

BOOK

3

KEY STAGE 3

# Mastering Mathematics

Develop  
and  
Secure

Practice Book

Frances Carr

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# Powers and indices

● Fluency

● Reasoning

● Problem solving

## 1.1 Index notation

1 Work out:

- a  $4^2$       b  $3^2$       c  $9^2$   
d  $2^3$       e  $10^3$       f  $2^4$

Remember that  $5 \times 5$  is written as  $5^2$ . You say that 5 is the **base** and 2 is the **index** or **power**.

In the same way, repeated multiplications can be written using **index notation** like this:

$$\underbrace{5 \times 5 \times 5}_{3 \text{ fives}} = 5^3 \quad \underbrace{5 \times 5 \times 5 \times 5}_{4 \text{ fives}} = 5^4$$

In general,

$$\underbrace{a \times a \times a \times \dots \times a \times a}_{n \text{ times}} = a^n$$

2 Write each expression as a power of 8.

- a  $8 \times 8 \times 8$   
b  $8 \times 8 \times 8 \times 8 \times 8$   
c  $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$   
d  $8 \times 8 \times 8 \times 8$

3 Write each expression as a single power.

- a  $9 \times 9$   
b  $4 \times 4 \times 4 \times 4 \times 4$   
c  $11 \times 11 \times 11$   
d  $57 \times 57 \times 57 \times 57 \times 57 \times 57$   
e  $0.6 \times 0.6 \times 0.6$   
f  $18.2 \times 18.2 \times 18.2 \times 18.2$

- 4 a Work out the value of  $2^6$ .  
b Find the value of  $8^2 + 10^3 + 2^4$ .  
c Calculate the difference between  $2^3$  and  $2^5$ .

5 Write the correct symbol  $<$ ,  $>$  or  $=$  between each pair of numbers.

- a  $9^2$    $7^2$       b  $10^2$    $10^3$   
c  $2^6$    $6^2$       d  $3^3$    $5^2$   
e  $1^8$    $4^2$       f  $4^3$    $8^2$

To write  $3^5 \times 3^4$  as a single power of 3:

$$3^5 \times 3^4 = (3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3) = 3^9$$

The powers have been added:  
 $5 + 4 = 9$

6 Write each of these as a single power.

- a  $11^4 \times 11^2$   
b  $4^7 \times 4^3$   
c  $7^5 \times 7^{10} \times 7^6$

To write  $3^6 \div 3^4$  as a single power of 3:

$$3^6 \div 3^4 = \frac{3^6}{3^4} = \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3} = 3 \times 3 = 3^2$$

The powers have been subtracted:  $6 - 4 = 2$

7 Write each of these as a single power.

- a  $5^7 \div 5^3$       b  $12^{11} \div 12^8$       c  $\frac{8^9}{8^2}$

In the previous examples, the base was 3, but the rules are true for any base number.

Here are the general rules for multiplying and dividing powers:

$$a^m \times a^n = a^{m+n} \quad \text{and} \quad a^m \div a^n = a^{m-n}$$

Take care! The bases must be the same – you can't combine powers if the bases are different.

8 Where possible, write each of these as a single power.

- a  $7^9 \div 7^3$       b  $4^5 \times 4^8$       c  $10^2 + 10^3$   
d  $\frac{6^{15}}{6^4}$       e  $23^9 \times 23^7$       f  $4^5 \div 9^3$

9 Eleni says that  $3^5 \times 3^7$  is equal to  $9^{12}$ .

Eleni is wrong. What mistake has she made? What is the correct answer?

