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Core and Extended

Fifth edition

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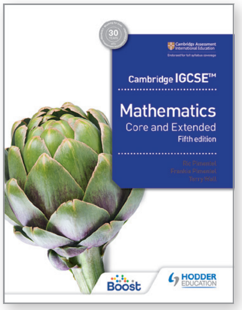

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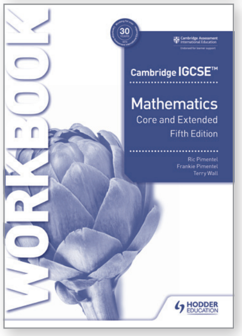

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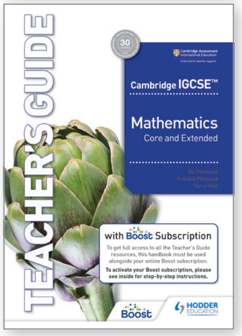

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1 Number and language

Vocabulary for sets of numbers

A **square** can be classified in many different ways. It is a quadrilateral but it is also a polygon and a two-dimensional shape. Just as shapes can be classified in many ways, so can numbers. Below is a description of some of the more common types of numbers.

Natural numbers

A child learns to count 'one, two, three, four, …'. These are sometimes called the counting numbers or whole numbers.

The child will say 'I am three', or 'I live at number 73'.

If we include the number 0, then we have the set of numbers called the **natural numbers**.

The set of natural numbers $\mathbb{N} = \{0, 1, 2, 3, 4, \ldots\}$.

Students should practise converting between values expressed in numbers and values expressed in words. For example, 12 014 is twelve thousand and fourteen; 1 745 233 is one million, seven hundred and forty-five thousand, two hundred and thirty-three.

Integers

On a cold day, the temperature may be 4 °C at 10 p.m. If the temperature drops by a further 6 °C, then the temperature is 'below zero'; it is –2 °C.

If you are overdrawn at the bank by \$200, this might be shown as –\$200.

The set of **integers** $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.

$\mathbb{Z}$ is therefore an extension of $\mathbb{N}$. Every natural number is an integer.

Reciprocal

The **reciprocal** of a number is obtained when 1 is divided by that number. The reciprocal of 5 is $\frac{1}{5}$, the reciprocal of $\frac{2}{5}$ is $\frac{1}{\frac{2}{5}}$ which simplifies to $\frac{5}{2}$.

Exercise 1.1

1 In a 10 by 10 square, write the numbers 1 to 100.
Cross out number 1.
Cross out all the even numbers after 2 (these have 2 as a factor).
Cross out every third number after 3 (these have 3 as a factor).
Continue with 5, 7, 11 and 13, then list all the prime numbers less than 100.

Exercise 1.1 (cont)

2 Write the reciprocal of each of the following:

a $\frac{1}{8}$ b $\frac{7}{12}$ c $\frac{3}{5}$

d $1\frac{1}{2}$ e $3\frac{3}{4}$ f 6

Square numbers

The number 1 can be written as 1×1 or 1^2. This can be read as '1 squared' or '1 raised to the **power** of 2'.

The number 4 can be written as 2×2 or 2^2.

9 can be written as 3×3 or 3^2.

16 can be written as 4×4 or 4^2.

When an integer (whole number) is multiplied by itself, the result is a **square number**. In the examples above, 1, 4, 9 and 16 are all square numbers.

Cube numbers

The number 1 can be written as $1 \times 1 \times 1$ or 1^3. This can be read as '1 cubed' or '1 raised to the power of 3'.

The number 8 can be written as $2 \times 2 \times 2$ or 2^3.

27 can be written as $3 \times 3 \times 3$ or 3^3.

64 can be written as $4 \times 4 \times 4$ or 4^3.

When an integer is multiplied by itself and then by itself again, the result is a **cube number**. In the examples above, 1, 8, 27 and 64 are all cube numbers.

Factors

The factors of 12 are all the numbers which will divide exactly into 12, i.e. 1, 2, 3, 4, 6 and 12.

