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Answers

Practice-for-exam questions

Rohan Kakade

Cepheid variable stars

1

a

$$L = \sigma 4\pi R^2 T^4$$

$$R = 1.22 \times 696000 \times 10^3 \text{ m} = 8.4912 \times 10^8 \text{ m}$$

$$T = (5780 + 10.0) \text{ K} = 5790.0 \text{ K}$$

$$L = (5.67 \times 10^{-8}) \times 4\pi \times R^2 \cdot T^4$$

$$L = (5.67 \times 10^{-8}) \times 4\pi \times (8.4912 \times 10^8)^2 \times (5790)^4 \text{ W} = 5.77 \times 10^{26} \text{ W}$$

$$L = \frac{(5.77 \times 10^{26})}{(3.85 \times 10^{26})} \approx 1.50 L_{\odot} \text{ (3sf)}$$

b

$$b = \frac{L}{4\pi d^2}$$

$$b = \frac{5.77 \times 10^{26}}{4\pi \times (4.34 \times 9.461 \times 10^{15})^2} \text{ W m}^{-2}$$

$$b = \frac{5.77 \times 10^{26}}{2.1187 \times 10^{34}} \text{ W m}^{-2}$$

$$b = 2.72 \times 10^{-8} \text{ W m}^{-2}$$

$$b = \frac{2.72 \times 10^{-8}}{1370} = 1.98 \times 10^{-11} b_{\odot} \text{ (3sf)}$$

c

$$d = m - M \sim 5 \log \left(\frac{d}{32.6} \right)$$

$$d = 5 \log \left(\frac{4.34}{32.6} \right) = -4.37 \text{ (3 sf)}$$

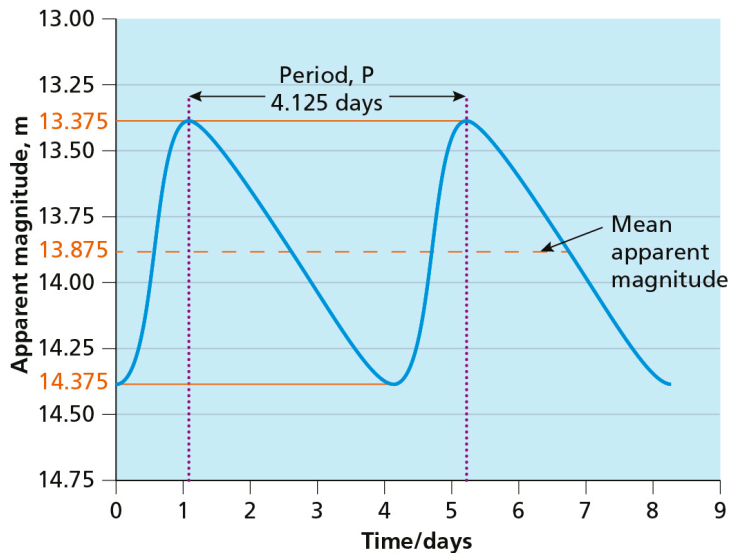
2

a

From the annotations overlaid on the graph below, the pulsation period (P) and mean apparent magnitude (m) is determined to be:

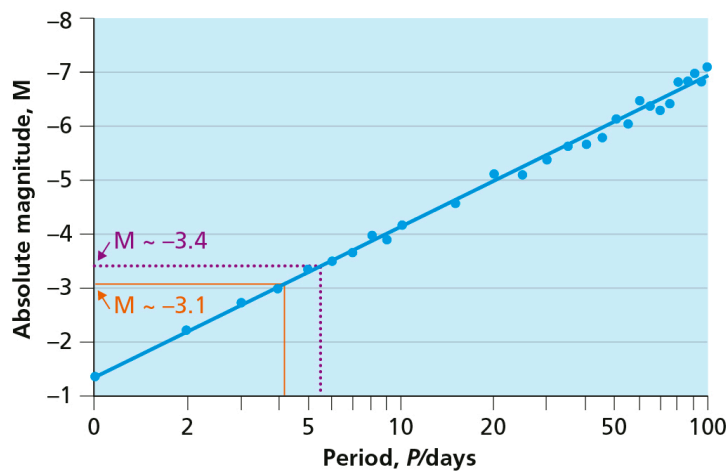
$$P = 4.125 \text{ days (4 sf)}$$

$$m = 13.875 = 13.88 \text{ (4 sf)}$$



b

For a period of 4.125 days, the mean absolute magnitude can be determined to be $M = -3.1$ by interpolating (see graph below) the period-luminosity plot shown in Figure 3.2 of the article.



