FOR THE IB DIPLOMA PROGRAMME

SECOND EDITION

Mathematics

ANALYSIS AND APPROACHES SL

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Ben Woolley



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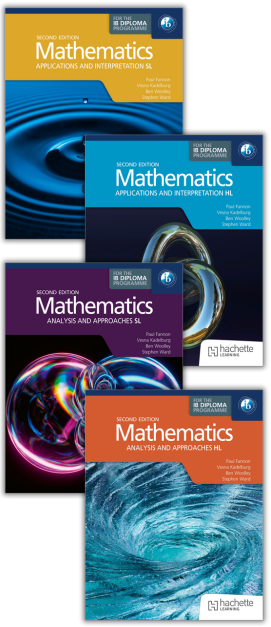
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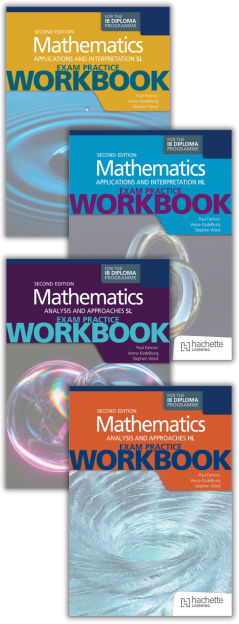


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Contents

Introduction viii

Prior knowledge check xi

Chapter 1 Exponents and logarithms 2
1A Laws of exponents 4
1B Logarithms 13
1C Logarithms with other bases 18

Chapter 2 Sequences 28
2A Arithmetic sequences 29
2B Geometric sequences 36
2C The sum of infinite convergent geometric sequences 42
2D The binomial expansion 45

Chapter 3 Functions 56
3A Concept of a function 58
3B Composite functions 65
3C Inverse functions 67
3D Using technology to sketch and solve 74
3E Transformations of graphs 79

Chapter 4 Further functions and equations 100
4A Equations of straight lines in two dimensions 102
4B Quadratics functions and their graphs 112
4C Solving quadratic equations and inequalities 125
4D The discriminant 129
4E Rational functions 133
4F Exponential and logarithmic functions 136
4G Solving equations analytically 143
4H Solving equations and inequalities using technology 147

Chapter 5 Trigonometric functions and equations 158
5A Radian measure and definitions of trigonometric functions 160
5B Graphs of trigonometric functions 168

5C Trigonometric identities . . . 175
5D Trigonometric equations . . . 181

Chapter 6 Geometry and trigonometry . . . 192
6A Volumes and surface areas of three-dimensional solids . . . 193
6B Trigonometry in triangles . . . 201
6C Applications of trigonometry . . . 213
6D Arcs and sectors of circles . . . 221

Chapter 7 Univariate data . . . 234
7A Summarizing data . . . 235
7B Presenting data . . . 248

Chapter 8 Probability . . . 260
8A Introduction to probability . . . 262
8B Probability techniques . . . 266
8C Conditional probability . . . 277
8D Reversing conditional probability . . . 280

Chapter 9 Probability distributions . . . 290
9A Discrete random variables . . . 292
9B Binomial distribution . . . 297
9C The normal distribution . . . 302
9D Normal distribution . . . 306

Chapter 10 Differential calculus . . . 318
10A Limits and derivatives . . . 320
10B Graphical interpretation of derivatives . . . 326
10C The derivatives of common functions . . . 335
10D Equations of tangents and normals at a given point . . . 342
10E The chain rule for composite functions . . . 349
10F The product and quotient rules . . . 351
10G The second derivative . . . 355
10H Local maximum and minimum points . . . 361
10I Points of inflection with zero and non-zero gradients . . . 365

Chapter 11 Integral calculus . . . 376
11A Indefinite integration . . . 377
11B Definite integration and the area under a curve . . . 385
11C Kinematics . . . 399

Chapter 12 Proof 410
12A The structure of mathematical proof 411

Chapter 13 Internal Assessments 416
Types of exploration 416
Marking criteria 416
Evaluating the sample answers 430

Review Exercise 432

Analysis and approaches SL: Practice Paper 1 448

Analysis and approaches SL: Practice Paper 2 452

Answers 455

Glossary 544

Acknowledgements 546

Index 548

Introduction

Welcome to your coursebook for Mathematics for the IB Diploma: analysis and approaches SL. The structure and content of this coursebook follow the structure and content of the 2027 IB Mathematics: analysis and approaches guide, with headings that correspond directly with the content areas listed in the guide.

This is also the first book required by students taking the higher level course. Students should be familiar with the content of this book before moving on to Mathematics for the IB Diploma: HL.

Using this book

The book begins with a Prior knowledge check, which is a diagnostic test that will help remind you of the mathematical concepts and techniques with which you should be familiar before starting the course. The following twelve chapters in this book then cover the entirety of the content for the Mathematics: analysis and approaches course at Standard Level. Please note that students studying the course at Higher Level should complete this book and also the supplementary material in the Higher Level textbook. The final chapter contains advice on how to complete your mathematical exploration.

