CURRICULUM FOR WALES

MATHEMATICS AND NUMERACY

Mastering Mathematics

FOR 11–14 YEARS

BOOK 2

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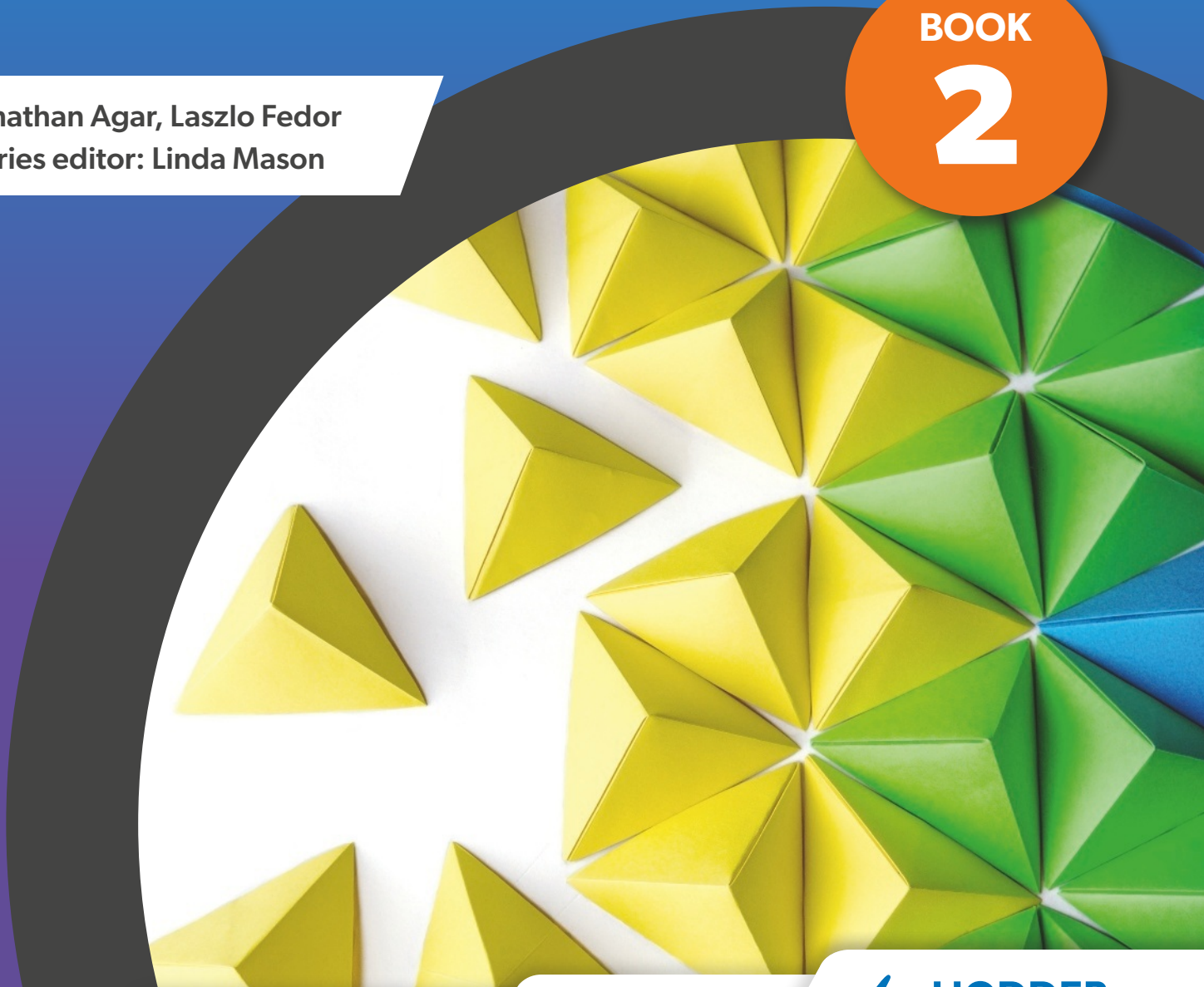
Series editor: Linda Mason



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Contents

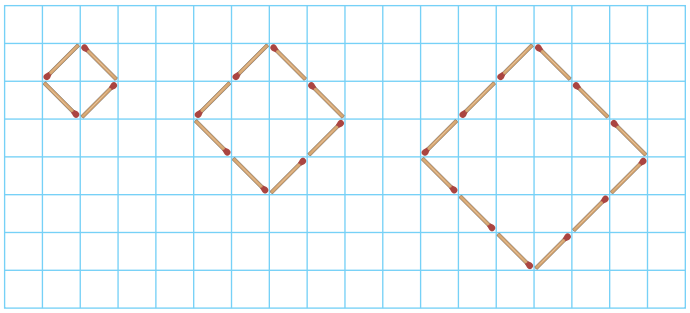
1 Sequences

Coming up...

