Page 95 Test yourself on prior knowledge

1 Time period = 1/50 Hz = 0.02 s

2 Frequency = 1/ 2 x 10^{-3} s = 500 Hz

3 One cycle has a phase difference of 2π, so the phase change here is $2\pi \times 2.75 = 5.5\pi$, which reduces to a phase difference of 1.5π.

4 The particles in both parts of the wave move at the same speed in the same direction.

5 The size of the hill is the same order of magnitude as the radio waves so the waves can diffract around the hill.

6 Sound diffracts around the corner as its wavelength is a similar order of magnitude to the size of the building. However, the wavelength of light is many times smaller which means that it does not spread around the corner.

7 Speed = 3×10^3 m × 100×10^3 Hz = 3×10^8 m s^{-1}

Page 101 Test yourself

1 **a)** The interference is constructive as the path difference is a whole number of wavelengths (and the waves were in phase to start with).

b) The interference is destructive. The path difference from the aerial is a whole number of wavelengths, so the waves are out of phase – as they were at the aerial.

c) Destructive interference because the path difference is an odd number of half wavelengths.

d) Constructive interference. The path difference from the speaker is a whole number of wavelengths plus a half wavelength, so the waves are in phase when they reach the microphone.

2 Superposition and interference both occur when waves of the same type travelling in the same medium overlap and their displacements add up.
Interference is a special case of superposition that occurs when the waves are coherent. (It produces an interference pattern – bright and dark fringes of light, or loud and quiet regions of sound).

3 Coherent waves have a fixed phase difference, the same frequency and are of the same type; e.g. two sources of light of wavelength 600 nm with a phase difference of 180°.

4 **a)** He detects a signal that varies in intensity at regular intervals. It is strong in regions of constructive interference and is weakest in regions of destructive interference. The maxima are separated by a distance of 10 m.

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{1\times10^9} = 0.3m$$

$$w = \frac{\lambda D}{s} = \frac{0.3\times200}{6} = 10m$$

b) The position of the maxima shift by 10 m ÷ 4 = 2.5 m
If he walks one way, he will reach a maximum after 2.5 m, but if he goes the other way, he will need to walk 7.5 m to find a maximum.

5 a) Laser light is monochromatic and intense so clear fringes are visible.

b) w = $\frac{\lambda D}{s} = \frac{(600\times10^{-9}\text{ m}\times 3.0\text{ m})}{(2\times10^{-4}\text{m})} = 9.0\text{mm}$

c) Increase the distance between slits and screen, increase the wavelength of light or reduce the separation of the slits.

6 $w = \frac{\lambda D}{s} = \frac{(0.028\text{ m})\times(0.30\text{ m})}{(0.058\text{ m})} = 0.14\text{ m}$

7 $s = \frac{\lambda D}{w}$

There are 5 fringe spacings between 6 bright fringes so the width of one fringe is 3×10^{-3} m ÷ 5 = 6×10^{-4} m

So s = $\frac{(600\times10^{-9}\text{ m}\times0.36\text{ m})}{(6\times10^{-4}\text{m})} = 0.36\text{mm}$

8 The string is at A at its equilibrium position for one second then both waves arrive in phase and the string starts moving down. It reaches a maximum negative displacement of 2m at 1.5s. The string then moves up through the equilibrium position, reaching its maximum positive displacement of 2m at 2.5s. The string then moves down again for 1s, then up for 1s, then down for 0.5s . It reaches the equilibrium position after 5s.

The wave from the right arrives at B after 0.5 s and the string begins to move down. The wave from the left arrives after 1.5s out of phase with the wave from the right. The string returns to, and remains at its equilibrium position while the wave from the left and the wave from the right superpose. After 4.5 s, only the wave from the left is passing point B, and the string moves up and then reverts to the equilibrium position at 5s

Pages 105–106 Required practical 1

Investigation into the variation of the frequency of stationary waves on a string with length, tension and mass per unit length of the string

1 Passing the string over the pulley removes friction, thus ensuring that the tension in the string is the same as the weight.

