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Answers

Practice-for-exam questions

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Thinking about motion: particles and planets (part 1)

- 1 a Area under the curve will yield the total distance travelled in 24 hrs. To do this, let us divide the area into triangles, trapeziums and rectangles. We will then add up their areas.

Triangle formed under segment AB:

$$Area_{AB} = \frac{1}{2} \times (2 - 0) \times 80 = 80.0 \text{ km}$$

Trapezium formed under segment BC:

$$Area_{BC} = \frac{1}{2} \times (80 + 50) \times (5 - 2) = 195 \text{ km}$$

Rectangle formed under segment CD:

$$Area_{CD} = 50 \times (8 - 5) = 150 \text{ km}$$

Trapezium formed under segment DE:

$$Area_{DE} = \frac{1}{2} \times (50 + 100) \times (16 - 8) = 600 \text{ km}$$

Trapezium formed under segment EF:

$$Area_{EF} = \frac{1}{2} \times (100 + 87.5) \times (20 - 16) = 375 \text{ km}$$

Triangle formed under segment FG:

$$Area_{FG} = \frac{1}{2} \times (24 - 20) \times 87.5 = 175 \text{ km}$$

Thus, the total distance travelled by the car in 24 hrs is:

$$D = Area_{AB} + Area_{BC} + Area_{CD} + Area_{DE} + Area_{EF} + Area_{FG}$$

$$D = 80.0 + 195 + 150 + 600 + 375 + 175 = 1575 \text{ km} = \mathbf{1580 \text{ km}} \text{ (3 s.f.)}$$

- b Average speed in km/h:

$$S_{km/h} = \frac{D}{24} = 65.63 \text{ km/h}$$

Average speed in m/s:

$$S_{m/s} = \frac{1575 \times 1000}{24 \times 60 \times 60} = 18.23 \text{ m/s} = \mathbf{18.2 \text{ m/s}} \text{ (3 s.f.)}$$

c Car accelerates when its speed increases and this happens in sections AB and DE.

d During the section CD the car travels at a constant speed of 50 km/h.

e The car would accelerate or decelerate the most when the rate of change of speed is the greatest.

Section FG has a gradient m given by:

$$m = \frac{\Delta \text{speed}}{\Delta \text{time}} = \frac{0 - 87.5}{24 - 20} = -\frac{87.5}{4} = -21.875 \text{ km/h}^2$$

Section AB has a gradient given by:

$$m = \frac{\Delta \text{speed}}{\Delta \text{time}} = \frac{80 - 0}{2 - 0} = \frac{80}{2} = 40 \text{ km/h}^2$$

Section BC has a gradient given by:

$$m = \frac{\Delta \text{speed}}{\Delta \text{time}} = \frac{50 - 80}{5 - 2} = \frac{30}{3} = 10 \text{ km/h}^2$$

Section DE has a gradient given by:

$$m = \frac{\Delta \text{speed}}{\Delta \text{time}} = \frac{100 - 50}{16 - 8} = \frac{50}{8} = 6.25 \text{ km/h}^2$$

Section EF has a gradient given by:

$$m = \frac{\Delta \text{speed}}{\Delta \text{time}} = \frac{87.5 - 100}{20 - 16} = -\frac{12.5}{4} = -3.125 \text{ km/h}^2$$

Section FG of the journey is when the car decelerates the most.

f Looking back at the gradients in part (e), we can say that the highest acceleration was 40 km/h^2 and lowest acceleration was 6.25 km/h^2 .

2

a Using the suvat equation:

$$v = u + at$$

With u as 0 ms^{-1} , v as 6.0 ms^{-1} and t as 10.0 s , we get:

$$6 = 0 + a \times 10$$

$$a = \mathbf{0.600 \text{ ms}^{-2}} \text{ (3 s.f.)}$$

b Using the suvat equation:

$$s = ut + \frac{1}{2}at^2$$

With u as 0ms^{-1} , a as 0.600ms^{-2} and t as 10.0s , we get:

$$s = 0 + \frac{1}{2} \times (0.600 \times 10^2)$$

$$s = \mathbf{30.0\text{ m}} \text{ (3 s.f.)}$$

c Using the suvat equation:

