Page 133 Test yourself on prior knowledge

1 The total distance travelled is: (40 km h^{-1} × 2 h) + (60 km h^{-1} × 4 h) = 320 km

Average speed $= \frac{320 \text{ km}}{6 \text{ h}}$

= 53 km h^{-1}

2 Acceleration $= \frac{\text{change of velocity}}{\text{time}}$

$= \frac{30 \text{ m s}^{-1} - 12 \text{ m s}^{-1}}{6 \text{ s}}$

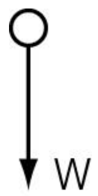
= 3 m s^{-2} (or you might write - 3 m s^{-2} to show a deceleration).

3 54 km h^{-1} = $\frac{54000 \text{ m}}{3600 \text{ s}}$ = 15 m s^{-1}

4 v_h = 12 cos 35° = 9.8 m s^{-1}

v_v = 12 sin 35° = 6.9 m s^{-1}

5 The only force is the ball's weight – the pull of gravity. It is a common mistake to think that there is a forwards force; there is not. You should have drawn the ball with a downwards arrow showing the weight.



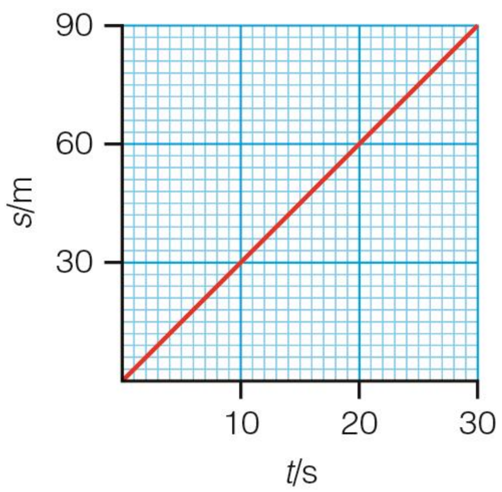
Page 135 Test yourself

1 a) A moving object gradually slows down and stops.

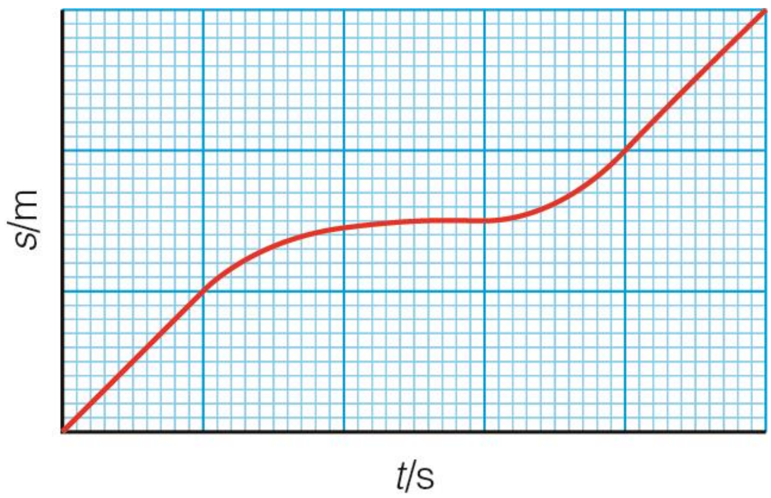
b) An object moves at a constant velocity in one direction; then instantaneously reverses direction and moves at a much greater constant speed, going an equal distance beyond its starting point; it then moves in the original direction at the original constant velocity.

c) An object moves at 2 m s^{-1} for 10 s, then increases its velocity to 8 m s^{-1} for 5 s.

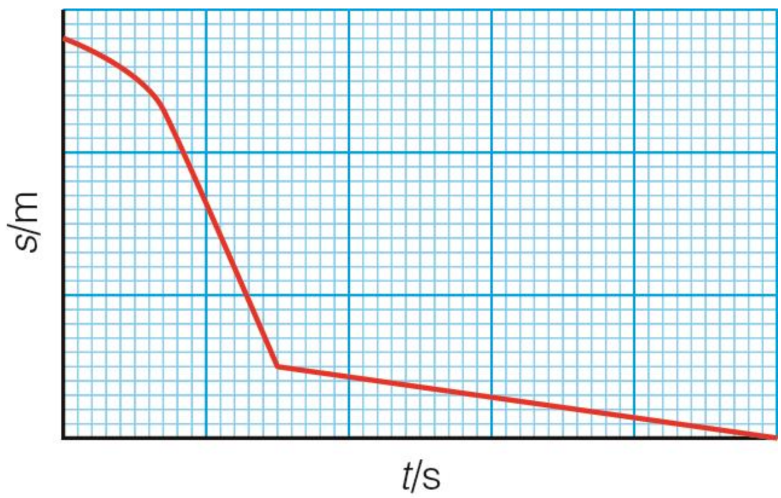
2 a)



