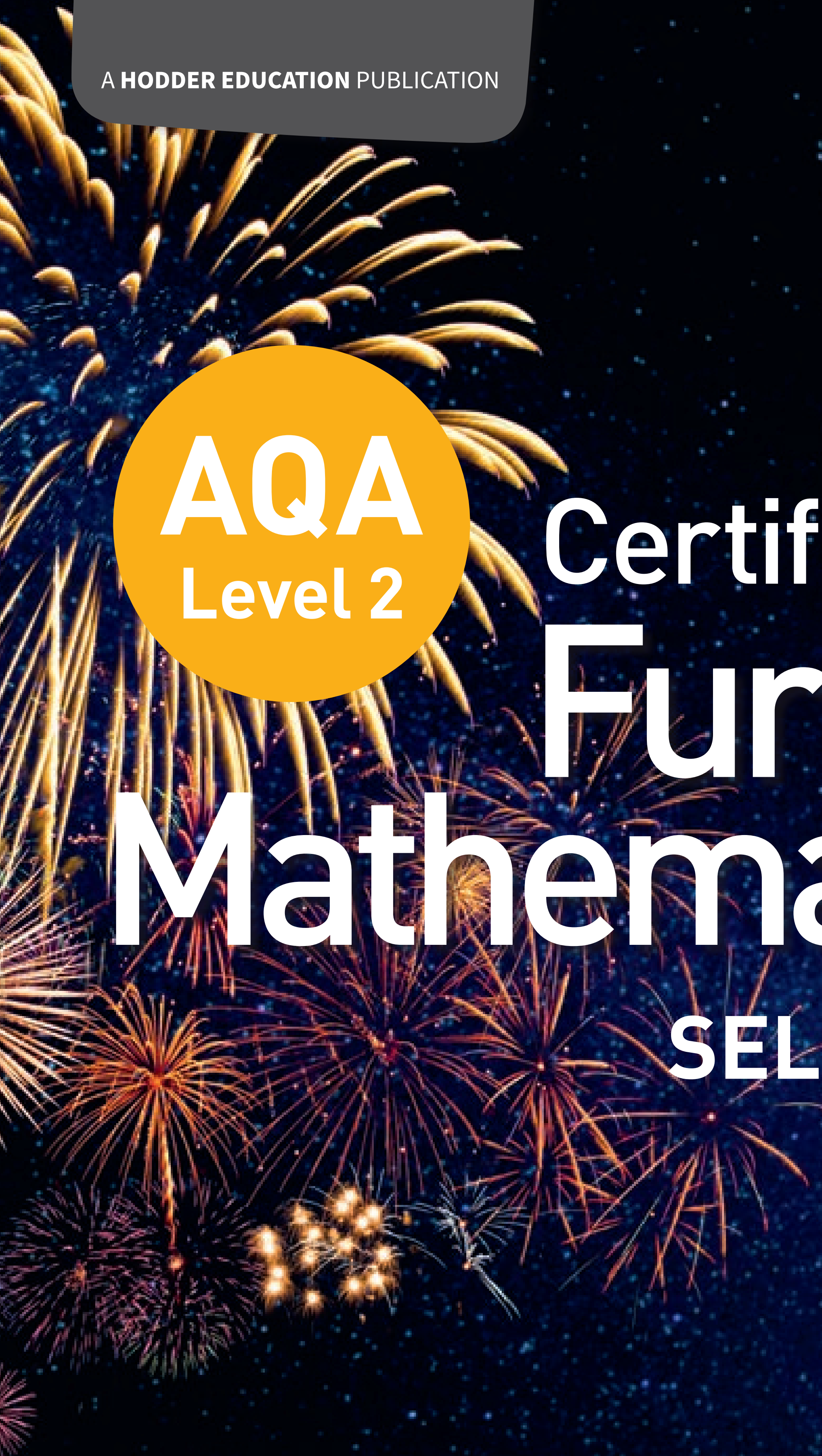
A HODDER EDUCATION PUBLICATION

AQA Approved



AQA Level 2 Certificate in Further Mathematics

SELF-STUDY

Author
Andrew Ginty

hachette LEARNING

Contents

Introduction v

Section 1 Level 2 Further Mathematics

1 Limiting value of a sequence 01
1.1 Limiting value 01

2 Binomial expansion 04
2.1 Pascal's triangle 04

3 Simultaneous equations 08
3.1 Simultaneous equations in three unknowns 08

4 Lines and circles 13
4.1 Pythagorean triples 13
4.2 Dividing a line by a given ratio 14
4.3 Equation of a circle 15

5 Factorising 18
5.1 Common factors 18
5.2 Quadratic expressions involving two letters 19
5.3 Difference of two squares 19
5.4 Factor theorem 20

