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Answers

Practice-for-exam questions

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Thinking about motion: orbiting objects (part 2)

1 The force between the two masses is given by Newton's law of gravitation:

$$F = G\frac{m^2}{r^2}$$

The bodies rotate around their common centre of mass in circular orbits of radius $r/2$, with angular velocity, ω, so they have a centripetal acceleration $\omega^2 r/2$, so:

$$G\frac{m^2}{r^2} = m\frac{\omega^2 r}{2}$$

Rearranging this gives:

$$\omega = \sqrt{\frac{2Gm}{r^3}}$$

The period, T, of the rotation is given by:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{r^3}{2Gm}} = \sqrt{\frac{2\pi^2 r^3}{Gm}}$$

2 From Box 1, Equation 1.5, we have (remembering that the radius of the orbit is $r/2$):

$$L = m\left(\frac{r}{2}\right)^2 \omega$$

The total angular momentum of the system is therefore:

$$L_{total} = 2 \times m\left(\frac{r}{2}\right)^2 \omega = \frac{1}{2}\, m\, r^2 \omega$$

3 Using the reduced mass, we can model the system as a single body of reduced mass μ rotating about the centre of one of the two masses in an orbit of radius r. The reduced mass is given by Equation 7 in the article:

$$\mu = \frac{m^2}{2m} = \frac{m}{2}$$

Hence, using Equation 1.5 again:

$$L_{total} = \mu r^2 \omega = \left(\frac{m}{2}\right) r^2 \omega = \frac{1}{2} m r^2 \omega$$

This is the same expression as in the answer to question 2.

4 Equation 5 of the article gives the radius of the orbit of the Earth:

$$r_M = r \frac{m}{m + M}$$

Using the data given:

$$r_M = 384000 \times 10^3 \mathrm{m} \times \left(\frac{7.35 \times 10^{22} \mathrm{kg}}{7.35 \times 10^{22} \mathrm{kg} + 5.97 \times 10^{24} \mathrm{kg}}\right) = 4.670 \times 10^6 \mathrm{m}$$

Therefore, using the given figure for the radius of the Earth, the common centre of mass of the Earth–Moon system is:

$6.380 \times 10^6 \mathrm{m} - 4.670 \times 10^6 \mathrm{m} = 1710 \mathrm{km}$ below sea level.

This agrees with the figure given in the article, to 3 significant figures.

To calculate the period of rotation of the Earth–Moon system, use:

$$F = M r_M \omega^2$$

$$G \frac{M m}{r^2} = M r_M \omega^2$$

Note that on the left-hand side, Newton's law of gravitation depends on the total distance, r, between the centres of mass, whereas on the right-hand side, we consider the orbit of the Earth, which has radius r_M. Cancelling M on both sides and rearranging, we get:

$$\omega = \sqrt{\frac{G m}{r^2 r_M}}$$

Hence:

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{r^2 r_M}{G m}}$$

$$= 2\pi \sqrt{\frac{(384\,000 \times 10^3 \mathrm{~m})^2 \times 4.670 \times 10^6 \mathrm{~m}}{6.67 \times 10^{-11} N \mathrm{~m}^2 \mathrm{~kg}^{-2} \times 7.35 \times 10^{22} \mathrm{~kg}}}$$

$$= 2.35 \times 10^6 \mathrm{~s} = 27.26 \text{ days}$$

Mathskit: standard form and SI prefixes

1

$$a_0 = \frac{4\pi \times 8.85 \times 10^{-12}\, C^2\, kg^{-1}\, m^{-3}\, s^2 \times (1.05 \times 10^{-34}\, J\, s)^2}{(1.60 \times 10^{-19}\, C)^2 \times 9.11 \times 10^{-31} kg}$$

$$= 5.26 \times 10^{-11}\, m$$

2 Ignoring powers of ten and units, the numerical value of this expression is:

$$\frac{4\pi \times 8.85 \times 1.05^2}{1.60^2 \times 9.11} = 5.26$$

The powers of ten in the calculation are as follows:

$$\frac{10^{-12} \times (10^{-34})^2}{(10^{-19})^2 \times 10^{-31}}$$

Using the rules for combining powers, the power of ten is given by:

$$[(-12) + (2 \times -34)] - [(2 \times -19) + (-31)] = -11$$

Hence, we obtain the same answer as in question 1.