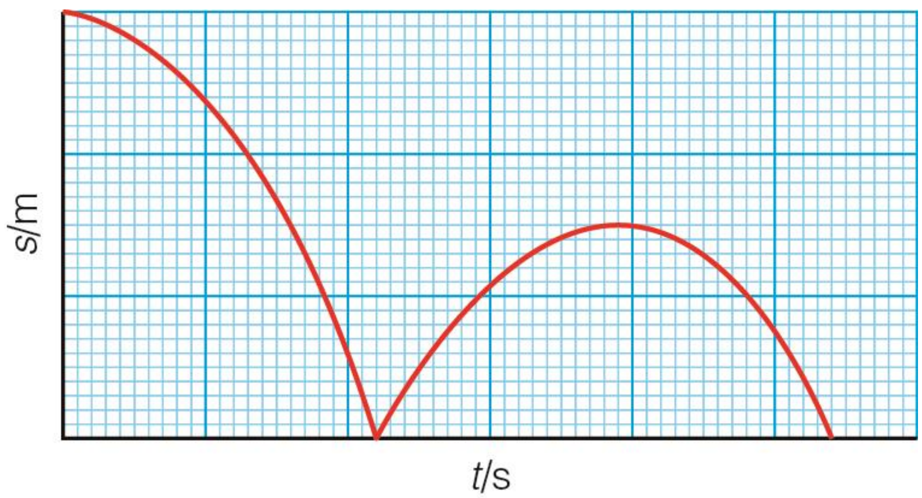
b)



c)

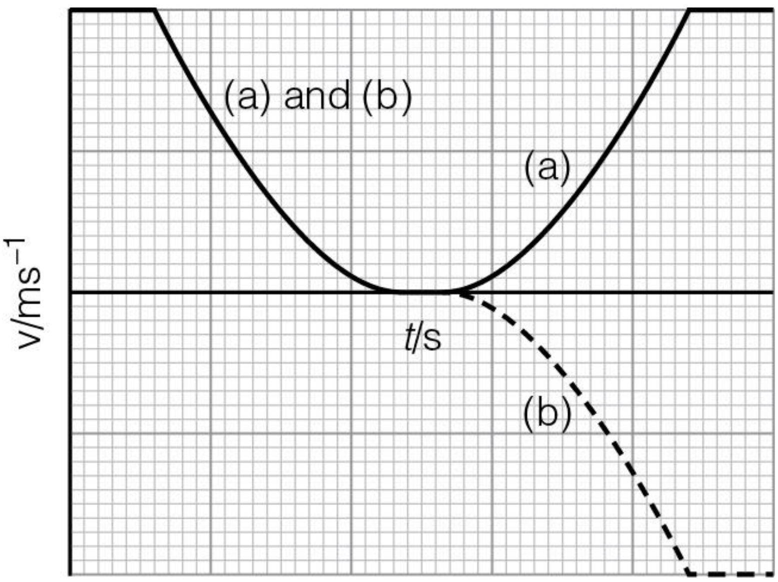


d)



Page 136 Test yourself

3 a) and b)



4 a) acceleration = $\frac{v-u}{t}$

$= \frac{120 \text{ m s}^{-1}}{170 \text{ s}}$

$= 0.71 \text{ m s}^{-2}$

b) distance = speed × time

$= 120 \text{ m s}^{-1} \times 50 \text{ s}$

= 6 000 m or 6 km

c) distance = area under the graph

$= (½ \times 120 \text{ m s}^{-1} \times 170 \text{ s}) + 6\,000 \text{ m} + (½ \times 120 \text{ m s}^{-1} \times 220 \text{ s})$

= 29 400 m or 29.4 km

d) average speed $= \frac{\text{distance}}{\text{time}}$

$= \frac{29\,400 \text{ m}}{440 \text{ s}}$

$= 66.8 \text{ m s}^{-1}$ or 241 km h^{-1}

5 a) The gradient is equal to the acceleration of the rocket. As the rocket loses mass, the acceleration increases.

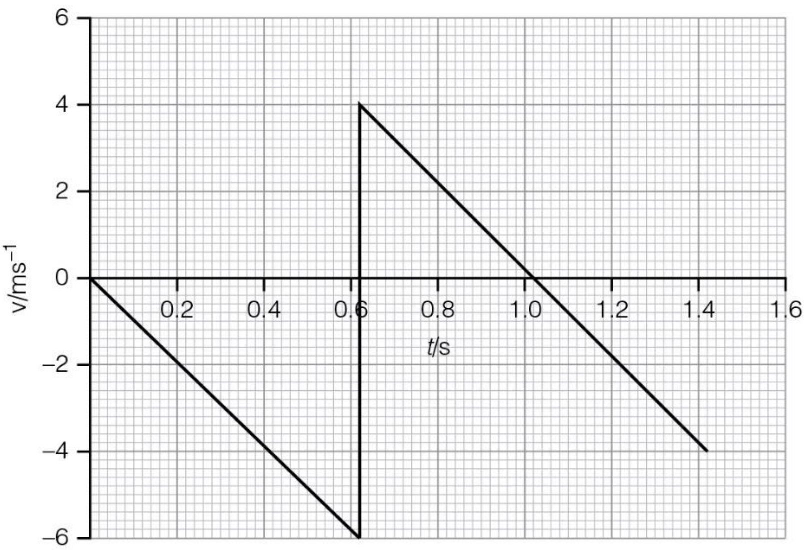
b) C

c) This is because the only force acting is the pull of gravity; this is 10 N kg^{-1} which causes a deceleration of 10 ms^{-2}.

d) The falling rocket reaches its terminal speed, when the drag force balances the pull of gravity (its weight). Since the velocity is constant, the acceleration is zero, and the gradient is zero.

e) The area above the line is the displacement upwards; the area below the line is the displacement downwards. These are equal.

