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Mathematics

Second edition

Michael Handbury, Jean Matthews
Series editor: Brian Seager
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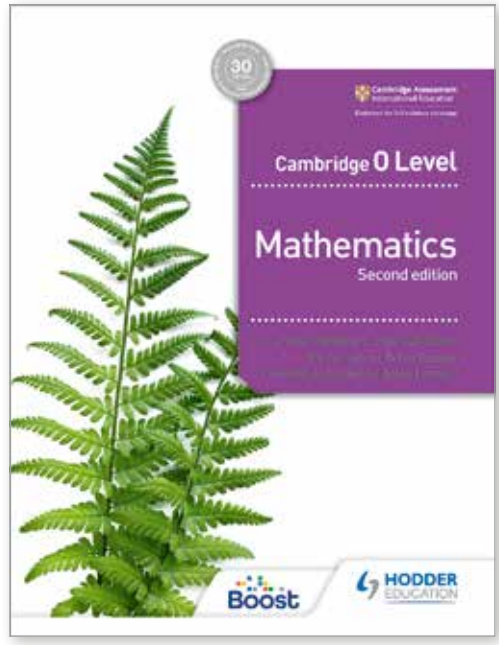

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CONTENTS

41 PYTHAGORAS' THEOREM AND TRIGONOMETRY

CHECK YOU CAN:

- use a ruler and a protractor
- find the area of a triangle
- use your calculator to find squares and square roots
- solve simple equations.

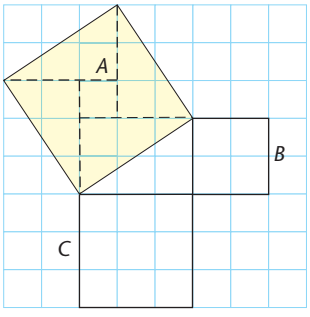
BY THE END OF THIS CHAPTER YOU WILL BE ABLE TO:

- know and use Pythagoras' theorem
- know and use the sine, cosine and tangent ratios for acute angles in calculations involving sides and angles of a right-angled triangle
- solve problems in two dimensions using Pythagoras' theorem and trigonometry
- know that the perpendicular distance from a point to a line is the shortest distance to the line
- carry out calculations involving angles of elevation and depression
- use the sine and cosine rules in calculations involving lengths and angles for any triangle
- use the formula

  $$\text{Area of triangle} = \frac{1}{2}ab\sin C$$

- carry out calculations and solve problems in three dimensions using Pythagoras' theorem and trigonometry, including calculating the angle between a line and a plane.

Pythagoras' theorem



Look at square A.

Area of square A = areas of the four triangles + small square

$$= 4\left(\frac{1}{2} \times 2 \times 3\right) + 1$$

$$= 13$$

Area of square B + area of square $C = 4 + 9 = 13$

This is an example of the rule linking the areas of squares around a right-angled triangle, known as Pythagoras' theorem.

The largest square will always be on the longest side of the triangle – this is called the **hypotenuse** of the right-angled triangle.

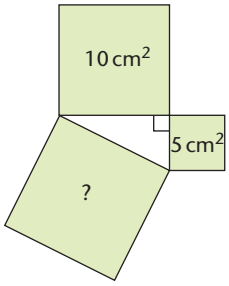
Pythagoras' theorem can be stated like this.

The area of the square on the hypotenuse = the sum of the areas of the squares on the other two sides.

Exercise 41.1

Calculate the missing area in each of these diagrams.

1



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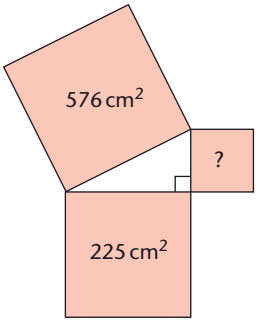

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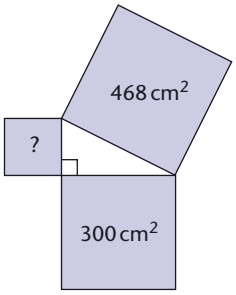