- Using function machines
- Understanding arithmetic sequences
- Sequences in diagrams

Patterns

The diagram below shows three shapes in a pattern made using matchsticks.



Without drawing the shape, work out how many matchsticks will be needed to make the next shape in the pattern.

Draw the next shape to see whether you were correct or not.

The number of matchsticks in each shape in this pattern forms an arithmetic sequence, which you will learn about in Section 1.1.

1.1 Function machines

Skill checker

① Complete each of the calculations below.

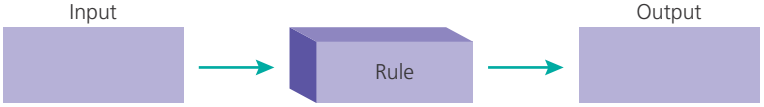
a $16 + 27$ b $96 - 39$ c 18×6

② Find the unknown values in each of the calculations below.

a $18 + \square = 39$ b $\square - 13 = 58$ c $\square \times 9 = 63$

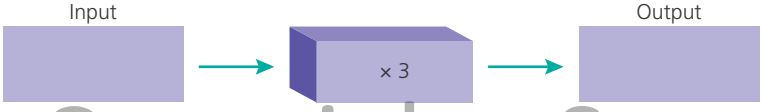
Function machines

A function machine applies a **rule** to any numbers which are **input** into the machine.

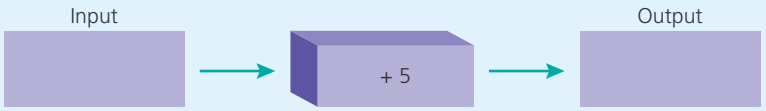


The **output** of the function machine is determined by what operation the machine performs. This is called its **rule**.

This function machine multiplies the **input** number by 3.



Worked example



a What is the output of the function machine when the input number is 7?

b What is the output of the function machine when the input number is 31?

c What is the input for the function machine when the output number is 16?

Solution

a $7 + 5 = 12$

b $31 + 5 = 36$

c $16 - 5 = 11$

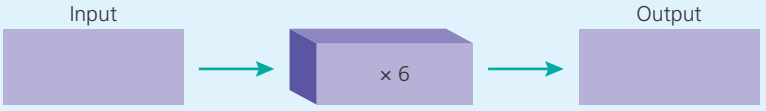
To work out the input number when you know the output, just apply the opposite operation of the rule.

The opposite operation of addition is subtraction.

+ 5 becomes − 5.

Conceptual understanding

Worked example



a What is the output of the function machine when the input number is 12?

b What is the output of the function machine when the input number is 0.7?

c What is the input for the function machine when the output number is 54?

Solution

a $12 \times 6 = 72$

b $0.7 \times 6 = 4.2$

c $54 \div 6 = 9$

The opposite operation of multiplication is division.

× 6 becomes ÷ 6.

Activity

① Owain thinks of a number.
He adds 17 to his number and gets an answer of 25.
What number did Owain first think of?

② Elin thinks of a number.
She multiplies her number by 5 and gets an answer of 60.
What number did Elin first think of?

③ Tomos thinks of a number.
He divides his number by 4 and gets an answer of 6.
What number did Tomos first think of?