Sometimes powers involve brackets. To write  $(3^2)^4$  as a single power of 3:

$$(3^2)^4 = (3^2) \times (3^2) \times (3^2) \times (3^2) = (3 \times 3) \times (3 \times 3) \times (3 \times 3) \times (3 \times 3) = 3^8$$

The powers have been multiplied:  $2 \times 4 = 8$

The general rule for a power of a power is:

$$(a^m)^n = a^{m \times n}$$

10 Write each of these as a single power.

- a  $(5^3)^2$       b  $(9^2)^7$       c  $(16^5)^6$

11 Write each of these calculations as a single power.

- a  $(4^9)^3$       b  $3^8 \div 3^6$   
c  $9^5 \times 9^{11}$       d  $13^7 \div 13^4$   
e  $(25^2)^9$       f  $17^3 \times 17^4 \div 17^5$



You now know that

$$8^3 \div 8^2 = 8^1$$

The powers have been subtracted:  $3 - 2 = 1$

Writing this calculation as a

$$\text{fraction: } \frac{8 \times \cancel{8} \times \cancel{8}}{\cancel{8} \times \cancel{8}} = 8$$

This shows that  $8^1 = 8$ .

Here is a general rule: any number to the power 1 is itself.

$$a^1 = a$$

- 12 Write each of these as a single power.

a  $3^7 \div 3$

b  $6 \times 6^{11}$

c  $8^5 \times 8$

d  $9^5 \times 9 \times 9^{12}$

e  $14^8 \times 14^2 \times 14$

f  $2^5 \div 2 \times 2^2$

You now also know

$$\text{that } 7^3 \div 7^3 = 7^0$$

The powers have been subtracted:  $3 - 3 = 0$

Writing this calculation

$$\text{as a fraction: } \frac{\cancel{7} \times \cancel{7} \times \cancel{7}}{\cancel{7} \times \cancel{7} \times \cancel{7}} = 1$$

This shows that  $7^0 = 1$ .

Here is another general rule: any number to the power 0 is 1.

$$a^0 = 1$$

- 13 Work out the value of:

a  $5^0$

b  $23^0$

c  $1.9^0$

d  $(-4)^0$

## 1.2 Standard form

- 1 Write these numbers as powers of 10.

a 100

b 10 000

c 10 000 000

d 1000

e 10

- 2 Work out these powers of 10.

a  $10^5$

b  $10^2$

c  $10^1$

d  $10^6$

e  $10^0$

- 3 a Write down ten thousand as:

i an ordinary number

ii a power of 10.

b Write down ten million as:

i an ordinary number

ii a power of 10.

- 4 a Copy and complete this pattern.

$$4.5 \times 10^0 = 4.5$$

$$4.5 \times 10^1 = 4.5 \times 10 = \square$$

$$4.5 \times 10^2 = 4.5 \times 100 = \square$$

$$4.5 \times 10^3 = 4.5 \times \square = \square$$

$$4.5 \times 10^4 = 4.5 \times \square = \square$$

b Work out the value of  $4.5 \times 10^6$ .

c Copy and complete this calculation:

$$4\,500\,000\,000 = 4.5 \times 10^{\square}$$

In maths and science, large and small numbers are usually written using powers of 10.

**Standard form** is a way of writing down large numbers without writing down all the zeros.

A number is in standard form when it is written as a number between 1 and 10 multiplied by a power of 10.

In symbols this is written as  $A \times 10^n$  where  $A$  can be any number from 1 up to 10 [ $1 \leq A < 10$ ] and  $n$  must be an integer [whole number]

- 5 Which of these numbers is in standard form? Explain the mistakes in the other answers.

a  $0.5 \times 10^7$

b  $4.08 \times 10^{2.6}$

c  $9.2 \times 10^5$

d  $31.5 \times 10^2$

A forest covers an area of 83 000 m<sup>2</sup>.

- 1 To write this number in standard form, first find the value of  $A$ .

$$83\,000 \text{ in standard form is } 8.3 \times 10^{\square}$$

$A$  is always between 1 and 10.

- 2 Use a place value diagram to help you work out what the power of 10 should be.

T	Th	H	T	U		t
				8	.	3
8	3	0	0	0	.	

$$83\,000 = 8.3 \times 10\,000$$

$$= 8.3 \times 10^4$$

You have to multiply 8.3 by ten 4 times to get 83 000.

So, the forest covers an area of  $8.3 \times 10^4$  m<sup>2</sup>.

