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Answers

Practice-for-exam questions

James Lees

Understanding Feynman diagrams

1

a The upwards pointing arrow indicates a particle and a downwards pointing arrow an antiparticle. Straight, solid lines represent fermions.

b W Bosons: weak force

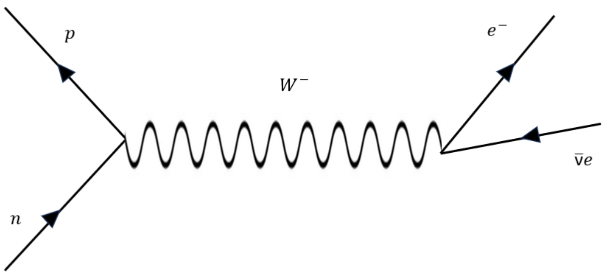
Z Bosons: weak force

Gluons: strong force

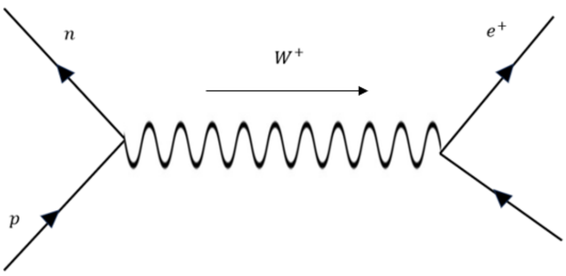
Photons: electromagnetic force

2

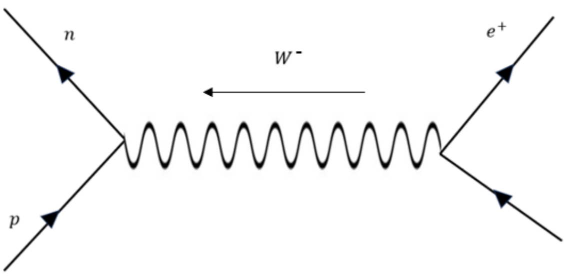
a



b



or



Note: the mathematics of quantum field theory includes both these diagrams automatically so that, in practice, when carrying out calculations, a single diagram would be drawn to represent a W without specifying the charge or direction.

Observing gravitational waves

1 $1AU = 1.50 \times 10^{11} m, G = 6.67 \times 10^{-11}\ N\ m^2\ kg^{-2}, 1\ M_{\odot} = 1.99 \times 10^{30} kg$

a

$$F = \frac{GMm}{r^2}$$

$$F = \frac{6.67 \times 10^{-11} N\ m^2 kg^{-2} \times (45 \times 1.99 \times 10^{30} kg) \times (20 \times 1.99 \times 10^{30} kg)}{\left((80 + 30) \times\ 1.5 \times 10^{11} m\right)^2}$$

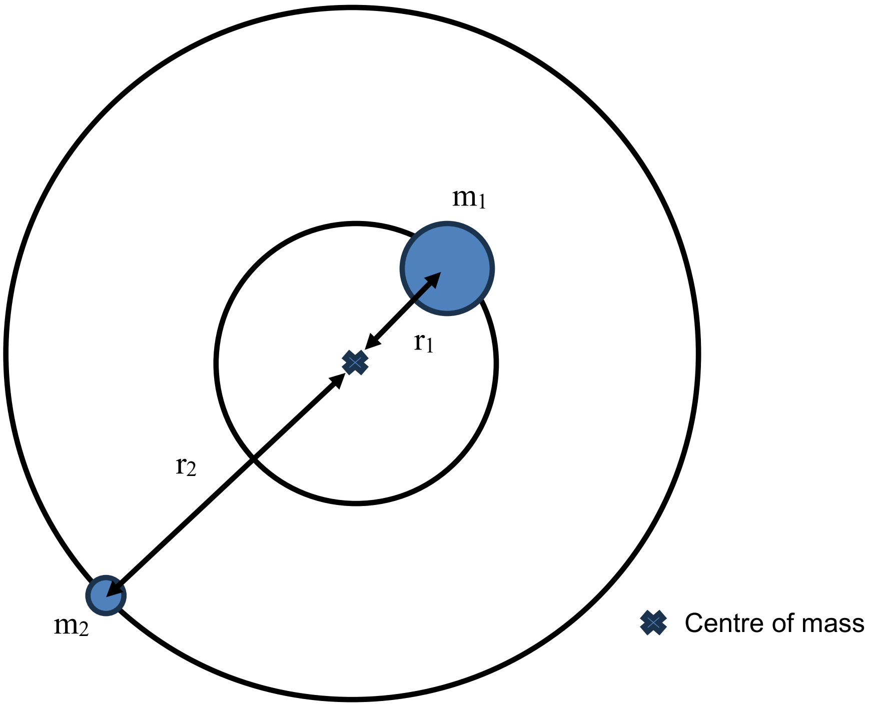
$$F = 8.73 \times 10^{26} N$$

b

$\frac{GMm}{r^2} = \frac{mv^2}{r}$ and $v = \frac{2\pi r}{T}$

$$\therefore T = \sqrt{\frac{4\pi^2 r^3}{GM}} = 2\pi\sqrt{\frac{r^3}{GM}}$$

For binary systems, we need to modify this expression:



