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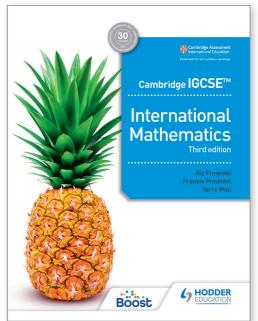
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International Mathematics

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First published in 2023 by Hodder Education, An Hachette UK Company Carmelite House 50 Victoria Embankment London EC4Y 0DZ www.hoddereducation.com

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Illustrations by Integra Software Services

Typeset in Times Ten LT Std Roman 10/12 by Integra Software Services Pvt. Ltd., Pondicherry, India

Printed in

A catalogue record for this title is available from the British Library.

ISBN: 978 1 3983 7985 5

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Solving quadratic equations

An equation of the form $y = ax^2 + bx + c$, in which the highest power of the variable x is x^2 , is known as a **quadratic equation**. The following are all types of quadratic equations:

 $y = x^{2} + 2x - 4$ $y = -3x^{2} + x + 2$ $y = x^{2}$ $y = \frac{1}{2}x^{2} + 2$

There are a number of ways to solve quadratic equations and the most efficient method to use is largely dependent on the type of quadratic equation given. The main methods are explained later in this section; however, you can also use your graphics calculator to solve quadratic equations and therefore check your answers.

Note: You should always show some working as to how you solve a quadratic equation; therefore you should use your calculator only as a tool for checking your answer.

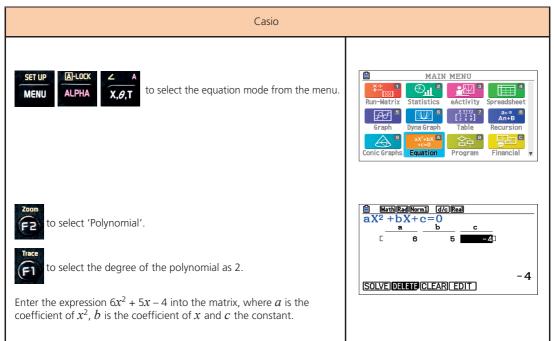
Worked example

18

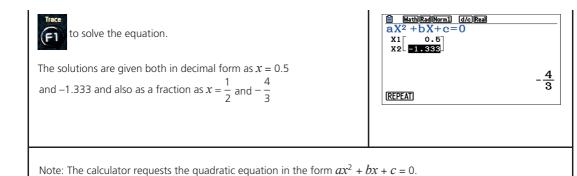
Using your graphics calculator, solve the quadratic equation $6x^2 = 4 - 5x$.

First, the equation needs to be rearranged in the form $ax^2 + bx + c = 0$. Therefore $6x^2 = 4 - 5x$ is rearranged to become $6x^2 + 5x - 4 = 0$.

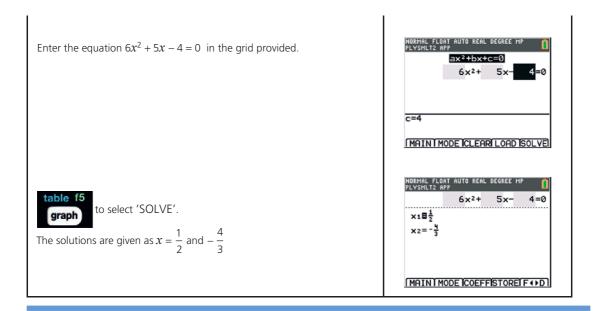
You should only use the following method to check your answers.



18 SOLVING QUADRATIC EQUATIONS



Texas NORMAL FLOAT AUTO REAL RADIAN MP APPLICATIONS angle B 1:Finance… 2:CabriJr to select the 'PlySmlt2' mode. apps 3:CelSheet 4:Conics 5:EasyData 6:Inequalz 7:Periodic 8:PlySmlt2 9↓Prob Sim NORMAL FLOAT AUTO REAL RADIAN MP Plysmlt2 app LEPOLYNOMIAL ROOT FINDER 2:SIMULTANEOUS EQN SOLVER 3:ABOUT to select the 'POLYNOMIAL ROOT FINDER' option. 4:POLY ROOT FINDER HELP 5:SIMULT EQN SOLVER HELP 6:QUIT APP NORMAL FLOAT AUTO REAL DEGREE MP Plysmlt2 app Check that the order of the polynomial is '2'. POLY ROOT FINDER MODE POLY ROUTE FINDER NUDE ORDER 1 2 3 4 5 6 7 8 9 10 REAL a+bi re^(0i) RUTO DEC NORMAL SCI ENG FLOAT 0 1 2 3 4 5 6 7 8 9 RADIAN DEGREE table f5 to select 'NEXT'. graph (MAIN) [HELP NEXT]



Solving quadratic equations by factorising

 $x^2 - 3x - 10 = 0$ is a quadratic equation which when factorised can be written as (x - 5)(x + 2) = 0.