Special features of the chapters include:

Key point

Important mathematical rules and formulae are presented as Key Points, making them easy to locate and refer back to when necessary.

ESSENTIAL UNDERSTANDINGS

Each chapter begins with a summary of the key ideas to be explored and a list of the knowledge and skills you will learn. These are revisited in a checklist at the end of each chapter.

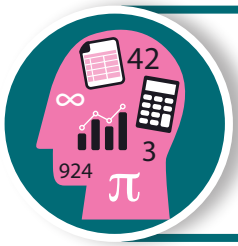
Prior knowledge

At the start of each chapter, you will find a list of key questions that include mathematical concepts and skills you should already be familiar with before starting the chapter. If you can complete the questions, you are ready to tackle the new material. If something looks unfamiliar, it is your cue to review syllabus topics before you begin. Mastering these prerequisites first will allow you to focus on your new International Baccalaureate learning.

WORKED EXAMPLES

There are many Worked Examples in each chapter, demonstrating how the Key Points and mathematical content described can be put into practice. Each Worked Example comprises two columns:

On the left, how to **think** about the problem and what tools or methods will be needed at each step.	On the right, what to **write**, prompted by the left column, to produce a formal solution to the question.



Prior knowledge check

These questions will help remind you of the mathematical concepts and techniques with which you should be familiar before starting the course. It contains one question for each of the 38 bullet points listed in the 'Prior knowledge topics' section of the DP guide.

Number and algebra

1 *[Number systems]* For each of these numbers, list all the number sets, out of $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, to which it belongs:

a $\sqrt{2}$ **b** $1\frac{3}{4}$ **c** -5 **d** 3 **e** $0.\dot{6}$ **f** 0

2 *[SI units]* Convert a speed of 72 km/h into m/s.

3 *[Rounding and significant figures]* Round the number 0.05492 to two significant figures.

4 *[Absolute value]* Evaluate without a calculator: $|-3| + |4 - 7|$.

5 *[Operations and order of operations]* Evaluate without a calculator: $\frac{3}{4} + 2 \times (1.5 - 0.5)$.

6 *[Prime numbers, factors and multiples]*

a List all the distinct prime factors of 60.

b List all the factors of 25.

c List the first four positive multiples of 3.

7 *[Greatest common factor and least common multiple (HL only)]* Find the greatest common factor (GCF) and least common multiple (LCM) of 12 and 18.

8 *[Ratio, percentage and proportion]* A shirt costing 40 GBP is discounted by 15%. Find the new price.

9 *[Manipulation and factorization of algebraic expressions]*

a Expand and simplify $(2x - 3)(x + 4)$.

b Factorize fully $4x^2 - 12x - 40$.

10 *[Rearranging formulae]* Rearrange $V = \frac{1}{3}\pi r^2 h$ to make h the subject.

11 *[Evaluating expressions by substitution]* Evaluate $x^2 - 3y$ when $x = -2$ and $y = 4$.

12 *[Positive integer exponents]* Evaluate $2^3 \times 3^2$ without a calculator.

13 *[Rational exponents (HL only)]* Evaluate $16^{\frac{3}{4}}$ without a calculator.

14 *[Inequalities and intervals]* List all the integers which satisfy the inequality $-2 \leqslant x < 5$.

15 *[Simplification of roots (surds or radicals)]* Write $\sqrt{45}$ in the form $a\sqrt{b}$.

16 *[Rationalizing the denominator (HL only)]* Rationalize the denominator of $\frac{6}{\sqrt{3}}$ and simplify.

17 *[Standard form / Scientific notation]* Write 34 500 in the form $a \times 10^k$, where $1 \leqslant a < 10$ and $k \in \mathbb{Z}$.

18 *[Familiarity with world currencies]* Convert 50 USD to EUR, given an exchange rate of 1 USD = 0.85 EUR.

19 *[Solution of linear equations]* Solve the equation $3(x - 2) = 15$.

20 *[Quadratic equations by factorization]* Solve $x^2 - x - 20 = 0$ by factorization.

21 *[Quadratic equations using the formula (HL only)]* Solve $2x^2 - 4x - 1 = 0$ using the quadratic formula, giving exact answers.

22 *[Solving systems of linear equations]* Solve the simultaneous equations: $x + 2y = 5$ and $3x - y = 1$.

1 Exponents and logarithms

ESSENTIAL UNDERSTANDINGS

- Number and algebra allow us to represent patterns, show equivalences and make generalizations which enable us to model real-world situations.
- Algebra is an abstraction of numerical concepts and employs variables to solve mathematical problems.

In this chapter you will learn …

- how to use the laws of exponents with integer and rational exponents
- about the number e
- about logarithms and how to use logarithms to any (positive) base
- how to use the laws of logarithms
- how to change the base of a logarithm
- how to solve exponential equations.

Tip

The notation $k \in \mathbb{Z}$ means that k is an integer.