- 6 Copy and complete these by filling in the missing numbers.

a  $\square \times 10^3 = 9000$

b  $\square \times 10^2 = 400$

c  $2 \times 10^{\square} = 200\,000$

d  $8.3 \times 10^{\square} = 830$

e  $\square \times 10^5 = 560\,000$

f  $\square \times 10^4 = 13000$

- 7 Write these numbers in standard form.

a 500

b 30 000

c 8000

d 670 000

e 9400

f 4 250 000

g 801 000 000 h 36 000

i 71 400 j 12

- 8 Write these numbers as powers of 10.

a 0.01

b 0.0001

c 0.1

d 0.000 01

- 9 Work out these powers of 10.

a  $10^3$ b  $10^{-1}$ c  $10^5$ d  $10^{-4}$ e  $10^{-2}$ 

The mass of a grain of rice is approximately 0.029 g.

- 1 To write this number in standard form, first find the value of *A*.

0.029 in standard form is  $2.9 \times 10^{\square}$

*A* is always between 1 and 10.

- 2 Use a place value diagram to help you work out what the power of 10 should be.

U		T	h	th
2	.	9		
0	.	0	2	9

$$0.029 = 2.9 \times 100$$

$$= 2.9 \times 10^{-2}$$

Dividing by 10 is the same as multiplying by  $10^{-1}$ .  
Dividing by 100 is the same as multiplying by  $10^{-2}$ .

So, the mass of a grain of rice is  $2.9 \times 10^{-2}$  g

- 10 Write these numbers in standard form.

a 0.004

b 0.000 03

c 0.9

d 0.07

e 0.082

f 0.000 75

g 0.000 001 02

h 0.000 934

i 0.0051

j 0.308

- 11 Write the numbers in the statements in standard form.

- a The circumference of the Earth is approximately 40 000 kilometres.
- b A red blood cell has a diameter of 0.000 007 metres.
- c In 2018, England had an estimated population of 56 000 000.
- d The mass of a grain of sand is 0.011 grams.

To convert from standard form back to ordinary numbers, remember that when the power of 10

► is positive then the number is BIG

► is negative then the number is SMALL.

To convert numbers from standard form to ordinary numbers, first think about the power of 10:

$$2.7 \times 10^3 = 2.7 \times 1000 = 2700$$

$$2.7 \times 10^{-3} = 2.7 \times 0.001 = 0.0027$$

- 12 Write each of these as an ordinary number.

a  $5 \times 10^3$ b  $3 \times 10^2$ c  $9 \times 10^5$ d  $4.2 \times 10^4$ e  $8.7 \times 10^6$ f  $1.5 \times 10^1$ g  $7 \times 10^{-3}$ h  $4 \times 10^{-4}$ i  $8 \times 10^{-2}$ j  $1.9 \times 10^{-6}$ k  $2.6 \times 10^{-1}$ l  $6.8 \times 10^{-5}$ 

## 1.3 Prime factorisation

- 1 Look at these numbers:

10 7 6 36 25 1 3 2 4 24 8

Write down all the numbers that are:

a multiples of 5

b factors of 12

c square numbers

d cube numbers.

- 2 Prime numbers are numbers that have exactly 2 factors: 1 and the number itself.

Which of the numbers in the box in question 1 are prime?

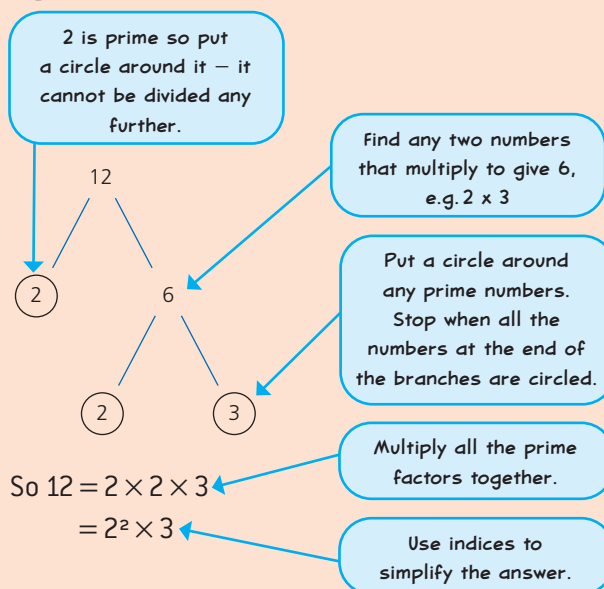
Every number can be made by multiplying together prime numbers.