$$v^2 = u^2 + 2as$$

With v as 0ms^{-1} , u as 6ms^{-1} , and s as 15.0m , we get:

$$6^2 = 0^2 + (2a \times 15)$$

$$a = -\frac{36}{30} = -\frac{6}{5} = -\mathbf{1.20\text{ ms}^{-2}} \text{ (3 s.f.)}$$

d Now using another suvat equation:

$$v = u + at$$

We can find time t :

$$t = \frac{v - u}{a} = \frac{0 - 6}{-1.20} = \mathbf{5.00\text{ s}} \text{ (3 s.f.)}$$

e Total distance travelled by the cyclist, S_{Total} :

$$S_{\text{Total}} = S_A + S_B + S_C$$

Where,

S_A is the distance during acceleration, which has determined to be 30.0 m (part (a))

S_B is the distance during constant velocity:

$$S_B = vt = 6.00 \times 30 = 180\text{ m}$$

S_C is the distance during deceleration, given to be 15.0 m

Thus,

$$S_{\text{Total}} = 30.0 + 180 + 15.0 = \mathbf{225\text{ m}} \text{ (3.s.f.)}$$

- 3 a Total momentum before the explosion:** Given that the shell reaches its maximum height before the explosion, its vertical velocity at that point is zero.

$$p_{\text{initial}} = m_{\text{shell}} \times v_{\text{shell}}$$

Given m_{shell} as 10.0kg and v_{shell} as 0ms^{-1} , the initial momentum p_{initial} is 0kg ms^{-1} .

b Using conservation of momentum: The total momentum before the explosion must equal the total momentum after the explosion, and thus:

$$p_{\text{initial}} = p_{\text{final}}$$

$$0 = m_A v_A + m_B v_B$$

Given m_A as 5.00 kg and v_A as -10.0 ms^{-1} (since it is falling straight down) and m_B as 5.00 kg, we can find v_B :

$$0 = (5.00 \times -10.0) + (5.00 \times v_B)$$

$$v_B = \mathbf{10.0 \text{ ms}^{-1}} \text{ (3 s.f.)}$$

Thus, the second fragment (Fragment B) moves upwards with a velocity of 10.0 ms^{-1} .

c Total kinetic energy of the fragments immediately after the explosion: The kinetic energy imparted by the explosion adds to the initial kinetic energy of the fragments.

$$KE_{Total} = KE_A + KE_B$$

Where,

$$KE_A = \frac{1}{2} m_A v_A^2$$

$$KE_A = \frac{1}{2} \times 5.00 \times (-10.0)^2 = 250 \text{ J}$$

And,

$$KE_B = \frac{1}{2} m_B v_B^2$$

$$KE_B = \frac{1}{2} \times 5.00 \times (10.0)^2 = 250 \text{ J}$$

Thus, the total kinetic energy of the fragments immediately after the explosion is:

$$KE_{Total} = 250 \text{ J} + 250 \text{ J} = \mathbf{500 \text{ J}} \text{ (3 s.f.)}$$

d Significance of the conservation of momentum: The principle of conservation of momentum states that the total momentum of an isolated system remains constant if no external forces act on it. In explosive events, this principle allows us to predict the motion of fragments after the explosion. Despite the internal forces acting during the explosion, the total momentum before and after the explosion must be equal. In this situation, since the shell is initially at rest (maximum height), the momenta of the two fragments must cancel each other out, ensuring the total momentum remains zero. There is an external force acting here (the force of gravity), but it has no effect on the initial velocities of the explosive fragments and so we are justified in using the principle of the conservation of momentum.