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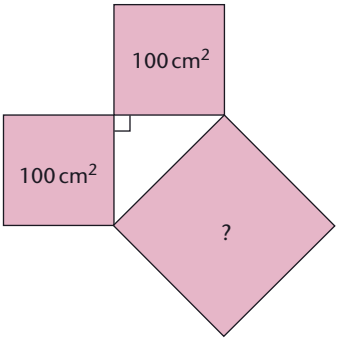
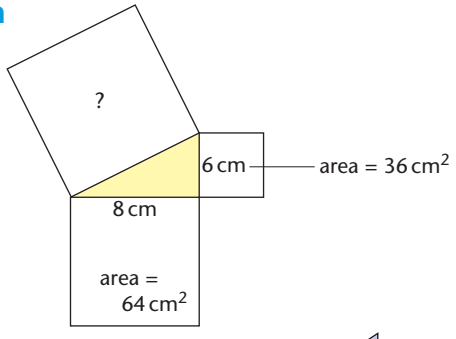


Using Pythagoras' theorem

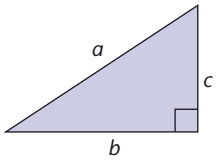
If you know the lengths of two sides of a right-angled triangle you can use Pythagoras' theorem to find the length of the third side.



The unknown area = 64 + 36 = 100 cm^2.

This means that the sides of the unknown square have a length of $\sqrt{100} = 10$ cm.

When using Pythagoras' theorem, you don't need to draw the squares − you can simply use the rule.

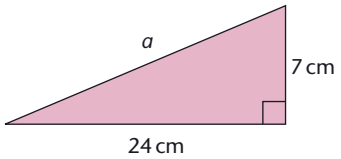


$a^2 = b^2 + c^2$

Example 41.1

Question

Find the length a in the diagram.



Solution

$a^2 = 7^2 + 24^2$

$a^2 = 49 + 576$

$a^2 = 625$

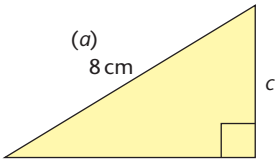
$a = \sqrt{625}$

$a = 25$ cm

Example 41.2

Question

Find the length c in the diagram.



Solution

$a^2 = b^2 + c^2$

$8^2 = 6^2 + c^2$

$64 = 36 + c^2$

$c^2 = 64 - 36$

$c = \sqrt{28}$

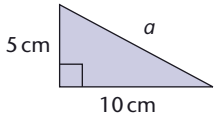
$c = 5.29$ cm (to 2 decimal places)

Exercise 41.2

For each of the triangles in this exercise, find the length of the side marked with a letter.

Give your answers either exactly, or correct to 2 decimal places.

1



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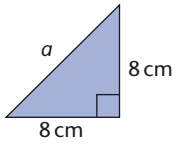

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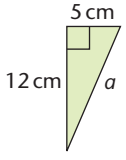

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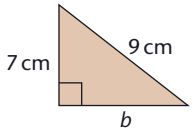

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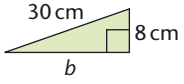

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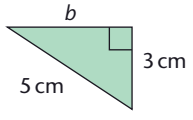

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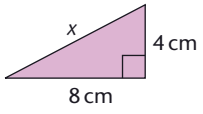

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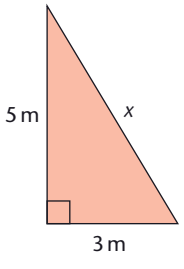

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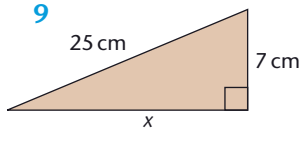

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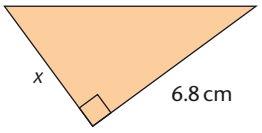

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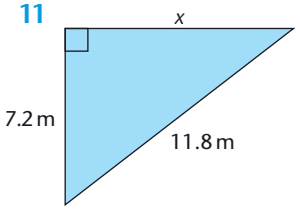

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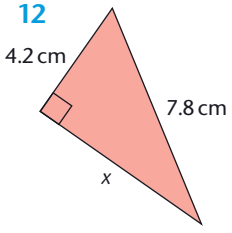


Using Pythagoras' theorem to solve problems

You can use Pythagoras' theorem to solve problems.

It is a good idea to draw a sketch if a diagram isn't given.

Try to draw it roughly to scale and mark on it any lengths you know.

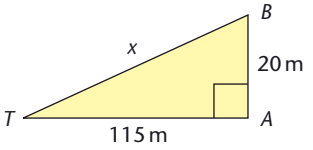
Example 41.3

Question

Tao is standing 115 m from a vertical tower.

The tower is 20 m tall.

Work out the distance from Tao directly to the top of the tower.



Solution

$x^2 = 115^2 + 20^2$

$= 13\,625$

$x = \sqrt{13\,625}$

$= 116.7$ m (to 1 decimal place)

Tao is 116.7 m (to 1 decimal place) from the top of the tower.

