CURRICULUM FOR WALES

MATHEMATICS AND NUMERACY

Mastering Mathematics

FOR 11–14 YEARS

BOOK 3

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Boost

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Contents

1 Powers and indices

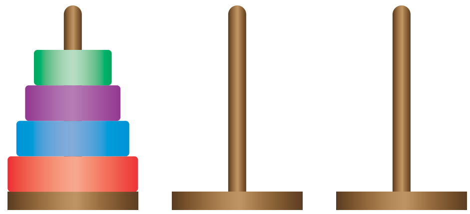
Coming up...

- Index notation and the laws of indices
- Using standard form
- Writing integers as a product of prime factors
- Finding the least common multiple and highest common factor of two numbers

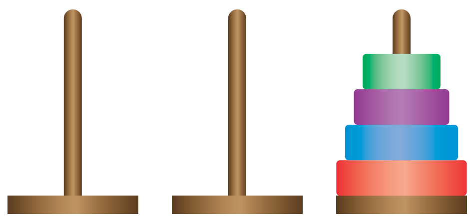
The tower of Hanoi

This puzzle is called 'The tower of Hanoi'.

You have 3 poles and 4 coloured discs of different sizes set on one of the poles.

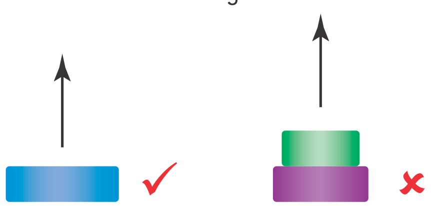


The aim is to move the discs onto one of the other poles.

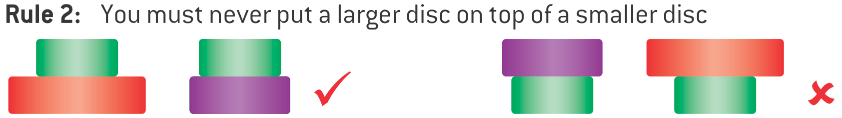


There are two rules for moving the discs.

Rule 1: You can only move one disc at a time



Rule 2: You must never put a larger disc on top of a smaller disc



① Make your own Tower of Hanoi.
- Cut out 4 circles of card of different diameters to use as your discs.
- Make 3 large square bases for your towers.
- Put the circles on one of your towers in order of size.

② What is the smallest number of moves needed to move all your discs to another tower?

③ Investigate further for different numbers of discs.

Find a rule for the smallest number of moves, m, needed to move d discs from one tower to another.

Find out about the history and myths of the Tower of Hanoi puzzle.

1.1 Index notation

Skill checker

Make a copy of this cross-number and then solve the clues.

Across

1 $7^3 - 5^3$

4 $3^2 + 4^2$

5 2×3^3

6 $4^2 \times \sqrt{36}$

10 $2 \times \sqrt{100}$

11 $\frac{5^2 \times 4^3}{\sqrt{4}}$

Down

1 5^2

2 $10^3 - 5^3$

3 $\sqrt{256}$

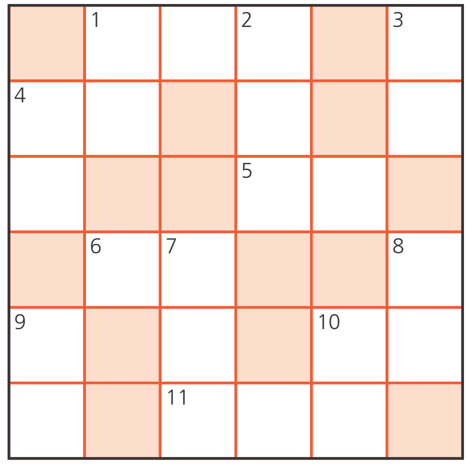
4 $6^2 - \sqrt{100}$

7 3×6^3

8 $\frac{\sqrt{10\,000}}{2}$

9 $\frac{6^3}{2^3}$

10 5×2^2



Communication using symbols

▶ Using indices

Remember that 5×5 is written as 5^2. ← Say 5 squared.

You say that 5 is the **base** and 2 is the **index** or **power**.

In the same way, repeated multiplications can be written using **index notation** like this:

$\underbrace{5 \times 5 \times 5}_{3 \text{ fives}} = 5^3$ ← Say 5 cubed.

$\underbrace{5 \times 5 \times 5 \times 5}_{4 \text{ fives}} = 5^4$ ← Say 5 to the power of 4.

Note

You have a power button on your calculator; it may look like $x^{\blacksquare}$ or x^y. Make sure you know how to use it: press 5 $x^{\blacksquare}$ 4 = and check you get an answer of 625.

You can have any number as the base.

For example, $\underbrace{7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7}_{8 \text{ sevens}} = 7^8$

Check you can use your calculator to work out 7^8. You should get 5 764 801.

In general, $\underbrace{a \times a \times a \times \ldots \times a \times a}_{n \text{ times}} = a^n$

The next examples show you how to multiply and divide numbers written using index notation.

Communication using symbols

Worked example

Write $3^5 \times 3^4$ as a single power of 3.

Solution

$3^5 \times 3^4$

$= (3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3)$

There are 5 threes multiplied together multiplied by 4 threes multiplied together which makes 9 threes multiplied together.

$= 3^9$

The powers have been added: 5 + 4 = 9.

Worked example

Write $3^6 \div 3^4$ as a single power of 3.

Solution

$3^6 \div 3^4 = \frac{3^6}{3^4}$

It is easier to write this as a fraction.

$= \frac{3 \times 3 \times \not{3} \times \not{3} \times \not{3} \times \not{3}}{\not{3} \times \not{3} \times \not{3} \times \not{3}}$

If you multiply by 3 and then divide by 3 it cancels out, so 4 of the 3s cancel from the top and bottom.

$= 3 \times 3$

$= 3^2$

The powers have been subtracted; 6 − 4 = 2.

In the examples the base was 3, but the same would be true for any base.