- 4 a** Initial acceleration of the centre of mass: using Newton's second law of motion.

$$F = ma$$

Given $F = 30.0 \text{ N}$ and $m = 0.160 \text{ kg}$:

$$30 = 0.160 \times a$$

$$a = 187.5 \text{ ms}^{-2} = \mathbf{188 \text{ ms}^{-2}} \text{ (3 s.f.)}$$

b Torque about the centre of mass: torque τ is given by:

$$\tau = F \times r$$

Given $r = 0.02$ m and $F = 30.0$ N:

$$\tau = 30.0 \times 0.02 = \mathbf{0.600 \text{ Nm}} \text{ (3 s.f.)}$$

c Angular acceleration: using the formula for the moment of inertia of a solid sphere:

$$I = \frac{2}{5}mr^2$$

Substituting the known values:

$$I = \frac{2}{5} \times 0.160 \times (0.02)^2 = 7.84 \times 10^{-5} \text{ kg m}^2$$

Angular acceleration α is related to moment of inertia I by:

$$\tau = I \times \alpha$$

Thus,

$$\alpha = \frac{0.60}{7.84 \times 10^{-5}} = 7653 \text{ rad s}^{-2} = \mathbf{7650 \text{ rad s}^{-2}} \text{ (3 s.f.)}$$

d The position of the applied force relative to the centre of mass significantly affects the motion of the cricket ball. If the force is applied directly through the centre of mass, the ball will experience pure translational motion without rotation. However, when the force is applied off-centre (as in this case), it generates both translational and rotational motion. The distance from the centre of mass to the point of application of the force creates a torque, causing the ball to spin. This combined motion influences the trajectory, stability, and overall behaviour of the ball during its flight.

The physics of falling

1 a Potential energy of each object:

$$PE_A = m_A \cdot g \cdot h$$

$$PE_B = m_B \cdot g \cdot h$$

$$PE_C = m_C \cdot g \cdot h$$

Where m_A , m_B and m_C are the mass of each object in kg and h is the drop height (height above the hard surface). Ratio of these is:

$$PE_A : PE_B : PE_C = m_A : m_B : m_C$$

$$PE_A : PE_B : PE_C = 5.00 : 50.0 : 500$$

$$PE_A : PE_B : PE_C = 1 : 10 : 100$$

b In motion, the PE is converted to kinetic energy (KE):

$$PE = KE$$

$$m \cdot g \cdot h = \frac{1}{2} m \cdot v^2$$

$$g \cdot h = \frac{1}{2} v^2$$

$$v = \sqrt{2gh}$$

Where v is the impact velocity in ms^{-1} . Note that this is independent of the object mass.

Thus, each object will have the same impact velocity:

$$v = \sqrt{2 \times 9.81 \times 10.0} = 14.01 = \mathbf{14.0 \text{ ms}^{-1}} \text{ (3 s.f.)}$$

c Using the suvat equation below, we can find the travel time:

$$S = u \cdot t + \frac{1}{2} a \cdot t^2$$

Here the initial velocity u is 0 ms^{-1} , a is the value of 'g' and S is 'h'.

And so:

$$h = \frac{1}{2} g \cdot t^2$$

$$t = \sqrt{\frac{2h}{g}}$$

Substituting the values of g and h , we get:

$$t = \sqrt{\frac{2 \times 10.0}{9.81}} = \mathbf{1.43 \text{ s}} \text{ (3 s.f.)}$$

d Assuming a perfectly inelastic collision and a deceleration time of 0.1 s, using Newton's second law of motion, the force acting upon each object will be:

$$F_A = m_A \cdot a$$

$$F_A = m_B \cdot a$$

$$F_A = m_C \cdot a$$

Deceleration for each is given by:

$$a = \frac{\text{change in velocity}}{\text{deceleration time}}$$

which is :

$$a = \frac{v}{0.1} = \frac{14.01}{0.1} = 140.1 \text{ ms}^{-2}$$

Thus, the force acting upon each object will be:

$$\begin{aligned} F_A &= 5.00 \times 140.1 = 700.5 = 701 \text{ N (3.s.f)} \\ F_B &= 50.0 \times 140.1 = 7005 = 7010 \text{ N (3.s.f)} \\ F_C &= 500.0 \times 140.1 = 70050 = 70100 \text{ N (3.s.f)} \end{aligned}$$

e With the use of the padded mat, the deceleration time increases to:

$$\Delta t_{\text{new}} = 1.1 \times 0.1 = 0.11 \text{ s}$$

The deceleration will now be:

$$a = \frac{14.01}{0.11} = 127.36 = 127 \text{ ms}^{-2}$$

Recalculated the forces, using the same formula in part (d), we get:

$$\begin{aligned} F_A &= 5.00 \times \frac{14.01}{0.11} = 637 \text{ N (3.s.f)} \\ F_B &= 50.0 \times \frac{14.01}{0.11} = 6370 \text{ N (3.s.f)} \\ F_C &= 500.0 \times \frac{14.01}{0.11} = 63700 \text{ N (3.s.f)} \end{aligned}$$

f We need to determine the new deceleration time $\Delta t_{\text{required}}$ for when:

$$F_C \leq 40 \text{ kN}$$

$$\text{As } F_C = m_C \cdot a \text{ and } a = v / \Delta t_{\text{required}}$$

$$m_C \cdot \frac{v}{\Delta t_{\text{required}}} \leq 40 \text{ kN}$$

$$500.0 \times \frac{14.01}{\Delta t_{\text{required}}} \leq 40 \text{ kN}$$

$$\Delta t_{\text{required}} \geq 500.0 \times \frac{14.01}{40 \times 10^3}$$

$$\Delta t_{\text{required}} \geq \mathbf{0.175 \text{ s (3 s.f.)}}$$

The deceleration time will need to be at least 0.175s to ensure the force no greater than (or equal to) acts on Object C when it falls from a height of 10.0m above a hard surface.

2 a New force acting on the car is given by:

$$F_{\text{net}} = 4000 \text{ N} - 1000 \text{ N} = 3000 \text{ N}$$

Using Newton's second law:

$$F_{net} = ma$$

Given mass of the car as 1500 kg, the acceleration is calculated as:

$$a = \frac{F_{net}}{m} = \frac{3000}{1500} = 2.00 \text{ ms}^{-2} \text{ (3 s.f.)}$$

b Using the suvat equation below we can find the time taken to reach the speed of 20.0 ms^{-1} :

$$v = u + at$$

Here u and v represent the initial and final car velocities, a represents the car's acceleration and t represents the time taken for the car to go from u to v .

Substituting u as 0 ms^{-1} (as the car is at rest initially), a as calculated previously and v as 20 ms^{-1} , we get:

$$20 = 0 + (2 \times t)$$

$$t = 10.0 \text{ s (3 s.f.)}$$

c Using Newton's second law, we can first determine the deceleration when the brakes are applied:

$$a_s = \frac{F_d}{m}$$

Here F_d is the decelerative force when the brakes are applied, which is given as 5000 N.

Substituting this and the given mass of the car (1500 kg), we get:

$$a_s = \frac{-5000}{1500} = -3.33 \text{ ms}^{-2} \text{ (3 s.f.)}$$

The negative sign indicates the force acts against the direction of travel of the car. The car is thus slowing down at the rate 3.33 ms^{-2} .

Using the suvat equation:

$$v^2 = u^2 + 2a_s s$$

where v represents the final velocity of the car (which is 0 ms^{-1}), u represents the velocity of the car before the brake was applied, which is 20 ms^{-1} , and s represents the distance travelled by the car before coming to a stop.

Substituting the values we get:

$$0 = (20)^2 + \left(2 \times -\frac{5000}{1500} \times s\right)$$

$$\frac{10000}{1500} \times s = 400$$

$$s = \frac{600000}{10000} = 60.0 \text{ m (3 s.f.)}$$

d Newton's third law states that for every action, there is an equal and opposite reaction.

When the car accelerates, the car exerts a force on the road, and the road exerts an equal and opposite force on the car (providing forward motion).

When the brakes are applied, the brakes exert a force on the wheels (resulting in deceleration), and the wheels exert an equal and opposite force on the brakes.