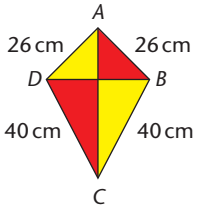
Exercise 41.3

1 A rectangular field is 225 m long and 110 m wide.
Find the length of the diagonal path across it.

2 A rectangular field is 25 m long.
A footpath 38.0 m long crosses the field diagonally.
Find the width of the field.

3 A ladder is 7 m long.
It is resting against a wall, with the top of the ladder 5 m above the ground.
How far from the wall is the base of the ladder?

4 Haleef is making a kite for his sister.
This is his diagram of the kite.
The kite is 30 cm wide.
Haleef needs to buy some cane to make the struts *AC* and *DB*.
What length of cane does he need to buy?



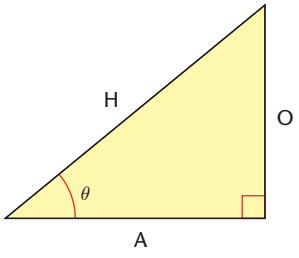
Note

Label the sides in the order hypotenuse, opposite, adjacent.

To identify the opposite side, go straight out from the middle of the angle. The side you hit is the opposite.

You can shorten the labels to 'H', 'O' and 'A'.

θ is the Greek letter 'theta'.



Note

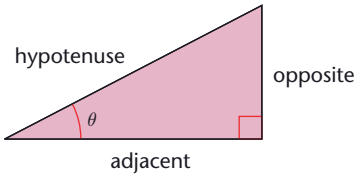
Notice that the ratio of the lengths is written as a fraction, $\frac{\text{Opposite}}{\text{Hypotenuse}}$, rather than a ratio, Opposite : Hypotenuse.

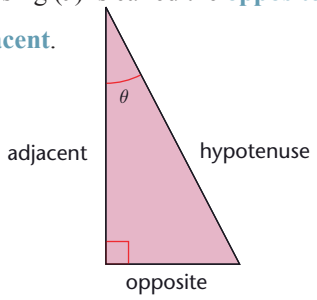
Trigonometry

You already know that the longest side of a right-angled triangle is called the **hypotenuse**.

The side opposite the angle you are using (θ) is called the **opposite**.

The remaining side is called the **adjacent**.





For a given angle $\theta°$, all right-angled triangles with an angle $\theta°$ will be similar (angles in each triangle of 90°, $\theta°$ and $(90 - \theta°)$). It follows that the ratios of the sides will be constant for that value of θ.

The ratio $\frac{\text{Opposite}}{\text{Hypotenuse}}$ is called the **sine** of the angle.

This is often shortened to 'sin'.

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

The ratio $\frac{\text{Adjacent}}{\text{Hypotenuse}}$ is called the **cosine** of the angle.

This is often shortened to 'cos'.

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

The ratio $\frac{\text{Opposite}}{\text{Adjacent}}$ is called the **tangent** of the angle.

This is often shortened to 'tan'.

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

Note

You need to learn the three ratios.

$\sin\theta = \frac{O}{H}$

$\cos\theta = \frac{A}{H}$

$\tan\theta = \frac{O}{A}$

There are various ways of remembering these, but one of the most popular is to learn the 'word' 'SOHCAHTOA'.

Using the ratios 1

When you need to solve a problem using one of the ratios, you should follow these steps.

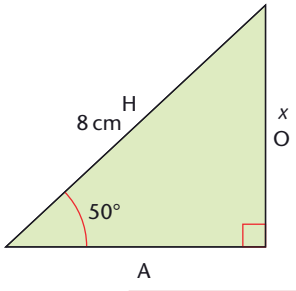
- Draw a clearly labelled diagram.
- Label the sides H, O and A.
- Decide which ratio you need to use.
- Solve the equation.

In one type of problem you will encounter, you are required to find the numerator (top) of the fraction. This is demonstrated in the following examples.

Example 41.4

Question

Find the length marked x.



Solution

Since you know the hypotenuse (H) and want to find the opposite (O), you use the sine ratio.

$\sin 50° = \frac{O}{H}$

$\sin 50° = \frac{x}{8}$

$8 \times \sin 50° = x$ Multiply both sides by 8.

$x = 6.128\,35\ldots = 6.13$ cm correct to 3 significant figures.

Note

Press these keys on your calculator to find x.

