SAMPLE MATERIAL

We are working with Cambridge Assessment International Education to gain endorsement for this forthcoming title.

Working for over 30 YEARS with Cambridge Assessment International Education

Cambridge Assessment International Education

Endorsed for full syllabus coverage

Cambridge IGCSE™

International Mathematics

Third edition

Ric Pimentel
Frankie Pimentel
Terry Wall

Boost

HODDER EDUCATION

Please note this is a sample and not a full chapter

We are working with Cambridge Assessment International Education to gain endorsement for this forthcoming title.

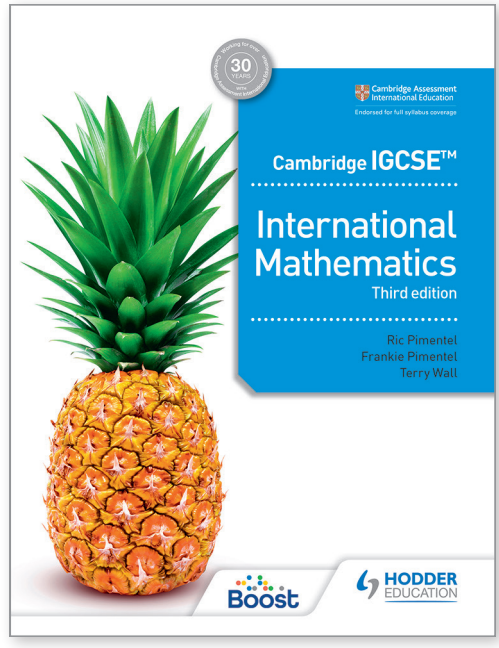
Build students' confidence in using technology to develop mathematical skills.

Cambridge IGCSE International Mathematics Third edition	Cambridge IGCSE International Mathematics Third edition Boost eBook
£36 March 2023 9781398373945	Boost £18 for 1-year access March 2023 9781398373761

Boost eBooks – interactive, engaging and completely flexible

Boost eBooks use the latest research and technologies to provide the very best learning experience for students. They can be downloaded onto any device and used in the classroom, at home or on the move.

- **Interactive:** Packed with features such as notes, links, highlights, bookmarks, formative quizzes, flashcards, videos and interactive revision.
- **Accessible:** Effortlessly support different learning styles with text-to-speech function.
- **Flexible:** Seamlessly switch between the printed page view and interactive view.



For more information, visit **www.hoddereducation.com/cambridge-igcse-maths**

We're here to help!
To find out more, please contact us at **international.sales@hoddereducation.com**

Cambridge IGCSE™

International Mathematics

Third Edition

Ric Pimentel
Frankie Pimentel
Terry Wall

Boost

HODDER EDUCATION
AN HACHETTE UK COMPANY

Acknowledgements

Every effort has been made to trace all copyright holders, but if any have been inadvertently overlooked, the Publishers will be pleased to make the necessary arrangements at the first opportunity. All exam-style questions and sample answers in this title were written by the authors.

Although every effort has been made to ensure that website addresses are correct at time of going to press, Hodder Education cannot be held responsible for the content of any website mentioned in this book. It is sometimes possible to find a relocated web page by typing in the address of the home page for a website in the URL window of your browser.

Hachette UK's policy is to use papers that are natural, renewable and recyclable products and made from wood grown in well-managed forests and other controlled sources. The logging and manufacturing processes are expected to conform to the environmental regulations of the country of origin.

Orders: please contact Hachette UK Distribution, Hely Hutchinson Centre, Milton Road, Didcot, Oxfordshire, OX11 7HH. Telephone: +44 (0)1235 827827. Email education@hachette.co.uk Lines are open from 9 a.m. to 5 p.m., Monday to Friday. You can also order through our website: www.hoddereducation.com

First published in 2023 by
Hodder Education,
An Hachette UK Company
Carmelite House
50 Victoria Embankment
London EC4Y 0DZ
www.hoddereducation.com

Illustrations by Integra Software Services

Typeset in Times Ten LT Std Roman 10/12 by Integra Software Services Pvt. Ltd., Pondicherry, India

Printed in

A catalogue record for this title is available from the British Library.