We can now use the values of 'm' and 'M' to determine astronomical distance, 'd' of the Cepheid variable star as:

$$d = 10^{\frac{m-M+5}{5}}$$

$$d = 10^{\frac{13.875 - (-3.1) + 5}{5}}$$

$$d = 10^{\frac{13.875 - (-3.1) + 5}{5}}$$

$$d = 10^{\frac{21.975}{5}}$$

$$d = 10^{4.395}$$

$$d = 24831 \text{ pc} = 2.5 \times 10^4 \text{ pc (2 sf)}$$

$$\text{or, } d = 80988 \text{ ly} = 8.1 \times 10^5 \text{ ly (2sf)}$$

Cryo-electron microscopy

1 a

As explained in Box 2 of the article, the de-Broglie wavelength λ is related to the applied potential difference by:

$$\lambda = \frac{h}{\sqrt{2mE}} \quad (1)$$

$$E = \frac{\left(\frac{h}{\lambda}\right)^2}{2m} \quad (2)$$

We know that electron energy is related to applied potential difference by:

$$E = eV \quad (3)$$

Comparing equation (2) and equation (3):

$$eV = \frac{\left(\frac{h}{\lambda}\right)^2}{2m}$$

$$V = \frac{\left(\frac{h}{\lambda}\right)^2}{2me}$$

$$2meV = \left(\frac{h}{\lambda}\right)^2$$

$$\sqrt{2meV} = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 1.60 \times 10^{-19} \times \sqrt{V}}}$$

$$\lambda = \frac{1.23 \times 10^{-9}}{\sqrt{V}} = \frac{1.23}{\sqrt{V}} \text{ nm} \quad (4)$$

b

With $\lambda = 0.020 \text{ nm}$

$$0.02 \text{ nm} = \frac{1.23 \text{ nm}}{\sqrt{V}}$$

$$V = \left(\frac{1.23}{0.02}\right)^2 = 3782.25 \text{ V} = 3.78 \text{ kV (3 sf)}$$

c

Kinetic energy of the electrons is given by:

$$E = \frac{1}{2} m \cdot v^2$$

Where 'm' is the mass of the electron and 'v' is the velocity of the electron. Re-arranging the above equation:

$$v = \sqrt{\frac{2E}{m}}$$

Substituting equation (3) in above we get:

$$v = \sqrt{\frac{2eV}{m}}$$

$$v = \sqrt{\frac{2 \times 1.60 \times 10^{-19} \times 3782.25}{9.11 \times 10^{-31}}}$$

$$v = 3.65 \times 10^7 \text{ ms}^{-1} \text{ (3 sf)}$$

2

a

The de-Broglie wavelength λ_e of the electron (with mass m_e and charge q_e) when accelerated through a potential difference V is given by:

$$\lambda_e = \frac{h}{\sqrt{2m_e q_e V}} \quad (1)$$

Similarly, the de-Broglie wavelength λ_p of a proton (with mass m_p and charge q_p) when accelerated through the same potential difference V is given by,

$$\lambda_p = \frac{h}{\sqrt{2m_p q_p V}} \quad (2)$$

Dividing equation (1) by equation (2), we get

$$\frac{\lambda_e}{\lambda_p} = \frac{\frac{h}{\sqrt{2m_e q_e V}}}{\frac{h}{\sqrt{2m_p q_p V}}} \quad (3)$$

As the magnitude of charge on an electron and a proton are equal ($q_e = q_p$), we can simplify the equation (3) as:

$$\frac{\lambda_e}{\lambda_p} = \frac{\frac{1}{\sqrt{2m_e}}}{\frac{1}{\sqrt{2m_p}}} = \frac{\sqrt{2m_p}}{\sqrt{2m_e}} = \frac{\sqrt{m_p}}{\sqrt{m_e}}$$