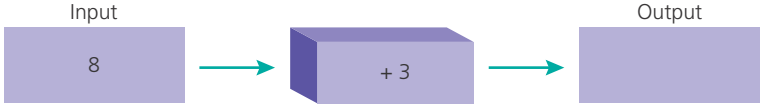
1.1 Now try these

Band 1 questions

Fluency

1 Work out the output for each of these function machines.

a



b

c

d

e

f

g

h

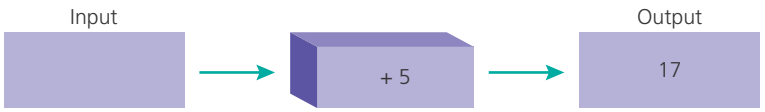
i

j

Strategic competence

2 Work out the input for each of these function machines.

a



b

c

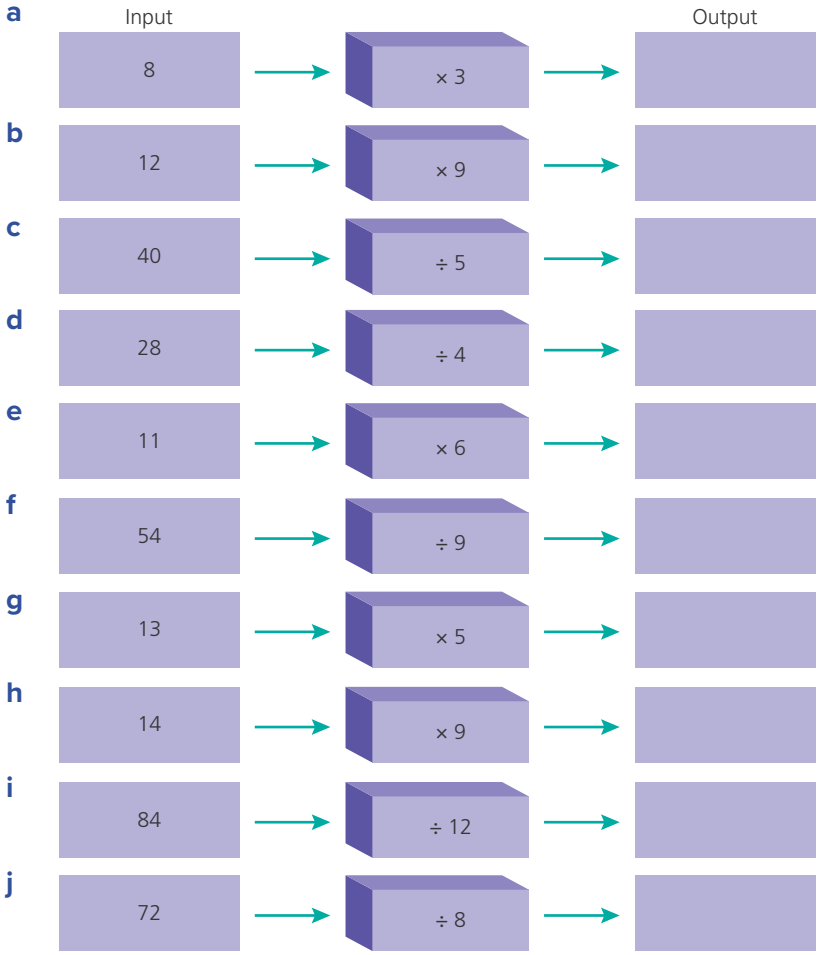
d

e

Band 2 questions

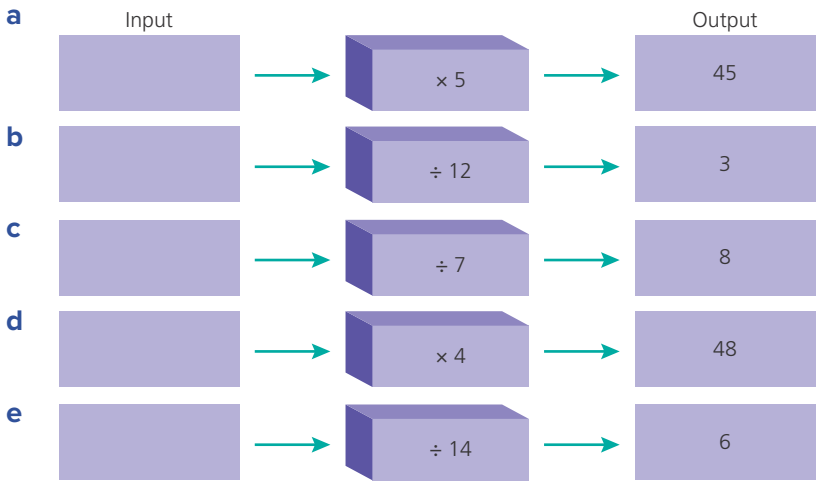
Fluency

3 Work out the output for each of these function machines.



Strategic competence

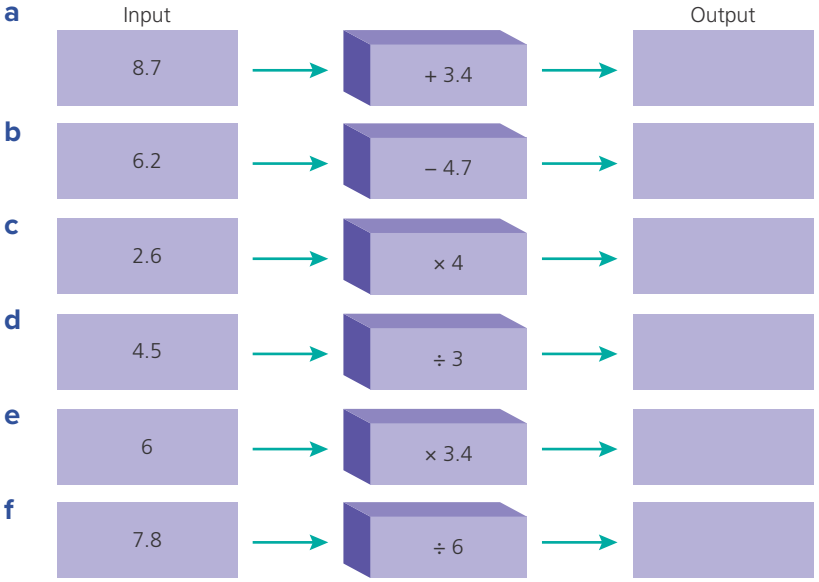
4 Work out the input for each of these function machines.