6



Page 139 Test yourself

7 a) $v = u + at$

$= 0 + 1.8\ \text{m s}^{-2} \times 40\ \text{s}$

$= 72\ \text{m s}^{-1}$

b) $s = 0 + \frac{1}{2} a t^2$

$= \frac{1}{2} \times 1.8\ \text{m s}^{-2} \times (40\ \text{s})^2$

$= 1440\ \text{m}$

8 $v^2 = u^2 + 2as$

$\Rightarrow 30^2 = 22^2 + 2a \times 120$

$\Rightarrow 30^2 - 22^2 = 240a$

$\Rightarrow a = 1.73\ \text{m s}^{-2}$

9 a) $a = \Delta v/\Delta t = 55\ \text{m s}^{-1}/3.6\ \text{s} = 15.3\ \text{m s}^{-2}$

b) s = average speed × time = $\frac{1}{2} \times 55\ \text{m s}^{-1} \times 3.6\ \text{s} = 99\ \text{m}$

10 s = average speed × time

= ½ (50 + 30) m s^{-1} × 180 s

= 7200 m

OR

s = ut + ½ a t^2

= 50 m s^{-1} × 180 s - ½ {(- 20 m s^{-1})/ (180 s)} × $(180\ s)^2$

= 9000 m – 1800 m

= 7200 m

11 a) The cars are level when their displacements are again equal:

i.e. when s_1 = ½ a_1t^2 = s_2= u_2t

so when ½ a_1t = u_2

½ × 2 m s^{-2} × t = 20 m s^{-1}

t = 20 s

b) v = u + at

= 0 + 2 m s^{-2} × 20 s

= 40 m s^{-1}

c) s_1 = ½ at^2

= ½ × 2 m s^{-2} × $(20\ s)^2$

= 400 m

12 Take down as positive and use s = ut + ½ a t^2

-10 = -20 t + ½(9.8) t^2

4.9 t^2 – 20 t + 10 = 0

$t = \frac{20 \pm \sqrt{20^2 - 4 \times 4.9 \times 10}}{2 \times 4.9}$

$t = \frac{20 \pm 14.3}{9.8}$

t = 0.6 s or 3.5 s

Page 140 Required practical 3

Determination of *g* by a free-fall method

1 Experiment 2

The velocity of the ball as it falls through gate A: $v_A = \frac{0.0132\text{ m}}{0.0067\text{ s}}$ = 1.97 m s^{-1}

The velocity of the ball as it falls through gate B: $v_B = \frac{0.0132\text{ m}}{0.0031\text{ s}}$ = 4.26 m s^{-1}

$$g = \frac{v_B - v_A}{t}$$

$$= \frac{4.26\text{ m s}^{-1} - 1.97\text{ m s}^{-1}}{0.2197\text{ s}}$$

= 10.4 m s^{-2}

Experiment 3

The speed of the ball as it falls through gate A: $v_A = \frac{0.0132\text{ m}}{0.0097\text{ s}}$ = 1.36 m s^{-1}

The speed of the ball as it falls through gate B: $v_B = \frac{0.0132\text{ m}}{0.0052\text{ s}}$ = 2.54 m s^{-1}

$$g = \frac{v_B - v_A}{t}$$

$$= \frac{2.54\text{ m s}^{-1} - 1.36\text{ m s}^{-1}}{0.1212\text{ s}}$$

= 9.7 m s^{-2}

2 s = ½ (u+ v)t

Inserting the numbers: s_1 = 1.00 m; s_2 = 0.68 m; s_3 = 0.24 m

3 The light gates measure times, and allow the speed at the start and end of a time interval to be calculated (given the diameter of the ball). Since acceleration is the change speeds divided by time, the distance travelled in that time is not required.

4 If you drop the ball from a greater height, the ball travels faster. This does not matter as the computer measures the speeds and the time interval (but see below).

5 Here are two possible sources of error:

- A significant error can occur in the timing. It looks really accurate when the computer records a time of 3.1 ms – but it could have been 3.2 ms or 3.0 ms. This gives an error of 3%. The error can be reduced by dropping the ball from a smaller distance above the first light gate (so it is travelling more slowly through the first gate) and putting the gates closer together (so it has less time to accelerate between the gates, meaning it is travelling more slowly through the second gate) making the time to pass through each gate longer and reducing the error in these measurements. (Moving the gates closer will increase the error in the time to travel from one gate to another, but this is >0.1% in all examples given – much less than that for the times used to calculate speeds.)

- A further error is introduced if the ball does not fall accurately though the centre of each light gate. This can be reduced by careful alignment of equipment (e.g. using a level and plumbline or/and dropping the ball through a clear tube). Although the use of electronic equipment and computers speeds up the process of calculating 'g', care still has to be taken to ensure an accurate outcome to the experiment.

