

**BGE S1-S3**



# Mathematics & Numeracy

**Fourth Level  
bridging to National 5**

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# Introduction to BGE Mathematics

Mathematics is the richest language in the world. Learning mathematics is one of the most important things you can do to boost your brain power, now and throughout your life. Mathematicians understand, describe and influence the world around us.

This book covers all the BGE Benchmarks for Mathematics at Fourth Level. It will also extend you where appropriate in preparation for your transition to national qualifications.

The chapters take you on a journey which will both support and challenge you. As you focus and work hard on each section you will gain knowledge, understanding and the ability to solve problems and communicate your solutions to others.

Working through this book will equip you to be the best mathematician you can be. Every topic is explained with rigour, taking no short cuts, and has an abundance of practice and challenge to suit every learner. The book is ambitious, encouraging you to aim high and build the skills you need for future success.

The book is enjoyable and easy to read. Every topic includes a clear, 'straight-to-the-point' method set out using bullet points in concise and simple language. It is full of worked examples and helpful hints and contains proven methods of mastering difficult concepts. This book has been designed both as a classroom aid and for your personal study.

Each lesson starts with an explanation and memorable method to which you can refer.

Each lesson has planned and progressive worked examples which prepare you to tackle the problem set.

## 11 Further algebra

### Expanding double brackets

- To expand double brackets, reduce the problem to the sum of single brackets.
- Split up the first bracket and then multiply everything in the second bracket by each term from the first bracket (see working in red in the Worked examples below).
  - Expand the resulting single brackets and simplify by gathering like terms.

Worked examples		Expand and simplify:	
1	$(x+3)(x+4)$	2	$(x+5)(x-1)$
$= x(x+4) + 3(x+4)$		$= x(x-1) + 5(x-1)$	
$= x^2 + 4x + 3x + 12$		$= x^2 - x + 5x - 5$	
$= x^2 + 7x + 12$		$= x^2 + 4x - 5$	
4	$(k-10)(k-10)$	5	$(5y-3)(9y+7)$
$= (k-10)(k-10)$		$= 5y(9y+7) - 3(9y+7)$	
$= k^2 - 10k - 10k + 100$		$= 45y^2 + 35y - 27y - 21$	
$= k^2 - 20k + 100$		$= 45y^2 + 8y - 21$	
3	$(y-7)(y+7)$	6	$(3x-4)(3x-4)$
$= y(y+7) - 7(y+7)$		$= (3x-4)(3x-4)$	
$= y^2 + 7y - 7y - 49$		$= 3x(3x-4) - 4(3x-4)$	
$= y^2 - 49$		$= 9x^2 - 12x - 12x + 16$	
		$= 9x^2 - 24x + 16$	

With practice, you should be able to go straight to the penultimate line and then simplify to your final answer. Some learners like the visual aid of adding lines to show the expansion order, for example  $2x$  multiplying the second bracket (blue lines) and then  $+5$  multiplying the second bracket (red lines).

$$(2x+5)(x+1) = 2x^2 + 2x + 5x + 5$$

#### Exercise 11A

- Expand and simplify:  
a)  $(x+2)(x+3)$  b)  $(x+6)(x+2)$  c)  $(y+7)(y+1)$  d)  $(y+9)^2$   
e)  $(x+4)(x-2)$  f)  $(x+5)(x-4)$  g)  $(y+6)(y-7)$  h)  $(y+2)(y-8)$
- Expand and simplify:  
a)  $(x-8)(x+1)$  b)  $(x-7)(x+4)$  c)  $(y-1)(y+9)$  d)  $(y-3)(y+3)$   
e)  $(x-2)(x-9)$  f)  $(x-1)(x-9)$  g)  $(y-5)(y-8)$  h)  $(y-4)^2$   
i)  $(x-9)(x+2)$  j)  $(x-3)^2$  k)  $(y+1)(y-1)$  l)  $(y+6)(y+5)$
- Expand and simplify:  
a)  $(x+4)(2x+3)$  b)  $(x+2)(3x+9)$  c)  $(x+7)(4x-1)$  d)  $(x+6)(5x-2)$   
e)  $(y-8)(3x+5)$  f)  $(y-10)(2y+5)$  g)  $(m-3)(6m-1)$  h)  $(10h-7)^2$   
i)  $(7x+5)(3x+4)$  j)  $(9m+2)(2m+3)$  k)  $(4p+3)(5p-2)$  l)  $(8k+1)(k-6)$   
m)  $(3x-4)(2x+7)$  n)  $(5x-2)(3x-1)$  o)  $(7w-2)^2$  p)  $(3a-8)(4a-1)$

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Every chapter has a check-up exercise. Use these to bring your ideas together and boost your memory of key methods.

