33

Engineering Physics

PRIOR KNOWLEDGE

Before you start, make sure that you are confident in your knowledge and understanding of the following points:

- Vector quantities have magnitude and direction whereas scalar quantities have magnitude only.
- Velocity = $\frac{\text{displacement}}{\text{time}}$
- Acceleration = $\frac{\text{change in velocity}}{\text{time}}$
- The four equations of uniform acceleration for linear motion.
- Radians can be used to measure angles, $s = r\theta$. 2π rad = 360°
- Torque is the turning effect (moment) of a force and is calculated by multiplying the force by the perpendicular distance between the line of action of the force and the axis of rotation (pivot).
- Angular velocity = $\frac{\text{angular displacement}}{\text{time}} = \frac{2\pi}{T} = 2\pi f$
- Kinetic energy, $E_K = \frac{1}{2} m v^2$
- Energy is the capacity to do work where work done = force × distance moved in direction of the force = $F \times d$
- Power = $\frac{\text{work done}}{\text{time}}$
- Impulse = change in momentum and $F t = \Delta p$
- Momentum is always conserved when bodies interact provided there is no external force acting.
- Heat = transfer of energy due to a temperature difference.
- Temperature is a measure of the average kinetic energy of the particles in a gas.
- Internal energy is the sum of the random kinetic and potential energies of the particles in a gas.
- The ideal gas equation, $p V = n R T$ and gas laws (Boyle's Law $p V$ = constant, Charles' Law $\frac{V}{T}$ = constant, Pressure Law $\frac{p}{T}$ = constant) enable the pressure, volume and temperature of gases to be investigated and calculated.
- 1 mol of a gas contains 6.02×10^{23} particles.
- The rms speed of particles in a gas is their average speed.
- The kinetic theory equation links the macroscopic properties (pressure, volume, temperature) of a gas with the movement and energy of the particles, $p V = \frac{1}{3} N m (c_{rms})^2$ and $E_K = \frac{3}{2} k T$

TEST YOURSELF ON PRIOR KNOWLEDGE

1 A bird of prey can spot a 5 cm long mouse on the ground from a height of 100 m. What angle in radians is subtended at the bird's eyes?

2 The Moon orbits the Earth once every 29 days with a radius of orbit of 380 000 km. What is the angular velocity of the Moon?

3 Baking powder is added to cake mixture and releases carbon dioxide when activated. Use the gas laws to explain why a cake rises when baked in the oven.

4 A deep-sea diver is working at a depth where the pressure is 2.8 atm. She is breathing out air bubbles each of volume 2 cm^3. If she moves up to a depth of 10 m where the pressure is only 1.9 atm what would now be the volume of each bubble? You should assume that each bubble contains the same mass of air and that the temperature is not changing.

Rotational dynamics

A large part of Year 1 of the A-level course involved the study of the dynamics of objects in linear motion. The first half of the Engineering Physics optional topic studies the forces and motion involved in rotating objects. Some examples of rotational motion are:

- a gymnast performing a somersault
- the Earth spinning on its axis, taking almost 24 hours to complete one full rotation
- a car wheel rotating on its axis
- a ball turning as it rolls down a hill.

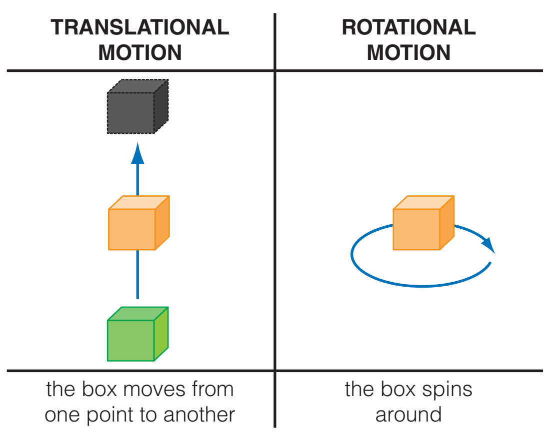


Figure 33.1 Comparing translational and rotational motion.

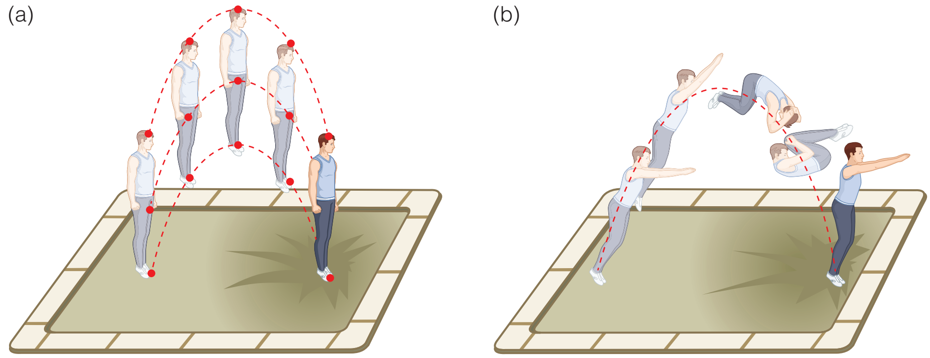


Figure 33.2 (a) In translational motion, all points of the object travel along parallel paths. (b) Most motion is a combination of translation and rotation.

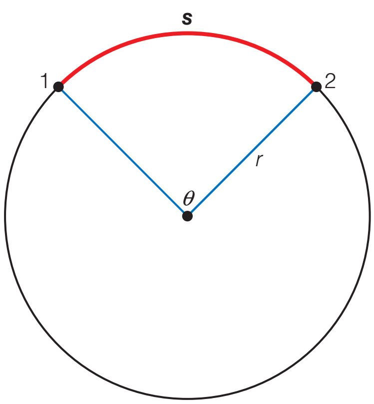


Figure 33.3

Rotational motion

Displacement, velocity and acceleration

Angular displacement

Angular displacement, θ, of a body is the angle in radians (degrees or revolutions) through which a point or line has been rotated about an axis.

By definition, when we work in radians, the angle θ is defined by:

$$\theta = \frac{s}{r}$$

where s is known as the arclength, and r is the distance of the point from the axis of rotation.

When we go right around the circle, s becomes the circumference $2\pi r$ so

$$\theta = \frac{2\pi r}{r} = 2\pi$$

Here we see that 2π radians = 360°.

Since we can go clockwise or anticlockwise, we need to define a direction for angular displacement.

EXAMPLE

The Tour de France is a cycle race and involves cyclists cycling 3500 km over 2 weeks. Calculate the number of complete rotations made by a cyclist's wheel during the race.

Answer

The radius of a bike wheel is 0.35 m.

$$s = 3500 \times 10^3 \text{ m}$$

$$r = 0.35 \text{ m}$$

$$\theta = \frac{s}{r} = \frac{3500 \times 10^3}{0.35} = 10 \times 10^6 \text{ rad}$$

number of rotations $= \dfrac{\theta}{2\pi}$

$= \dfrac{10 \times 10^6 \text{ rad}}{2\pi}$

$= 1\,591\,549$ rotations $\approx 1.6 \times 10^6$ rotations

This is the same as dividing the total distance travelled by the circumference ($2\pi r$) of the wheel (i.e. $\dfrac{3\,500\,000}{(2\pi \times 0.35)} \approx 1.6 \times 10^6$ rotations).

TIP

Radians are usually used when dealing with rotational motion. Figure 33.3 on page 3 shows the definition of a radian and how it is useful to work in radians when calculating the length of an arc. This is because the arc length, *s*, is equal to the distance travelled by a point as an object rotates.

Angular velocity

Angular velocity is analogous to linear velocity.

Velocity $= \dfrac{\text{displacement}}{\text{time}}$

or

$$v = \frac{s}{t}$$

And so angular velocity $= \dfrac{\text{angular displacement}}{\text{time}}$

or

$$\omega = \frac{\Delta\theta}{\Delta t}$$

A rotating object therefore has **angular velocity**, ω. Where the direction is not specified this may be referred to as **angular speed** or **angular frequency**.

We can express angular velocity in terms of the time taken to complete one complete rotation.

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi f$$

where θ = angular displacement and T = period of rotation $= \dfrac{1}{f}$ where f = frequency of rotation.

The units of angular velocity are radians per second (rad s^{-1}) although angular speed is often expressed in terms of revolutions per minute (rpm or rev s^{-1}) or revolutions per second (rps or rev s^{-1}).

TIP

There are 2π radians in a complete rotation and 60 seconds in a minute so to convert from rpm to rad s^{-1} multiply by $\dfrac{2\pi}{60}$. To convert from rad s^{-1} to rpm multiply by 60 and divide by 2π.

