Cambridge checkpoint

SECOND EDITION

Lower Secondary Mathematics

REVISION GUIDE

FOR THE SECONDARY 1 TEST

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Boost

HODDER EDUCATION

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Introduction

This Revision Guide helps you to recall the key mathematical content that you have covered in Mathematics through Stages 7–9 to prepare you for the Cambridge Secondary 1 Checkpoint test (Stage 9).

The Cambridge Secondary 1 Checkpoint exam consists of two papers:

- Paper 1: Non-calculator (1 hour)
- Paper 2: Calculator (1 hour)

You can use this Revision Guide to check your understanding of each of the topic areas. It will also ensure that you are familiar with the types of questions that you will be asked in the test. This Revision Guide includes a number of features to help you to prepare:

- **Skill check** – these exercises enable you to check your knowledge and understanding of a topic.
- **Remember** – boxes containing hints and reminders to aid your understanding.
- **Tips for success** – useful pointers of what the test is looking to check your understanding on.
- **Worked examples** – exam-style questions and full worked solutions.
- **Spotlight exercises** – exam-style questions to help prepare you for the exam itself.
- **Answers** are provided in the back of this revision guide.

Thinking and working mathematically (TWM) skills are used through out this book. You will need to make sure that you can use the eight key skills of: specialising, generalising, conjecturing, convincing, characterising, classifying, critiquing and improving.

It is hoped that this Revision Guide will help you to feel fully prepared for your Cambridge Secondary 1 Checkpoint tests.

General revision tips

Find somewhere quiet to revise. Sit on a comfortable chair at a table and have a pen or pencil and some sheets of paper as well as this book. Plan what you will revise in your revision session. Remember to take a break perhaps every twenty minutes or half hour to let your mind rest.

When using a calculator, write down the calculation you are intending to do first, before entering it into the calculator.

Just reading through the text is not always the best way of learning. It is better to make your revision more active. You should use a variety of active ways to make your learning secure. Here are some of them:

- Cover up the solution to a worked example on the topic you are revising. Write out your own solution, and then check it against the worked solution. In addition to getting the right answer, make sure that all your key steps of working are shown clearly.
- Continue your active revision by completing the Spotlight sections. Write your answers on paper.
- Study the Tips for success; write up some of them on revision cards.

If you get stuck on a question try:

- drawing a diagram
- highlighting the key words in the question
- writing down the information you know, and thinking about what you can work out using this
- looking back at a similar question or example.

Section 1 Number

Chapter 1 Integers, powers and roots

1.1 Calculations

LINKS
- Stage 7 Unit 1
- Stage 8 Unit 1

REMEMBER
Adjust the calculation to make it easier to do.

REMEMBER
$\frac{2044}{28}$ means

$2044 \div 28$

Multiplication and division are **inverse operations**.

So, multiplying by 28 'undoes' dividing by 28.

KEY POINTS

- You can use strategies to help you work out a calculation.

Example Work out 32×40.

Solution 32×40 is 10 times greater than 32×4

$32 \times 4 = 32 \times 2 \times 2 = 64 \times 2 = 128$

So, $32 \times 40 = 128 \times 10 = 1280$

Example Work out $1296 - 407$

Solution $1296 - 407$ is close to $1300 - 400 = 900$

1300 is 4 more than 1296 so the answer should be 4 less.

400 is 7 less than 407 so the answer should be 7 less (as you've taken away too little).

So, $1296 - 407 = 900 - 4 - 7 = 889$

- You can make **generalisations** to help you carry out calculations.

Example You can use the fact that $\frac{2044}{28} = 73$ to also say that:

$73 \times 28 = 2044$ and $28 \times 73 = 2044$

WATCH OUT!
You can multiply or add in any order. This means you can change the order of a multiplication or addition to make it easier to do.

So, $6 \times 11 \times 5 = 11 \times 6 \times 5$
$= 11 \times 30$
$= 11 \times 3 \times 10$
$= 330$

But the order matters when you divide or subtract.

So, $5 - 2 \neq 2 - 5$

and $5 \div 2 \neq 2 \div 5$

Skill check

1 Use a strategy to help you work out each of these as quickly as you can.
  - a $37 + 102 + 63 + 38$
  - b $1803 - 799$
  - c $398 + 403$
  - d $2996 - 1307$
  - e $37 \times 2 \times 5$
  - f $22 \times 5 \times 6$
  - g $3 \times 5 \times 6 \times 9$
  - h $5 \times 8 \times 15 \times 2$

2 Look for patterns to help you work out these.
  - a i $624 \div 2$ ii $624 \div 4$ iii $624 \div 8$
  - b i $3620 \div 10$ ii $3620 \div 20$ iii $3620 \div 5$
  - c i $720 \div 3$ ii $720 \div 6$ iii $720 \div 12$

TIPS FOR SUCCESS

Make sure you use the = sign properly. You can only use it to connect two statements which are equal. You must not use it to mean 'and then'.

This is correct:

$6 + 4 = 10$ ✓

$10 \times 2 = 20$ ✓

But this is wrong:

$6 + 4 = 10 \times 2 = 20$ ✗

WORKED EXAMPLE

The table shows how much Amir is paid for his job.

Monday–Friday	\$9 per hour
Saturday	\$12 per hour

Last week Amir was paid \$339.

He worked 8 hours on Saturday.

How many hours did he work altogether?

Solution

On Saturday Amir was paid $8 \times \$12 = \$96$

During the week Amir earns $\$339 - \$96 = \$243$

Amir worked $243 \div 9 = 27$ *hours from Monday to Friday.*

So, altogether Amir worked 27 hours + 8 hours = 35 hours.

HINT

An equation is a statement that says that two expressions are equal.

HINT

Look at the last digit in each answer. Think about how it could be made.

HINT

Start by working out how many seats the cinema has. How many child tickets were sold?

