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THIRD EDITION

Lower Secondary Mathematics 8

SAMPLE MATERIAL

We are working with Cambridge Assessment International Education to gain endorsement for this forthcoming title.

Ric Pimentel
Frankie Pimentel
Terry Wall

Boost

HODDER EDUCATION

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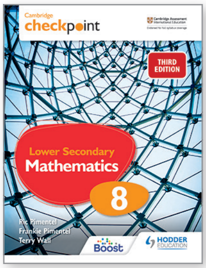

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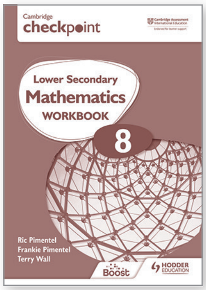

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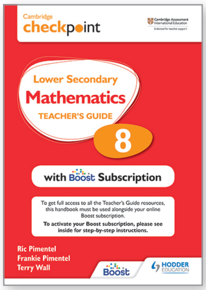

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Contents

The units in this book have been arranged to match the Cambridge Lower Secondary Mathematics curriculum framework. Each unit is colour coded according to the area of the syllabus it covers:

- Number
- Geometry & Measure
- Statistics & Probability
- Algebra

8 Properties of three-dimensional shapes

- Understand and use Euler's formula to connect number of vertices, faces and edges of 3D shapes.
- Use knowledge of area and volume to derive the formula for the volume of a triangular prism. Use the formula to calculate the volume of triangular prisms.
- Use knowledge of area, and properties of cubes, cuboids, triangular prisms and pyramids to calculate their surface area.

Euler's formula

Leonard Euler (1707–1783) was a famous Swiss mathematician. He is often considered one of the greatest mathematicians, along with Sir Isaac Newton.

He studied many areas of mathematics, but also in Physics, Astronomy and Engineering.

This section looks at one of the beautiful formulae he discovered to do with the properties of **polyhedra**.

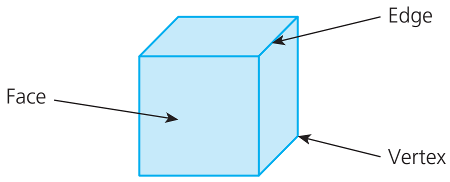
> Polyhedra means more than one polyhedron.

> Remember, a polygon is a 2D shape made up of straight edges. Examples include quadrilaterals, pentagons, hexagons etc.

A **polyhedron** is a three-dimensional shape, characterised by the fact that all its **faces** are flat and a type of **polygon**.

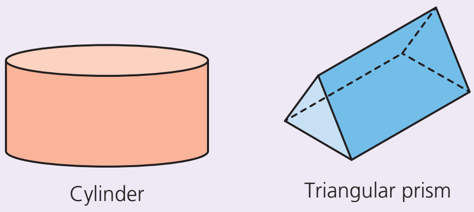
Different parts of a polyhedron have specific names as shown in the diagram of a cube below.

> Remember, more than one vertex are known as vertices.



Worked example

Of the two shapes below, decide whether both, one or none are polyhedral. Justify your choice.



The cylinder is not a polyhedron whilst the triangular prism is. This is because all the faces of a polyhedron must be a 2D polygon. The cylinder has two circular faces (not polygons) and a curved face.

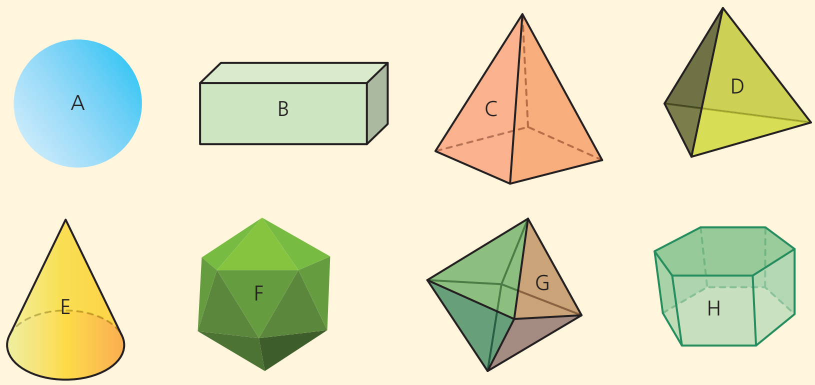
Exercise 8.1

1 Complete the following sentences:

a The corners of a polyhedron are called ....

b The flat side of a polyhedron is called a ....

c Two corners are joined by an ....