- 3 a** The rod's centre of mass is located at a height of $L/2$ meters above the surface. Thus, its PE is given by:

$$PE = mg \frac{L}{2}$$

Assuming inelastic collision, by the law of conservation of energy, this potential energy converts to kinetic rotational energy when the rod's bottom end makes contact with the surface. This KE is given by:

$$KE_{\text{rotational}} = \frac{1}{2} I \omega^2$$

where,

$$I = \frac{1}{3} mL^2$$

Substituting above in the KE equation, and then equating it to the PE equation we get:

$$mg \frac{L}{2} = \frac{1}{2} \times \left(\frac{1}{3} mL^2 \right) \omega^2$$

$$gL = \left(\frac{1}{3} L^2 \right) \omega^2$$

$$g = \left(\frac{1}{3} L \right) \omega^2$$

$$\omega^2 = \frac{3g}{L} = \frac{3 \times 9.81}{2} = 14.715$$

$$\omega = \sqrt{14.715} = 3.836 = \mathbf{3.84 \text{ rad/s}} \text{ (3 s.f.)}$$

- b** The linear velocity of the centre of mass (COM) is the product of angular velocity and the distance from the COM to the bottom end of the rod (viz., $L/2$):

$$v = \omega \times \frac{L}{2} = \sqrt{14.715} \times 1 = \mathbf{3.84 \text{ ms}^{-1}} \text{ (3 s.f.)}$$

- c** Using Newton's second law of motion, the force exerted on the surface is:

$$F = m \cdot a$$

$$F = m \cdot \frac{\Delta v}{\Delta t}$$

$$F = 10 \times \frac{3.84}{0.1} = \mathbf{384 \text{ N}} \text{ (3 s.f.)}$$

By Newton's third law of motion, this will also be the force exerted by the surface on the rod on impact.

d New deceleration time $\Delta t_{new} = 0.1 \times 1.1 = 0.11 \text{ s}$

Thus, new average force exerted by the surface on the rod and vice-versa will be:

$$F_{new} = m \cdot \frac{\Delta v}{\Delta t_{new}} = 10 \times \frac{3.84}{0.11} = 348.18 \text{ N} = \mathbf{348 \text{ N}} \text{ (3 s.f.)}$$

Speedy stars

1 a Kepler's first law: The orbit of S2 is an ellipse with Sagittarius A* located at one of the two foci. This implies that Sagittarius A* is not at the centre of the ellipse but at a focal point, leading to varying distances between S2 and Sagittarius A* throughout the orbit.

b Kepler's second law: This law, also known as the law of equal areas, states that a line segment joining a star and the black hole sweeps out equal areas during equal intervals of time. Thus, S2 moves faster when it is closer to Sagittarius A* (at periapsis) and slower when it is farther away (at aphelion), resulting in varying velocities along its orbit.

c Kepler's third law: substituting $T = 16.0$ years and $a = 5.50$ light-years in the equation below and re-arranging for M , we can solve for M . Note that 1 light-year in seconds equals the speed of light multiplied by the number of seconds in one day ($1 \text{ day} = 24 \times 60 \times 60 = 86400 \text{ s}$).

$$T^2 = \frac{4\pi^2 a^3}{GM}$$

$$M = \frac{4\pi^2 a^3}{GT^2}$$

$$M = \frac{4\pi^2 \times (5.50 \times 3 \times 10^8 \times 24 \times 60 \times 60)^3}{(6.67 \times 10^{-11}) \times (16.0 \times 365 \times 24 \times 60 \times 60)^2}$$

$$M = \frac{4\pi^2 \times (1.4256 \times 10^{14})^3}{(6.67 \times 10^{-11}) \times (5.0456 \times 10^8)^2}$$

$$M = \frac{4\pi^2 \times 2.8972 \times 10^{42}}{6.67 \times 10^{-11} \times 2.5458 \times 10^{17}}$$

$$M = \frac{1.14378 \times 10^{44}}{1.698 \times 10^8}$$

$$M = \mathbf{6.75 \times 10^{36} \text{ kg}}$$

- 2 a Orbital velocity of the moon:** Using the formula for orbital velocity (circular orbit):

$$v_{\text{moon}} = \sqrt{\frac{GM}{r}}$$

Where G is the universal gravitational constant (given), M is the mass of the Earth (body that is being orbited) and r is the distance between the Moon (the orbiter) and the Earth. Substituting the known values:

$$v_{\text{moon}} = \sqrt{\frac{(6.67 \times 10^{-11}) \times (6.00 \times 10^{24})}{384400 \times 10^3}}$$