We hope that you enjoy using this book as much as we have enjoyed creating the content, questions and activities for you!

### Expanding double brackets

#### Worked examples

- $(3x+4)(2x^2+7x-8)$   
 $= 3x(2x^2+7x-8) + 4(2x^2+7x-8)$   
 $= 6x^3 + 21x^2 - 24x + 8x^2 + 28x - 32$   
 $= 6x^3 + 29x^2 + 4x - 32$
  - $(x+2)(x+3) + (5x+1)(x-4)$   
 $= x(x+3) + 2(x+3) + 5x(x-4) + 1(x-4)$   
 $= x^2 + 3x + 2x + 6 + 5x^2 - 20x - 4x - 4$   
 $= 6x^2 - 14x + 2$
- Notice the pattern this time:  $(3x+4)(2x^2+7x-8) = 6x^3 + 21x^2 - 24x + 8x^2 + 28x - 32$
- $(3x+5)(2x-7) - (4x-3)(x+6)$   
 $= 3x(2x-7) + 5(2x-7) - [4x(x+6) - 3(x+6)]$   
 $= 6x^2 - 21x + 10x - 35 - [4x^2 + 24x - 3x - 18]$   
 $= 6x^2 - 11x - 35 - [4x^2 + 21x - 18]$   
 $= 6x^2 - 11x - 35 - 4x^2 - 21x + 18$   
 $= 2x^2 - 32x - 17$

Be careful here! You MUST subtract all terms from the expansion of  $(4x-3)(x+6)$ . Introducing new brackets around this expansion will help. We have used square brackets to stand out from the round brackets everywhere else.

#### 4 Expand and simplify:

- $(x+5)(x^2+3x+1)$
- $(x+6)(x^2-5x+4)$
- $(x-4)(5x-2) - (x+1)(x+5)$
- $(x^2+7x-1)(5x-3)$
- $(x^2-2x-3)(7x-8)$
- $(x+9)(x^2+4x-2)$
- $(3x-1)(x^2+2x+3)$
- $(2x-4)(x^2-2x+8)$
- $(x^2-6x+9)(4x-5)$

#### 5 Expand and simplify:

- $(2x+1)(x+6) + (3x-1)(x+2)$
- $(2x-3)(x+4) + (3x-5)(x+1)$
- $(3x-7)(5x+3) + (2x-4)(x+9)$
- $(3x-1)(9x+2) + (2x-10)(4x-1)$
- $(x-4)(5x-2) - (x+1)(x+5)$
- $(2x-3)(x+6) - (x+4)(x-7)$
- $(3x+5)(2x-6) - (2x-1)(x+8)$
- $(7x+4)(x-2) - (5x+1)(3x-5)$

For each of the following shapes, find an expression without brackets for their area. All lengths are in centimetres.



7 The two rectangles below have the same area. All lengths are in centimetres.



- Obtain an expression without brackets for the area of each rectangle.
- Form an equation and solve it to find the dimensions of each rectangle.

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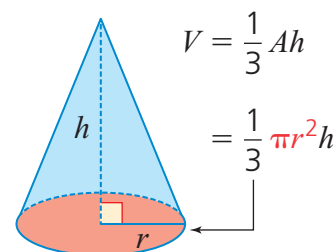
Work through the exercise to practise and consolidate your skills. There are solutions at the end of the book, so check your work as you go to boost your confidence.

Each exercise gets progressively harder. Try to finish the exercises and test your skills with the most challenging problems.

## ► Volume of a cone

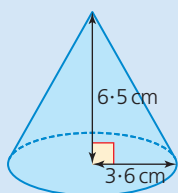
A cone is a circle-based pyramid. To find the volume of a cone:

- Clearly state the formula  $V = \frac{1}{3}\pi r^2 h$ .
- Substitute the correct values for the radius of the base  $r$  and the perpendicular height  $h$  of the cone.
- Calculate and include cubic units.



### Worked example

Calculate the volume of the cone. Give your answer correct to three significant figures.



$$\begin{aligned} V &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \times \pi \times 3.6^2 \times 6.5 \\ &= 88.21 \dots \\ &= 88.2 \text{ cm}^3 \text{ to 3 s.f.} \end{aligned}$$

Make use of the fraction button on your calculator.

