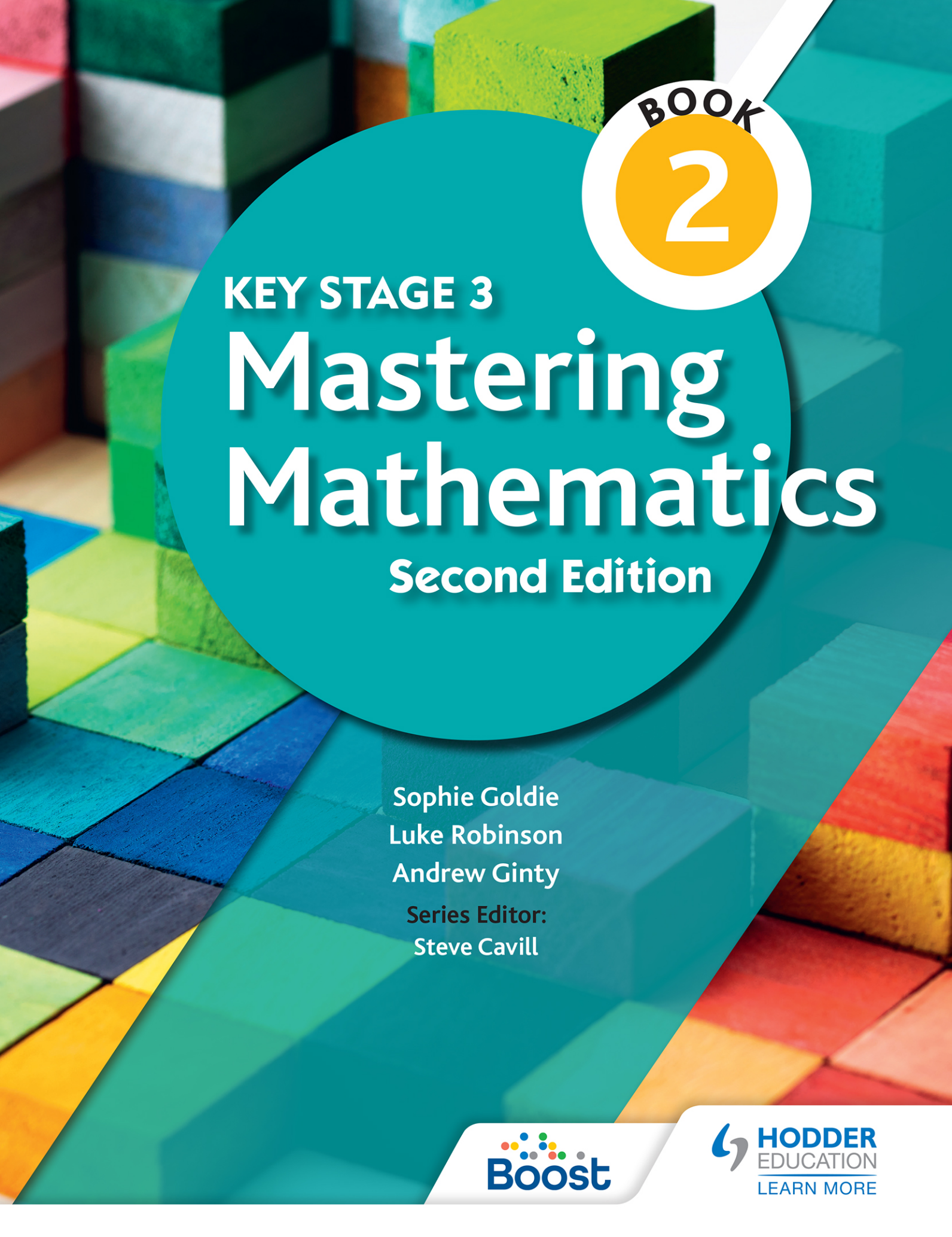
BOOK
2
KEY STAGE 3
Mastering Mathematics
Second Edition
Sophie Goldie
Luke Robinson
Andrew Ginty
Series Editor:
Steve Cavill
Boost
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Contents

CHAPTER 1

Sequences

Coming up...

- Arithmetic sequences
- Geometric sequences
- Sequences involving square numbers
- Sequences involving triangle numbers
- Fibonacci style sequences

Fibonacci squares

Cut out two 1 cm by 1 cm squares and place them side-by-side to form a rectangle.

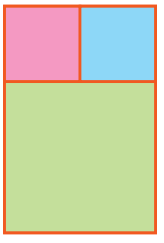
The longest side of this rectangle is 2 cm.

You could also do this by drawing the squares.

Cut out a 2 cm square and place it against the long edge of the rectangle to form a larger rectangle.

Repeat the process.

Look at the longest side of the new rectangle.



What size square do you need to cut out now? Place this square against the longest side of the rectangle to form a new rectangle.

Write down the length of each square's sides as a sequence:

1, 1, 2, ...

Continue the pattern until you have eight squares.

Hint: your eighth square should have a side length of 21 cm.

The numbers form a very famous sequence called the Fibonacci sequence, which you will learn about in Section 1.2.

1.1 Arithmetic sequences

Skill checker

① Continue these sequences for three more terms:

a 3, 6, 9, 12, ... **b** 45, 50, 55, 60, ... **c** 13, 23, 33, 43, ... **d** 11, 22, 33, 44, ...

② Substitute $n = 1$, $n = 2$ and $n = 3$ into these expressions:

a $3n$ **b** $n + 4$ **c** $5n - 1$ **d** $100n - 2$

Arithmetic sequences

Look at this sequence of numbers:

3, 6, 9, 12 ...

This sequence is based on the three times table.

The 1st term is $3 \times 1 = 3$.

The 4th term is $3 \times 4 = 12$.

The 10th term is $3 \times 10 = 30$.

The nth term is $3 \times n = 3n$.

$3n$ is called the formula or the position-to-term formula for the sequence. It is also sometimes called the formula for the nth term.

You can use the position-to-term formula to find any term of the sequence.

For example, to find the 20th term:

$3 \times 20 = 60$.

A sequence in which you add or subtract a fixed amount to move from one term to the next is called an **arithmetic sequence**.

In this arithmetic sequence the **term-to-term rule** is 'add 3'.

Worked example

Look at this sequence:

4, 7, 10, 13, ...

a What is the term-to-term rule?

b Find the position-to-term formula for the sequence.

c What is the 50th term of the sequence?

d Which term of the sequence is 100?

e What type of sequence is this?

Solution

a The term-to-term rule is 'add 3'.

b Each term is 1 more than the three times table so the **position-to-term formula** is $3n + 1$.

The sequence goes up in 3s, so is based on the three times table.

Look at this comparison with the previous example.