8 × sin 5 0 =

Note

Make sure that your calculator is set to degrees.

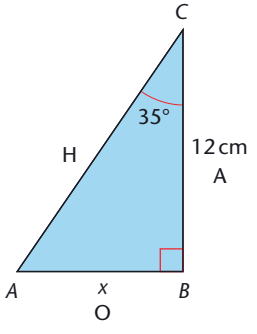
This is the default setting but, if you see 'rad' or 'R' or 'grad' or 'G' in the window, change the setting using the key labelled 'mode' or 'DRG' or 'set up'.

Example 41.5

Question

In triangle ABC, $BC = 12$ cm , angle $B = 90°$ and angle $C = 35°$.

Find the length AB.



Solution

Draw the triangle and label the sides.

Since you know the adjacent (A) and want to find the opposite (O), you use the tangent ratio.

$\tan 35° = \frac{O}{A}$

$\tan 35° = \frac{x}{12}$

$12 \times \tan 35° = x$ Multiply both sides by 12.

$x = 8.402\,49\ldots = 8.40$ cm correct to 3 significant figures.

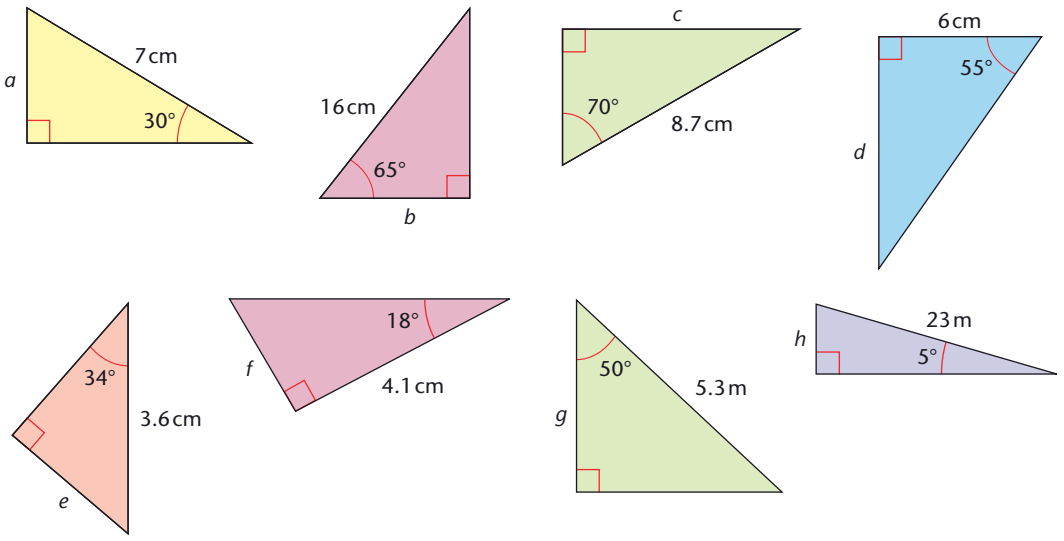
Note

Press these keys on your calculator to find x.

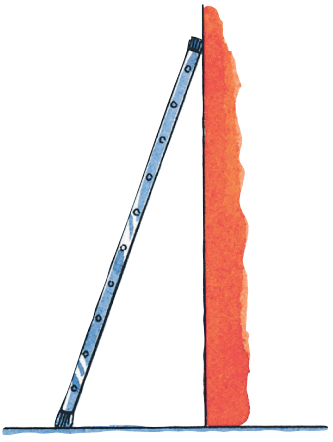
1 2 × tan 3 5 =

Exercise 41.4

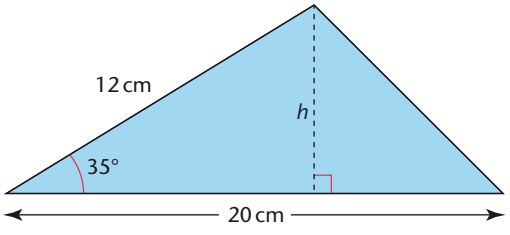
1 In these diagrams find the lengths marked a, b, c, d, e, f, g and h.



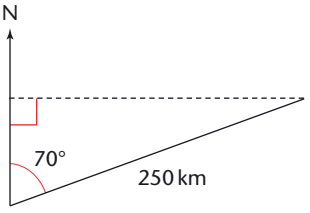
2 The ladder shown here is 6 metres long.
The angle between the ladder and the ground is 70°.
How far from the wall is the foot of the ladder?