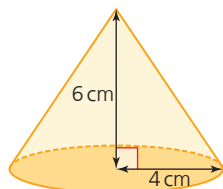
### Exercise 4M

Throughout this exercise give your answers correct to three significant figures.

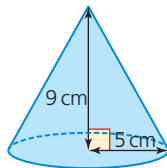


- 1 Calculate the volume of each cone.

a)



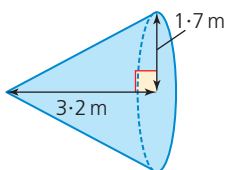
b)



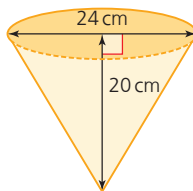
c)



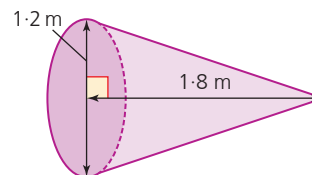
d)



e) Be careful.

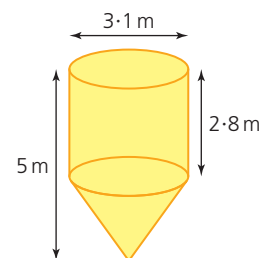


f)



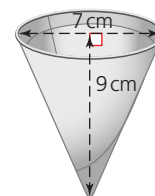
- 2 The diagram shows a silo for ready-mixed mortar on a building site.

- Find the volume of the cylinder to 2 decimal places.
- Find the height of the cone.
- Calculate the total volume of the silo.



- 3 A water station has a 5-litre bottle of water and small paper cone cups with the dimensions shown.

Two friends attempt to calculate the number of full cups that can be filled from the bottle. Both have made a big mistake and both are wrong. Read through each attempt and describe the mistakes.



a) Attempt 1:

$$V = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \times \pi \times 7^2 \times 9$$

$$= 461.81...$$

$$5 \text{ litre} = 5000 \text{ ml} = 5000 \text{ cm}^3$$

$$5000 \div 461.81$$

$$= 10.82$$

$$= 10 \text{ cups}$$

(can't round up to fill 11)

**X**

b) Attempt 2:

$$V = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \times \pi \times 3.5^2 \times 9$$

$$= 115.45...$$

$$= 100 \text{ cm}^3 \text{ to 1 s.f.}$$

$$5000 \div 100$$

$$= 50 \text{ cups}$$

**X**

c) Show that the correct answer is 43 full cups.

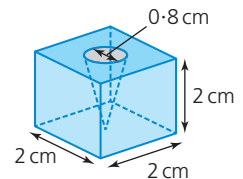
- 4 Dominic is making jelly for a party. He wants to fill 20 identical cone-shaped moulds each with radius 3 cm and height 5.5 cm. Will 1 litre of jelly be enough? Show working to explain your answer.

Remember 1 litre = 1000 cm<sup>3</sup>.

- 5 A plastic pencil sharpener is made from a cube with a cone removed.

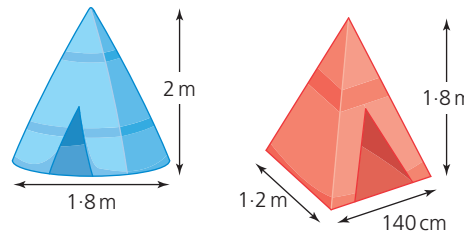
Calculate the volume available for the pencil sharpenings.

You may ignore the blade for sharpening the pencils.

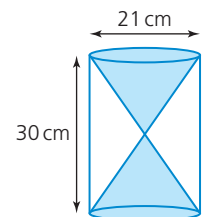


- 6 Two designs for a tent are shown. One is a cone and the other is a rectangle-based pyramid.

Calculate the difference between the volume of the tents.

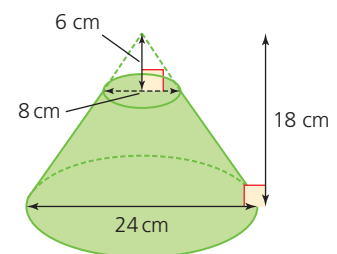


- 7 The structure of an ornament is shown. It is a cylinder enclosing two cones. Calculate the volume of the cylinder which is not occupied by the cones.