ISBN: 978 1 3983 7985 5

Contents

18 Solving quadratic equations

An equation of the form $y = ax^2 + bx + c$, in which the highest power of the variable x is x^2, is known as a **quadratic equation**. The following are all types of quadratic equations:

$$y = x^2 + 2x - 4 \qquad y = -3x^2 + x + 2 \qquad y = x^2 \qquad y = \frac{1}{2}x^2 + 2$$

There are a number of ways to solve quadratic equations and the most efficient method to use is largely dependent on the type of quadratic equation given. The main methods are explained later in this section; however, you can also use your graphics calculator to solve quadratic equations and therefore check your answers.

Note: You should always show some working as to how you solve a quadratic equation; therefore you should use your calculator only as a tool for checking your answer.

Worked example

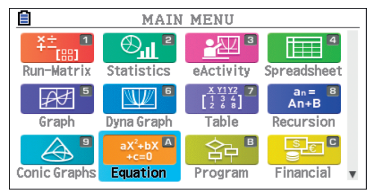
Using your graphics calculator, solve the quadratic equation $6x^2 = 4 - 5x$.

First, the equation needs to be rearranged in the form $ax^2 + bx + c = 0$. Therefore $6x^2 = 4 - 5x$ is rearranged to become $6x^2 + 5x - 4 = 0$.

You should only use the following method to check your answers.

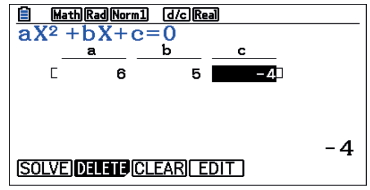
Casio

to select the equation mode from the menu.



to select 'Polynomial'.

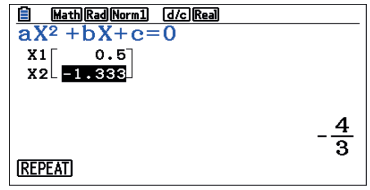
to select the degree of the polynomial as 2.



Enter the expression $6x^2 + 5x - 4$ into the matrix, where a is the coefficient of x^2, b is the coefficient of x and c the constant.

Trace F1 to solve the equation.

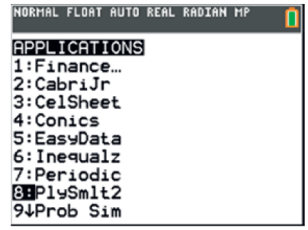
The solutions are given both in decimal form as $x = 0.5$ and -1.333 and also as a fraction as $x = \frac{1}{2}$ and $-\frac{4}{3}$



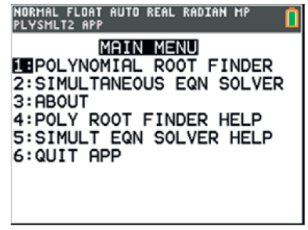
Note: The calculator requests the quadratic equation in the form $ax^2 + bx + c = 0$.

Texas

angle B apps v P 8 to select the 'PlySmlt2' mode.

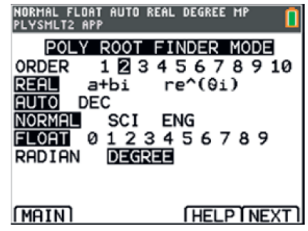


L1 Y 1 to select the 'POLYNOMIAL ROOT FINDER' option.



Check that the order of the polynomial is '2'.

table f5 graph to select 'NEXT'.



Enter the equation $6x^2 + 5x - 4 = 0$ in the grid provided.

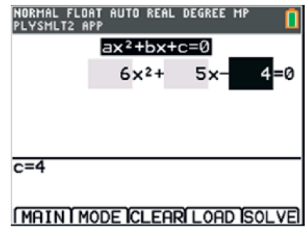
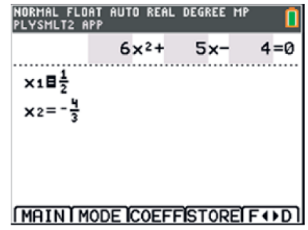


table f5 graph to select 'SOLVE'.