SPOTLIGHT

1 Fill in the boxes to make each equation true.

a $\square \times 12 \times 4 = 144$

b $\square + 96 - 39 = 93 + 47$

c $5 \times \square + 27 = 193 - 76$

2 Fill in the boxes with the missing digits in each of these calculations:

a $5\square + \square 3 = 100$

b $\square 3 \square 8 - 76\square = \square 83$

c $1\square \times \square 3 = 3\square 2$

3 Look at the equation $918 \div 27 = 34$.

Use the equation to write down the answers to the following:

a 27×34

b $9180 \div 34$

c $\dfrac{91800}{270}$

4 Star Cinema has 18 rows of seats.

Each row has 20 seats.

Last night Star Cinema sold tickets for all of its seats.

Star Cinema

Adult ticket	\$12
Child ticket	\$7

116 tickets were for adults and the rest were child tickets.

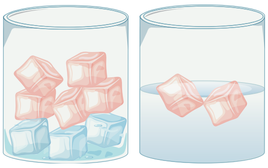
How much money did the cinema make from selling tickets last night?

1.2 Negative numbers

LINKS
- Stage 7 Unit 1
- Stage 8 Unit 1

REMEMBER

Imagine adding 'cold cubes' and 'hot cubes' to a glass.



These glasses show $-3+5=\mathbf{2}$

When you **add cold** cubes, the temperature goes **down**.

When you **take away cold cubes** the temperature goes **up**.

REMEMBER

When a number does not have a sign, then it is positive.

So, $(-3)\times 5=-15$

is the same as $(-3)\times(+5)=-15$

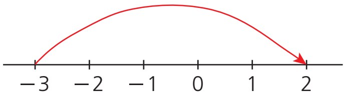
WATCH OUT

Remember the order doesn't matter when you multiply.
For example:
$(-2)\times(-7)\times(-5)$
$=(+10)\times(-7)$
$=-70$

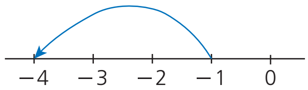
KEY POINTS

When you **add**:

- a **positive number**, move **RIGHT** on the number line
- a **negative number**, move **LEFT** on the number line.



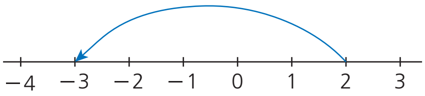
$-3+5=2$



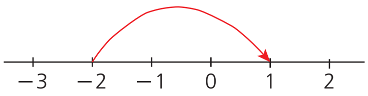
$-1+(-3)=-4$

When you **subtract**:

- a **positive number**, move **LEFT** on the number line
- a **negative number**, move **RIGHT** on the number line.



$2-5=-3$



$-2-(-3)=1$

When you multiply or divide two numbers with the:

- **SAME** signs, the answer is **positive**
- **DIFFERENT** signs, the answer is **negative.**

Examples

Same signs	Different signs
$3\times 5=15$	$(-3)\times 5=-15$
$(-3)\times(-5)=15$	$3\times(-5)=-15$
$12\div 2=6$	$(-12)\div 2=-6$
$(-12)\div(-2)=6$	$12\div(-2)=-6$

Skill check

Calculate these.

1 $(-2)+2$
2 $-1-(-4)$
3 $(-3)-(-3)$
4 $1+(-2)+3+(-4)$
5 $1-(-2)-3-(-4)$
6 $12\div(-4)$
7 $(-6)\div(-2)$
8 $2\times 3\times(-4)$
9 $(-2)\times 3\times(-4)\times 5$
10 $(-5)\times 11\times(-4)\times(-2)$

TIPS FOR SUCCESS

Remember range is the difference between the highest and lowest values.

To find the mean you add up all the values and divide by how many there are.

Don't forget the units for your answers!

WORKED EXAMPLE

The table shows the temperature in Skardu, Pakistan, in January taken at midnight each night for six nights.

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Temperature (°C)	–5	–7	–4	–2	3	–1	2

Calculate:

a the range in temperature

b the mean temperature.

Solution

a Range = highest temperature – lowest temperature

$= 3°C - (-7°C) = 10°C$

b Mean $= \dfrac{(-5)+(-7)+(-4)+(-2)+3+(-1)+2}{7}$

$= \dfrac{-14}{7} = -2$

So the mean is –2°C

SPOTLIGHT

1 Insert the correct symbol (< or >) into each box.

a $-635 \ \square \ -563$

b $(-24) - (-59) \ \square \ (-24) + (-59)$

c $(-124) + 48 \ \square \ (-124) - 48$

2 Classify these calculations into two groups with the same answers.

$17 \times (-19)$	$16 \ -19 \times 16$	$(-19) \times 17$
$(-17) \times 18 - 17$	$(-16) \times 19 + 16$	$(-16) \times 19 - 19$

HINT

You can **classify** the calculations without actually working any of them out. Can you see how?

3 Jamal knows that $3 \times 6 \times 12 = 216$.

Write down the answers Jamal should find when he uses his equation to work out these:

a $18 \times (-12)$

b $-216 \div 12$

c $-216 \div (-18)$

d $(-3) \times (-6) \times (-12)$

HINT

Think carefully about your signs.

4 Write a number in each box to make each calculation correct.

a $(-4) + 2 + \square = 0$

b $(-4) + \square = (-3) - (-2)$

c $(-2) \times \square \times 4 = 40$

d $(-12) \div \square = 2 \times 3$

HINT

Start by working out the total of all the numbers.

The cards might not be in order of size.

5 Each of these cards has a number on it.

- The mean of the numbers is –4.
- The range of the numbers is 14.

What are the missing numbers on the cards?

? **4** **?**

LINKS
- Stage 7 Unit 5
- Stage 8 Unit 5

REMEMBER
If the operations are all the same type (+ and – or × and ÷) then the order doesn't matter.
$9-6+2=9+2-6$ and $6\times2\div3=6\div3\times2$

REMEMBER
Division is often written as a fraction.
So $12\div3$ is written as $\frac{12}{3}$.
You can think of the fraction line as 'divided by'.