For example,  $60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$ . This is called a **product of prime factors**.

One method of finding prime factors is to use a **factor tree**.

For example, to write 12 as a product of prime factors, start with 12 at the top.

Look for any two numbers that multiply to give 12, e.g.  $2 \times 6$



$$\text{So } 12 = 2 \times 2 \times 3$$

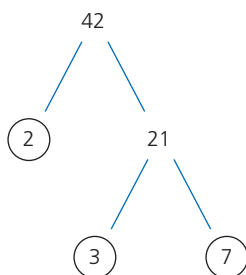
$$= 2^2 \times 3$$

There is only one way to write any number as a product of prime factors.

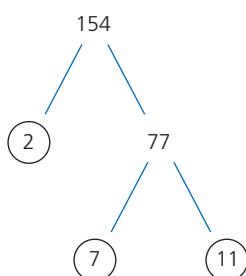
There may be different factor trees, but they will always give the same product of prime factors.

- 3 Use the factor trees to write each number as a product of prime factors.

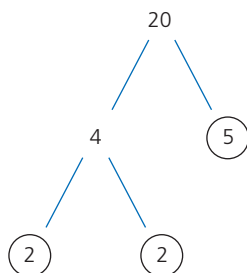
a



b



c



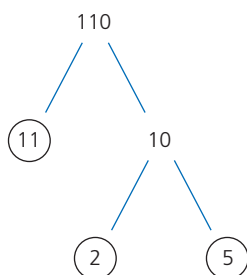
- 4 Draw factor trees to write these numbers as products of the prime factors.

a 30      b 70      c 66  
d 130      e 105

- 5 Write each of the numbers as a product of its prime factors, using indices.

a 18      b 44      c 75  
d 56      e 100

- 6 Jessica uses a factor tree to write 110 as a product of prime factors.



Jessica writes:

$$110 = 2 + 5 + 11$$

Jessica is wrong. What mistake has she made?

The **highest common factor (HCF)** of two numbers is the largest factor that they share.

You can find the HCF of two numbers by listing their factors.

**Factors of 20:**

**1, 2, 4, 5, 10, 20**

**Factors of 30:**

**1, 2, 3, 5, 6, 10, 15, 30**

The highest common factor of 20 and 30 is 10.

- 7 a What are the factors of 10?  
b What are the factors of 12?  
c What is the highest common factor (HCF) of 10 and 12?
- 8 Find the highest common factor (HCF) of each pair of numbers.  
a 35 and 50  
b 16 and 24  
c 45 and 36

The **lowest common multiple (LCM)** of two numbers is the lowest multiple that they share.

You can find the LCM of two numbers by listing their multiples.

**Multiples of 8:**

**8, 16, 24, 32, 40, ...**

**Multiples of 3:**

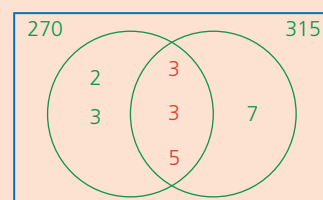
**3, 6, 9, 12, 15, 18, 21, 24, 27, ..**

The lowest common multiple of 8 and 3 is 24.

- 9 a What are the first ten multiples of 4?  
b What are the first ten multiples of 9?  
c What is the lowest common multiple (LCM) of 4 and 9?
- 10 Find the lowest common multiple (LCM) of each pair of numbers.  
a 5 and 7      b 6 and 10  
c 8 and 12

A **Venn diagram** can be used to find the HCF and LCM of two numbers.