Therefore either x - 5 = 0 or x + 2 = 0 since, if two things multiply to make zero, then one of them must be zero.

x-5=0 or x+2=0x=5 or x=-2

Worked examples

Solve the following equations to give two solutions for *x*:

```
a x^2 - x - 12 = 0

(x - 4)(x + 3) = 0

so either x - 4 = 0 or x + 3 = 0

x = 4 or x = -3

b x^2 + 2x = 24

This becomes x^2 + 2x - 24 = 0

(x + 6)(x - 4) = 0

so either x + 6 = 0 or x - 4 = 0

x = -6 or x = 4

c x^2 - 6x = 0

x(x - 6) = 0

so either x = 0 or x - 6 = 0

or x = 6
```

18 SOLVING QUADRATIC EQUATIONS

d $x^2 - 4 = 0$ (x-2)(x+2) = 0so either x - 2 = 0 or x + 2 = 0x = 2 or x = -2

.

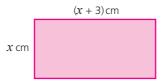
Exercise 2.24	Solve the following quadratic equations by factorising. Check your solutions using a calculator.				
	1 a $x^2 + 7x + 12 = 0$	b $x^2 + 8x + 12 = 0$	c $x^2 + 13x + 12 = 0$		
	d $x^2 - 7x + 10 = 0$	e $x^2 - 5x + 6 = 0$	f $x^2 - 6x + 8 = 0$		
	2 a $x^2 + 3x - 10 = 0$ d $x^2 - 5x - 14 = 0$	b $x^2 - 3x - 10 = 0$ e $x^2 + 2x - 15 = 0$			
	3 a $x^2 + 5x = -6$	b $x^2 + 6x = -9$	c $x^2 + 11x = -24$		
	d $x^2 - 10x = -24$	e $x^2 + x = 12$	f $x^2 - 4x = 12$		
	4 a $x^2 - 2x = 8$ d $x^2 - x = 42$		c $x^2 + x = 30$ f $x^2 + 3x = 54$		
Exercise 2.25	Solve the following quadratic equations. Check your solutions using a calculator.				
	1 a $x^2 - 9 = 0$	b $x^2 - 16 = 0$	c $x^2 = 25$		
	d $x^2 = 121$	e $x^2 - 144 = 0$	f $x^2 - 220 = 5$		
	2 a $4x^2 - 25 = 0$	b $9x^2 - 36 = 0$	c $25x^2 = 64$		
	d $x^2 = \frac{1}{4}$	e $x^2 - \frac{1}{9} = 0$	f $16x^2 - \frac{1}{25} = 0$		
	3 a $x^2 + 5x + 4 = 0$ d $x^2 - 6x + 8 = 0$	b $x^2 + 7x + 10 = 0$ e $x^2 - 7x + 10 = 0$			
	4 a $x^2 - 3x - 10 = 0$ d $x^2 + 3x - 18 = 0$	b $x^2 + 3x - 10 = 0$ e $x^2 - 2x - 24 = 0$			
	5 a $x^2 + x = 12$	b $x^2 + 8x = -12$	c $x^2 + 5x = 36$		
	d $x^2 + 2x = -1$	e $x^2 + 4x = -4$	f $x^2 + 17x = -72$		
	6 a $x^2 - 8x = 0$	b $x^2 - 7x = 0$	c $x^2 + 3x = 0$		
	d $x^2 + 4x = 0$	e $x^2 - 9x = 0$	f $4x^2 - 16x = 0$		
	7 a $2x^2 + 5x + 3 = 0$ d $2x^2 + 11x + 5 = 0$	b $2x^2 - 3x - 5 = 0$ e $2x^2 - 13x + 15 = 0$			
	8 a $x^2 + 12x = 0$	b $x^2 + 12x + 27 = 0$	c $x^2 + 4x = 32$		
	d $x^2 + 5x = 14$	e $2x^2 = 72$	f $3x^2 - 12 = 288$		

Exercise 2.26 In the following questions, construct equations from the information given and then solve them to find the unknown.