Here are the rules for multiplying and dividing powers.

$$a^m \times a^n = a^{m+n} \qquad a^m \div a^n = a^{m-n}$$

For example, $9^3 \times 9^8 = 9^{3+8} = 9^{11}$

and $\frac{4^{12}}{4^7}$ or $4^{12} \div 4^7$ is $4^{12-7} = 4^5$

Take care! The bases must be the same – you can't combine powers if the bases are different. There is no way to simplify $3^7 \times 5^4$ any further.

Sometimes powers involve brackets like in this next example.

Communication using symbols

Worked example

Write $(3^2)^4$ as a single power of 3.

Solution

$(3^2)^4$ means $\underbrace{(3^2) \times (3^2) \times (3^2) \times (3^2)}_{4 \text{ times}}$

So $(3^2) \times (3^2) \times (3^2) \times (3^2) = (3 \times 3) \times (3 \times 3) \times (3 \times 3) \times (3 \times 3)$

There are 2 threes multiplied together 4 times.

$= 3^8$

The powers have been multiplied; 2 x 4 = 8.

$(3^2)^4$ is sometimes called a power of a power. Again, the example would have worked for any base, not just 3.

Here is the rule for a power of a power.

$$(a^m)^n = a^{m \times n}$$

For example, $(11^4)^5 = 11^{4 \times 5} = 11^{20}$

Activity

a Work out these powers of 2:

i 2^1 ii 2^2 iii 2^3

iv 2^4 v 2^5 vi 2^6

b Using only your answers to part **a**, complete these sums.

The first one has been done for you.

You can only use each number once!

You are writing each number as a sum of powers of 2.

i 21 = [16] + [4] + [1]

ii 7 = [ ] + [ ] + [ ]

iii 10 = [ ] + [ ]

iv 31 = [ ] + [ ] + [ ] + [ ] + [ ]

v 51 = [ ] + [ ] + [ ] + [ ]

c The table shows how you can write the numbers 1 to 8 as sums of powers of 2.

- 1 means that the power of 2 is needed.
- 0 means that the power of 2 is NOT needed.
- You don't need to write in any zeros ***before*** the first 1.

	Powers of 2					
Number	**32**	**16**	**8**	**4**	**2**	**1**
1						1
2					1	0
3					1	1
4				1	0	0
5				1	0	1
6				1	1	0
7				1	1	1
8			1	0	0	0

5 = 4 + 1

7 = 4 + 2 + 1

Carry on the table for the numbers up to 32.

Can all numbers be written using powers of 2?

d The table in part **c** is used to write numbers in **binary**.

In binary, 7 is written as 111 and 8 is 1000.

Convert these binary numbers back to ordinary numbers.

i 100000 ii 101010

iii 1101011 iv 10000000

Each binary digit is called a 'bit'. The number 10000000 uses 8 bits, and 8 bits is called a byte. A kilobyte is 1024 bytes and a megabyte is roughly 1 million bytes. Binary numbers are used in computers to store information. Binary is a very powerful tool as data can be represented as strings of 0s and 1s which are represented as 'on/off' signals.

e Find out more about binary numbers and how they are used in computing.

Can every number be represented as a binary number?

How do you think text is converted to binary?

Powers of 1 and 0

Activity

1. Complete these.
   - **a** Use your calculator to work out
     - **i** $3^7 \div 3^6$ **ii** $4^8 \div 4^7$ **iii** $9^{11} \div 9^{10}$
   - **b** Use the laws of indices to write these as a single power.
     - **i** $3^7 \div 3^6 = 3^{\square}$ **ii** $4^8 \div 4^7 = 4^{\square}$ **iii** $9^{11} \div 9^{10} = 9^{\square}$
   - **c** What do you notice?
     Write down the value of 56^1.
2. Complete these.
   - **a** Use your calculator to work out
     - **i** $2^9 \div 2^9$ **ii** $5^8 \div 5^8$ **iii** $19^3 \div 19^3$
   - **b** Use the laws of indices to write these as a single power.
     - **i** $2^9 \div 2^9 = 2^{\square}$ **ii** $5^8 \div 5^8 = 5^{\square}$ **iii** $19^3 \div 19^3 = 19^{\square}$
   - **c** What do you notice?
     Write down the value of 56^0.

In the activity you found that

- any number to the power 1 is itself
- any number divided by itself is 1. ← The only exception to this rule is 0.

Using indices these are written as

$$a^1 = a \qquad a^0 = 1$$

Negative powers

Look at this pattern.

$$2^3 = 2 \times 2 \times 2 = 8$$
÷ 2
$$2^2 = 2 \times 2 = 4$$
÷ 2
$$2^1 = 2$$
÷ 2
$$2^0 = 1$$
÷ 2
$$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$$
÷ 2
$$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$
÷ 2
$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

You can carry on the pattern so $2^{-8} = \frac{1}{2^8}$ and $2^{-n} = \frac{1}{2^n}$

You could have used any base so you can say

$$a^{-n} = \frac{1}{a^n}$$ ← A negative power means '1 over'.

Worked example

Write 10^{-3} as a decimal.

Solution

$$10^{-3} = \frac{1}{10 \times 10 \times 10}$$
$$= \frac{1}{1000}$$
Find 1 ÷ 1000.
$$= 0.001$$

Communication using symbols

Roots

Remember that you can take roots to 'undo' squaring and cubing.

For example, $3^2 = 9$ so $\sqrt{9} = 3$ ← Say the square root of 9 is 3.

and $5^3 = 125$ so $\sqrt[3]{125} = 5$ ← Say the cube root of 125 is 5.

In the same way you can take roots to 'undo' higher powers.

For example, $4^6 = 4096$

so $\sqrt[6]{4096} = 4$ ← Say the sixth root of 4096 is 4.

You have a root button on your calculator; it may look like [$\sqrt[\square]{\square}$] or [$\sqrt[x]{\ }$]. You may have to use the SHIFT key or 2nd key to reach it. Make sure you know how to use the root button: press 6 [$\sqrt[\square]{\square}$] 4096 = and check you get an answer of 4.