3 The Bohr radius can also be written as:

$$5.26 \times 10^{-11}\, m = 5.26 \times 10^{-11} \times 10^2\, cm = 5.26 \times 10^{-9}\, cm$$

$$5.26 \times 10^{-11}\, m = 5.26 \times 10^{-11} \times 10^3\, mm = 5.26 \times 10^{-8}\, mm$$

$$5.26 \times 10^{-11}\, m = 5.26 \times 10^{-11} \times 10^6\, \mu m = 5.26 \times 10^{-5}\, \mu m$$

$$5.26 \times 10^{-11}\, m = 5.26 \times 10^{-11} \times 10^9\, nm = 5.26 \times 10^{-2}\, nm$$

$$5.26 \times 10^{-11}\, m = 5.26 \times 10^{-11} \times 10^{12}\, pm = 5.26 \times 10^1\, pm$$

$$5.26 \times 10^{-11}\, m = 5.26 \times 10^{-11} \times 10^{-3}\, km = 5.26 \times 10^{-14}\, km$$

Everyday quantum effects

1

$$\frac{m_e e^4}{8h^2\epsilon_0^2} = \frac{9.11 \times 10^{-31}\,\mathrm{kg} \times (1.60 \times 10^{-19}\,\mathrm{C})^4}{8 \times (6.63 \times 10^{-34}\,\mathrm{Js})^2 \times (8.85 \times 10^{-12}\,\mathrm{C^2\,kg^{-1}\,m^{-3}\,s^2})^2}$$

$$2.17 \times 10^{-18}\,\mathrm{J}$$

as required.

The units are calculated as follows:

$$\frac{\mathrm{kg\,C^4}}{\mathrm{J^2\,s^2\,C^4\,kg^{-2}\,m^{-6}\,s^4}} = \mathrm{kg^3\,m^6\,J^{-2}\,s^{-6}} = \mathrm{kg^3\,m^6\,(kg\,m^2\,s^{-2})^{-2}\,s^{-6}}$$

In the final step, we have used the fact that

$$1\text{J} = 1\text{N m} = 1\text{kg m}^2\text{ s}^{-2}$$

Now we simplify:

$$\text{kg}^3\text{ m}^6\text{ (kg m}^2\text{ s}^{-2})^{-2}\text{ s}^{-6} = \text{kg}^3\text{ m}^6(\text{kg}^{-2}\text{ m}^{-4}\text{ s}^4)\text{ s}^{-6} = \text{kg m}^2\text{ s}^{-2} = \text{J}$$

2 Using the given equation, the energy of electromagnetic radiation emitted in the transition is given by:

$$E_{4\to 2} = -2.17 \times 10^{-18}\text{J} \times \left(\frac{1}{4^2} - \frac{1}{2^2}\right) = 4.07 \times 10^{-19}\text{J}$$

Using $E = hf$, the frequency is given by:

$$f = \frac{E}{h} = \frac{4.07 \times 10^{-19}\text{J}}{6.63 \times 10^{-34}\text{ J s}} = 6.14 \times 10^{14}\text{ Hz}$$

We obtain the wavelength from $c = f\lambda$:

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8\text{ m s}^{-1}}{6.14 \times 10^{14}\text{ Hz}} = 489\text{nm}$$

This is blue light.

3 A power of 10 W corresponds to 10 J of energy emitted per second. Each photon has an energy of 4.07×10^{-19} J. Hence, the number of photons emitted per second is:

$$\frac{10\text{J}}{4.07 \times 10^{-19}\text{J}} = 2.46 \times 10^{19}$$

Powerful connections

1 To find the rms value of the voltage we use:

$$V_{rms} = \frac{V_{peak}}{\sqrt{2}} = \frac{325\text{V}}{\sqrt{2}} = 230\text{V to 3 significant figures (3sf).}$$