Figure 33.4

For small angular changes in small times we can write

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Also, since $s = r\,\theta$ and $v = \frac{s}{t}$

$$v = \frac{r\,\theta}{t}$$
$$= \omega\, r$$

or

$$\omega = \frac{v}{r}$$

This enables the linear velocity of a point to be related to the angular velocity. Maximum linear velocity is achieved in sports such as golf, football, or volleyball by maximising the distance to the axis of rotation.

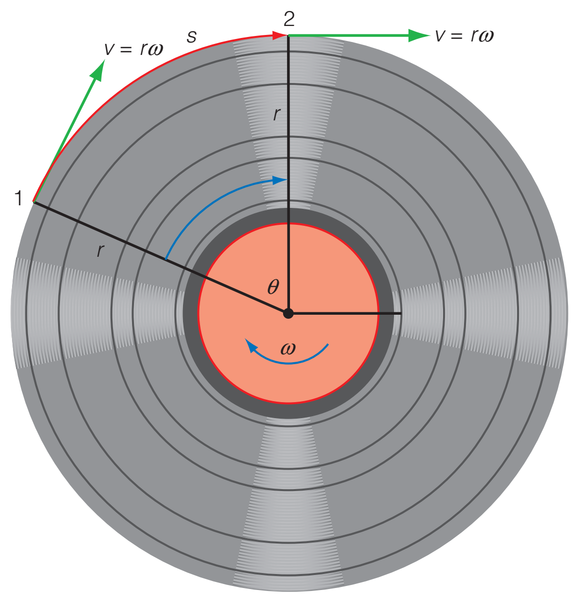


Figure 33.5 A vinyl record has angular velocity, ω, when spinning clockwise on a record player. It rotates from position 1 to position 2 in time, t. Its angular displacement in this time is θ. Every point on the record has the same angular velocity, ω, but the linear velocity depends on how far the point is from the axis of rotation as $v = r\,\omega$.

EXAMPLE

1 A bike wheel tyre with a diameter of 0.8 m is spinning with an angular velocity, $\omega = 3.8$ rad s^{-1}.

a) Calculate the linear velocity of a point on the tyre.

Answer

$v = r\,\omega = 0.4 \times 3.8$ rad s^{-1} $= 1.52$ m s^{-1}

b) Calculate the number of rotations the tyre completes in 1 minute.

Answer

$$f = \frac{\omega}{2\pi}$$
$$= \frac{1.52 \text{ rad s}^{-1}}{2\pi}$$
$$= 0.24 \text{ rps}$$

0.24 rps × 60 s = 8.7 rpm

2 Convert the following into rps.

a) 2 rad s^{-1}

b) 15 rad s^{-1}

Answer

$\frac{2}{2\pi}$ = 0.32 rps

$\frac{15}{2\pi}$ = 2.39 rps

3 Convert the following into rpm.

a) 2 rad s^{-1}

b) 15 rad s^{-1}

Answer

0.32 rps × 60 = 19.2 rpm

2.39 rps × 60 = 143 rpm

4 Convert the following into rad s^{-1}.

a) 2500 rpm

b) 10 rpm

Answer

2500 rpm × $\left(\frac{2\pi}{60\,\text{s}}\right)$ = 262 rad s^{-1}

10 rpm × $\left(\frac{2\pi}{60\,\text{s}}\right)$ = 1.05 rad s^{-1}

TIP

To calculate the number of rotations, divide the angular displacement by 2π.

Number of rotations = $\frac{\theta}{2\pi}$

Angular acceleration

We call the change in an object's linear velocity, acceleration. Linear acceleration is calculated using:

$$a = \frac{\text{change in velocity}}{\text{time}}$$
$$= \frac{v - u}{t}$$

where the velocity changes from an initial velocity u to a final velocity v in a time t.

Similarly, the **angular velocity** of an object can change – its angular velocity can increase or decrease.

The **angular acceleration** is analogous to linear acceleration:

$$\text{angular acceleration} = \frac{\text{change in angular velocity}}{\text{time}}$$
$$\alpha = \frac{\Delta\omega}{\Delta t}$$
$$= \frac{\omega_2 - \omega_1}{t}$$

Angular acceleration, α, is the rate of change of angular velocity or the change of angular velocity per second.

We use ω_1 to signify the initial angular velocity and ω_2 for the final angular velocity.

The units of angular acceleration are rad s^{-2} $\left(\frac{\text{rad s}^{-1}}{\text{s}}\right)$.

This equation is the rotational equivalent of $v = u + a\,t$ and it can be rearranged to give:

$$\omega_2 = \omega_1 + \alpha t$$

This is our first rotational equivalent of the uniform acceleration formulae.

We saw above in the angular velocity section (and in the core circular motion topic) that the linear speed of a point a distance r from the axis of rotation is $v = \omega\,r$, so $\omega = \frac{v}{r}$.

We can substitute this into the equation for linear acceleration to express the acceleration of a point a distance r from the axis of rotation in terms of angular acceleration (α).

$$a = \frac{(v - u)}{t}$$

$$= \frac{(\omega_2\,r - \omega_1\,r)}{t}$$

$$a = \alpha\,r$$

TIP

Angular acceleration of a point = linear acceleration divided by the distance of the point from the axis of rotation: $\alpha = \frac{a}{r}$

EXAMPLE

1 The angular velocity of a spinning ball increases from 8 rad s^{-1} to 23 rad s^{-1} in 10 seconds. Calculate the angular acceleration.

Answer

$\omega_1 = 8$ rad s^{-1}

$\omega_2 = 23$ rad s^{-1}

$$\alpha = \frac{\omega_2 - \omega_1}{t}$$

$$\alpha = \frac{23 - 8}{10} = \frac{15}{10} = 1.5 \text{ rad s}^{-2}$$

2 A car is accelerating from the slip road of a motorway from an initial speed of 50 km h^{-1} to 90 km h^{-1} over a time of 5 seconds. The diameter of the wheels is 45 cm.

a) Determine the change in angular speed of a point on the edge of a wheel in rad s^{-1} and rpm.

Answer

$$u = \frac{50\,000\text{ m}}{3600\text{ s}} = 13.9 \text{ m s}^{-1}$$

$$\omega_1 = \frac{u}{r} = \frac{13.9\text{ m s}^{-1}}{0.225\text{ m}} = 61.8 \text{ rad s}^{-1}$$

$$v = \frac{90\,000\text{ m}}{3600\text{ s}} = 25 \text{ m s}^{-1}$$

$$\omega_2 = \frac{v}{r} = \frac{25\text{ m s}^{-1}}{0.225\text{ m}} = 111.1 \text{ rad s}^{-1}$$

$$\omega_2 - \omega_1 = 49.3 \text{ rad s}^{-1}$$

$$\frac{49.3}{2\,\pi} \times 60\text{ s} = 471 \text{ rpm}$$

b) Determine the angular acceleration of the wheels.

Answer

$$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{(111.1 - 61.8)}{5} = 9.9 \text{ rad s}^{-2}$$

Equations of motion for uniform angular acceleration

On pages 6 and 7, the rotational equivalent of $v = u + a\,t$ was shown to be:

$$\omega_2 = \omega_1 + \alpha\,t \qquad (1)$$

The other three equations of uniform angular acceleration can be obtained in a similar way to those for linear uniform acceleration. (See pages 136–137 of the Student's Book for details of these derivations.)

For uniform angular acceleration, average angular velocity = $\dfrac{(\omega_2 + \omega_1)}{2}$

and angular displacement = average angular velocity × time, so

$$\theta = \frac{(\omega_2 + \omega_1)\,t}{2} \qquad (2)$$

This is the second equation of uniform rotational acceleration.

To obtain the third equation of uniform rotational acceleration we can substitute ω_2 from the first equation into the second equation to obtain:

$$\theta = \frac{(\omega_1 + \omega_1 + \alpha t) \times t}{2}$$

$$\theta = \omega_1\,t + \frac{\alpha\,t^2}{2} \qquad (3)$$

The linear motion equivalent of this is $s = u\,t + \frac{1}{2}\,a\,t^2$.

MATHS BOX

The fourth equation of uniform rotational acceleration may be derived as follows.

$$\omega_2 = \omega_1 + \alpha\,t$$

$$\rightarrow \quad (\omega_2)^2 = (\omega_1 + \alpha\,t)^2$$

$$= (\omega_1)^2 + 2\,\omega_1\,\alpha\,t + \alpha^2\,t^2$$

$$= (\omega_1)^2 + 2\,\alpha\,(\omega_1\,t + \frac{1}{2}\,\alpha t^2)$$

Using equation (3),

$$\theta = \omega_1\,t + \frac{1}{2}\,\alpha\,t^2$$

$$(\omega_2)^2 = (\omega_1)^2 + 2\,\alpha\theta \qquad (4)$$

The derivation of the fourth equation is shown in the Maths box and by analogy with the linear motion equivalent $v^2 = u^2 + 2as$ we can see it will be

$$(\omega_2)^2 = (\omega_1)^2 + 2\,\alpha\theta \qquad (4)$$

EXAMPLE

1 A wheel is rotating at 100 rev min^{-1} (rpm). If it is brought to rest in 15 revolutions what is the angular acceleration?