6 Trigonometry 23
6.1 Trigonometric equations 23
6.2 The ambiguous case of the sine rule 25
6.3 Trigonometric identities 26
6.4 Disguised quadratic equations 27

7 Exponential graphs and functions 29
7.1 Exponential graphs 29
7.2 Domain of a function 32
7.3 Range of a function 32
7.4 Multi-part functions 35

8 Matrices 38
8.1 Introduction to matrices 38
8.2 Multiplying a matrix by a scalar 39
8.3 Multiplying two or more matrices 39
8.4 The identity matrix 41
8.5 Transformations 42
8.6 Special transformations 43
8.7 Forming a transformation matrix 43
8.8 Combining transformations 45

9 Calculus 48
9.1 Differentiation 48
9.2 Tangents and normal 51
9.3 Increasing and decreasing functions 52
9.4 Second derivative 53
9.5 Stationary points 54
9.6 Sketching graphs 57
9.7 Maxima and minima problems 58

Section 2 Beyond Level 2 Further Mathematics

10 Extension: Limiting value of a sequence 61
10.1 Proof of $\frac{an+c}{bn+d} \to \frac{a}{b}$ 61
10.2 Rational functions 61
10.3 Euler's number 63

11 Extension: Binomial expansion 64
11.1 Factorial function 64
11.2 Permutations and combinations 66
11.3 Pascal's triangle 68
11.4 Binomial distribution 74
11.5 Trinomial expansions 76
11.6 'Anagrams' 77

12 Extension: Simultaneous equations 79
12.1 Substitution 79
12.2 Gaussian elimination 81
12.3 No solution or multiple solutions 88

Contents

13 Extension: Lines and circles 93

13.1 Pythagorean triples 93
13.2 Proof of Pythagoras' theorem 94
13.3 Fermat's Last Theorem 95
13.4 Proof of the line division formula 96
13.5 Proof of the circle equation 97

14 Extension: Factorising 98

14.1 Long division 98
14.2 Grid method 1 102
14.3 Grid method 2 104
14.4 Remainder theorem 105
14.5 Useful factorisations 107

15 Extension: Trigonometry 108

15.1 Proof of the sine rule 108
15.2 Proof of the cosine rule 108
15.3 Area of a triangle 109
15.4 Extension of the sine rule 109
15.5 Radians 111
15.6 Arc-length and sector area 112

16 Extension: Exponential graphs and functions 114

16.1 Set notation 114
16.2 Complex numbers 114
16.3 The exponential function with base e 117

17 Extension: Matrices 120

17.1 Adding and subtracting matrices 120
17.2 Multiplying matrices 121
17.3 Inverse matrices 122
17.4 Determination of a matrix 124
17.5 'Dividing' matrices 125
17.6 Transpose of a matrix 126
17.7 Determinant of a 3×3 matrix 127
17.8 3×3 inverse matrix 129
17.9 Simultaneous equations 130

18 Extension: Calculus 134

18.1 Non-integer indices 134
18.2 Differentiation from first principles 134
18.3 Gradient of $y = x^n$ 137
18.4 Integration 138
18.5 Definite integrals 140

Mixed Questions *142*

Common command words used in questions *148*

Answers *149*

Introduction

This resource has been written to support students who do not have access to Further Mathematics lessons.

Chapters 1 to 9 cover all the Further Mathematics topics that are not covered in the GCSE Mathematics specification. Chapters 10 to 18 are not assessed in the Level 2 Certificate in Further Mathematics, and they are for interest only.

Complete Chapter 1 before attempting this chapter

These latter chapters include proofs of the rules learned in Chapters 1 to 9, and extension material that learners may find interesting. Students who go on to study mathematics at A-Level or beyond will benefit from reading Chapters 10 to 18. Each of these chapters can be accessed once the corresponding chapter from 1 to 9 has been completed.