Watch out!
9 has two square roots, +3 and −3, as (−3) x (−3) = 9 and 3 x 3 = 9.
However, the symbol $\sqrt{\ }$ means the positive square root only, so $\sqrt{9} = 3$.

Worked example

Calculate

a $5^4 + 4^5$

b $\sqrt[5]{343} - \sqrt[6]{64}$

Solution

a $5^4 + 4^5 = 625 + 1024$

$= 1649$

b $\sqrt[5]{243} - \sqrt[6]{64} = 3 - 2$

$= 1$

Watch out! You can't combine sums and differences into a single power because the base number is different, so you just need to use your calculator to work these out.

1.1 Now try these

Band 1 questions

Fluency

1. Write each expression as a power of 2.
   - **a** $2 \times 2 \times 2 \times 2$
   - **b** $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
   - **c** $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
   - **d** $2 \times 2 \times 2 \times 2 \times 2 \times 2$
2. Write each expression as a single power.
   - **a** $7 \times 7 \times 7$
   - **b** $5 \times 5 \times 5 \times 5 \times 5$
   - **c** $14 \times 14 \times 14 \times 14 \times 14 \times 14 \times 14$
   - **d** $12 \times 12 \times 12 \times 12 \times 12 \times 12$
   - **e** $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$
   - **f** $33 \times 33 \times 33 \times 33 \times 33 \times 33 \times 33 \times 33 \times 33 \times 33 \times 33$
3. **a** Calculate the difference between 3^2 and 2^3.
   b Find the value of $3^3 + 4^2 + 2^4$.

Strategic competence

4. 5^4 can be written in several ways.
   Here are some.

$5 \times 5 \times 5 \times 5$	$5^2 \times 5^2$	$5^3 \times 5^1$

   Write 4^5 in as many different ways as you can.

Logical reasoning

5. Copy and complete these.
   - **a** $4^2 = \square$
   - **b** $\square^3 = 125$
   - **c** $2^{\square} = 32$

Band 2 questions

6. Write the correct symbol <, > or = between each pair of numbers.
   - **a** $2^5 \ \square \ 6^2$
   - **b** $2^7 \ \square \ 5^3$
   - **c** $4^3 \ \square \ 3^4$
   - **d** $2^9 \ \square \ 8^3$
   - **e** $10^3 \ \square \ 2^{10}$
   - **f** $12^5 \ \square \ 3^8$

Strategic competence

7. The expressions on these 12 cards can be matched into six pairs.
   All the missing numbers are the same.

$4^{\square}$	$3 \times \square$	9	32	$\square^3$	6	10	$\square \times 5$	16	$3^{\square}$	8	$\square^5$

   - **a** Match the cards into pairs.
   - **b** What is the missing number?

Fluency

8. Write each of these as a single power.
   - **a** $4^5 \times 4^3$
   - **b** $6^{12} \times 6^4$
   - **c** $5^9 \times 5^3$
9. Write each of these as a single power.
   - **a** $3^5 \div 3^3$
   - **b** $7^{14} \div 7^8$
   - **c** $8^9 \div 8^5$
   - **d** $\frac{2^7}{2^3}$
   - **e** $\frac{10^8}{10^3}$
   - **f** $\frac{20^6}{20}$
10. Write each of these as a single power.
    - **a** $(6^2)^3$
    - **b** $(2^4)^5$
    - **c** $(13^6)^3$

Logical reasoning

11 **a** Write down the value of $49 \div 49$.

b Write 49 as a power of 7.

c Write $7^2 \div 7^2$

i as a power of 7 **ii** as a number.

d What is the value of 7^0?

e What are the values of

i 2^0 **ii** 3^0 **iii** 17^0?

f State a general value for the power zero.

Fluency

12 Write each of these calculations as a single power.

a $(13^7)^8$ **b** $7^9 \div 7^2$ **c** $4^5 \times 4^9$

d $3^6 \div 3^4$ **e** $(12^4)^6$ **f** $11^3 \times 11^4 \times 11^5$

13 Work out the value of these. Give each of your answers as a decimal.

a 2^{-1} **b** 10^{-1} **c** 5^{-1} **d** 4^{-1} **e** 100^{-1} **f** 8^{-1}

Logical reasoning

14 Jerin says that $3^4 \times 2^2$ is equal to 6^6.

Show that Jerin is wrong.

What mistake has she made?

Band 3 questions

Fluency

15 Work out the value of these. Give each of your answers as a decimal.

a 2^{-2} **b** 10^{-2} **c** 5^{-2} **d** 2^{-3} **e** 10^{-3} **f** 5^{-3}

16 Work out the value of these.

a $\sqrt[5]{243}$ **b** $\sqrt[7]{128}$ **c** $\sqrt[4]{625}$

17 Write these as a single power of 5.

a $5^3 \times 5^{-2}$ **b** $5^3 \div 5^{-2}$ **c** $(5^{-2})^3$

d $5^{-4} \times 5^8$ **e** $5^{-1} \times 5$ **f** $5^{-2} \div 5^{-2}$

Strategic competence

18 Find the missing digits.

a $7^{\square} = 7^5 \div 7^2$ **b** $7^3 = 7^3 \times 7^{\square}$ **c** $(7^{\square})^4 = 7^8$ **d** $7^6 \div 7^2 = 7^{\square} \times 7^3$

19 Write each of these as a single power.

a $\dfrac{3^5 \times 3^4}{3^6}$ **b** $(2^4 \times 2^2)^3$ **c** $\dfrac{(2^2)^3}{(2^4)^2}$

d $\dfrac{6^{-7}}{6^{-3}}$ **e** $(5^{-3} \times 5^4)^5$ **f** $\dfrac{9^3 \times 9^2}{9^{-2} \times 9^{-1}}$

Logical reasoning

20 **a** Priya says that $(5^3)^2$ is the same as $(5^2)^3$.

Show that Priya is right.

b Show that $(a^m)^n$ is the same as $(a^n)^m$.

c Is it **always true, sometimes true or never true** that $a^m \div a^n$ is the same as $a^n \div a^m$?

Explain your answer fully.

