BGE S1-S3

Mathematics & Numeracy

Third Level

Dr Helen Kelly
Kathleen McQuillan
Dr Alan Taylor

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Contents

Introduction to BGE Mathematics

How to get the most from this book

Mathematics is the richest language in the world. Learning mathematics is one of the most important things you can do to boost your brain power, now and throughout your life. Mathematicians understand, describe and influence the world around us.

This book covers all of the BGE Benchmarks for Mathematics at Third Level. The chapters take you on a journey which will both support and challenge you. Extension to Fourth Level material is included where appropriate. As you focus and work hard on each section you will gain knowledge, understanding and the ability to solve problems and communicate your solutions to others.

Working through this book will equip you to be the best mathematician you can be. Every topic is explained with rigour, taking no short cuts, and has an abundance of practice and challenge to suit every learner. The book is ambitious, encouraging you to aim high and build the skills you need for future success.

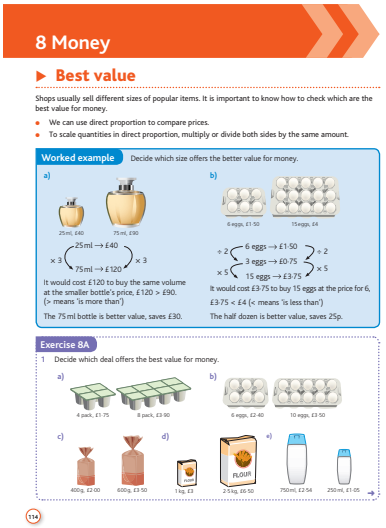
The book is enjoyable and easy to read. Every topic includes a clear, 'straight-to-the-point' method set out using bullet points in concise and simple language. It is full of worked examples and helpful hints and contains proven methods for mastering difficult concepts. This book has been designed both as a classroom aid and for your personal study.

Build your skills and understanding

To get the most from this book, read the explanations and think about the ideas. Follow the worked examples, paying close attention to the details.

Each lesson starts with an explanation and memorable method to which you can refer.

Each lesson has planned and progressive worked examples which prepare you to tackle the problem set.


8 Money

Best value

Shops usually sell different sizes of popular items. It is important to know how to check which are the best value for money.

- We can use direct proportion to compare prices.
- To scale quantities in direct proportion, multiply or divide both sides by the same amount.

Worked example — Decide which size offers the better value for money.

a) 25ml, £40 — 75 ml, £90

25ml → £40 (× 3) 75ml → £120 (× 3)

It would cost £120 to buy the same volume at the smaller bottle's price, £120 > £90. (> means 'is more than')

The 75 ml bottle is better value, saves £30.

b) 6 eggs, £1·50 — 15eggs, £4

6 eggs → £1·50 (× 2) 3 eggs → £0·75 (× 5) 15 eggs → £3·75

It would cost £3·75 to buy 15 eggs at the price for 6, £3·75 < £4 (< means 'is less than')

The half dozen is better value, saves 25p.

Exercise 8A

1 Decide which deal offers the best value for money.

a) 4 pack, £1·75 — 8 pack, £3·90

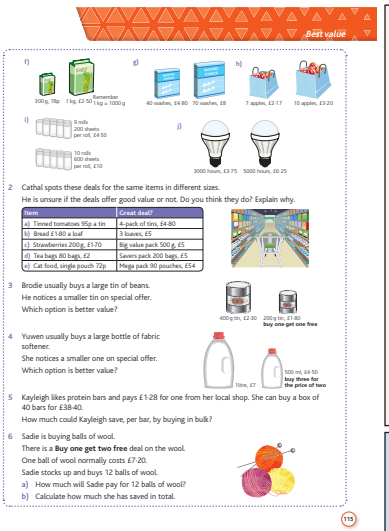
b) 6 eggs, £2·40 — 10 eggs, £3·50

c) 400g, £2·00 — 600g, £3·50

d) 1 kg, £3

e) 2·5 kg, £6·50 — 750ml, £2·54 — 250ml, £1·05

114


Best value

f) 300g, 78p — 1 kg, £2·50 — Remember 1 kg = 1000 g

g) 40 washes, £4·80 — 70 washes, £8

h) 7 apples, £2·17 — 10 apples, £3·20

i) 9 rolls 200 sheets per roll, £4·50 — 10 rolls 600 sheets per roll, £10

j) 3000 hours, £3·75 — 5000 hours, £6·25

2 Cathal spots these deals for the same items in different sizes. He is unsure if the deals offer good value or not. Do you think they do? Explain why.