6 The average acceleration is:

a = ⅓ (9.7 + 9.8 + 10.4)

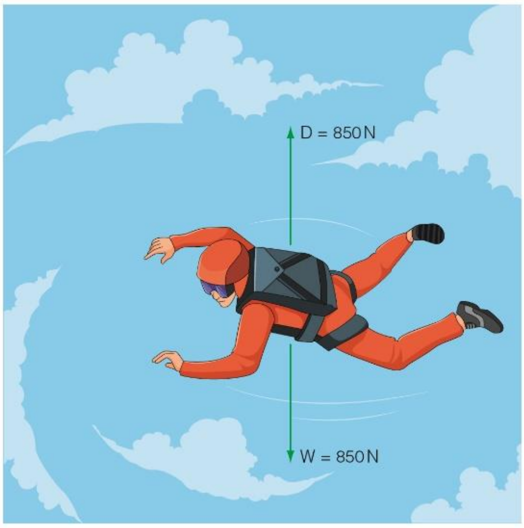
= 10.0 $\pm$ 0.4 m s^{-2}

This is a percentage uncertainty of about 4%.

If you are unsure about calculating uncertainties, refer to the chapter on practical physics.

Pages 141–142 Test yourself

13 a)



b) When the parachutist first begins to fall, the drag increases with speed and is initially less than their weight, so they accelerate until they reach their first terminal velocity.
When they first open the parachute, the drag suddenly increases (due to the greater surface area of the parachute) so the overall force is upwards.
The parachutist decelerates and, as they do so, the drag force decreases until it is once again equal to the weight.
The parachutist will then again travel at a constant velocity, but this will be less than the one at which they were travelling before the parachute opened.

14 a) As the ping-pong ball increases its speed, drag increases; the drag becomes comparable to the ball's weight, so the rate of acceleration decreases.
The drag on the steel ball is always much less than its weight, so it accelerates at a rate close to g.
The average speed of the steel ball is greater, so it hits the ground first.

b) Ignoring the effect of air resistance, the steel ball takes 1 s to reach the ground:

$s = ½ gt^2$

$5 = ½(9.8)t^2$

$t = 1.0$ s

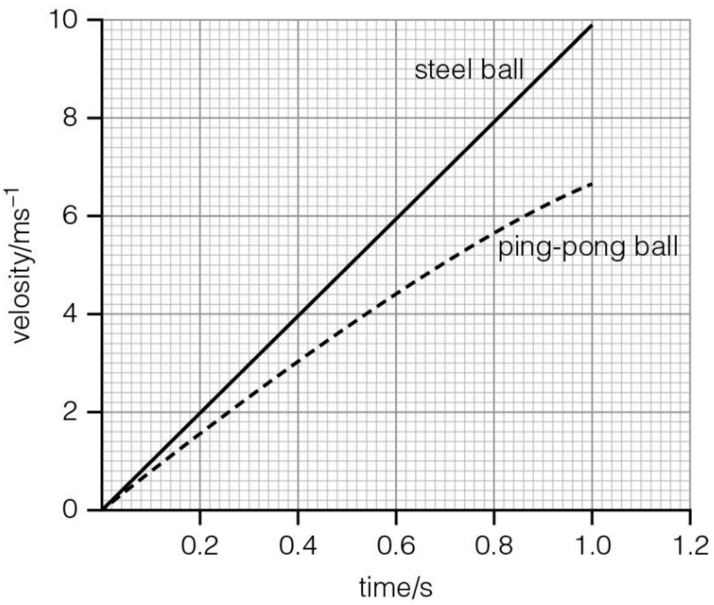
And is travelling at a speed of

$v^2 = u^2 + 2as$

$v^2 = 2 \times 9.8 \times 5$

$v = 9.9$ ms^{-1}

When sketching the graph, do not forget to add times and speeds.

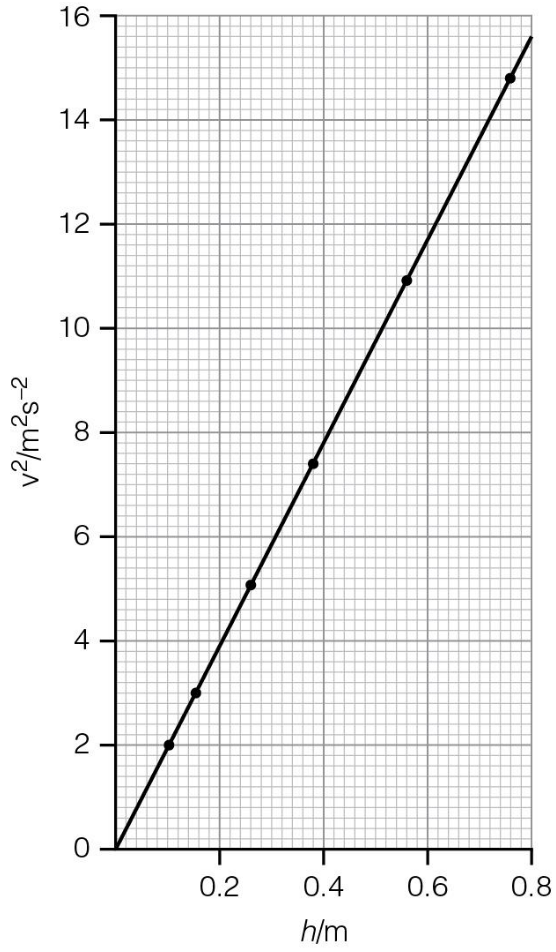


15 a)