Band 3 questions

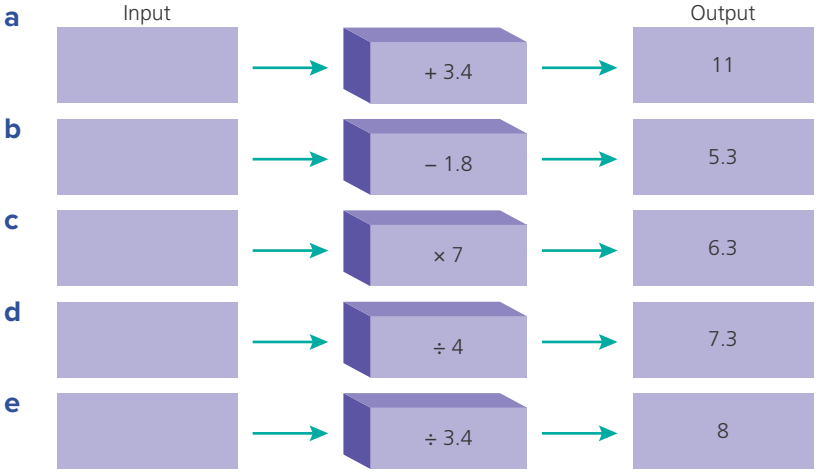
Fluency

5 Work out the output for each of these function machines.



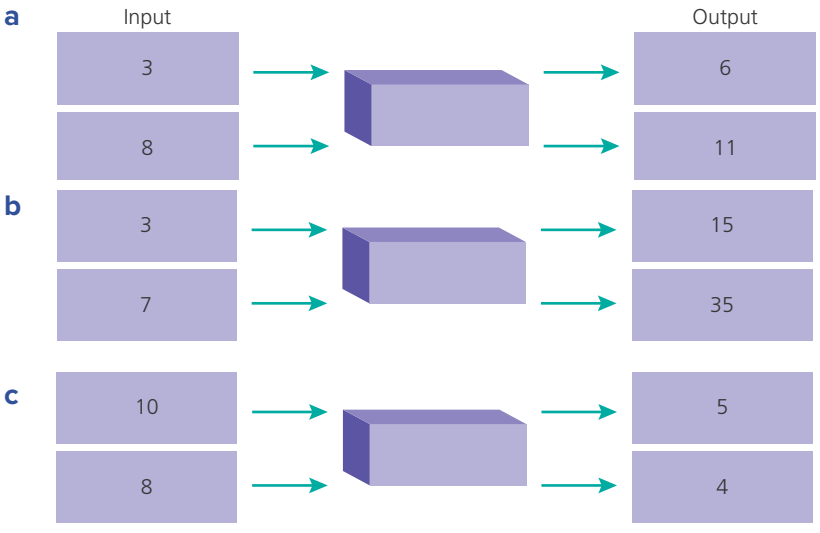
Strategic competence

6 Work out the input for each of these function machines.



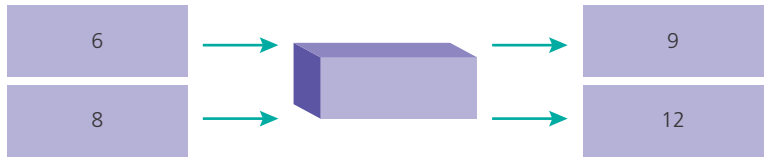
Logical reasoning

7 Work out the rule for each of these function machines.

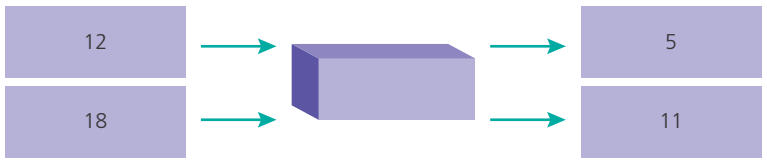


Logical reasoning

d



e



8 Dewi input 3 into a function machine and got an output of 12.
Angharad input 1.5 into the same function machine and got an output of 6.
What is the rule of the function machine?

9 Aled input 3 into a function machine.
He got an output number which was five times as big as the input number.
What are the two possible rules of the function machine?

10 Bethan input 6 into a function machine and the output was a single-digit square number.
Catrin input 11 into the function machine and the output was also a single-digit square number.
What is the rule of the function machine?

1.2 Arithmetic sequences

Skill checker

① Continue these sequences for three more terms.

a 3, 6, 9, 12, ... b 45, 50, 55, 60, ... c 13, 23, 33, 43, ... d 11, 22, 33, 44, ...

② Substitute $n = 1$, $n = 2$ and $n = 3$ into these expressions.

a $3n$ b $n + 4$ c $5n - 1$ d $100n - 2$

Communication using symbols

▶ Arithmetic sequences

Look at this sequence of numbers:

3, 6, 9, 12, ...

This sequence is based on the three times table.

The 1st term is $3 \times 1 = 3$.

The 4th term is $3 \times 4 = 12$.

The 10th term is $3 \times 10 = 30$.

The nth term is $3 \times n = 3n$.

$3n$ is called the 'position-to-term rule' for the sequence. It is also sometimes called the nth term of the sequence.

You can use the **position-to-term rule** to find any term of the sequence.

For example, to find the 20th term:

$3 \times 20 = 60$.

A sequence in which you add or subtract a fixed amount to move from one term to the next is called an **arithmetic sequence**.