Exercise 1.2

1 List all the factors of the following numbers:

a 6 b 9 c 7 d 15 e 24

f 36 g 35 h 25 i 42 j 100

Prime numbers

A **prime number** is one whose only **factors** are 1 and itself. (Note: 1 is not a prime number.)

Prime factors

The factors of 12 are 1, 2, 3, 4, 6 and 12.

Of these, 2 and 3 are prime numbers, so 2 and 3 are the **prime factors** of 12.

Exercise 1.3

1 List the prime factors of the following numbers:

a 15	**b** 18	**c** 24	**d** 16	**e** 20
f 13	**g** 33	**h** 35	**i** 70	**j** 56

An easy way to find prime factors is to divide by the prime numbers in order, smallest first.

Worked examples

a Find the prime factors of 18 and express it as a product of prime numbers:

	18
2	9
3	3
3	1

$18 = 2 \times 3 \times 3$ or 2×3^2

b Find the prime factors of 24 and express it as a product of prime numbers:

	24
2	12
2	6
2	3
3	1

$24 = 2 \times 2 \times 2 \times 3$ or $2^3 \times 3$

c Find the prime factors of 75 and express it as a product of prime numbers:

	75
3	25
5	5
5	1

$75 = 3 \times 5 \times 5$ or 3×5^2

Exercise 1.4

1 Find the prime factors of the following numbers and express them as a product of prime numbers:

a 12	b 32	c 36	d 40	e 44
f 56	g 45	h 39	i 231	j 63

Highest common factor

The factors of 12 can be listed as 1, 2, 3, 4, 6, 12.

The factors of 18 can be listed as 1, 2, 3, 6, 9, 18.

As can be seen, the factors 1, 2, 3 and 6 are common to both numbers. They are known as **common factors**. As 6 is the largest of the common factors, it is called the **highest common factor (HCF)** of 12 and 18.

The prime factors of 12 are $2 \times 2 \times 3$.

The prime factors of 18 are $2 \times 3 \times 3$.

So the highest common factor can be seen by inspection to be 2×3, i.e. 6.

Multiples

Multiples of 2 are 2, 4, 6, 8, 10, etc.

Multiples of 3 are 3, 6, 9, 12, 15 etc.

The numbers 6, 12, 18, 24 etc., are **common multiples** as these appear in both lists.

The **lowest common multiple (LCM)** of 2 and 3 is 6, since 6 is the smallest number divisible by 2 and 3.

The LCM of 3 and 5 is 15.

The LCM of 6 and 10 is 30.

Exercise 1.5

1 Find the HCF of the following numbers:

a 8, 12	b 10, 25	c 12, 18, 24
d 15, 21, 27	e 36, 63, 108	f 22, 110
g 32, 56, 72	h 39, 52	i 34, 51, 68
j 60, 144		

2 Find the LCM of the following:

a 6, 14	b 4, 15	c 2, 7, 10
d 3, 9, 10	e 6, 8, 20	f 3, 5, 7
g 4, 5, 10	h 3, 7, 11	i 6, 10, 16
j 25, 40, 100		

Rational and irrational numbers

A **rational number** is any number that can be expressed as a fraction. Examples of some rational numbers and how they can be expressed as a fraction are shown below:

$0.2 = \frac{1}{5}$ $\quad 0.3 = \frac{3}{10}$ $\quad 7 = \frac{7}{1}$ $\quad 1.53 = \frac{153}{100}$ $\quad 0.\dot{2} = \frac{2}{9}$

An **irrational number** cannot be expressed as a fraction. Examples of irrational numbers include:

$\sqrt{2}, \quad \sqrt{5}, \quad 6 - \sqrt{3}, \quad \pi$

In summary:

Rational numbers include:
- whole numbers,
- fractions,
- recurring decimals,
- terminating decimals.

Irrational numbers include:
- the **square root** of any number other than square numbers,
- a decimal which does not repeat or terminate (e.g. π).

Real numbers

The set of rational and irrational numbers together form the set of **real numbers** $\mathbb{R}$.

