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Answers

Practice-for-exam questions

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Making the perfect cup of tea

1

$$\Delta Q = mc\Delta\theta$$
$$0.15\text{ kg} \times 4200\text{Jkg}^{-1}\text{K}^{-1} \times (85°\text{C} - 65°\text{C}) = m \times 4000\text{Jkg}^{-1}\text{K}^{-1} \times (65°\text{C} - 5°\text{C})$$
$$m = \frac{0.15\text{ kg} \times 4200\text{Jkg}^{-1}\text{K}^{-1} \times (85°\text{C} - 65°\text{C})}{4000\text{Jkg}^{-1}\text{K}^{-1} \times (65°\text{C} - 5°\text{C})}$$
$$m = \frac{12600\text{J}}{240000\text{Jkg}^{-1}}$$
$$\underline{m = 0.0525\text{kg or } 53\text{g}}$$

Note that strictly speaking, the temperature here should be in kelvin but because we need to calculate only a temperature difference, we can use Celsius. This is because converting one temperature scale to the other requires only the addition of a constant.

2 The volume of tea in both beakers is the same. As the porcelain beaker has a greater diameter, the depth of tea in the beaker will be smaller.

The rate of transfer of energy from the tea to the surroundings by conduction depends on the thermal conductivity of the material of the beaker, the surface area of the tea in contact with the material, the thickness of the beaker, and the temperature difference between the tea and the surroundings. These last two factors are the same for both beakers.

The surface area of tea in contact with the beaker is given by the area of the base of the beaker, πr^2, where r is the radius of the beaker, added to the surface area of the cylinder, $2\pi rd$, where d is the depth of tea in the beaker. The total surface area is therefore $\pi r^2 + 2\pi rd$, which is different for the two beakers as the diameter is different. The volume of tea in both beakers, given by $\pi r^2 d$, is the same for both beakers.

The depth of the tea in the Pyrex beaker is given by (using the subscript g for glass):

$$d_g = \frac{V}{\pi\, r_g^2} = \frac{300\text{ cm}^3}{\pi \times 3.00^2\text{cm}^2} = 10.6\text{ cm}$$

Similarly, the depth of tea in the Pyrex beaker is 5.97 cm.

The total surface area of tea in contact with the Pyrex beaker is therefore:

$$A_g = \pi\, r_g^2 + 2\pi\, r_g d_g = \pi \times 3.00^2\text{cm}^2 + 2 \times \pi \times 3.00\text{ cm} \times 10.6\text{ cm} = 228\text{ cm}^2$$

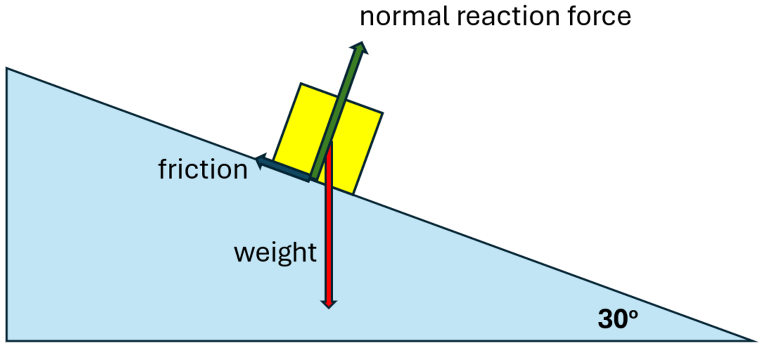
For the porcelain beaker,

$$A_p = 200\text{ cm}^2$$

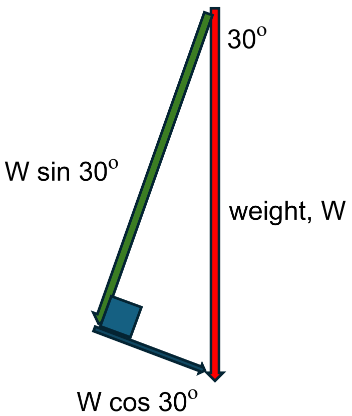
The area of tea in contact with the porcelain beaker is smaller, but porcelain has a greater thermal conductivity. The thermal conductivity of porcelain is 33% greater than that of Pyrex, and the surface area in contact with the tea is 12% smaller than in the Pyrex beaker. Therefore, the tea in the porcelain beaker will cool faster due to conduction because the thermal conductivity is the more significant factor.