For two bodies orbiting around their common centre of mass,

$m_1 r_1 = m_2 r_2$

The force between the two bodies is given by Newton's law of universal gravitation:

$$F = \frac{G m_1 m_2}{(r_1 + r_2)^2}$$

The centripetal acceleration of m_1 is given by:

$a_1 = r_1 \omega^2$

where $\omega = \frac{2\pi}{T}$

and T is the orbital period (which is the same for both bodies). Hence, we have:

$$\frac{G m_1 m_2}{(r_1 + r_2)^2} = m_1 r_1 \omega^2$$

Substituting in the expression for ω and rearranging gives:

$$T = 2\pi \sqrt{\frac{r_1 (r_1 + r_2)^2}{G m_2}}$$

Using $m_1 r_1 = m_2 r_2$, we can write:

$$m_1 + m_2 = \frac{m_2 r_2}{r_1} + m_2 = \frac{m_2 (r_1 + r_2)}{r_1}$$

and so:

$$m_2 = \frac{M r_1}{R}$$

where M is the total mass $(= m_1 + m_2)$ and R is the distance between the two bodies $(= r_1 + r_2)$.

Substituting this into the expression for T, we have:

$$T = 2\pi \sqrt{\frac{r_1 (r_1 + r_2)^2}{G m_2}} = 2\pi \sqrt{\frac{R^3}{GM}}$$

This expression is the usual one for Kepler's third law, but with the total mass of the two stars and the total distance between the two stars replacing the mass and orbital radius of a single object.

Hence, in this case:

$$T = 2\pi\sqrt{\frac{(r_1 + r_2)^3}{G(M + m)}}$$

$$T = 2\pi\sqrt{\frac{((75.0 + 30.0) \times\ 1.50 \times 10^{11} m)^3}{6.67 \times 10^{-11} N\ m^2 kg^{-2} \times ((20.0 + 50.0) \times 1.99 \times 10^{30} kg)}}$$

$$T = 2\pi\sqrt{\frac{((105) \times\ 1.5 \times 10^{11} m)^3}{6.67 \times 10^{-11} N\ m^2 kg^{-2} \times (70.0 * 1.99 \times 10^{30} kg)}}$$

$T = 4.07 \times 10^9$ seconds $= 129$ years

c

$$\frac{9.50 \times 10^9 \times 1.50 \times 10^{11}\text{m}}{3 \times 10^8 \text{m s}^{-1}} = 4.75 \times 10^{12} s$$

$t \approx 150\ 000$ years

Dating the Earth

1

a The decay constant of Tc-99m is given by:

$$\lambda = \frac{\ln(2)}{T_{\frac{1}{2}}}$$

$$\lambda = \frac{\ln(2)}{6.04 \times 60 \times 60\ s}$$

$$\lambda = 3.19 \times 10^{-5}\ s^{-1}$$

b

$$N_t = N_0 e^{-\lambda t}$$

$$\frac{N_t}{N_0} = e^{-3.19\times10^{-5}\ s^{-1}\times(24\times60\times60)s} = e^{-3.19\times10^{-5}\times(86400)} = e^{-2.757}$$

$$\frac{N_t}{N_0} = 0.0635$$

$$= 6.35\%$$

c

$$N_t = N_0 e^{-\lambda t}$$

$$t = -\frac{\ln\left(\frac{N_t}{N_0}\right)}{\lambda}$$

$$t = -\frac{\ln\left(\frac{0.001}{1}\right)}{3.21 \times 10^{-5} s^{-1}} = -\frac{\ln(0.001)}{3.21 \times 10^{-5} s^{-1}}$$

$t = 215194\text{s} = 2.50$ days

d

$$T_{\frac{1}{2}} = 210\,000 years = (210\,000 \times 365.25 \times 24 \times 60 \times 60)s = 6.63 \times 10^{12} s$$

$$\lambda = \frac{\ln(2)}{T_{\frac{1}{2}}} = \frac{\ln(2)}{6.63 \times 10^{12} s} = 1.05 \times 10^{-13}\ s^{-1}$$

$$N = \frac{m}{m_M} \times N_A$$

$$N_{Tc-99m} = \frac{50.0 \times 10^{-6}\ g}{98\ g\ mol^{-1}} \times 6.02 \times 10^{23} mol^{-1} = 3.07 \times 10^{17}$$

$$N_{Tc-99} = N_{Tc-99m} \times 0.99999 = 3.07 \times 10^{17} \times 0.99999 = 3.07 \times 10^{17}$$