1.2 Standard form

Skill checker

1. Work out these powers of 10.
   a 10^1
   b 10^2
   c 10^3
   d 10^4
   e 10^5
   f 10^6

2. a Write down 1 million as
   i an ordinary number
   ii a power of 10.
   b Write down 1 billion as
   i an ordinary number
   ii a power of 10.
   c A googol is 10^{100}.
   Aeron writes a googol down as an ordinary number.
   How many zeros should follow the 1?

3. a Copy and complete this pattern.

$2.7 \times 10 = \square$

$2.7 \times 10^2 = 2.7 \times 100 = \square$

$2.7 \times 10^3 = 2.7 \times 1000 = \square$

$2.7 \times 10^4 = 2.7 \times \square = \square$

$2.7 \times 10^5 = 2.7 \times \square = \square$

$2.7 \times \square = 2.7 \times \square = 2\,700\,000$

   b Work out the value of 2.7×10^9
   Which of these is the correct way to say this number?

2.7 billion	**27 hundred million**
27 thousand million	**2 thousand 7 hundred million**

Using standard form for large numbers

Communication using symbols

You often see headlines in newspapers that involve large numbers. For example, there are more than 400 billion plastic toy bricks in the world or there are 8.9 million school children in the UK.

In newspapers, large numbers are usually written using the words 'billion' and 'million' rather than being written out in full like this:

400 000 000 000 Lego bricks or 8 900 000 school children

In Maths and Science, large numbers are usually written using powers of 10.

For example, 4×10^{11} Lego bricks or 8.9×10^6 school children.

Standard form is a way of writing down large numbers without writing down all the zeros.

A number is in standard form when it is written as

a number between 1 and 10 multiplied by a power of 10

In symbols this is written as

$A \times 10^n$

- A can be any number from 1 up to 10 (but not 10). ← $1 \leq A < 10$
- n must be an integer.

Cross-curricular activity

This painting is called 'Salvator Mundi' and it was painted by Leonardo da Vinci.

It was bought for over $450 million dollars, making it one of the world's most expensive paintings (at the time of writing, this is still the case!).

How would you write this figure in standard form?

Find out about other expensive paintings and the artists who painted them.

Communication using symbols

Worked example

The Pacific Ocean has a surface area of 168 000 000 km^2.

Write this number in standard form.

Solution

168 000 000 in standard form is $1.68 \times 10^?$ ← A is always between 1 and 10.

Use a place value diagram to help you work out what the power of 10 should be.

H M	T M	M	H Th	T Th	Th	H	T	O	.	t	h
								1	.	6	8
1	6	8	0	0	0	0	0	0	.		

$168\,000\,000 = 1.68 \times 10\,000\,000$

$= 1.68 \times 10^8$ ← You have to multiply 1.68 by ten 8 times to get 168 000 000.

So the Pacific Ocean has a surface area of 1.68×10^8 km^2

You don't need to draw a place value diagram each time. The digits move 8 places so that the decimal point is now between the 1 and the 6, so you multiply by 10^8.

$168\,000\,000. = 1.68 \times 10^8$ ← Count the arrows. They tell you the power of 10.

▶ Using standard form for small numbers

Activity

① Complete this pattern.

÷10, ÷10, ÷10

$10^2 = 100$

$10^1 = 10$

$10^0 = $ ____

$10^{-1} = 0.1$

$10^{-2} = $ ____

$10^{-3} = $ ____

$10^{-4} = $ ____

÷10, ÷10, ÷10

② **a** Work these out.

i 2×10^{-1} 2×10^{-2} 2×10^{-3}

ii 84×10^{-1} 84×10^{-2} 84×10^{-3}

iii 7.9×10^{-1} 7.9×10^{-2} 7.9×10^{-3}

b Complete each statement.

Multiplying by 10^{-1} is the same as dividing by ____ once

Multiplying by 10^{-2} is the same as dividing by 10 ____

Multiplying by 10^{-3} is the same as dividing by 10 ____

Communication using symbols

Worked example

A flea weighs around 0.000 087 kg.

Write this number in standard form.

Solution

A is always between 1 and 10.

0.000 087 in standard form is $8.7 \times 10^?$

Use a place value diagram to help you work out what the power of 10 should be.

O	.	t	h	th	t th	h th	m
8	.	7					
0	.	0	0	0	0	8	7

$0.000\,087 = 8.7 \div 10 \div 10 \div 10 \div 10 \div 10$

$= 8.7 \times 10^{-5}$

So the flea weighs 8.7×10^{-5} kg

Remember: dividing by 10 is the same as multiplying by 10^{-1}.
You have to divide 8.7 by ten five times to get 0.000 087.
Dividing by 10 five times is the same as multiplying by 10^{-5}.
You don't need to draw a place value diagram each time.
You have moved the digits 5 places to get the decimal point between the 8 and the 7, so you multiply by 10^{-5}.

$0.000\,087 = 8.7 \times 10^{-5}$

▶ Converting numbers from standard form

You also need to be able to convert from standard form back to ordinary numbers.

Remember when the power of 10:

- is positive, then the number is BIG
- is negative, then the number is SMALL.

Conceptual understanding

Worked example

Convert these numbers from standard form to ordinary numbers.

a 5.67×10^4 **b** 3.08×10^{-6}

Solution

a 5.67×10^4 means you multiply 5.67 by 10 four times.

The digits move 4 places.

56 700.

So $5.67 \times 10^4 = 56\,700$

A positive power means the number is big!

b 3.08×10^{-6} means you divide 3.08 by 10 six times.

The digits move 6 places.

0.00000308

So $3.08 \times 10^{-6} = 0.000\,003\,08$

A negative power means the number is small!

Activity

1. Are there any tall buildings or structures in Wales? Write down their heights in standard form.
2. **a** What is Wales's current estimated population in standard form?
   b How many times bigger is China's population? Write this in standard form.

Maths in context

Standard form, also known as scientific notation, was invented by Muhammad Al-Khwarizmi. He was a Persian mathematician and astronomer in the 9th century. Al-Khwarizmi was also responsible for introducing the 10 digits that we still use today (0, 1, 2, 3, 4, 5, 6, 7, 8, 9).