2 Average readings are: 138 Hz; 198 Hz; 240 Hz; 279 Hz; 310 Hz; 340 Hz; 367 Hz; 391 Hz

3 Using the equation to calculate the frequency for a mass of 0.20 kg:

$$T = mg = 0.20 \times 9.8 = 1.96\, N$$

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{2\times0.40}\sqrt{\frac{1.96}{0.16\times10^{-3}}} = 138\text{ Hz}$$ = 140 Hz (2sf)

Working in the same way, the frequencies for the remaining lengths are: 200 Hz; 240 Hz; 280 Hz; 310 Hz; 340 Hz; 370 Hz; 390 Hz

4 Maximum uncertainty in masses is likely to be 0.005 kg, so % uncertainty for smallest mass (0.20 kg) is 2.5%.

Resolution of metre ruler is 1mm, but errors of up to 1 cm are possible in this set up so uncertainty in measurement of 0.4 m could be 2.5%.

We are not given the % uncertainty of the mass per unit length, but it could be up to 0.005 g/m giving an uncertainty of 3%.

Since tension (proportional to mass on hanger) and mass per unit length are to the power of ½, their % uncertainty is halved.

Maximum % uncertainty in calculated frequencies would be the % uncertainty in each value added together, giving 2.5 % + 1.25% + 1.5% = 4.4%.

Page 106 Activity

Measuring the wavelength of sound waves

1 Powder gathers at the nodes of the stationary wave within the tube. The distance between nodes is equal to half a wavelength of the stationary wave. So for a wavelength of 2 m, the distance between the nodes (and thus piles) would be 100 cm. This happens when the frequency is 170 Hz.

2 and 3

Pitch of note/Hz	Number of wavelengths	Wavelength /m	Speed = frequency × length/m s^{-1}
57	0.25	6	342
170	0.75	2	340
283	1.25	1.2	340
395	1.75	0.857	339

4 The calculated speeds vary. This may be because is difficult to measure accurately the distance between the centres of the piles of powder.

Page 107 Activity

Interference patterns from microwaves

1 Suppose the probe is placed at a position of minimum intensity. If it is moved towards the plate by a distance of λ/2, the distance from the transmitter increased by λ/2, but the distance from the reflecting late decreases by λ/2. So, the change in path difference is λ and there will again be a minimum. Thus the wavelength is twice the distance between adjacent minima: λ = 2.8 cm.

2 The uncertainty is reduced by measuring over several minimums – eg if the receiver is moved from one minimum to the 7th, a distance of 3 wavelength has been measured. Now the uncertainty has been reduced.

3 Waves travelling directly from the transmitter meet waves travelling from the opposite direction, having been reflected from the metal plate. When two progressive waves meet, stationary waves are set up. The minima are nodes, and the maxima are the antinodes. Because the reflected waves have a smaller amplitude than the waves which travel directly to the receiver, the nodes will not be places of exactly zero intensity.

Pages 107–108 Test yourself

9 a) A progressive wave transfers energy over long distances whereas the energy in a stationary wave is confined to a region (but energy must be supplied to keep the stationary wave moving – for example, in a wind instrument by blowing into it, because energy leaves the instrument in the form of progressive sound waves); a stationary wave has fixed nodes and anti-nodes but a progressive wave does not.

b) Particles at a node do not oscillate but particles at an antinode oscillate with the maximum amplitude.

10 Three conditions that are needed for a stationary wave: the waves forming the stationary wave must have the same frequency; they must also have the same amplitude; they must be travelling in opposite directions in the same region.

11 The amplitude of an anti-node is double the amplitude of the original waves because it is produced when the waves are superposed and amplitudes add during superposition.

12 The displacement of a particle at an antinode changes from the undisturbed point at t = 0 to twice the original peak amplitude at t = T/4, then back to the undisturbed point at t = T/2, then to twice the original trough amplitude at t = 3T/4, then back to the undisturbed position at t = T.