Answer

$\omega_1 = 100 \text{ rpm} \times \left(\frac{2\pi}{60}\right)$

$= \frac{10\pi}{3} \text{ rad s}^{-1}$

$= 10.47 \text{ rad s}^{-1}$

$\theta = 15 \times 2\pi$

$= 30\pi = 94.2 \text{ rad}$

$\omega_2 = 0 \text{ rad s}^{-1}$

$(\omega_2)^2 = (\omega_1)^2 + 2\alpha\theta$

$0 = (10.47 \text{ rad s}^{-1})^2 + (2\alpha \times 94.2 \text{ rad})$

$\alpha = -\frac{109.7}{188.5}$

$= -0.58 \text{ rad s}^{-2}$

2 An electric drill bit rotating clockwise takes 3.0 s to speed up to 2500 rpm from rest.

a) Calculate the drill's angular acceleration.

Answer

$\omega_1 = 0 \text{ rad s}^{-1}$

$\omega_2 = 2500 \times \left(\frac{2\pi}{60}\right)$

$= 261.8 \text{ rad s}^{-1}$

$t = 3.0 \text{ s}$

$\alpha = \frac{\omega_2 - \omega_1}{t}$

$= \frac{261.8 - 0 \text{ rad s}^{-1}}{3.0 \text{ s}} = 87.3 \text{ rad s}^{-2}$

(b) Calculate how many revolutions the drill makes before it reaches top speed, assuming a constant acceleration.

Answer

We can use any of the uniform acceleration equations to calculate angular displacement.

Using, $(\omega_2)^2 = (\omega_1)^2 + 2\alpha\theta$

$\frac{(261.8^2 - 0)}{(2 \times 87.3)} = 392.6 \text{ radians}$

Number of rotations $= \frac{\text{angular displacement}}{2\pi}$

$= \frac{392.6 \text{ radians}}{2\pi}$

$= 62.5$ rotations

$= 62$ complete rotations

Graphs of angular motion

Angular displacement, angular velocity and angular acceleration can be plotted on graphs to show how they change with time.

Displacement–time graphs

The minute hand moves around an analogue clock face at a steady rate. Its angular displacement can be plotted on a graph as it takes 1 hour to complete a whole rotation.

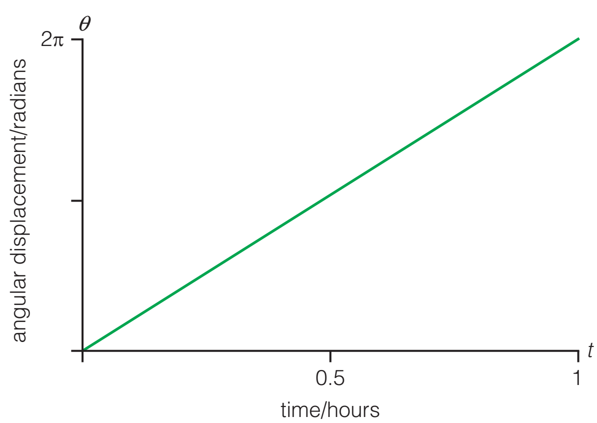


Figure 33.6 Angular velocity against time for the minute hand of a clock.

The angular velocity can be calculated from the gradient of the angular displacement against time graph.

For the minute hand of a clock the angular velocity = $\dfrac{\Delta y}{\Delta x}$

$$= \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{3600} = 1.75 \times 10^{-3} \text{ rad s}^{-1}$$

If the graph is a curve then drawing a tangent to the curve at that point gives the instantaneous angular velocity at that moment in time.

Velocity–time graphs

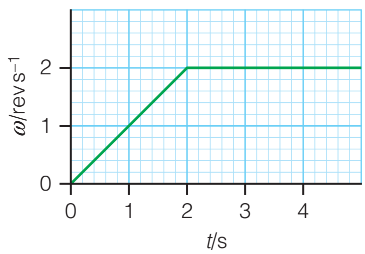


Figure 33.7

Figure 33.7 shows the angular velocity of a spinning roulette wheel.

We can use the graph to describe the motion of the wheel. For the first 2 seconds, the angular velocity increases at a constant rate from 0 rev s^{-1} to 2 rev s^{-1}. The roulette wheel accelerates uniformly. Between 2 and 4 seconds the wheel spins at a constant angular velocity of 2 rev s^{-1}.

We can use the graph to calculate the angular acceleration of the wheel as angular acceleration is equal to the gradient of the graph:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\Delta y}{\Delta x}$$

To calculate the angular acceleration for the first 2 seconds we convert rev s^{-1} to rad s^{-1}:

$$2 \text{ rev s}^{-1} = (2 \times 2\pi) = 12.6 \text{ rad s}^{-1}$$

$$\text{gradient} = \frac{\Delta y}{\Delta x}$$

$$= \frac{12.6 \text{ rad s}^{-1}}{2 \text{ s}} = 6.3 \text{ rad s}^{-2}$$

TIP

When calculating the gradient use the largest triangle possible. Examiners prefer you to use a triangle with a base of at least 8 cm where at all possible.

We can also use the graph to calculate the number of complete rotations the wheel makes in 4 seconds as we know that angular displacement = area under the graph as $\theta = \omega t$.

First we calculate the area from $t = 0$ s to $t = 2$ s

$$= \frac{1}{2} \times 2 \times 2 = 2 \text{ revolutions}$$

Now we calculate the area from $t = 2$ s to $t = 4$ s

$$= 2 \times 2 = 4 \text{ revolutions}$$

Total number of revolutions completed in the first 4 seconds

$$= 2 + 4 = 6 \text{ revolutions.}$$

TIP

The gradient of an angular velocity against time graph gives the angular acceleration. If the gradient is negative the object is decelerating.

If the graph is a curve then drawing a tangent to the curve at that point gives the instantaneous angular acceleration at that moment in time.

EXAMPLE

1 a) Describe the motion shown by the graph in Figure 33.8 as fully as you can.

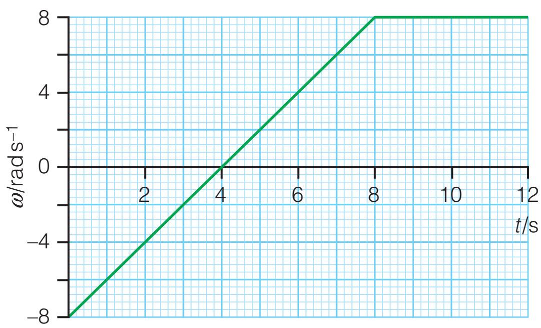


Figure 33.8 Angular velocity–time graph for a ship's propeller.

Answer

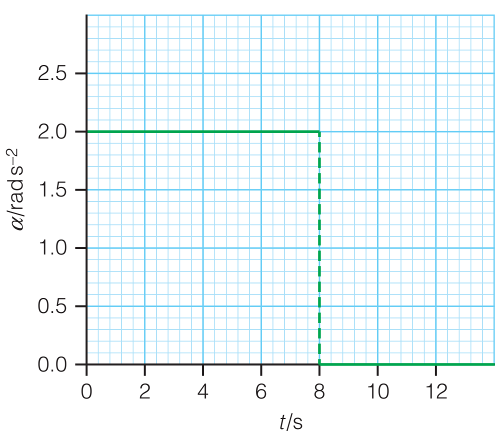
The propeller is slowing down at a uniform rate until at 4 s it comes momentarily to a stop. It then continues to accelerate with the same uniform acceleration until it is rotating at a constant angular velocity of 8 rad s^{-1} in the opposite direction. Between 0 s and 8 s the graph shows uniform angular acceleration because it is a straight line and therefore has a constant gradient.

(b) Sketch the corresponding angular acceleration–time graph for this motion.

Answer

Gradient of straight line section of the ω-t graph

$$= \frac{16 \text{ rad s}^{-1}}{8 \text{ s}} = 2 \text{ rad s}^{-2}$$



2 Figure 33.9 shows how angular displacement changes with time.

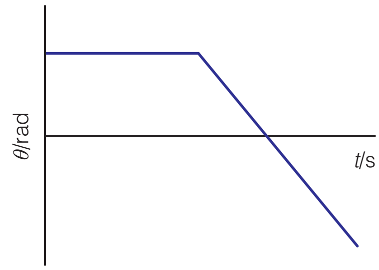
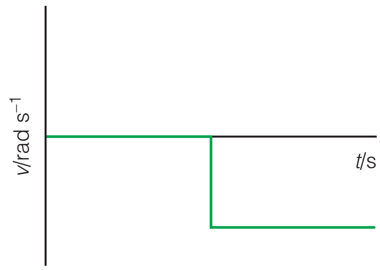


Figure 33.9

Sketch a graph to show how the velocity changes with time for this object.

