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Answers

Practice-for-exam questions

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Thinking about motion: orbiting objects (part 2)

- 1 The force between the two masses is given by Newton's law of gravitation:

$$F = G \frac{m^2}{r^2}$$

The bodies rotate around their common centre of mass in circular orbits of radius $r/2$, with angular velocity, ω , so they have a centripetal acceleration $\omega^2 r/2$, so:

$$G \frac{m^2}{r^2} = m \frac{\omega^2 r}{2}$$

Rearranging this gives:

$$\omega = \sqrt{\frac{2Gm}{r^3}}$$

The period, T , of the rotation is given by:

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{r^3}{2Gm}} = \sqrt{\frac{2\pi^2 r^3}{Gm}}$$

- 2 From Box 1, Equation 1.5, we have (remembering that the radius of the orbit is $r/2$):

$$L = m \left(\frac{r}{2}\right)^2 \omega$$

The total angular momentum of the system is therefore:

$$L_{total} = 2 \times m \left(\frac{r}{2}\right)^2 \omega = \frac{1}{2} m r^2 \omega$$

- 3 Using the reduced mass, we can model the system as a single body of reduced mass μ rotating about the centre of one of the two masses in an orbit of radius r . The reduced mass is given by Equation 7 in the article:

$$\mu = \frac{m^2}{2m} = \frac{m}{2}$$

Hence, using Equation 1.5 again:

$$L_{total} = \mu r^2 \omega = \left(\frac{m}{2}\right) r^2 \omega = \frac{1}{2} m r^2 \omega$$

This is the same expression as in the answer to question 2.

- 4 Equation 5 of the article gives the radius of the orbit of the Earth:

$$r_M = r \frac{m}{m + M}$$

Using the data given:

$$r_M = 384000 \times 10^3 \text{ m} \times \left(\frac{7.35 \times 10^{22} \text{ kg}}{7.35 \times 10^{22} \text{ kg} + 5.97 \times 10^{24} \text{ kg}} \right) = 4.670 \times 10^6 \text{ m}$$

Therefore, using the given figure for the radius of the Earth, the common centre of mass of the Earth–Moon system is:

$$6.380 \times 10^6 \text{ m} - 4.670 \times 10^6 \text{ m} = 1710 \text{ km below sea level.}$$

This agrees with the figure given in the article, to 3 significant figures.

To calculate the period of rotation of the Earth–Moon system, use:

$$F = M r_M \omega^2$$

$$G \frac{M m}{r^2} = M r_M \omega^2$$

Note that on the left-hand side, Newton's law of gravitation depends on the total distance, r , between the centres of mass, whereas on the right-hand side, we consider the orbit of the Earth, which has radius r_M . Cancelling M on both sides and rearranging, we get:

$$\omega = \sqrt{\frac{G m}{r^2 r_M}}$$

Hence:

$$\begin{aligned} T &= \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{r^2 r_M}{G m}} \\ &= 2\pi \sqrt{\frac{(384\,000 \times 10^3 \text{ m})^2 \times 4.670 \times 10^6 \text{ m}}{6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \times 7.35 \times 10^{22} \text{ kg}}} \\ &= 2.35 \times 10^6 \text{ s} = 27.26 \text{ days} \end{aligned}$$

Mathskit: standard form and SI prefixes

1

$$a_0 = \frac{4\pi \times 8.85 \times 10^{-12} \text{ C}^2 \text{ kg}^{-1} \text{ m}^{-3} \text{ s}^2 \times (1.05 \times 10^{-34} \text{ J s})^2}{(1.60 \times 10^{-19} \text{ C})^2 \times 9.11 \times 10^{-31} \text{ kg}}$$

$$= 5.26 \times 10^{-11} \text{ m}$$

2 Ignoring powers of ten and units, the numerical value of this expression is:

$$\frac{4\pi \times 8.85 \times 1.05^2}{1.60^2 \times 9.11} = 5.26$$

The powers of ten in the calculation are as follows:

$$\frac{10^{-12} \times (10^{-34})^2}{(10^{-19})^2 \times 10^{-31}}$$

Using the rules for combining powers, the power of ten is given by:

$$[(-12) + (2 \times -34)] - [(2 \times -19) + (-31)] = -11$$

Hence, we obtain the same answer as in question 1.