Cone	Square-based pyramid	Icosahedron	Sphere
Cuboid	Tetrahedron	Hexagonal prism	Octahedron

2 The diagrams above show several 3D shapes and names.

a Using the internet as a resource if necessary, match each of the shapes to their correct name.

b Which of the shapes are polyhedral?

3 a Draw a polyhedron not already drawn in Q1 and 2 above.

b What is the name of your polyhedron?

4 a For each of the polyhedra in questions 1–3, count the number of faces, edges and vertices and enter the results in a table similar to the one below. One is completed for you.

Name	Number of faces	Number of vertices	Number of edges
Triangular prism	5	6	9

b Can you spot a rule linking the number of faces, vertices and edges for each of the polyhedra in your table? If so, describe it in words.

LET'S TALK

Many of these shapes belong to other families of shapes. Discuss what these might be.

c Using F for the number of faces, V for the number of vertices and E for the number of edges express your rule described in part b) as a formula using algebra.

5 a Using a book or the internet, find two more polyhedra. Write down their names.

b Work out the numbers of faces, vertices and edges for your two polyhedra.

c Check that the results you wrote for part b) fit the formula you worked out in question 4.

6 Explain why your rule will not work for a cylinder or a cone. Use the words 'faces', 'edges' and 'vertices' in your explanation.

Convex polyhedra

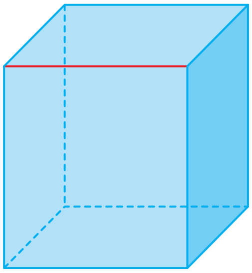
You found in the previous section that there is a relationship between the number of faces (F), vertices (V) and edges (E) of polyhedra. As a formula this can be written as:

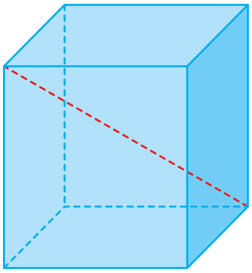
$$V + F - E = 2$$

This is known as **Euler's formula** and is true for most types of polyhedron. It does not, however, work for other 3D shapes.

However, Euler's formula does not work for every type of polyhedron either. All the polyhedra so far have been examples of **convex polyhedra**.

For a polyhedron to be convex, a line joining any two vertices falls completely on or within the shape itself. Looking at the cuboids shown below, in the first case the line joining two vertices runs along the edge of the cuboid, whilst in the second example it passes through the cuboid.





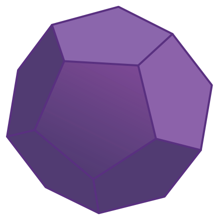
KEY INFORMATION

A pentagonal-based pyramid is simply a pyramid with its base in the shape of a pentagon.

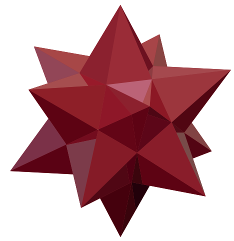
This is not the case for all polyhedra.

The word stellated means star shaped.

The diagram below shows a **dodecahedron**, which is a convex polyhedron with twelve faces. Each face of the dodecahedron is a **regular pentagon**. If a **pentagonal-based pyramid** is added to each face, the shape produced is called a **stellated dodecahedron**.



Dodecahedron



Stellated dodecahedron

LET'S TALK

How can we show that a stellated dodecahedron will not fit Euler's formula without having to count all the faces, vertices and edges?

If a line is drawn between the two of the peaks of the stellated dodecahedron it will not pass through the shape itself.

More analysis of these types of polyhedra is beyond the syllabus for this book.

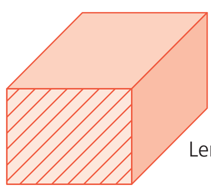
LET'S TALK

What other regular, non-convex polyhedra are there? Research this with a friend.