3 **a** Find the height, h, of the triangle.
b Use the height you found in part **a** to find the area of the triangle.

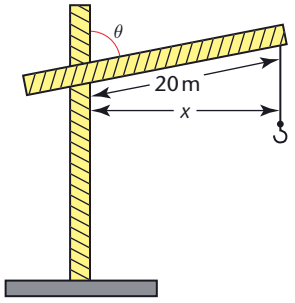


4 A ship sails on a bearing of 070° for 250 km.
- **a** Find how far north the ship has travelled.
- **b** Find how far east the ship has travelled.



Note
You can read more about bearings in Chapter 34.

5 The diagram shows a crane.
The length of the crane's arm is 20 metres.
The crane can operate with the arm anywhere between 15° and 80° to the vertical.
Calculate the minimum and maximum values of x, the distance from the crane at which a load can be lowered.



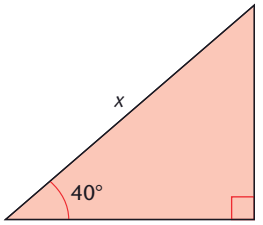
Using the ratios 2

In the second type of problem you will encounter, you are required to find the denominator (bottom) of the fraction. This is demonstrated in the following example.

Example 41.6

Question

Find the length marked x.



Solution

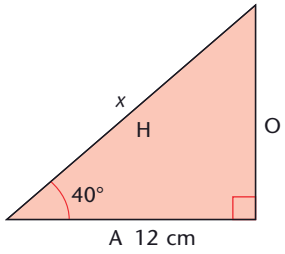
Since you know A and want to find H, you use the cosine ratio.

$\cos 40° = \frac{A}{H}$

$\cos 40° = \frac{12}{x}$

$x = \frac{12}{\cos 40°}$

$x = 15.664\,88\ldots = 15.7$ cm correct to 3 significant figures.



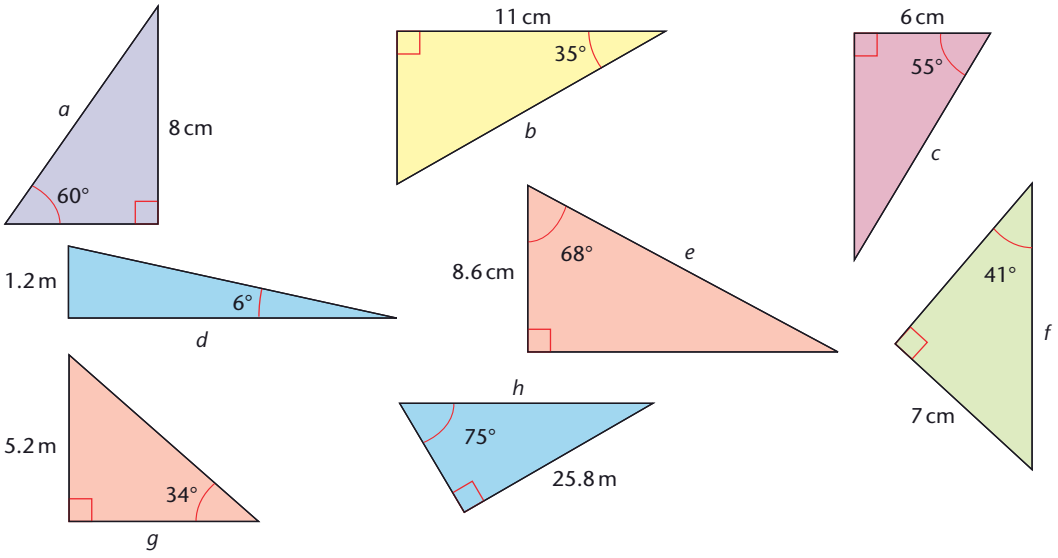
Note

Always look to see whether the length you are trying to find should be longer or shorter than the one you are given.

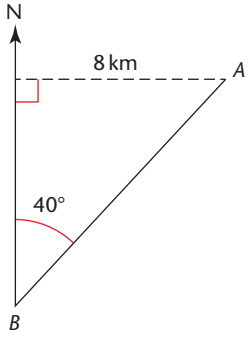
If your answer is obviously wrong, you have probably multiplied instead of divided.

Exercise 41.5

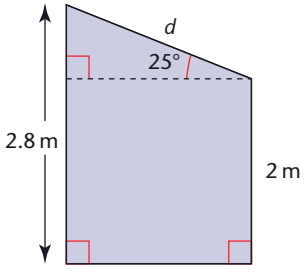
1 In these diagrams find the lengths marked a, b, c, d, e, f, g and h.