13 a) The first harmonic occurs when half a wave ‘fits’ on the string, so wavelength = 2L = 6 m.

b) For the third harmonic, there are three half waves on the string so wavelength = 2L/3 = 2 m.

14 a) i) The first harmonic occurs when there is only one node between the antinodes at each end of the pipe: L, = λ/2 = 2.8 m.

ii) The wavelength of the second harmonic is L = 1.4 m.

b) i) One end of the pipe is a node and the other an antinode, so L = λ/4 = 1.4 m, so λ = 5.6 m.

ii) This is next possible when L = 3λ/4 = 1.4 m, so λ = 1.87 m.

15 a) The bottle acts as a pipe closed at one end so for the first harmonic is λ = 4L = 0.8 m.

b) Frequency = c/λ = 340 m s^{-1}/0.8 m = 425 Hz.

c) The wavelength is halved, so the frequency doubles to 850 Hz – the note has a higher pitch.

16 a) $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

Therefore $T = 4l^2f^2\mu = 4 \times (0.9\ \text{m})^2 \times (170\ \text{s}^{-1})^2 \times 5 \times 10^{-3}\text{kg m}^{-3} = 470\ \text{N}$

b) Since the frequency is proportional to the square root of tension, when the tension is quadrupled the frequency of the first harmonic is doubled to 340 Hz.

For a string, the second harmonic has twice the frequency of the first harmonic. The frequency of the second harmonic is, therefore, also doubled from 340 Hz to 680 Hz.

17 a) $\lambda = 2/3 \times 63$ cm = 42 cm

b) $c = f\lambda = 83$ Hz $\times$ 0.42 m = 34.9 m s^{-1}

c) $T = mg = 4$ kg $\times$ 9.8 N kg^{-1} = 39.2 N

d) $c^2 = T/\mu$

$\mu = T/c^2 = 39.2$ N/(34.9 m $s^{-1})^2$ = 0.032 kg m^{-1}

e) For a cylinder, density = $\frac{m}{lA} = \frac{\mu}{A}$

0.032 kg m^{-1}= density × area

A = 0.032 kg m^{-1}/(7800 kg m^{-3}) = 4.1 × 10^{-6} m^2

πr^2 = 4.1 x 10^{-6} m^2 so r = 1.14 mm and d = 2.3 mm

Page 111 Test yourself

18 Diffraction of light best seen using very narrow slits because diffraction is most pronounced when slit size and wavelength are similar and the wavelength of light is very small (400 nm – 700 nm).

19 The intensity of the fringes decreases as angle increases.
(The fringes have constant separation as the minima occur where sin θ = λ/a, 2λ/a, 3λ/a etc. and the small angle approximation angles can be used)

20 Making the slit narrower, the width of the central maximum gets wider.

21 The spacing of the grating, d, is $1 \times \frac{10^{-3}}{500} = 2.00 \times 10^{-6}\text{m}$

$$\sin\theta = \frac{\text{n}\lambda}{\text{d}} = \frac{2 \times 500 \times 10^{-9}\text{m}}{2.00 \times 10^{-6}\text{m}} = 0.5$$

θ = $\sin^{-1}$ 0.5 = 30°

22 $\text{d} = \frac{\text{n}\lambda}{\sin\theta}$

Substituting gives $d = \frac{(2\times400\times10^{-9}\text{m})}{\sin 20} = \frac{(2\times400\times10^{-9}\text{m})}{0.342} = 2.3 \times 10^{-6}$ m

23 a) The central maximum is white. The first maximum produces the colours of the rainbow with blue closest to the central maximum, and red furthest out. Second and third order maxima are broader and begin to overlap (see part b).

b) Since d sin θ = nλ

(1 × 10^{-3}/400) sin θ = 2 λ_r = 2 × 690 x 10^{-9} m = 1.38 x 10^{-6}m (which is also 3 λ_b)

sin θ = 0.552

θ = 34°

c) The maximum value of sin θ is 1 and $n = \frac{d \sin\theta}{\lambda}$

For red light $n = \frac{2.5\times10^{-6}\times1}{690\times10^{-9}} = 3.6$, so n cannot be greater than 3.