3 The Bohr radius can also be written as:

$$5.26 \times 10^{-11} \text{ m} = 5.26 \times 10^{-11} \times 10^2 \text{ cm} = 5.26 \times 10^{-9} \text{ cm}$$

$$5.26 \times 10^{-11} \text{ m} = 5.26 \times 10^{-11} \times 10^3 \text{ mm} = 5.26 \times 10^{-8} \text{ mm}$$

$$5.26 \times 10^{-11} \text{ m} = 5.26 \times 10^{-11} \times 10^6 \mu\text{m} = 5.26 \times 10^{-5} \mu\text{m}$$

$$5.26 \times 10^{-11} \text{ m} = 5.26 \times 10^{-11} \times 10^9 \text{ nm} = 5.26 \times 10^{-2} \text{ nm}$$

$$5.26 \times 10^{-11} \text{ m} = 5.26 \times 10^{-11} \times 10^{12} \text{ pm} = 5.26 \times 10^1 \text{ pm}$$

$$5.26 \times 10^{-11} \text{ m} = 5.26 \times 10^{-11} \times 10^{-3} \text{ km} = 5.26 \times 10^{-14} \text{ km}$$

Everyday quantum effects

1

$$\frac{m_e e^4}{8h^2 \epsilon_0^2} = \frac{9.11 \times 10^{-31} \text{ kg} \times (1.60 \times 10^{-19} \text{ C})^4}{8 \times (6.63 \times 10^{-34} \text{ Js})^2 \times (8.85 \times 10^{-12} \text{ C}^2 \text{ kg}^{-1} \text{ m}^{-3} \text{ s}^2)^2}$$

$$2.17 \times 10^{-18} \text{ J}$$

as required.

The units are calculated as follows:

$$\frac{\text{kg C}^4}{\text{J}^2 \text{ s}^2 \text{ C}^4 \text{ kg}^{-2} \text{ m}^{-6} \text{ s}^4} = \text{kg}^3 \text{ m}^6 \text{ J}^{-2} \text{ s}^{-6} = \text{kg}^3 \text{ m}^6 (\text{kg m}^2 \text{ s}^{-2})^{-2} \text{ s}^{-6}$$

In the final step, we have used the fact that

$$1\text{J} = 1\text{N m} = 1\text{kg m}^2 \text{s}^{-2}$$

Now we simplify:

$$\text{kg}^3 \text{m}^6 (\text{kg m}^2 \text{s}^{-2})^{-2} \text{s}^{-6} = \text{kg}^3 \text{m}^6 (\text{kg}^{-2} \text{m}^{-4} \text{s}^4) \text{s}^{-6} = \text{kg m}^2 \text{s}^{-2} = \text{J}$$

- 2 Using the given equation, the energy of electromagnetic radiation emitted in the transition is given by:

$$E_{4 \rightarrow 2} = -2.17 \times 10^{-18} \text{J} \times \left(\frac{1}{4^2} - \frac{1}{2^2} \right) = 4.07 \times 10^{-19} \text{J}$$

Using $E = hf$, the frequency is given by:

$$f = \frac{E}{h} = \frac{4.07 \times 10^{-19} \text{J}}{6.63 \times 10^{-34} \text{J s}} = 6.14 \times 10^{14} \text{Hz}$$

We obtain the wavelength from $c = f\lambda$:

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{m s}^{-1}}{6.14 \times 10^{14} \text{Hz}} = 489 \text{nm}$$

This is blue light.

- 3 A power of 10 W corresponds to 10 J of energy emitted per second. Each photon has an energy of $4.07 \times 10^{-19} \text{J}$. Hence, the number of photons emitted per second is:

$$\frac{10\text{J}}{4.07 \times 10^{-19} \text{J}} = 2.46 \times 10^{19}$$

Powerful connections

- 1 To find the rms value of the voltage we use:

$$V_{rms} = \frac{V_{peak}}{\sqrt{2}} = \frac{325\text{V}}{\sqrt{2}} = 230\text{V to 3 significant figures (3sf).}$$

2

$$\begin{aligned} P &= I_{rms} \times V_{rms} = \frac{I_{peak}}{\sqrt{2}} \times \frac{V_{peak}}{\sqrt{2}} = \frac{I_{peak} \times V_{peak}}{2} \\ &= \frac{325\text{V} \times 15\text{A}}{2} = 2440\text{W to 3sf} \end{aligned}$$