- 8 This diagram shows a truncated cone. A smaller cone has been removed from the top. Shapes like these, where a solid shape is cut with a line parallel to the base, are also called **frustrums**.

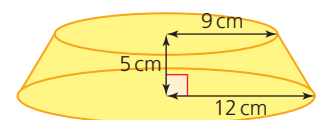
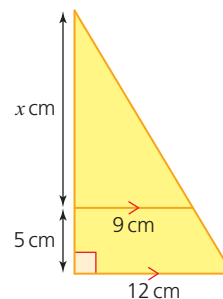
Calculate the volume of the frustrum.



- 9 Libbie forms the similar triangle shown to help find the volume of this frustrum.

a) Find the length marked  $x$  on the similar triangle.

b) Hence calculate the volume of the frustrum.



## ► Which formula?

You will not always be told which formula to use. Sometimes you must decide based on the information you have about a given triangle. We can use the acronym "SOH CAH TOA" to help us decide which formula to use and to help memorise the formulae themselves. Each part is an acronym of a trig ratio:

**SOH**

$$\sin x^\circ = \frac{\text{opposite}}{\text{hypotenuse}}$$

**CAH**

$$\cos x^\circ = \frac{\text{adjacent}}{\text{hypotenuse}}$$

**TOA**

$$\tan x^\circ = \frac{\text{opposite}}{\text{adjacent}}$$

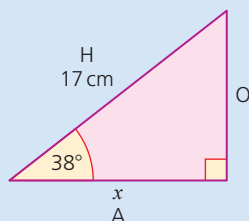
To decide which formula to use:

- Write out the acronym SOH CAH TOA.
- Tick off the sides you know or want to find.
- The part of the acronym that has two letters ticked tells you which trig ratio to use.

### Worked examples

In each triangle, calculate the size of the marked side or angle, giving your answers correct to three significant figures.

1



Write out the acronym.

We know the **hypotenuse** so we tick 'H'. We want the **adjacent** so we tick 'A'.

'CAH' has two letters ticked so we must use the cosine ratio.

S   O   **H**   C   **A**   **H**   T   O   **A**

$$\cos x^\circ = \frac{A}{H}$$

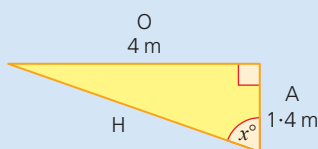
$$\cos 38^\circ = \frac{x}{17}$$

$$x = 17 \times \cos 38^\circ$$

$$= 13.39\dots$$

$$= 13.4 \text{ cm to 3 s.f.}$$

2



This time we are trying to find an angle. We know the **adjacent** and the **opposite** so we tick off 'A' and 'O'.

'TOA' has two letters ticked so we must use the tangent ratio.

S   **O**   H   C   **A**   H   T   **O**   **A**

$$\tan x^\circ = \frac{O}{A}$$

$$= \frac{1.4}{4}$$

$$x^\circ = \tan^{-1}\left(\frac{1.4}{4}\right)$$

$$= 70.70\dots$$

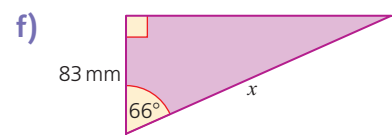
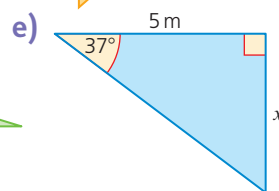
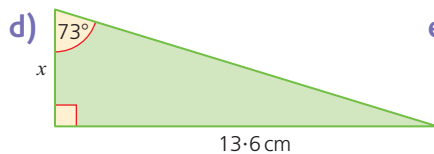
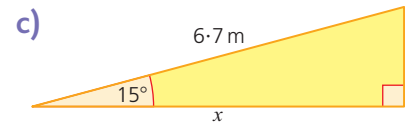
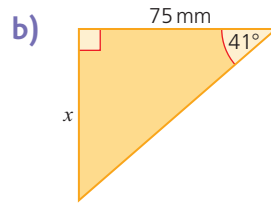
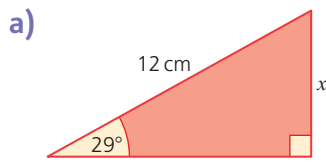
$$= 70.7^\circ \text{ to 3 s.f.}$$