Why do you think standard form is important in the world of an astronomer?

1.2 Now try these

Band 1 questions

Fluency

1. Write each of these numbers as a power of 10.
   a 1 000 000 **b** 100 000 000 **c** 10
   d 0.01 **e** 0.001 **f** one hundred thousand
   g one thousand million **h** one ten thousandth
2. Write each of these as an ordinary number.
   a 10^5 **b** 10^{10} **c** 10^{-1} **d** 10^{-2} **e** 10^{-3}
3. Work out these multiplications. Part **a** has been answered for you.
   a $2.6 \times 10^3 = 2.6 \times 1000 = 2600$ **b** 4×10^2
   c 4.8×10^5 **d** 1.3×10^4
   e 2.4×10^7 **f** 9.3×10^6

Strategic competence

4. Copy and complete these. Fill in the missing numbers.
   a $\square \times 10^2 = 500$ **b** $\square \times 10^3 = 3000$ **c** $6 \times 10^{\square} = 6\,000\,000$
   d $\square \times 10^2 = 350$ **e** $4.2 \times 10^{\square} = 42\,000$ **f** $\square \times 10^5 = 450\,000$

Band 2 questions

Fluency

5. These numbers are in standard form. Write them as ordinary numbers.
   a 2×10^3 **b** 7×10^6 **c** 4.2×10^5 **d** 7.1×10^7 **e** 8.6×10^9
6. Write these numbers in standard form.
   a 200 **b** 5000 **c** 7 000 000 **d** 3600 **e** 7 200 000
7. Work out these multiplications.
   a 3×10^{-1} **b** 4×10^{-2} **c** 5×10^{-3}
8. Copy and complete these.
   Fill in the missing numbers.
   a $4 \times 10^{\square} = 0.4$ **b** $\square \times 10^{-3} = 0.006$ **c** $3 \times 10^{\square} = 0.03$

Strategic competence

9 Write these as ordinary numbers.

a The length of a human chromosome is 5×10^{-6} m.

b The CN tower in Toronto is 5.53×10^{2} m tall.

c The mass of an electron is 9.11×10^{-31} kg.

10 Write these numbers in standard form.

a The distance between the Earth and the Moon is 239 000 miles.

b A £5 note is 0.000 22 m thick.

c Quartz fibre has a diameter of 0.000 001 m.

Band 3 questions

Logical reasoning

11 **a** Write the correct inequality symbol < or > between each of these pairs of numbers.

i 3×10^{4} ☐ 3×10^{5}

ii 4.6×10^{6} ☐ 5.8×10^{6}

iii 7×10^{9} ☐ 5.3×10^{9}

iv 6×10^{5} ☐ 1 million

b Explain how you can compare the sizes of numbers written in standard form.

Strategic competence

12 Write these numbers in order, starting with the smallest.

4.56×10^{5} **3.4×10^{4}** **563 000** **7.4×10^{6}** **820 000**

13 Correct each of the pieces of homework below.

For each question say who has got it right and explain where the other has gone wrong.

Seren

Write 3800 in standard form

3.8×10^{2}

Write 0.000 006 78 in standard form

6.78×10^{-6}

Write 5×10^{7} as an ordinary number

50 000 000

Write 9.6×10^{-5} as an ordinary number

9 600 000

Put in order of size, small to big:

2.6×10^{4}, 2.9×10^{2}, 2.7×10^{-3}

2.6×10^{4}, 2.9×10^{2}, 2.7×10^{-3}

Double 6.7×10^{5}

13 400 000 000

Bryn

Write 3800 in standard form

3.8×10^{3}

Write 0.000 006 78 in standard form

6.78×10^{-8}

Write 5×10^{7} as an ordinary number

500 000

Write 9.6×10^{-5} as an ordinary number

0.000 096

Put in order of size, small to big:

2.6×10^{4}, 2.9×10^{2}, 2.7×10^{-3}

2.7×10^{-3}, 2.9×10^{2}, 2.6×10^{4}

Double 6.7×10^{5}

1.34×10^{6}

Logical reasoning

14 **a** **i** Explain how to work out 2000×300 in your head.

ii Now write your answer in standard form.

b **i** Write the answer to $(2 \times 10^{3}) \times (3 \times 10^{2})$ in standard form.

Look carefully at your answer.

ii How is the number worked out?

iii How is the power of 10 worked out?

c Work out

i $(4 \times 10^{2}) \times (2 \times 10^{3})$

ii $(1.2 \times 10^{4}) \times (3 \times 10^{2})$

d Explain how to multiply numbers when they are written in standard form.

e Use your method to work out $(3 \times 10^{-2}) \times (4 \times 10^{-3})$.

Make sure you write your answer in standard form.

Strategic competence

15 The problems below can be solved either by multiplying or by dividing.
Choose the correct operation for each one and then answer the question.

- **a** A mouse weighs 1.5×10^{-2} kg.
  An owl eats 1000 mice in a year.
  What weight of mice is this?
- **b** The speed of sound is 3.3×10^{2} metres per second.
  How far does sound travel in an hour?
- **c** A grain of salt weighs 2×10^{-5} grams.
  How many grains of salt are there in a 750 gram packet?
- **d** A packet of 500 sheets of paper is 55 mm thick.
  How thick is each sheet of paper?
- **e** The average number of clover leaves in a square metre of lawn is 1.5×10^{3}.
  Estimate the number of clover leaves in a park with 5×10^{4} m^2 of lawn.

16 The galaxy that the Earth is in is called the Milky Way.
The Milky Way is about 120 000 light years across.
The Earth is around 27 000 light years from the centre of the Milky Way.
A light year is the distance that light can travel in a year and it is 9.46×10^{15} metres.

- **a** Imagine a spaceship that can travel at half the speed of light.
  It sets off from the Earth towards the centre of the Milky Way. How long would it take?
- **b** How many generations of astronauts do you think it would take?
- **c** Assuming an original crew of 20 people, how many people do you think would arrive at the centre of the Milky Way?