3	6	9	12	Formula $3n$
+1	+1	+1	+1	+1
4	7	10	13	Formula $3n + 1$

You can **check** this position-to-term formula works. Use $n = 1, 2, 3$ and 4 in the formula.

n	1	2	3	4
$3n + 1$	$3 \times 1 + 1 = \mathbf{4}$	$3 \times 2 + 1 = \mathbf{7}$	$3 \times 3 + 1 = \mathbf{10}$	$3 \times 4 + 1 = \mathbf{13}$

The table shows that the position-to-term formula gives the first four terms of the sequence.

c You can find the 50th term using the formula.

nth term $= 3n + 1$

50th term $= 3 \times 50 + 1$

50th term $= 151$

d To find which term of the sequence is 100, use the formula again.

Solve the equation to find n.

nth term $= 3n + 1$

$100 = 3n + 1$

$100 - 1 = 3n$

$3n = 99$

$n = 33$

The 33rd term of the sequence is 100.

e To move from one term to the next you add 3. This is an arithmetic sequence.

Remember

The difference tells you which times table the sequence is based on. For example, if there is a difference of 3 between each term, then $3n$ will appear in the position-to-term formula.

Worked example

a Find the **position-to-term formulas** for these sequences:

i $-2, -4, -6, -8, \ldots$

ii $-1, -3, -5, -7, \ldots$

b What type of sequence are these?

Solution

a i This is the -2 times table so the position-to-term formula is $-2n$.

ii Each term is one more than the terms in the -2 times table.

The position-to-term formula is $-2n + 1$.

Another way to write this is $1 - 2n$.

b In both sequences you subtract 2 to move from one term to the next.

They are both arithmetic sequences.

Activity

Read this newspaper report about Steel City. Somebody has spilt coffee on the page and some of the numbers are unreadable.

STEEL CITY BOOM TOWN!

It's a new year and the residents of Steel City are waking up this morning to the sound of yet more construction work.

Work has begun on the huge SC Tower, which will become the tallest building so far in Steel City.

We have contacted the city planners. They say that there are currently 14 skyscrapers in Steel City and there will be 44 skyscrapers in the city in five years' time.

They have given permission for [coffee] new skyscrapers EVERY YEAR!

If the city's plans become a reality, by the end of this year there will be [coffee] skyscrapers.

If construction work in Steel City continues at this rate, there will be 62 skyscrapers in [coffee] years' time.

In ten years' time the city will have [coffee] new skyscrapers!

Can the residents of Steel City put up with this much noise? Does Steel City have enough sandwich shops to feed all these hungry office workers? Let us know what you think.

a Can you replace the coffee marks with the correct numbers?

b What is the term-to-term rule for the sequence of the number of skyscrapers?

1.1 Now try these

Band 1 questions

Fluency

1 Find the next three terms in these arithmetic sequences.

a 2, 3, 4, 5, ...

b 0, 4, 8, 12, ...

c $-4, -2, 0, 2, \ldots$

d $-5, -8, -11, -14, \ldots$

e $4, 0, -4, -8, -12, \ldots$

f $0, -2, -4, -6, \ldots$

g $-0.1, -3.1, -6.1, -9.1, \ldots$

Fluency

2 Find the missing number or numbers in the arithmetic sequences below.

a 2, 4, 6, 8, ☐, 12, ...
b 5, 9, 13, ☐, 21, ...
c −3, −6, ☐, −12, −15, ...
d 21, 25.5, ☐, 34.5, 39, ...
e 0, −2, −4, ☐, −8, ...
f −1.5, −3, −4.5, ☐, −7.5, ...
g 30, 27.5, 25, ☐, 20, ☐, ...
h 8, ☐, 24, ☐, 40, ...
i 6, ☐, 18, ☐, 30, ...
j ☐, −15, ☐, −25, −30, ...

3 Find the formula for the nth term of these sequences.

In each case the formula is ☐n.

a 2, 4, 6, 8, 10, ...
b 4, 8, 12, 16, ...
c 5, 10, 15, 20, ...
d 1, 2, 3, 4, 5, ...
e 100, 200, 300, 400, 500, ...
f 10, 20, 30, 40, 50, ...
g 8, 16, 24, 32, ...

4 Find the formula for the nth term of these sequences.

In each case the formula is ☐$n + 1$.

a 6, 11, 16, 21, 26, ...
b 11, 21, 31, 41, 51, ...
c 101, 201, 301, 401, 501, ...
d 5, 9, 13, 17, ...

5 Find the formula for the nth term of these sequences.

In each case the formula is ☐$n - 1$.

a 99, 199, 299, 399, ...
b 9, 19, 29, 39, 49, ...
c 4, 9, 14, 19, 24, ...
d 7, 15, 23, 31, ...
e 3, 7, 11, 15, ...

Reasoning

6 Copy this spiral pattern onto isometric paper.

Start with the red dot near the centre of the page.

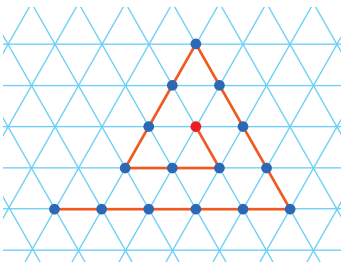
a Count the number of dots on each line that you draw.

Write them as a sequence: 2, 3, ...

b Predict how many dots will be on the next line that you draw.

Check by drawing and counting.

c Describe the pattern in the number of dots.



Fluency

7 Find the first five terms of these sequences, given their position-to-term formulas. Use the tables to help you.

a The sequence with position-to-term formula $n + 4$.

n	1	2	3	4	5
$n + 4$	1 + 4 = 5	2 + 4 = _			

b The sequence with position-to-term formula $n - 5$.

n	1	2	3	4	5
$n - 5$	1 − 5 = −4	2 − 5 = _			

c The sequence with position-to-term formula $2n$.

n	1	2	3	4	5
$2n$	2 × 1 = 5	2 × 2 = _			

d The sequence with position-to-term formula $-3n$.

n	1	2	3	4	5
$-3n$	−3 × 1 = _				

e The sequence with position-to-term formula $2n + 3$.

n	1	2	3	4	5
$2n + 3$	2 × 1 + 3 = 5	2 × 2 + 3 = _			

Fluency

f The sequence with position-to-term formula $2n - 4$.

n	1	2	3	4	5
$2n - 4$					

Reasoning

8 Ahmed has £16 in his money box at the start of the year.
He gets £5 pocket each week and puts it into his money box without spending any money.

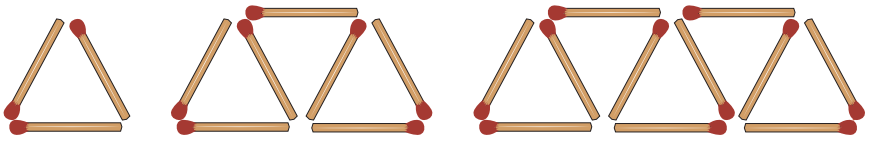
a Beginning with £16, write down the amount of money in Ahmed's money box as a sequence for 5 weeks.

b What is the term-to-term rule for this sequence?

c What is the position-to-term formula?