Substituting the values of the masses (given in the question):

$$\frac{\lambda_e}{\lambda_p} = \frac{\sqrt{1.67 \times 10^{-27}}}{\sqrt{9.11 \times 10^{-31}}} = 42.815 = 42.8 \text{ (3 sf)}$$

Therefore, the de Broglie wavelength of the electron is about 40 times of a proton when each is accelerated through the same potential difference.

b

De-Broglie wavelength of the electron is related to the electron energy E by,

$$\lambda_e = \frac{h}{\sqrt{2m_e E_e}} \quad (4)$$

Squaring both sides of equation (4) and rearranging we get:

$$E_e = \frac{h^2}{2m_e(\lambda_e)^2}$$

$$E_e = \frac{(6.63 \times 10^{-34})^2}{2 \times (9.11 \times 10^{-31}) \times (1.23 \times 10^{-9})^2} = \frac{4.395 \times 10^{-67}}{2.756 \times 10^{-48}}$$

$$E_e = 1.5947 \times 10^{-19} \text{ J}$$

Since both the electron and the proton are being accelerated through the same potential difference V , they will have the same kinetic energy (Recall that $E = eV$) associated with them.

$$E_p = E_e$$

$$E_p = 1.59 \times 10^{-19} \text{ J (3 sf)} \approx 1 \text{ eV}$$

3

a

Substituting equation (4) in given equation we get,

$$r = \frac{0.610}{NA} \cdot \frac{1.23 \times 10^{-9}}{\sqrt{V}}$$

$$r = \frac{0.750}{NA} \cdot \frac{1}{\sqrt{V}} \text{ nm (3 sf)}$$

b

If $NA = 10^{-2} = 0.01$, then for $V = 3782.25 \text{ V}$, the theoretical TEM resolution will be,

$$r = \frac{0.750}{NA} \cdot \frac{1}{\sqrt{V}} \text{ nm} = 1.22 \text{ nm (3 sf)}$$

Electrical wires do not transfer energy

1 a

Since Avogadro's number represents the number of atoms in one mole of a substance, we can calculate the mass of one gold atom as follows:

$$m = \frac{\text{Gold's Molar mass}}{\text{Avagadro's number}}$$

$$m = \frac{197}{6.02 \times 10^{23}} = 3.272 \times 10^{-22} \text{ g/atom}$$

Converting to kg/atom:

$$m = 3.272 \times 10^{-25} \text{ kg/atom}$$

Next, we can calculate the number of gold atoms in 1 cubic meter of gold by dividing the total mass of gold in 1 cubic meter by the mass of one gold atom:

$$n = \frac{\text{Mass of 1 m}^3 \text{ of Gold}}{\text{Mass of 1 Gold atom}}$$

Mass of 1 cubic metre of Gold can be found by multiplying its density into its known volume (1.00 m³).

$$m_{1\text{m}^3} = \text{Density of Gold} \times 1.00 \text{ m}^3$$

$$m_{1\text{m}^3} = 19320 \frac{\text{kg}}{\text{m}^3} \times 1.00 \text{ m}^3 = 19320 \text{ kg}$$

Therefore 'n' is:

$$n = \frac{m_{1\text{m}^3}}{m} = \frac{19320 \text{ kg}}{3.27 \times 10^{-25} \text{ kg/atom}} = 5.908 \times 10^{28} = \mathbf{5.91 \times 10^{28} \text{ atoms}} \text{ (3 sf)}$$

Therefore, there are approximately 5.91×10^{28} atoms contained within 1 cubic metre of gold.

b

Each gold atom has 1 free (valence) electron, and thus the total number of free electrons in 1 cubic metre of gold is 5.91×10^{28} .