In this arithmetic sequence, the **term-to-term rule** is 'add 3'.

Worked example

Look at this sequence:

4, 7, 10, 13, ...

a What is the term-to-term rule?

b Find the position-to-term rule for the sequence.

c What is the 50th term of the sequence?

d Which term of the sequence is 100?

e What type of sequence is this?

Solution

a The term-to-term rule is 'add 3'.

b Each term is 1 more than the three times table, so the position-to-term rule is $3n + 1$.

The sequence goes up in 3s, so is based on the three times table.

Look at this comparison with the previous example.

$3n$	3	6	9	12
+1	+1	+1	+1	+1
$3n + 1$	4	7	10	13

You can **check** this position-to-term rule works. Use $n = 1, 2, 3$ and 4 in the rule.

n	1	2	3	4
$3n + 1$	$3 \times 1 + 1 =$ **4**	$3 \times 2 + 1 =$ **7**	$3 \times 3 + 1 =$ **10**	$3 \times 4 + 1 =$ **13**

The table shows that the position-to-term rule gives the first four terms of the sequence.

c You can find the 50th term using the rule.

nth term $= 3n + 1$

50th term $= 3 \times 50 + 1$

$= 151$

d To find which term of the sequence is 100, use the rule again.

Solve the equation to find n.

n**th term** $= 3n + 1$

$100 = 3n + 1$

$100 - 1 = 3n$

$3n = 99$

$n = 33$

The 33rd term of the sequence is 100.

e To move from one term to the next, you add 3. This is an arithmetic sequence.

Remember

The difference tells you which times table the sequence is based on. For example, if there is a difference of 3 between each term, then $3n$ will appear in the position-to-term rule.

Worked example

a Find the position-to-term rules for these sequences:

i $-2, -4, -6, -8, \ldots$

ii $-1, -3, -5, -7, \ldots$

b What type of sequence are these?

Solution

a **i** This is the -2 times table, so the position-to-term rule is $-2n$.

ii Each term is one more than the terms in the -2 times table.

The position-to-term rule is $-2n + 1$.

Another way to write this is $1 - 2n$.

b In both sequences you subtract 2 to move from one term to the next.
They are both arithmetic sequences.