REMEMBER
$4^2=4\times4=16$ and $\sqrt{16}=4$.
You say 4 squared is 16 and the square root of 16 is 4.
Also, remember that $4^3=4\times4\times4=64$ and $\sqrt[3]{64}=4$.
You say 4 cubed is 64 and the cube root of 64 is 4.

WATCH OUT!
If you change the order, then make sure you keep each operation with its number.
For example, $2-6\neq6-2$

WATCH OUT
Remember the order doesn't matter when you multiply.
For example:
$(-2)\times(-7)\times(-5)$
$=(+10)\times(-7)$
$=-70$

1.3 Order of operations

KEY POINTS

- To write down a calculation you need to use the **operations**: addition (+), subtraction (–), multiplication (×) and division (÷)
- You can also use **brackets** and **indices** (powers and roots). These operations are carried out in a certain order.

Brackets – always work out any brackets first.

Indices – then work out any powers such as 2^3 or roots such as $\sqrt{16}$.

Division and **M**ultiplication – Next work out any multiplications or divisions.

Addition and **S**ubtraction – Last work out any additions or subtractions.

The shorthand '**BIDMAS**' can help you to remember the order in which to carry out operations.

Example Work out $6+10\times9-5$.

Solution Using BIDMAS, carry out the multiplication first.

$6+\underbrace{10\times9}_{90}-5$
$=6+90-5$
$=96-5$
$=91$

Example Work out $4^2-\sqrt{9}\times(7-2)$.

Solution Using BIDMAS, work out the brackets first.

$4^2-\sqrt{9}\times(7-2)=4^2-\sqrt{9}\times5$

Then any indices (powers or roots)

$4^2-\sqrt{9}\times5=16-3\times5$

Then multiply and finally subtract.

$16-3\times5$
$=16-15$
$=1$

So, $4^2-\sqrt{9}\times(7-2)=1$

Skill check

1 Use BIDMAS to work out the following calculations.

a $5+2\times3$
b $5-2\times3$
c $(5-2)\times3$
d $24\div4+2$
e $24\div(4+2)$
f $5-2\times3+24\div4+2$

2 Use BIDMAS to work out the following calculations.

a $15+4\times2^2$
b $15-4\times2^2$
c $(15-4)\times2^2$
d $15^2-4\times2$
e $(15^2-4)\times2$
f $(15-4)^2\times2$

TIPS FOR SUCCESS

Watch out for hidden brackets or multiplication signs.

$\frac{7+3}{6-4}$ means $\frac{(7+3)}{(6-4)} = \frac{10}{2} = 5$

and $(7+3)(6-4)$ means

$(7+3) \times (6-4)$
$= 10 \times 2 = 20$

WORKED EXAMPLE

Calculate $\sqrt{100} - \frac{6+5\times3}{7-2^2}$

Solution

Using BIDMAS: $\sqrt{100} - \frac{6+5\times3}{7-2^2}$

$= \sqrt{100} - \frac{(6+5\times3)}{(7-2^2)}$

$= 10 - \frac{(6+15)}{(7-4)}$

$= 10 - \frac{21}{3}$

$= 10 - 7$

$= 3$

HINT

Try out the brackets in different places.

SPOTLIGHT

1 Add brackets ( ) to make each statement correct.

You may use more than one pair of brackets in each statement.

a $8+2^2\times7-5=16$

b $8+2^2\times7-5=79$

c $8+2^2\times7-5=31$

d $8+2^2\times7-5=695$

e $8+2^2\times7-5=24$

One of the statements did not need brackets. Which one?

2 Johan says that $3+2\times5^2$ is 125.

Maria says that $3+2\times5^2$ is 103.

Zac says that $3+2\times5^2$ is 53.

a Who is correct?

Give a **convincing** reason for your answer.

b Show how you can add brackets to the other two calculations to make their answers correct.

3 Decide whether each of these statements is True or False.

If the statement is False, write down the correct answer.

a $\sqrt{3^2+4^2} = 3+4=7$

b $20-15\div5 = 1$

c $\frac{2\times10-8}{3\times2^2} = 1$

1.4 Multiples, factors and primes

LINKS
- Stage 7 Unit 9
- Stage 8 Unit 9

REMEMBER
Factors come in pairs:

$1 \times 24 = 24$

$2 \times 24 = 24$

$3 \times 8 = 24$

$4 \times 6 = 24$

KEY POINTS

- A **factor** of a number is a whole number which is divisible exactly into that number.
- The **highest common factor (HCF)** is the *highest* number which is a *common factor* of two or more numbers.

Example Factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.

Factors of 30 are 1, 2, 3, 5, 6, 10, 15, 30

6 is the highest number that appears on both lists.

So, the HCF of 24 and 30 is 6.

- You can use **divisibility tests** to help you find factors.

Divisible by ...	Divisibility test
2	The units digit is even
3	The sum of the digits is divisible by 3
5	The units digit is 0 or 5
6	Divisible by 2 and 3
9	The sum of the digits is divisible by 9

REMEMBER
A prime factor is a prime number that is a factor.

- A **prime number** is a number with only two factors: 1 and the number itself.

Example The first five prime numbers are 2, 3, 5, 7 and 11.

- Any whole number can be written as a product of prime factors.

Example $30 = 2 \times 3 \times 5$ and $24 = 2 \times 2 \times 2 \times 3$ or $2^3 \times 3$

WATCH OUT!
2 is the only even prime number.

1 is not a prime number because it only has one factor – itself!

- A **multiple** of a number is a whole number that is in that number's times table.
- The **lowest common multiple (LCM)** is the *lowest* number which is a *common multiple* of two or more numbers.

Example Multiples of 24 are 24, 48, 72, 96, 120, 144, ...

Multiples of 30 are 30, 60, 90, 120, 150, ...

120 is the lowest number that appears on both lists.

So, the LCM of 24 and 30 is 120.