- 1 First, write both numbers as a product of their prime factors.  
 $270 = 2 \times 3 \times 3 \times 3 \times 5$   
 $315 = 3 \times 3 \times 5 \times 7$
- 2 Next, show the prime factors in a Venn diagram.



3, 3 and 5 are common factors so they go in the intersection.

The highest common factor is found by multiplying the numbers in the intersection.

$$3 \times 3 \times 5 = 45$$

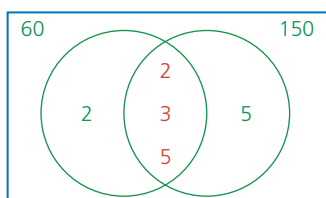
So, the HCF of 270 and 315 is 45.

The lowest common multiple is found by multiplying all of the numbers in the Venn diagram.

$$2 \times 3 \times 3 \times 3 \times 5 \times 7 = 1890$$

So, the LCM of 270 and 315 is 1890.

- 11** Here is a Venn diagram showing the prime factors of 60 and 150.



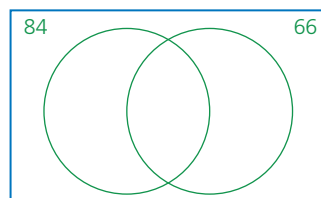
- What is the highest common factor of 60 and 150?
- What is lowest common multiple of 60 and 150?

- 12** Here are 84 and 66, written as products of prime factors:

$$84 = 2 \times 2 \times 3 \times 7$$

$$66 = 2 \times 3 \times 11$$

- Copy and complete the Venn diagram showing the prime factors of 84 and 66.



- Find the highest common factor of 84 and 66.
  - Find the lowest common multiple of 84 and 66.
- 13** Find the highest common factor and lowest common multiple of these pairs of numbers by first finding their prime factors.
- |                     |                     |
|---------------------|---------------------|
| <b>a</b> 56 and 20  | <b>b</b> 42 and 60  |
| <b>c</b> 20 and 140 | <b>d</b> 66 and 154 |



● Fluency

● Reasoning

● Problem solving

## 2.1 Fractions review

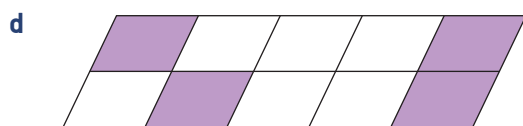
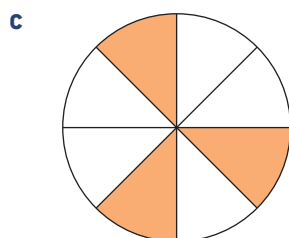
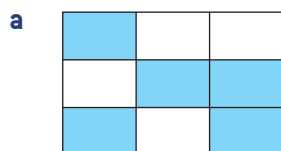
- 1 Find the lowest common multiple of these pairs of numbers.

a 3 and 5                      b 4 and 10  
c 2 and 7                      d 6 and 9

- 2 Find the highest common factor of these pairs of numbers.

a 10 and 14                      b 35 and 20  
c 11 and 15                      d 24 and 16

- 3 What fraction of each of the shapes is shaded?



You can change a fraction into a decimal by either writing an equivalent fraction with a denominator of 10 or 100 and using place value, or by dividing the numerator by the denominator.

For example,  $\frac{4}{5} = \frac{8}{10} = 0.8$  and  $\frac{3}{8} = 3 \div 8 = 0.375$ .

- 4 Write these fractions as decimals.

a  $\frac{7}{10}$                       b  $\frac{1}{2}$                       c  $\frac{11}{20}$   
d  $\frac{3}{4}$                       e  $\frac{8}{25}$                       f  $\frac{7}{8}$

You can find **equivalent fractions** by multiplying or dividing **both the numerator and denominator** by the same number.

**Equivalent fractions** simplify to give the same fraction in its lowest terms.

So,  $\frac{3}{8} = \frac{6}{16}$  and  $\frac{12}{32} = \frac{3}{8}$

You can't simplify  $\frac{3}{8}$  any further, so you say that  $\frac{3}{8}$  is in its **simplest form**.