Different schools structure the teaching of GCSE Mathematics in different ways. For this reason, Chapters 1 to 9 are designed to be independent of each other so they can be studied in any order. If you have already completed GCSE Maths, you can access any of the chapters. If you have not yet completed GCSE Maths, then consider each of the diagnostic tests. If you score 100% on a test, then you can access the corresponding chapter. If you cannot complete a diagnostic test, then wait until you have completed the necessary topic(s) in your GCSE Maths lessons.

Each chapter is divided into short learning sections. Each section introduces the knowledge required, followed by worked examples:

Example 1.3

The nth term of a sequence is A_n where $A_n = \dfrac{5+2n^2}{n^3-6n+8}$

Write down the limiting value of A_n as $n \to \infty$

Solution

The expression could be rewritten as $\dfrac{0n^3+2n^2+5}{1n^3-6n+8}$

The highest power of n is 3

So the limiting value is $\dfrac{0n^3}{1n^3} = 0$

Exercises are available for students to test their understanding of each section, with questions progressively increasing in difficulty.

The key points are summarised at the end of each section:

KEY POINTS

- Each term in the expansion $(a+b)^n$ is of the form Pa^qb^r
- P is from the nth row of Pascal's triangle. Learn the 3rd row of Pascal's triangle and then find the others by adding adjacent numbers.
- Note: You are not expected to know Pascal's triangle beyond the 5th row.
- The sum of the indices q and r is always n

Regular notes are included to provide extra explanation, exam advice and points of interest:

Example 6.2

Note

This question could be on a non-calculator paper as you are expected to know that sin 30 is 0.5

Solve $4\sin x = 2$ for $0° \leqslant x \leqslant 360°$

Solution

Rearrange the equation: $\sin x = \frac{2}{4} = 0.5$

Step 1 $\sin^{-1}(0.5) = 30$

Step 2 $180 - 30 = 150$

$180 - \alpha$ is used because the previous step was $\sin^{-1}$

Step 3 There are other solutions, but they are either less than 0 or greater than 360

So the answer is $x = 30°,\ 150°$

PS This indicates a problem-solving question. These questions will sometimes involve more than one topic area.

At the end of this resource, you will find a mixed exercise with questions to help you practise the skills learned in Chapters 1 to 9.

1 Limiting value of a sequence

DIAGNOSTIC TEST

The online diagnostic test for this topic is at this link:

When you score 100% on the diagnostic test, you will be ready to complete this chapter.

Objective

In this section, you will learn how to:

➜ find the limiting value of a sequence.

1 Limiting value

As some sequences progress, the terms gradually approach a particular value, called a **limit** or **limiting value**, but never reach it.

In this section you will learn how to find the limit of such a sequence.

For example, the sequence given by the formula $n\text{th term} = \frac{n+5}{2n-1}$

has the terms $6, \frac{7}{3}, \frac{8}{5}, \frac{9}{7}, \frac{10}{9}, 1, \frac{12}{13}, \frac{13}{15}, \frac{14}{17}, \frac{15}{19}, \ldots$

Consider the nth term as n gets very large:

$$100\text{th term} = \frac{105}{199} = 0.527\,638\,19 \text{ (8 d.p.)}$$

$$1000\text{th term} = \frac{1005}{1999} = 0.502\,751\,38 \text{ (8 d.p.)}$$

$$1\,000\,000\text{th term} = \frac{1\,000\,005}{1\,999\,999} = 0.500\,002\,75 \text{(8 d.p.)}$$

Try some even larger values of n

As n increases, the nth term of the sequence gets closer to 0.5

As n approaches infinity, 0.5 is the limiting value of the sequence given by $n\text{th term} = \frac{n+5}{2n-1}$

This can be written as

$$\frac{n+5}{2n-1} \to 0.5 \quad \text{as} \quad n \to \infty$$

We often use an arrow to replace the word 'approaches'.

Note

$n \to \infty$ is an abbreviation for 'n approaches infinity'

It would be tedious (and unreliable) to substitute very large numbers into an nth term formula in order to work out the limiting value of the sequence.