Skill check

1 Write down the factors of each of these numbers.

a 10 b 35
c 42 d 60
e 100 f 125

2 Write down the first five multiples of each of these numbers.

a 5 b 7
c 20 d 40
e 100 f 1

3 Write down all the prime numbers between:

a 10 and 20 b 30 and 40
c 50 and 60 d 80 and 90

TIPS FOR SUCCESS

- You can use a factor tree to help you to write larger numbers as a product of prime factors.
  Look for multiplication facts to make the number at the start of each branch.
  Circle each prime number.
- You can write the products using powers.
  For example, $60 = 2^2 \times 3 \times 5$ and $72 = 2^3 \times 3^2$.
- Use multiplication to check your answers are right.

HINT

To find the LCM, you multiply together all the numbers in the Venn diagram.

WORKED EXAMPLE

Two neighbourhood dogs bark regularly.

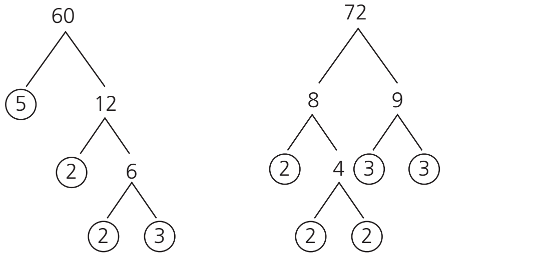
One dog barks every 60 minutes, and the other dog barks every 72 minutes.

Both dogs bark together at 12 noon.

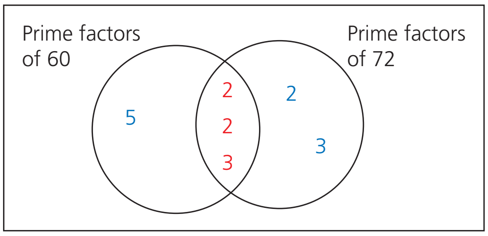
When do the dogs next bark at the same time?

Solution

Use factors trees to write both numbers as a product of prime factors.



So $60 = 2\times2\times3\times5$ *and* $72 = 2\times2\times2\times3\times3$



HINT

A Venn **diagram** is useful for finding the HCF and LCM.

The intersection gives the factors in common, so you can multiply the numbers in the intersection to find the HCF.

So, the HCF of 60 and 72 is $2 \times 2 \times 3 = 12$

The LCM of 60 and 72 is $5\times2\times2\times3\times2\times3 = 360$

So, they next bark together 360 minutes later, which is at 6 pm.

SPOTLIGHT

1 Write a **different** digit in each box to make each statement true.
   a 247 ☐ is divisible by 5
   b 247 ☐ is divisible by 3
   c 247 ☐ is divisible by 6
   d 247 ☐ is divisible by 9

2 Gemma writes down a **conjecture**.

   91 is prime number.

   Is Gemma correct?

   Give a **convincing** reason for your answer.

3 Romesh writes down a **conjecture**.

   The number of factors of any number is even.

   Is Romesh correct?

   Give a **convincing** reason for your answer.

4 a Write 240 as a product of prime factors.
   b Find the lowest common multiple of 240 and 210.
   c Find the highest common factor of 240 and 210.

5 m and n are integers, where m is greater than n.

   The lowest common multiple of m and n is 300.

   The highest common factor of m and n is 15.

   Work out the value of m and the value of n.

   Find **two** possible answers.

HINT

Draw a Venn diagram to help you.

Fill in the intersection first.

1.5 Types of number

LINKS
- Stage 7 Unit 26
- Stage 8 Unit 26
- Stage 9 Units 9 and 26

REMEMBER
The 5th square number is
$5^2 = 5 \times 5 = 25$
You say, '5 squared is 25.'

HINT
See the next section for more on indices and powers.

KEY POINTS

- **Squaring** means multiplying *any number* by itself.
  An **index** or **power** of 2 is used to show a squaring.
  A **square number** is made by multiplying an *integer* by itself.
  The first 10 square numbers are 1, 4, 9, 16, 25, 36, 49, 64, 81 and 100
- The inverse operation of squaring a number is **square rooting**.
  $(-3) \times (-3) = 9$ and $3 \times 3 = 9$ so the square roots of 9 are 3 and –3.
  The square root symbol $\sqrt{\ }$ means the **positive square root**, so $\sqrt{9} = 3$.
- **Cubing** means multiplying a number by itself twice.
  An **index** or **power** of 3 is used to show a cubing.
  The first 5 cube numbers are 1, 8, 27, 64 and 125
- The inverse operation of cubing a number is **cube rooting**.
  The square root symbol $\sqrt[3]{\ }$ means cube root.

Example $\sqrt[3]{216} = 6$ because $6^3 = 6 \times 6 \times 6 = 216$

- **Natural numbers** are the counting numbers 1, 2, 3, ...
- **Integers** are any whole number: positive or negative and zero.
- A **rational number** is any number that can be written as a fraction.
  Any integer and terminating or recurring decimal can be written as a fraction.

Example 17, 0.43 and 2.7 are all rational numbers.

Reason $17 = \frac{34}{2}$, $0.43 = \frac{43}{100}$ and $2.\dot{7} = \frac{25}{9}$

- Any number which cannot be written as a fraction is **irrational**.
- A **surd** is a type of irrational number that has a square or cube root.
  Surds cannot be simplified into whole numbers or rational numbers.

Example 3 is not a square number so $\sqrt{3}$ is a surd,

But $\sqrt{4}$ is not a surd as $\sqrt{4} = 2$.

- You can **estimate** the value of a surd.

Example 110 is between 10^2 (= 100) and 11^2 (= 121)

So, $\sqrt{110}$ is between 10 and 11.

You can write this as an inequality: $10 < \sqrt{110} < 11$

REMEMBER
π is an irrational number.

REMEMBER
The square root of any integer that is not a square number is a surd.
For example, $\sqrt{3} = 1.73205...$, the decimal part continues forever and never repeats.

Skill check

1 Write down the answer to these.
a 6^2 b $(-7)^2$ c 20^2
d 2^3 e 10^3 f $(-12)^3$

2 Write down the square roots of each of these numbers.
a 144 b 1 c 169 d 225

3 Complete these inequalities by writing a whole number in each box.
Each surd lies between two consecutive whole numbers.
a $\square < \sqrt{60} < \square$
b $\square < \sqrt{200} < \square$
c $\square < \sqrt[3]{10} < \square$
d $\square < \sqrt[3]{100} < \square$

WATCH OUT!
When you square a number, the result is always positive.
The cube of a negative number is also negative.