Item	Great deal?
a) Tinned tomatoes 95p a tin	4-pack of tins, £4·80
b) Bread £1·80 a loaf	3 loaves, £5
c) Strawberries 200g, £1·70	Big value pack 500 g, £5
d) Tea bags 80 bags, £2	Savers pack 200 bags, £5
e) Cat food, single pouch 72p	Mega pack 90 pouches, £54

3 Brodie usually buys a large tin of beans. He notices a smaller tin on special offer. Which option is better value?

400 g tin, £2·30 — 200 g tin, £1·80 buy one get one free

4 Yuwen usually buys a large bottle of fabric softener. She notices a smaller one on special offer. Which option is better value?

1 litre, £7 — 500 ml, £4·50 buy three for the price of two

5 Kayleigh likes protein bars and pays £1·28 for one from her local shop. She can buy a box of 40 bars for £38·40. How much could Kayleigh save, per bar, by buying in bulk?

6 Sadie is buying balls of wool. There is a **Buy one get two free** deal on the wool. One ball of wool normally costs £7·20. Sadie stocks up and buys 12 balls of wool.

a) How much will Sadie pay for 12 balls of wool?

b) Calculate how much she has saved in total.

115

Every chapter has a check-up exercise. Use these to bring your ideas together and boost your memory of key methods.

Work through the exercise to practise and consolidate your skills. There are solutions at the end of the book, so check your work as you go to boost your confidence.

Each exercise gets progressively harder. Try to finish the exercises and test your skills with the most challenging problems.

We hope that you enjoy using this book as much as we have enjoyed creating the content, questions and activities for you!

Sharing in a given ratio

We can share a total quantity into amounts according to a given ratio.

- Add the parts of the ratio.
- Divide the total by the sum of the parts to find the value of one share.
- Multiply each part by the value of one share.

Worked examples

1 A man wants to share his savings between two charities, Wildlife UK and Animal Rescue. He has £4000 to donate and wants to share the money in the ratio 3:2.

$3 + 2 = 5$

$$5\overline{)4^{4}000} = 800$$

$800 \times 3 = 2400$

$800 \times 2 = 1600$

Wildlife UK : Animal Rescue

3 : 2

× 800 → £2400 : £1600 ← × 800

Check 2400 + 1600 = £4000

2 White, yellow and blue paint is mixed in the ratio 4:3:1. The mixed paint is green and is sold in 3-litre tins. How much white, yellow and blue paint is needed to make one tin of green?

$4 + 3 + 1 = 8$

$$8\overline{)3^{3}0^{6}0^{4}0} = 375$$

$375 \times 4 = 1500$

$375 \times 3 = 1125$

$375 \times 1 = 375$

white : yellow : blue

4 : 3 : 1 × 375

1500 ml : 1125 ml : 375 ml

Check 1500 + 1125 + 375 = 3000 ml

Exercise 4C

1 Share each amount in the given ratio.

a) 50 in the ratio 2:3
b) 54 in the ratio 2:7
c) 63 in the ratio 5:2
d) 720 in the ratio 4:5
e) 270 in the ratio 7:2
f) 121 in the ratio 6:5
g) 260 in the ratio 7:3
h) 1080 in the ratio 5:1

2 Two friends share £49 in the ratio 2:5. How much will each receive?

3 Two friends split a 280 g treat bar in the ratio 3:4. How much did each receive?

4 An athlete splits her time into training and rest days in the ratio 3:2. Calculate how many training days she will have over a 365-day year.

5 Two farmers split 108 kg of animal feed in the ratio 7:5. How much feed will each farmer have?

6 An oil worker works for 13 days in a row and then has 5 days off. Calculate how many working and holiday days she will have over a contract lasting 180 days.