The solutions are given as $x = \frac{1}{2}$ and $-\frac{4}{3}$



Solving quadratic equations by factorising

$x^2 - 3x - 10 = 0$ is a quadratic equation which when factorised can be written as $(x - 5)(x + 2) = 0$.

Therefore either $x - 5 = 0$ or $x + 2 = 0$ since, if two things multiply to make zero, then one of them must be zero.

$x - 5 = 0$ or $x + 2 = 0$
$x = 5$ or $x = -2$

Worked examples

Solve the following equations to give two solutions for x:

a $x^2 - x - 12 = 0$
$(x - 4)(x + 3) = 0$
so either $x - 4 = 0$ or $x + 3 = 0$
$x = 4$ or $x = -3$

b $x^2 + 2x = 24$
This becomes $x^2 + 2x - 24 = 0$
$(x + 6)(x - 4) = 0$
so either $x + 6 = 0$ or $x - 4 = 0$
$x = -6$ or $x = 4$

c $x^2 - 6x = 0$
$x(x - 6) = 0$
so either $x = 0$ or $x - 6 = 0$
or $x = 6$

d $x^2 - 4 = 0$
$(x - 2)(x + 2) = 0$
so either $x - 2 = 0$ or $x + 2 = 0$
$x = 2$ or $x = -2$

Exercise 2.24

Solve the following quadratic equations by factorising. Check your solutions using a calculator.

1 **a** $x^2 + 7x + 12 = 0$ **b** $x^2 + 8x + 12 = 0$ **c** $x^2 + 13x + 12 = 0$
d $x^2 - 7x + 10 = 0$ **e** $x^2 - 5x + 6 = 0$ **f** $x^2 - 6x + 8 = 0$

2 **a** $x^2 + 3x - 10 = 0$ **b** $x^2 - 3x - 10 = 0$ **c** $x^2 + 5x - 14 = 0$
d $x^2 - 5x - 14 = 0$ **e** $x^2 + 2x - 15 = 0$ **f** $x^2 - 2x - 15 = 0$

3 **a** $x^2 + 5x = -6$ **b** $x^2 + 6x = -9$ **c** $x^2 + 11x = -24$
d $x^2 - 10x = -24$ **e** $x^2 + x = 12$ **f** $x^2 - 4x = 12$

4 **a** $x^2 - 2x = 8$ **b** $x^2 - x = 20$ **c** $x^2 + x = 30$
d $x^2 - x = 42$ **e** $x^2 - 2x = 63$ **f** $x^2 + 3x = 54$

Exercise 2.25

Solve the following quadratic equations. Check your solutions using a calculator.

1 **a** $x^2 - 9 = 0$ **b** $x^2 - 16 = 0$ **c** $x^2 = 25$
d $x^2 = 121$ **e** $x^2 - 144 = 0$ **f** $x^2 - 220 = 5$

2 **a** $4x^2 - 25 = 0$ **b** $9x^2 - 36 = 0$ **c** $25x^2 = 64$
d $x^2 = \frac{1}{4}$ **e** $x^2 - \frac{1}{9} = 0$ **f** $16x^2 - \frac{1}{25} = 0$

3 **a** $x^2 + 5x + 4 = 0$ **b** $x^2 + 7x + 10 = 0$ **c** $x^2 + 6x + 8 = 0$
d $x^2 - 6x + 8 = 0$ **e** $x^2 - 7x + 10 = 0$ **f** $x^2 + 2x - 8 = 0$

4 **a** $x^2 - 3x - 10 = 0$ **b** $x^2 + 3x - 10 = 0$ **c** $x^2 - 3x - 18 = 0$
d $x^2 + 3x - 18 = 0$ **e** $x^2 - 2x - 24 = 0$ **f** $x^2 - 2x - 48 = 0$

5 **a** $x^2 + x = 12$ **b** $x^2 + 8x = -12$ **c** $x^2 + 5x = 36$
d $x^2 + 2x = -1$ **e** $x^2 + 4x = -4$ **f** $x^2 + 17x = -72$

6 **a** $x^2 - 8x = 0$ **b** $x^2 - 7x = 0$ **c** $x^2 + 3x = 0$
d $x^2 + 4x = 0$ **e** $x^2 - 9x = 0$ **f** $4x^2 - 16x = 0$