For blue light, $n = \frac{2.5\times10^{-6}\times1}{460\times10^{-9}} = 5.4$ so n cannot be greater than 5.

Pages 112–114 Practice questions

1 B

2 A

3 D

4 C

5 A

6 B

7 B

8 C

9 a) d = 1 × 10^{-3}/500 = 2 × 10^{-6} m [1]

b) sin θ = nλ/d [1]

sin θ = 3 × 633 × 10^{-9}/2 × 10^{-6} = 0.9495 [1]

θ = 71.7° [1]

c) 4 × 633 × 10^{-9}/2 × 10^{-6} > 1 [1]

Since sin θ has a maximum value of 1, this is not possible. [1]

10 a) Separation of adjacent fringes, *w* = λD/s [1]

w = 2 × 557 × 10^{-9} / 0.3 × 10^{-3} = 3.71 × 10^{-3} m [1]

So 10*w* = 3.71 cm [1]

b) The student could:
increase the separation of the screen and slits (but this reduces the intensity) [1]
increase the wavelength using a different source of light [1]
decrease the separation of the slits. [1]

11 a) Spacing between or thickness of eyelashes needs to be similar to wavelength of light [1]
the light must contain more than one colour for a spectrum to be seen [1]

b) d = 0.1 mm and sin θ = nλ/d [1]

sin θ_{4r} = 4 × 400 × 10^{-9}/0.1 × 10^{-3} = 0.016 [1]

sin θ_{4b} = 4 × 700 × 10^{-9}/0.1 × 10^{-3} = 0.028 [1]

The range of angles that the fourth order spectrum will be see is 0.92° – 1.6° [1]

c) Any two from:
- These angles are so small that it is unlikely the spectrum can be viewed close up (the linear separation will be tiny and the resolving power of the eye insufficient)
- the angles are so small, this order of spectrum is likely to overlap with those from adjacent orders and so appear white
- the coloured fringes are so close the colours are likely to appear to blend into each other

12 a) A stationary wave – any 2:
- has nodes and antinodes in fixed positions
- is a formed from the superposition of two waves travelling in opposite directions
- in a bounded space

b) i) $\lambda = c/f = 3 \times 10^8$ ms^{-1}/ 2.4×10^9 m [1]

$\lambda = 0.125$ m [1]

ii) Distance between antinodes = $\lambda/2 = 0.0625$ m [1]

c) The cheese melts at the antinodes where the microwaves have maximum amplitude and in between, at nodes, the cheese is unmelted. [1]
They could measure the distance, d, between places where the cheese melts and calculate frequency using $f = c/\lambda = 2c/d$. [1]

13

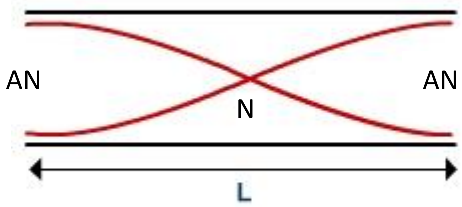


Diagram shows first harmonic [1]

Diagram marks node at the centre of the pipe and antinodes are at each end of the pipe. [1]

Stationary wave formed by superposition of waves travelling in opposite directions [1]

if the waves have equal speed, amplitude and frequency [1]

and the length of the open pipe is an odd multiple of $\lambda/2$. [1]

Nodes form where there is destructive superposition (the travelling waves cancel out) and antinodes form where there is constructive superposition. [1]

QWC [2]

14 a) It demonstrated that light waves can interfere with each other and therefore evidence for the wave nature of light. [1]

b) Coherent waves have a fixed phase difference, and are of the same frequency [1]
they can therefore interfere in a straightforward way to produce a stable (and therefore visible) fringe pattern. [1]

c) $\lambda = ws/D \Rightarrow s/D = \lambda/w$ [1]
$\lambda/w = 460 \times 10^{-9}$ m$/1.8 \times 10^{-3}$ m $= 2.6 \times 10^{-4}$ [1]
So the ratio of slit width to screen distance is approximately 1:4000 [1]