Answer



3 Figure 33.10 shows how the angular acceleration of an object varies with time. The initial angular velocity is 10 rad s^{-1}. Draw the shape of the corresponding graph which shows how velocity changes with time.

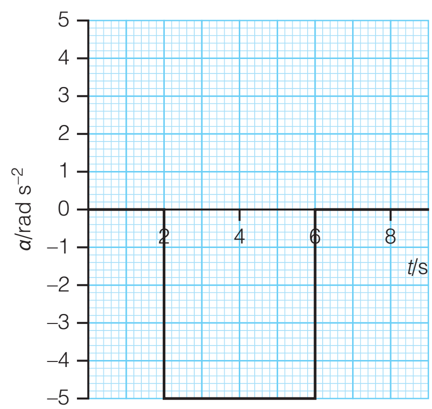
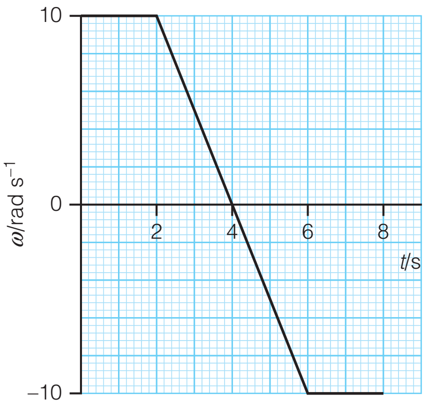


Figure 33.10

Answer



4 An object accelerates from rest with a uniform angular acceleration of 2 rad s^{-2}.

a) Plot a graph to show how the object's

i) displacement

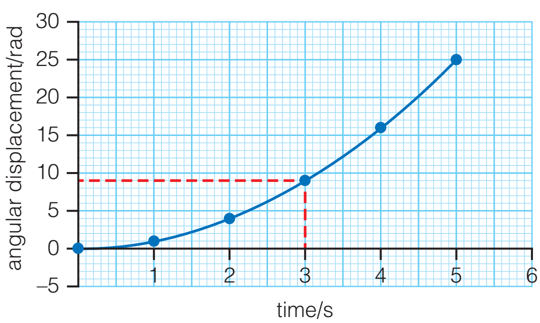
ii) velocity varies for the first 5 seconds of motion.

Answer

i) Angular displacement is given by

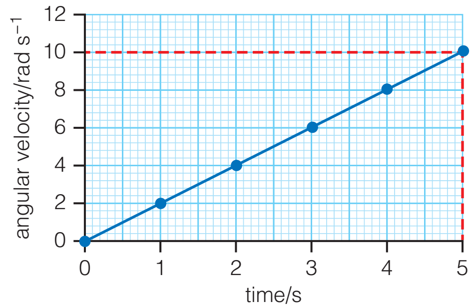
$\theta = \omega_1 t + \frac{1}{2} \alpha t^2$

$\omega_1 = 0$ and $\alpha = 2$ rad s^{-2} so the graph will be a parabola with an increasing gradient.



ii) $\omega_2 = \omega_1 + \alpha t$

so the graph will be a straight line through the origin with a gradient of 2 rad s^{-2}.



b) Determine the angular velocity after 5 seconds.

Answer

Reading off the graph when $t = 5$ s, $\omega = 10$ rad s–1

c) Calculate the number of complete rotations the object has made after 3 seconds.

Answer

Reading off the graph when $t = 3$ s, $\theta = 9$ radians.

Number of rotations $= \frac{\theta}{2\pi} = \frac{9}{2\pi} = 1.43$

d) Add a line to show the displacement–time graph for an object which has only half the acceleration ($\alpha = 1$ rad s^{-2}).

Answer

Curve starting at (0, 0) and passing through (2, 2), (4, 8), and so on.

TEST YOURSELF

Rotational motion

1 Two children sit on a carousel horse ride at the funfair. One sits on a horse near the centre of the ride and the other sits on a horse at the outer edge. Which child has

a) the greater linear velocity

b) the greater angular velocity?

2 A playground roundabout is initially at rest when it is given a constant angular acceleration of 0.05 rad s^{-2} for 12 seconds. Calculate

a) the angular velocity of the roundabout after 12 seconds

b) the linear velocity of a child sitting 2.00 m from the axis of rotation after 12 seconds.

3 A centrifuge in a hospital lab is able to separate blood into its constituent parts. It accelerates uniformly from rest to 30 000 rpm in 20 seconds. Calculate

a) the average angular acceleration

b) the number of revolutions it has completed 60 seconds after being switched on.

4 Figure 33.11 shows how the angular velocity varies for a passenger on a roundabout.

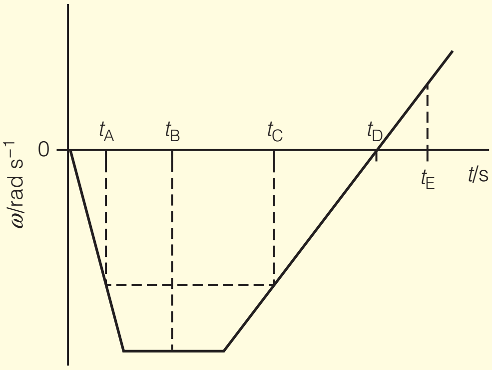


Figure 33.11

At what time is the angular acceleration experienced by the roundabout passenger

a) greatest

b) least

5 A flywheel is a heavy rotating disc used to store kinetic energy. A 2 m diameter flywheel is mounted on a bus and is used to supply additional kinetic energy when the bus is going uphill. If the flywheel is accelerated from rest for 10 seconds at a rate of 0.21 rad s^{-2}, calculate

a) the angular speed of a point on the rim of the flywheel after 10 seconds

b) the angle through which that point will have rotated.

6 A pulsar is a rapidly rotating neutron star that emits a radio beam, emitting a radio pulse for each rotation of the star. The period of rotation is found by measuring the time period between pulses. A pulsar in the Crab nebula has a period of rotation of $T = 0.033$ s that is increasing at the rate of 1.26×10^{-5} seconds per year.

a) What is the pulsar's angular acceleration?

b) In how many years from now will the pulsar stop rotating?

Torque and angular acceleration

Figure 33.12

To accelerate an object in a straight line, a resultant force must be applied. Therefore to make a rotating object spin faster, a **torque** needs to be applied.

The turning effect of a force or **torque** is calculated by multiplying the force by the perpendicular distance between the line of action of the force and the axis of rotation.

$$T = F r$$

The units of torque are N m.

The amount of torque depends on where and in what direction the force is applied, as well as the axis of rotation.

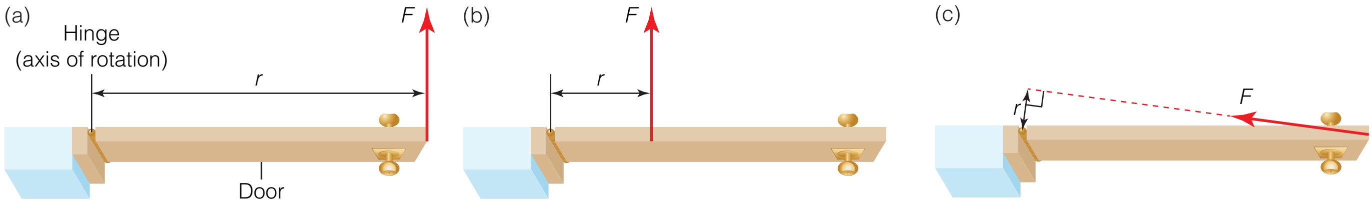


Figure 33.13 (a) results in the largest torque as the force is applied furthest from the axis of rotation (the hinge). (b) results in a smaller torque than in (a) as the line of action of the force is closer to the axis of rotation. (c) results in the smallest torque as the force acts at an angle and so the perpendicular distance between the line of action of the force and the axis of rotation is very small.

EXAMPLE

A series of spanners of different lengths is used to loosen a bolt, as shown in Figure 33.14. Which combination of spanner length and force applies the greatest torque to the bolt?

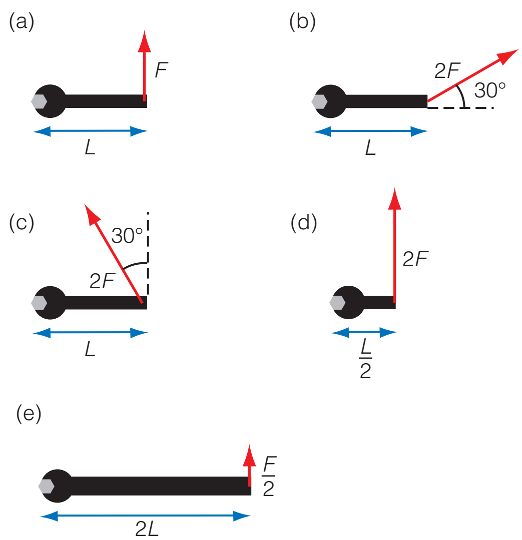


Figure 33.14 Torque on a bolt.