7 A business woman keeps and reinvests her profits in the ratio 4:3. Her company made a total profit of £47250 last year. Calculate the amount of money which was reinvested.

8 Share each amount in the given ratios. Begin by adding all three parts of the ratio.

a) 121 m in the ratio 2:4:5

b) 360 g in the ratio 3:1:4

c) £24 in the ratio 9:1:10

d) 8 kg in the ratio 5:3:2

e) 280 mm in the ratio 7:2:5

f) 1 day in the ratio 4:3:1

g) 1 hour in the ratio 8:3:1

h) 3 hours in the ratio 4:3:8

9 Share 90 m^2 of shop floor space amongst shoes, bags and hats in the ratio 9:4:2. How much space will each have?

10 A scone mix weighs 1720 g and contains flour, butter and sugar in the ratio 5:2:1. Calculate how much of each ingredient the mix contains.

11 A manufacturer sells to Scotland, Wales and England in the ratio 19:1:20. The total value of sales one year was £216000. Calculate the value of sales in each part of the UK.

12 The table shows the basic ratios used to mix different colours of paint.

Vermillion	red : yellow 2 :1
Magenta	red : blue 2 :1
Green	red : blue : yellow 1 :4 :2

a) How much red paint is needed to make 1·5 litres of Vermillion?

b) How much red paint is needed to make 4·5 litres of Magenta?

c) How much blue paint is needed to make 1·82 litres of Green?

d) How much yellow paint is in 2·94 litres of Green?

In these questions think carefully about the information you have been asked for.

13 In a school with 570 pupils the ratio of junior to senior pupils is 10:9. How many more juniors than seniors are there?

14 A building brick castle contains 1440 bricks. The bricks are grey, black and white in the ratio 4:3:5. Calculate the total number of grey and black bricks used.

15 A shopping centre has a total of 800 parking spaces. The ratio of standard, disability and family spaces is 15:3:2. How many more standard than family spaces are there?

16 The tables below show the basic ratios used to make different kinds of coffee and the sizes available.

	Coffee	Milk	Water	Foam
Americano	1	0	4 hot	0
Cappuccino	2	1 hot	0	2
Latte	2	3 hot	0	1
Frappe	1	2 cold	2 cold	1

Small	180 ml
Regular	330 ml
Large	450 ml

a) Calculate the volume of coffee and water in a small Americano.

b) Write out the recipe for a large Frappe. Calculate the volume of each ingredient.

c) How much coffee is needed for an order of 3 regular Lattes?

d) What contains more foam, a regular Cappuccino or a large Frappe?

e) Which regular drink will contain the most coffee: Americano, Cappuccino, Latte or Frappe? Explain your answer.

Calculating time

We can calculate the time taken for any journey from the formula time = $\frac{\text{distance}}{\text{speed}}$, which we can write as $T = \frac{D}{S}$.

You should already be familiar with these time conversions.

Time	Decimal hour	Minutes
Half an hour	0·5	30
Quarter of an hour	0·25	15
Three quarters of an hour	0·75	45

Worked examples

1 A train travels 475 km at a speed of 100 km/h. How long does the train journey take?

$T = \frac{D}{S}$

$= \frac{475}{100}$

= 4·75 hours = 4 hours and 45 minutes

2 A snake slithers at 8 mph.

How long did the snake take to slither 12 miles?

$T = \frac{D}{S}$

$= \frac{12}{8}$

= 1·5 hours = 1 hour and 30 minutes

3 Qasim competes in open water swimming competitions. In his last competition, his average speed was 2·5 km/h. How long will it take Qasim to complete a 10000 metre race if he can maintain the same average speed?

Distance has to be in kilometres to be consistent with the unit for speed.

10 000 metres = 10 km

$T = \frac{D}{S}$

$= \frac{10}{2{\cdot}5}$

= 4 hours

Exercise 5D

1 Calculate the time taken for each journey. Be sure to include units with your answer. Convert any answers that include a decimal part of an hour to hours and minutes.

	a)	b)	c)	d)	e)	f)	g)
Distance	250 miles	78 metres	420 km	1170 m	8778 km	370 miles	4175 km
Speed	10 mph	6 m/s	5 km/h	9 m/s	7 km/h	40 mph	50 km/h

2 Hannah has a hoverboard. She completed a 35-metre obstacle course at an average speed of 5 m/s.

How long did it take Hannah to complete the course?