$$v_{\text{moon}} = 1020.34 \text{ ms}^{-1} = \mathbf{1020 \text{ ms}^{-1}} \text{ (3 s.f.)}$$

b Orbital velocity of a satellite: For a satellite in a circular orbit at an altitude of 300km. Total distance from the centre of the Earth r_{sat} is the sum of the Earth's radius R_{Earth} and the altitude of the satellite h_{sat} :

$$r_{\text{sat}} = R_{\text{Earth}} + h_{\text{sat}}$$

$$r_{\text{sat}} = 6370 + 300 = 6670 \text{ km}$$

Using the same formula:

$$v_{\text{sat}} = \sqrt{\frac{(6.67 \times 10^{-11}) \times (6.00 \times 10^{24})}{6670 \times 10^3}}$$

$$v_{\text{sat}} = 7745.96 \text{ ms}^{-1} = \mathbf{7750 \text{ ms}^{-1}} \text{ (3 s.f.)}$$

c Orbital velocity is the speed required for an object to stay in a stable orbit around a planet or star. For artificial satellites, achieving the correct orbital velocity ensures that they do not fall back to Earth or drift off into space.

If a satellite's velocity is too high, it may escape the Earth's gravity and move into a higher orbit or into space. If the velocity is too low, the satellite might enter the Earth's atmosphere and burn up. Correct orbital velocity is critical for maintaining the desired orbit and for the satellite to function as intended, whether for communication, navigation or scientific research.

- 3** For a highly elliptical orbit where periapsis distance r (closest distance between the star and the black hole) is much smaller than the semi-major axis distance a , and so $2/r \gg 1/a$. And so, the vis-viva equation can be approximated to:

$$v \approx \sqrt{\frac{2GM}{r}}$$

M in kg can be calculated to be:

$$M = 4.30 \times 10^6 \text{ solar mass}$$

$$M = 4.30 \times 10^6 \times 1.99 \times 10^{30} \text{ kg}$$

$$M = 8.56 \times 10^{36} \text{ kg (3 s.f.)}$$

r in metres is given as:

$$r = 1.90 \times 10^{12} \text{ m}$$

Substituting values of M and r into the orbital velocity equation derived above we get:

$$v = \sqrt{\frac{2 \times (6.67 \times 10^{-11}) \times (4.30 \times 10^6 \times 1.99 \times 10^{30})}{1.90 \times 10^{12}}}$$

$$v = \sqrt{6.008 \times 10^{14}}$$

$$v = 2.45 \times 10^7 \text{ ms}^{-1} \text{ (3 s.f.)}$$

As a percentage of speed of light:

$$\frac{v}{c} \% = \frac{(2.45 \times 10^6)}{3.00 \times 10^8} \times 100 = 8.17\% \approx 8\%$$

Michelson's stellar interferometer

- 1 a Fringe separation (fringe width) is given by:

$$w = \frac{\lambda D}{d}$$

Substituting the give values:

$$w = \frac{(600 \times 10^{-9}) \times (1.50)}{0.200 \times 10^{-3}} = 4.50 \text{ mm (3 s.f.)}$$

- b With the new slit separation, d_{new} of 0.300mm:

$$w_{\text{new}} = \frac{\lambda D}{d_{\text{new}}}$$

$$w_{\text{new}} = \frac{(600 \times 10^{-9}) \times (1.50)}{0.300 \times 10^{-3}}$$

$$w_{\text{new}} = 3.00 \text{ mm (3 s.f.)}$$

- c With the new wavelength λ_{new} as 500nm and using the original 0.200mm slit separation:

$$w_{\text{new2}} = \frac{\lambda_{\text{new}} D}{d}$$

$$w_{\text{new2}} = \frac{(500 \times 10^{-9}) \times (1.50)}{0.200 \times 10^{-3}}$$

$$w_{\text{new2}} = 3.75 \text{ mm (3 s.f.)}$$

The fringe separation decreases as the wavelength of light used decreases. This is because fringe separation is directly proportional to the wavelength, and a decrease in wavelength results in a smaller fringe separation.