Fluency

9 Sean is making patterns from matchsticks.



Pattern 1 Pattern 2 Pattern 3

a Copy and complete this table for the number of matchsticks used in each pattern.
Include the number of matchsticks Sean would use if he continued to Pattern 4.

Pattern number n	1	2	3	4
Number of matchsticks	3			

b If the pattern is continued, which pattern will use 23 matchsticks?

c Find the term-to-term rule for this sequence.

d Find the position-to-term formula for the number of matchsticks.

Hint: you add four matchsticks to move from one pattern to the next. This means that $4n$ will be in your position-to-term formula.

Band 2 questions

Fluency

10 Find the formula for the nth term of these sequences.

a 4, 8, 12, 16, ...

b 5, 9, 13, 17, ...

c 3, 7, 11, 15, ...

d 6, 10, 14, 18, ...

e 9, 13, 17, 21, ...

Hint: this is the four times table.

Hint: these numbers are all one more than the numbers in the four times table.

Hint: these numbers are all one less than the numbers in the four times table.

11 Find the formula for the nth term of these sequences.

a 2, 4, 6, 8, 10, ...

b 4, 6, 8, 10, 12, ...

c 1, 3, 5, 7, 9, ...

Hint: this is the two times table.

Hint: these numbers are all two more than the numbers in the two times table.

12 Find the formula for the nth term of these sequences.

a 3, 6, 9, 12, 15, ... **b** 6, 9, 12, 15, 18, ... **c** −2, 1, 4, 7, 10, ...

13 Find the formula for the nth term of these sequences.

a 3, 4, 5, 6, 7, ... **b** −1, 3, 7, 11, 15, ... **c** −1, −2, −3, −4, −5, ... **d** −3, −6, −9, −12, −15, ...

Problem solving

14 The Olympic Games takes place every four years. The first Olympic Games took place in 1896.
A position-to-term formula to find the year of the nth Olympic Games is

$4n + 1892$

The table shows you how to use the position-to-term formula to find the year of the 2nd Olympic Games.

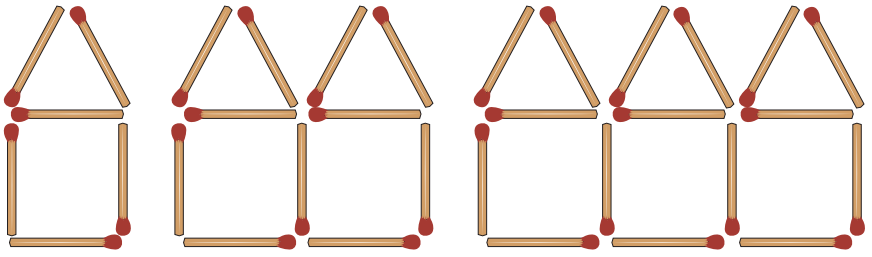
a Copy and complete the table to find the years of the 3rd, 5th and 10th Olympic Games.

n	2	3	5	10
Year	$4 \times 2 + 1892$ $= 1900$	$4 \times 3 + 1892$ $=$		

Some Olympic Games did not actually take place. For example the 13th Olympic Games in 1944 was cancelled because of World War II, but the 1948 Games was still called the 14th Games. The 2020 Games were postponed to 2021 because of the Covid-19 pandemic.

b Use the formula to find what number Olympics Games took place in London in 2012.

15 Christophe is making patterns of houses from matchsticks.



Pattern 1 Pattern 2 Pattern 3

a Copy and complete this table for the number of matchsticks used in each pattern.
Include the number of matchsticks Christophe would use if he continued to Pattern 4.

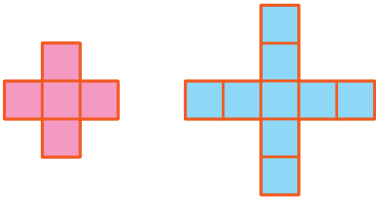
Pattern number n	1	2	3	4
Number of matchsticks	6			

b If the pattern is continued, which pattern will use:

i 26 matchsticks? **ii** 201 matchsticks?

c What is the term-to-term rule for this sequence?

16 Look at the patterns made up from square tiles.
Each pattern is in the shape of a cross.
Each tile measures 1 cm by 1 cm. The area of one tile is 1 cm^2.
The total area of Pattern 1 is 5 cm^2.
The total area of Pattern 2 is 9 cm^2.



Pattern 1 Pattern 2

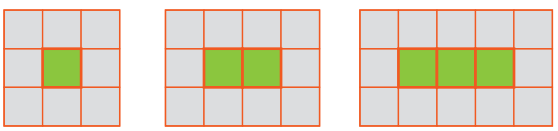
a If the pattern is continued, what is the area of Pattern 4?

b Which pattern will have an area of 101 cm^2?
The perimeter of Pattern 1 is 12 cm.
The perimeter of Pattern 2 is 20 cm.

c If the pattern is continued, what is the perimeter of Pattern 4?

d Which pattern will have a perimeter of 100 cm?

17 Peter is making patterns with square tiles. Some of the tiles are green, some are shaded.



Pattern 1 Pattern 2 Pattern 3

In Pattern 1, there is 1 green tile and there are 8 shaded tiles.
In Pattern 2, there are 2 green tiles and 10 shaded tiles.
In Pattern 3, there are 3 green tiles and 12 shaded tiles.

Reasoning

a How many **green tiles** will there be in Pattern 6?

b How many **shaded tiles** will there be in Pattern 6?

c How many tiles will there be in Pattern 6 **in total**?

d If Peter continued these patterns, which pattern would contain 80 shaded tiles?
How many green tiles would this pattern have?

Band 3 questions

Fluency

18 Find the formula for the nth term of these sequences.

a $-1, -3, -5, -7, -9, \ldots$

b $3, 2, 1, 0, -1, ..$

c $-6, -8, -10, -12, -14, \ldots$

d $-8, -13, -18, -23, -28, \ldots$

e $-5, -8, -11, -14, -17, \ldots$

f $-4, -9, -14, -19, -24, \ldots$

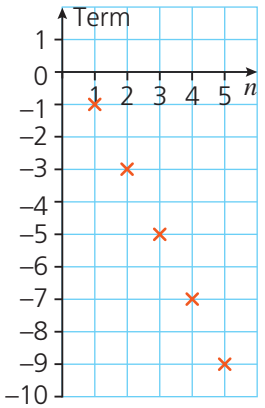
g $0, -1, -2, -3, -4, \ldots$

19 For each of the sequences in question 18:

a Enter the terms of the sequence into a table, like the one below.
Part **a** has been done for you:
The terms are $-1, -3, -5, -7, -9$.