Triangular prisms

If a 3D shape is the same all the way through it is said to have a **constant cross-sectional area**.

You already know that a cuboid can look similar to this:



All faces are either squares or rectangles.

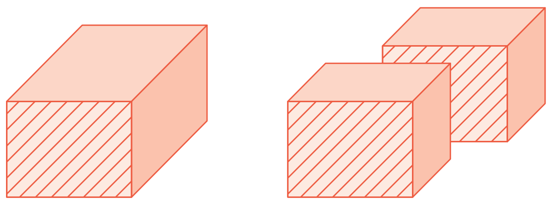
To calculate its volume, the length is multiplied by the width and height:

$$\text{Volume of cuboid} = \text{Length} \times \text{Width} \times \text{Height}$$

From the diagram we can see that $\text{Width} \times \text{Height}$ gives the area of the shaded end face.

Therefore, $\text{Volume of cuboid} = \text{Area of end face} \times \text{Length}$

If the cuboid was sliced vertically, the shape of the end face would be the same all the way through.



LET'S TALK

What other 3D shapes belong to the family of prisms? Can you sketch them?

This is known as the **cross-section** of the cuboid. A 3D shape with a constant cross-sectional area is called a **prism**. A cuboid therefore is a type of prism.

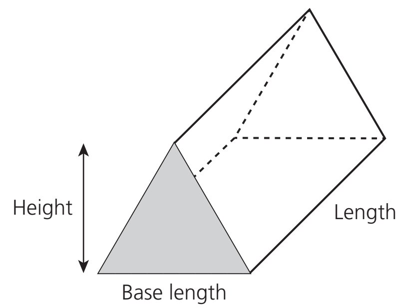
A type of prism you will have seen before is a **triangular prism**.

The cross-section is a triangle.

Volume of triangular prism

$= \text{Area of cross-section} \times \text{Length}$

$= \frac{1}{2} \times \text{Base length} \times \text{Height} \times \text{Length}$



Worked example

a The triangular prism below has dimensions as shown. Calculate its volume.

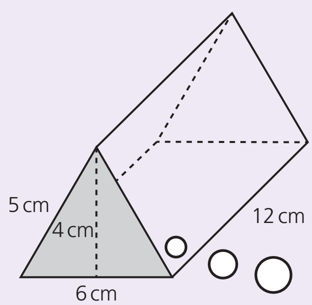
Volume of triangular prism
$= \text{Area of cross-section} \times \text{Length}$

The area of the cross-section is the area of the triangular end face:

$= \frac{1}{2} \times \text{Base length} \times \text{Height}$

$= \frac{1}{2} \times 6 \times 4 = 12\ \text{cm}^2$

Therefore, volume is $12 \times 12 = 144\ \text{cm}^3$



Note that to calculate the area of a triangle the perpendicular height is needed and not the length of the sloping edge. The 5cm length is therefore not needed.

b The volume of a triangular prism below is 270 cm³. Calculate its height.

The formula needs to be rearranged to make the height, *h*, the subject.

Volume = Area of cross-section × Length

270 = Area of cross-section × 12

Therefore, rearranging the formula gives:

$$\text{Area of cross-section} = \frac{270}{12} = 22.5\ \text{cm}^2$$

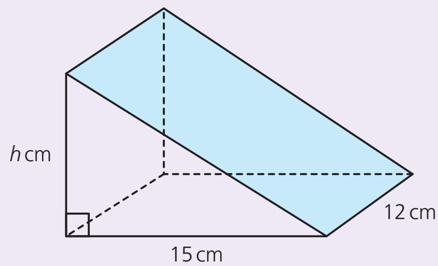
$$\text{Area of cross-section} = \frac{1}{2} \times \text{Base length} \times \text{Height}$$

$$22.5 = \frac{1}{2} \times 15 \times h$$

$45 = 15 \times h$ (multiplying both sides of the formula by 2)

$\frac{45}{15} = h$ (dividing both sides by 15)

