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Additional Mathematics

Second edition

Val Hanrahan, Jeanette Powell, Stephen Wrigley

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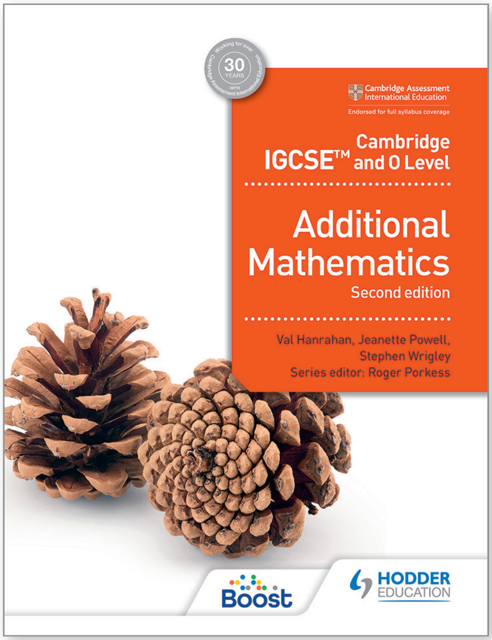

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ISBN: 9781398373952

First published in 2016
This edition published in 2023 by
Hodder Education,
An Hachette UK Company
Carmelite House
50 Victoria Embankment
London EC4Y 0DZ

www.hoddereducation.com

Cover photo © mimadeo - stock.adobe.com

Illustrations by Integra Software Services

Typeset by Integra Software Services

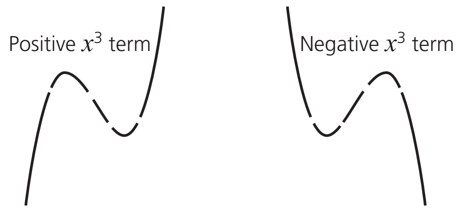
Printed in

A catalogue record for this title is available from the British Library.

Contents

Using graphs to solve cubic inequalities

Cubic graphs have distinctive shapes determined by the coefficient of x^3.



The centre part of each of these curves may not have two distinct turning points like those shown above, but may instead 'flatten out' to give a **point of inflection.** When the modulus of a cubic function is required, any part of the curve below the x-axis is reflected in that axis.

Worked example

You are asked for a sketch graph, so although it must show the main features, it does not need to be absolutely accurate. You may find it easier to draw the curve first, with the positive x^3 term determining the shape of the curve, and then position the x-axis so that the distance between the first and second intersections is about half that between the second and third, since these are 3 and 6 units, respectively.

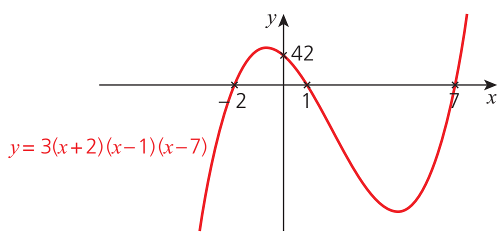
a Sketch the graph of $y = 3(x + 2)(x - 1)(x - 7)$. Identify the points where the curve cuts the axes.

b Sketch the graph of $y = |3(x + 2)(x - 1)(x - 7)|$.

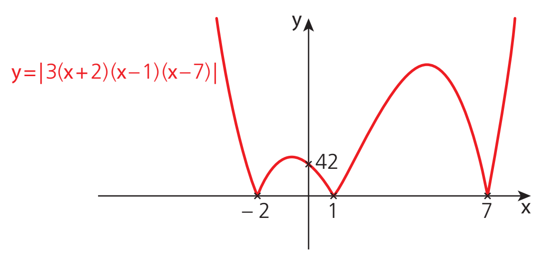
Solution

a The curve crosses the x-axis at –2, 1 and 7. Notice that the distance between consecutive points is 3 and 6 units, respectively, so the y-axis is between the points –2 and 1 on the x-axis, but closer to the 1.

The curve crosses the y-axis when $x = 0$, i.e. when $y = 3(2)(-1)(-7) = 42$.



b To obtain a sketch of the modulus curve, reflect any part of the curve that is below the x-axis in the x-axis.



Worked example

Solve the inequality $3(x + 2)(x - 1)(x - 7) \leqslant -100$ graphically.