REMEMBER
Consecutive means 'following on from each other'.

TIPS FOR SUCCESS

When you estimate the value of a surd, look for the closest square or cube numbers to the number under the root.

This question is about a cube, so you need to look for cube numbers.

HINT

Remember that a cube has 6 faces.

The area of one face is the side length squared.

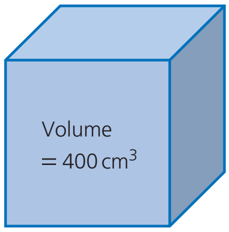
WORKED EXAMPLE

The volume of a cube is $400\,\text{cm}^3$.

Find the surface area, A, of the cube.

Give your answer in the form $m < A < n$

where m and n are the closest integer values to A.



Solution

The closest cube numbers to 400 are $7^3 = 343$ and $8^3 = 512$

So, $7\,\text{cm} < \text{sidelength} < 8\,\text{cm}$.

A cube of side 7 cm has a surface area of $6 \times 7^2 = 294\,\text{cm}^2$

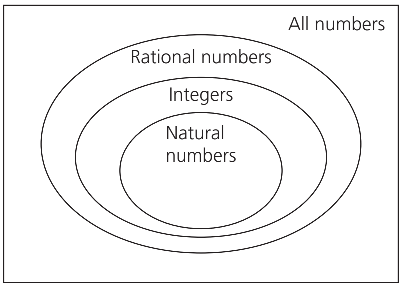
A cube of side 8 cm has a surface area of $6 \times 8^2 = 384\,\text{cm}^2$

So, $294 < A < 384$.

SPOTLIGHT

1 Classify these numbers by writing them in the correct place on a copy of the Venn diagram.

π 5 0 -4.5 $\sqrt{5}$ $\sqrt{16}$ $\sqrt[3]{3}$ -9

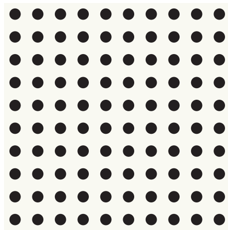


HINT

Look for two square numbers which have a sum of 100.

2 Jaz arranges 100 counters to make a square pattern.

Show that Jaz can rearrange the 100 counters to make a pattern of two smaller squares without any counters left over.



3 Benji has 160 cube shaped bricks.

Benji arranges the bricks to make a larger cube.

a What is the largest cube that Benji can make?

Benji uses all 160 bricks to make 3 cubes.

b What size cubes does Benji make?

4 Meena estimates that the perimeter of this square is 50 cm.

a Show that Meena's estimate is sensible.

b Give an **improved** estimate for the perimeter of the square.

HINT

You need to use your **specialising** skills to choose appropriate numbers and check whether the **conjectures** are true.

5 **a** Parvati writes down a **conjecture**:

All integers are natural numbers.

Is Parvati correct? Give a **convincing** reason for your answer.

b Jack writes down a **conjecture**.

All natural numbers are also rational numbers.

Is Jake correct? Give a **convincing** reason for your answer.

6 Luca estimates that $\sqrt{50}$ is 7.5.

Improve Luca's estimate.

LINKS
- Stage 8 Unit 1
- Stage 9 Unit 1

WATCH OUT!
Take care when the power is 1 or 0. Remember that $5^1 = 5$ and $5^0 = 1$.

1.6 Powers and indices

KEY POINTS

- When you multiply the same number by itself several times you can use powers to write the calculation in a shorter way.

Example $5 \times 5 \times 5 \times 5$ can be written as 5^4.

- 5 is called the base and 4 is the power or index.

In general, $a^n = \underbrace{a \times a \times \ldots \times a \times a}_{n \text{ times}}$

- Any number to the power 1 is equal to itself.

$$a^1 = a$$

- Any number to the power 0 is equal to 1.

$$a^0 = 1$$

- You can use the **laws of indices** to multiply or divide powers of the same base number.

$$a^m \times a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{m \times n}$$

Examples $5^3 \times 5^4 = 5^{3+4} = 5^7$

$5^9 \div 5^8 = 5^{9-8} = 5^1 = 5$

$(5^2)^3 = 5^{2 \times 3} = 5^6$

- A **negative power** means '1 divided by' or '1 over'.

$$a^{-n} = \frac{1}{a^n}$$

Example $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

Skill check

1 Write each of the following as a single power.

a $4 \times 4 \times 4 \times 4 \times 4$
b $7 \times 7 \times 7$
c $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$
d $2 \times 2 \times 2 \times 2$

2 Write each of the following as a single power.

a $3^7 \times 3^4$
b $2^3 \times 2^5$
c $5^3 \times 5^{-1}$
d $9^7 \div 9^3$
e $11^4 \div 11^{-6}$
f $(15^4)^3$

3 Write each of the following as a single power.

a $\dfrac{6^4 \times 6^8}{(6^2)^3}$
b $\dfrac{7^3 \times (7^3)^5}{7^{-4}}$

REMEMBER
Make sure you know how to use your calculator to work out powers.

Use your calculator to check your answers to the Skill check exercise.

TIPS FOR SUCCESS

Remember you can only use the rules of indices if the base numbers are the same.

HINT

You can rewrite $9^3 \times 3^5$ as $(3^2)^3 \times 3^5$

WORKED EXAMPLE

a Write a number in the box to make this statement true:

$9^3 \times 3^5 = 3^{\square}$

b Without working out either power, show that $8^2 > 2^5$

Solution

a Remember that $9 = 3^2$, so $9^3 = (3^2)^3$

$$(3^2)^3 \times 3^5 = 3^{2\times3} \times 3^5$$
$$= 3^6 \times 3^5$$
$$= 3^{6+5}$$
$$= 3^{11}$$

b Remember that $8 = 2^3$, so $8^2 = (2^3)^2$

Writing as a power of 2 gives: $(2^3)^2 = 2^{3\times2} = 2^6$

$2^6 > 2^5$ because $6 > 5$ and the base numbers are both 2.

