarrow{PS}$ represents $4\mathbf{i}+4\mathbf{j}+8\mathbf{k}$
$\overrightarrow{PV}$ represents $6\mathbf{i}+11\mathbf{j}+9\mathbf{k}$
Find $\overrightarrow{VT}$ in component form.



4 Two circles, with centres A and B, have equations $x^2 + y^2 - 4x - 60 = 0$ and $x^2 + y^2 - 20x - 30y + 300 = 0$, respectively.



a) Find the distance between the centres of the two circles. **3**

Question	Answer	Marks	Commentary, hints and tips	Grade demand
13	**Method 1** $\log_e y = \log_e be^{ax}$ $\log_e y = \log_e b + \log_e e^{ax}$ $\log_e y = ax + \log_e b$ $a = \frac{4.5}{3} = 1.5$ $\log_e b = 4.5$ $\Rightarrow b = e^{4.5} = 90$ **Method 2** $\log_e y = 1.5x + 4.5$ $y = e^{1.5x+4.5}$ $y = e^{1.5x} \times e^{4.5}$ $y = 90e^{1.5x}$ $a = 1.5, b = 90$	**Method 1** Take logs of both sides of equation. Use $\log_e xy = \log_e x + \log_e y$ and $\log_e e^{ax} = ax \log_e e = ax$. Find a. Find b. **Method 2** State linear equation. Convert to exponential form. Use law of indices. Obtain result. **4 marks**	Use laws of logarithms to rearrange $y = be^{ax}$ into the form $\log_e y = ax + \log_e b$. This is the equation of the straight line with gradient a and y-intercept $(0, \log_e b)$. Remember $\log_e b = y \Leftrightarrow b = e^y$. The straight line has equation $\log_e y = mx + c$. Use laws of logarithms to rearrange this equation into the form $y = be^{ax}$. Remember $\log_e y = x \Leftrightarrow y = e^x$.	> C
14 **a)**	$a = v'(t)$ $= 5 - 2t$	Know to differentiate. Differentiate.	Acceleration is the rate of change of velocity with respect to time, i.e. $a = v'(t)$.	> C
b)	$5 - 2t < 0$ $t > 2.5$ After 2.5 seconds,	Set up inequality. Solve inequality and interpret result.		> C
c)	$s(t) = \int v(t)\,dt$ $= t + \frac{5}{2}t^2 - \frac{1}{3}t^3 + c$ $s(0) = 2 \Rightarrow c = 2$ $\Rightarrow s(t) = t + \frac{5}{2}t^2 - \frac{1}{3}t^3 + 2$	Know to integrate. Integrate. Determine constant and state $s(t)$. **7 marks**	Velocity is the rate of change of displacement with respect to time, i.e. $v = s'(t)$ so $s = \int v(t)dt$. Remember to add c, the constant of integration. Substitute $s = 2$ and $t = 0$ into the result of the integration to find the value of c.	> C
15	$2\cos^2\theta - 1 + \cos\theta = 0$ $2\cos^2\theta + \cos\theta - 1 = 0$ $(2\cos\theta - 1)(\cos\theta + 1) = 0$ $\cos\theta = \frac{1}{2}, \cos\theta = -1$ $\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \pi$	Use double angle formula. Rearrange and factorise. Solve for $\cos\theta$. Solve for θ. **4 marks**	Use the substitution $\cos 2\theta = 2\cos^2\theta - 1$ to obtain an equation with terms in $\cos\theta$ only. Use exact values to give the answers in terms of π.	> C