2 The bearing of A from B is 040°.
A is 8 kilometres east of B.
Calculate how far A is north of B.



3 The diagram shows a shed.



a Find the length d.

b The length of the shed is 2.5 m.
Find the area of the roof.

4 A ship sailed on a bearing of 140°.
It was then 90 km south of its original position.

a Draw a diagram to show the ship's journey.

b How far east of its original position is it?

5 Mrs Khan wants to buy a ladder.
Her house is 5.3 metres high and she needs to reach the top.
The ladders are in two sections, each section being the same length.
When extended, there must be an overlap of 1.5 metres between the two sections.
The safe operating angle is 76°.
Calculate the length of each of the sections of ladder she needs to buy.

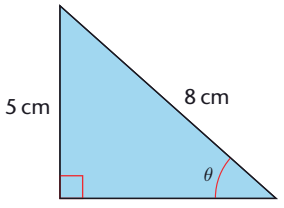
Using the ratios 3

In the third type of problem you will encounter, you are given the value of two sides and are required to find the angle. This is demonstrated in the following examples.

Example 41.7

Question

Find the angle θ.



Solution

This time, look at the two sides you know.

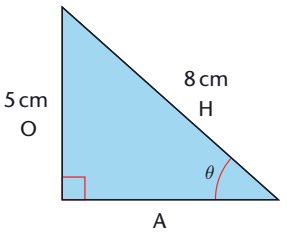
Since they are O and H, you use the sine ratio.

$\sin\theta = \frac{O}{H}$ $\sin\theta = \frac{5}{8}$

Work out 5 ÷ 8 = 0.625 on your calculator and leave this number in the display.

Now use the $\sin^{-1}$ function (the inverse of sine).

With 0.625 still in the display, press SHIFT sin =, or the equivalent on your calculator.



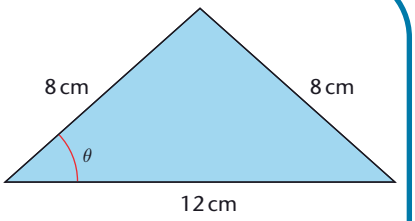
You should see 38.68218… .

So $\theta = 38.7°$ correct to 3 significant figures, or 39° correct to the nearest degree.

Example 41.8

Question

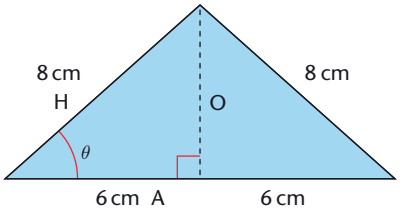
Find the angle θ.



Solution

Since this is an isosceles triangle, not a right-angled triangle, you need to use the fact that the line of symmetry splits an isosceles triangle into two equal right-angled triangles.

The sides you know are A and H so you use the cosine ratio.



$\cos\theta = \frac{A}{H} = \frac{6}{8}$

$\theta = \cos^{-1}\frac{6}{8}$

$\theta = 41.4°$ correct to 3 significant figures.

Note

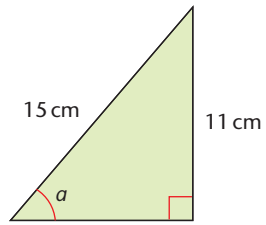
Example 41.8 shows how to deal with isosceles triangles.

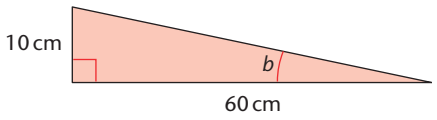
You use the line of symmetry to split the triangle into two equal right-angled triangles.

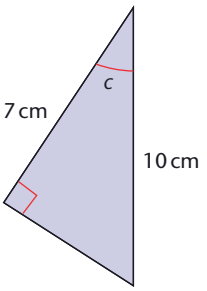
This works only with isosceles triangles, because they have a line of symmetry.

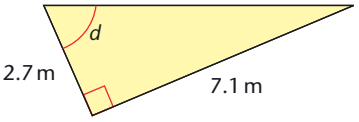
Exercise 41.6

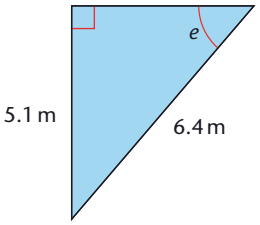
1 In these diagrams find the angles marked a, b, c, d, e, f, g and h.