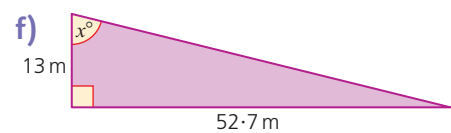
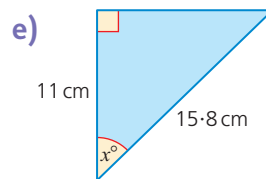
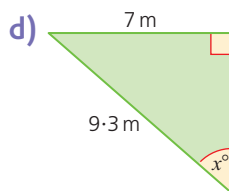
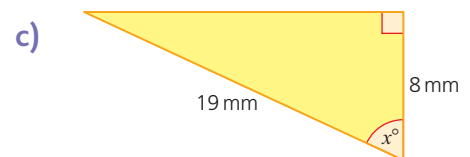
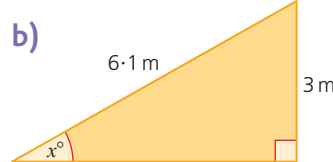
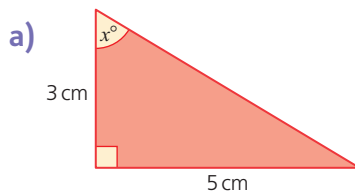
## Exercise 9N



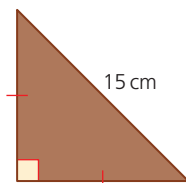
- 1 Calculate the lengths of the marked sides, giving your answers correct to three significant figures.



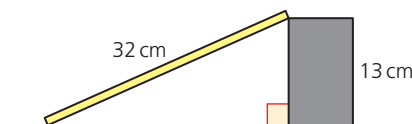
- 2 Calculate the sizes of the marked angles, giving your answers correct to three significant figures.



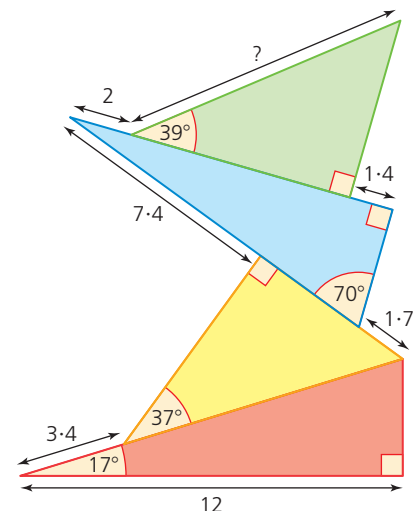
- 3 A chocolatier designs a chocolate slab in the shape of a right isosceles triangle. The longest edge of the slab is to be 15 cm. How long must the other two edges be?



- 4 Callum is building a ramp for his remote-controlled car. He uses a piece of plywood 32 cm long and props it up on a 13 cm high paperweight as shown. What is the angle of elevation of Callum's ramp?



- 5 Calculate the length of the marked side at the top of the stack of triangles. All lengths are in centimetres. Work to two decimal places throughout.



# 11 Further algebra

## ► Expanding double brackets

To expand double brackets, reduce the problem to the sum of single brackets.

- Split up the first bracket and then multiply everything in the second bracket by each term from the first bracket (see working in red in the Worked examples below).
- Expand the resulting single brackets and simplify by gathering like terms.

### Worked examples

Expand and simplify:

$$\begin{aligned} 1 \quad (x + 3)(x + 4) \\ &= x(x + 4) + 3(x + 4) \\ &= x^2 + 4x + 3x + 12 \\ &= x^2 + 7x + 12 \end{aligned}$$

$$\begin{aligned} 2 \quad (x + 5)(x - 1) \\ &= x(x - 1) + 5(x - 1) \\ &= x^2 - x + 5x - 5 \\ &= x^2 + 4x - 5 \end{aligned}$$

$$\begin{aligned} 3 \quad (y - 7)(y + 7) \\ &= y(y + 7) - 7(y + 7) \\ &= y^2 + 7y - 7y - 49 \\ &= y^2 - 49 \end{aligned}$$

$$\begin{aligned} 4 \quad (k - 10)^2 \\ &= (k - 10)(k - 10) \\ &= k(k - 10) - 10(k - 10) \\ &= k^2 - 10k - 10k + 100 \\ &= k^2 - 20k + 100 \end{aligned}$$

$$\begin{aligned} 5 \quad (5y - 3)(9y + 7) \\ &= 5y(9y + 7) - 3(9y + 7) \\ &= 45y^2 + 35y - 27y - 21 \\ &= 45y^2 + 8y - 21 \end{aligned}$$

$$\begin{aligned} 6 \quad (3x - 4)^2 \\ &= (3x - 4)(3x - 4) \\ &= 3x(3x - 4) - 4(3x - 4) \\ &= 9x^2 - 12x - 12x + 16 \\ &= 9x^2 - 24x + 16 \end{aligned}$$