Answer

The answer is (c).

Torque, the 'turning effect' produced by a force applied is calculated using $T = F \times r$. Each combination of spanner length and force produces a net torque of LF except for answer (c):

$$T = L \times 2F \times \cos 30 = \sqrt{3}LF$$

TIP

Torques are usually couples, i.e. two forces separated by a distance. The second force has its reaction at the pivot and so does not result in a moment. Without this second reaction force the object would accelerate linearly.

TIP

The moment of inertia describes the tendency of a body to resist angular acceleration. It is a measure of how difficult it is to change the angular velocity of an object for a given torque. Moment of inertia is a measure of how hard it is to set a body rotating. For a given torque, an object with a large moment of inertia experiences a smaller angular acceleration than a body with a low moment of inertia.

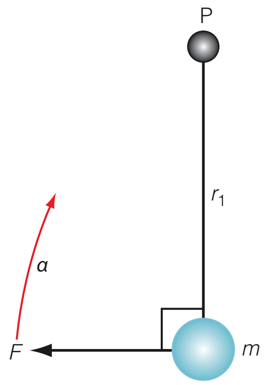


Figure 33.15

I is called the **moment of inertia** of the object about the axis of rotation and depends on the mass and distribution and position of the masses relative to the axis of rotation. The unit of I is kg m^2.

Moment of inertia

All objects have a resistance to having their motion changed – this is called **inertia**. Newton's first law of motion is known as the law of inertia: 'An object will remain at rest or continue to move with constant velocity unless acted on by a resultant force'. Similarly, objects have resistance to having their rotational motion changed. This is called **rotational inertia**.

Formulae for moment of inertia

In Figure 33.15 a point mass is set rotating about point P. The force, F, causes an acceleration but we want to calculate the angular acceleration.

From Newton's second law

$$\Sigma F = m\,a$$

and using torque to calculate the angular acceleration

$$Fr = m\,r\,a$$

but

$$a = r\,\alpha$$

so

$$Fr = mr\,(r\,\alpha)$$

$$= mr^2\,\alpha$$

mr^2 is called the **moment of inertia**, I, for a point mass, m, rotating about a point at a distance r. The unit of moment of inertia is kg m^2.

However, we usually are dealing with solid masses and so we have to sum up all the point moments of inertia.

EXAMPLE

In Figure 33.16, two 1 kg masses rotate about point P. The masses are connected by a very strong light rod (i.e. we can neglect its mass). Calculate the moment of inertia.

Answer

$$I = I_1 + I_2$$

$$= m_1\,(r_1)^2 + m_2\,(r_2)^2$$

$$= (1 \times 1^2) + (1 \times 2^2)$$

$$= 5\text{ kg m}^2$$

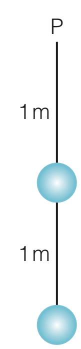


Figure 33.16

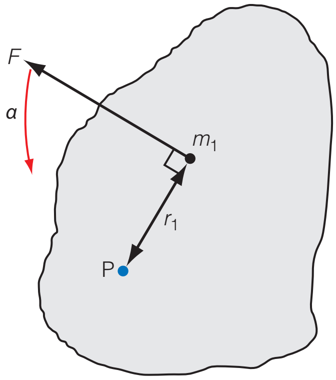


Figure 33.17 Moment of inertia of a solid body.

The torque form of Newton's Second Law

A body is rotating about a point P. The object can be thought of as being made of an infinite collection of small mass elements. A torque is applied due to the force F acting. At the instant shown in Figure 33.17, the small mass m_1 will have a linear speed v of ωr_1 (as $v = r\omega$).

If the object accelerates then m_1 also accelerates with an acceleration of $a_1 = \alpha r_1$.

Newton's Second Law of motion, $F = ma$, tells us the resultant force needed to accelerate a mass and so

$$F = m_1 a = m_1 \alpha r_1$$

The **resultant torque**, T about point P, due to this force $= F \times r_1 = m_1 (r_1)^2 \alpha$.

The total torque can be calculated by adding up all the torques on all the mass elements which make up the object:

$$\text{total torque} = m_1 (r_1)^2 \alpha + m_2 (r_2)^2 \alpha + m_3 (r_3)^2 \alpha + \dots + m_n (r_n)^2 \alpha = I\alpha$$

$$T = \sum_{i=1}^{n} \left(m_i r_i^2\right)\alpha = I\alpha$$

TIP

For a point mass m a distance r from the axis of rotation, the moment of inertia would be calculated using $I = m r^2$.
For an extended object the 'sigma m r squared', $I = \Sigma m r^2$, summation should be completed using integration.

Resultant torque (N m) = moment of inertia (kg m^2) × angular acceleration (rad s^{-2})

This is the rotational equivalent of Newton's Second Law of motion: 'A resultant torque causes angular acceleration. Torque causes a change in rotational motion.'

$$\Sigma T = I\alpha$$

This is sometimes called the torque form of Newton's Second Law.

Figure 33.18 shows the moment of inertia of some common shapes. You are not expected to remember these formulae but refer to these to answer the Test yourself questions on page 18.

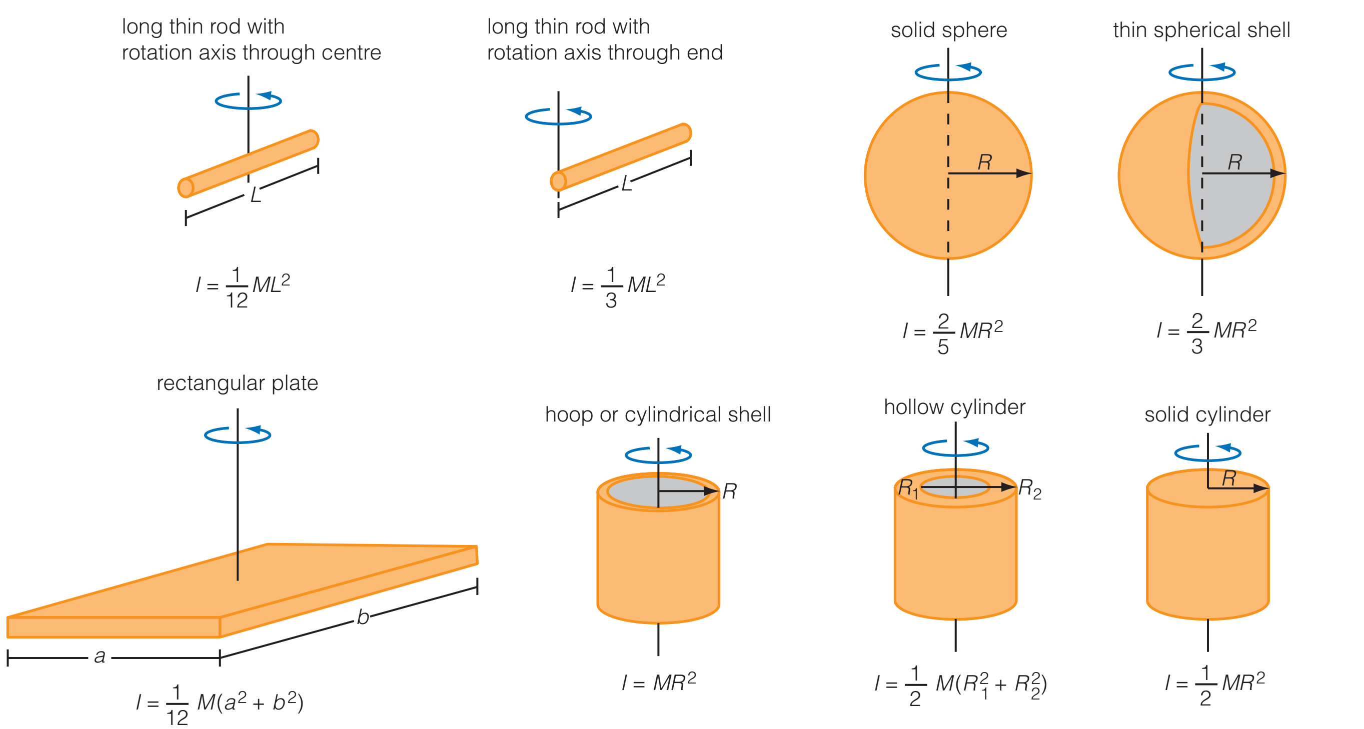


Figure 33.18 Moment of inertia of common shapes.

TIP
You will not be expected to derive $I = \sum m r^2$.

MATHS BOX

The derivation of the moment of inertia of a uniform rod rotating about its end is shown here for the interested mathematician.