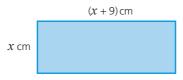
- 1 When a number x is added to its square, the total is 12. Find two possible values for *x*.
- **2** A number *x* is equal to its own square minus 42. Find two possible values for x.

Exercise 2.26 (cont)

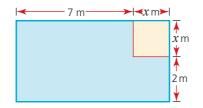
3 If the area of the rectangle below is 10 cm², calculate the only possible value for *x*.



4 If the area of the rectangle is 52 cm^2 , calculate the only possible value for x.



- 5 A triangle has a base length of 2x cm and a height of (x 3) cm. If its area is 18 cm^2 , calculate its height and base length.
- 6 A triangle has a base length of (x 8) cm and a height of 2x cm. If its area is 20 cm², calculate its height and base length.
- 7 A right-angled triangle has a base length of x cm and a height of (x-1) cm. If its area is 15 cm^2 , calculate the base length and height.
- 8 A rectangular garden has a square flowerbed of side length *x* m in one of its corners. The remainder of the garden consists of lawn and has dimensions as shown.



If the total area of the lawn is 50 m²:

- **a** form an equation in terms of *x*
- **b** solve the equation
- c calculate the length and width of the whole garden.

The quadratic formula

In general, a quadratic equation takes the form $ax^2 + bx + c = 0$, where *a*, *b* and *c* are integers. Quadratic equations can be solved by the use of the quadratic formula, which states that:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

18 SOLVING QUADRATIC EQUATIONS

Worked examples

a = 1, b = 7 and c = 3

```
a Solve the quadratic equation x^2 + 7x + 3 = 0.
```

The answers -0.46 and -6.54 are only approximations correct to 2 d.p.

$$x = \frac{-7 \pm \sqrt{7^2 - 4 \times 1 \times 3}}{2 \times 1}$$

$$x = \frac{-7 \pm \sqrt{49 - 12}}{2}$$

$$x = \frac{-7 \pm \sqrt{37}}{2}$$
Therefore $x = \frac{-7 \pm 6.08}{2}$ or $x = \frac{-7 - 6.08}{2}$

Substituting these values into the quadratic formula gives:

x = -0.46 (2 d.p.) or x = -6.54 (2 d.p.)

b Solve the quadratic equation $x^2 - 4x - 2 = 0$.

$$a = 1, b = -4$$
 and $c = -2$

Substituting these values into the quadratic formula gives:

If the exact answer is wanted, then $\frac{4 \pm \sqrt{24}}{2}$ needs to be given in its simplest form. $\frac{4 \pm \sqrt{24}}{2} = \frac{4 \pm 2\sqrt{6}}{2}$ $= 2 \pm \sqrt{6}$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - (4 \times 1 \times (-2))}}{2 \times 1}$$

$$x = \frac{4 \pm \sqrt{16 + 8}}{2}$$

$$x = \frac{4 \pm \sqrt{24}}{2}$$

Therefore $x = \frac{4 + 4.90}{2}$ or $x = \frac{4 - 4.90}{2}$
 $x = 4.45$ (2 d.p.) or $x = -0.45$ (2 d.p.)

Completing the square

Although the method of completing the square will not be assessed directly, this method often simplifies problems involving quadratics and their graphs.

Quadratics can also be solved by writing them in terms of a perfect square. We look once again at the quadratic $x^2 - 4x - 2 = 0$.

The perfect square $(x - 2)^2$ can be expanded to give $x^2 - 4x + 4$. Notice that the x^2 and x terms are the same as those in the original quadratic.

Therefore $(x-2)^2 - 6 = x^2 - 4x - 2$ and can be used to solve the quadratic.

$$(x-2)^{2}-6 = 0$$

(x-2)^{2} = 6
x-2 = \pm \sqrt{6}
x = 2 ± $\sqrt{6}$
x = 4.45 or x = -0.45

The quadratic formula stated earlier can be derived using the method of completing the square as shown:

Solve $ax^2 + bx + c = 0$.

Divide all terms by $a: x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

Complete the square: $\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$

Rearrange: $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$

Arrange both fractions on the right-hand side with a common denominator of $4a^2$:

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$$

Simplify: $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4a}{4a^2}$

Take the square root of both sides: $x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$

Simplify: $x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

Rearrange: $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$ Simplify to give the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Exercise 2.27

Solve the following quadratic equations using either the quadratic formula or by completing the square. Give your answers to 2 d.p.