With practice, you should be able to go straight to the penultimate line and then simplify to your final answer. Some learners like the visual aid of adding lines to show the expansion order, for example  $2x$  multiplying the second bracket (blue lines) and then  $+ 5$  multiplying the second bracket (red lines).

$$(2x + 5)(x + 1) = 2x^2 + 2x + 5x + 5$$

### Exercise 11A

1 Expand and simplify:

a)  $(x + 2)(x + 3)$

b)  $(x + 6)(x + 2)$

c)  $(y + 7)(y + 1)$

d)  $(y + 9)^2$

e)  $(x + 4)(x - 2)$

f)  $(x + 5)(x - 4)$

g)  $(y + 6)(y - 7)$

h)  $(y + 2)(y - 8)$

2 Expand and simplify:

a)  $(x - 8)(x + 1)$

b)  $(x - 7)(x + 4)$

c)  $(y - 1)(y + 9)$

d)  $(y - 3)(y + 3)$

e)  $(x - 2)(x - 4)$

f)  $(x - 1)(x - 9)$

g)  $(y - 5)(y - 8)$

h)  $(y - 4)^2$

i)  $(x - 9)(x + 2)$

j)  $(x - 3)^2$

k)  $(y + 1)(y - 1)$

l)  $(y + 6)(y + 5)$

3 Expand and simplify:

a)  $(x + 4)(2x + 3)$

b)  $(x + 2)(3x + 9)$

c)  $(x + 7)(4x - 1)$

d)  $(x + 6)(5x - 2)$

e)  $(y - 8)(3y + 5)$

f)  $(y - 10)(2y + 5)$

g)  $(m - 3)(6m - 1)$

h)  $(10h - 7)^2$

i)  $(7x + 5)(3x + 4)$

j)  $(9m + 2)(2m + 3)$

k)  $(4p + 3)(5p - 2)$

l)  $(8k + 1)(k - 6)$

m)  $(3x - 4)(2x + 7)$

n)  $(5x - 2)(9x - 1)$

o)  $(7w - 2)^2$

p)  $(3a - 8)(4a - 1)$

**Worked examples** Expand and simplify:

$$\begin{aligned}
 1 \quad & (3x + 4)(2x^2 + 7x - 8) \\
 &= 3x(2x^2 + 7x - 8) + 4(2x^2 + 7x - 8) \\
 &= 6x^3 + 21x^2 - 24x + 8x^2 + 28x - 32 \\
 &= 6x^3 + 29x^2 + 4x - 32
 \end{aligned}$$

$$\begin{aligned}
 2 \quad & (x + 2)(x + 3) + (5x + 1)(x - 4) \\
 &= x(x + 3) + 2(x + 3) + 5x(x - 4) + 1(x - 4) \\
 &= x^2 + 3x + 2x + 6 + 5x^2 - 20x + x - 4 \\
 &= 6x^2 - 14x + 2
 \end{aligned}$$

Notice the pattern this time:  $(3x + 4)(2x^2 + 7x - 8) = 6x^3 + 21x^2 - 24x + 8x^2 + 28x - 32$

$$\begin{aligned}
 3 \quad & (3x + 5)(2x - 7) - (4x - 3)(x + 6) \\
 &= 3x(2x - 7) + 5(2x - 7) - [4x(x + 6) - 3(x + 6)] \\
 &= 6x^2 - 21x + 10x - 35 - [4x^2 + 24x - 3x - 18] \\
 &= 6x^2 - 11x - 35 - [4x^2 + 21x - 18] \\
 &= 6x^2 - 11x - 35 - 4x^2 - 21x + 18 \\
 &= 2x^2 - 32x - 17
 \end{aligned}$$

**Be careful here!** You **MUST** subtract all terms from the expansion of  $(4x - 3)(x + 6)$ .

Introducing new brackets around this expansion will help. We have used square brackets to stand out from the round brackets everywhere else.