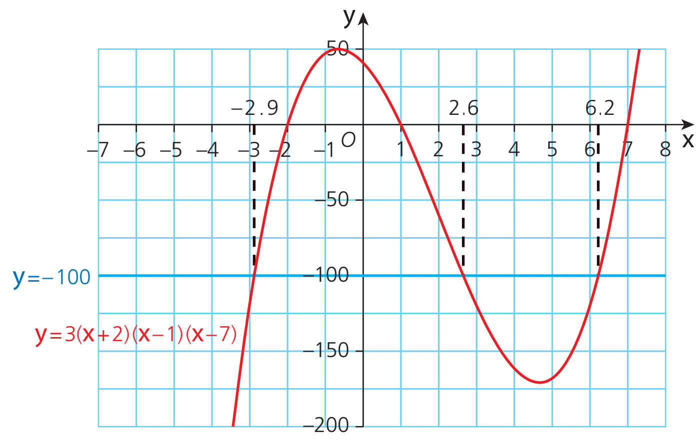
Solution

Because you are solving the inequality graphically, you will need to draw the curve as accurately as possible on graph paper, so start by drawing up a table of values.

$$y = 3(x + 2)(x - 1)(x - 7)$$

x	–3	–2	–1	0	1	2	3	4	5	6	7	8
$(x + 2)$	–1	0	1	2	3	4	5	6	7	8	9	10
$(x - 1)$	–4	–3	–2	–1	0	1	2	3	4	5	6	7
$(x - 7)$	–10	–9	–8	–7	–6	–5	–4	–3	–2	–1	0	1
x	–120	0	48	42	0	–60	–120	–162	–168	–120	0	210

The solution is given by the values of x that correspond to the parts of the curve on or below the line $y = -100$.



From the graph, the solution is $x \leqslant -2.9$ or $2.6 \leqslant x \leqslant 6.2$.

Exercise 4.3

Remember: $\sqrt{(x)}$ means the positive square root of x.

1 Where possible, use the substitution $x = u^2$ to solve the following equations:

a $x - 4\sqrt{x} = -4$ **b** $x + 2\sqrt{x} = 8$

c $x - 2\sqrt{x} = 15$ **d** $x + 6\sqrt{x} = -5$

2 Use the substitution $x = u^3$ to solve the equation $x^{\frac{2}{3}} + 3x^{\frac{1}{3}} = 4$.

3 Use the substitution $x = u^{\frac{3}{2}}$ to solve the equation $x^{\frac{4}{3}} - 10x^{\frac{2}{3}} = -9$.

4 Using a suitable substitution, solve the following equations:

a $x - 7\sqrt{x} = -12$ **b** $x - 2\sqrt{x} + 1 = 0$

c $x^{\frac{2}{3}} + 3x^{\frac{1}{3}} = 10$

5 **a** Use the substitution $x = u^{\frac{1}{2}}$ to solve the equation $x^4 - 5x^2 + 4 = 0$.

b Using the same substitution, show that the equation $x^4 + 5x^2 + 4 = 0$ has no solution.

c Solve where possible:

i $x^{\frac{4}{3}} - 5x^{\frac{2}{3}} + 4 = 0$ **ii** $x^{\frac{4}{3}} + 5x^{\frac{2}{3}} + 4 = 0$

6 Sketch the following graphs, indicating the points where they cross the x-axis:

a $y = x(x - 2)(x + 2)$ **b** $y = |x(x - 2)(x + 2)|$

c $y = 3(2x - 1)(x + 1)(x + 3)$ **d** $y = |3(2x - 1)(x + 1)(x + 3)|$

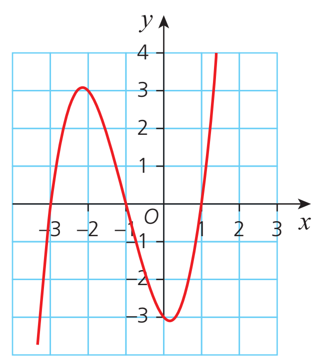
7 Solve the following equations graphically. You will need to use graph paper.

a $x(x + 2)(x - 3) \geqslant 1$ **b** $x(x + 2)(x - 3) \leqslant -1$

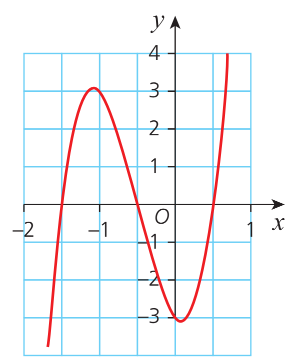
c $(x + 2)(x - 1)(x - 3) > 2$ **d** $(x + 2)(x - 1)(x - 3) < -2$