Logical reasoning

17 Which of these numbers are not in standard form? Why not?
Where possible re-write the number so it is in standard form.

a 0.3×10^{4}	**b** 7×10^{-3}	**c** 10.1×10^{-4}
d $4 \times 10^{0.5}$	**e** 10^{9}	**f** 9.9×10^{100}

1.3 Prime factorisation

Skill checker

1 Look at the numbers in this box.

9	7	51	36	20
16	37	2	27	24
11	21	100	1	42
8	64	6	12	5

Write down all the numbers from the box that are

- **a** square
- **b** cubes
- **c** factors of 36
- **d** prime
- **e** multiples of 6
- **f** factors of 100 and prime.

2 Write down the first ten prime numbers.

Remember

Prime numbers are numbers that have exactly 2 factors: 1 and the number itself.

Writing numbers as a product of prime factors

Remember that the **factors** of a number divide into it exactly.

For example, the factors of 12 are 1, 2, 3, 4, 6 and 12

A **prime factor** is a factor of a number that is also prime.

The prime factors of 12 are 2 and 3.

Remember

1 is not a prime number.

Activity

Show that every number between 2 and 20 is either prime or can be made by multiplying prime numbers together.

Number	Answer
2	2 (prime)
3	3 (prime)
4	2 × 2
5	5 (prime)
6	2 × 3
7	

In the activity you found that every number up to 20 is either prime or can be made by multiplying together prime numbers. In fact, every whole number above 1 is either prime or can be made by multiplying together prime numbers.

This is known as the Fundamental Theorem of Arithmetic.

For example, $60 = 2 \times 2 \times 3 \times 5$

$= 2^2 \times 3 \times 5$

This is called a product of prime factors. Remember: product means multiply.

The next example shows you how to use a factor tree to help you write a number as a product of prime factors.

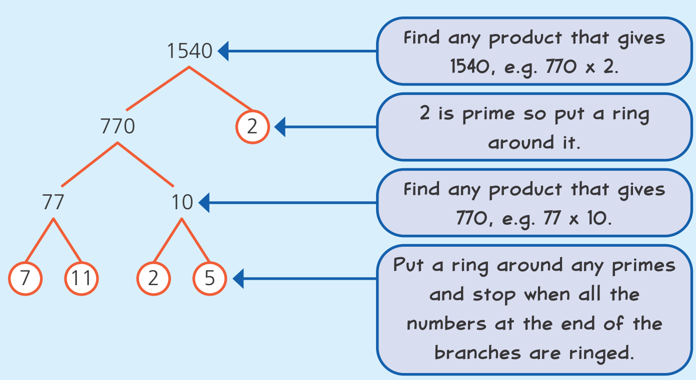
Communication using symbols

Worked example

Write 1540 as a product of prime factors.

Solution

Using a factor tree:



Note

Every whole number greater than 1 has one unique way to rewrite it as a product of prime factors. A different product of prime factors will give a different number!

So $1540 = 7 \times 11 \times 2 \times 5 \times 2$ ← Multiply all the numbers in the rings together.

$= 2^2 \times 5 \times 7 \times 11$ ← You can multiply in any order. 2×2 can be written as 2^2.

If you use a different factor tree, is the final answer always the same?

▶ Finding the HCF and LCM of two numbers

The highest common factor (HCF) of two numbers is the largest factor that they share.

You can find the HCF of two numbers by listing their factors, as in this example for 20 and 30.

Factors of 20: 1 2 4 5 10 20

Factors of 30: 1 2 3 5 6 10 15 30

The highest number in both lists is 10 so this is the HCF of 20 and 30.

> The HCF is often quite a small number.

The lowest common multiple (LCM) of two numbers is the lowest multiple that they share.

You can find the LCM of two numbers by listing their multiples, as in this example, again using 20 and 30.

Multiples of 20: 20 40 60 80 100 . . .

Multiples of 30: 30 60 90 120 150 . . .

The lowest number in both lists is 60 so this is the LCM of 20 and 30.

The next example shows you how to use a Venn diagram to help you find the HCF and LCM of two numbers.

Conceptual understanding

Worked example

a Show that 270 written as a product of prime factors is $2 \times 3^3 \times 5$.

b Write 315 as a product of prime factors.

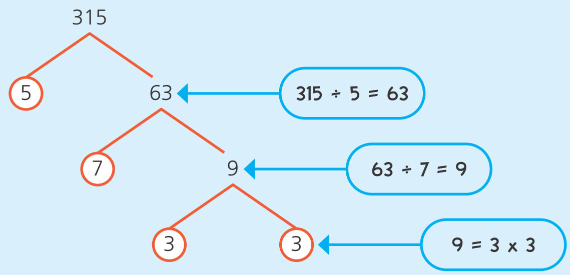
c Find the HCF and LCM of 270 and 315.

Solution

a $2 \times 3^3 \times 5 = 2 \times 27 \times 5$

$= 10 \times 27 = 270$

b



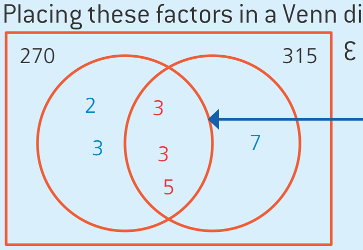
Multiply together all the prime (circled) numbers in the factor tree:

$315 = 5 \times 7 \times 3 \times 3$

$= 3^2 \times 5 \times 7$

c $270 = 2 \times 3 \times 3 \times 3 \times 5$ and $315 = 3 \times 3 \times 5 \times 7$

Placing these factors in a Venn diagram gives



> The epsilon sign ℰ represents the universal set.

> 3, 3 and 5 are common factors so they go in the intersection. The intersection is the middle/crossover of the two circles.

The HCF is found by multiplying the numbers in the intersection:

$3 \times 3 \times 5 = 45$

> These are the common factors of both numbers so when you multiply them together you'll get the highest common factor.

So the HCF of 270 and 315 is 45.