7 **a** $2x^2 + 5x + 3 = 0$ **b** $2x^2 - 3x - 5 = 0$ **c** $3x^2 + 2x - 1 = 0$
d $2x^2 + 11x + 5 = 0$ **e** $2x^2 - 13x + 15 = 0$ **f** $12x^2 + 10x - 8 = 0$

8 **a** $x^2 + 12x = 0$ **b** $x^2 + 12x + 27 = 0$ **c** $x^2 + 4x = 32$
d $x^2 + 5x = 14$ **e** $2x^2 = 72$ **f** $3x^2 - 12 = 288$

Exercise 2.26

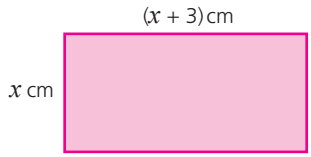
In the following questions, construct equations from the information given and then solve them to find the unknown.

1 When a number x is added to its square, the total is 12. Find two possible values for x.

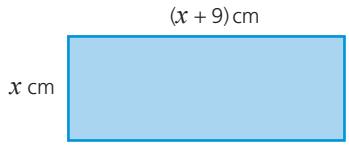
2 A number x is equal to its own square minus 42. Find two possible values for x.

Exercise 2.26 (cont)

3 If the area of the rectangle below is 10 cm^2, calculate the only possible value for x.



4 If the area of the rectangle is 52 cm^2, calculate the only possible value for x.

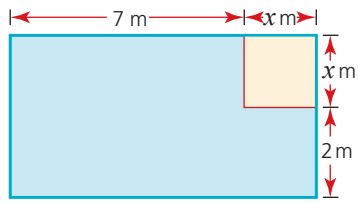


5 A triangle has a base length of $2x$ cm and a height of $(x - 3)$ cm. If its area is 18 cm^2, calculate its height and base length.

6 A triangle has a base length of $(x - 8)$ cm and a height of $2x$ cm. If its area is 20 cm^2, calculate its height and base length.

7 A right-angled triangle has a base length of x cm and a height of $(x - 1)$ cm. If its area is 15 cm^2, calculate the base length and height.

8 A rectangular garden has a square flowerbed of side length x m in one of its corners. The remainder of the garden consists of lawn and has dimensions as shown.



If the total area of the lawn is 50 m^2:

a form an equation in terms of x

b solve the equation

c calculate the length and width of the whole garden.

The quadratic formula

In general, a quadratic equation takes the form $ax^2 + bx + c = 0$, where a, b and c are integers. Quadratic equations can be solved by the use of the quadratic formula, which states that:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Worked examples

a Solve the quadratic equation $x^2 + 7x + 3 = 0$.

$a = 1$, $b = 7$ and $c = 3$

Substituting these values into the quadratic formula gives:

$$x = \frac{-7 \pm \sqrt{7^2 - 4 \times 1 \times 3}}{2 \times 1}$$

$$x = \frac{-7 \pm \sqrt{49 - 12}}{2}$$

$$x = \frac{-7 \pm \sqrt{37}}{2}$$

The answers −0.46 and −6.54 are only approximations correct to 2 d.p.

The answer $\frac{-7 \pm \sqrt{37}}{2}$ is the exact solution.

Therefore $x = \frac{-7 + 6.08}{2}$ or $x = \frac{-7 - 6.08}{2}$

$x = -0.46$ (2 d.p.) or $x = -6.54$ (2 d.p.)

b Solve the quadratic equation $x^2 - 4x - 2 = 0$.

$a = 1$, $b = -4$ and $c = -2$

Substituting these values into the quadratic formula gives:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - (4 \times 1 \times (-2))}}{2 \times 1}$$

$$x = \frac{4 \pm \sqrt{16 + 8}}{2}$$

$$x = \frac{4 \pm \sqrt{24}}{2}$$

If the exact answer is wanted, then $\frac{4 \pm \sqrt{24}}{2}$ needs to be given in its simplest form.

$\frac{4 \pm \sqrt{24}}{2} = \frac{4 \pm 2\sqrt{6}}{2}$

$= 2 \pm \sqrt{6}$

Therefore $x = \frac{4 + 4.90}{2}$ or $x = \frac{4 - 4.90}{2}$

$x = 4.45$ (2 d.p.) or $x = -0.45$ (2 d.p.)