KEY POINT

- If the nth term of a sequence is in the form of a fraction, the limiting value can be calculated by dividing the coefficients of the highest power of n
  All other coefficients can be ignored.

Extension

For an explanation of this rule, see the EXTENSION resources.

Example 1.1

Write down the limiting value of $\frac{2n-3}{7n+1}$ as $n \to \infty$

Solution

The highest power of n is 1, so the limiting value is $\frac{2n^1}{7n^1} = \frac{2}{7}$

Example 1.2

The nth term of a sequence is $\frac{7-8n^2}{5n^2+n-2}$

Write down the limiting value of the sequence as $n \to \infty$

Solution

The highest power of n is 2, so the limiting value is $\frac{-8n^2}{5n^2} = -\frac{8}{5}$ (or -1.6)

Example 1.3

The nth term of a sequence is A_n where $A_n = \frac{5+2n^2}{n^3-6n+8}$

Write down the limiting value of A_n as $n \to \infty$

Solution

The highest power of n is 3

The expression could be rewritten as $\frac{0n^3+2n^2+5}{1n^3-6n+8}$

So the limiting value is $\frac{0n^3}{1n^3} = 0$

Exercise 1A

① The nth term of a sequence is $\frac{4n-1}{3n-5}$

(a) Find the 5th term of the sequence.

(b) Write down the limiting value of the sequence as $n \to \infty$

② A sequence is given by $u_n = \frac{3n+8}{2+5n}$ where u_n is the nth term.

(a) Given that $u_k = 1$, find the value of k

(b) Write down the limiting value of the sequence as $n \to \infty$

③ The nth term of a sequence is $\frac{n^2-3n+5}{3n^2+n-7}$

(a) Find the 1st, 2nd and 3rd terms of the sequence.

(b) Write down its limiting value as $n \to \infty$

④ The nth term of a sequence is $\frac{3+2n^4+7n^2}{8+5n-3n^4}$

Write down its limiting value as $n \to \infty$

⑤ A sequence is given by $u_n = \frac{2-n^3}{7n^4+n^2}$ where u_n is the nth term.

(a) Find the values of u_1 and u_2

(b) Write down the limiting value of u_n as $n \to \infty$

PS ⑥ $t_n = \frac{2n+1}{5n-4} + \frac{n+8}{3-7n}$

Calculate the limiting value of t_n as $n \to \infty$

PS ⑦ A sequence is given by nth term $= \frac{an^2+5n-1}{4n^2-3n+c}$ where a and c are constants.

The 1st term of the sequence is $\frac{2}{3}$ and the limiting value as $n \to \infty$ is $\frac{1}{2}$

Calculate the sum of the first two terms of the sequence.

PS ⑧ The nth terms of two different sequences are given by

$s_n = \frac{an-1}{3n+5}$ and $t_n = \frac{bn-3}{6n-2}$ where a and b are constants.

As $n \to \infty$ the two sequences have the same limiting value.

Also, $t_1 = 2s_1$

Calculate the value of t_2

2 Binomial expansion

DIAGNOSTIC TEST

The online diagnostic test for this topic is at this link:

When you score 100% on the diagnostic test, you will be ready to complete this chapter.

Objectives

In this section, you will learn about:

- expansions of $(a + b)^n$
- individual terms in the expansion of $(a + b)^n$

1 Pascal's triangle

Note

Check these for yourself by expanding $(a + b)(a + b)$

Then multiply by $(a + b)$ to find $(a + b)^3$

Continue by multiplying each expansion by $(a + b)$

Consider the first few simplified expansions of $(a + b)^n$ where n is a non-negative integer:

$(a + b)^0 = 1$

$(a + b)^1 = a + b$

$(a + b)^2 = a^2 + 2ab + b^2$

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

You need to remember the above expansions.

This may seem a daunting task, but it is easy once you spot the patterns.

