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Answers

Practice-for-exam questions

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Dirigible planetary exploration

1 **a** The volume of the balloon is the sum of the volumes of the central cylindrical section (call it V_{cyl}) and the volume of the left-hand side half-sphere (call it V_{left}) and the volume of the right-hand side half-sphere (call it V_{right}). That is:

$$V_{balloon} = V_{cyl} + V_{left} + V_{right} \qquad \text{(Eq. 1)}$$

As both the half-spheres have the same radius, together they represent a single sphere of radius $r = 2$ m. The volume of this sphere (call it V_{sphere}) is given by:

$$V_{sphere} = V_{left} + V_{right} = \frac{4}{3}\pi r^3 \qquad \text{(Eq. 2)}$$

The volume of the central cylinder is given by multiplying its length (7 m) by its cross-sectional area:

$$V_{cyl} = \pi r^2 l \qquad \text{(Eq. 3)}$$

Substituting equations 3 and 2 into 1:

$$V_{balloon} = \pi r^2 l + \frac{4}{3}\pi r^3 \qquad \text{(Eq. 4)}$$

$$V_{balloon} = \pi.(2^2)\text{m}^2.7\text{m} + \frac{4}{3}\pi.(2)^3\text{m}^3 = 121.47\text{ m}^3 = \mathbf{121\ m^3}\ (3\text{ s.f.}) \qquad \text{(Eq. 5)}$$

b The buoyancy force acting on the balloon is:

$$F_{upward} = \rho_{atmosphere\ on\ X} \times V_{balloon} \times g_{planet\ X} \qquad \text{(Eq. 6)}$$

$$F_{upward} = (15 \times 1.23)\text{kg m}^{-3} \times 121\text{m}^3 \times (0.5 \times 9.81)\text{N kg}^{-1} \qquad \text{(Eq. 7)}$$

$$F_{upward} = 10950\ N = \mathbf{11000\ N}\ (3\text{ s.f.}) \qquad \text{(Eq. 8)}$$

$$g = \frac{f}{l} = \frac{44.15m}{5m}$$
$\underline{g = 8.83}$

c The downward force acting on the balloon due to its skin is 1200 N. The uplift force will thus be:

$$F_{uplift} = 11000\text{ N} - 1200\text{ N} = 9800\text{ N }(3\ s.f.)$$

Thus, hardware weight of less than 9.8 kN would be permissible. This would translate to:

$$m_{hardware} = \frac{F_{uplift}}{g_{planet\ X}} = \frac{9800\text{N}}{0.5 \times 9.81\text{Nkg}^{-1}} = 1998\text{ kg} = \mathbf{2000\ kg}\ (3\text{ s.f.})$$

2 Ideal gas law states that:

$$p\,V = n\,R\,T \qquad \text{(Eq. 9)}$$

$$1 \times 10^5\text{Nm}^{-2} \times V = 8.80 \times 8.314\text{ J K}^{-1} \times (273 + 25)\text{K}$$

$$V = 0.218\text{ m}^3$$

As 1 m^3 equals 1000 L, we get the volume of the balloon in litres to be:
$V = 0.218 \times 1000 = \mathbf{218\ L\ (3\ s.f.)}$

3 By Archimedes principle, 200 kg of air will be displaced by the balloon.

Ideal gas law can be used to determine the volume of air displaced:

$$pV = nRT$$

$$p = \frac{nRT}{V}$$

Multiplying both sides of the above equation by mass of displaced air *m*, we get:

$$mp = \frac{m \times nRT}{V} = \left(\frac{m}{V}\right) \times nRT \qquad \text{(Eq. 10)}$$

Dividing both sides of the equation by *n*, we get:

$$\left(\frac{m}{n}\right)p = \left(\frac{m}{V}\right) \times RT$$

(*m/n*) is the molar mass of air, which is given to be 30 g mol^{-1} or 0.03 kg mol^{-1}.