Conceptual understanding

Activity

Read this newspaper report about Steel City.
Somebody has spilt coffee on the page and some of the numbers are unreadable.

STEEL CITY BOOM TOWN!

It's a new year and the residents of Steel City are waking up this morning to the sound of yet more construction work.

Work has begun on the huge SC Tower, which will become the tallest building so far in Steel City. We have contacted the city planners. They say that there are currently 14 skyscrapers in Steel City and there will be 44 skyscrapers in the city in five years' time.

They have given permission for [redacted] new skyscrapers EVERY YEAR!

If the city's plans become a reality, by the end of this year there will be [redacted] skyscrapers.

If construction work in Steel City continues at this rate, there will be 62 skyscrapers in [redacted] years' time.

In ten years' time the city will have [redacted] new skyscrapers!

Can the residents of Steel City put up with this much noise? Does Steel City have enough sandwich shops to feed all of these hungry office workers? Let us know what you think.

a Can you replace the coffee marks with the correct numbers?

b What is the term-to-term rule for the sequence of the number of skyscrapers?

1.2 Now try these

Band 1 questions

Fluency

1 Find the next three terms in these arithmetic sequences.

a 2, 3, 4, 5, ...

b 0, 4, 8, 12, ...

c $-4, -2, 0, 2, \ldots$

d $-5, -8, -11, -14, \ldots$

e $4, 0, -4, -8, -12, \ldots$

f $0, -2, -4, -6, \ldots$

g $-0.1, -3.1, -6.1, -9.1, \ldots$

Fluency

2 Find the missing numbers in the arithmetic sequences below.

a 2, 4, 6, 8, ☐, 12, ...
b 5, 9, 13, ☐, 21, ...
c −3, −6, ☐, −12, −15, ...
d 21, 25.5, ☐, 34.5, 39, ...
e 0, −2, −4, ☐, −8, ...
f −1.5, −3, −4.5, ☐, −7.5, ...
g 30, 27.5, 25, ☐, 20, ☐, ...
h 8, ☐, 24, ☐, 40, ...
i 6, ☐, 18, ☐, 30, ...
j ☐, −15, ☐, −25, −30, ...

3 Find the nth term of these sequences.

In each case the rule is ☐n.

a 2, 4, 6, 8, 10, ...
b 4, 8, 12, 16, ...
c 5, 10, 15, 20, ...
d 1, 2, 3, 4, 5, ...
e 100, 200, 300, 400, 500, ...
f 10, 20, 30, 40, 50, ...
g 8, 16, 24, 32, ...

4 Find the nth term of these sequences.

In each case the rule is ☐$n + 1$.

a 6, 11, 16, 21, 26, ...
b 11, 21, 31, 41, 51, ...
c 101, 201, 301, 401, 501, ...
d 5, 9, 13, 17, ...

5 Find the nth term of these sequences.

In each case the rule is ☐$n - 1$.

a 99, 199, 299, 399, ...
b 9, 19, 29, 39, 49, ...
c 4, 9, 14, 19, 24, ...
d 7, 15, 23, 31, ...
e 3, 7, 11, 15, ...

Logical reasoning

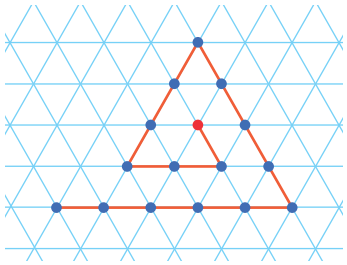
6 Copy this spiral pattern onto isometric paper.

Start with the red dot near the centre of the page.

a Count the number of dots on each line that you draw.
Write them as a sequence: 2, 3, ...

b Predict how many dots will be on the next line that you draw.
Check by drawing and counting.

c Describe the pattern in the number of dots.



Fluency

7 Find the first five terms of these sequences, given their position-to-term rules. Use the tables to help you.

a The sequence with position-to-term rule $n + 4$.

n	1	2	3	4	5
$n + 4$	1 + 4 = 5	2 + 4 = _			

b The sequence with position-to-term rule $n - 5$.

n	1	2	3	4	5
$n - 5$	1 − 5 = −4	2 − 5 = _			

c The sequence with position-to-term rule $2n$.

n	1	2	3	4	5
$2n$	2 × 1 = 5	2 × 2 = _			

d The sequence with position-to-term rule $-3n$.

n	1	2	3	4	5
$-3n$	−3 × 1 = _				

e The sequence with position-to-term rule $2n + 3$.

n	1	2	3	4	5
$2n + 3$	2 × 1 + 3 = 5	2 × 2 + 3 = _			

Fluency

f The sequence with position-to-term rule $2n - 4$.

n	1	2	3	4	5
$2n - 4$					

Logical reasoning

8 Ahmed has £16 in his money box at the start of the year.

He gets £5 pocket money each week and puts it into his money box without spending any of it.