Height fallen: h/m	Speed: v/m s $^{-1}$	v^2/m^2s^{-2}
0.10	1.40	1.96
0.15	1.72	2.96
0.26	2.26	5.10
0.38	2.73	7.45
0.55	3.28	10.76
0.75	3.84	14.75

15 b) $v^2 = 2gh$

$\Rightarrow$ gradient of graph with h on *x*-axis and v^2 on *y*-axis = 2g



gradient = 10/0.51 = 19.6 m s^{-2}

$\Rightarrow$ g = 9.8 m s^{-2}

16 a) a $= \frac{\Delta v}{\Delta t}$

a $= \frac{200 \text{ m s}^{-1}}{20 \text{ s}}$

= 10 m s^{-2}

b) As Felix increased his speed the drag forces increased. Initially the drag was low because the atmosphere is of very low density at high altitude. But as he fell, the density of the air increased and this also contributed to increased drag. At C, he reached a terminal, when the drag forces balanced his weight.

c) The density of the atmosphere affects the terminal speed. As Felix reached lower altitudes the density of the air increased and the drag for a given speed became larger. So the speed at which his weight was balanced by drag – the terminal speed – kept decreasing.

d) Each square of the graph has an 'area' of 50 m s^{-1} × 20 s = 1000 m (1 km). The area under the graph is about 35 squares, or 35 km.

e) Average speed $= \frac{d}{t}$

$$= \frac{35\,000\text{ m}}{260\text{ s}}$$

= 135 m s^{-1} (485 km h^{-1})

f) Felix's maximum speed was about 350 m s^{-1}.

So 350 = 1.2 v_s

v_s = 292 m s^{-1}.

The speed of sound depends on the density of the air, which is lower at high altitudes.

Pages 144–145 Activity

Projectile paths

1 The balls fall at the same rate – and therefore take the same time to reach the ground – regardless of their initial sideways speed. There is no sideways force, only their weight which accelerates them downwards.

2 Consider the horizontal motion of Ball B as shown by B_1 and B_2:

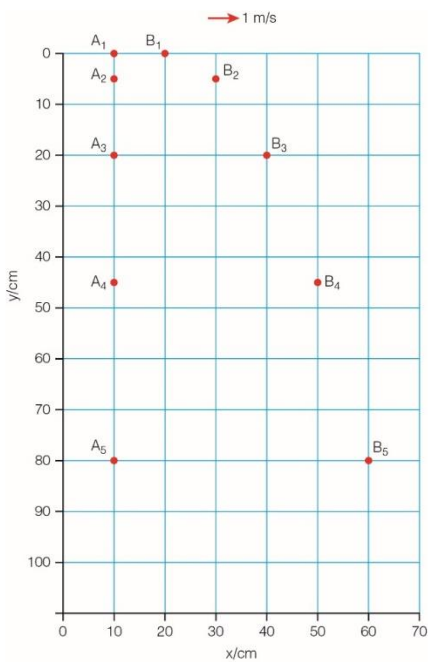
$t = \frac{d}{v}$

$$= \frac{0.1\text{ m}}{1\text{ m s}^{-1}}$$

= 0.1 s

3 B_3 = (40, 20)

B_4 = (50, 45)



4 $v_v = u + gt$

$= 0 + 9.8 \text{ m s}^{-2} \times 0.4 \text{ s}$

$= 3.9 \text{ m s}^{-1}$

5 The vertical component of the velocity is the same as that of ball A at this time, and the horizontal component is still 1ms^{-1}

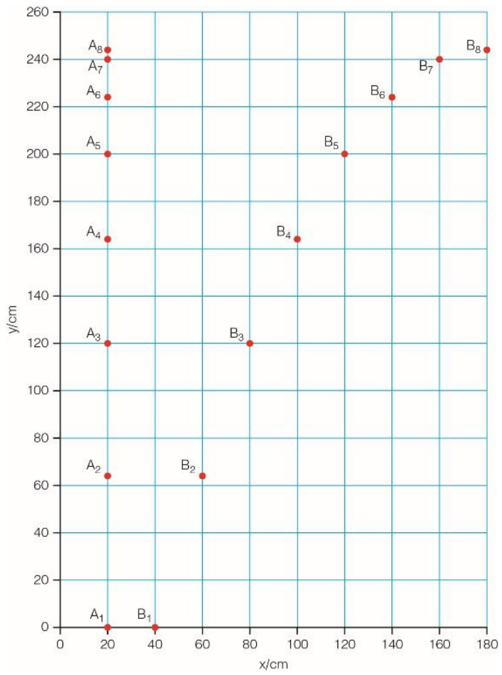
$v = (3.9^2 + 1^2)^{1/2} \text{ m s}^{-1}$