$(a + b)^0 = 1a^0b^0$

$(a + b)^1 = 1a^1b^0 + 1a^0b^1$

$(a + b)^2 = 1a^2b^0 + 2a^1b^1 + 1a^0b^2$

$(a + b)^3 = 1a^3b^0 + 3a^2b^1 + 3a^1b^2 + 1a^0b^3$

$(a + b)^4 = 1a^4b^0 + 4a^3b^1 + 6a^2b^2 + 4a^1b^3 + 1a^0b^4$

$(a + b)^5 = 1a^5b^0 + 5a^4b^1 + 10a^3b^2 + 10a^2b^3 + 5a^1b^4 + 1a^0b^5$

Each term in the expansion of $(a + b)^n$ is in the form

$$Pa^qb^r$$

where q and r are integers such that $q + r = n$

In each expansion, reading from left to right, the powers of a decrease while the powers of b increase.

The coefficients, P, form **Pascal's triangle**:

					1					
				1		1				
			1		2		1			
		1		3		3		1		
	1		4		6		4		1	
1		5		10		10		5		1

The top line of Pascal's triangle is referred to as the 0th row.

This line is the 4th row of Pascal's triangle.

The first and last numbers in each row are always 1

The other numbers in each row are the sum of two numbers from the row above. For example, the 10 in the 5th row is the sum of 4 and 6 in the 4th row:

$$\begin{matrix} 4 & & 6 \\ & \searrow\ \swarrow & \\ & 10 & \end{matrix}$$

Hint: Learn the 3rd row of Pascal's triangle. Find the subsequent rows by adding adjacent numbers of the previous row. Remember that the second half of each row is the same as the first half reversed.

Example 2.1

(a) Write down the simplified expansion of $(a + b)^4$

(b) Hence write down the simplified expansion of $(1 + x)^4$

Note

The instruction 'write down' suggests that very little work is required. However, you may show working lines if you prefer.

Solution

(a) $$(a+b)^4 = 1a^4b^0 + 4a^3b^1 + 6a^2b^2 + 4a^1b^3 + 1a^0b^4$$
$$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

Consider the three parts of each term separately:

1	a^4	b^0
4	a^3	b^1
6	a^2	b^2
4	a^1	b^3
1	a^0	b^4

(b) In the expansion of $(a + b)^4$ replace a with 1 and b with x

$$(1+x)^4 = 1^4 + 4\times 1^3 \times x + 6\times 1^2 \times x^2 + 4\times 1\times x^3 + x^4$$
$$= 1 + 4x + 6x^2 + 4x^3 + x^4$$

Example 2.2

Expand and simplify $(3p-2)^5$

Note

$(3p-2)^5$ is expanded as $(3p+(-2))^5$

Solution

Calculate each term separately:

$$\begin{aligned}
1\ (3p)^5(-2)^0 &= 1 \times 243p^5 \times 1 = 243p^5\\
5\ (3p)^4(-2)^1 &= 5 \times 81p^4 \times -2 = -810p^4\\
10\ (3p)^3(-2)^2 &= 10 \times 27p^3 \times 4 = 1080p^3\\
10\ (3p)^2(-2)^3 &= 10 \times 9p^2 \times -8 = -720p^2\\
5\ (3p)^1(-2)^4 &= 5 \times 3p \times 16 = 240p\\
1\ (3p)^0(-2)^5 &= 1 \times 1 \times -32 = -32
\end{aligned}$$

So $(3p-2)^5 = 243p^5 - 810p^4 + 1080p^3 - 720p^2 + 240p - 32$

Example 2.3

Find the coefficient of x^4 in the expansion of $(2x-7)^6$

Solution

Each term in the expansion is of the form $P(2x)^q(-7)^r$

Step 1: Identify the power of $2x$

The question has specified the x^4 term, so $(2x)^4$ is required.

Step 2: Identify the power of -7

The sum of the powers in each term is 6, so $(-7)^2$ is required. [$6-4=2$]

Step 3: Identify the binomial coefficient.

The 5th row of Pascal's triangle is 1, 5, 10, 10, 5, 1

So the 6th row is 1, 6, 15, 20, 15, 6, 1

The 1st number of any row is 1
The 2nd number of the 6th row is 6
The 3rd number is $5+10$ (from the 5th row)
The 4th number is $10+10$ (from the 5th row)
The last three are the same as the first three reversed.