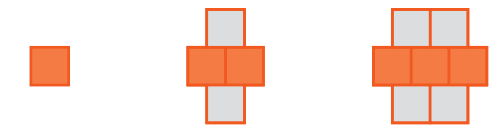
n	1	2	3	4	5
Term	-1	-3	-5	-7	-9

b Plot the first five terms in the sequence against its position number n on a graph.
For part **a**, plot the points $(1, -1), (2, -3), (3, -5), (4, -7), (5, -9)$.
What do you notice about all six of the graphs you have drawn?



Reasoning

20 Dayah is making patterns with square tiles. Some of the tiles are red, some are grey.



Pattern 1 Pattern 2 Pattern 3

a How many red tiles will there be in Pattern 6?

b How many grey tiles will there be in Pattern 6?

c If the patterns are continued, which pattern will have a total of 118 tiles?

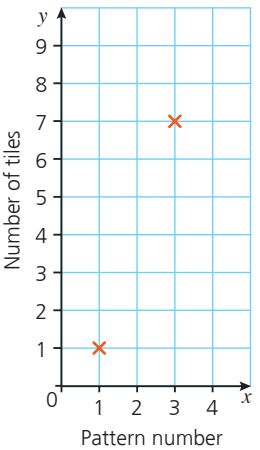
d Fill in the blank space below with a number:
The first __ patterns could be made using 30 grey and 21 red tiles.

e Dayah draws a graph to show the total number of tiles used for each pattern.
Copy and complete Dayah's graph.

f What do you notice about the points?

g Is Dayah's sequence arithmetic?

h Do you think she should join the points?
Why/why not?



Problem solving

21 Mr Bryant is driving from London to Manchester.
The total distance is 220 miles.
He drives at 55 miles per hour.

a Copy and complete this table.

Time taken (hours)	0	1	2		
Distance travelled (miles)	0	55			
Distance left (miles)	220				

Problem solving

b How many hours does it take Mr Bryant to drive to Manchester?

c **i** Write down the numbers in the second row of the table as a sequence.

ii What is the term-to-term rule for this sequence?

iii What is the position-to-term formula?

d **i** Write down the numbers in the third row of the table as a sequence.

ii What is the term-to-term rule for this sequence?

iii What is the position-to-term formula?

22 Mr Coulston has 500 bars of chocolate in his fridge, after buying them in a special offer from his local supermarket. He is now trying to give them away.

a On Day 1 Mr Coulston gives away 40 bars of chocolate. He has 460 bars of chocolate left.

On Day 2 Mr Coulston gives away another 40 bars of chocolate. How many bars of chocolate does he have left?

b He continues giving away 40 bars of chocolate each day. Copy and complete this table.

Day n	1	2	3	4	5	6	7
Number of bars of chocolate left in fridge	460						

c The number of bars of chocolate left forms a sequence. Find the term-to-term rule for the sequence.

d Find the position-to-term formula for the number of bars of chocolate.

Reasoning

23 **a** Make as many arithmetic sequences with five terms as you can from these numbers:

1 3 5 6 10 15 21 20 25 9 16 8 2 32 12 4

b For each sequence you have made, work out the position-to-term formula.

Problem solving

24 Purvi's clock has gone crazy!

At 12:01 pm it said 5:32 pm.

One minute later it said 5:34 pm.

One minute after that it said 5:36 pm.

It seems to be moving forward 2 minutes every minute!

a Copy and complete this table.

Real time	12:01 pm	12:02 pm	12:03 pm			
Time shown on clock	5:32 pm	5:34 pm				

b Look at just the minutes part of the times shown on the clock.

They form this sequence:

32, 34, 36, ...

What is the term-to-term rule for this sequence?

c What is the position-to-term formula?

d What time is shown on the clock when the real time is 12:14 pm?

e What is the real time when the clock shows 5:50 pm?

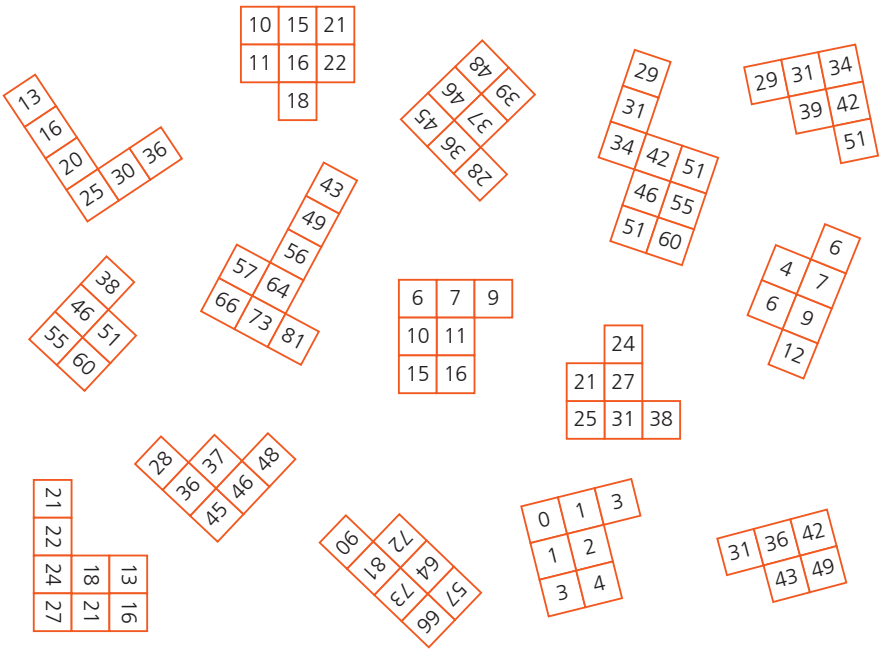
1.2 Other sequences

Skill checker

① Make a copy of these jigsaw pieces.

a Can you put them together to make a square?

b What number sequences can you see in the square?



Other sequences

In Section 1.1 you learnt about **arithmetic sequences**.

There are many other types of sequence.

Worked example

Look at this sequence:

1, 2, 4, 8, 16, ...

a What is the term-to-term rule to move from one term to the next?

b Find the next two terms.

Solution

a In this sequence you **multiply by 2** to move from one term to the next.

This is called a **geometric sequence**.

b The next two terms are 32 and 64.

Worked example

a Look at this special sequence of numbers.

1, 4, 9, 16, 25, ...

i What are these numbers?

ii What is the position-to-term formula for this sequence?

b Now look at this sequence:

2, 5, 10, 17, 26, ...

What is the position-to-term formula for this sequence?

Solution

a **i** This is a sequence of the square numbers.

ii The position-to-term formula for the sequence is n^2

b Each term in this sequence is 1 more than the square numbers.