Hence, the activity, A $\left(= -\frac{dN}{dt}\right)$ is given by:

$A = \lambda N = 1.05 \times 10^{-13}\ s^{-1} \times 3.07 \times 10^{17}$

$A = 3.32 \times 10^{4} s^{-1}$

This is still quite a high level of activity, but far lower than that of the Tc-99m. As the Tc-99 decays by beta emission the ionising effect of the beta radiation is greater than that of the gamma radiation from Tc-99m. However, the Tc-99 is eliminated from the body quickly, with a biological half-life of about 1 day.

2

$$N_t = N_0 e^{-\lambda t}$$

$$t = -\frac{\ln\left(\frac{N_t}{N_0}\right)}{\lambda} \text{ and } \lambda = \frac{\ln(2)}{T_{\frac{1}{2}}}$$

$$t = -\frac{T_{\frac{1}{2}} \ln\left(\frac{N_t}{N_0}\right)}{\ln(2)}$$

$$t = -\frac{5730y \times \ln\left(\frac{0.0005}{1}\right)}{\ln(2)} = -\frac{5730\ y \times \ln(0.0005)}{\ln(2)}$$

$$t = 62\,833 \text{ years} \approx 63\,000 \text{ years old}$$

Methods for measuring *g* with a falling object

1

Method 1: time taken to fall

Allow an object at rest (i.e. $u=0$) to fall a set distance measuring the time it takes to fall. Repeat for different distances. Use $s = ut + \frac{1}{2}at^2$, with $u=0$, so that $a=2s/t^2$. The acceleration, *a*, is the acceleration due to gravity we wish to find.

If we plot a graph of *s* (the independent variable, on the *x*-axis) against t^2 (the dependent variable, on the *y*-axis), we can rearrange the equation to give $t^2 = (2/a)\ s$. Hence the graph should be a straight line through the origin, with gradient $2/a$, so we can find the acceleration due to gravity from *2/gradient*.

By drawing steepest and shallowest lines through the points, we can calculate a maximum and minimum value for the acceleration and so calculate uncertainties.

Method 2: speed after falling a set distance

Allow an object initially at rest to fall a set distance and measure its final velocity. Use $v^2 = u^2 + 2as$

with $u=0$ so that $v^2=2as$. Repeat the measurements for different values of s and plot graph of *s* (x-axis) against v^2 (y-axis). The equation shows that the graph should be a straight line through the origin with gradient $2a$, so that the acceleration due to gravity is given by *gradient/2.*

Drawing steepest and shallowest lines gives the uncertainty on the experimental value of the acceleration due to gravity.

Method 3: measure the change in speed over a set time

Allow an object to fall measuring its speed at two different points, and the time taken to travel between them. Use the equation $v = u + at$ so that the acceleration due to gravity can be calculated from

$$a = \frac{v-u}{t}$$

Vary the distance between the two points. Plot a graph of $(v-u)$ on the *y*-axis against *t* on the *x*-axis. This gives a straight line through the origin with gradient *a.*

Drawing steepest and shallowest lines gives the uncertainty on the experimental value of the acceleration due to gravity.

2 There is often more than one way of calculating the correct answer when using the constant acceleration equations. Some different methods are indicated below, and it is worth trying them all to check you get the right answer, and also to give you more practice in using these equations.

a Use $s = u\,t + \frac{1}{2}\,a\,t^2$ to find a, and then use $v = u + at$. This gives:

$v = 8.0\ \mathrm{m\,s^{-1}}$ and $a = 0.32\ \mathrm{m\,s^{-2}}$

Alternatively, use $s = \frac{1}{2}\,(u + v)\,t$ to find v, and then use $a = (v - u)/t$. Both methods must give the same answer, of course.

b Use $v = u + a\,t$ to find u, and then use $s = u\,t + \frac{1}{2}\,a\,t^2$, or $s = \frac{1}{2}\,(u + v)\,t$ to find *s*.

$u = 5\ \mathrm{m\,s^{-1}}$ and $s = 37.5\ \mathrm{m}$

c Use $s = \frac{1}{2}\,(u + v)\,t$ to find *t*. The easiest way to calculate *a* is using $a = (v - u)/t$, but you could also use $s = u\,t + \frac{1}{2}\,a\,t^2$ once you have found *a*.

$t = 0.625\mathrm{s}$ and $a = 12.8\mathrm{ms^{-2}}$