SPOTLIGHT

1 Joshua says that $2^5 \times 4^4 = 8^9$.

a Is Josh correct? Give a **convincing** reason for your answer.

b Write a number in the box to make this statement true.

$2^5 \times 4^4 = 2^{\square}$

2 Match each expression in the blue rectangle with the equivalent expression in a coloured shape.

HINT

Start by simplifying each expression.

$\frac{x^5}{x^2}$	$x^2 \times x^3$	$x \times x^3$	$\frac{x^4 \times x^{-2}}{x^2}$	$(x^4)^2$	$x^7 \div x^6$
x^4	$x \times x \times x$	x	x^8	x^5	1

3 Write a number in each box to make each statement true.

a $5^3 \times 5^{\square} = 5^8$

b $5^3 \div 5^{\square} = 5$

c $(5^3)^{\square} = 5^8 \div 5^2$

d $5^3 \times 5^{\square} = 1$

4 **a** Work out the answer to these calculations.

i Square the number 4 and then cube your answer.

ii Cube the number 4 and then square your answer.

What do you notice?

b Amir writes down a **conjecture**.

Cubing a number and then squaring the result always gives the same answer as squaring a number and then cubing the result.

Is Amir correct?

Give a **convincing** reason for your answer.

HINT

Use the laws of indices.

Chapter 2 Place value, ordering and rounding

2.1 Multiplying and dividing by powers of 10

LINKS
- Stage 7 Unit 11
- Stage 8 Unit 11

KEY POINTS

- A place value table is used to show the value of each digit in a number. This table shows the number 932.58

Thousands	Hundreds	Units •	Tenths	Hundredths
9	3	2 •	5	8

- When you **multiply** a number by a power of 10 each digit moves to the **LEFT**.
  When you **divide** a number by a power of 10 each digit moves to the **RIGHT**.

Multiplying /dividing by...	Digits move...
10	1 place **LEFT**/**RIGHT**
100	2 places **LEFT**/**RIGHT**
1000	3 places **LEFT**/**RIGHT**

Example $634.7 \times 100 = 63\,470$ (the answer is 100 times greater)

$634.7 \div 100 = 6.347$ (the answer is 100 times less)

- Powers of 10 can also be less than 1.

$10^{-1} = \frac{1}{10} = 0.1$ $10^{-2} = \frac{1}{100} = 0.01$ $10^{-3} = \frac{1}{1000} = 0.001$

So, **multiplying by 0.1** is the same as **dividing by 10**

and **dividing by 0.1** is the same as **multiplying by 10** and so on.

Example $34.5 \times 0.01 = 34.5 \div 100 = 0.345$

REMEMBER
You can write powers of 10 using indices.

So, $10^3 = 1000$

$10^2 = 100$

$10^1 = 10$

$10^0 = 1$

$10^{-1} = 0.1$

$10^{-2} = 0.01$

and so on.

WATCH OUT!
Make sure you think about your answers and check they make sense!

Skill check

1 Work out these.

a i 73×10 ii $73 \div 10$
b i 42×100 ii $42 \div 100$
c i 351×1000 ii $351 \div 1000$

2 Work out these.

a i 0.53×10 ii $0.53 \div 10$
b i 0.076×100 ii $0.076 \div 100$
c i 12.9×1000 ii $12.9 \div 1000$

3 Work out these.

a 2.5×0.01
b $396 \div 0.001$
c 0.0056×0.001
d 32.9×0.1
e $1\,383\,000 \times 0.0001$
f $0.093 \div 0.1$

TIPS FOR SUCCESS

- Test out the conjecture with some numbers. You need to test both positive and negative powers.
- You only need to find one example that doesn't work to show that a conjecture is not **always** true.
- Make sure you finish off the question by saying clearly what you have shown.

WORKED EXAMPLE

Rory writes down this **conjecture**.

When you multiply a number by a power of 10, the answer is always greater than the number you started with.

Is Rory right?

Give a **convincing** reason for your answer.

Solution

$3 \times 10^2 = 3 \times 100 = 300$

300 is greater than 3.

$3 \times 10^{-2} = 3 \times \frac{1}{100} = 3 \div 100 = 0.03$

0.03 is less than 3.

This shows that Rory is not correct.

When you multiply by a number that is less than 1, the answer is less than your original number.

So, when you multiply by a negative power of 10 like 10^{-2}, the answer is less than the number you started with.

HINT

Make sure your answer makes sense. Should the answer be bigger or smaller than the starting number?

WATCH OUT

Take care to move each digit the right number of places.

SPOTLIGHT

1 Complete these number chains.

$\times 0.1$ $\times 0.01$ $\times 0.001$ $\div 0.1$ $\div 0.01$ $\div 0.001$

a 45 [ ] 4.5 [ ] 450

b 0.216 [ ] 2.16 [ ] 216 [ ] 0.216

c 0.039 [ ] 0.39 [ ] 0.0039 [ ] 3.9 [ ] 0.39

2 Connect the calculations which have the same answers.

67×0.01	$0.067 \div 0.1$
$0.067 \div 0.0001$	$0.0067 \div 0.1$
$6.7 \div 10$	$670\,000 \div 1000$
0.67×10	670×0.01
0.67×100	6700×0.01

2.2 Standard form

LINKS
- Stage 9 Unit 1

WATCH OUT!
n can be positive or negative, but it must be a whole number.

REMEMBER
You need to count how many places you need to move the '5' so that it is in the units place.

It needs to move 8 places, so you need to multiply 5.4 by 10 eight times to get 540 000 000.

REMEMBER
You need to count how many places you need to move the '7' so that it is in the units place.

It needs to move 4 places so you need to divide 7.08 by 10 four times to get 0.000 708.