The position-to-term formula for this sequence is $n^2 + 1$

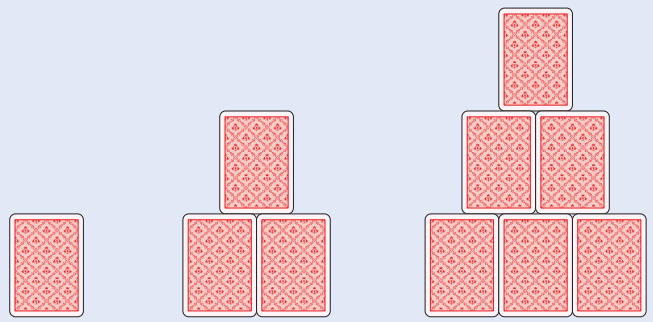
Worked example

Samantha lays one playing card on the table.

For her second pattern, she adds an extra row of two cards, as shown below.

This makes a triangle pattern.

For her third pattern, she adds an extra row of three cards, as shown below.



Pattern 1 Pattern 2 Pattern 3

She carries on making triangular patterns by adding an extra row at the bottom.

a How many playing cards will there be in Pattern 4?

b Which pattern will contain 21 playing cards?

c Write down the number of cards in the first six patterns as a sequence.

Solution

a In Pattern 4 there will be an extra row of 4 cards.

In total there will be 10 cards.

b In Pattern 5 there will be an extra row of 5 cards, making 15 in total.

In Pattern 6 there will be an extra row of 6 cards, making 21 in total.

Pattern 6 has 21 cards.

c The number of cards in each pattern forms this sequence:

1, 3, 6, 10, 15, 21, ...

The sequence of numbers in the previous example are called **triangle numbers**.

This formula helps you to work out triangle numbers:

$$T = \frac{n(n+1)}{2}$$

For example, to work out the fourth triangle number:

$$T = \frac{4(4+1)}{2} = 10$$

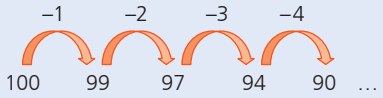
Worked example

a What number comes next in this sequence?

100, 99, 97, 94, 90, ...

b How many positive terms are there in this sequence?

Solution



a The term-to-term rule is subtract 1, 2, 3, 4, 5, ...

The next term is 85.

b Continuing the sequence:

100, 99, 97, 94, 90, 85, 79, 72, 64, 55, 45, 34, 22, 9, –5, ...

There are 14 positive terms.

Worked example

a Look at this sequence:

1, 1, 2, 3, 5, 8, 13, ...

Each term in this sequence is the sum of the two numbers before it.

This sequence is called the **Fibonacci sequence**.

Find the next three terms.

b Other sequences can be formed in the same way.

Find the next two terms in this Fibonacci-type sequence:

4, 1, 5, 6, 11, ...

Solution

a $8 + 13 = 21$

$13 + 21 = 34$

$21 + 34 = 55$

The next three terms are 21, 34, 55.

b Since this is a Fibonacci-type sequence, a term can be found be adding the two terms before it:

$6 + 11 = 17$

$11 + 17 = 28$

The next two terms are 17 and 28.

Fibonacci sequences frequently appear in natural and unexpected places, such as in the patterns of seeds in pine cones and sunflowers.

1.2 Now try these

Band 1 questions

Fluency

1 Find the next three terms in these geometric sequences.

a 1.5, 3, 6, 12, ...
b 0.2, 0.6, 1.8, 5.4, ...
c −1, −3, −9, −27, ...
d 10, 40, 160, ...
e 100, 50, 25, ...

2 Find the next three terms in these Fibonacci-type sequences.

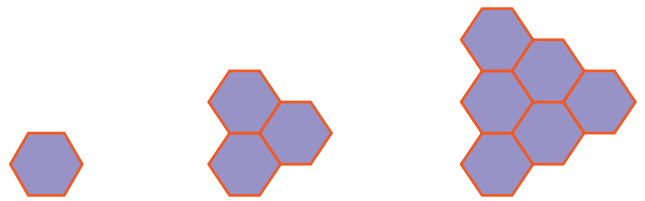
a 1, 3, 4, 7, ...
b 2, 2, 4, 6, ...
c 6, 4, 10, 14, ...
d 3, 1, 4, 5, ...
e 10, 20, 30, 50, ...

3 Are these sequences arithmetic or geometric? In each case, what is the term-to-term rule?

a −2, 1, 4, 7, 10, 13
b 20, 10, 5, 2.5, 1.25, 0.625
c 20, 16, 12, 8, 4, 0
d −3, 6, −12, 24, −48, 96
e −8, −10, −12, −14, −16, −18

4 Hattie is making patterns with hexagonal tiles.

a How many hexagons will Hattie use in Pattern 4?
b Write down the number of hexagons in each pattern as a sequence.
What is the name of this sequence?



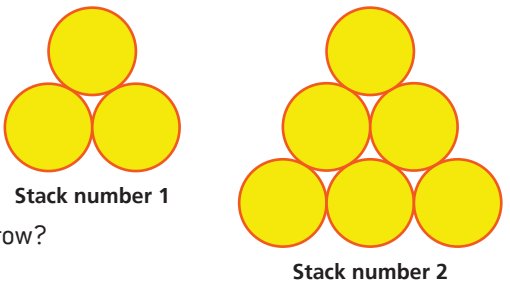
Reasoning

5 Bronagh is making patterns from dots.
Look at her first three patterns below.

a How many dots will there be in Pattern 5?
b What special name is given to this sequence?

Problem solving

6 In a carpet showroom, they stack the rolls of carpet in triangular shapes. The first and second stacks are shown on the right.
Each triangular stack is bigger than the last, with one more roll of carpet in the bottom row each time.



a Which number stack will have five rolls of carpet in the bottom row?
b Which number stack will contain 28 rolls of carpet?

Band 2 questions

Fluency

7 Look at the sequences below. Copy and complete the sequences by writing the missing numbers in the spaces.

a 14, 17, ☐, 26, 32, 39, ☐, 56
b 30, 29, 27, 24, ☐, 15, 9, ☐, −6
c 40, 37, 33, 28, ☐, 15
d 26, ☐, 35, 41, 48, 56, ☐, 75
e 0, 3, 8, 15, ☐, ☐, 48

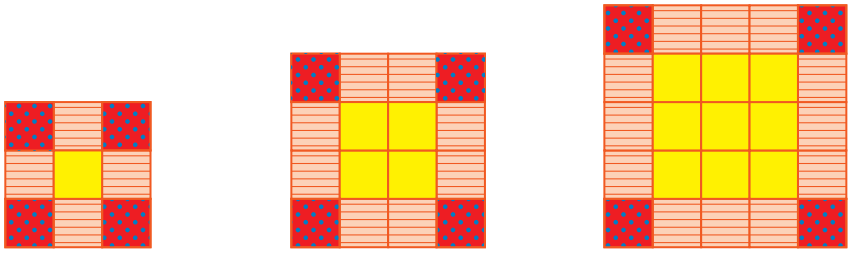
Reasoning

8 Fill in the blanks in these Fibonacci-type sequences:

a 1, 1, ☐, 3, ☐, ☐, 13
b 13, 21, ☐, 55, ☐, 144
c 2, 6, 8, ☐, 22, 36, ☐, 94, 152
d 4, ☐, 11, ☐, 29, 47, 76, 123
e ☐, 7, 10, ☐, 27, 44, 71, ☐, 186
f ☐, 4, ☐, 11, 18, 29, 47, 76, 123
g ☐, ☐, 7, ☐, 19, 31, 50, 81