- 5 Simplify these fractions.

a  $\frac{12}{15}$                       b  $\frac{30}{45}$                       c  $\frac{24}{28}$   
d  $\frac{44}{77}$                       e  $\frac{12}{36}$                       f  $\frac{63}{81}$

- 6 Look at these fractions.

$\frac{2}{8}$     $\frac{6}{12}$     $\frac{4}{6}$     $\frac{5}{20}$     $\frac{14}{21}$     $\frac{9}{18}$     $\frac{6}{9}$     $\frac{3}{12}$     $\frac{5}{10}$

Write down all the fractions that are equivalent to:

a  $\frac{1}{2}$                       b  $\frac{1}{4}$                       c  $\frac{2}{3}$

You can use equivalent fractions to rewrite a decimal as a fraction.

For example, 0.42 means 42 hundredths, which is  $\frac{42}{100}$

Simplifying the fraction gives  $\frac{42}{100} = \frac{21}{50}$  so  $0.42 = \frac{21}{50}$

- 7 Write each of these decimals as a fraction in its simplest form.

a 0.8                      b 0.14  
c 0.128                      d 0.625

Remember: you can only add or subtract fractions when they have the **same denominator**.

To work out  $\frac{3}{4} + \frac{1}{5}$ :

- 1 Find the lowest common multiple of the denominators.

The LCM of 4 and 5 = 20

- 2 Write equivalent fractions with this denominator.

$\frac{3}{4} = \frac{15}{20}$  and  $\frac{1}{5} = \frac{4}{20}$

- 3 Add or subtract the numerators.

$\frac{3}{4} + \frac{1}{5} = \frac{15}{20} + \frac{4}{20} = \frac{19}{20}$

- 8 In each pair, which fraction is bigger?

a  $\frac{1}{4}$  or  $\frac{1}{5}$                       b  $\frac{3}{4}$  or  $\frac{3}{7}$   
c  $\frac{2}{5}$  or  $\frac{3}{5}$                       d  $\frac{2}{9}$  or  $\frac{1}{4}$

- 9 Work these out giving each answer as a fraction in its simplest form.

a  $\frac{7}{11} - \frac{5}{11}$       b  $\frac{3}{4} + \frac{1}{8}$       c  $\frac{1}{3} + \frac{5}{12}$

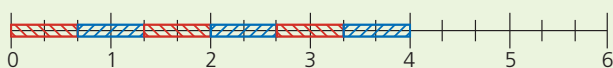
d  $\frac{1}{2} - \frac{1}{6}$       e  $\frac{2}{5} + \frac{1}{3}$       f  $\frac{5}{6} - \frac{3}{4}$

g  $\frac{3}{4} - \frac{1}{3}$       h  $\frac{1}{2} + \frac{5}{11}$

Multiplication is commutative, which means that it doesn't matter in which order you multiply numbers.

So, 6 lots of  $\frac{2}{3} = 6 \times \frac{2}{3} = \frac{2}{3} \times 6 = \frac{2}{3}$  of 6

This number line shows that  $6 \times \frac{2}{3} = 4$



To find fractions of amounts, first divide by the denominator and then multiply by the numerator.

For example, to find  $\frac{2}{3}$  of 6:

$6 \div 3 = 2$        $\frac{1}{3}$  of 6 = 2

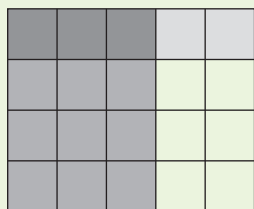
$2 \times 2 = 4$        $\frac{2}{3}$  of 6 = 4

- 10 Work out:

a  $\frac{1}{4}$  of 24      b  $\frac{1}{9} \times 36$       c  $66 \times \frac{1}{6}$

d  $\frac{3}{8} \times 40$       e  $\frac{2}{5}$  of 35      f  $14 \times \frac{4}{7}$

You can also multiply fractions together.



For example,  $\frac{1}{4} \times \frac{3}{5}$  means  $\frac{1}{4}$  of  $\frac{3}{5}$ .