Each electron has a charge of $1.60 \times 10^{-19} \text{ C}$, and thus total charge contained within 1 cubic metre of gold is,

$$Q_{1\text{m}^3} = 1.6 \times 10^{-19} \times 5.91 \times 10^{28} = 9.453 \times 10^9 \text{ C}$$

Given circular cross-sectional radius of 1.00 mm (half of the given diameter), the length of the wire,

$$l = \frac{\text{Volume}}{\pi r^2} = \frac{1\text{m}^3}{\pi(10^{-3})^2} = 3.183 \times 10^5 \text{ m}$$

This the length of gold wire containing $Q_{1\text{m}^3}$ of charge. Therefore 1 C of charge is contained within a length of,

$$l_{1c} = \frac{3.183 \times 10^5}{9.453 \times 10^9} = 3.367 \times 10^{-5} \text{ m} = 3.37 \times 10^{-5} \text{ m (3 sf)}$$

This means that a current of 1 A, corresponding to the flow of 1 C/s is achieved by a flow of electrons at a speed of $3.36 \times 10^{-5} \text{ m/s}$ along a gold wire 2mm in diameter.

c

For a 2.00 mm diameter cross-sectional area:

Speed of electron flow in copper (as derived in Box 1 of the article) is $2.3 \times 10^{-5} \text{ m/s}$

Speed of electron flow in gold (as derived in Box 1 of the article) is $3.36 \times 10^{-5} \text{ m/s}$

Electron flow in gold is $3.36 \div 2.3 = 1.46 = 1.5$ times (2 sf) faster than that in copper.

2

Individual electron velocity, electron drift velocity, and signal velocity are all related to the movement of electrons in a conductor, but they represent different aspects of this movement.

Individual electron velocity (sometimes called *Fermi velocity*) refers to the speed at which each electron is moving, on average, as it travels through the conductor. Within a conductor, each free electron is constantly moving in a straight line under its own momentum while colliding with an atom and changing direction as a result. They then continue moving in a straight line again until the next collision. If a metal wire is left to itself, the free electrons inside constantly fly about and collide into atoms in a random fashion. Macroscopically, we call the random motion of small particles 'heat'. This speed can be quite high, on the order of millions of meters per second, but it is not the same as the electron drift velocity or the signal velocity.

Electron drift velocity, on the other hand, refers to the average speed at which electrons move in a particular direction in a conductor, under the influence of an electric field. This speed is much slower than the individual electron velocity, typically on the order of millimetres per second. The reason for this is that electrons in a conductor are constantly colliding with atoms and other electrons, which slows down their overall movement. The electric current in the wire consists of the ordered portion of the electrons' motion, whereas the random portion of the motion still just constitutes the heat in the wire.

Signal velocity refers to the speed at which an electrical signal or wave travels through a conductor. This velocity depends on the properties of the conductor, and it can be much faster than the electron drift velocity, typically on the order of millions or billions of meters per second. The signal velocity is determined by the interactions between the electromagnetic fields created by the signal and the conductor.

In general, the signal velocity is somewhat close to the speed of light in vacuum, the individual electron speed is about 2 orders of magnitude slower than this and the electron drift velocity is very small (few mm/s)

Physics online: Upthrust

1 a

Total mass of wooden block containing the lead,

$$M_{\text{block with lead}} = (\rho_{\text{wood}} \times \text{Volume}_{\text{block without lead}}) + (\rho_{\text{lead}} \times \text{Volume}_{\text{lead}})$$

$$M_{\text{block with lead}} = 0.500 \times (32.00 - 2.00) + 11.0 \times 2.0$$

$$M_{\text{block with lead}} = 37.0 \text{ g (3sf)}$$

b

New density of wooden block containing the lead, $\rho_{\text{block with lead}}$

$$\rho_{\text{block with lead}} = \frac{M}{\text{Volume}_{\text{block with lead}}}$$

$$\rho_{\text{block with lead}} = \frac{37}{32} = 1.156 = 1.16 \text{ g cm}^{-3} \text{ (3 sf)}$$

c

As the $\rho_{\text{block with lead}} > \rho_{\text{water}}$, the block will sink.

d

For the block to just about float/sink, its density will need to equal that of water.i.e.