9 Rowan is making patterns with square tiles. The tiles are:

Red and spotty  Orange and stripy  Yellow 



Pattern 1 **Pattern 2** **Pattern 3**

Each pattern is in the shape of a square. Only the corners of each pattern are red.

a Write down the first three terms in the sequence for the number of yellow tiles.

b What special name does this sequence have?

c Write down the first three terms in the sequence for the number of orange tiles.

d Write down the first three terms in the sequence for the **total** number of tiles.

e How many yellow tiles will there be in Pattern 5?

f How many orange tiles will there be in Pattern 5?

g How many tiles will there be in Pattern 5 **in total**?

10 James is making patterns from black beads and red beads. Look at his first three patterns below.

In Pattern 1, there is 1 black bead.

In Pattern 2, there are 4 black beads and 1 red bead.

In Pattern 3, there are 9 black beads and 4 red beads.

Pattern 1 **Pattern 2** **Pattern 3**

a How many black beads will be used in Pattern 5?

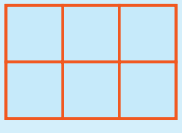
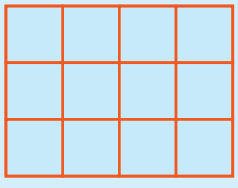
b How many red beads will be used in Pattern 5?

11 Fill in the blanks in these Fibonacci-type sequences:

a 2, 7, ☐, 16, ☐, 41 **b** ☐, 8, 9, 17, ☐, 43 **c** 8, ☐, ☐, ☐, 52 **d** 8, ☐, ☐, 52

12 Here are some rectangles.

a Copy and complete the table, giving the length, width and area of each rectangle.

Shape	Length (cm)	Width (cm)	Area (cm²)
	2	1	2
			
			

Reasoning

b Add one more row to your table. Draw the next rectangle and fill in its length, width and area.

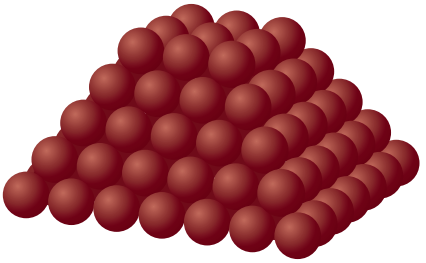
c The numbers in the area column form a sequence.

It is closely related to one of the sequences you have already learnt about.

What is the sequence?

Problem solving

13 'The Ambassador is spoiling us!' said Mrs Pelligrew.



Mrs Pelligrew was one of the guests at the Ambassador's party for Very Important People.

She had seen the Ambassador's butler coming towards her and her group of friends. He was carrying a tray stacked with round chocolates.
The entire stack was in the shape of a pyramid, although the top part of the pyramid was missing.

There were five layers of chocolates on the tray, each layer in the shape of a square.

The layer at the bottom of the pyramid was a square seven chocolates long and seven chocolates wide.

Above that was a square measuring six chocolates by six chocolates, and so on.

The butler had already offered chocolates to several guests, and this is why some of the top layers of the pyramid were missing.

a How many chocolates are in the bottom layer?

b How many chocolates are in the next layer up?

c Write down a sequence of five terms for the number of chocolates in each layer.

d How many layers are missing?

e How many chocolates would have been in those missing layers?

Band 3 questions

Fluency

14 Find the first four terms of the sequences with these formulas.

a $n^2 + 2$ **b** $n^2 + 10$ **c** $100 - n^2$ **d** $n^2 - 5$ **e** $16 - n^2$

Reasoning

15 Is the following statement always true, never true or sometimes true?

Adding two consecutive triangle numbers always gives a square number.

Explain your answer.

Consecutive means next to each other.
For example:
- 7 and 9 are consecutive odd numbers.
- 40 and 50 are consecutive multiples of 10.

Problem solving

16 Joanne is organising a raffle for her youth club.

To win a prize, the number on the ticket must be a triangle number.

a Do these tickets win prizes?

17	20	1	9
13	45	50	7

b Norman thinks he knows a way to cheat in the raffle.

He knows the formula for the triangle numbers:

$$T = \frac{n(n+1)}{2}$$

The tickets for the first seven triangle numbers have already been sold.

Norman works out the 8th triangle number and asks if he can buy this ticket.

What number ticket does he ask for?

Reasoning

17 Use the formula for the nth triangle number

$$T = \frac{n(n+1)}{2}$$

to work out:

a The sum of the first 10 positive integers.

b The sum of the first 20 positive integers.

c The sum of the first 100 positive integers.

18 On Cow Island, all cows are female and they never die.

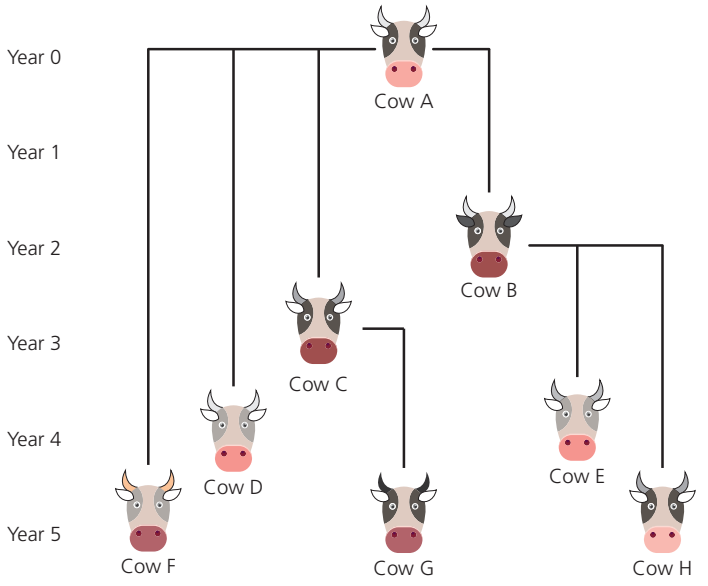
A cow produces her first female calf at the age of two years.

After that she produces one female calf every year.

The diagram shows the cow family tree.

Cow A is the first cow on the island in the year 0.

At two years old, she gives birth to Cow B, and then to Cow C in Year 3.