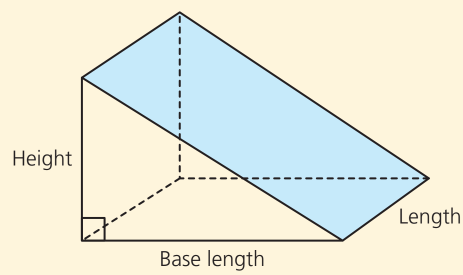
$h = 3$ cm



Exercise 8.2

1 Calculate the missing quantities in each of the following triangular prisms.

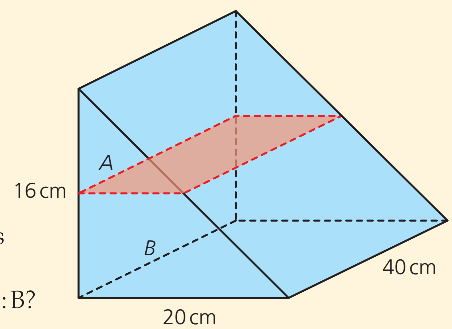
	Base length (cm)	Height (cm)	Length (cm)	Cross-sectional area (cm²)	Volume (cm³)
a	10	6	8		
b	4	15	6		
c		12	6	48	
d	10	7			700
e		10		25	125



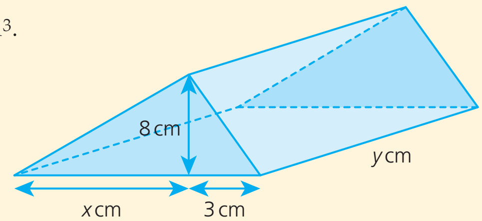
2 **a** Calculate the volume of the whole triangular prism.

A horizontal cut is made half-way down the height of the prism as shown by the shaded slice. It divides the original prism into two pieces, A and B.

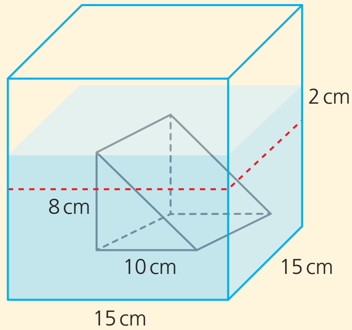
b What is the ratio of the volume of the two pieces, A : B? Give your answer in its simplest form.



3 The volume of the triangular prism shown is 560 cm^3.
The area of the triangular cross-section is 40 cm^2.
Calculate the values of x and y.



4 A water tank is in the shape of a cuboid.
When the triangular prism shown is lowered into the water, the water level rises by 2 cm.
Calculate the length of the prism.



Surface area

The surface area of a 3D shape is the total area of all of its faces.

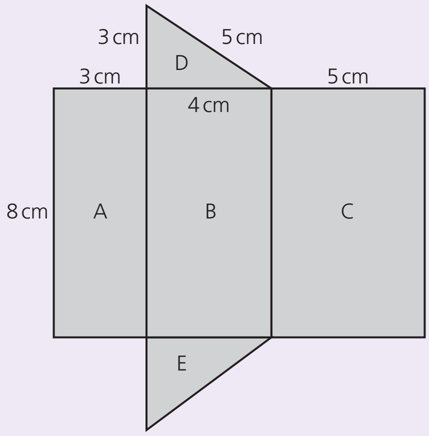
Therefore, knowledge of what types of face make up the 3D shape is important.

A net of an object is a 2D representation of that object. When a **net** is folded up, it makes that 3D object.

To do this it is sometimes helpful to visualise the net of the 3D shape.

Worked example

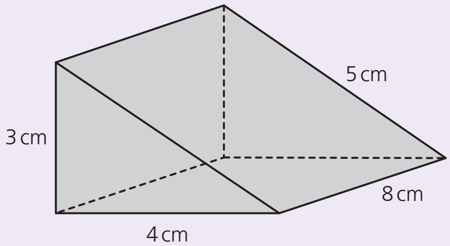
The net of a 3D object is given on the right.



LET'S TALK

Six dimensions have been included in the diagram. Was it necessary to write all six to know the dimensions of the shape? Which ones are necessary?

a Draw and name the 3D shape.



A triangular prism

b Calculate its total surface area.

From the net we can see that there are five faces: A, B, C, D and E that make up the total surface area of the triangular prism.