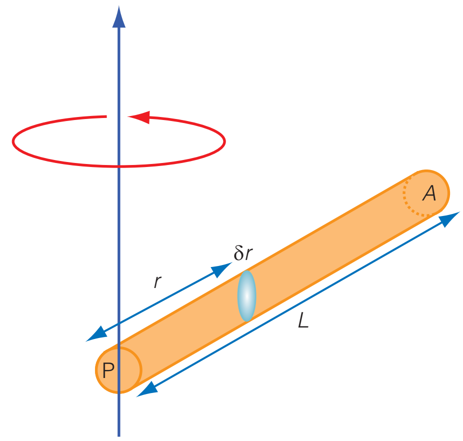


Figure 33.19

The rod has a density ρ, cross-sectional area A and length L. So

$$M = \rho A L$$

The moment of inertia about P, ΔI for the small section Δr is

$$\Delta I = \rho A \Delta r \times r^2$$

So the total moment of inertia is the sum of all the small mass elements:

$$I = \int_0^L \rho A r^2 \delta r = \frac{1}{3}\rho A L^3 = \frac{1}{3} M L^2$$

EXAMPLE

1 A constant force of 50 N acts on a wheel of radius 20 cm which rotates about its centre. The moment of inertia of the wheel is 9.0 kg m^2.

a) Calculate the resultant torque acting on the wheel.

Answer

$T = F r$

$= 50\,\text{N} \times 0.2\,\text{m}$

$= 10\,\text{N m}$

b) Calculate the acceleration of the wheel caused by the torque and state any assumptions you are making.

Answer

$$\alpha = \frac{T}{I} = \frac{10}{9} = 1.11\ \text{rad s}^{-2}$$

We are assuming the force is applied tangentially.

2 An external torque of 40 N m is applied to a wheel rotating on bearings against a constant frictional force of 12 N m for 20 seconds after which it is removed. The wheel is initially stationary and reaches an angular speed of 760 rpm after 20 seconds.

a) Calculate the moment of inertia of the wheel.

Answer

Resultant torque = 40 N m − 12 N m = 28 N m

$\omega_1 = 0\ \text{rad s}^{-1}$

$\omega_2 = 760\ \text{rpm}$

$\omega_2 = 760\ \text{rpm} \times \left(\frac{2\pi}{60\,\text{s}}\right) = 79.6\ \text{rad s}^{-1}$

$$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{(79.6\ \text{rad s}^{-1} - 0)}{20\,\text{s}}$$

$= 3.98\ \text{rad s}^{-2}$

$$I = \frac{T}{\alpha} = \frac{28\ \text{N m}}{3.98\ \text{rad s}^{-2}}$$

$= 7.0\ \text{kg m}^2$

b) Calculate how long the wheel takes to come to rest after the external torque is removed.

Answer

When the external torque is removed the wheel slows down as there is a resultant torque of 12 N m acting against its motion due to friction between the axle and bearings.

$$\alpha = \frac{T}{I} = \frac{-12\text{ N m}}{7.0\text{ kg m}^2}$$

$$= -1.7\text{ rad s}^{-2}$$

Using

$$\omega_2 = \omega_1 + \alpha t$$

$$0\text{ rad s}^{-1} = 79.6\text{ rad s}^{-1} - 1.7\text{ rad s}^{-2}\, t$$

$$t = \frac{79.6\text{ rad s}^{-1}}{1.7\text{ rad s}^{-2}}$$

$$= 46.8\text{ s}$$

TEST YOURSELF

Torque and moment of inertia

1 Find the moment of inertia for

a) a 2 kg mass on the end of a massless rope that is 3 m long

b) a helicopter rotor which is made of three blades (treated as rods), each of mass = 30 kg and length = 7.0 m. The blades are fixed at an angle of 120° relative to each other.

$$I_{rod} = \frac{1}{3}MR^2$$

2 A shelf is supported by a metal rod which is pivoted at one end. The rod support has a mass of 0.5 kg and is 50 cm long. When released from the horizontal position it accelerates downwards towards a vertical position. The moment of inertia of a uniform rod is $\frac{1}{3}ML^2$. Calculate

a) the angular acceleration of the rod shelf support

b) the linear acceleration of the right-hand tip of the rod. Comment on your answer.

3 Two 10 kg masses are placed on opposite ends of a 2 m long bar at a gym.
Calculate the moment of inertia if

a) the bar is rotated at the midpoint

b) the bar is rotated a distance of 50 cm from the left-hand mass.

You may ignore the mass of the bar.

4 A 20 N force is applied to a pulley which has a mass of 5 kg and a radius of 45 cm. The force accelerates the pulley from rest to an angular speed of 40.0 rad s^{-1} in 5.0 s. Frictional forces apply a torque of 1.5 N m about the axle. Calculate the moment of inertia of the pulley.

5 What continuous force would need to be applied at the equator to reduce the Earth's rotation to zero in 12 hours?

$$I_{sphere} = \frac{2}{5}MR^2$$

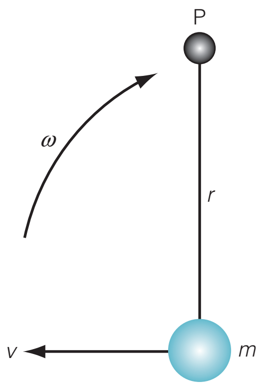


Figure 33.20

Rotational kinetic energy

An object which is rotating will have kinetic energy as it has angular velocity. A rotating object can do work or transfer energy to another body.

The kinetic energy E_K of a moving mass m which is rotating about point P is

$$E_K = \frac{1}{2} m v^2$$
$$= \frac{1}{2} m r^2 \omega^2$$
$$= \frac{1}{2} I \omega^2$$

So, in general we can say that the rotational kinetic energy of a body is

$$E_K = \frac{1}{2} I \omega^2$$

Objects which are rotating may also be moving through space. The total E_K will be the sum of the rotational E_K and translational E_K.

MATHS BOX

We can calculate the total rotational E_k of a body by summing the contribution from each mass element.

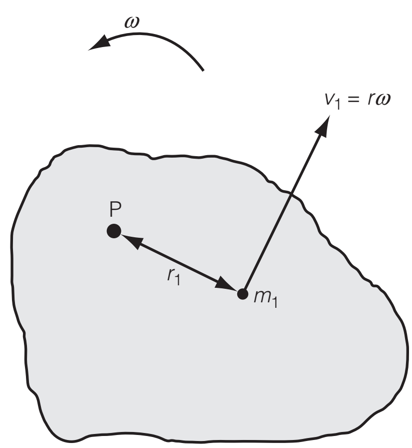


Figure 33.21 Rotational kinetic energy.

The kinetic energy of the mass element m_1 shown in Figure 33.21 is $\frac{1}{2} m (v_1)^2$.

The total kinetic energy of the object will be equal to the sum of the kinetic energy of the individual mass elements.

$$\text{Total kinetic energy} = \frac{1}{2} m(v_1)^2 + \frac{1}{2} m(v_2)^2 + \frac{1}{2} m(v_3)^2 + \dots + \frac{1}{2} m(v_n)^2$$

We can write this in terms of angular velocity as

$$v = r\,\omega$$

So

$$\text{total kinetic energy} = \frac{1}{2} m(r_1\omega)^2 + \frac{1}{2} m(r_2\omega)^2 + \frac{1}{2} m(r_3\omega)^2 + \dots + \frac{1}{2} m(r_n\omega)^2$$

and as was shown in the moment of inertia section (see page 16)

$$I = \Sigma m\, r^2$$

Therefore

$$\text{total rotational kinetic energy} = \frac{1}{2} I\, \omega^2$$

EXAMPLE

1 Calculate the angular speed of a spinning basketball if its moment of inertia is 12 kg m^2 and its rotational kinetic energy is 105 J.

Answer

$$E_K = \frac{1}{2} I \omega^2$$

$$\omega = \sqrt{\frac{2 E_K}{I}}$$

$$= \sqrt{\frac{2 \times 105}{12}} = 4.2 \text{ rad s}^{-1}$$

2 Calculate the kinetic energy of a cylinder of mass 15 kg and radius 0.3 m if it is rolling with a translational velocity of 0.5 m s^{-1}.

$I_{cylinder} = 0.3$ kg m^2

Answer

$$\omega = \frac{v}{r} = \frac{0.5 \text{ ms}^{-1}}{0.3 \text{ m}}$$

$$= 1.67 \text{ rad s}^{-1}$$

total E_K = translational E_K + rotational E_K

$$= \frac{1}{2}mv^2 + \frac{1}{2} I \omega^2$$

$$= \left(\frac{1}{2} \times 15 \times 0.5^2\right) + \left(\frac{1}{2} \times 0.3 \times 1.67^2\right)$$

$$= 2.29 \text{ J}$$

Flywheels

Rotational kinetic energy is put to use by machines which use flywheels. These are usually large and heavy and so have a large moment of inertia.