Exercise 1.6

1 For each of the numbers shown below, state whether it is rational or irrational:

a 1.3 **b** $0.\dot{6}$ **c** $\sqrt{3}$

d $-2\frac{3}{5}$ **e** $\sqrt{25}$ **f** $\sqrt[3]{8}$

g $\sqrt{7}$ **h** 0.625 **i** $0.\dot{1}\dot{1}$

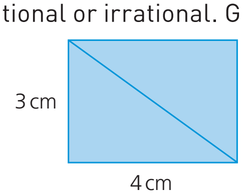
2 For each of the numbers shown below, state whether it is rational or irrational:

a $\sqrt{2} \times \sqrt{3}$ **b** $\sqrt{2} + \sqrt{3}$ **c** $(\sqrt{2} \times \sqrt{3})^2$

d $\frac{\sqrt{8}}{\sqrt{2}}$ **e** $\frac{2\sqrt{5}}{2\sqrt{20}}$ **f** $4 + (\sqrt{9} - 4)$

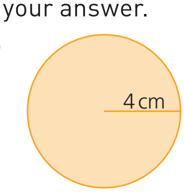
3 In each of the following decide whether the quantity required is rational or irrational. Give reasons for your answer.

a



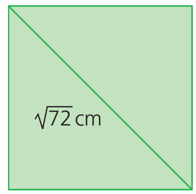
The length of the diagonal

b



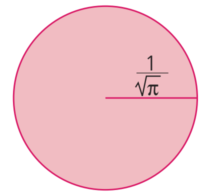
The circumference of the circle

c



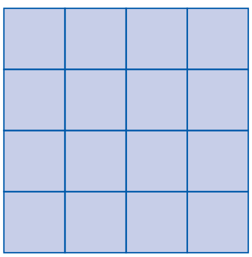
The side length of the square

d



The area of the circle

Square roots



The square shown contains 16 squares. It has sides of length 4 units.

So the square root of 16 is 4.

This can be written as $\sqrt{16} = 4$.

Note that 4×4 is 16, so 4 is the square root of 16.

However, -4×-4 is also 16, so -4 is also the square root of 16.

By convention, $\sqrt{16}$ means 'the positive square root of 16', so $\sqrt{16} = 4$ but the square root of 16 is ±4 i.e. $+4$ or -4.

Note: -16 has no square root since any integer squared is positive.

Exercise 1.7

1 Find the following:

a $\sqrt{25}$ **b** $\sqrt{9}$ **c** $\sqrt{49}$ **d** $\sqrt{100}$

e $\sqrt{121}$ **f** $\sqrt{169}$ **g** $\sqrt{0.01}$ **h** $\sqrt{0.04}$

i $\sqrt{0.09}$ **j** $\sqrt{0.25}$

2 Use the $\sqrt{\ }$ key on your calculator to check your answers to question 1.

3 Calculate the following:

a $\sqrt{\frac{1}{9}}$ **b** $\sqrt{\frac{1}{16}}$ **c** $\sqrt{\frac{1}{25}}$ **d** $\sqrt{\frac{1}{49}}$

e $\sqrt{\frac{1}{100}}$ **f** $\sqrt{\frac{4}{9}}$ **g** $\sqrt{\frac{9}{100}}$ **h** $\sqrt{\frac{49}{81}}$

i $\sqrt{2\frac{7}{9}}$ **j** $\sqrt{6\frac{1}{4}}$

Using a graph

Exercise 1.8

1 Copy and complete the table below for the equation $y = \sqrt{x}$.

x	0	1	4	9	16	25	36	49	64	81	100
y											

Plot the graph of $y = \sqrt{x}$.

Use your graph to find the approximate values of the following:

a $\sqrt{35}$ **b** $\sqrt{45}$ **c** $\sqrt{55}$ **d** $\sqrt{60}$ **e** $\sqrt{2}$

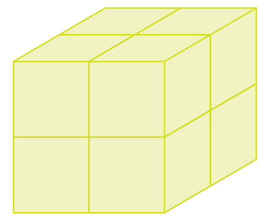
Exercise 1.8 (cont)

2 Check your answers to question 1 above by using the $\sqrt{\ }$ key on a calculator.

Cube roots

The cube shown has sides of 2 units and occupies 8 cubic units of space. (That is, $2 \times 2 \times 2$.)