The LCM is found by multiplying all of the numbers in the Venn diagram:

315

$2 \times 3 \times 3 \times 3 \times 5 \times 7 = 1890$

270

> Check: 1890 = 6 x 315 which means 1890 is a multiple of 315 and 1890 = 270 x 7 which means 1890 is also a multiple of 270. Can you see why this method works?

Remember

- Writing a number as a product of its prime factors means writing all of the prime factors as a multiplication.
- Use index notation to write a product of prime factors neatly.

Note

Venn diagrams are covered in full in chapter 15 (unit 15.2).

1.3 Now try these

Band 1 questions

Fluency

1 Work out these.

a $3 \times 5 \times 7$ **b** $3 \times 7 \times 11$ **c** $2 \times 3 \times 5$ **d** $2 \times 3 \times 11$ **e** 2^3

f 2^4 **g** $3^2 \times 5$ **h** $2^2 \times 3$ **i** $2^2 \times 5^2$

2 **a** What are the first ten multiples of 5?

b What are the first ten multiples of 6?

c What is the lowest common multiple (LCM) of 5 and 6?

3 **a** List the first eight multiples of 30.

b List the first eight multiples of 24.

c What is the lowest common multiple (LCM) of 30 and 24?

4 Find the lowest common multiple (LCM) of each pair of numbers.

a 3 and 5 **b** 4 and 6 **c** 9 and 12

5 **a** What are the factors of 12?

b What are the factors of 16?

c What is the highest common factor (HCF) of 12 and 16?

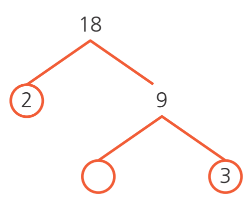
6 Find the HCF of each pair of numbers.

a 15 and 20 **b** 18 and 24

Band 2 questions

Fluency

7 **a** Copy and complete this diagram to find the prime factors of 18.



b Write 18 as the product of prime factors.

c Rewrite your answer using indices.

8 **a** Complete this list of the factors of 60.

1, 2, 3, ..., 60

b Write a list of the prime factors of 60.

c Write 60 as a product of prime factors.

9 Draw factor trees to write these numbers as products of prime factors.

a 12 **b** 8 **c** 15 **d** 20 **e** 30

10 Write each of these numbers as a product of its prime factors.

Write your answers using indices.

a 50 **b** 140 **c** 84 **d** 36 **e** 200

11 Find the **i** LCM and **ii** HCF of these pairs of numbers by first finding their prime factors and then using a Venn diagram.

a 64 and 72 **b** 20 and 35 **c** 16 and 28

Logical reasoning

12 **a** Find the HCF of 90 and 360.

b Find the LCM of 90 and 360.

c What do you notice about your answers to parts **a** and **b**?

Find another pair of numbers with this pattern.

13 **a** Find a number with prime factors of only 13, 17 and 19.

b Find a number between 100 and 200 with prime factors that are all even.

c Find a pair of numbers that have prime factors of only 2, 3 and 5.

Band 3 questions

Strategic competence

14 **a** Show the prime factors of 24 and 90 on a Venn diagram.

b Use the Venn diagram to find the HCF and LCM of 24 and 90.

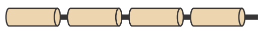
15 Pedr makes rosewood jewellery.

Pieces of wood are joined together to make bracelets, necklaces and anklets.

All the pieces of wood are the same length.

Look at the poster.

What is the greatest possible length for one of the pieces of wood?



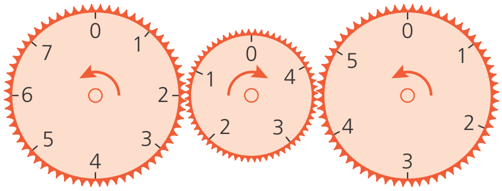
Pedr's jewellery

Bracelets	*18 cm*
Short Necklaces	*48 cm*
Long Necklaces	*54 cm*
Anklets	*24 cm*

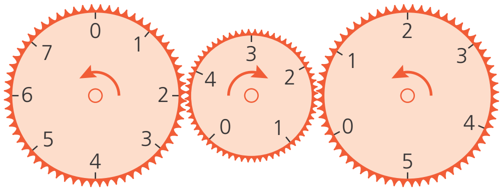
GUARANTEED TO BRING YOU LUCK

16 Find the LCM and HCF of 90, 75 and 60.

17 These dials are set at 0:



After the left dial has been turned through one complete turn they look like this:

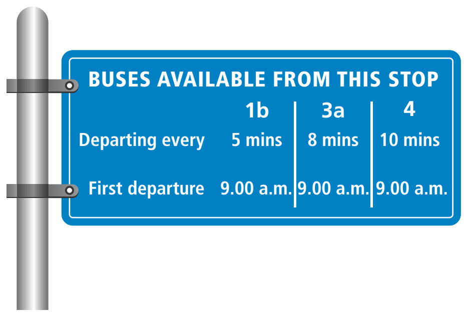


a Draw a diagram to show what they will look like after two complete turns of the left dial.

b How many complete turns of the left dial are needed before the first two dials are both set to 0?

c How many complete turns of the left dial are needed before all three dials are again set to 0?

18



All three buses leave together at 9.00 a.m.

When is the next time that all three buses leave together?

Key words

Here is a list of the key words you met in this chapter.

Cube	Cube root	Factor	Highest common factor (HCF)
Indices	Lowest common multiple (LCM)	Multiple	Power
Prime	Product	Square	Square root
Standard form	Venn diagram		

Use the glossary at the back of this book to check any you are unsure about.

Review exercise: powers and indices

Band 1 questions

Fluency

1 Multiply out these.

a $2^2 \times 3^2$ **b** 3×5^2 **c** 2×7^2 **d** $2^3 \times 3^3$

Logical reasoning

2 Write 64 as a power of

a 2 **b** 4 **c** 8 **d** 64

Strategic competence

3 Write the number in each statement as a power of 10.

a There are 100 steps to the top of the tower.

b The winner won by just 0.1 of a second.

c The car costs £10 000.