3 A satellite with a high earth orbit travels at 10000 mph.

Calculate the time taken for the satellite to travel 5000 miles. Give your answer in minutes.

4 A delivery driver has to make two deliveries. She starts in Stirling and drives 120 miles to Carlisle at 40 mph. She then stays in Carlisle for 45 minutes before driving 125 miles to Port Glasgow at 50 mph. If the driver left Stirling at 10:50 am, when did she reach Port Glasgow?

5 John cycles at 8 m/s. How long will it take him to cycle 1·8 kilometres? Give your answer in minutes and seconds.

A mixture!

You may find this memory aid helpful.

To find the correct formula, cover the letter you need to calculate. If you are asked to calculate the distance, for example, cover the 'D' with your finger to leave S × T.

Remember to:

- Show all working.
- Make sure units are consistent and include them in your final answer.

Exercise 5E

1 Calculate the missing entry for each journey.

	a)	b)	c)	d)	e)	f)	g)
Distance	60 metres		360 km	728 km		1470 km	30 km
Speed		14 mph	90 km/h		2·8 m/s		12 km/h
Time	6 seconds	3 hours		8 hours	40 seconds	5 hours	

2 The speed limit on a motorway is 70 mph. Caleb has 3 hours to drive 189 miles.
Can he make the journey without exceeding the speed limit? Justify your answer.

3 Louise rode her horse at an average speed of 7·5 mph. How far did she travel in 2 hours?

4 A small plane flies at an average speed of 200 km/h.
Calculate the time it takes to fly 650 km. Give your answer in hours and minutes.

5 At a fixed temperature, the speed of sound in air is 343 metres per second. Thunder can be heard 15 seconds after seeing the lightning strikes. How far away is the lightning? Give your answer in kilometres.

6 A train left the station at 17:00 and was due to arrive at its destination at 18:50.
Unfortunately, the train was 10 minutes late in arriving due to a fault on the line.
If the train travelled 184 000 metres, what was its average speed in kilometres per hour?

7 At football training, the team are told to run around the perimeter of the pitch three times.
James has an average speed of 4 m/s and completes the run in 2 minutes.
Calculate the length of the perimeter of the pitch.

8 Kitty competed in a triathlon. She

- swam 1500 metres at 20 metres per minute
- cycled 40 km at 16 km/h
- ran 10 km at 8 km per hour
- took 2 minutes to transition from swimming to cycling
- took 1 minute to transition from cycling to running.

a) If Kitty started the race at 11 am, when did she finish it?

b) She aims to compete in another triathlon in two months. She thinks she can reduce her swimming time to 50 minutes and maintain all her other times.
How much faster will she have to swim to achieve this?

9 Patterns and relationships

Sequences

A **sequence** is a set of numbers linked by a rule. A particular number in a sequence is a **term**. In the sequence 4, 7, 10, 13, 16, 19 ... the first term is 4, the second term is 7, the third term is 10 and so on.

To describe a sequence, state the first term and write down the rule that connects one term to the next.

Worked examples

1 These sequences involve addition or subtraction. Write down the next three terms and write down a rule that describes the sequence.

a) 12, 19, 26, 33, 40, 47, 54 ...

Start at 12 and add 7 each time.

b) 45, 41, 37, 33, 29, 25, 21 ...

Start at 45 and subtract 4 each time.

2 These sequences involve multiplication or division. Write down the next three terms and write down a rule that describes the sequence.

a) 4, 8, 16, 32, 64, 128, 256 ...

Start at 4 and multiply by 2 each time.

b) 2187, 729, 243, 81, 27, 9, 3 ...

Start at 2187 and divide by 3 each time.

3 The first term of a sequence is 12 and the rule is 'add 7 each time'. What is the fourth term in the sequence?

2nd term: 12 + 7 = 19 3rd term: 19 + 7 = 26 4th term: 26 + 7 = 33.