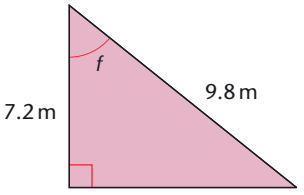


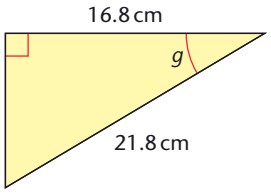


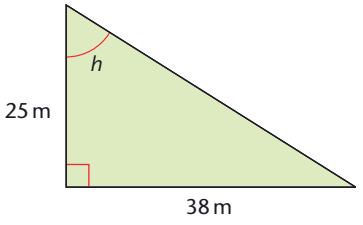




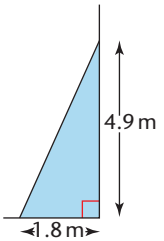




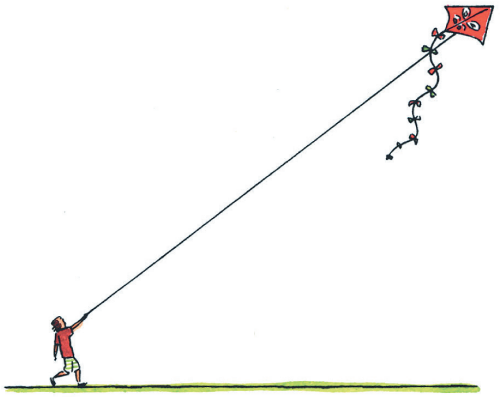




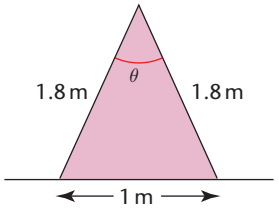
2 The diagram represents a ladder leaning against a wall.
Find the angle the ladder makes with the horizontal.



3 In the picture, the kite is 15 metres above the girl.
The string is 25 metres long.
Find the angle the string makes with the horizontal.



4 The diagram represents a pair of step ladders standing on a horizontal floor.
Find the angle, θ, between the two parts of the step ladder.

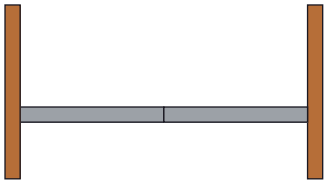


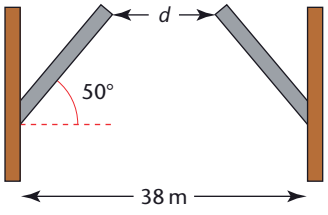
5 An aircraft flies 180 km due east from A to B.
It then flies 115 km due south from B to C.
a Draw a diagram to show the positions of A, B and C.
b Calculate the bearing of C from A.

6 A television mast is 54 metres high and stands on horizontal ground.
Six guy wires keep the mast upright.
Three of these are attached to the top and a point on the ground.
These three make an angle of 16.5° with the vertical.
a Calculate the total length of these three wires.
The other three wires are attached $\frac{2}{3}$ of the way up the mast.
They are attached to the same points on the ground as the previous three.
b Calculate the angle these make with the vertical.

7 A ship sails for 150 km on a bearing of 115° from A to B.
It then sails for 250 km on a bearing of 230° from B to C.
Calculate the distance AC and the bearing of C from A.

8 The diagrams show a bridge in the shut and open positions.

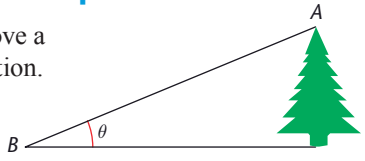




When opened, the bridge sections raise by 50°.
The distance between the supports is 38 metres.
Calculate the distance d.

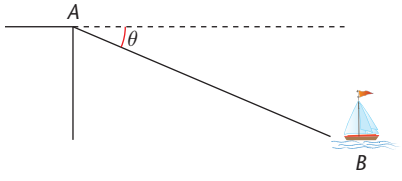
Angle of elevation and angle of depression

The angle that a line to an object makes above a horizontal plane is called the angle of elevation.



The angle of elevation of the top of this tree, A, from B, is θ.

The angle a line makes to an object below a horizontal plane is called the angle of depression.



The angle of depression of the boat, B, from A, is θ.