KEY POINTS

- **Standard form** is a way of writing very large or very small numbers.
- A number is in standard form when it is written in the form

$$a \times 10^n$$

where a is a number greater or equal to 1 but less than 10,

and n is an integer.

Example Write 540 000 000 in standard form.

Solution $540\,000\,000 = 5.4 \times 10^8$

Example Write 4.81×10^5 as an ordinary number.

Solution Multiply 4.81 by 10 five times:
$4.81 \times 10^5 = 481\,000$

- You can also write small numbers in standard form.

Example Write 0.000 708 in standard form.

Solution $0.000\,708 = 7.08 \times 10^{-4}$

Example Write 3.7×10^{-4} as an ordinary number.

Solution Divide 3.7 by 10 four times:
$3.7 \times 10^{-4} = 0.000\,37$

- You can use the laws of indices to help you work out calculations in standard form.

Example Work out $(4 \times 10^6) \times (3 \times 10^7)$

Solution Rewrite the calculation: $4 \times 3 \times 10^6 \times 10^7$

Use the laws of indices: 12×10^{13}

Rewrite answer so it is standard form: 1.2×10^{14}

Skill check

1 Write the following numbers in standard form.
 a 4 560 000 000 b 98 300
 c 0.00032 d 0.000 000 15

2 Write the following as ordinary numbers.
 a 2.04×10^9 b 1.3×10^5
 c 6×10^{-5} d 2.8×10^{-7}

3 Work out the following calculations.
Give your answers in standard form.
 a $(2 \times 10^5) \times (4 \times 10^3)$
 b $(9 \times 10^8) \div (2 \times 10^6)$
 c $(3 \times 10^{-4}) + (5 \times 10^{-4})$
 d $(9 \times 10^{-11}) - (2 \times 10^{-11})$

WATCH OUT!
Make sure you know how to use your calculator to work out calculations involving standard form.

Use your calculator to check your answer to question 3.

TIPS FOR SUCCESS

- You need to rewrite numbers in standard form so that they have the same powers before you add or subtract them.
- You only need the laws of indices when you multiply or divide numbers in standard form.
- Make sure you understand how to multiply and divide by powers of 10.

HINT

Compare the powers first.

Remember: 10^7 is greater than 10^6,

but 10^{-7} is less than 10^{-6}.

HINT

Think about the place value of each digit.

HINT

4.35 billion + 0.65 billion is 5 billion.

2 lots of 5 billion is 10 billion.

WORKED EXAMPLE

A space probe X travels 4.35×10^9 km from Earth to fly past Neptune.

It then travels a further 6.5×10^8 km before it crashes into a passing comet.

a How far does the space probe X travel altogether?
b Another space probe Y travels twice as far as space probe X.
How far does space probe Y travel?

Give your answers in standard form.

Solution

a Rewrite the distance the probe travels: 6.5×10^8 km $= 0.65 \times 10^9$ km

4.35×10^9 km $+ 0.65 \times 10^9$ km $= 5 \times 10^9$ km

b Probe Y travels $2 \times (5 \times 10^9) = 10 \times 10^9$ km

In standard form this is 1×10^{10} km

SPOTLIGHT

1 Write these numbers in order of size, from smallest to largest.

8.1×10^{-6} 9.7×10^{-7} 4.2×10^{-7} 3.9×10^{-6}

2 **Classify** these numbers into the correct column in the table.

10^6 $9 \times 10^{-0.5}$ 1×10^{-15}

2.3×10^{-6} 0.4×10^8 10×10^4

Numbers in standard form	Numbers NOT in standard form

3 Danya says that 9×10^6 is a square number.
Is Danya correct?
Give a **convincing** reason for your answer.

4 Fill in the boxes to boxes to make this statement true.

$(1 \times 10^{\square}) + (2 \times 10^{\square}) + (3 \times 10^{\square}) = 100200.003$

5 Given $a = 6 \times 10^{12}$, $b = 5 \times 10^{11}$ and $c = 2 \times 10^5$

Giving your answers in standard form, calculate the value of:

a $a + b$
b $b \times c$
c $a \div c$

2.3 Rounding

LINKS
- Stage 7 Unit 11
- Stage 8 Unit 11
- Stage 9 Unit 11

WATCH OUT!
When you round to 2 or more decimal places, you must give 2 digits after the decimal point even if one (or both) of them is 0.

REMEMBER
The most significant figure is the first non-zero digit because that is the digit that tells you the most about the size of a number.

KEY POINTS

- You often need to round an answer rather than giving it exactly.
- To round to a number to 2 decimal places (2 d.p.):
  Look at the digit in the 3rd decimal place, if it is:

  4 or less → round **down**

  5 or more → round **up**

  You can use this method to round to any number of decimal places.

Example 3.41719 rounded to 2 d.p. is 3.42

17.1902 rounded to 3 d.p. is 17.910

- You can also round numbers to a certain number of **significant figures (s.f.)**.
  The **1st significant figure** in a number is the first non-zero digit in that number.
  The **2nd significant figure** is the digit immediately to the right of the 1st s.f.
  The next digit is the **3rd significant figure** and so on.
- To round to 3 significant figures:
  Step 1: Underline the first 3 significant figures.
  Step 2: Look at the next significant figure, if it is
  4 or less, leave the 3rd significant figure **as it is**
  5 or more, round the 3rd significant figure **UP**
  Step 3: Replace any other digits BEFORE the decimal point with a 0

Examples 304 524.6 rounded to 3 s.f. is 305 000

0.007019 rounded to 2 s.f. is 0.0070

- You can use rounding to 1 s.f. to help you estimate answers to check calculations.

Example $204.9 \times 72 \approx 200 \times 70 \approx 14000$

- A rounded value lies halfway between the **lower** and **upper limit** for the original number.

Example The number of students, N, at a school is 1200 correct to the nearest 100.

The actual number of students, N, at the school could be any number from 1150 up to 1250.

As an inequality this is: $1150 \leq N < 1250$.

WATCH OUT!
Estimation is a useful way to check your answer is roughly correct. For example, if you worked out the exact answer to 204.9×72 you would expect the answer to be about 14 000.