8 Identify the following cubic graphs:

a

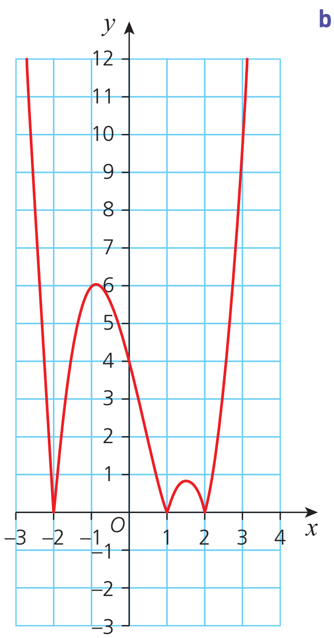


b



9 Identify these graphs. (They are the moduli of cubic graphs.)

a



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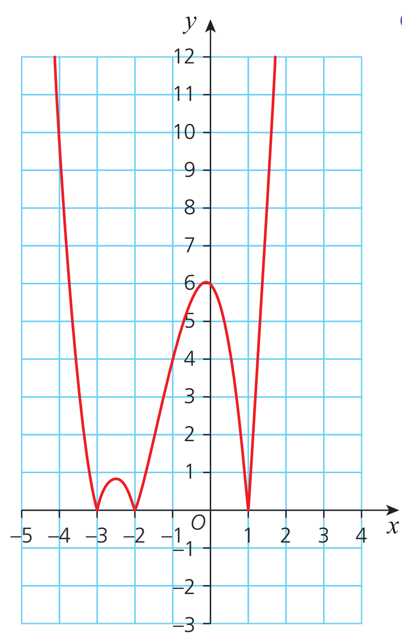


c

Past-paper questions

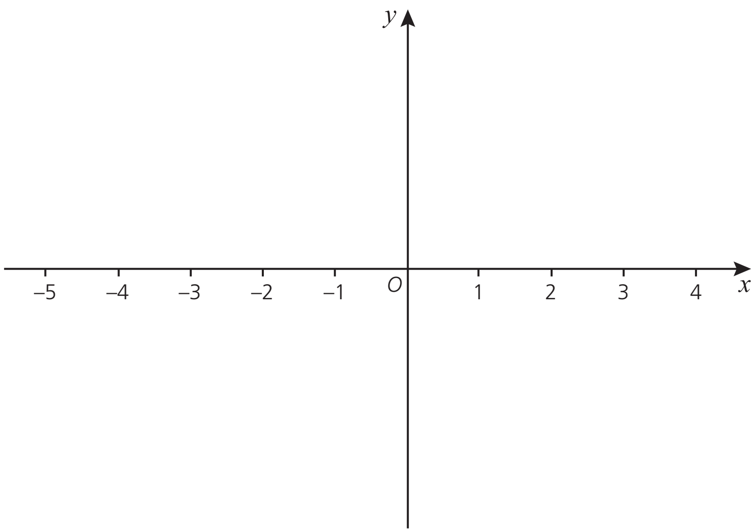
1 **(i)** Sketch the graph of $y = |(2x + 3)(2x - 7)|$. [4]

(ii) How many values of x satisfy the equation $|(2x + 3)(2x - 7)| = 2x$? [2]

Cambridge O Level Additional Mathematics 4037 Paper 23 Q6 November 2011

Cambridge IGCSE Additional Mathematics 0606 Paper 23 Q6 November 2011

2 **(i)** On a grid like the one below, sketch the graph of $y = |(x - 2)(x + 3)|$ for $-5 \leqslant x \leqslant 4$, and state the coordinates of the points where the curve meets the coordinate axes. [4]



(ii) Find the coordinates of the stationary point on the curve $y = |(x - 2)(x + 3)|$. [2]

(iii) Given that k is a positive constant, state the set of values of k for which $|(x - 2)(x + 3)| = k$ has 2 solutions only. [1]

Cambridge O Level Additional Mathematics 4037 Paper 12 Q8 November 2013

Cambridge IGCSE Additional Mathematics 0606 Paper 12 Q8 November 2013

3 Solve the inequality $9x^2 + 2x - 1 < (x + 1)^2$. [3]

Cambridge O Level Additional Mathematics 4037 Paper 22 Q2 November 2014
Cambridge IGCSE Additional Mathematics 0606 Paper 22 Q2 November 2014