- 3 The transmission line has a resistance of 3 ohms per 100 km. Therefore, a 500 km length of line has a resistance of $5 \times 3 \Omega = 15 \Omega$. Therefore:

$$P_{dissipated} = (I_{rms})^2 \times R = \left(\frac{I_{peak}}{\sqrt{2}} \right)^2 \times R = \frac{I_{peak}^2}{2} \times R$$

$$\frac{(15A)^2}{2} \times 15\Omega = 1690W$$

To calculate the energy dissipated in an hour, we use:

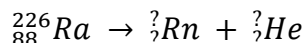
$$Power = \frac{energy}{time}$$

Therefore:

$$E = P \times t = 1690W \times (60 \times 60)s = 6.08 \times 10^6J = 6.08MJ$$

Radiometric age dating of rocks and fossils

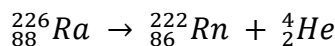
1



An alpha particle is a helium nucleus containing 2 protons and 2 neutrons, so we can write this as ${}_2^4He$, as in Box 1 of the article. As the total number of nucleons and protons must balance on each side of the equation, the missing numbers for radon (Rn) are:

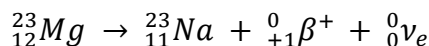
$$\begin{aligned} 226 - 4 &= 222 \\ 88 - 2 &= 86 \end{aligned}$$

The decay equation can thus be written as:



2

A positron has zero mass and a positive charge and so can be written as: ${}_{+1}^0\beta$. An electron neutrino has neither charge nor mass, so can be written: ${}_0^0\nu$. Again, the total number of nucleons and the total charge must balance on both sides of the equation so that the decay can be written as:



3

The constant k is the reciprocal of the half-life, so:

$$k = \frac{1}{12}s^{-1} = 0.833s^{-1} \text{ to 3 sf}$$

The mass of magnesium-23 remaining at any time is proportional to the number of nuclei of Mg-23, so:

$$m(t = 10\text{mins}) = m_0 \times 0.5^{kt} = 2g \times 0.5^{\left(\frac{1}{12}s^{-1} \times (10 \times 60)s\right)} = 1.78 \times 10^{-15}g$$

4

Using the standard equation, we can write:

$$m(t = 10\text{mins}) = m_0 e^{-\lambda t} = 2g \times \exp(-\lambda t)$$

The decay constant, λ , is given by:

$$\lambda = \frac{\ln 2}{t_{\frac{1}{2}}} = \frac{\ln 2}{12s} = 0.0578s^{-1}$$

Hence:

$$\begin{aligned} m(t = 10\text{mins}) &= m_0 e^{-\lambda t} = 2\text{g} \times \exp(-\lambda t) \\ &= 2\text{g} \times \exp\left(-\frac{\ln 2}{12\text{s}} \times (10 \times 60)\text{s}\right) \\ &= 1.78 \times 10^{-15}\text{g to 3sf, as in question 3.} \end{aligned}$$

5 Starting from:

$$N(t) = N_0 \times 0.5^{kt},$$

take natural logarithms of both sides to get:

$$\ln N(t) = \ln N_0 + kt \ln 0.5 = \ln N_0 - kt \ln 2$$

We cannot measure N directly, but Box 2 of the article shows that the number of decays per second, $\frac{\Delta N}{\Delta t}$ (called the activity, A), is proportional to N , so that:

$$\ln A(t) = \ln A_0 + kt \ln 0.5 = \ln A_0 - kt \ln 2$$

Experimentally, therefore, we would measure the decay rate as a function of time.

This is the equation of a straight-line graph. Comparing with $y = mx + c$, plotting $\ln A(t)$ on the y -axis and t on the x -axis, we can see that the intercept of the graph is $\ln A_0$, where A_0 is the initial activity at time $t = 0$, and the gradient is $k \ln 0.5$. We can simplify this expression:

$$\text{gradient} = k \ln 0.5 = \frac{1}{t_{\frac{1}{2}}} \times \ln \frac{1}{2} = \frac{1}{t_{\frac{1}{2}}} \times -\ln 2 = -\lambda$$

The gradient of the graph therefore tells us the (negative of) the decay constant. It is measured in units of $(\text{seconds})^{-1}$.

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