15 a) The light has a single frequency or wavelength (do not accept a single colour). [1]

b) The central peak is broader [1]
The fringes either side of the central peak are further apart [1]
(the graph remains symmetrical; the central maximum is again wider than the subsidiary fringes)

c) Lasers can damage cells in the eye as laser light is very intense [risk and reason = 1]
To reduce risk: do not shine lasers directly at people; do not direct reflections into eyes; use warning signs or laser goggles; stand behind the laser [any 1]

d) Any two:

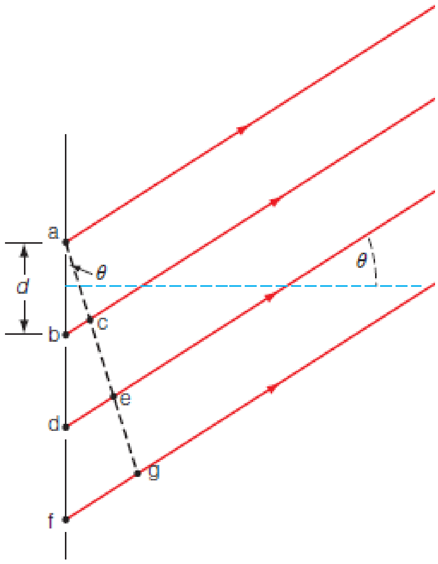
- The central maximum is white; fringes either side are separated into a spectrum
- Outer fringes have red furthest from the centre
- outer fringes of each colour are narrower than central band
- fringe spacing for each individual colour increases with wavelength

16 a) First-order maximum occurs at angle θ_1 when the path difference for light travelling from adjacent slits, a distance d apart, is λ. [1]

Sin θ_1 is opposite/hypotenuse: the hypotenuse is the slit spacing and the opposite side is the path difference so, substituting the condition for constructive interference, sin $\theta_1 = \lambda/d$. [1]

For second order fringes, the path difference is 2λ and light travels at angle θ_2 to the direction of incident light. The equation becomes: sin $\theta_2 = 2\lambda/d$. [1]

Extending to higher orders and rearranging gives $d\sin\theta_n = n\lambda$ where n is a whole number. [1]



b) As λ increases, sin θ increases so:

- second- and higher-order maxima move further apart
- The number of maxima is reduced as sin θ cannot be more than 1

c) $\lambda = d\sin\theta_n/n = \lambda$ [1]
$\lambda = 1 \times 10^{-6}$ m × (sin 70)/3 [1]
$\lambda = 3.1 \times 10^{-7}$ m [1]

17 $d\sin\theta_n = n\lambda$ [1]

$n\lambda = 2 \times 10^{-6}$ m × sin42 = 1.338×10^{-6} m [1]

If n= 2, λ = 669 nm [1]

If n=3, λ = 446 nm [1]

The red light is 2nd order, the blue light is 3rd order. [1]

18 a) Slit spacing = 1×10^{-3} m/300 = 3.3×10^{-6} m [1]

b) (sin θ)/n = λ /d = 7 × 10^{-7}/3.3 × 10^{-6} = 0.21 [1]

sin θ = 0.21 × n, and cannot be more than 1 so n can be 0, 1, 2, 3, 4 – so 4 orders visible [1]

Possible values of sin θ are 0, 0.21, 0.42, 0.63 and 0.84 [1]

This gives maxima at angles of 0° (central maximum); 12° (1st order); 25° (2nd order); 39° (3rd order); 57° (4th order) [1]

Pages 114–115 Stretch and challenge

19 All methods result in a fringe pattern. Dark fringes are due to destructive interference (path difference (n+1/2)λ), and bright fringes due to constructive interference (path difference nλ) of light from parts of the slit or individual slits.

In Young's experiment, the fringes have approximately equal intensities in the central region, but the intensity of the fringes diminishes at higher order due to the effects of diffraction form each single slit.

