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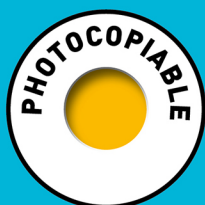


KEY STAGE

3

MATHEMATICS

Questions to support NCETM
Teaching for Mastery



BOOK 1

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Get the most from this book

This book contains questions which are designed to support teachers who are implementing aspects of the NCETM Teaching for Mastery approach in their classrooms. The questions utilise aspects of the five big ideas of mastery:

- Coherence
- Representation and structure
- Mathematical thinking
- Fluency
- Variation

The book consists of a series of topics, each of which have been broken down into Key Learning Points (KLPs). These have been designed to aid progression through topics in a series of well-planned, small steps that all students can work through together. Both the order of topics and KLPs are also aligned with the Kangaroo Schemes of Work.

A complete list of references links each KLP to the relevant NCETM and National Curriculum Key Stage 3 and GCSE content. This can be accessed for free online at www.hoddereducation.co.uk/masteryquestions.

The questions have been written based on a lesson design structure from Steve Lomax (@MaxtheMaths). The twelve questions for each Key Learning Point have been broken down into three types of exercise: Do It, Secure It, and Deepen It. These sections also align with the Key Stage 3 Programme of Study aims of Fluency, Reasoning and Problem Solving.

Within the Key Learning Points, questions have been crafted to support students in developing a **coherent** understanding of each KLP and the connections between them.

The first 'Do It' section of questions comprises of standard questions, many of which have subsections in which principles of **variation** theory have been applied. In these questions, students are encouraged to notice what is the same, and what is different. By spotting the connections between the series of sub-questions, students can explore and understand the underlying mathematical structures.

These are followed by 2 non-standard questions. These include questions written in a more unusual format, or those that require a higher level of number skills such as the use of negatives, decimals or fractions.

The 'Secure It' section usually contains one question that addresses misconceptions and one that specifically requires mathematical reasoning. These may be a multiple choice format, reasoning about and correcting mistakes, or answering more open questions such as whether statements are 'always, sometimes or never true'.

By considering 'what it is' alongside 'what it is not' students are supported to develop procedural **fluency** alongside a conceptual understanding of the structures involved.

The final 'Deepen It' section contains questions that encourage students to make connections between areas of mathematics, applying skills to different contexts or considering real life applications. These questions may also include open-ended problems, or algebraic generalisations to test whether students have developed a deep understanding of the KLP. In this section, students must think deeply about the mathematics involved, developing their **mathematical thinking** skills.

These questions are not designed as a worksheet to be completed in one sitting, but be broken up and used throughout lessons following teacher input phases. Some questions require specific **representations and structures** to be used, but more often it is left up to the teacher to decide. This flexibility allows teachers to select and demonstrate the representations which they prefer their students to use. Students should be encouraged to use representations and structures throughout their work, regardless of whether questions specifically demand them.

The answers for all questions can be found online at www.hoddereducation.co.uk/masteryquestions to allow for ease of marking and immediate feedback.

- 1 Ben is trying to guess what quadrilateral Meena is thinking of. Their conversation is below. What type of quadrilateral is Meena thinking of?

Ben: Does it have two pairs of parallel sides?

Meena: Yes.

Ben: In that case, are all the angles equal?

Meena: Yes.

Ben: Finally, are all its sides equal?

Meena: Yes.

- 2 a) Pete draws a quadrilateral with four right angles. What types of quadrilateral can it be?

b) Dolores draws a quadrilateral with opposite sides parallel.
What types of quadrilateral can it be?

- 3 Complete this table.

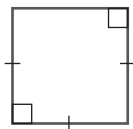
	Name of Shape	How many pairs of parallel sides?	How many pairs of equal sides?
			
	Square	Two pairs	
			
			
			
		None	
			

- 4 Describe these shapes as fully and accurately as you can.

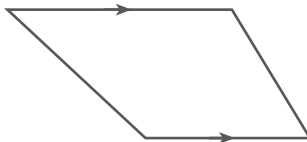
a)



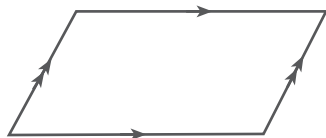
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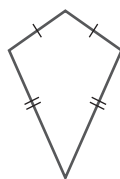
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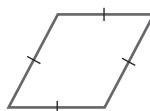
d)



e)



f)

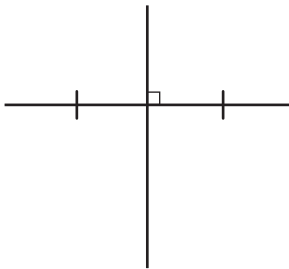


5 a) Which quadrilaterals have diagonals that bisect each other?