Trigonometry

1



To find the frictional force, resolve the weight into two components parallel to and perpendicular to the slope.



As the box is at rest, the total force parallel to the slope is zero, and so the frictional force acting up the slope is:

$W \cos 30^{\circ} = 5.00 \text{ kg} \times 9.81 \text{ N kg}^{-1} \times 0.866 = 42.5 \text{ N}$

Another way of seeing the same thing is to note that as the box is at rest, the total force acting is zero, so that a closed triangle of forces can be drawn to show that the resultant force is zero, as shown here:

This looks almost identical to the triangle drawn above when resolving the weight, but note the directions of the arrows. The calculation is identical to that shown above.

If the box slides down the slope at a steady speed, the total force acting is still zero as the acceleration is zero. Therefore the frictional force acting is the same.

Internal resistance and emf in a battery

1 When the switch is closed, the pd across the terminals of the cell is 1.5 V. This is the same as the pd across the external resistance. The current flowing through the circuit is therefore:

$$I = \frac{V}{R} = \frac{1.5\ \mathrm{V}}{5.0\ \Omega} = 0.3\ \mathrm{A}$$

The pd 'lost' across the internal resistance of the cell is 0.5 V, so the internal resistance is:

$$r = \frac{V}{I} = \frac{0.5\ \mathrm{V}}{0.3\ \mathrm{A}} = 1.67\ \Omega$$

The cell gets warm because energy is dissipated in the internal resistance. Energy in the chemical store of the cell is transferred into thermal energy. The power dissipated is:

$$P = IV = 0.3\mathrm{A}\ \times 0.5\ \mathrm{V} = 0.15\ \mathrm{W}$$

The useful power transferred to the external resistance is:

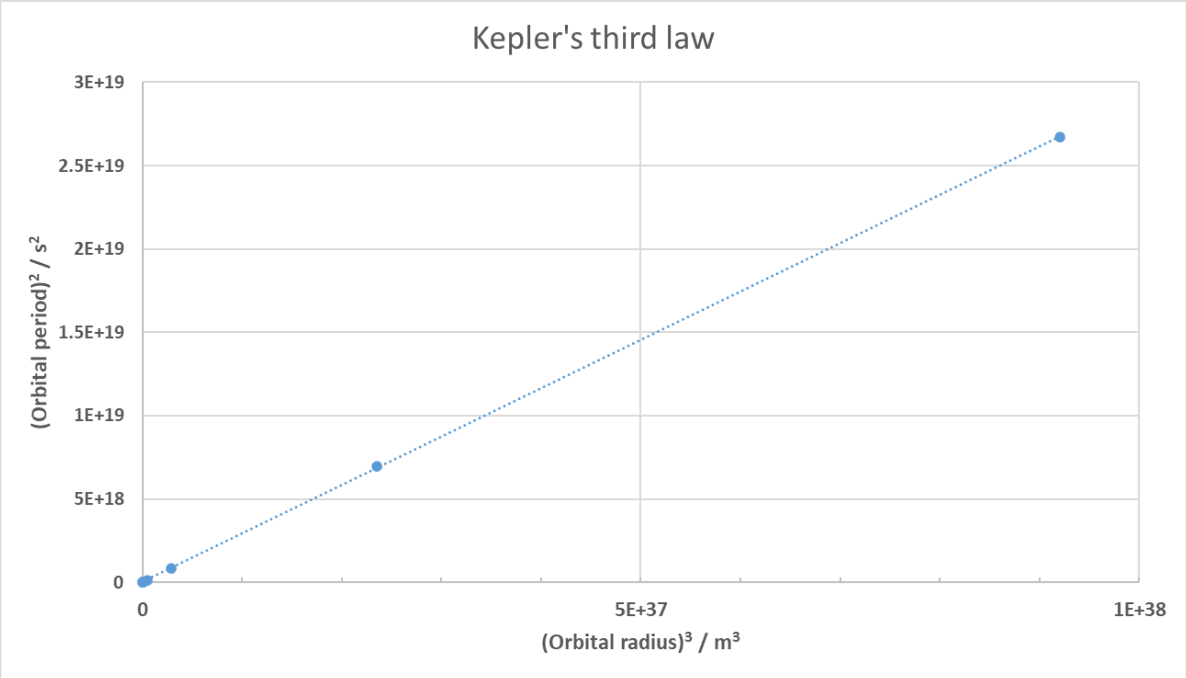
$$P = IV = 0.3\mathrm{A}\ \times 1.5\ \mathrm{V} = 0.45\ \mathrm{W}$$

Hence the efficiency is:

$$\frac{0.45}{0.45 + 0.15} = \frac{0.45}{0.60} = 75\%$$

Orbital motion

1 To verify Kepler's third law, $T^2=kr^3$, plot a graph of T^2 on the y-axis against r^3 on the x-axis. This gives a straight line, with gradient *k*.