Figure 33.22 A rim type flywheel forming part of an early steam engine. This design gave a larger moment of inertia than for a solid disc design of the same mass as the mass is positioned further from the axis of rotation.

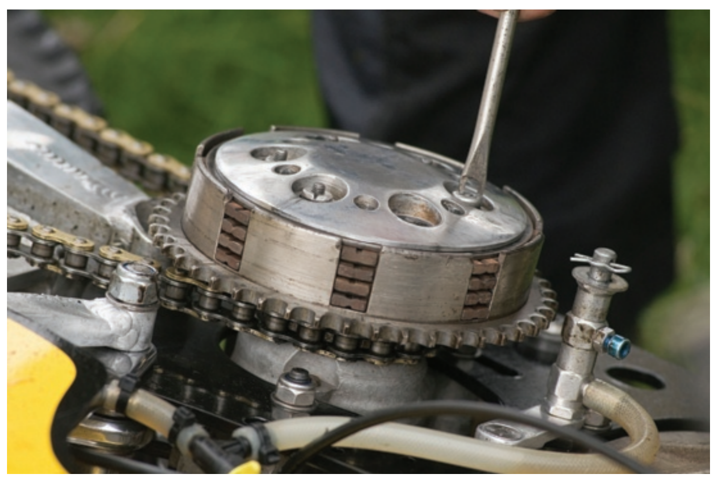


Figure 33.23 A scooter flywheel helps the engine to idle smoothly. The flywheel may also contain an integral fan to cool the engine. It also rotates about magnets which use electromagnetic induction to produce electrical power.

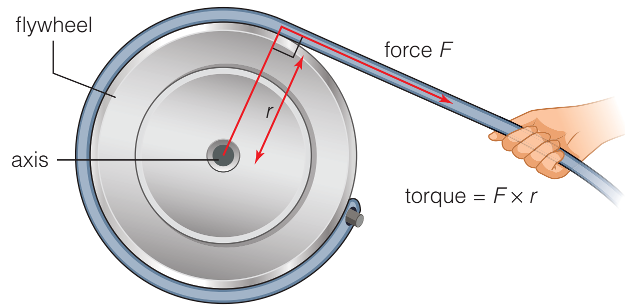


Figure 33.24 A torque applied to a flywheel.

Flywheels are used to smooth out torque and angular speed and can store energy, releasing it when the vehicle or machine requires additional energy to do work. The flywheel helps steady the rotation of the shaft when a fluctuating torque is exerted on it by the pistons. It smooths out the pulsing from the engine.

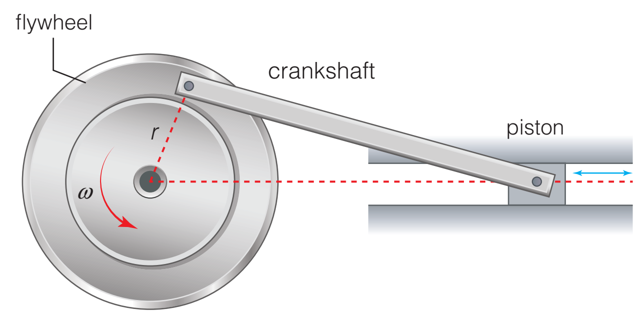


Figure 33.25 To smooth torque a flywheel is attached to the crankshaft.

The crankshaft translates the linear oscillations of the pistons into rotational energy to drive the wheels. The heavy flywheel stores rotational kinetic energy and releases it when the engine pistons are not delivering power. This helps to ensure the wheels turn smoothly and can also help the wheels turn when the vehicle is moving downhill and the engine is not connected (i.e. idling).

Some vehicles use flywheels to store kinetic energy when braking. Instead of this energy being wasted as thermal energy in the brake blocks and surroundings, the translational kinetic energy of the vehicle can be transferred to the rotational E_K of the flywheel. The linear E_K can then be returned to the vehicle when the driver wishes to accelerate.

Factors affecting the energy stored by a flywheel

The mass and shape of the flywheel both affect the moment of inertia and so affect the amount of energy stored, as total rotational kinetic energy $= \frac{1}{2} I \omega^2$.

The angular speed is limited by the breaking stress that the material from which the flywheel is made can experience. Developments in new composite materials mean that flywheels have now been developed which can rotate at speeds over 50 000 rpm.

EXAMPLE

1 A flywheel is made of a solid disc of thickness t, density ρ and radius R. By what factor does the E_K stored increase if the flywheel's radius and thickness are doubled and it is spun at half the speed?

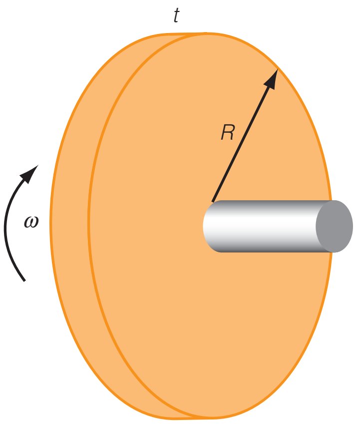


Figure 33.26

Answer

$$E_K = \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} \times \frac{1}{2} MR^2\omega^2$$

$$= \frac{1}{4} MR^2\omega^2$$

$$M = \rho\pi R^2 t$$

Therefore

$$E_K = \frac{1}{4}\left(\rho \pi R^2 t\right) R^2\omega^2$$

$$= \frac{1}{4}\rho\pi R^4 t\,\omega^2$$

Wheel 1: R, t, ω

Wheel 2: $2R$, $2t$, $\frac{\omega}{2}$

So

$$\frac{E_{K2}}{E_{K1}} = \frac{\frac{1}{4}\rho\pi(2R)^4(2t)\left(\frac{\omega}{2}\right)^2}{\frac{1}{4}\rho\pi R^4(2t)\omega^2}$$

$$\frac{E_{K2}}{E_{K1}} = 2^4 \times 2 \times \frac{1}{4}$$

$$= 8$$

2 How does the energy stored in a spoked flywheel compare with one made from a solid disc? Calculate the angular velocity a solid disc flywheel would need to rotate at in order to store the same energy as a spoked flywheel of the same mass.

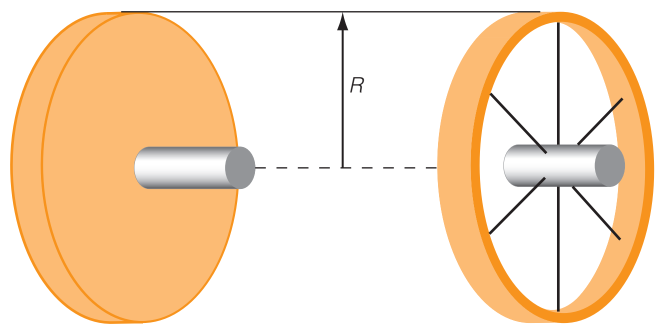


Figure 33.27

Answer

For the spoked flywheel, as the spokes are relatively thin we can neglect their mass and assume that all of the mass lies in the rim. The moment of inertia of the spoked wheel is therefore twice that of the solid disc and so for the same mass and angular velocity it would store twice the E_K.

$$I_{spoke} = 2 \times I_{solid}$$

but to have the same E_K

$$\frac{1}{2} I_{spoke}\,\omega_{spoke}^2 = \frac{1}{2} I_{solid}\,\omega_{solid}^2$$

so

$$2 \times I_{solid} \times \omega_{spoke}^2 = I_{solid}\,\omega_{solid}^2$$

$$\omega_{solid} = \sqrt{2}\,\omega_{spoke}$$

3 State and explain two ways to increase the energy storage capacity of a flywheel.

Answer

Rotational E_K is proportional to the moment of inertia and the angular velocity squared. If the mass of the flywheel is increased then the E_K will increase. By moving more mass to the edge of the flywheel the moment of inertia would increase. If the angular speed is doubled then the E_K stored will increase by a factor of 4. This means that even small increases in the angular velocity of the flywheel will result in a large increase in the E_K stored.

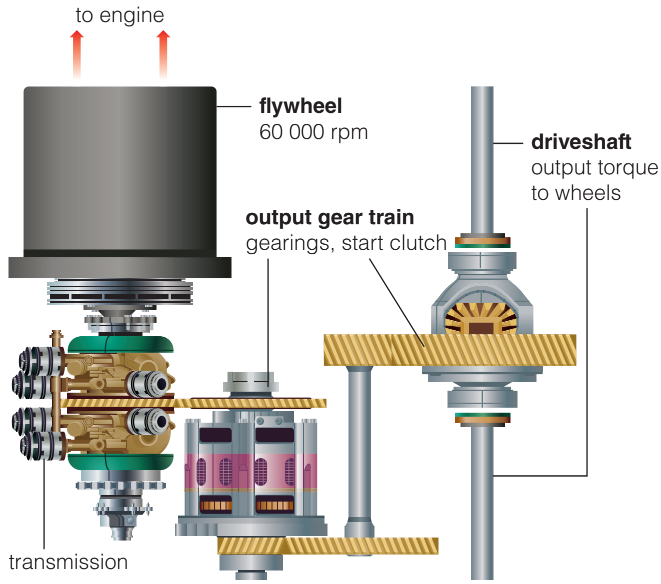


Figure 33.28 KERS flywheel system layout.