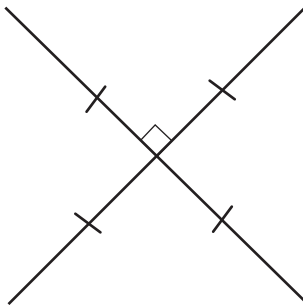
b) Which quadrilaterals have diagonals that intersect at right angles?

6 Identify these quadrilaterals by considering their diagonals.

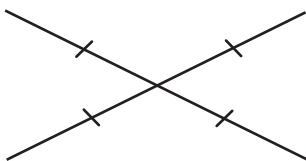
a)



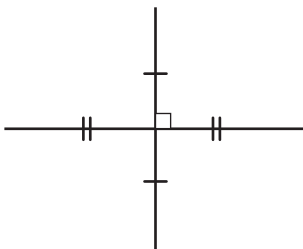
b)



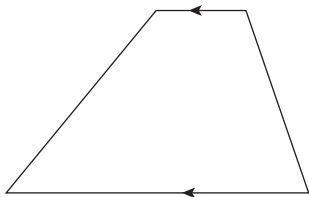
c)



d)



1 Mandy says there is only one type of trapezium. She draws this scalene trapezium.



Nithin says he can draw three types of trapezia.

He draws a scalene trapezium, an isosceles trapezium and a right-angled trapezium.

Draw Nithin's trapezia. Use geometric notation to show their properties.

2 Say whether each of these statements is true or false. Explain your answer.

a) A parallelogram with a right angle is a rectangle.

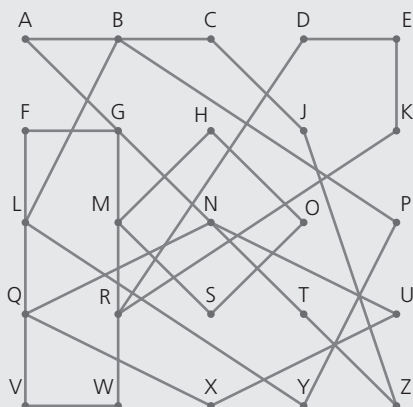
b) A trapezium with a right angle is a rectangle.

c) A rectangle with equal sides is a square.

d) Every kite is also a rhombus.

e) An arrowhead cannot have an obtuse angle.

1 Look at this pattern:



Find a square, a trapezium, a rhombus, a rectangle, a kite and a parallelogram.

For example, HOSM is a square.

No letter is used more than once.

Which letter is not used?

2 a) Place these various types of quadrilateral in this two-way table.

Square Rectangle Rhombus Parallelogram Kite Arrowhead Trapezium

Angles \ Parallel sides	None equal	One pair equal	Two pairs equal	All angles equal
None				
One pair parallel				
Two pairs parallel				

b) You should find that three of the boxes contain two quadrilaterals each. In each of these cases, give a further test to tell them apart.

- 3 The points A(1,6), B(5,4) and C(5,2) are plotted on a grid.

Write down the coordinates of point D when ABCD is:

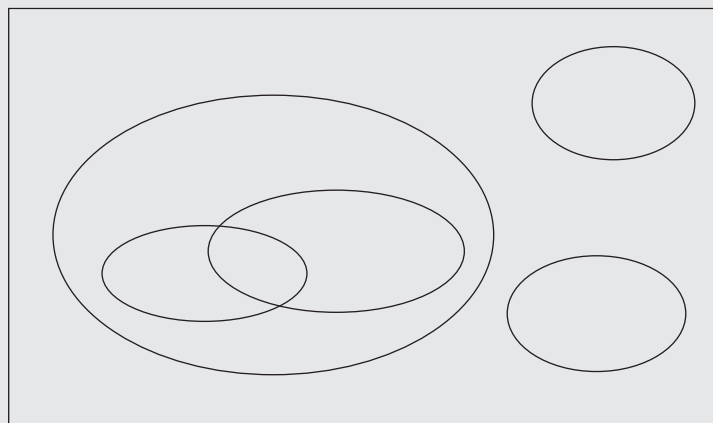
a) a parallelogram

b) a kite

c) a trapezium containing a right angle.

- 4 Place these words into the Venn diagram:

Kite, parallelogram, quadrilaterals, rectangle, rhombus, square, trapezium.





1 Calculate:

a) $\frac{9}{4} \div 3$

b) $\frac{9}{4} \div 4$

c) $\frac{9}{4} \div 5$

2 Calculate, giving answers as mixed numbers:

a) $\frac{8}{3} \div \frac{3}{5}$

b) $\frac{8}{3} \div \frac{4}{5}$

c) $\frac{8}{3} \div \frac{5}{5}$

3 Calculate, giving answers as mixed numbers:

a) $\frac{7}{3} \div \frac{9}{5}$

b) $\frac{7}{3} \div \frac{8}{5}$

c) $\frac{7}{3} \div \frac{7}{5}$

4 Calculate, giving answers as mixed numbers when appropriate:

a) $\frac{5}{4} \div \frac{9}{8}$

b) $\frac{5}{4} \div \frac{10}{8}$

c) $\frac{5}{4} \div \frac{11}{8}$

5 Carolyn has $\frac{24}{9}$ m of ribbon.How many pieces of length $\frac{2}{3}$ m can be cut from it?

6 Simplify $\frac{a}{b} \div \frac{c}{d}$

1 Rob is calculating $\frac{5}{3} \div \frac{5}{2}$
Here is his working:

$$\frac{5}{3} \div \frac{5}{2} = \frac{3}{5} \times \frac{5}{2} = \frac{3}{2}$$

What mistake has Rob made?