Completing the square

Although the method of completing the square will not be assessed directly, this method often simplifies problems involving quadratics and their graphs.

Quadratics can also be solved by writing them in terms of a perfect square. We look once again at the quadratic $x^2 - 4x - 2 = 0$.

The perfect square $(x - 2)^2$ can be expanded to give $x^2 - 4x + 4$. Notice that the x^2 and x terms are the same as those in the original quadratic.

Therefore $(x - 2)^2 - 6 = x^2 - 4x - 2$ and can be used to solve the quadratic.

$$(x - 2)^2 - 6 = 0$$
$$(x - 2)^2 = 6$$
$$x - 2 = \pm\sqrt{6}$$
$$x = 2 \pm \sqrt{6}$$
$$x = 4.45 \quad \text{or} \quad x = -0.45$$

The quadratic formula stated earlier can be derived using the method of completing the square as shown:

Solve $ax^2 + bx + c = 0$.

Divide all terms by a: $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

Complete the square: $\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$

Rearrange: $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$

Arrange both fractions on the right-hand side with a common denominator of $4a^2$:

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$$

Simplify: $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$

Take the square root of both sides: $x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$

Simplify: $x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$

Rearrange: $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

Simplify to give the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Exercise 2.27

Solve the following quadratic equations using either the quadratic formula or by completing the square. Give your answers to 2 d.p.

1 **a** $x^2 - x - 13 = 0$ **b** $x^2 + 4x - 11 = 0$ **c** $x^2 + 5x - 7 = 0$
d $x^2 + 6x + 6 = 0$ **e** $x^2 + 5x - 13 = 0$ **f** $x^2 - 9x + 19 = 0$

2 **a** $x^2 + 7x + 9 = 0$ **b** $x^2 - 35 = 0$ **c** $x^2 + 3x - 3 = 0$
d $x^2 - 5x - 7 = 0$ **e** $x^2 + x - 18 = 0$ **f** $x^2 - 8 = 0$

3 **a** $x^2 - 2x - 2 = 0$ **b** $x^2 - 4x - 11 = 0$ **c** $x^2 - x - 5 = 0$
d $x^2 + 2x - 7 = 0$ **e** $x^2 - 3x + 1 = 0$ **f** $x^2 - 8x + 3 = 0$

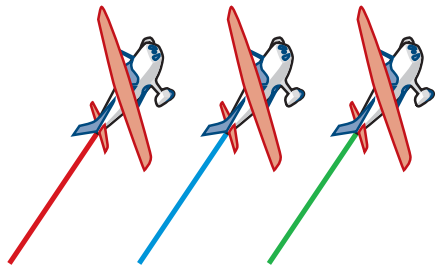
4 **a** $2x^2 - 3x - 4 = 0$ **b** $4x^2 + 2x - 5 = 0$ **c** $5x^2 - 8x + 1 = 0$
d $-2x^2 - 5x - 2 = 0$ **e** $3x^2 - 4x - 2 = 0$ **f** $-7x^2 - x + 15 = 0$

40 Investigations, modelling and ICT

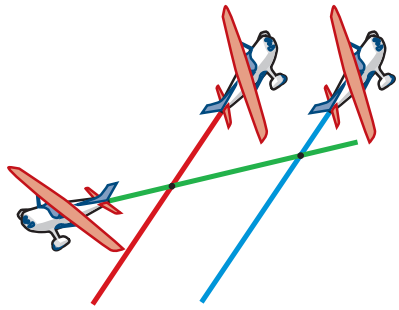
Plane trails

In an aircraft show, planes often fly with a coloured smoke trail. Depending on the formation of the planes, the trails can intersect in different ways.

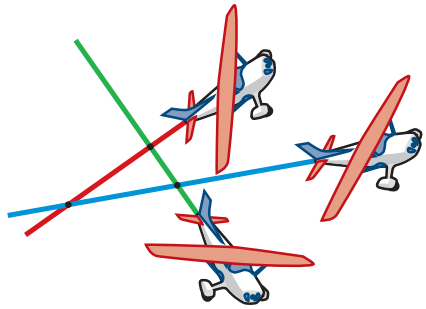
In the diagram below, the three smoke trails do not cross as they are parallel.