Prior knowledge

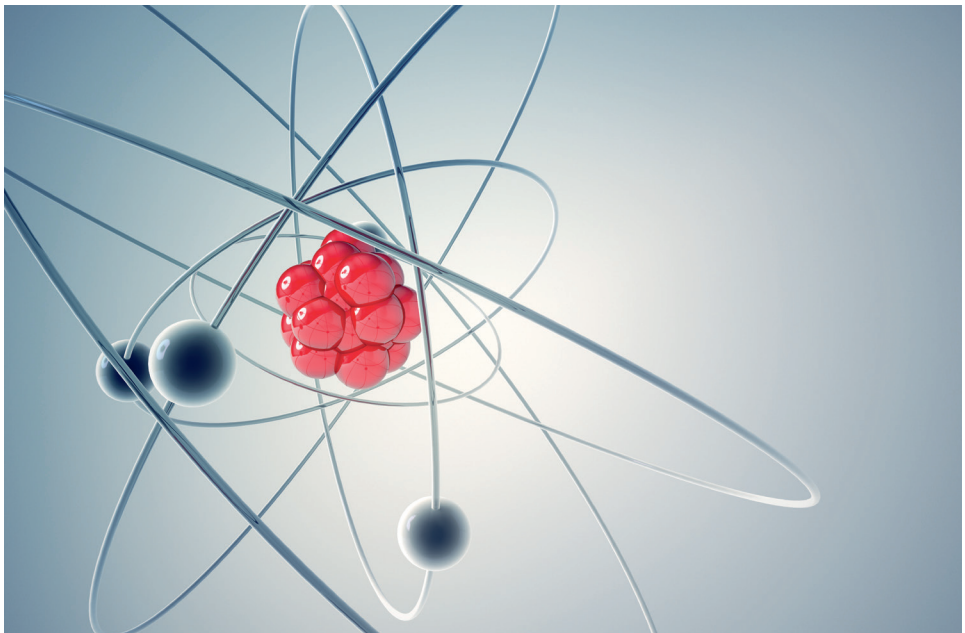
Before starting this chapter, you should already be able to complete the following:

1 Evaluate the following:

a 3^4 **b** 5×2^3

2 Express 64 in the form 2^k, where $k \in \mathbb{Z}$.

Figure 1.1 How do we compare very large numbers?





When you first study maths, it is easy to think it is a subject that is all about numbers. But actually, the study of numbers is just one area in which we apply the logic of maths. Once you have experience of numbers, you will be looking to make links: for example, $3^2 + 4^2 = 5^2$, but is this true in general for consecutive numbers? Of course the answer is 'no', but $3^2 \times 4^2 = 12^2$ gives you a rule that works when applied to multiplications.

In this chapter we will be generalizing some patterns you are probably already aware of. You will then be applying these rules to numbers represented in different ways. You might wonder why we need a different form for writing numbers, but try writing down the number of atoms in the sun. When numbers are very large or very small, it is useful to have a more convenient form to express them in.

Finally, you will see how to solve equations where the unknown is in the exponent, such as $10^x = 7$. In so doing you will meet a number that has wide-ranging applications in fields including science, economics and engineering.

STARTER ACTIVITY

Look at the pictures in Figure 1.1. In small groups discuss whether there are more atoms in a jug of water than there are jugs of water in the Atlantic Ocean.

Now look at this problem:

A model suggests that the level of carbon dioxide (CO_2) in the atmosphere in parts per million is given by 400×1.05^n, where n is the number of years after 2018.

Use a spreadsheet to estimate when this model predicts the level of CO_2 in the atmosphere will reach 1000 parts per million.

Learner Profile – Inquirers

Is mathematics invented or discovered? Is mathematics designed to mirror reality?





1A Laws of exponents

TOK Link

◆ In expressions like a^n, a is called the base and n is called the exponent, although you may see it referred to as power, index or indice. Does having a label help you understand it? Would it be better if everybody used the same label?

Tip

Be aware that you will often have to use these rules 'backwards'. For example, if you see 2^{12} you can rewrite it as $(2^3)^4$ or as $(2^4)^3$.

In your previous work you have may have noticed that $2^3 \times 2^2 = (2 \times 2 \times 2) \times (2 \times 2) = 2^5$. Based on specific examples such as this you can generalize to a formula.

Key point 1.1

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$

To formally prove these rules requires a method called mathematical induction, which you will encounter if you are studying the HL course.

If we set m and n equal in the second law in Key Point 1.1 then it follows that $a^0 = 1$ for any (non-zero) value of a.

TOOLKIT: Problem Solving

Anything raised to the power 0 is 1, but 0 to any power is 0. So, what is the value of 0^0?

WORKED EXAMPLE 1.1

Simplify $x^6 \times x^3$.

Use $a^m \times a^n = a^{m+n}$ ..... $x^6 \times x^3 = x^{6+3}$

$= x^9$

WORKED EXAMPLE 1.2

Simplify $\frac{y^8}{y^2}$.

Use $\frac{a^m}{a^n} = a^{m-n}$ ..... $\frac{y^8}{y^2} = y^{8-2}$

$= y^6$

WORKED EXAMPLE 1.3

Simplify $(7^3)^5$. You do not need to evaluate your result.