2 $\frac{3}{4} \div \frac{\square}{5} < 1$

Fill in the blank using an integer.

Find another integer that makes the inequality true.

How many solutions are there?

Explain how you know.

DEEPEEN IT

- 1 Asher has a bag of sweets weighing $\frac{15}{8}$ of a kilogram.

He wants to share them equally between himself and 4 friends.

What weight of sweets will they each get?

- 2 Mrs Jackson's garden path is $\frac{49}{6}$ m long.

Each of her steps is $\frac{7}{9}$ m long.

How many steps does it take for Mrs Jackson to walk down her garden path?

- 3 The area of a rectangle is $\frac{7}{3}$ m².

The length of the rectangle is $\frac{5}{2}$ m.

What is the width of the rectangle?

4 $a = \frac{9}{5}$

$b = \frac{3}{10}$

How many times bigger is a than b ?



- 1 Find the range of these data sets:

a) 14, 18, 13, 11, 16, 14

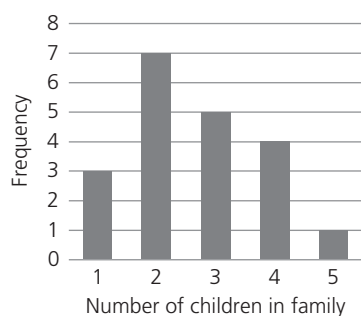
b) 209, 201, 203.5, 212.5, 206

c) 4.3, 4.26, 4.5, 4.15, 4.32, 4.61

- 2 Find the range of this data set:

-2, 5, 8, 3, 0, -4, 1

- 3 What is the range of the number of children in the families surveyed?



- 4 Three athletes enter the long jump trial for their club.

Athlete	Length of jump (metres)									
John Wells	3.45	4.12	3.56	3.75	2.96	2.77	3.98	3.25	3.78	3.44
Dipesh Raj	2.91	3.46	3.75	2.88	3.24	3.51	3.63	2.99	3.33	3.22
Zak Trent	4.02	3.55	4.06	4.03	3.70	3.84	3.99	4.03	3.24	3.53

- a) How many jumps does each athlete complete?

- b) Find the median length of jump for each athlete.

- c) Find the range for each athlete.

- d) Which athlete has the longest jump?

- e) Which athlete would you choose to represent the club?

Give a reason for your choice.

- 5 Here is a data set:

9, 4, 6, 5, 8,

If the range of the data is 7, find two possible values for the missing number.



- 6 Three penguins, Bertie, Bill and Gertrude, were weighed at the zoo one week ago.

	Bertie	Bill	Gertrude
Weight (pounds)	26	19	33

- a) Put the weights in order, starting with the smallest.

- b) What is the median weight?

- c) What is the range?

Over the next four days, Gertrude lays 3 eggs and now weighs 25 pounds.

- d) What is the new median weight of the three penguins?

- e) How has the range changed?

- 1 Here are the number of goals scored by the school football team so far this season:

3, 1, 2, 0, 4, 8

Rob calculates the range to be 5.

Rob says a high range shows the team are consistent.

Rob is wrong.

Explain why.

- 2 Dawn and James both choose five cabbage plants.

They count the numbers of caterpillars on them. Their results are shown below.

Dawn 2, 0, 5, 14, 10

James 7, 2, 5, 3, 1

Whose data is more spread out?

- 1 Caroline is taking a typing test.

She types for one minute and the number of correct words is recorded.

She does this six times. Here are her results:

46, 49, 44, 43, 45, 49

She must have a range of less than 7 and a median of over 45 in order to pass. Does she pass?

2 Janice plays cricket for an under-12 team. These are her scores in eight games:

2, 4, 10, 9, 20, 5, 30, 2

a) Say whether each statement is true or false for this data set and explain how you know:

i) The mode is 2.

ii) The median is 7.5.

iii) The range is 16.

b) Is Janice good at cricket? Use the correct figures from part a in your answer.

3 Mike has two potential sites for his hot dog stand.

He spends a week at each site and records the number of hot dogs he sells each day.

Site A 56, 45, 32, 29, 49, 25, 61

Site B 12, 74, 35, 22, 69, 10, 44

a) Find the median and the range for each site.

Mike chooses site A.

b) Do you agree with his choice? Explain your answer.

4 Find 4 numbers that satisfy these criteria:

mode < median < mean < range