$= 4.0 \text{ m s}^{-1}$

It is travelling at an angle θ to the vertical where $\tan \theta = 1/3.9 = 0.256$

$\Rightarrow \theta = 14°$

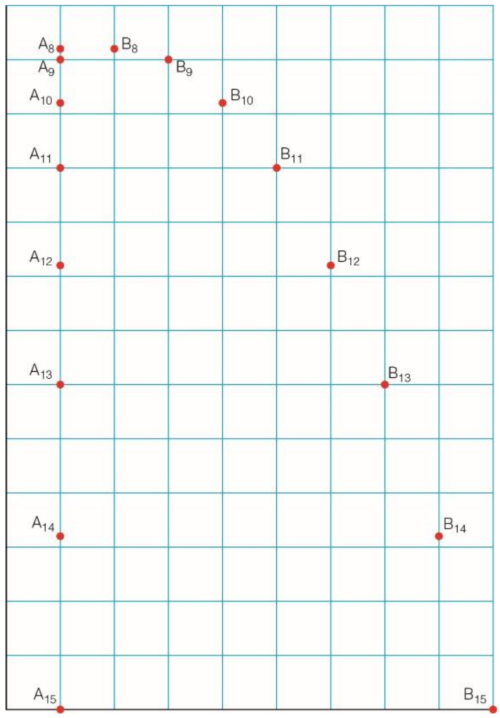
6 $B_4 = (100, 165)$
$B_5 = (120, 200)$
$B_6 = (140, 225)$
$B_7 = (160, 240)$
$B_8 = (180, 245)$



7 **a)** The velocity of A at $A_8 = 0$

b) The velocity of B at B_8 is 2 m s^{-1} horizontally to the right (the vertical component of velocity is zero).

8



Page 145 Test yourself

17 Statement c) is correct: the pull of gravity depends only on the mass of the object (and distance from the centre of the earth)

a) is false because horizontal and vertical motion are independent.

b) is incorrect – it continues moving forward because there is no force to stop it doing so (or until the deceleration produced by such a force reduces its horizontal speed to zero).

18 a) $t = \frac{d}{v} = \frac{160\ \text{m}}{800\ \text{m s}^{-1}} = 0.2\ \text{s}$

b) $s = ut + \frac{1}{2} g t^2$

$= 0 + \frac{1}{2}\ 9.8\ \text{m s}^{-2} \times (0.2\ \text{s})^2$

$= 0.20\ \text{m}$

c) i) The distance to the target, and therefore the time taken, is three times greater.

The distance the bullet falls is proportional to the square of the time taken, so it increases by a factor of $3^2 = 9$ to $9 \times 0.20\ \text{m} = 1.8\ \text{m}$

ii) The sights must be adjusted to take account of the additional distance the bullet will fall.

19 Considering the vertical motion of the ball: $s = \frac{1}{2} gt^2$

$1.3 \text{ m} = \frac{1}{2} \times 9.8 \text{ ms}^{-2} \times t^2$

$t^2 = 1.3/4.9$

$t = 0.51 \text{ s}$

Considering horizontal motion: $d = 30 \text{ ms}^{-1} \times 0.51 \text{ s} = 15 \text{ m}$

Pages 146–149 Practice questions

1 C

2 A

3 B

4 C

5 C

6 B

7 A

8 B

9 A

10 C

11 a) The real ball does not travel as far horizontally as the ideal ball, nor does it go as high.

The descent of the real ball is much steeper than that of the ideal ball.

b) The drag has both vertical and horizontal components.

It therefore causes the ball to decelerate horizontally, so reducing the range.

And increases the downward force on the ball, so reducing the height it reaches.

12 a) $v^2 = u^2 + 2as = 2gs$

$v = (2 \times 9.8 \text{ m s}^{-2} \times 19 \text{ m})^{\frac{1}{2}} = 19 \text{ m s}^{-1}$ (2 sig figs)

b) $s = u t + \frac{1}{2} g t^2$

Taking up as positive: $s = 19 \times 3 - \frac{1}{2} \times 9.8 \times 3^2 = 13$ m (2 sig figs)

c) The effects of drag have been ignored: the assumption made is that there is no drag.

13 a) i) $a = \frac{v-u}{t} = \frac{10-0}{1}$ [1]

$a = 10 \text{ m s}^{-2}$ downwards [1]

ii) u = 10 ms^{-1}, v = –8 ms^{-1}, t = 0.05 s (values from graph with correct signs) [1]

a $= \frac{-8\ \text{m s}^{-1} - 10\ \text{m s}^{-1}}{0.05\ \text{s}}$ [1]

= 360 m s^{-2} upwards (as down is positive)[1]

b) i) d = area under the graph = $\frac{1}{2}$ × 10 m s^{-1} × 1 s [1]

= 5 m [1]

ii) d = area under the graph = $\frac{1}{2}$ × 8 m s^{-1} × (2.7 – 1.8)s [1]

= 3.6 m [1]