Use $(a^m)^n = a^{mn}$ ..... $(7^3)^5 = 7^{3\times 5}$

$= 7^{15}$

If the expression has numbers as well as algebraic values, just multiply or divide the numbers separately. This follows because we can do multiplication in any convenient order.

WORKED EXAMPLE 1.4

Simplify $2b^3 \times 7b^4$.

You can reorder the multiplication as $2 \times 7 \times b^3 \times b^4$ ..... $2b^3 \times 7b^4 = 14b^3b^4$

Then $b^3 \times b^4 = b^{3+4}$ ..... $= 14b^7$

WORKED EXAMPLE 1.5

Simplify $\frac{6c^9}{2c^4}$.

You can use your knowledge of fractions to write the expression as a numeric fraction multiplied by an algebraic fraction ..... $\frac{6c^9}{2c^4} = \frac{6}{2} \times \frac{c^9}{c^4}$

Use $\frac{c^9}{c^4} = c^{9-4}$ ..... $= 3c^5$

The laws in Key Point 1.1 work for negative as well as positive integers. But what is the meaning of a negative exponent? In the second law in Key Point 1.1 we can set $m = 0$ and use the fact that $a^0 = 1$ to deduce a very important rule.

Key point 1.2

- $a^{-n} = \frac{1}{a^n}$

WORKED EXAMPLE 1.6

Without a calculator write 2^{-4} as a fraction in its simplest terms.

Use $a^{-n} = \frac{1}{a^n}$ ..... $2^{-4} = \frac{1}{2^4}$

$= \frac{1}{16}$

Exercise 1A

For questions 1 to 3, use the method demonstrated in Worked Example 1.1 to simplify the expressions. If numerical, you do not need to evaluate your result.

1 **a** $x^2 \times x^4$ **b** $x^5 \times x^7$

2 **a** $y^3 \times y^3$ **b** $z^5 \times z^5$

3 **a** $5^7 \times 5^{10}$ **b** $2^{12} \times 2^{12}$

For questions 4 to 6, use the method demonstrated in Worked Example 1.2 to simplify the expressions. If numerical, you do not need to evaluate your result.

4 **a** $\frac{x^4}{x^3}$ **b** $\frac{x^8}{x^5}$

5 **a** $y^8 \div y^4$ **b** $z^9 \div z^3$

6 **a** $11^{12} \div 11^4$ **b** $7^{10} \div 7^5$

For questions 7 to 9, use the method demonstrated in Worked Example 1.3 to simplify the expressions. If numerical, you do not need to evaluate your result.

7 **a** $(x^3)^5$ **b** $(x^4)^8$

8 **a** $(y^4)^4$ **b** $(z^5)^5$

9 **a** $(3^5)^{10}$ **b** $(13^7)^4$

For questions 10 and 11, use the method demonstrated in Worked Example 1.4 to simplify the expressions.

10 **a** $12x^2 \times 4x^5$ **b** $3x^4 \times 5x^3$

11 **a** $5x^2yz \times 4x^3y^2$ **b** $6x^7yz^2 \times 2xz^3$

For questions 12 to 14, use the method demonstrated in Worked Example 1.5 to simplify the expressions.

12 **a** $\frac{10x^{10}}{5x^5}$ **b** $\frac{9x^9}{3x^3}$

13 **a** $15x^2 \div 9x^4$ **b** $21x^5 \div 28x^3$

14 **a** $14x^3y^5 \div 2xy^2$ **b** $6x^5yz^2 \div 3x^2y^2z$

For questions 15 to 18, use the method demonstrated in Worked Example 1.6 to write the expression as a fraction in its simplest terms.

15 **a** 10^{-1} **b** 7^{-1}

16 **a** 3^{-3} **b** 5^{-2}

17 **a** $\left(\frac{3}{4}\right)^{-1}$ **b** $\left(\frac{5}{7}\right)^{-1}$

18 **a** $\left(\frac{2}{3}\right)^{-2}$ **b** $\left(\frac{2}{5}\right)^{-3}$

For questions 19 to 21, use the method demonstrated in Worked Example 1.7 to write the expression as a fraction in its simplest terms.

19 **a** 7×3^{-2} **b** 5×2^{-4}

20 **a** $6x^{-1}$ **b** $10x^{-4}$

21 **a** $3^{-2} \div 2^{-3}$ **b** $4^{-3} \div 3^{-4}$

For questions 22 to 24, use the method demonstrated in Worked Example 1.8 to simplify each expression.

22 **a** $(3u^{-2})^3$ **b** $(2v^{-3})^5$

23 **a** $(2a^{-5})^{-2}$ **b** $(3b^{-7})^{-3}$

24 **a** $(5x^2y^3)^2$ **b** $(3a^2b^{-2})^4$

For questions 25 to 27, use the method demonstrated in Worked Example 1.9 to simplify each expression.

25 **a** $\left(\frac{x}{3}\right)^3$ **b** $\left(\frac{5}{x^2}\right)^2$

26 **a** $\left(\frac{3x^2}{2y^3}\right)^3$ **b** $\left(\frac{5uv^3}{7b^4}\right)^2$

27 **a** $\left(\frac{3u}{4v^2}\right)^{-2}$ **b** $\left(\frac{2a^3}{3b^2}\right)^{-3}$

For questions 28 to 30, use the method demonstrated in Worked Example 1.10 to simplify each expression.