- 2 a** Angular resolution (θ) is given by the Rayleigh criterion:

$$\theta = 1.22 \frac{\lambda}{D}$$

where D is 100m and λ is 21.0cm or 0.21m. Substituting these values, we get:

$$\theta = 1.22 \times \frac{0.21}{100} \text{ radians}$$

Multiplying by 2.06×10^5 we can convert the angular resolution in arcseconds (as):

$$\theta = 1.22 \times \frac{0.21}{100} \times 2.06 \times 10^5 = \mathbf{528 \text{ as}} \text{ (3. s. f.)}$$

b The angular resolution improves (i.e. θ becomes smaller) when the wavelength (λ) of the observed radiation decreases or when the diameter (D) of the telescope increases. Thus, smaller wavelengths and larger telescope diameters yield better angular resolutions.

c For a smaller optical telescope with $\lambda_{\text{optical}} = 500 \text{ nm}$ and diameter $D = 1.00\text{m}$:

$$\theta_{\text{optical}} = 1.22 \frac{\lambda_{\text{optical}}}{D}$$

$$\theta_{\text{optical}} = 1.22 \times \frac{(500 \times 10^{-9})}{1.00} \text{ radians}$$

Converting to arcseconds:

$$\theta_{\text{optical}} = 1.22 \times \frac{(500 \times 10^{-9})}{1.00} \times 2.06 \times 10^5 \text{ as}$$

$$\theta_{\text{optical}} = 0.12566 \text{ as}$$

$$\theta_{\text{optical}} = \mathbf{1.26 \times 10^{-2} \text{ as}} \text{ (3. s. f.)}$$

The optical telescope thus has a much better (smaller) angular resolution compared to the radio telescope.

d Angular resolution determines the ability of a telescope to distinguish between two closely spaced objects in the sky. Higher angular resolution means the telescope can distinguish between objects that are closer together.

Poor angular resolution results in blurry images and the inability to separate closely spaced objects. Good angular resolution provides clear, distinct images and allows astronomers to study fine details of celestial objects and structures.

- 3 a** Converting the distance d into km we get:

$$d = 700 \times 9.46 \times 10^{15} \text{ m} = 6.622 \times 10^{15} \text{ km}$$

Next, let us convert the angular diameter of the star as provided in the question to radians:

$$\theta = 0.047 \text{ as} = \frac{0.047}{2.06 \times 10^5} \text{ radians}$$

We can now calculate the diameter of Betelgeuse using the formula (as described in the article):

$$\theta = 2 \tan^{-1} \left(\frac{D}{2d} \right)$$

Using the small angle approximation ($\tan \theta \approx \theta$), we get:

$$\frac{\theta}{2} = \left(\frac{D}{2d} \right)$$

$$\theta = \left(\frac{D}{d} \right)$$

$$D = d \times \theta$$

Substituting the values of θ and d calculated above:

$$D = 700 \times 9.46 \times 10^{15} \times \frac{0.047}{2.06 \times 10^5}$$

$$D = 151.0845 \times 10^{10} \text{ m} = \mathbf{1.51 \times 10^9 \text{ km}} \text{ (3 s.f.)}$$

b Diameter of Betelgeuse in solar radii R_{\odot} :

$$D_{R_{\odot}} = \frac{D \text{ (in km)}}{1.39 \times 10^6 \text{ km}}$$

$$D_{R_{\odot}} = \frac{1.510845 \times 10^9 \text{ km}}{1.39 \times 10^6 \text{ km}} = 1.0869 \times 10^3 R_{\odot} = \mathbf{1090 R_{\odot}} \text{ (3 s.f.)}$$

c Betelgeuse is approximately 1090 times the diameter of the Sun, indicating it is an extremely large star. This comparison highlights the immense size of Betelgeuse relative to our Sun, (reflecting its classification as a red supergiant).

d If the distance from Earth to Betelgeuse were twice as far, its angular diameter would be halved. This is because angular diameter is inversely proportional to distance. To then achieve the same angular resolution, the telescope deployed for the measured would need to have its diameter significantly increased. The resolving power also depends inversely on the wavelength of light used. Observing Betelgeuse at shorter wavelengths (such as using ultraviolet or X-ray wavelengths) could also be undertaken for meeting the same angular resolution needs as when the star is as far as it is now.

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