4 Expand and simplify:

a)  $(x + 5)(x^2 + 3x + 1)$

b)  $(x + 7)(x^2 + 2x + 3)$

c)  $(x + 9)(x^2 + 4x - 2)$

d)  $(x + 6)(x^2 - 5x + 4)$

e)  $(2x - 4)(x^2 - 2x + 8)$

f)  $(3x - 1)(x^2 + 9x + 2)$

g)  $(x^2 + 7x - 1)(5x - 3)$

h)  $(x^2 - 2x - 3)(7x - 8)$

i)  $(x^2 - 6x + 9)(4x - 5)$

5 Expand and simplify:

a)  $(2x + 1)(x + 6) + (3x - 1)(x + 2)$

b)  $(2x - 3)(x + 4) + (3x - 5)(x + 1)$

c)  $(3x - 7)(5x + 3) + (2x - 4)(x + 9)$

d)  $(3x - 1)(9x + 2) + (2x - 10)(4x - 1)$

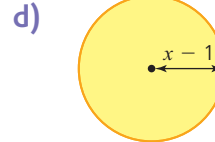
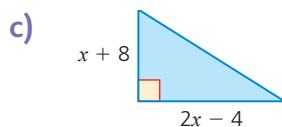
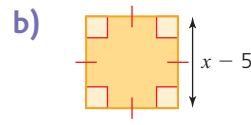
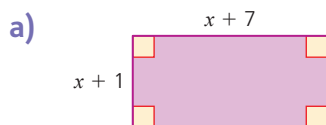
e)  $(x - 4)(5x - 2) - (x + 1)(x + 5)$

f)  $(2x - 3)(x + 6) - (x + 4)(x - 7)$

g)  $(3x + 5)(2x - 6) - (2x - 1)(x + 8)$

h)  $(7x + 4)(x - 2) - (5x + 1)(3x - 5)$

6 For each of the following shapes, find an expression without brackets for their area. All lengths are in centimetres.



7 The two rectangles below have the same area. All lengths are in centimetres.



a) Obtain an expression without brackets for the area of each rectangle.

b) Form an equation and solve it to find the dimensions of each rectangle.

## ► Scatter graph and line of best fit

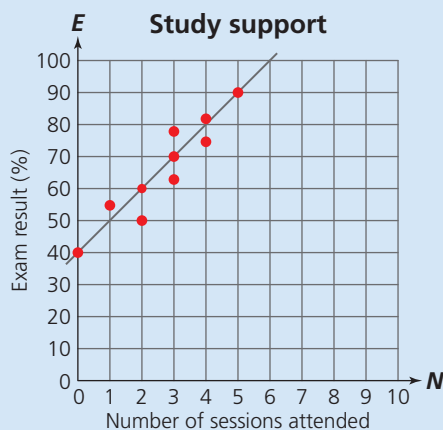
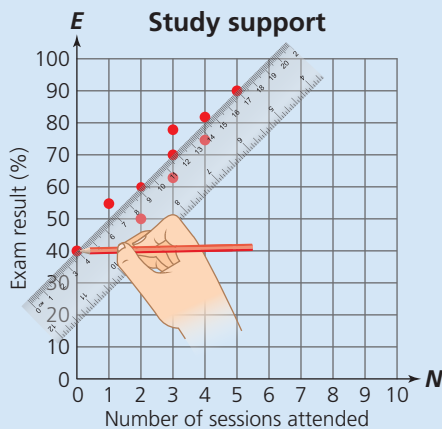
Scatter graphs display possible correlations between numerical data sets. We can add a **line of best fit**, which follows the trend of the points. The **equation of the line of best fit** gives us a formula for the connection between the data. This is called a **linear model**. To find the equation of the line of best fit:

- Use two points on the line of best fit to find its gradient.
- Use  $y = mx + c$  to find the equation of the line.
- Replace  $y$  and  $x$  with the correct variables for the context.

### Worked example

The scatter graph shows how many supported study sessions learners attended ( $N$ ) and their exam score ( $E$ ).

- Add a line of best fit to the scatter graph.
- Find the equation of the line in terms of  $E$  and  $N$ .
- Use your model to estimate the exam mark for someone who attended all six sessions.