28 **a** $\frac{6x^2 - 21x^7}{3x}$ **b** $\frac{15y^4 + 25y^6}{5y^3}$

29 **a** $\frac{15u^3v + 18u^2v^3}{3uv}$ **b** $\frac{20a^5b^7c^2 - 16a^4b^2c^3}{4a^3b^2c}$

30 **a** $\frac{10p^3q - 6pq^3}{2p^2q}$ **b** $\frac{14s^4t^3 + 21s^5t^7}{7s^2t^5}$

51 The population of penguins (P) after t years on two islands is modelled by:

first island: $P = 200 \times 1.1^t$

second island: $P = 100 \times 1.2^t$.

How many years are required before the population of penguins on both islands is equal?

52 Find the exact solution of the equation $\log_4 x = 6\log_x 8$.

53 Solve the equation $2^{5-3x} = 3^{2x-1}$, giving your answer in the form $\frac{\ln p}{\ln q}$, where p and q are integers.

54 Moore's law states that the density of transistors on an integrated circuit doubles every 2 years. Write an expression for the time taken for the density to multiply by 10 and use technology to evaluate this expression.

55 **a** If $\log_a(x^2) = b$ find the product of all possible values of x.

b If $(\log_a x)^2 = b$ find the product of all possible values of x.

Checklist

- You should know how to use the laws of exponents:
  - ☐ $a^m \times a^n = a^{m+n}$
  - ☐ $(ab)^n = a^n \times b^n$
  - ☐ $a^{-n} = \frac{1}{a^n}$
  - ☐ $(a^m)^n = a^{mn}$
  - ☐ $\frac{a^m}{a^n} = a^{m-n}$
  - ☐ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- You should be able to extend the laws of exponents to general rational exponents:
  - ☐ $a^{\frac{m}{n}} = \sqrt[n]{a^m}$
- You should know the definition of a logarithm and be able to work with logarithms with any positive base, including the value e:

  $a = b^x$ is equivalent to $\log_b a = x$
- You should know the cancellation property of exponentiation and logarithms:
  - ☐ $b^{\log_b x} = x$
  - ☐ $\log_b(b^x) = x$
- In particular, for base 10 and base e:
  - ☐ $\log_{10} 10^x = x$
  - ☐ $\ln e^x = x$
  - ☐ $10^{\log_{10} x} = x$
  - ☐ $e^{\ln x} = x$
- You should be able to work with the laws of logarithms:
  - ☐ $\log_a xy = \log_a x + \log_a y$ where $a, x > 0$ and $a \neq 1$
  - ☐ $\log_a \frac{x}{y} = \log_a x - \log_a y$
  - ☐ $\log_a x^m = m\log_a x$
- You should be able to change the base of a logarithm:
  - ☐ $\log_a x = \frac{\log_b x}{\log_b a}$
- You should be able to use logarithms to solve exponential equations.

Mixed practice

1 Simplify $\frac{(3xy^2)^2}{(xy)^3}$.

2 Write $(3x^2y^{-3})^{-2}$ without brackets or negative indices.

3 Zipf's law in geography is a model for how the population of a city (P) in a country depends on the rank of that city (R), that is, whether it is the largest ($R = 1$), second largest ($R = 2$), and so on. The suggested formula is $P = kR^{-1}$.

In a particular country the second largest city has a population of 2 000 000.

a Find the value of k.

b According to the model, find:

i the size of the country's fourth largest city

ii the rank of the city with population 250 000.

4 Find the exact solution of $8^x = 2^{x+6}$.

5 Find the exact solution of $\log_{10}(x+1) = 2$.

6 Find the exact solution to $\ln(2x) = 3$.

7 Use technology to solve $e^x = 2$.

8 Use technology to solve $5 \times 10^x = 17$.

9 Rearrange to make x the subject of $5e^x - 1 = y$.

10 **a** Find the exact value of $\left(\frac{4}{9}\right)^{-\frac{3}{2}}$.

b Find $\log_2\left(\frac{1}{8}\right)$.

11 Write $\ln 4 + 2\ln 3$ in the form $\ln k$.

12 Solve the equation $2 \times 3^{x-2} = 54$.

13 Use technology to solve $1.05^x = 2$.

14 Solve the equation $100^{x+1} = 10^{3x}$.

15 Find the value of x such that $\log_3(5x+1) = 2$.

16 Find the exact solution of the equation $3\ln x + 2 = 2(\ln x - 1)$.

17 Given that $a = \log_3 x$, $b = \log_3 y$ and $c = \log_3 z$, write down the following in terms of a, b and c:

a $\log_3(x^2y)$ **b** $\log_3\left(\frac{x}{yz^3}\right)$ **c** $\log_3\left(\sqrt{zx^3}\right)$

18 Find the value of each of the following, giving your answer as an integer.

a $\log_6 36$ **b** $\log_6 4 + \log_6 9$ **c** $\log_6 2 - \log_6 12$

Mathematics AA SL May 2015 Paper 1 TZ1 Q2

19 **a** Given that $2^m = 8$ and $2^n = 16$, write down the value of m and of n.

b Hence or otherwise solve $8^{2x+1} = 16^{2x-3}$.