The diagram shows  $\frac{1}{4}$  of  $\frac{3}{5}$ .

So  $\frac{1}{4} \times \frac{3}{5} = \frac{3}{20}$ .

Remember that to multiply fractions, you multiply the numerators and multiply the denominators.

Cancelling out common factors before multiplying means you do not need to simplify at the end.

$$\frac{3}{8} \times \frac{4}{9} = \frac{12}{72} = \frac{1}{6}$$

$\div 12$  (above 12 and 72)  
 $\div 12$  (below 12 and 72)

$$\frac{1}{2} \times \frac{4}{3} = \frac{4}{6} = \frac{2}{3}$$

$\div 2$  (above 4 and 6)  
 $\div 3$  (below 4 and 6)

- 11 Work these out, giving each answer as a fraction in its simplest form.

a  $\frac{1}{2} \times \frac{1}{7}$       b  $\frac{2}{5} \times \frac{4}{9}$       c  $\frac{3}{11} \times \frac{1}{2}$

d  $\frac{2}{9} \times \frac{1}{4}$       e  $\frac{5}{9} \times \frac{3}{5}$       f  $\frac{4}{9} \times \frac{9}{20}$

Remember that  $1 \div \frac{1}{5}$  means 'how many fifths are there in 1 whole?'

So,  $1 \div \frac{1}{5} = 5$

Dividing by  $\frac{1}{5}$  is the same as multiplying by 5

Every division by a fraction can be written as a multiplication by the **reciprocal**.

- 12 Write down the reciprocal of each of these:

a 4      b  $\frac{1}{2}$       c  $\frac{3}{5}$

d  $\frac{1}{8}$       e  $\frac{4}{9}$       f  $\frac{7}{3}$

- 13 Work out these division calculations.

a  $3 \div \frac{1}{4}$       b  $4 \div \frac{1}{3}$       c  $9 \div \frac{1}{2}$

d  $10 \div \frac{1}{6}$       e  $6 \div \frac{1}{5}$

- 14 Work out the following, giving each answer as a fraction in its simplest form.

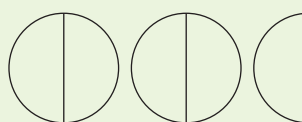
a  $\frac{1}{5} \div \frac{1}{4}$       b  $\frac{1}{7} \div \frac{1}{3}$       c  $\frac{1}{6} \div \frac{6}{7}$

d  $\frac{1}{2} \div \frac{3}{4}$       e  $\frac{2}{5} \div \frac{7}{8}$       f  $\frac{3}{5} \div \frac{9}{10}$

## 2.2 Mixed numbers

Fractions can be used to represent numbers which are greater than 1.

This diagram shows  $\frac{5}{2}$ .



There are 2 halves in one whole.

So, there are 4 halves in 2 wholes. 5 halves make 2 and a half.

Fractions like  $\frac{5}{2}$  are called top heavy or **improper fractions**.

They can also be written as **mixed numbers** (a mix of a whole number and a fraction).

For example,  $\frac{5}{2} = 2\frac{1}{2}$ .

Every improper fraction can be written as a mixed number.

- 1 Find the missing number in each statement.

a  $1 = \frac{\square}{3}$

b  $1 = \frac{\square}{8}$

c  $1 = \frac{\square}{10}$

d  $2 = \frac{\square}{3}$

e  $3 = \frac{\square}{4}$

f  $10 = \frac{\square}{5}$

g  $5 = \frac{\square}{6}$

h  $4 = \frac{\square}{2}$

i  $8 = \frac{\square}{10}$

To write  $\frac{19}{5}$  as a mixed number, change the fraction to a division.

$\frac{19}{5} = 19 \div 5$   $19 \div 5 = 3$  remainder 4 So  $\frac{19}{5} = 3\frac{4}{5}$ .