- The line of best fit follows the trend of the points. Aim to have as many points close to the line as you can. Try to have roughly the same number of points above and below the line. Extend the line to the  $y$ -axis. It is easier to calculate the gradient if your line of best fit goes through two clear coordinate points.

$$\begin{aligned}
 \text{b) } (0, 40) \quad (2, 60) \quad m &= \frac{y_2 - y_1}{x_2 - x_1} \\
 (x_1, y_1) \quad (x_2, y_2) \quad &= \frac{60 - 40}{2 - 0} \\
 &= \frac{20}{2} \\
 &= 10
 \end{aligned}$$

The  $y$ -intercept is  $(0, 40)$ ,  $c = 40$

$$y = mx + c$$

$$y = 10x + 40$$

Now replace  $y$  and  $x$  with the correct variables for the context. Exam score  $E$  is on the  $y$ -axis, so replace  $y$  with  $E$ . Number of sessions  $N$  is on the  $x$ -axis, so replace  $x$  with  $N$ .

$$E = 10N + 40$$

- For six supported study sessions  $N = 6$ ,  $E = 10N + 40$ 

$$= 10 \times 6 + 40$$

$$= 100$$

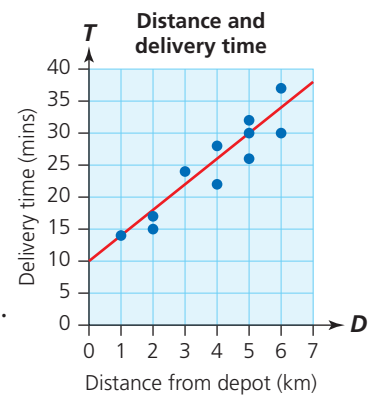
The model predicts an exam score of 100% for attending all six sessions.



## Exercise 13G

- 1 A courier company records the data shown. A line of best fit has been added to the graph for you.

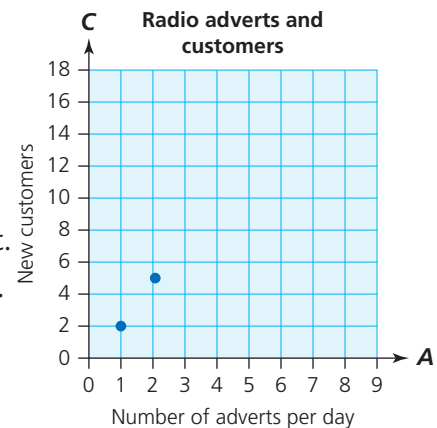
- The points (1, 14) and (5, 30) lie on the line of best fit. Use these points and the gradient formula to calculate the gradient  $m$ , of the line of best fit.
- Complete the coordinate of the  $y$ -intercept (0, ) and state the value of  $c$ .
- Using  $y = mx + c$ , write down the equation of the line of best fit.
- State the equation of the line of best fit in terms of  $T$  and  $D$ ,  $T = mD + c$ .
- Use your model to estimate the time to deliver a parcel 8km from the depot,  $D = 8$ .



- 2 A vegetable box delivery service pays for adverts on local radio. They run different numbers of adverts and record the number of new customers each day. The table shows the results.

Number of adverts per day (A)	1	2	3	4	5	6	7	8
Number of new customers (C)	2	5	6	7	12	10	14	16

- Draw a scatter graph to display the data.
- Add a line of best fit to your scatter graph.
- Find the gradient of the line of best fit and state the  $y$ -intercept.
- State the equation of the line of best fit in terms of  $C$  and  $A$ .
- Use your model to predict how many new customers the service would have if they ran 15 adverts per day.



- 3 A family records the daily minimum outside temperature  $T$  ( $^{\circ}\text{C}$ ) and their energy usage  $E$  (KWh).

Temperature $T(^{\circ}\text{C})$	10	9	1	7	2	8	7	3	6	4	5
Energy usage $E(\text{KWh})$	15	16	45	35	35	20	25	38	21	33	30

- Draw a scatter graph, display temperature  $T$  on the  $x$ -axis and energy usage  $E$  on the  $y$ -axis.
- Add a line of best fit to your scatter graph.
- Find the equation of the line of best fit in terms of  $E$  and  $T$ .
- Use your formula to predict the energy usage if the minimum outside temperature is  $-2^{\circ}\text{C}$ .

- 4 A group of friends recorded the data shown about the number of weeks they studied for an exam.

- Draw a scatter graph and add a line of best fit.
- Find a formula which will predict exam results based on weeks of study.
- One friend scored 70% in the exam. Estimate how many weeks they studied for.

Weeks of study (W)	1	2	3	4	5	4	3	6	5	8
Exam result (R)	32	45	50	61	53	65	48	85	80	95

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