So the **cube root** of 8 is 2.



This can be written as $\sqrt[3]{8} = 2$.

$\sqrt[3]{\ }$ is read as 'the cube root of ...'.

$\sqrt[3]{64}$ is 4, since $4 \times 4 \times 4 = 64$.

Note that $\sqrt[3]{64}$ is not -4

since $-4 \times -4 \times -4 = -64$

but $\sqrt[3]{-64}$ is -4.

Exercise 1.9

1 Find the following cube roots:

a	$\sqrt[3]{8}$	**b**	$\sqrt[3]{125}$	**c**	$\sqrt[3]{27}$	**d**	$\sqrt[3]{0.001}$
e	$\sqrt[3]{0.027}$	**f**	$\sqrt[3]{216}$	**g**	$\sqrt[3]{1000}$	**h**	$\sqrt[3]{1\,000\,000}$
i	$\sqrt[3]{-8}$	**j**	$\sqrt[3]{-27}$	**k**	$\sqrt[3]{-1000}$	**l**	$\sqrt[3]{-1}$

Further powers and roots

We have seen that the square of a number is the same as raising that number to the power of 2, for example, the square of 5 is written as 5^2 and means 5×5. Similarly, the cube of a number is the same as raising that number to the power of 3, for example, the cube of 5 is written as 5^3 and means $5 \times 5 \times 5$.

Numbers can be raised by other powers too. Therefore, 5 raised to the power of 6 can be written as 5^6 and means $5 \times 5 \times 5 \times 5 \times 5 \times 5$.

You will find a button on your calculator to help you to do this. On most calculators, it will look like y^x.

We have also seen that the square root of a number can be written using the $\sqrt{\ }$ symbol. Therefore, the square root of 16 is $\pm\sqrt{16}$ and is ± 4, because both $4 \times 4 = 16$ and $-4 \times -4 = 16$.

The cube root of a number can be written using the $\sqrt[3]{\ }$ symbol. Therefore, the cube root of 27 is written as $\sqrt[3]{27}$ and is 3, because $3 \times 3 \times 3 = 27$.

Other roots of numbers can also be found. The fourth root of a number can be written using the symbol $\sqrt[4]{\ }$. Therefore the fourth root of 625 can be expressed as $\pm\sqrt[4]{625}$ and is ± 5, because both $5 \times 5 \times 5 \times 5 = 625$ and $(-5) \times (-5) \times (-5) \times (-5) = 625$.

You will find a button on your calculator to help you to calculate with roots too. On most calculators, it will look like $\sqrt[x]{y}$.

Exercise 1.10

1 Work out the following:

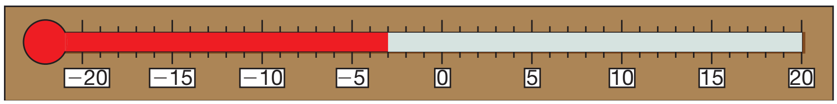
a 6^4 b $3^5 + 2^4$ c $(3^4)^2$

d $0.16 \div 0.01^4$ e $\sqrt[4]{2401}$ f $\sqrt[8]{256}$

g $(\sqrt[5]{243})^3$ h $(\sqrt[9]{36})^9$ i $2^7 \times \sqrt{\frac{1}{4}}$

j $\sqrt[6]{\frac{1}{64}} \times 2^7$ k $\sqrt[4]{5^4}$ l $(\sqrt[10]{59049})^2$

Directed numbers



Worked example

The diagram above shows the scale of a thermometer. The temperature at 04 00 was −3 °C. By 09 00, the temperature had risen by 8 °C. What was the temperature at 09 00?

$(-3)° + (8)° = (5)°$

Exercise 1.11

1 The highest temperature ever recorded was in Libya. It was 58 °C. The lowest temperature ever recorded was −88 °C in Antarctica. What is the temperature difference?