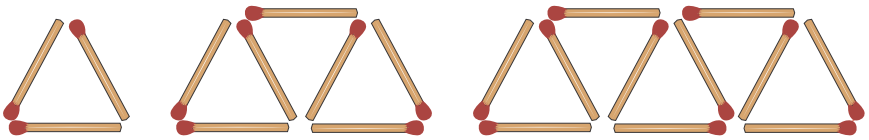
a Beginning with £16, write down the amount of money in Ahmed's money box as a sequence for 5 weeks.

b What is the term-to-term rule for this sequence?

c What is the position-to-term rule?

Fluency

9 Siôn is making patterns from matchsticks.



Pattern 1 Pattern 2 Pattern 3

a Copy and complete this table for the number of matchsticks used in each pattern.

Include the number of matchsticks Siôn would use if he continued to Pattern 4.

Pattern number, n	1	2	3	4
Number of matchsticks	3			

b If the pattern is continued, which pattern will use 23 matchsticks?

c Find the term-to-term rule for this sequence.

d Find the position-to-term rule for the number of matchsticks.

Hint

You add four matchsticks to move from one pattern to the next. This means that $4n$ will be in your position-to-term rule.

Band 2 questions

10 Find the nth term of these sequences.

a 4, 8, 12, 16, ... ← This is the four times table.

b 5, 9, 13, 17, ... ← These numbers are all one more than the numbers in the four times table.

c 3, 7, 11, 15, ...

d 6, 10, 14, 18, ... ← These numbers are all one less than the numbers in the four times table.

e 9, 13, 17, 21, ...

11 Find the nth term of these sequences.

a 2, 4, 6, 8, 10, ... ← This is the two times table.

b 4, 6, 8, 10, 12, ... ← These numbers are all two more than the numbers in the two times table.

c 1, 3, 5, 7, 9, ...

12 Find the nth term of these sequences.

a 3, 6, 9, 12, 15, ... **b** 6, 9, 12, 15, 18, ... **c** −2, 1, 4, 7, 10, ...

13 Find the nth term of these sequences.

a 3, 4, 5, 6, 7, ... **b** −1, 3, 7, 11, 15, ... **c** −1, −2, −3, −4, −5, ... **d** −3, −6, −9, −12, −15, ...

Strategic competence

14 The Olympic Games take place every four years. The first modern Olympic Games took place in 1896.

A position-to-term rule to find the year of the nth Olympic Games is:

$4n + 1892$

The table shows you how to use the position-to-term rule to find the year of the 2nd Olympic Games.

Some Olympic Games did not actually take place. For example, the 13th Olympic Games in 1944 was cancelled because of the Second World War, but the 1948 Games was still called the 14th Games. The 2020 Games were postponed until 2021 because of the Covid-19 pandemic.

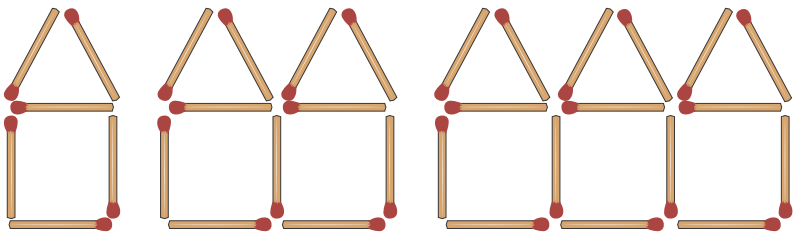
a Copy and complete the table to find the years of the 3rd, 5th and 10th Olympic Games.

n	2	3	5	10
Year	$4 \times 2 + 1892 = \mathbf{1900}$	$4 \times 3 + 1892 =$ ___		

b Use the rule to find what number Olympic Games took place in London in 2012.

Logical reasoning

15 Christophe is making patterns of houses from matchsticks.



Pattern 1 Pattern 2 Pattern 3

a Copy and complete this table for the number of matchsticks used in each pattern.
Include the number of matchsticks Christophe would use if he continued to Pattern 4.