Cows alive	Total
A	1
A	1
A, B	2
A, B, C	

a Copy and complete the table, with three columns: Year, Cows alive and Total.

b How many cows give birth in Year 6?

c How many cows will there be in total in Year 6?

Remember, a cow must be 2 years old or more to give birth.

d The total number of cows forms a sequence. What is the name of this sequence?

19 The inventor of the game of chess took his invention to the king. The king loved the game so much he told the inventor he could have anything he wished for.

'All I want,' said the inventor, 'Is one grain of rice on the first square of the chess board, two grains on the second square, four on the third, eight on the fourth and so on.'

The king thought this sounded like a fairly small reward. He knew there was a lot of rice in the royal palace stores.

Do you think the inventor got what he asked for?

You can use your calculator to help you answer this question. There are 64 squares on a chess board.

Key words

Here is a list of the key words you met in this chapter.

Arithmetic sequence
Fibonacci sequence
Fibonacci-type sequence
Geometric sequence
Position-to-term formula
Square numbers
Term
Term-to-term rule
Triangle numbers

Use the glossary at the back of this book to check any you are unsure about.

Review exercise: sequences

Band 1 questions

Fluency

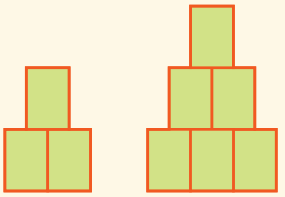
1 Toby is stacking tins of paint in a DIY shop.

He begins by making a triangle with the tins as shown in Pattern 1.

He then makes a second stack of tins as shown in Pattern 2.

Toby notices there are two tins in the bottom row of his first pattern and three tins in the bottom row of his second pattern.

The table below shows these results.



Pattern 1 Pattern 2

Pattern number	1	2
Number of tins in bottom row	2	3

a If Toby keeps making triangle patterns, **which pattern number** will give him **six tins in the bottom row**?

b Which pattern number will have a **total** of 15 tins?

2 Find the next term and the term-to-term rule for each of these sequences.

a 2, 5, 8, 11, 14, ... **b** 1, 10, 100, 1000, ... **c** 50, 47, 44, 41, ... **d** 9, $10\frac{1}{2}$, 12, $13\frac{1}{2}$, ...

e 1, 3, 7, 15, ... **f** 32, 16, 8, 4, 2, ... **g** 2, 6, 18, 54, ... **h** 64, 160, 400, ...

Reasoning

3 **a** A sequence has the position-to-term formula $2n - 6$.

Find the first five terms of this sequence. Use the table below to help you.

n	1	2	3	4	5
$2n - 6$	$2 \times 1 - 6 = -4$	$2 \times 2 - 6 = ...$			

b Match the position-to-term formulas on the left with the correct sequences on the right.

You can use tables like the one above to help you.

$2n - 6$	1, 4, 9, 16, 25, ...
$3n + 2$	5, 8, 11, 14, 17, ...
n^2	−4, −2, 0, 2, 4, ...
$4n$	2, 3, 4, 5, 6, ...
$n + 1$	4, 8, 12, 16, 20, ...

Problem solving

4 A car is travelling at 60 miles per hour along a straight road.

The driver sees a red traffic light ahead and applies the brakes.

The car slows down, losing 5 miles per hour of its speed every second.

a Copy and complete this table to show how fast the car was travelling throughout the following 7 seconds.

Number of seconds after braking	1	2	3	4	5	6	7
Speed (miles per hour)	55						

b How fast is the car going after 10 seconds?

Band 2 questions

Fluency

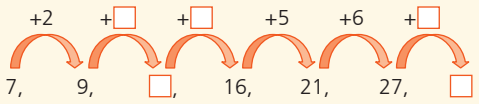
5 For each of these sequences:

i Copy the sequence, the arrows, the differences and boxes above the arrows.

ii Fill in the boxes above the arrows, showing the differences between the terms.

iii Fill in the missing terms in the sequence.

a



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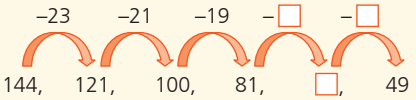

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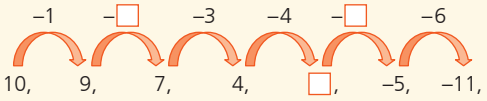

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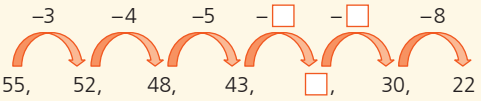

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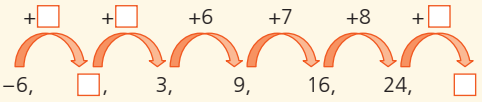

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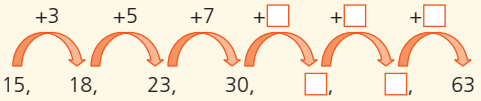

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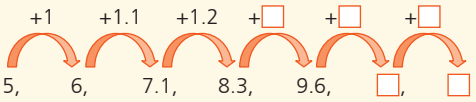


Problem solving

6 A boy drops a stone off a cliff towards the sea below.

Every second, the speed of the stone increases by 10 metres per second.

a Copy and complete the table below.

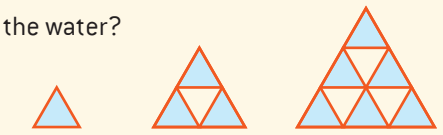
Number of seconds after stone was dropped	1	2	3
Speed (metres per second)	10		

b The stone takes 6 seconds to reach the sea.

How fast was the stone travelling when it hit the water?

Reasoning

7 Look at this sequence of triangular patterns.



a Draw the next two patterns.

b Copy and complete this table.

Pattern	A Number of blue triangles	B Number of white triangles	C Total number of triangles
1			
2			
3			
4			
5			

c What is the name for the sequence of numbers in columns A and B of your table?

d How many blue and white triangles do you think there would be in Pattern 6?

e Draw Pattern 6 to check your answers to part **d**.

f What is the name of the sequence of numbers in column C of your table?

g Without drawing it, how many triangles in total do you think there are in pattern 10?

8 The first two terms of a sequence are 1 and 4.

How many different term-to-term rules can you think of that would give this sequence?

Problem solving

9 Jake is climbing up the stairs at home. There are 21 stairs.

He starts at ground level, up 0 stairs.

Jake's sister Eva claps. Every time Eva claps, Jake climbs five steps higher, but then comes two steps back down.

a Starting with 0, write down the first six terms in the number sequence that describes Jake's height just before each clap.

b Eventually Jake reaches the top of the stairs. How many times did Eva clap?

Fluency

10 Look at the three term-to-term rules below.

i Multiply by 2

ii Add $1/2$

iii Add $1\,1/2$

Now look at these number sequences.

Match each of the sequences to one of the term-to-term rules above.