With single slit diffraction there is a brighter central maximum and higher-order maxima decrease in intensity.

A diffraction grating produces well defined, narrow maxima. The central (zeroth order maximum is brighter than higher orders.

In Young's double slits, the fringes are usually viewed from a large distance relative to the separation of the slits and so only very small viewing angles are considered (allowing the small angle approximation to be used to derive the equation: w = λD/s), whereas diffraction patterns are viewed from a range of angles.

The resolution (sharpness) of fringes is greatest with the diffraction grating (so it is used to separate out spectra).

If a diffraction grating with many slits is used, there are fewer fringes, more widely spaced than those from a single slit.

Increasing the number of slits increases the number of fringes visible. Increasing the width of the single slit increases the width of the central maxima. Different maxima are seen for different orders for diffraction sin θ = nλ /d.

Above we have given some brief answers; there is a great deal of research you could do into interference and diffraction from single, double and multiple slits.

20 The maxima for red light occur at greater angles than for violet light so, if first two orders overlap, the angle for the first-order red maxima will be greater than that for second-order violet maxima.

For red light, $\sin \theta_1 = \lambda/d = 700 \times 10^{-9}/d$
For violet light, $\sin \theta_2 = 2\lambda/d = 400 \times 10^{-9} \times 2/d = 800 \times 10^{-9}/d$
$700 \times 10^{-9}/d < 800 \times 10^{-9}/d$
So 1st and 2nd orders do not overlap.

Try second and third orders:
For red light, $\sin \theta_2 = 2\lambda/d = 700 \times 10^{-9} \times 2/d = 1400 \times 10^{-9}/d$
For violet light, $\sin \theta_3 = 3\lambda/d = 400 \times 10^{-9} \times 3/d = 1200 \times 10^{-9}/d$
$1400 \times 10^{-9}/d > 1200 \times 10^{-9}/d$
So second-order red maxima is at a greater angle than the third-order violet maxima meaning second and third orders overlap.

21 a) Path difference = AR – BR

b) There is an intensity maximum when the waves are in phase.
Since there is a phase change of 180° on reflection, the path difference must be (n+1/2) λ for waves that are initially in phase to also be in phase at R. So the condition is

$$200 \sin \theta = \left(n + \frac{1}{2}\right)\lambda$$

The optional mathematical bit:

BR = AR cos 2θ

So AR – BR = AR (1– cos 2θ) = $2\sin^2 \theta$ (because $\cos 2\theta = 1 - 2\sin^2 \theta$)

But AR = 100/sin θ

So AR – BR = (100/sin θ) × $2\sin^2 \theta$ = 200 sin θ

c) As the sun sets the angle of light coming from the sun changes so the angle θ changes.
The path difference continuously changes creating minima when it is equal to a whole number of wavelengths and maxima when it is an odd number of half wavelengths.

d) sin θ changes most rapidly for small values of θ, so the changes between maxima and minima will be most rapid when the sun is low in the sky and nearly set.

22 The answer is in the question. If the path difference between the first and second slit is 0.01 λ, the path difference between the first and fifty-first slit is λ/2. So, the light from each slit is cancelled by the light from a slit 50 below it.

Thus, the maxima are very sharp because the path difference between light from each slit must be very close to λ, 2λ etc. for there not to be destructive interference as a result of light from nearby slits.

23 a) The ray reflecting from the top face of the lower slide (as this reflection is at the boundary between two materials).

b) The waves have a path difference of (n + ½) λ, where n is a whole number.

c) Path difference for destructive interference = $(n + ½)\lambda = 2d + \lambda/2$ (allowing for the phase change on reflection) so $n\lambda = 2d$. That is, dark fringes will be seen above places where the separation of the slides is equal to a whole number of wavelengths.

d) i) The wavelength in the liquid is smaller than the wavelength in air, so the fringe spacing is reduced (there are a greater number of values of d that are equal to a whole number of wavelengths).

ii) If the wavelength increases, the fringe spacing increases.