Fluency

4 In the number 4.12×10^6, the first digit has a value of 4 million or 4 000 000.

The numbers in the table are written in standard form.

Copy the tables and fill in the value of the first digit for each number.

a

Number	Value of first digit
6.1×10^4	
3.52×10^4	
2.9×10^7	
1.352×10^7	
4.5×10^9	
1.236×10^9	

b

Number	Value of first digit
2×10^{-2}	
1.46×10^{-2}	
3×10^{-4}	
6.2×10^{-4}	
5×10^{-6}	
3.21×10^{-6}	

5 Find the LCM and HCF of these pairs of numbers by first finding their prime factors.

a 8 and 14 **b** 30 and 35 **c** 18 and 24

Band 2 questions

Fluency

6 Find the values of these.

a $(2^2)^3$ **b** $2^2 \times 2^3$ **c** $2^2 + 2^3$ **d** $2^3 \times 2^3$

e $2^3 \div 2^3$ **f** $2^5 \div 2^3$ **g** $2^{99} \div 2^{96}$ **h** $\dfrac{2^5}{(2\times2\times2\times2\times2)}$

7 Find the LCM and HCF of these pairs of numbers by first finding their prime factors.

a 80 and 100 **b** 210 and 240

Strategic competence

8 Work out the missing digits in these. There may be more than one answer.

a $\square^2 = 25$ **b** $\square\square^2 = \square\square5$

c $\square^3 = \square\square6$ **d** $\square^6 = \square^3$

Fluency

9 Write these numbers in standard form.

a 2000 **b** 32 000 **c** 1450 **d** 36 000 000

10 Write these numbers in standard form.

a 0.067 **b** 0.003 41 **c** 0.000 006 **d** 0.23

11 Write these as ordinary numbers.

a 2×10^3 **b** 1.4×10^2 **c** 4.56×10^4 **d** 5.6×10^5

e 3.576×10^{12} **f** 2.7×10^{-3} **g** 8.32×10^{-7} **h** 4.9×10^{-10}

Strategic competence

12 Write these numbers in standard form.

a China has an estimated population of 1 393 000 000.

b It takes 0.000 000 003 3 seconds for light to travel a distance of 1 metre.

c The world's longest river, the Nile, is 6695 km long.

d An amoeba is 0.0005 metres across.

e The Star Wars films had a worldwide box office gross of £10 320 000 000.

Fluency

13 Write each expression as a single power of 3 or 5.

a $3^4 \times 3^2$ **b** $3^5 \div 3^2$ **c** $(3^2)^4$ **d** $5^2 \div 5^2$ **e** $5^2 \div 5^4$ **f** $5^2 \div 5^3$

Band 3 questions

Logical reasoning

14 Lowri has got all of her homework wrong.

The numbers are correct, but they should be written in standard form.

Correct Lowri's homework.

a 100×10^5 ✗ b 0.3×10^9 ✗

c 0.5×10^{-3} ✗ d 200×10^{-6} ✗

15 Write these sets of numbers in order of size, starting with the smallest.

a 2.3×10^4 32 000 5.47×10^3 1.36×10^3 40 thousand

b 4×10^{-5} 3.7×10^{-4} 1.8×10^{-4} 0.000 65 0.000 03

16 Nomsa thinks she has found a counter-example to the rule that any number has only one set of prime factors. She says,

'My number is 24 871. I can factorise it in two ways. It can be 209 × 119 or it can be 187 × 133. I have found an exception to the rule!'

Is Nomsa correct?

17 The prime factors of a cube number can be grouped in threes like this:

$216 = (2 \times 2 \times 2) \times (3 \times 3 \times 3)$ is the cube of (2×3).

a $15 = 3 \times 5$. Write down the cube of 15.

b What is the cube root of $3^3 \times 7^3$?

$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$ is a square number and a cube number.

c Show that its factors can be grouped in pairs and in threes.

d Find two more numbers that are both square numbers and cube numbers.

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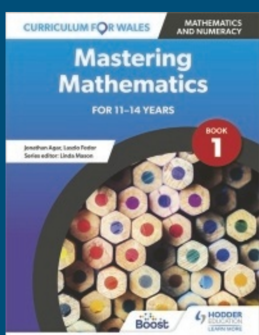

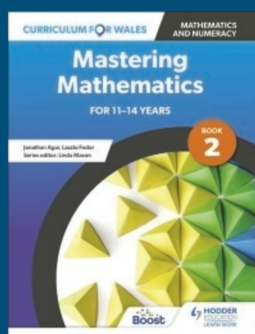

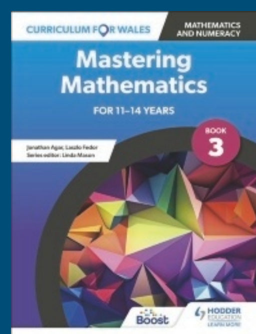

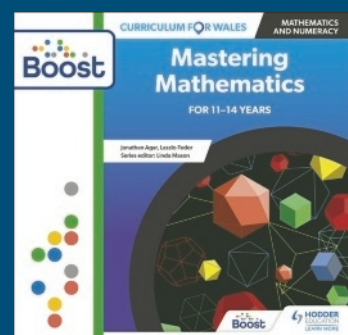

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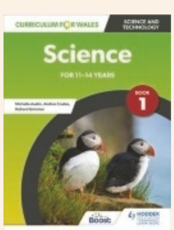

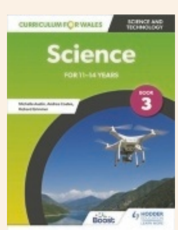

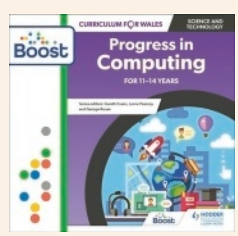

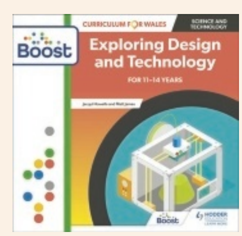

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