Two of the sequences do not match a rule. What are the rules for these ones?

a $1\,1/2, 2, 2\,1/2, \ldots$ Rule _____

b $1\,1/2, 2\,1/2, 4\,1/2, \ldots$ Rule _____

c $1\,1/2, 3, 6, 12, \ldots$ Rule _____

d $2\,1/2, 5\,1/2, 11\,1/2, \ldots$ Rule _____

e $2\,1/2, 4, 5\,1/2, \ldots$ Rule _____

Band 3 questions

Fluency

11 Complete this crossword with the clues below.

Across:

1 The third, fourth, fifth and sixth Fibonacci numbers

3 The first, second and third Fibonacci numbers

5 The eighth number in the sequence with position-to-term formula $11n$

7 The first three triangle numbers

8 The fifth number in the geometric sequence in 1 Down

9 The seventh square number

Down:

1 The first three terms in the geometric sequence with the term-to-term rule 'Multiply by 2' and first term 2

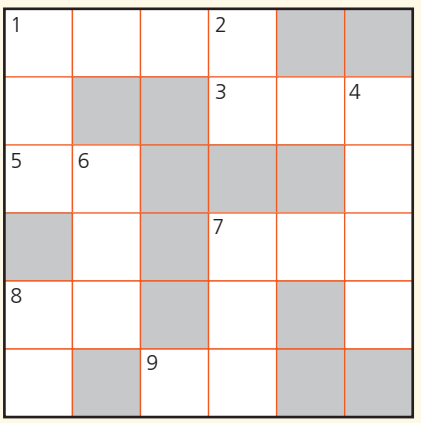
2 The ninth square number

4 The first four terms in the arithmetic sequence with the term-to-term rule 'Add 2' and first term 2

6 The first three terms in the sequence with position-to-term formula $-3n + 11$

7 The first three square numbers

8 The sixth square number



Problem solving

12 A bus leaves the city centre bus station with 80 people on board.

At each stop on its route it drops off five people and picks up one person.

a Write down the number of people on the bus after it leaves the first five stops.

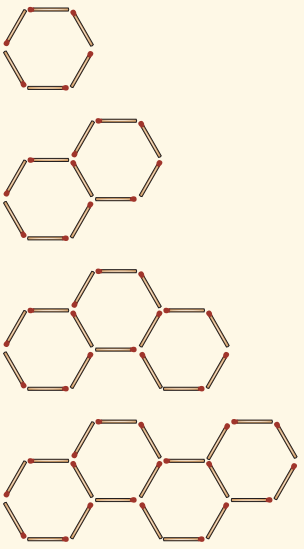
Begin with 80 and write the numbers as a sequence.

b How many people are left on the bus after nine stops, including the bus station.

c Find a formula for the number of people left on the bus after n stops.

d After how many stops are there 28 people on board?

13 Look at this sequence of hexagonal matchstick patterns.



a Draw the next pattern.

b Copy and complete this table for the first five patterns.

Number of hexagons	1	2	3	4	5
Number of matchsticks	6				

c Predict the number of matchsticks in six hexagons using the patterns in the table.

d What is the term-to-term rule for the number of matchsticks?

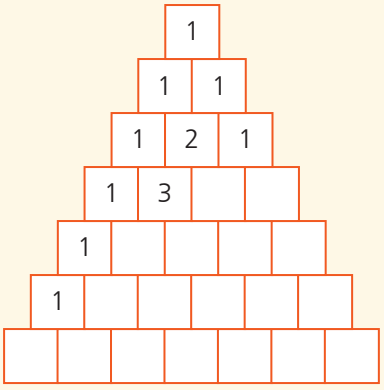
e What is the position-to-term formula?

f Zoe has 43 matchsticks.

Can she use all of them to make one pattern?

14 Sanjay has built the triangle of bricks below.

This is a famous triangle called Pascal's triangle.



Each brick is the sum of the two bricks above it.

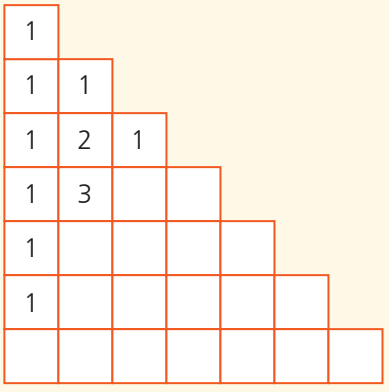
a Copy and complete the triangle.

Pascal's triangle is full of surprises.

b Can you find an arithmetic sequence?

c Can you spot another sequence discussed in this chapter?

Sanjay now slides all the bricks to the left to make a right-angled triangle.



d Copy and complete this diagram showing the new pattern of bricks.

e Sanjay says he can work out the Fibonacci numbers using this new diagram.

Can you see how he does it?

15 Square brackets around a number mean: round down to the next whole number.

For example:

$[3.5] = 3$

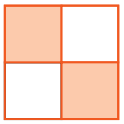
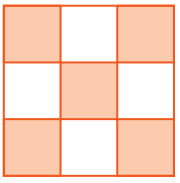
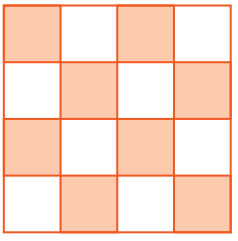
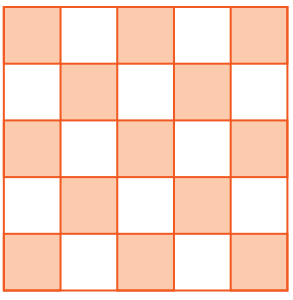
$[4] = 4$

$[10.95] = 10$ ← Integers don't change.

a If $n = 5$ what is:

i $\left[\frac{n}{2}\right]$ **ii** $\left[\frac{n^2}{2}\right]$

b Look at the sequence of patterns below.
Copy and complete the table.

Shape number	Shape	Number of orange squares	Number of white squares
1		1	0
2		2	2
3			
4			
5			

c If n is the shape number, which of these position-to-term formulas gives the number of white squares?
Which formula gives the number of orange squares?

$[2n]$ $\left[\frac{n}{2}\right]$ $[n^2]$ $\left[\frac{n^2}{2}\right]$ $\left[\frac{n^2}{2}\right]+1$

KEY STAGE 3

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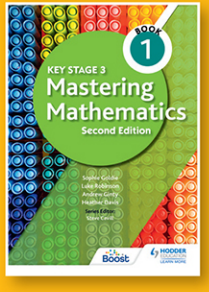

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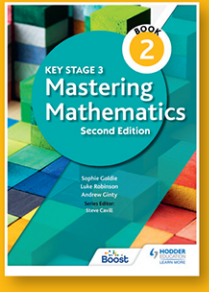

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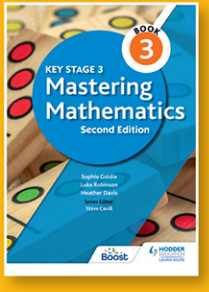

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