Find the coefficient linked with x^4

1	6	15	20	(15)	6	1
x^0	x^1	x^2	x^3	x^4	x^5	x^6

The x^4 term is $15(2x)^4(-7)^2$

$$= 15 \times 16x^4 \times 49$$

$$= 11760x^4$$

The coefficient of x^4 is 11 760

KEY POINTS

- Each term in the expansion of $(a + b)^n$ is of the form $Pa^q b^r$
- P is from the nth row of Pascal's triangle. Learn the 3rd row of Pascal's triangle and then find the others by adding adjacent numbers.
- Note: You are not expected to know Pascal's triangle beyond the 5th row.
- The sum of the indices q and r is always n

Extension

For more information on Pascal's triangle and how to use it, see the EXTENSION resources.

Exercise 2A

① (a) Write down the simplified expansion of $(a + b)^3$

(b) Hence write down the simplified expansion of $(x + 1)^3$

② Expand and simplify $(x + 2)^4$

③ Expand and simplify $(2m - 1)^5$

④ Expand and simplify $(5p + 2q)^3$

⑤ Find the coefficient of x^3 in the expansion of $(3x - 5)^4$

⑥ Simplify $(n + 3)^4 - n(n - 1)^3$

PS ⑦ In the expansion of $(ax + 3)^5$ the coefficient of x^2 is 2430

Find the possible values of a

PS ⑧ In the expansion of $(a + 2x)^5$ the coefficient of x^2 is double the coefficient of x^4

Given that a is positive, find its value.

PS ⑨ Find the coefficient of x^2y^4 in the expansion of $(3x - 5y)^6$

PS ⑩ Given that n is a positive integer, show that $(n + 3)^3 - n^3$ is a multiple of 9

3 Simultaneous equations

DIAGNOSTIC TEST

The online diagnostic test for this topic is at this link:

When you score 100% on the diagnostic test, you will be ready to complete this chapter.

Objective

In this section, you will learn how to:

→ solve simultaneous equations in three unknowns.

1 Simultaneous equations in three unknowns

In your GCSE Maths studies you will have learned how to solve simultaneous equation in two unknowns.

In this section you will learn how to solve simultaneous equations in three unknowns.

Follow these steps:
Step 1 Eliminate one of the unknowns by combining a pair of equations.
Step 2 Eliminate the same unknown by combining another pair of equations.
Step 3 Solve the resulting pair of simultaneous equations in two unknowns.
Step 4 Substitute the answers from Step 3 into one of the original equations to find the value of the third unknown.

Extension

For alternative methods see the EXTENSION resources.

Example 3.1

Solve these simultaneous equations:

$2x + 3y + z = 8$

$3x - y + z = 21$

$x + 4y + z = 1$

Note

In this example, the coefficients of z are all the same, so z can be eliminated by subtracting pairs of equations.

Solution

Subtracting the first two equations gives an equation in x and y

$$(2x + 3y + z) - (3x - y + z) = 8 - 21$$
$$\Rightarrow \quad -x + 4y = -13$$

Subtracting the second and third equations gives another equation in x and y

$$(3x - y + z) - (x + 4y + z) = 21 - 1$$
$$\Rightarrow \quad 2x - 5y = 20$$

You can then solve this pair of simultaneous equations in two unknowns:

$$-x + 4y = -13 \quad ①$$
$$2x - 5y = 20 \quad ②$$

$$① \times 2: \quad -2x + 8y = -26 \quad ③$$
$$② \times 1: \quad 2x - 5y = 20 \quad ④$$

$$③ + ④: \quad 8y + (-5y) = -26 + 20$$
$$\Rightarrow \quad 3y = -6$$
$$\Rightarrow \quad y = -2$$

Substituting $y = -2$ into equation ① gives $-x + 4 \times (-2) = -13$

$$\Rightarrow \quad -x - 8 = -13$$
$$\Rightarrow \quad -x = -5$$
$$\Rightarrow \quad x = 5$$

Finally, substitute $x = 5$ and $y = -2$ into $2x + 3y + z = 8$

$$2 \times 5 + 3 \times (-2) + z = 8$$
$$\Rightarrow \quad 4 + z = 8$$
$$\Rightarrow \quad z = 4$$

The final solution is $x = 5$, $y = -2$, $z = 4$

Note

Always check your answer by substituting it back into the original equations.