2

$$P = I_{rms} \times V_{rms} = \frac{I_{peak}}{\sqrt{2}} \times \frac{V_{peak}}{\sqrt{2}} = \frac{I_{peak} \times V_{peak}}{2}$$

$$= \frac{325\text{V}\times 15\text{A}}{2} = 2440\text{W to 3sf}$$

3 The transmission line has a resistance of 3 ohms per 100 km. Therefore, a 500 km length of line has a resistance of $5 \times 3\ \Omega = 15\ \Omega$. Therefore:

$$P_{dissipated} = (I_{rms})^2 \times R = \left(\frac{I_{peak}}{\sqrt{2}}\right)^2 \times R = \frac{I_{peak}^2}{2} \times R$$

$$\frac{(15\text{A})^2}{2} \times 15\Omega = 1690\text{W}$$

To calculate the energy dissipated in an hour, we use:

$$Power = \frac{energy}{time}$$

Therefore:

$$E = P \times t = 1690\text{W} \times (60 \times 60)\text{s} = 6.08 \times 10^6\text{J} = 6.08\text{MJ}$$

Radiometric age dating of rocks and fossils

1

$$^{226}_{88}Ra \rightarrow {}^{?}_{?}Rn + {}^{?}_{?}He$$

An alpha particle is a helium nucleus containing 2 protons and 2 neutrons, so we can write this is as $^{4}_{2}He$, as in Box 1 of the article. As the total number of nucleons and protons must balance on each side of the equation, the missing numbers for radon (Rn) are:

226 − 4 = 222
88 − 2 = 86

The decay equation can thus be written as:

$$^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}He$$

2 A positron has zero mass and a positive charge and so can be written as: $^{0}_{+1}\beta$. An electron neutrino has neither charge nor mass, so can be written: $^{0}_{0}\nu$. Again, the total number of nucleons and the total charge must balance on both sides of the equation so that the decay can be written as:

$$^{23}_{12}Mg \rightarrow {}^{23}_{11}Na + {}^{0}_{+1}\beta^{+} + {}^{0}_{0}\nu_e$$

3 The constant k is the reciprocal of the half-life, so:

$$k = \frac{1}{12}\text{s}^{-1} = 0.833\text{s}^{-1} \text{ to 3 sf}$$

The mass of magnesium-23 remaining at any time is proportional to the number of nuclei of Mg-23, so:

$$m(t = 10\text{mins}) = m_0 \times 0.5^{kt} = 2\text{g} \times 0.5^{\left(\frac{1}{12}\text{s}^{-1} \times (10 \times 60)\text{s}\right)} = 1.78 \times 10^{-15}\text{g}$$

4 Using the standard equation, we can write:

$$m(t = 10\text{mins}) = m_0 e^{-\lambda t} = 2\text{g} \times \exp(-\lambda t)$$

The decay constant, λ, is given by:

$$\lambda = \frac{\ln 2}{t_{\frac{1}{2}}} = \frac{\ln 2}{12\text{s}} = 0.0578\text{s}$$

Hence:

$$m(t = 10\text{mins}) = m_0 e^{-\lambda t} = 2\text{g} \times \exp(-\lambda t)$$
$$= 2\text{g} \times \exp\left(-\frac{\ln 2}{12\text{s}} \times (10 \times 60)\text{s}\right)$$
$$= 1.78 \times 10^{-15}\text{g to 3sf, as in question 3.}$$

5 Starting from:

$$N(t) = N_0 \times 0.5^{kt},$$

take natural logarithms of both sides to get:

$$\ln N(t) = \ln N_0 + kt \ln 0.5 = \ln N_0 - kt \ln 2$$

We cannot measure *N* directly, but Box 2 of the article shows that the number of decays per second, $\frac{\Delta N}{\Delta t}$ (called the activity, *A*), is proportional to *N*, so that:

$$\ln A(t) = \ln A_0 + kt \ln 0.5 = \ln A_0 - kt \ln 2$$

Experimentally, therefore, we would measure the decay rate as a function of time.

This is the equation of a straight-line graph. Comparing with $y = mx + c$, plotting ln*A(t)* on the *y*-axis and *t* on the *x*-axis, we can see that the intercept of the graph is $\ln A_0$, where A_0 is the initial activity at time $t = 0$, and the gradient is $k \ln 0.5$. We can simplify this expression:

$$gradient = k \ln 0.5 = \frac{1}{t_{\frac{1}{2}}} \times \ln\frac{1}{2} = \frac{1}{t_{\frac{1}{2}}} \times -\ln 2 = -\lambda$$

The gradient of the graph therefore tells us the (negative of) the decay constant. It is measured in units of (seconds)$^{-1}$.

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