Now you should be able to:

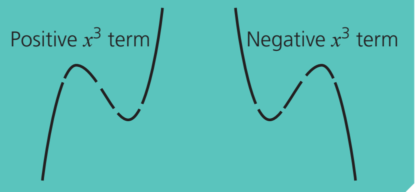
- solve graphically or algebraically equations of the type $|ax + b| = c\ (c \geqslant 0)$, $|ax + b| = cx + d$, $|ax + b| = |cx + d|$ and $|ax^2 + bx + c| = d$
- solve graphically or algebraically inequalities of the type $k|ax + b| > c\ (c \geqslant 0)$, $k|ax + b| \leqslant c\ (c \geqslant 0$, $k|ax + b| \leqslant |cx + d|$, where $k > 0$, $|ax + b| \leqslant cx + d$, $|ax^2 + bx + c| > d$ and $|ax^2 + bx + c| \leqslant d$
- use substitution to form and solve a quadratic equation in order to solve a related equation
- sketch the graphs of cubic polynomials and their moduli, when given in factorised form $y = k(x - a)(x - b)(x - c)$
- solve cubic inequalities of the form $k(x - a)(x - b)(x - c) \leqslant d$ graphically.

Key points

- For any real number x, the **modulus** of x is denoted by $|x|$ and is defined as:

  $|x| = x$ if $x \geqslant 0$

  $|x| = -x$ if $x < 0$.
- A modulus equation of the form $|ax + b| = b$ can be solved either graphically or algebraically.
- A modulus equation of the form $|ax + b| = |cx + d|$ can be solved graphically by first drawing both graphs on the same axes and then, if necessary, identifying the solution algebraically.
- A modulus inequality of the form $|x - a| < b$ is equivalent to the inequality $a - b < x < a + b$ and can be illustrated on a number line with an open circle marking the ends of the interval to show that these points are not included. For $|x - a| \leqslant b$, the interval is the same but the end points are marked with solid circles.
- A modulus inequality of the form $|x - a| > b$ or $|x - a| \geqslant b$ is represented by the parts of the line outside the intervals above.
- A modulus inequality in two dimensions is identified as a region on a graph, called the **feasible region.** It is common practice to shade out the region not required to keep the feasible region clear.
- It is sometimes possible to solve an equation involving both x and $\sqrt{x}$ by making a substitution of the form $x = u^2$. You must check all answers in the original equation.
- The graph of a cubic function has a distinctive shape determined by the coefficient of x^3.



10 Trigonometry

The laws of nature are written in the language of mathematics ... the symbols are triangles, circles and other geometrical figures without whose help it is impossible to comprehend a single word.

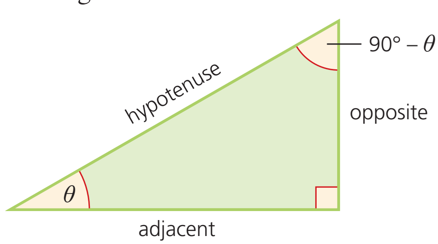
Galileo Galilei (1564–1642)

Discussion point

How can you estimate the angle that the sloping sides of this pyramid make with the horizontal?

Using trigonometry in right-angled triangles

The simplest definitions of the trigonometrical functions are given in terms of the ratios of the sides of a right-angled triangle, for values of the angle θ between 0° and 90°.



The Greek letter θ (theta) is often used to denote an angle. The Greek letters α (alpha) and β (beta) are also commonly used for this purpose.

Taking the first letters of each part gives the word 'sohcahtoa', which may help you to remember the formula.

In a right-angled triangle:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}.$$

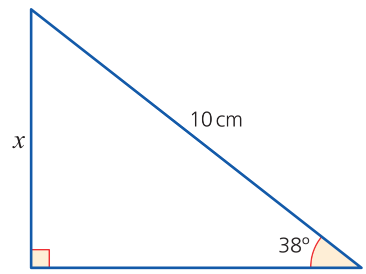
Sin is an abbreviation of sine, cos of cosine and tan of tangent. The previous diagram shows that:

$$\sin\theta = \cos(90° - \theta) \text{ and } \cos\theta = \sin(90° - \theta).$$

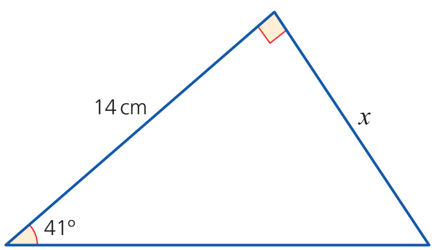
Worked example

Work out the length of x in each triangle. Give your answers correct to three significant figures.