In the following diagram, there are two crossing points.



By flying differently, the three planes can produce trails that cross at three points.



78 Student assessments

Student assessment 1

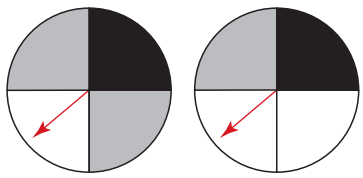
1 Calculate the theoretical probability of:
- **a** being born on a Saturday
- **b** being born on the 5th of a month in a non-leap year
- **c** being born on 20 June in a non-leap year
- **d** being born on 29 February.

2 When rolling an ordinary fair dice, calculate the theoretical probability of getting:
- **a** a 2
- **b** an even number
- **c** a 3 or more
- **d** less than 1.

3 A bag contains 12 white counters, 7 black counters and 1 red counter.
- **a** If, when a counter is taken out, it is not replaced, calculate the probability that:
  - **i** the first counter is white
  - **ii** the second counter removed is red, given that the first was black.
- **b** If, when a counter is picked, it is then put back in the bag, how many attempts will be needed before it is mathematically certain that a red counter will have been picked out?

4 A coin is tossed and an ordinary fair dice is rolled.
- **a** Draw a two-way table showing all the possible combinations.
- **b** Calculate the probability of getting:
  - **i** a head and a six
  - **ii** a tail and an odd number
  - **iii** a head and a prime number.

5 Two spinners A and B are split into quarters and coloured as shown. Both spinners are spun.



- **a** Draw a fully labelled tree diagram showing all the possible combinations on the two spinners. Write beside each branch the probability of each outcome.
- **b** Use your tree diagram to calculate the probability of getting:
  - **i** two blacks
  - **ii** two greys
  - **iii** a grey on spinner A and a white on spinner B.

6 A coin is tossed three times.
- **a** Draw a tree diagram to show all the possible outcomes.
- **b** Use your tree diagram to calculate the probability of getting:
  - **i** three tails
  - **ii** two heads
  - **iii** no tails
  - **iv** at least one tail.

7 A goalkeeper expects to save one penalty out of every three. Calculate the probability that he:
- **a** saves one penalty out of the next three
- **b** fails to save any of the next three penalties
- **c** saves two out of the next three penalties.

8 A board game uses a fair dice in the shape of a tetrahedron. The sides of the dice are numbered 1, 2, 3 and 4. Calculate the probability of:
- **a** not throwing a 4 in two throws
- **b** throwing two consecutive 1s
- **c** throwing a total of 5 in two throws.

9 A normal pack of 52 cards is shuffled and three cards are picked at random. Calculate the probability that all three cards are picture cards.

Develop students' mathematical skills by using technology to its full potential; trust a team of experienced authors to offer advice on how to advance knowledge of mathematical techniques covered in the latest Cambridge IGCSE™ International Mathematics syllabus (0607).

- Build confidence with fully updated and illustrated step-by-step instructions on the use of graphic display calculators.
- Trust an experienced team of authors offering advice on how to put theory into practice.
- Encourage practice with plenty of exercise questions, plus end-of-chapter assessments with full solutions available as well as numerical answers.
- Prepare for further study with a course that neatly leads into studying mathematics at a higher level.

This resource is endorsed by Cambridge Assessment International Education

- ✓ Supports the full syllabus for examination from 2025
- ✓ Has passed Cambridge International's rigorous quality-assurance process
- ✓ Developed by subject experts
- ✓ For Cambridge schools worldwide

We are working with Cambridge Assessment International Education to gain endorsement for this forthcoming series.

For over 30 years we have been trusted by Cambridge schools around the world to provide quality support for teaching and learning. For this reason we have been selected by Cambridge Assessment International Education as an official publisher of endorsed material for their syllabuses.

Working for over 30 YEARS with Cambridge Assessment International Education

HODDER EDUCATION
e: education@hachette.co.uk
w: hoddereducation.com

ISBN 978-1-3983-7985-5

FSC