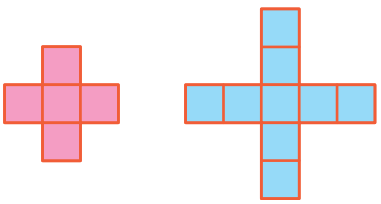
Pattern number, n	1	2	3	4
Number of matchsticks	6			

b If the pattern is continued, which pattern will use:

i 26 matchsticks **ii** 201 matchsticks?

c What is the term-to-term rule for this sequence?

16 Look at the patterns made up from square tiles.
Each pattern is in the shape of a cross.
Each tile measures 1 cm by 1 cm. The area of one tile is 1 cm^2.
The total area of Pattern 1 is 5 cm^2.
The total area of Pattern 2 is 9 cm^2.



Pattern 1 Pattern 2

a If the pattern is continued, what will be the area of Pattern 4?

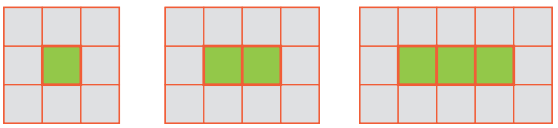
b Which pattern will have an area of 101 cm^2?

The perimeter of Pattern 1 is 12 cm.
The perimeter of Pattern 2 is 20 cm.

c If the pattern is continued, what will be the perimeter of Pattern 4?

d Which pattern will have a perimeter of 100 cm?

17 Pedr is making patterns with square tiles. Some of the tiles are green, some are shaded.
In Pattern 1, there is one green tile and there are eight shaded tiles.
In Pattern 2, there are two green tiles and ten shaded tiles.
In Pattern 3, there are three green tiles and twelve shaded tiles.



Pattern 1 Pattern 2 Pattern 3

a How many green tiles will there be in Pattern 6?

b How many shaded tiles will there be in Pattern 6?

c How many tiles will there be in Pattern 6 in total?

d If Pedr continued these patterns, which pattern would contain 80 shaded tiles?
How many green tiles would this pattern have?

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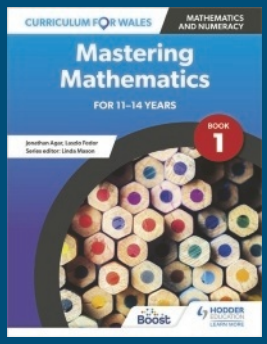

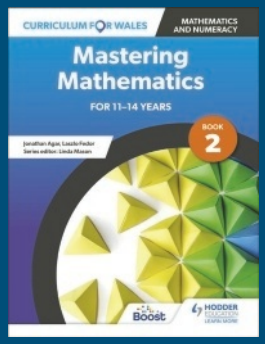

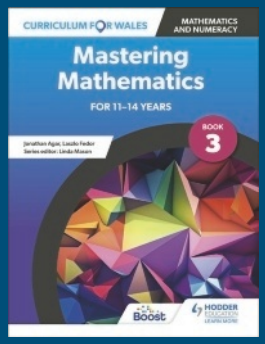

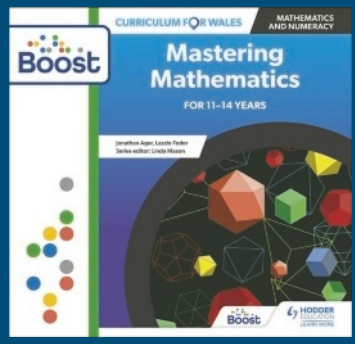

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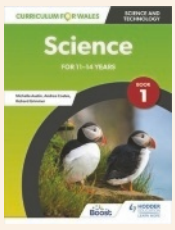

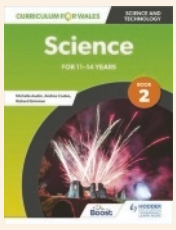

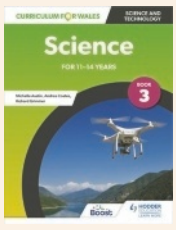

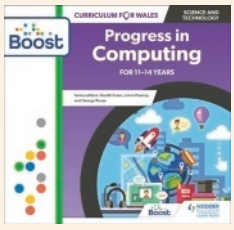

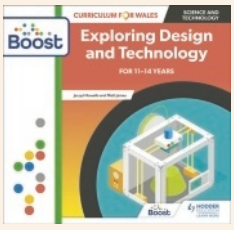

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