HINT
1150 is the smallest number that rounds to 1200 to the nearest 100, as 1149 rounds to 1100.

You write '< 1250' as the number of students must be less than 1250 as 1250 rounds up to 1300.

Skill check

1 Round each of these numbers as indicated.
  a 43.781 (to 1 d.p.)
  b 0.0723 (to 2 d.p.)
  c 4.5703 (to 3 d.p.)
  d 7.999 (to 2 d.p.)
2 Round each of these numbers as indicated.
  a 135 901 (to 3 s.f.)
  b 15.0343 (to 3 s.f.)
  c 0.002094 (to 1 s.f.)
  d 3.954 (to 2 s.f.)

TIPS FOR SUCCESS

- Make sure you read the accuracy carefully in the question. 'To the nearest metre' is the same as saying 'to the nearest whole number'.
- 10 centimetres is 0.1 metres so 'to the nearest 10 centimetres' is the same as saying 'to 1 decimal place'.
- Use a number line to help you work out the lower and upper limits.

WORKED EXAMPLE

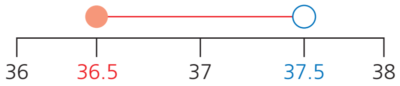
The length of a rectangular field is 37 m to the nearest metre.

The width of the field is 23.8 m to the nearest 10 centimetres.

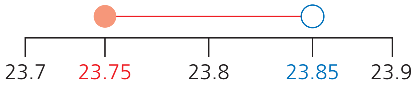
Find the lower and upper limits for the perimeter of the field.

Solution

Length of field is 37 m to the nearest metre.



Width of field is 23.8 m to the nearest 10 centimetres or 1 d.p.



The lower limit for the perimeter is $2(36.5+23.75)=120.5$ m

The upper limit for the perimeter is $2(37.5+23.85)=122.7$ m

WATCH OUT

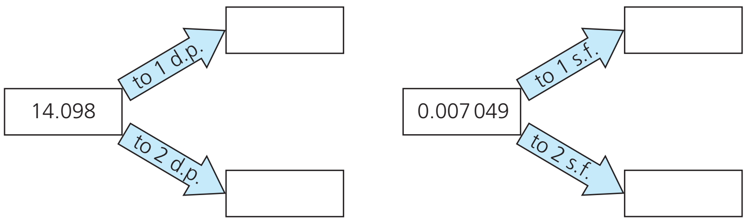
Sometimes you need to write the lower and upper limits as an inequality.

The inequality for the perimeter, P is $120.5 \le P < 122.7$

Take care! The perimeter cannot actually equal the upper bound because 37.5 and 23.85 would round up, so the inequality symbol is < and not ≤.

SPOTLIGHT

1 Round these numbers as shown.



2 Molly estimates the answer to a calculation.

$84.6 \times 7.34 \approx 100 \times 10$

So, $84.6 \times 7.34 \approx 1000$

Improve Molly's estimate.

HINT

Round each number to 1 significant figure.

3 Thea measures the length of a book.

It is 15 cm to the nearest centimetre.

Complete the inequality for the length, l, of Thea's book.

$\square \le l < \square$

4 Sanjay runs 100 metres, correct to the nearest metre.

Sanjay's stride length is 60 cm, to the nearest 10 centimetres.

What is the smallest possible number of steps that Sanjay takes?

5 Light travels 2.998×10^8 metres every second.

The sun is 1.496×10^{11} metres away from earth.

Estimate how many seconds it takes for light to travel from the Sun to Earth.

HINT

Round the numbers so that the calculation is easier to carry out.

Chapter 3 Fractions and decimals

LINKS
- Stage 7 Unit 15
- Stage 8 Unit 14
- Stage 9 Unit 14

REMEMBER
The top of a fraction is called the **numerator**.

The bottom of a fraction is called the **denominator**.

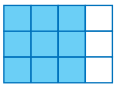
REMEMBER
To add or subtract fractions with different denominators, you first need to rewrite them with the same denominator. Look for the smallest number that is a multiple of both denominators – this is called the **lowest common denominator**.

REMEMBER
Mixed numbers are used for fractions that are greater than 1.

3.1 Adding and subtracting fractions

KEY POINTS

- A **fraction** is a way of writing a number that is not an integer.
  Sometimes you can think of a fraction as showing parts of a whole.

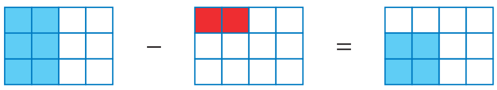
Example  This shows $\frac{9}{12}$ or $\frac{3}{4}$

- You can find **equivalent fractions** by multiplying or dividing the numerator and denominator by the same number.
- To simplify a fraction, divide the numerator and denominator by the same number.
  When a fraction can't be simplified any further it is in its **lowest terms**.

Example $\underset{\div 3}{\overset{\div 3}{\frac{9}{12} = \frac{3}{4}}}$ are equivalent fractions. $\frac{3}{4}$ is in its lowest terms.

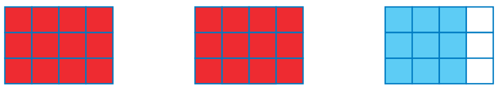
- Before you add or subtract fractions make sure they have same (common) denominator.

Example $\frac{1}{2} - \frac{1}{6} = \frac{6}{12} - \frac{2}{12} = \frac{4}{12} = \frac{1}{3}$



- A **mixed number** has a whole number part and a fraction part.

Example



shows the mixed number $2\frac{3}{4}$.

- Mixed numbers can be written as **improper** or **top-heavy fractions**.

Example $2\frac{3}{4} = 2 + \frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{11}{4}$

Skill check

1 Fill in the missing numbers.

a $3\frac{1}{6} = \frac{\square}{6}$

b $\frac{12}{15} = \frac{48}{\square} = \frac{\square}{10}$

2 Simplify these fractions.

a $\frac{25}{45}$

b $\frac{42}{49}$

c $\frac{24}{64}$

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ISBN 978-1-3983-4286-6