Exercise 3A

① Solve these sets of simultaneous equations by first eliminating x

(a) $x + 2y + 3z = 17$
$x + y + 2z = 12$
$x + 3y + z = 10$

(b) $3x + 2y + z = 10$
$3x + y + 3z = 16$
$3x + 4y + 2z = 23$

② Solve these sets of simultaneous equations by first eliminating y

(a) $x + 2y + 3z = 12$
$2x + 2y - z = 2$
$3x + 2y - 2z = 4$

(b) $2x - 3y + z = -1$
$3x + 3y - 2z = 3$
$4x + 3y - z = 7$

③ Solve these simultaneous equations.

(a) $2x + 3y + z = -6$
$x - 2y - z = -3$
$3x - y - z = -10$

(b) $x - 4y + 2z = 3$
$2x + 4y - z = -7$
$5x - 4y + 3z = -4$

In Example 3.1, the coefficients of the z term were all the same, making the initial step easy.

In the next example, the three equations do not have any terms with matching coefficients. In this case it is important to set out your work clearly and systematically.

Example 3.2

Solve these simultaneous equations:

$5x + y - 4z = -13$

$3x - 2y + 2z = 15$

$2x + 3y - 5z = -24$

Note

Label the equations. This will help you, and an examiner, to follow your work.

Solution

$$5x + y - 4z = -13 \quad ①$$
$$3x - 2y + 2z = 15 \quad ②$$
$$2x + 3y - 5z = -24 \quad ③$$

①×2: $10x + 2y - 8z = -26$ ④
②×1: $3x - 2y + 2z = 15$ ⑤
④+⑤: $13x - 6z = -11$ ⑥

①×3: $15x + 3y - 12z = -39$ ⑦
③×1: $2x + 3y - 5z = -24$ ⑧
⑦−⑧: $13x - 7z = -15$ ⑨

Note

Be careful when combining negative numbers. It is safer to include an extra step in your work rather than risk making a mistake.

⑥ − ⑨: $-6z - (-7z) = -11 - (-15) \Rightarrow z = 4$

Substitute $z = 4$ into ⑥: $13x - 6 \times 4 = -11 \Rightarrow 13x = 13 \Rightarrow x = 1$

Substitute $x = 1$ and $z = 4$ into ①: $5 \times 1 + y - 4 \times 4 = -13 \Rightarrow y = -2$

The solution is $x = 1$, $y = -2$, $z = 4$

Note

In the above example, y was eliminated first, but any letter could have been chosen. The equations could be combined in several different ways to eliminate a letter. Try to solve the example by first eliminating x or z instead.

Example 3.3

Find the nth term of the quadratic sequence 2, 2, 4, 8, 14, …

Solution

There are various methods for finding the nth term of a quadratic sequence. The method shown here uses three simultaneous equations in three unknowns.

Let nth term $= an^2 + bn + c$

The question states that the sequence is quadratic. So, we can assume the nth term is of the form $an^2 + bn + c$ where a, b and c are constants to be found.

AQA Level 2

Certificate in Further Mathematics

SELF-STUDY

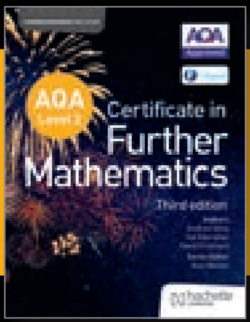
This title has been approved by AQA

This eBook is designed for students who are studying the course independently, or who require additional stretch beyond that supplied by the textbook. Focusing on the topics that go beyond GCSE, it is the perfect companion for self-study. Online diagnostic tests for each topic help students to target their studies, while comprehensive extension chapters provide ideal preparation for A level Maths and Further Maths.

- Build your understanding of mathematics with discussion points, thought-provoking activities and rigorous exercise questions.
- Develop your problem-solving skills and learn to use mathematical arguments with step-by-step worked examples.
- Check your understanding and target your study with an online diagnostic test at the start of each chapter.
- Focus on more advanced maths and prepare for A level with extension content.
- Can be used alongside the textbook and practice book

Author
Andrew Ginty has taught mathematics at Key Stages 3, 4 & 5 for over 30 years. He has extensive experience as an examiner and lead examiner at GCSE and A level.

Our AQA Level 2 Certificate in Further Mathematics textbook covers the entire course and is perfect for classroom use.



Visit us at hachettelearning.com

ISBN 978-1-0360-1166-6