Mathematics AA SL, Paper 1 May 2015 TZ1 Q3

20 The Henderson–Hasselbach equation predicts that the pH of blood is given by:

$$\text{pH} = 6.1 + \log_{10}\left(\frac{[HCO_3^-]}{[H_2CO_3]}\right)$$

where $[HCO_3^-]$ is the concentration of bicarbonate ions and $[H_2CO_3]$ is the concentration of carbonic acid (created by dissolved carbon dioxide). Given that the bicarbonate ion concentration is maintained at 0.579 moles per litre, find the range of concentrations of carbonic acid that will maintain blood pH at normal levels (which are between 7.35 and 7.45).

21 In attempting to set a new record, skydiver Felix Baumgartner jumped from close to the edge of the Earth's atmosphere. During his period of free fall, his predicted speed (v) in metres per second at a time t seconds after he jumped was modelled by: $v = 1350(1 - e^{-0.007t})$.

a Find the predicted speed after one second.

b Baumgartner's aim was to break the speed of sound ($300\,\text{m s}^{-1}$). Given that he was in free fall for 60 seconds, determine whether his speed ever exceeded $300\,\text{m s}^{-1}$.

22 The Richter scale measures the strength of earthquakes. The strength (S) is given by $S = \log_{10} A$, where A is the amplitude of the wave measured on a seismograph in micrometres.

a If the amplitude of the wave is 1000 micrometres, find the strength of the earthquake.

b Describe the effect on the strength of an earthquake if the amplitude on the seismograph multiplies by 10.

c The 1960 earthquake in Chile had a magnitude of 9.5 on the Richter scale. Find the amplitude of the seismograph reading for this earthquake.

13 Internal Assessments

The Internal Assessment is an opportunity to explore an area of mathematics more deeply, working through a problem (called an 'exploration') of your choice.

Types of exploration

Explorations tend to fall into three categories:

- modelling (creating a mathematical model for a situation)
- statistics (using statistics to answer a question or investigate a process)
- investigation into mathematics (exploring a mathematical problem or puzzle, exploring the historical development of a mathematical idea or the application of a mathematical method).

Here are some examples of the kind of problems that might fit into each of these categories:

	Real World	**Mathematical World**
Mathematical modelling	How can I model the spread of an epidemic? How do I find the shortest route to school?	What is the logistic map? What does stability mean in mathematical models?
Statistical thinking	Does screentime have an impact on school grades? Do home teams have an advantage in sport?	Are the digits of pi random? Is there a pattern to the gaps between prime numbers?
Investigation	What is the relationship between surface area and volume of different buildings? What is the best strategy in an auction?	How can Pick's theorem be applied to non-square grids? What is meant by a fractal dimension?

You will spend about 30 hours on the Internal Assessment, and it is worth 20% of the final mark. It should not exceed 2000 words, excluding mathematical notation, headings, table entries, diagrams, graphs, labels and the bibliography. Your work should generally use mathematics from your course – it certainly does not need to go beyond the course. The same marking criteria are used for Standard-Level and Higher-Level students.

Marking criteria

The Internal Assessment is judged on four different criteria:

- Problem specification
- Abstraction into mathematical form
- Computation of results
- Interpretation of results for their chosen inquiry.

Sample answers

The level descriptors can be illustrated using examples. Here are two sample answers that a student could have written as part of their exploration. Some comments have been added to point out important features.

Sample answer 1

How can we model the flow of traffic?

A: Problem Specification

A real-world motivation helps to add context.

I was travelling back home along a major road, and we were stuck in traffic due to a lane being closed. I noticed two interesting things:

- First, the traffic seemed to stop and start rather than just going consistently slowly.
- Secondly, I noticed that there were decreased speed limits leading up to the worst part of the traffic. It was reduced from 70 mph to 40 mph.

This made me wonder whether a mathematical model could help me to understand this behaviour.

It is very unlikely that you will be working on an entirely novel problem. It is good to understand what has already been done in the area you are interested in.

I found an interesting paper written by Sugiyama *et al* (2008)[1] which showed empirically that even when trying to move at constant speed, small deviations result in "phantom traffic jams". However, this paper did not give any mathematical explanation of this behaviour.

Most models are not totally original – that's OK. You should be clear if you are building on a standard model. If you can explain it in your own words, that's fine.

I found a book on mathematical modelling by Illner *et al*[2] that explored how to create a model for this. I've used this as a basis for the model I created, although I've adapted it to my own situation.

I want to:

a create a model that will be able to explain some aspects of traffic flow (this will be refined in section B).

b understand why traffic management authorities choose to reduce speed limits when they want to improve traffic flow and be able to advise my local authority about the appropriate speed limit they should use.

c understand why traffic might experience stop-start behaviour and be able to suggest ways to prevent this from happening.