$$\rho_{\text{block with lead}} = \rho_{\text{water}} = 1.00 \text{ g cm}^{-3}$$

As density = mass/Volume:

$$1.00 = \frac{\text{Required mass of wooden block}}{\text{Volume of the wooden block}} = \frac{M_{\text{new block with lead}}}{\text{Volume}_{\text{new block with lead}}}$$

$$1.00 = \frac{M_{\text{new block with lead}}}{32.00}$$

$$M_{\text{new block with lead}} = 32.00 \text{ g}$$

Using the equation in part a of the question:

$$M_{\text{new block with lead}} = (\rho_{\text{wood}} \times \text{Volume}_{\text{block without lead}}) + (\rho_{\text{lead}} \times \text{Volume}_{\text{lead in hole}})$$

$$32.00 = 0.5 \times (32.00 - \text{Volume}_{\text{lead in hole}}) + (11.0 \times \text{Volume}_{\text{lead in hole}})$$

$$32.00 = 0.5 \times (32.00 - \text{Volume}_{\text{lead in hole}}) + (11.0 \times \text{Volume}_{\text{lead in hole}})$$

$$16.00 = 10.5 \times \text{Volume}_{\text{lead in hole}}$$

$$\text{Volume}_{\text{lead in hole}} = 1.5238 = 1.52 \text{ cm}^3 \text{ (3 sf)}$$

Thus, a 1.52 cm^3 hole drilled (and filled with lead) into the original wooden block will result in a state where the block will just about sink/float on water.

2

a

Percentage volume of ice cube submerged in fresh water:

$$= \frac{\text{Density of ice}}{\text{density of fresh water}}$$

$$= \frac{0.90}{1.00} = 90\% \text{ (2sf)}$$

b

Percentage volume of ice cube submerged in sea water:

$$= \frac{\text{Density of ice}}{\text{density of sea water}}$$

$$= \frac{0.90}{1.30} = 69.23\% = 69\% \text{ (2sf)}$$

c

The iron sphere will sink because the density of water is less than the density of iron. This would mean the weight of the iron sphere would be more than the upthrust caused by water.

The density of water being more than the density of wood, the weight of the wooden sphere does not sink but floats with a volume immersed in water that is balanced by the upthrust due to water.

d

Volume of the ice cube can be calculated as:

$$V_{\text{cube}} = (\text{side length})^3$$

$$V_{\text{cube}} = (40 \text{ cm})^3 = 64000 \text{ cm}^3$$

Volume of the ice cube submerged in fresh water can then be found as:

$$V_{\text{cubeInFW}} = \text{Percentage volume of ice cube submerged in fresh water} \times V_{\text{cube}}$$

$$V_{\text{cubeInFW}} = 90\% \times V_{\text{cube}}$$

$$V_{\text{cubeInFW}} = 0.90 \times 64000 \text{ cm}^3$$

$$V_{\text{cubeInFW}} = 57600 \text{ cm}^3$$

The mass of the ice cube submerged in fresh water is calculated as:

$$m_{\text{cubeInFW}} = \rho_{\text{FW}} \times V_{\text{cubeInFW}}$$

$$m_{\text{cubeInFW}} = 1.00 \text{ g cm}^{-3} \times 57600 \text{ cm}^3 = 57600 \text{ g} = 57.6 \text{ kg}$$

Upthrust will equal the weight of the displaced fresh water:

$$U_{\text{cubeInFW}} = W_{\text{cubeInFW}}$$

$$U_{\text{cubeInFW}} = 57.6 \times 9.81 = 576.056 = 576 \text{ N (3 sf)}$$

e

Volume of the ice cube submerged in sea water,

$$V_{\text{cubeInSW}} = \text{Percentage volume of ice cube submerged in sea water} \times V_{\text{cube}}$$

$$V_{\text{cubeInSW}} = 69\% \times V_{\text{cube}}$$

$$V_{\text{cubeInSW}} = 0.69 \times 64000 \text{ cm}^3$$

$$V_{\text{cubeInSW}} = 39744 \text{ cm}^3$$

The mass of the cube submerged in fresh water is calculated as:

$$m_{\text{cubeInSW}} = \rho_{\text{SW}} \times V_{\text{cubeInSW}}$$

$$m_{\text{cubeInSW}} = 1.30 \text{ g cm}^{-3} \times 39744 \text{ cm}^3 = 51667.2 \text{ g} = 51.6672 \text{ kg}$$

Upthrust will equal the weight of the displaced sea water:

$$U_{\text{cubeInSW}} = W_{\text{cubeInSW}}$$

$$U_{\text{cubeInSW}} = 51.6672 \times 9.81 = 506.855 = 507 \text{ N (3 sf)}$$

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