Exercise 9A

1 These sequences involve addition or subtraction. For each one, write the next three terms and the rule.

a) 2, 5, 8, 11 ... **b)** 9, 15, 21, 27 ... **c)** 1, 14, 27, 40 ... **d)** 25, 23, 21, 19 ...

e) 63, 55, 47, 39 ... **f)** 86, 72, 58 ... **g)** 14, 8, 2, –4 ... **h)** –19, –7, 5 ...

i) 113, 94, 75 ... **j)** –3, –11, –19 ... **k)** 13, 27, 41 ... **l)** 7, 8·3, 9·6 ...

2 These sequences involve multiplication or division. For each one, write the next three terms and the rule.

a) 2, 4, 8, 16 ... **b)** 1, 3, 9, 27 ... **c)** 2, 10, 50 ... **d)** 400, 200, 100 ...

e) 3645,1215, 405, 135 ... **f)** 70 000, 7000, 700 ... **g)** 4, 20, 100 ... **h)** 288, 144, 72 ...

i) 7, 21, 63 ... **j)** 3125, 625, 125 ... **k)** 5120, 1280, 320 ... **l)** 4, 16, 64 ...

3 A sequence has first term 19 and rule 'add 18 each time'. What is the fourth term in this sequence?

4 A sequence has first term 23 and rule 'subtract 4·5 each time'. What is the third term in this sequence?

5 The first and third terms of a particular sequence are 5 and 45 respectively. The rule for this sequence involves multiplying by a positive number. What is the second term in the sequence?

6 The fourth and fifth terms of a sequence that involves subtraction are 97 and 78. What is the first term of the sequence?

➜

7 The Fibonacci sequence starts with two ones. After that, each term is the sum of the previous two terms. So the third term is 1 + 1 = 2, the fourth term is 1 + 2 = 3 and so on. The first five terms in the Fibonacci sequence are 1, 1, 2, 3, 5. Find the next five terms.

Square numbers

The sequence of **square numbers** can be formed by squaring the positive integers (1, 2, 3 ...) to give 1, 4, 9 ....

Exercise 9B

1 Copy and complete this table to list the first ten square numbers.

n	1	2	3	4	5	6	7	8	9	10
nth square number	1	4	9							

2 Note the differences between consecutive square numbers. What pattern can you spot?

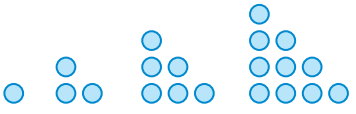
3 Draw a table to list the 11th to 20th square numbers; knowing these can be really helpful in mathematics.

4 Sharon says that the square of an odd number will always be odd. Is she correct? Why?

5 Look at your table and you will notice that $9 + 16 = 25$ ($3^2 + 4^2 = 5^2$). So 3, 4 and 5 are a **Pythagorean triple**. Find five more Pythagorean triples in your tables and write them in the form $a^2 + b^2 = c^2$.

Triangular numbers

This pattern shows the first four triangular numbers.



Exercise 9C

1 Copy the diagram above and continue the pattern up to the sixth triangular number.

2 Copy and complete the table below to list the first ten triangular numbers.

n	1	2	3	4	5	6	7	8	9	10
nth triangular number	1	3	6							

3 What is the difference between:

a) the 3rd and 4th triangular numbers
b) the 7th and 8th triangular numbers
c) the 9th and 10th triangular numbers
d) the 53rd and 54th triangular numbers?

4 Add together pairs of consecutive triangular numbers and note the answers. What do you notice? Try to use your diagrams from question 1 to demonstrate why this result is true. Two colours of pen may be helpful!

5 Using what you observed in question 4, write down the sum of

a) the 11th and 12th triangular numbers
b) the 19th and 20th triangular numbers.

6 James tries adding together the first square and triangular numbers, then the second and so on. He thinks that this will always result in a prime number. Is he correct? Give a reason for your answer.