It should be clear when the desired outcome is achieved, and breaking down the problem into sub-problems is a great way to make this clear. Here, (b) is fine, but (a) is currently too vague. However, it is OK to clarify it later once some abstraction has occurred.

A success will involve finding mathematical expressions for the traffic flow as well as estimating numerical values of speed limits, which will change observed behaviours.

1 Sugiyama, Y., Fukui, M., Kikuchi, M., Hasebe, K., Nakayama, A., Nishinari, K., Tadaki, S., and Yukawa, S., (2008), 'Traffic jams dynamics in a circular track experiment', New Journal of Physics, vol. 10, no 3)

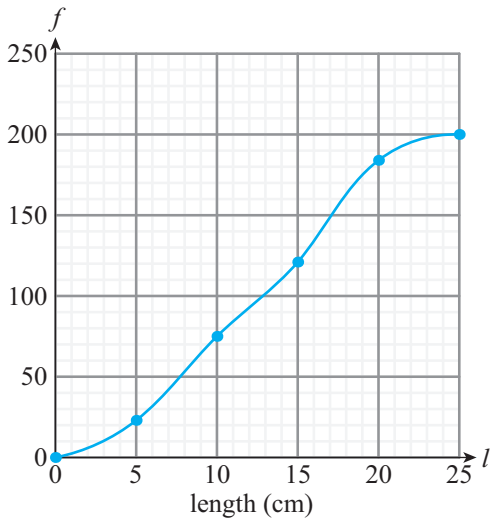
2 Illner, R., Bohun, C. S., McCollum, S., & van Roode, T. (2005). *Mathematical modelling: A case studies approach.* American Mathematical Society.

Practice Paper 1

Non–calculator.
1 hour 30 minutes, maximum mark for the paper [75 marks].

Section A

1 A Biology student measured the lengths of 200 leaves collected from the local park. The results are displayed in the cumulative frequency diagram.

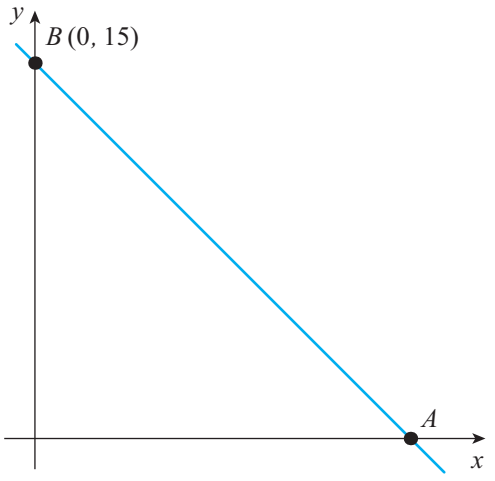


a Estimate the median leaf length.

b What is the probability that a randomly selected leaf is longer than 10 cm?

[5]

2 The line shown in the diagram has equation $y = -3x + c$.
It crosses the coordinate axes at points A and B.



a Write down the value of c.

b Find the coordinates of A.

c Find the coordinates of the midpoint of AB.

[5]

3 The table shows the number of students from two schools studying Maths SL and Maths HL.

	School 1	School 2
Maths SL	20	30
Maths HL	10	20

a How many students are there in the two schools together?

b What is the probability that a randomly selected student is from School 1 and studies Maths HL?

c Given that a student is from School 2, what is the probability that they study Maths SL?

[5]

Answers

Toolkit: Algebra Practice

1 **a** $\mathbb{R}$
b $\mathbb{Q}, \mathbb{R}$
c $\mathbb{Z}, \mathbb{Q}, \mathbb{R}$
d $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$
e $\mathbb{Q}, \mathbb{R}$
f $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$