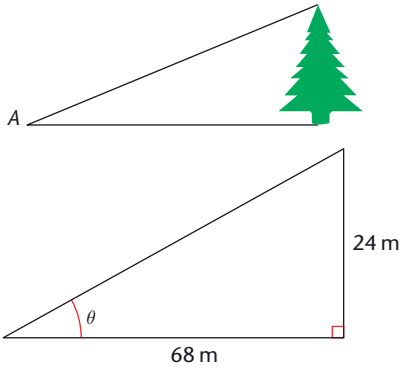
Note
Since alternate angles are equal, the angle of elevation of A from B equals the angle of depression of B from A.

Example 41.9

Question

The height of this tree is 24 m.
A is 68 m from the foot of the tree.

Find the angle of elevation of the top of the tree from A.



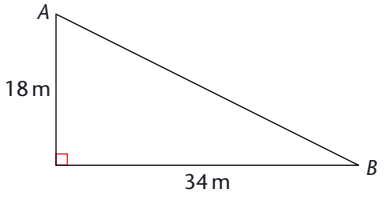
Solution

$\tan \theta = \frac{24}{68}$

$\theta = 19.4°$

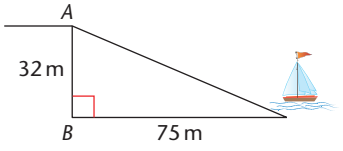
Exercise 41.7

1 Find the angle of elevation of A from B.

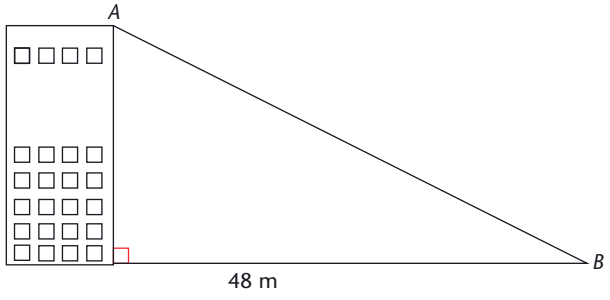


2 The angle of elevation of the top of a tree from A is 25°.
A is 38 m from the base of the tree.
Calculate the height of the tree.

3 AB is a vertical cliff 32 m high.
The boat is on the sea, 75 metres from the bottom of the cliff.
Calculate the angle of depression of the boat from the top of the cliff, A.



4 The diagram shows a tall building.
B is a point on the ground 48 m from the foot of the building.
The angle of depression of B from a point A, on the top of the building, is 28°.
Calculate the height of the building.

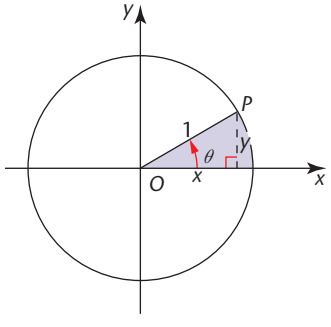


5 Adil is standing on horizontal ground looking at the top of a building 18 m high.
The angle of elevation of the top of the building from Adil's eyes is 24°.
Adil's eyes are 1.8 m above the ground.
Find how far away from the building Adil is standing.

The sine and cosine functions for obtuse angles

So far, you have only dealt with the sine and cosine functions for right-angled triangles, so the angles have all been acute.

However, your calculator will give you the sine and cosine of any angle.



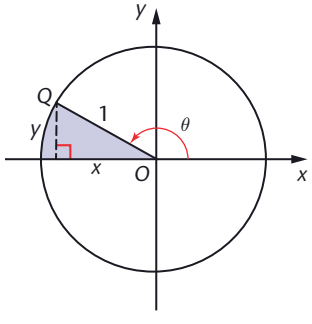
In this diagram you can see that for an acute angle

$\cos\theta = \frac{x}{1}$ so $x = \cos\theta$

$\sin\theta = \frac{y}{1}$ so $y = \sin\theta$

So P has coordinates $(\cos\theta, \sin\theta)$.

For other angles, the trigonometric functions are defined in a similar way, where the angle is measured anticlockwise from the x-axis.



By symmetry, Q has coordinates $(-\cos\theta, \sin\theta)$.

So

$\sin\theta = \sin(180 - \theta)$

$\cos\theta = -\cos(180° - \theta)$

By symmetry, $\sin\theta = y = \sin(180° - \theta)$

and $\cos\theta = x = -\cos(180° - \theta)$.

So for an obtuse angle θ, $\sin\theta$ is equal to $\sin(180° - \theta)$

and $\cos\theta$ takes the same value as $\cos(180° - \theta)$ but is negative.

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