- 2 Write these improper fractions as mixed numbers.

a  $\frac{7}{4} = 1\frac{\square}{4}$

b  $\frac{11}{5} = 2\frac{\square}{5}$

c  $\frac{13}{3} = \square\frac{\square}{3}$

d  $\frac{9}{7} = 1\frac{\square}{7}$

e  $\frac{63}{10} = \square\frac{3}{\square}$

f  $\frac{17}{6}$

g  $\frac{21}{8}$

h  $\frac{23}{4}$

i  $\frac{16}{3}$

j  $\frac{27}{5}$

To write  $2\frac{7}{8}$  as an improper fraction, first think about how many eighths are in 2 wholes.

$2 = \frac{16}{8}$

So,  $2\frac{7}{8} = \frac{16}{8} + \frac{7}{8} = \frac{23}{8}$ .

- 3 a How many fifths are there in 1 whole?

b Write  $1\frac{4}{5}$  as an improper fraction.

- 4 Write these mixed numbers as improper fractions.

a  $1\frac{3}{4} = \frac{\square}{4}$

b  $1\frac{5}{8} = \frac{\square}{8}$

c  $2\frac{1}{10} = \frac{\square}{10}$

d  $3\frac{2}{7} = \frac{\square}{7}$

e  $5\frac{2}{3} = \frac{17}{\square}$

f  $1\frac{2}{5}$

g  $2\frac{3}{4}$

h  $9\frac{1}{5}$

i  $10\frac{4}{9}$

j  $6\frac{3}{11}$

- 5 Work out these subtractions.

a  $1 - \frac{1}{5}$

b  $2 - \frac{1}{3}$

c  $4 - \frac{1}{2}$

d  $5 - \frac{2}{7}$

e  $3 - \frac{4}{9}$

There are two methods for adding and subtracting mixed numbers.

- Deal with the whole numbers first and then the fractions.
- Convert to improper fractions first and then add or subtract as normal.

For example, to work out:

a  $2\frac{1}{3} + 1\frac{4}{5}$

b  $2\frac{1}{3} - 1\frac{4}{5}$

Part a Method 1	Part a Method 2
$2\frac{1}{3} + 1\frac{4}{5} = 2\frac{5}{15} + 1\frac{12}{15}$ $= 2 + 1 + \frac{5}{15} + \frac{12}{15}$ $= 3 + \frac{17}{15}$ $= 3 + 1 + \frac{2}{15}$ $= 4\frac{2}{15}$	$2\frac{1}{3} + 1\frac{4}{5} = \frac{7}{3} + \frac{9}{5}$ $\frac{7}{3} + \frac{9}{5} = \frac{35}{15} + \frac{27}{15}$ $= \frac{62}{15}$ $= \frac{60}{15} + \frac{2}{15}$ $= 4\frac{2}{15}$
Part b Method 1	Part b Method 2
$2\frac{1}{3} - 1\frac{4}{5} = 2\frac{5}{15} - 1\frac{12}{15}$ $= 2 - 1 + \frac{5}{15} - \frac{12}{15}$ $= 1 - \frac{7}{15}$ $= \frac{8}{15}$	$2\frac{1}{3} - 1\frac{4}{5} = \frac{7}{3} - \frac{9}{5}$ $= \frac{35}{15} - \frac{27}{15}$ $= \frac{8}{15}$

- 6 Work these out giving your answers as mixed numbers.

a  $\frac{5}{7} + \frac{4}{7}$

b  $\frac{2}{3} + \frac{7}{9}$

c  $\frac{3}{4} + \frac{5}{6}$

d  $\frac{3}{2} + \frac{6}{5}$

e  $\frac{5}{4} + \frac{7}{3}$

f  $\frac{5}{2} + \frac{8}{3}$

- 7 Work these out giving your answers as mixed numbers.

a  $\frac{11}{3} - \frac{4}{3}$

b  $\frac{11}{4} - \frac{6}{5}$

c  $\frac{7}{2} - \frac{4}{3}$

d  $\frac{5}{2} - \frac{8}{7}$

e  $\frac{12}{5} - \frac{4}{3}$

f  $\frac{13}{2} - \frac{7}{6}$

BOOK

3

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