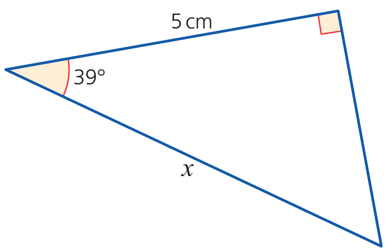
a



b



c



Solution

a $\frac{x}{10} = \sin 38°$

$\Rightarrow\ x = 10\sin 38°$

$x = 6.16$ cm

b $\frac{x}{14} = \tan 41°$

$\Rightarrow\ x = 14\tan 41°$

$\Rightarrow\ x = 12.2$ cm

c $\frac{5}{x} = \cos 39°$

$\Rightarrow\ 5 = x\cos 39°$

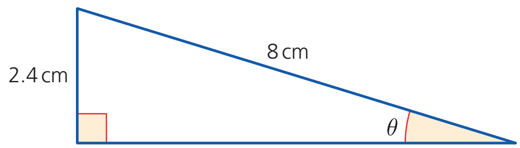
$\Rightarrow\ x = \frac{5}{\cos 39°}$

$\Rightarrow\ x = 6.43$ cm

Worked example

Work out the angle marked θ in each triangle. Give your answers correct to one decimal place.

a



Solution

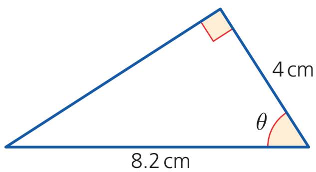
a $\sin\theta = \frac{2.4}{8}$

$\Rightarrow \theta = \sin^{-1} 0.3$

$\Rightarrow \theta = 17.5°$

$\sin^{-1} 0.3$ is shorthand notation for 'the angle θ where $\sin\theta = 0.3$'. $\cos^{-1} 0.3$ and $\tan^{-1} 0.3$ are similarly defined.

b

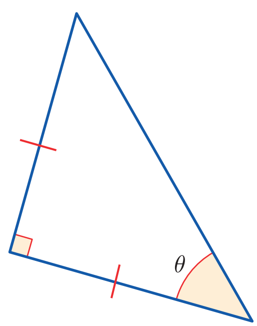


b $\cos\theta = \frac{4}{8.2}$

$\Rightarrow \theta = \cos^{-1}\frac{4}{8.2}$

$\Rightarrow \theta = 60.8°$

c



c The opposite and adjacent sides are equal, so $\tan\theta = 1$

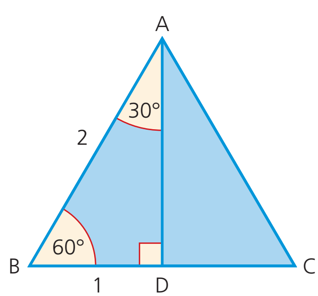
$\Rightarrow \theta = 45°$.

Special cases

Certain angles occur frequently in mathematics and you will find it helpful to know the value of their trigonometrical functions.

The angles 30° and 60°

Triangle ABC is an equilateral triangle with side length 2 units, and AD is a line of symmetry.



Using Pythagoras' theorem

$$AD^2 + 1^2 = 2^2 \Rightarrow AD = \sqrt{3}.$$

From triangle ABD,

$$\sin 60° = \frac{\sqrt{3}}{2}; \qquad \cos 60° = \frac{1}{2}; \qquad \tan 60° = \sqrt{3};$$

$$\sin 30° = \frac{1}{2}; \qquad \cos 30° = \frac{\sqrt{3}}{2}; \qquad \tan 30° = \frac{1}{\sqrt{3}}.$$

Worked example

Without using a calculator, find the value of $\sin^2 30° + \sin 60° \cos 30°$.
(Note that $\sin^2 30°$ means $(\sin 30°)^2$.)