1	$x^{2} - x - 13 = 0$ $x^{2} + 6x + 6 = 0$	b $x^2 + 4x - 11 = 0$ e $x^2 + 5x - 13 = 0$	c $x^2 + 5x - 7 = 0$ f $x^2 - 9x + 19 = 0$
2	$x^{2} + 7x + 9 = 0$ $x^{2} - 5x - 7 = 0$	b $x^2 - 35 = 0$ e $x^2 + x - 18 = 0$	c $x^2 + 3x - 3 = 0$ f $x^2 - 8 = 0$
3	$x^{2} - 2x - 2 = 0$ $x^{2} + 2x - 7 = 0$	b $x^2 - 4x - 11 = 0$ e $x^2 - 3x + 1 = 0$	c $x^2 - x - 5 = 0$ f $x^2 - 8x + 3 = 0$
4	$2x^2 - 3x - 4 = 0$ $-2x^2 - 5x - 2 = 0$	b $4x^2 + 2x - 5 = 0$ e $3x^2 - 4x - 2 = 0$	c $5x^2 - 8x + 1 = 0$ f $-7x^2 - x + 15 = 0$

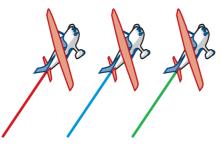
Investigations, modelling and ICT

Plane trails

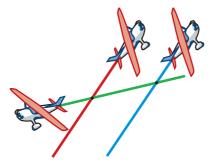
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In an aircraft show, planes often fly with a coloured smoke trail. Depending on the formation of the planes, the trails can intersect in different ways.

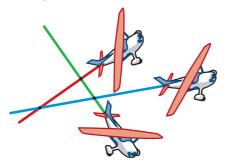
In the diagram below, the three smoke trails do not cross as they are parallel.



In the following diagram, there are two crossing points.



By flying differently, the three planes can produce trails that cross at three points.



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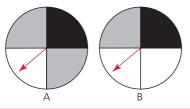
Student assessments

Student assessment 1

- 1 Calculate the theoretical probability of:
 - a being born on a Saturday
 - **b** being born on the 5th of a month in a non-leap year
 - c being born on 20 June in a non-leap year
 - d being born on 29 February.
- 2 When rolling an ordinary fair dice, calculate the theoretical probability of getting:
 - a a 2
- **b** an even number d less than 1.
- **c** a 3 or more
- 3 A bag contains 12 white counters. 7 black counters and 1 red counter.
 - a If, when a counter is taken out, it is not replaced, calculate the probability that: i the first counter is white

 - ii the second counter removed is red. given that the first was black.
 - **b** If, when a counter is picked, it is then put back in the bag, how many attempts will be needed before it is mathematically certain that a red counter will have been picked out?
- 4 A coin is tossed and an ordinary fair dice is rolled.
 - a Draw a two-way table showing all the possible combinations.
 - **b** Calculate the probability of getting: i a head and a six

 - ii a tail and an odd number
 - iii a head and a prime number.
- **5** Two spinners A and B are split into guarters and coloured as shown. Both spinners are spun.



- a Draw a fully labelled tree diagram showing all the possible combinations on the two spinners. Write beside each branch the probability of each outcome.
- **b** Use your tree diagram to calculate the probability of getting:
 - i two blacks
 - ii two greys
 - iii a grey on spinner A and a white on spinner B.
- 6 A coin is tossed three times.
 - a Draw a tree diagram to show all the possible outcomes.
 - **b** Use your tree diagram to calculate the probability of getting:
 - i three tails
 - ii two heads
 - iii no tails
 - iv at least one tail.
- 7 A goalkeeper expects to save one penalty out of every three. Calculate the probability that he:
 - a saves one penalty out of the next three
 - **b** fails to save any of the next three penalties
 - **c** saves two out of the next three penalties.
- 8 A board game uses a fair dice in the shape of a tetrahedron. The sides of the dice are numbered 1. 2. 3 and 4. Calculate the probability of:
 - a not throwing a 4 in two throws
 - **b** throwing two consecutive 1s
 - **c** throwing a total of 5 in two throws.
- **9** A normal pack of 52 cards is shuffled and three cards are picked at random. Calculate the probability that all three cards are picture cards.

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