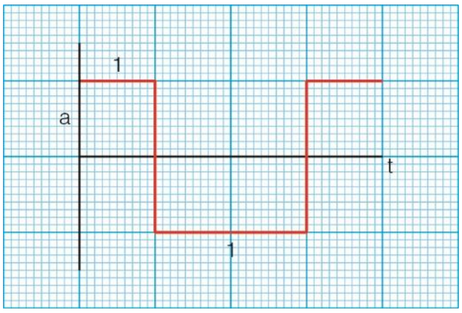
14 a) i) The gradient gives the acceleration [1]

which is positive, then negative, then positive. [1]

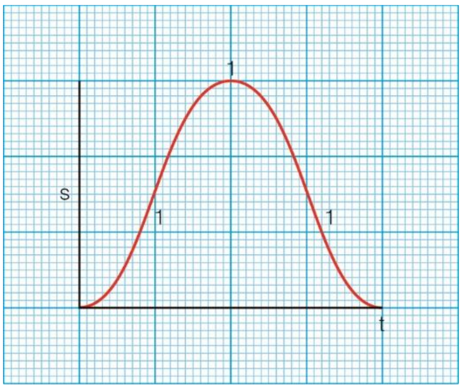
ii) Any two points: [2]

- The area under the graph gives displacement
- so for the first half of the movement, displacement increases (area above the axis) and when the velocity is negative, it decreases
- Since the area between the line and axis is the same in both cases, the object returns to its starting place: the displacement is zero by the end of the cycle.

b) Numbers on graph indicate mark allocation



c) Numbers on graph indicate mark allocation



15 a) a $= \frac{v-u}{t}$ [1]

$= \frac{20\ \text{m s}^{-1}}{2\ \text{s}} = 10\ \text{m s}^{-2}$ [1]

b) i) When v = 0 the jumper begins to change direction 3.2 s [1]

ii) s = area under the graph ≈ 40 m [1]

iii) The 'area' CDE is less than. [1]

c) i) At point B, the graph is horizontal so a = 0 [1]

16 a) s $= ut + \frac{1}{2}gt^2$

$93\ \text{m} = 0 + \frac{1}{2} \times 9.8\ \text{m s}^{-2} \times t^2$

$\Rightarrow t = (93/4.9)^{1/2}\ \text{s} = 4.36\ \text{s}$ i.e. 4.4 s to 2 sf.

b) $v_h = \frac{1250\ \text{m}}{4.4\ \text{s}}$

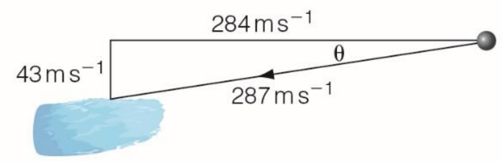
$v_h = 284\ \text{m s}^{-1}$

c) $v_v = u + at$

$v_v = 0 + 9.8\ \text{ms}^{-2} \times 4.4\ \text{s} = 43\ \text{ms}^{-1}$

d) v $= (284^2 + 43)^{1/2} = 2897\ \text{ms}^{-1}$

$\tan\theta = 43/284 = 0.15 \Rightarrow \theta = 8.6^{\circ}$



Suitable scale [1]; Arrows in correct positions [1]; Angle measured [1]

e) To increase the range of the ball, the barrel of the cannon is tilted upwards so that the ball goes upwards slightly so taking longer to reach sea level and thus travelling a greater horizontal distance.

17 a) The average values are: 0.14 s; 0.20 s; 0.25 s; 0.29 s; 0.32 s; 0.34 s.

b) i) about 7% – 1 part in 14 [1]

ii) about 3% – 1 part in 34 [1]

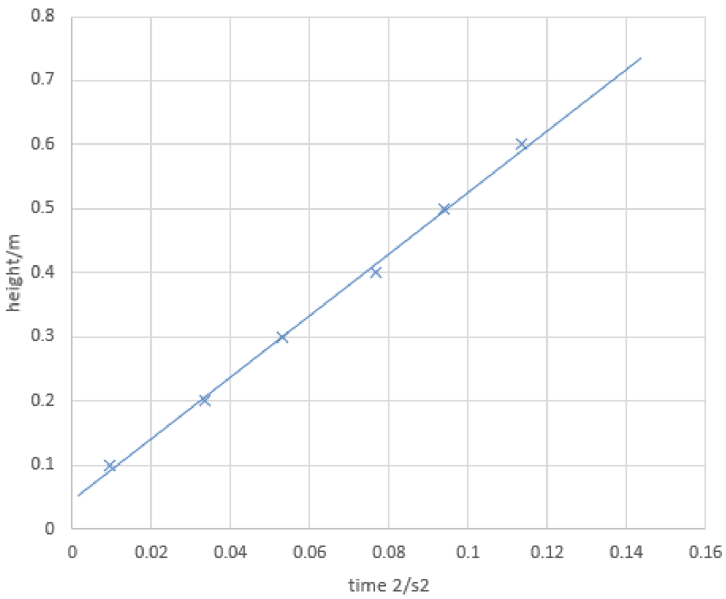
(Though errors are reduced by repeating the measurements.)

c) This could be as low as 1 part in 100, but there are likely to be errors of ± 1 mm at either end giving an error of 2%. [1]

d) Any two points:

- It is unlikely that the ball bearing will be in exactly the same place each time it is released
- so it may have a small downwards speed as it passes the first gate.
- Since this changes during each measurement, it introduces a random error.

e) $s = \frac{1}{2}gt^2$ so the gradient of the graph is ½ g. [1]



Graph plotted on sensible scales [1]; Good line of best fit drawn [1]; Gradient determined using suitable points and used to determine g (about 9.9 ± 0.5 ms^{-2}) [1]; The final error is quite hard to judge as it is related to the uncertainty in the gradient of the graph. [1]

18 a) $s = \frac{1}{2}gt^2$ [1]

$= \frac{1}{2} \times 9.8 \text{ m s}^{-1} \times (3.7 \text{ s})^2$ [1]

$= 67$ m [1]

b) $s = v \times t$ [1]

$= 54 \text{ m s}^{-1} \times 3.7 \text{ s} = 200$ m (2 sig figs) [1]

c) i) $v_v = u + at = gt$ [1]

$= 9.8 \text{ m s}^{-2} \times 3.7 \text{ s} = 36 \text{ m s}^{-1}$ [1]

ii) $v_{mag} = (54^2 + 36^2)^{\frac{1}{2}} \text{ m s}^{-1}$ [1]

$= 65 \text{ m s}^{-1}$ [1]

Pages 149–150 Stretch and challenge questions

19 When the boat catches the sandwich box, they have the same displacement (after time t).

Displacement of box = ut

u = river speed; t = time for the boat to catch the box.

v = speed of boat relative to the water

Taking the direction of flow of the river as positive,

displacement of boat at time t = (u – v) × 10 + (u + v)(t – 10) = ut + vt – 20v

When they meet, the displacement of the boat equals the displacement of the lunch box.

So ut = ut + vt – 20v

$\Rightarrow$ 20 v = v t

t = 20 min (= $\frac{1}{3}$ h)

So 1 km = u × $\frac{1}{3}$ h

u = 3 km h^{-1}

20 Time to move from A to B is:

$$t = \frac{L - x}{3} + \frac{(x^2 + y^2)^{\frac{1}{2}}}{1}$$

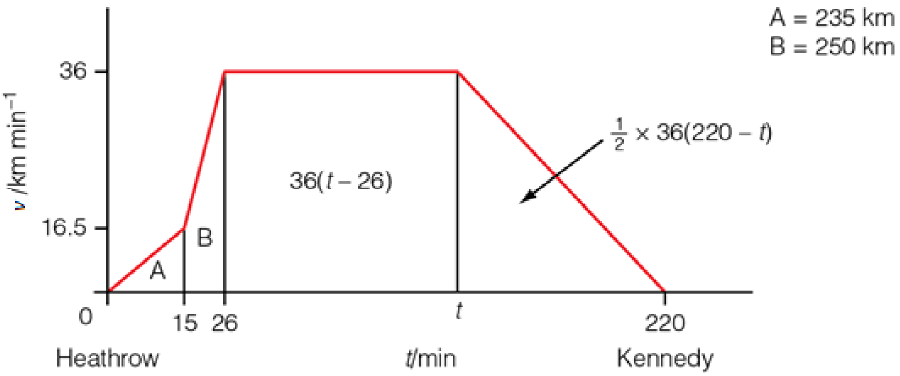
$$\frac{dt}{dx} = -\frac{1}{3} + \frac{\frac{1}{2}.2x}{(x^2 + y^2)^{\frac{1}{2}}}$$

The minimum time occurs when $\frac{dt}{dx} = 0 \Rightarrow \frac{1}{3} = \frac{x}{(x^2 + y^2)^{\frac{1}{2}}}$

but $\frac{x}{(x^2 + y^2)^{\frac{1}{2}}} = \sin\theta$

So minimum time is when $\theta = \sin^{-1}(1/3)$

21 By 26 minutes the distance travelled is 235 km + 250 km = 485 km. (This is the area marked A and B on the sketched graph. The exact shape of these areas does not matter – the important point in solving the problem is that the plane has travelled 485 km after 26 minutes.)
There are 5800 km – 485 km = 5315 km remaining to travel to Kennedy airport.
This distance, 5315 km, is the area under the graph from 26 minutes to 220 minutes.
If the plane begins to decelerate after t minutes, the velocity time graph looks like this (velocity in km min^{-1})



Working with the speed in km min^{-1}: $v = \frac{2160 \text{ km}}{60 \text{ min}}$ = 36 km min^{-1}

From the graph:

$5315 = 36(t - 26) + \frac{1}{2} . 36(220 - t)$

$\Rightarrow 5315 = 36t - 936 + 3960 - 18t$

$\Rightarrow 18t = 2291$

t = 127.3 min

So the plane begins to decelerate after 127.3 min.

12.30 p.m. is 120 min after take off and, at this time, the plane is still travelling at 36 km min^{-1}.

The distance travelled between 26 min and 120 min is:

d = 36 km min^{-1} × (120 – 26) min = 3384 km

So the total distance from Heathrow at 12.30 pm = 3384 km + 485 km = 3869 km