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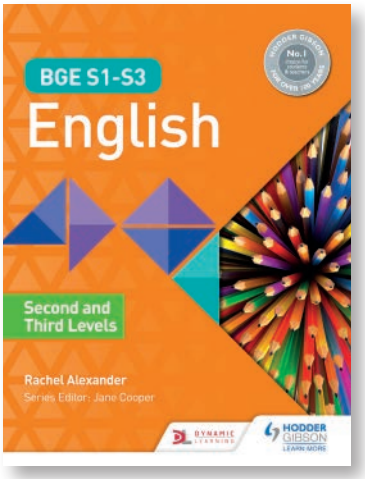

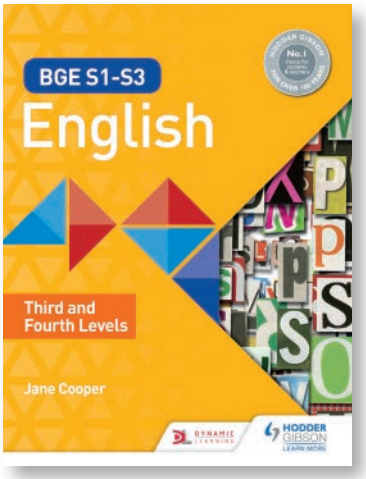

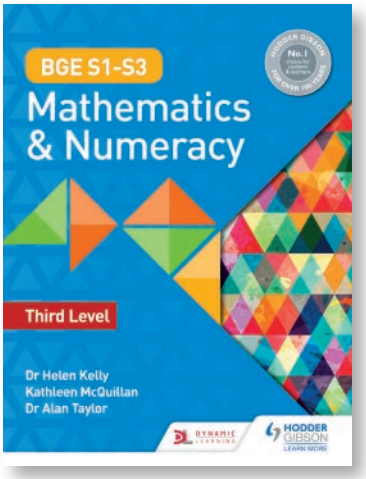

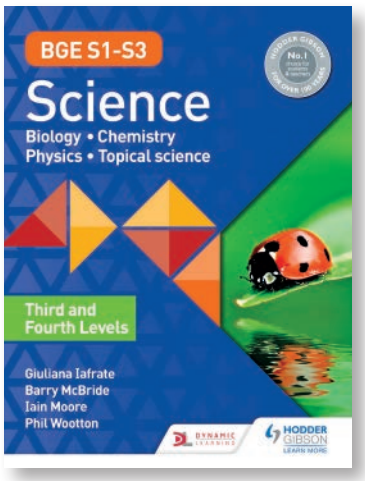

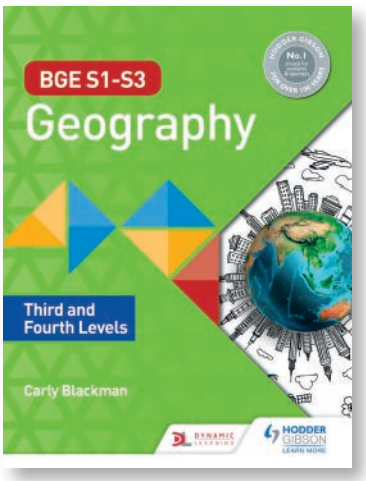

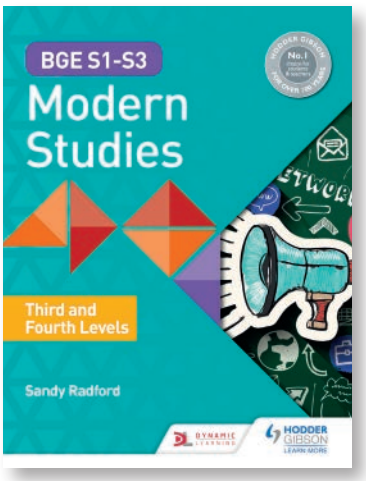

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Dr Helen Kelly, Kathleen McQuillan and Dr Alan Taylor are all experienced mathematics teachers who work within Scottish schools. They have taught pupils from upper Primary to Secondary 6 and strongly believe it is essential that a mathematical education should be rigorous, motivating and engaging. Their aim is to provide the solid foundation in core mathematics and numeracy skills that is essential for successful progression through National Qualifications and beyond.

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