Solution

$$\begin{aligned}\sin^2 30° + \sin 60° \cos 30° &= \left(\frac{1}{2}\right)^2 + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} \\ &= \frac{1}{4} + \frac{3}{4} \\ &= 1\end{aligned}$$

Note

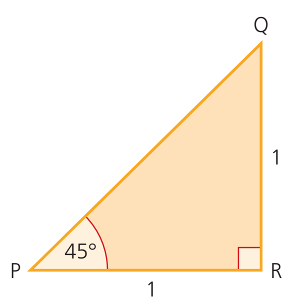
The equivalent results using radians are

$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}; \qquad \cos\frac{\pi}{3} = \frac{1}{2}; \qquad \tan\frac{\pi}{3} = \sqrt{3}$$

$$\sin\frac{\pi}{6} = \frac{1}{2}; \qquad \cos\frac{\pi}{3} = \frac{\sqrt{3}}{2}; \qquad \tan\frac{\pi}{3} = \frac{1}{\sqrt{3}}$$

The angle 45°

PQR is a right-angled isosceles triangle with equal sides of length 1 unit.



Using Pythagoras' theorem, $PQ = \sqrt{2}$.

This gives

$$\sin 45° = \frac{1}{\sqrt{2}}; \qquad \cos 45° = \frac{1}{\sqrt{2}}; \qquad \tan 45° = 1.$$

Note

In radians

$$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}};$$

$$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}};$$

$$\tan\frac{\pi}{4} = 1$$

Review Exercise 3

Cambridge O Level Additional Mathematics (4037) Paper 22 Q5, February/March 2019
Cambridge IGCSE Additional Mathematics (0606) Paper 22 Q5, February/March 2019

Chapter 7

1 **Solutions to this question by accurate drawing will not be accepted.**
The points $A(3, 2)$, $B(7, -4)$, $C(2, -3)$ and $D(K, 3)$ are such that CD is perpendicular to AB. Find the equation of the perpendicular bisector of CD. **[6]**

Chapter 7

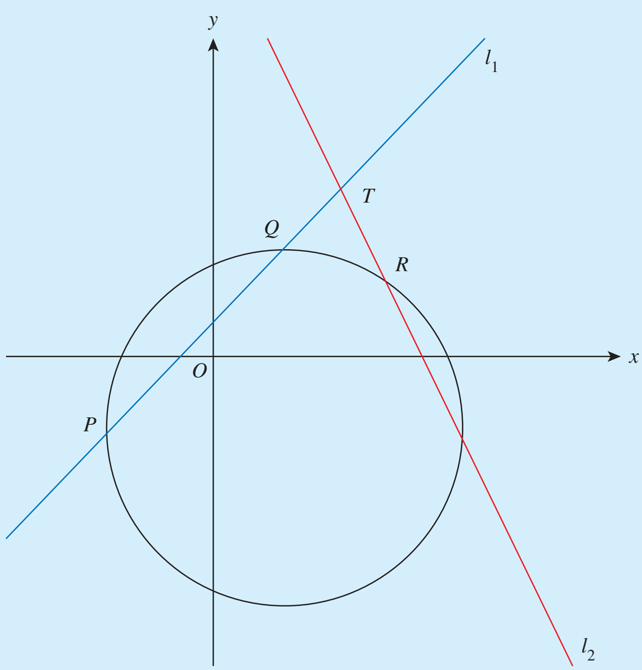
2 It is thought that the relationship $y = ax^n$, where a and n are constants, connects the variables x and y. An experiment was carried out recording the values of y for certain values of x.

a) Transform the relationship $y = ax^n$ into straight line form. **[2]**
The values of ln x and ln y were plotted and a line of best fit was drawn. It is given that the line of best fit passes through the points with coordinates (1.35, 4.81) and (5.55, 2.29).

b) Find an estimate of the values of the constants a and n. **[4]**

Chapter 2
Chapter 5
Chapter 7
Chapter 8

3 The diagram shows the circle $x^2 + y^2 - 4x + 4y - 17 = 0$ and the lines l^1, $y = x + 1$, and l^2. The line l^1 intersects the circle at points P and Q and the line l^2 intersects the circle at points $R(5, 2)$ and $S(7, -2)$. The lines intersect at point T.



a Find the coordinates of the point of intersection of l^1 and l^2 **[5]**
b Give the coordinates of the points P and Q. **[4]**
c Find the area of the triangle PST. **[2]**

Chapter 8

4 Two circles with equations $x^2 + y^2 + 6x - 8y + 9 =$ and $x^2 + y^2 - 2x - 15 = 0$ intersect at points A and B.

a Find the coordinates of the points A and B. **[4]**
b State the equation of the line that passes through the points A and B. **[1]**

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