The graph shown here was created with Excel.

Excel represents numbers such as 1.5×10^{19} as 1.5E+19.

In order to calculate the mass of the Sun from the gradient, the period and radius of the orbits need to be in seconds and metres respectively, or the gradient needs to be converted into appropriate units. With the units being seconds and metres, the gradient is approximately 2.90×10^{-19} s^2 m^{-3}. Note the units that you can see come from the graph.

To find the mass of the Sun, use the expression given for the constant in the article:

$$k = \frac{4\,\pi^2}{GM}$$

So:

$$M = \frac{4\pi^2}{Gk} = \frac{4\,\pi^2}{6.67 \times 10^{-11}\ \mathrm{N\ m^2 kg^{-2}} \times 2.90\ \times 10^{-19}\ \mathrm{s^2\ m^{-3}}} = 2.04 \times 10^{30}\ \mathrm{kg}$$

2 The weight of an object on the Earth is mg, where *m* is its mass, and *g* is the gravitational field strength. Hence:

$$mg = \frac{GmM}{r^2}$$

Cancelling out *m* and rearranging:

$$M = \frac{gr^2}{G} = \frac{9.81\ \mathrm{N\ kg^{-1}}\ \times (6.378 \times 10^6 \mathrm{m})^2}{6.67 \times 10^{-11} \mathrm{N\ m^2\ kg^{-2}}} = 5.98 \times 10^{24}\ \mathrm{kg}$$

3 For an object in orbit:

$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

Hence using the value for the mass of the Earth from question 2:

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67 \times 10^{-11}\ \mathrm{N\ m^2\ kg^{-2}}\ \times 5.98 \times 10^{24}\ \mathrm{kg}}{(6.378 \times 10^6 + 4.15 \times 10^5)\ \mathrm{m}}} = 7.66\ \mathrm{km\ s^{-1}}$$

The total energy of the ISS is the sum of the gravitational and kinetic energies:

$$E = -\frac{GMm}{r} + \frac{1}{2}mv^2$$

Rather than carrying out calculations 'per kilogram', as in the article, we shall leave the mass of the ISS, *m*, in the calculations so that you can see how it cancels out.

In its original orbit, the total energy is:

$$E = m\left(-\frac{GM}{r} + \frac{1}{2}v^2\right) = -2.94 \times 10^7\ m\ \mathrm{J\ kg^{-1}}$$

The unit here looks odd because m stands for the mass of the ISS and includes the unit, kg, so that the product, m J $\mathrm{kg^{-1}}$, has units of J, as required.

In the lower orbit, the gravitational energy is

$$E_{grav} = -\frac{GmM}{r} = -5.92 \times 10^7 m\ \mathrm{J\ kg^{-1}},$$

using $r = 6.378 \times 10^6\ m + 3.50 \times 10^5 m.$

Hence, the kinetic energy in the new orbit is:

$$\frac{1}{2}mv^2 = -2.94 \times 10^7 m\ \mathrm{J} - -5.92 \times 10^7 m\ \mathrm{J} = 2.98 \times 10^7 m\ \mathrm{J\ kg^{-1}}$$

Cancelling out the mass and rearranging:

$$v = \sqrt{2 \times 2.98 \times 10^7}\ \mathrm{m\ s^{-1}} = 7.72\ \mathrm{km\ s^{-1}}$$

[To check this final answer, we can calculate the orbital speed in this lower orbit as in the first part of the question:

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67 \times 10^{-11}\ \mathrm{N\ m^2\ kg^{-2}}\ \times 5.98 \times 10^{24}\ \mathrm{kg}}{(6.378 \times 10^6 + 3.50 \times 10^5)\ \mathrm{m}}} = 7.70\ \mathrm{km\ s^{-1}}$$

The difference to the answer above is due to rounding errors.]