Area A $= 3 \times 8 = 24\,\text{cm}^2$

Area B $= 4 \times 8 = 32\,\text{cm}^2$

Area C $= 5 \times 8 = 40\,\text{cm}^2$

Area D & E $= \frac{1}{2} \times 4 \times 3 \times 2 = 12\,\text{cm}^2$

Total surface area $= 24 + 32 + 40 + 12 = 108\,\text{cm}^2$

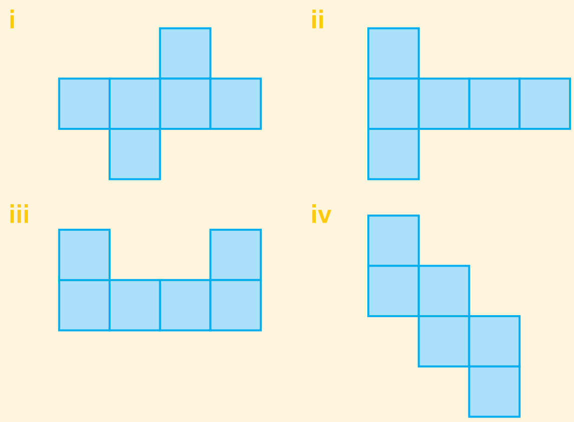
As the triangles D & E are the same, the area of one of them is calculated and then the result just doubled.

Exercise 8.3

1 **a** Four nets are shown below. Some of them fold up to make a cube. Which one(s) do not make a cube? Justify your answer.

b Draw another net for a cube different to the ones given above.

c If the face of the cube has an edge length of 5 cm, calculate the total surface area of the cube.

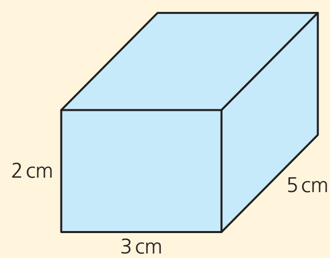


Cut out the nets from squared paper to check if necessary.

2 A cuboid is shown.

a Draw two possible nets for the cuboid.

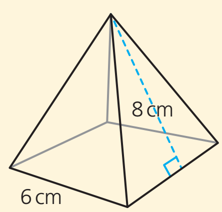
b Calculate the total surface area of the cuboid.



3 A square-based pyramid is shown.

a Calculate the area of one of its triangular faces.

b Calculate the total surface area of the pyramid.

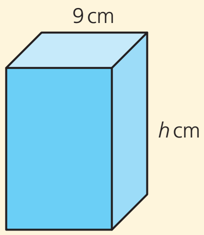


4 The cuboid shown has a square cross-section and a height h cm.

a If the total surface area is 594 cm², show that it can be given by the equation:

$$594 = 162 + 36h$$

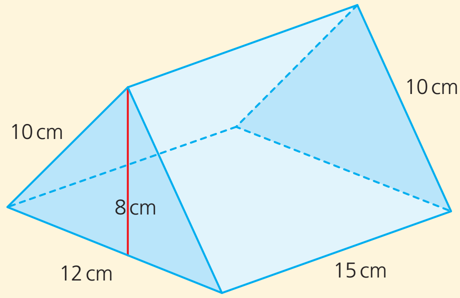
b Calculate the value of h.



5 A triangular prism is shown on the right.

a Draw and label a possible net for the triangular prism.

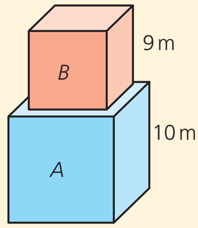
b Calculate its total surface area.



6 A modern art sculpture is being assembled. Its base is a cube A of edge length 10 m. Only five of its faces are visible as the one in contact with the ground cannot be seen. Another cube B of edge length 9 m is placed on top as shown.

a Calculate the total area of the visible surfaces of the combined sculpture of A and B.

b Successive cubes are placed on top of each other. The edge length of each decreasing by 1 m each time. How many cubes is the final sculpture made from if the total area of the visible surfaces is 1420 m²?



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- Provide activities to increase student's subject knowledge and develop the skills necessary to think and work mathematically.
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