2 My bank account shows a credit balance of \$105. Describe my balance as a positive or negative number after each of these transactions is made in sequence:

a rent \$140   b car insurance \$283

c 1 week's salary \$230   d food bill \$72

e credit transfer \$250

3 The roof of an apartment block is 130 m above ground level. The car park beneath the apartment is 35 m below ground level. How high is the roof above the floor of the car park?

4 A submarine is at a depth of 165 m. If the ocean floor is 860 m from the surface, how far is the submarine from the ocean floor?

Student assessment 1

1 State whether the following numbers are rational or irrational:

a 1.5 b $\sqrt{7}$ c $0.\dot{7}$

d $0.\dot{7}\dot{3}$ e $\sqrt{121}$ f π

2 Show, by expressing them as fractions or whole numbers, that the following numbers are rational:

a 0.625 b $\sqrt[3]{27}$ c 0.44

3 Find the value of:

a 9^2 **b** 15^2

c $(0.2)^2$ **d** $(0.7)^2$

4 Calculate:

a $(3.5)^2$ **b** $(4.1)^2$ **c** $(0.15)^2$

5 Without using a calculator, find:

a $\sqrt{225}$ **b** $\sqrt{0.01}$ **c** $\sqrt{0.81}$

d $\sqrt{\frac{9}{25}}$ **e** $\sqrt{5\frac{4}{9}}$ **f** $\sqrt{2\frac{23}{49}}$

6 Without using a calculator, find:

a 4^3 **b** $(0.1)^3$ **c** $\left(\frac{2}{3}\right)^3$

7 Without using a calculator, find:

a $\sqrt[3]{27}$ **b** $\sqrt[3]{1\,000\,000}$ **c** $\sqrt[3]{\frac{64}{125}}$

8 My bank statement for seven days in October is shown below:

Date	Payments ($)	Receipts ($)	Balance ($)
01/10			200
02/10	284		(a)
03/10		175	(b)
04/10	(c)		46
05/10		(d)	120
06/10	163		(e)
07/10		28	(f)

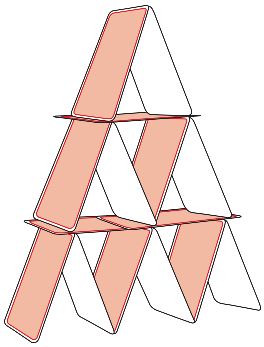
Copy and complete the statement by entering the amounts (a) to (f).

9 Using a calculator if necessary, work out:

a $2^6 \div 2^8$ **b** $4^5 \times \sqrt[6]{64}$ **c** $\sqrt[4]{81} \times 4^3$

Mathematical investigations and ICT 2

House of cards

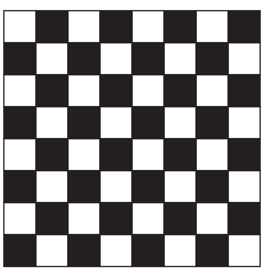


The drawing shows a house of cards 3 layers high. Fifteen cards are needed to construct it.

1 How many cards are needed to construct a house 10 layers high?
2 The world record is for a house 75 layers high. How many cards are needed to construct this house of cards?
3 Show that the general formula for a house n layers high requiring c cards is:

$c = \frac{1}{2}n(3n + 1)$

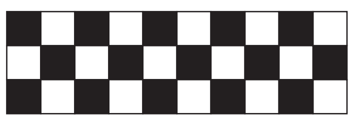
Chequered boards



A chessboard is an 8 × 8 square grid consisting of alternating black and white squares as shown.

There are 64 unit squares of which 32 are black and 32 are white.

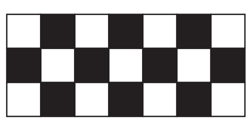
Consider boards of different sizes. The examples below show rectangular boards, each consisting of alternating black and white unit squares.



Total number of unit squares is 30.

Number of black squares is 15.

Number of white squares is 15.



Total number of unit squares is 21.

Number of black squares is 10.

Number of white squares is 11.

1 Investigate the number of black and white unit squares on different rectangular boards.
Note: For consistency you may find it helpful to always keep the bottom right-hand square the same colour.
2 What are the numbers of black and white squares on a board $m \times n$ units?

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