Atmospheric pressure *p* is 1 atm which is 105 N m^{-2}, the molar gas constant *R* is 8.314 J mol^{-1} K^{-1} and atmospheric temperature *T* is given to be 27 degrees Celsius which is 273 + 27 in Kelvin scale. Substituting these values into the above equation we get:

$$(0.030)\text{kg mol}^{-1} \times 10^5\text{N m}^{-2} = \left(\frac{200\text{ kg}}{V}\right) \times 8.314\text{ J kg}^{-1}\text{mol}^{-1} \times (273 + 27)\text{K}$$

$$V = \frac{498840}{(0.030) \times 10^5}\text{m}^3 = \mathbf{166.28\ m^3}$$

Being a spherical balloon, its volume is given by:

$$V = \frac{4}{3}\pi r^3 \qquad \text{(Eq. 11)}$$

Thus, radius of the balloon would need to be:

$$166.28\ \text{m}^3 = \frac{4}{3}\pi r^3$$

$$r = \sqrt[3]{\frac{3 \times 166.28}{4\pi}}\ \text{m} = \mathbf{3.41\ m\ (3.s.f.)}$$

How cold is space?

1 The 'Big Bang' theory provides an explanation for how the universe began. It says the universe originated ~13.7 billion years ago, as an infinitely hot and dense single point (called the singularity). The hypothesis is that this single point inflated and stretched in an event described as the 'Big Bang'. It says that this rapid expansion initially took place at an unimaginable speed and that our universe is still expanding but is now doing so at a more measured rated (but still very fast).

The expansion of the universe can be seen by observing the emission spectra of distant starts. Stars emit a range of wavelengths, but chemical elements in the star absorb some of these, resulting in dark lines appearing when their atomic emission spectrum is analysed in a spectrometer. Astronomers have observed that receding stars (stars that are moving away from us) feature dark lines shifted towards the red side of the emission spectrum due to wavelengths being stretched as the space through which the radiation is travelling expands. Astronomers were able to correlate the amount of this shift in dark lines (the red-shift), to the distance of the star from the observatory (point of observation). They are also able to ascertain the acceleration with which stars are moving away from us by calculating the change in red-shift over a period of time. The observation that distant stars are moving away from us at an increasing recessional velocity is evidence for the existence of dark energy, sometimes referred to as the 'cosmological constant'.

2 Shortly after the Big Bang, the universe was composed of elementary particles such as electrons, quarks and neutrinos, and photons. Charged particles interact with photons and thus light and matter were tightly coupled, restricting the ability of light to travel long distances in a straight line. This rendered the universe opaque in the sense that photons were continually scattered. After the Big Bang, it took about 380,000 years for the temperature of the universe to cool down to temperatures at which atoms can form (making the universe transparent as photons could now travel without scattering). Light could now travel freely and did so initially with a range of wavelengths but with a peak at X-ray wavelengths. This radiation, a relic of the Big Bang, is referred to as cosmic microwave background (CMB) radiation because the expansion of the universe has stretched the X-ray light to become microwaves. CMB, undetectable to our human eyes, can be detected using specially designed detectors. The CMB takes astronomers as close as possible to the Big Bang and is currently one of the most promising ways we have of understanding the birth and evolution of the universe in which we live.

3

$$\Delta\lambda = \frac{\lambda_{emitted} \times Speed_{galaxy}}{Speed_{light}} \qquad \text{(Eq. 12)}$$

$$\Delta\lambda = \frac{(600\ \times 10^{-9})\text{m} \times (2 \times 10^6)\text{m s}^{-1}}{3 \times 10^8\ \text{m s}^{-1}}$$

$$\Delta\lambda = \frac{(1200 \times 10^{-3})}{3 \times 10^{8}} \text{m} = 400 \times 10^{-11} \text{m} = 4.00 \times 10^{-9} \text{m} = \mathbf{4.00\ nm}\ (3\ \text{s.f.})$$

Supercapacitors

1 **a** The Farad (F) is the SI unit of capacitance and is determined by the distance between the capacitor plates, the area of these plates and the ability of the dielectric material to support electrostatic forces.