2 20 m/s

3 0.055

4 6

5 2.75 or $\frac{11}{4}$

6 **a** 2, 3, 5
b 1, 5, 25
c 3, 6, 9, 12

7 GCF 6, LCM = 36

8 34 GBP

9 **a** $2x^2 + 5x - 12$
b $4(x - 5)(x + 2)$

10 $h = \frac{3V}{\pi r^2}$

11 –8

12 72

13 8

14 –2, –1, 0, 1, 2, 3, 4

15 $3\sqrt{5}$

16 $2\sqrt{3}$

17 3.45×10^4

18 42.50 EUR

19 $x = 7$

20 $x = 5, x = -4$

21 $x = \frac{2 \pm \sqrt{6}}{2}$

22 $x = 1, y = 2$

23 {1, 2, 3, 4, 5}

24 $\frac{5x + 2}{x(x + 1)}$

25 Shape: parabola opening downwards, maximum point at $x = 0.75$

26

Input	–2	0	1	2
Output	5	0	3	3

27 **a** 10 cm
b $4^2 + 6^2 \neq 8^2$

28 **a** (2,1)
b $\sqrt{50}$ or $5\sqrt{2}$

29 A line

30 135°

31 75°

32 5

33 A reflection in the y-axis

34 **a** A chord
b 25π cm^2

35 25 cm^2

36 A cone

37 80π cm^3

38 48 km/h

Chapter 1 Prior knowledge

1 **a** 81
b 40

2 2^6

Be the examiner 1.1

Solution 3

Exercise 1A

1 **a** x^6
b x^{12}

2 **a** y^6
b z^{10}

3 **a** 5^{17}
b 2^{24}

4 **a** x
b x^3

5 **a** y^4
b z^6

6 **a** 11^8
b 7^5

7 **a** x^{15}
b x^{32}

8 **a** y^{16}
b z^{25}

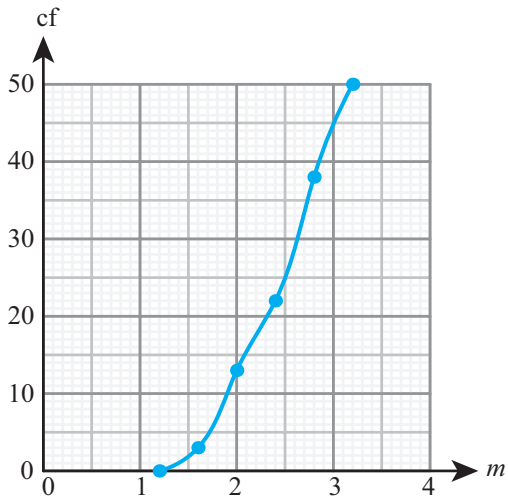
9 **a** 3^{50}
b 13^{28}

10 **a** $48x^7$
b $15x^7$

Chapter 7 Mixed practice

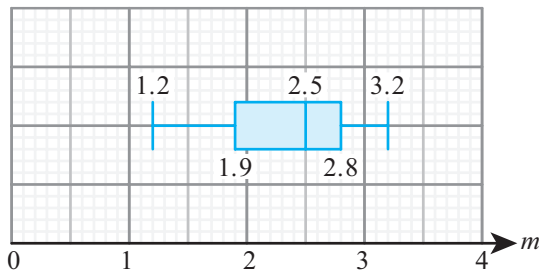
1 **a** 2.38 kg

b



c 2.5 kg, 0.9 kg

d



2 **a** discrete

b 0

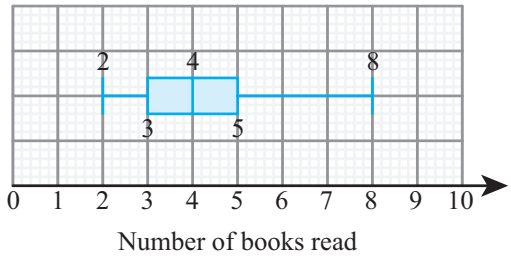
c **i** 1.47

ii 1.5

iii 1.25

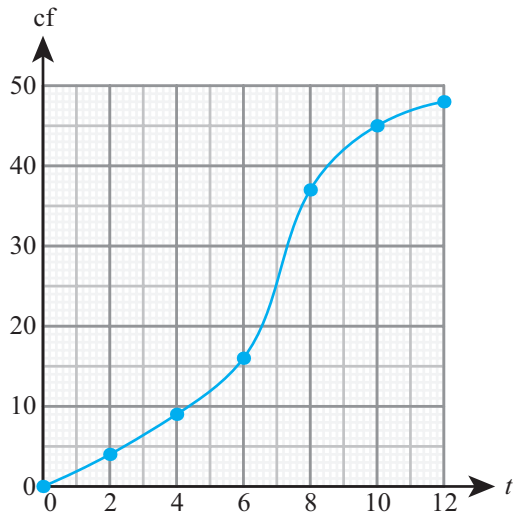
3 **a** 4

b



c 10

4 **a**



b **i** 6.9 minutes

ii 2.9 minutes

iii 9.3 minutes

c

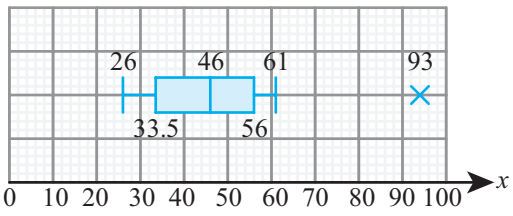
Time	Freq
$0 \leqslant t < 2$	4
$2 \leqslant t < 4$	5
$4 \leqslant t < 6$	7
$6 \leqslant t < 8$	21
$8 \leqslant t < 10$	8
$10 \leqslant t < 12$	3

d 6.38 minutes

5 **a** median = 46, Q_1 = 33.5, Q_3 = 56

b yes (93)

c



6 **a** 121 cm, 22.9 cm^2

b 156 cm, 22.9 cm^2

7 mean = \$55.02, sd = \$43.13

8 $x = 6$

Chapter 8 Prior knowledge

1 **a** {2, 3, 4, 5, 6, 7, 8, 9}

b {5, 8}

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About the authors

The authors are all University of Cambridge graduates and have a wide range of expertise in pure mathematics and in applications of mathematics, including economics, epidemiology, linguistics, philosophy and natural sciences. They have considerable experience of teaching IB Diploma Mathematics at Standard and Higher Level, and three of them currently teach the course - one at the University of Cambridge.

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