b Capacitance C, charge Q and applied voltage V are related by:

$$C = \frac{Q}{V} \qquad \text{(Eq. 13)}$$

And so, for Capacitor A:

$$C_A = \frac{10.0\ \mu\text{C}}{5.00\ \text{V}} = \mathbf{2.00\ \mu F}\ (3\ \text{s.f.})$$

c Energy stored in a capacitor is given by:

$$E = \frac{1}{2} C V^2 \qquad \text{(Eq. 14)}$$

$$E_A = \frac{1}{2} (2.00 \times 10^{-6}) \text{C} \times 9.00^2 \text{V}^2$$

$$E_A = \mathbf{0.0810\ mJ}\ (3\ \text{s.f.})$$

2 **a** Area of each metal plate:

$$A = 0.250 \times 0.100\ \text{m}^2 = \mathbf{0.0250\ m^2}\ (3\ \text{s.f.})$$

b Capacitance of a capacitor is given by:

$$C = \frac{\epsilon A}{d} \qquad \text{(Eq. 15)}$$

$$C = \frac{\epsilon_0 \epsilon_r A}{d} \qquad \text{(Eq. 16)}$$

Re-arranging for d:

$$d = \frac{\epsilon_0 \epsilon_r A}{C}$$

Substituting known values:

$$d = \frac{(8.854 \times 10^{-12}) \times 2.30 \times 0.0250}{(370 \times 10^{-12})} \text{m}$$

$$d = \mathbf{1.38\ mm}\ (3\ \text{s.f.})$$

c When the polythene is removed, the new capacitance of the capacitor will be:

$$C_{new} = \frac{\epsilon_0 \epsilon_r A}{d}$$

$$C_{new} = \frac{(8.854 \times 10^{-12}) \times 1 \times 0.0250}{(1.38 \times 10^{-3})} \text{F}$$

$$C_{new} = 161 \text{ pF} \ (3\ s.f.)$$

The 13 nC of charge will persist when the supply is disconnected, and the polythene sheet is removed. Knowing this, we can find the new potential difference across the plates to be:

$$V_{new} = \frac{Q}{C_{new}} = \frac{13.0 \times 10^{-9}\text{C}}{161 \times 10^{-12}\text{F}} = \mathbf{80.8\ V}\ (3\ \text{s.f.})$$

d When the dielectric is removed, the charge Q remains unchanged, but the potential difference between the plates increases (as worked out in part (c)). The energy stored in the capacitor when the dielectric is removed can be determined using:

$$E_{new} = \frac{1}{2} Q V_{new} = \frac{1}{2} \times (13.0 \times 10^{-9}\ \text{C}) \times 80.8\ \text{V} = 5.252\ \times 10^{-7}\ J$$

$$\therefore E_{new} = \mathbf{0.525\ \mu J}\ (3\ \text{s.f.})$$

The increase in energy comes from the mechanical work done in removing the polythene dielectric, which involves overcoming the electrostatic force between each plate and the dielectric.

3 **a** Energy density, measured in watt-hours per kilogram (Wh kg^{-1}) refers to how much energy a device can deliver per unit mass.

Power density, measured in watts per kilogram (W kg^{-1}) refers to how quickly a device can deliver energy per unit pass.

b

Table 1: ED and PD comparison between a standard capacitor, a standard rechargeable li-ion battery and a standard electrostatic double-layer type supercapacitor

Component	Energy density (Wh kg^{-1})	Power density (W kg^{-1})
Standard capacitor	10^{-2} to 10^{-1}	10^3 to 10^6
Rechargeable li-ion Battery	20 to 200	20 to 1000
EDL supercapacitor	10^{-1} to 50	10^2 to 10^5

c To a comparably sized li-ion rechargeable battery, a supercapacitor cannot store as much energy, but can be charged and discharged much faster.

d Regenerative breaking in automobiles to reduce fuel consumption and stabilising voltage supplies to provide a steady voltage supply are two popular practical applications of supercapacitors.