Uses of flywheels

Flywheels are used in a wide range of applications.

KERS (Kinetic Energy Recovery System)

KERS is a system for recovering a moving vehicle's kinetic energy during braking. The recovered energy is stored in a reservoir (for example a flywheel or high voltage batteries) for later use when accelerating. This can add more power whilst also increasing efficiency. These systems have been used on Formula 1 cars and are now even used on some London buses.

Flywheels as mechanical batteries

Flywheel energy storage can be used to overcome the limitations of intermittent energy supplies, such as solar photovoltaics or wind turbines that are not able to produce electricity at a constant rate as wind turbines rotate at variable speeds depending on the wind.

A flywheel energy storage system can be thought of as a mechanical battery, converting and storing the energy as kinetic energy until it is needed. Electricity can then be generated from the spinning flywheel when required.

By developing low friction bearings, energy losses can be reduced further. Such mechanical batteries could be a replacement for the chemical batteries currently used to store electricity.

Figure 33.29 A metal press uses a flywheel to assist the motor.

Machine tools

Heavy machinery often requires a large work output in a small time. Machines to work sheet metal for car body parts or for punching out parts are driven by high pressure liquid or air. They incorporate a flywheel to assist the motor and supply energy when required for the heavy, short duration task. If a motor was used by itself it would stall as the energy requirement is too great.

EXAMPLE

A prototype solid disc flywheel has a mass of 10 kg, a diameter of 40 cm and spins at a rate of 100 000 rpm.

a Calculate the rotational speed of a point on the rim of the flywheel.

Answer

$$f = \frac{100\,000}{60} = 1666.7 \text{ rps}$$

so

$$\omega = 2\pi \times 1666.7 = 10\,472 \text{ rad s}^{-1}$$

$v = r\,\omega = 0.2 \times 10\,472 = 2100\ \mathrm{ms^{-1}}$

(this is about six times the speed of sound in air).

b Comment on your answer to part **(a)**.

Answer

At such a high speed the wheel would be placed under enormous stress and could break apart. The bonds holding the solid together may be unable to provide the large centripetal force required. New, composite materials are being developed which are able to withstand such large 'centrifugal' forces.

c Calculate the energy stored by this flywheel in J and in kWh.

Answer

$$I = \frac{1}{2}MR^2$$
$$= \frac{1}{2} \times 10 \times 0.2^2$$
$$= 0.2\ \mathrm{kg\ m^2}$$
$$E_K = \frac{1}{2}I\,\omega^2$$
$$= \frac{1}{2} \times 0.2 \times 10\,472^2$$
$$= 10.97 \times 10^6\ \mathrm{J} = 11\ \mathrm{MJ}$$

1kWh = 3 600 000 J

therefore

$$\text{energy stored} = \frac{11 \times 10^6}{3.6 \times 10^6} = 3\ \mathrm{kWh}$$

Measuring the moment of inertia of a flywheel

One method of measuring the moment of inertia of a flywheel is by allowing it to be accelerated by a falling mass. The loss in gravitational potential energy of the falling mass is converted to rotational and translational kinetic energy.

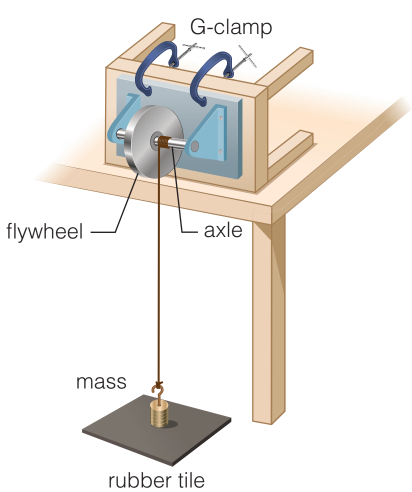


Figure 33.30 Measuring the moment of inertia of a flywheel.

Measurements

The flywheel is rotated anticlockwise and as it rotates the masses are lifted up and the string becomes wrapped around the axle of the flywheel.

When the masses reach the top they are released. The string begins to unwind as the masses fall and the flywheel accelerates. The time taken for the masses to fall a measured height is recorded.

The gravitational potential energy of the falling masses can now be calculated.

The final angular velocity is needed in order to be able to calculate the rotational kinetic energy gained by the flywheel.

The number of complete rotations of the flywheel is used to determine its average angular speed. The number of complete rotations can be determined by knowing the distance fallen by the masses and the circumference of the axle. The string unravels as the masses fall and so

$$\text{number of rotations made by the flywheel} = \frac{\text{distance fallen by masses}}{\text{circumference of axle}}$$

The number of rotations can then be converted into angular displacement by multiplying by 2π.

$$\text{average angular velocity} = \frac{\text{angular displacement}}{\text{time}}$$

so if it is assumed that the flywheel has uniform angular acceleration the final angular velocity can be calculated as

$\omega_2 = 2 \times$ average angular velocity

This can also be used to determine the final linear velocity as $v = r\,\omega$.

Therefore the radius of the axle where the string was wrapped around should also be measured.

EXAMPLE

Use the following data to determine the moment of inertia of the flywheel and comment on the answer.

A 0.5 kg mass hanging from a string is used to accelerate a flywheel of radius 6 cm from rest.

The mass falls through a distance of 2 metres in 9.6 seconds.

The diameter of the axle is 7.5 mm.

You will need to calculate:

1 Number of rotations of the flywheel
2 Average angular velocity
3 Maximum angular velocity
4 Maximum linear speed of falling mass (you cannot use the linear equations of uniform acceleration as the acceleration of the mass is not 9.81 ms^{-2})
5 Loss in gravitational potential energy of the masses.

Answers

Number of rotations made by the flywheel = distance fallen by mass × circumference of axle

$$n = \frac{2\,\text{m}}{\pi \times 7.5 \times 10^{-3}} = 84.9 \text{ rotations}$$

$$\text{Average angular speed} = \frac{\text{angular displacement}}{\text{time}}$$

$$= \frac{84.9 \times 2\pi}{9.6\,\text{s}} = 55.6 \text{ rad s}^{-1}$$

Wheel accelerates uniformly so maximum angular velocity

$\omega_2 = 2 \times$ average angular velocity

$\omega_2 = 55.6 \times 2 = 111.2 \text{ rad s}^{-1}$

Maximum linear speed of falling mass

$v = r\,\omega_2 = 3.75 \times 10^{-3} \text{ m} \times 111.2 \text{ rad s}^{-1}$

$= 0.417 \text{ ms}^{-1}$

Loss in GPE of falling mass = gain in KE of mass + gain in rotational KE of flywheel

$$mg\Delta h = \left(\frac{1}{2}m\,v^2\right) + \left(\frac{1}{2}\,I\,\omega^2\right)$$

$$0.5 \times 9.81 \times 2 = (0.5 \times 0.5 \times 0.417^2) + (0.5\,I \times 111.2^2)$$

$$I = \frac{9.77}{6183}$$

$$= \mathbf{1.56 \times 10^{-3}\ kg\ m^2}$$

Evaluation

The moment of inertia is actually smaller than this value as we have neglected the effect of friction between the bearings and axle and also any effects of drag. The angular acceleration of the flywheel will not be uniform as frictional forces will increase with velocity. The string is likely to double up on top of itself as it wraps around the axle. This will decrease the actual number of rotations completed by the flywheel.

EXTENSION TASK – MEASURING THE MOMENT OF INERTIA OF A YO-YO

Could you design an experiment to measure the moment of inertia of a yo-yo? What measurements would you make and what simplifying assumptions would be required?

Work, energy and power

Work has to be done on an object to cause it to rotate about an axis. Examples where large amounts of work (and correspondingly high torques) are required are swing bridges, rotating cranes and some fairground or adventure-park rides. Figure 33.31 shows a constant force, *F*, being applied at a tangent to the rim of a wheel.

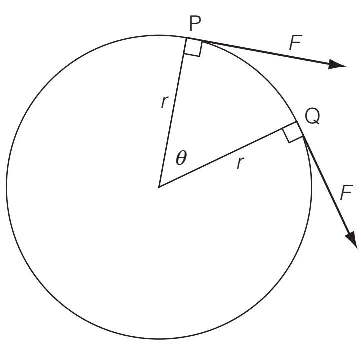


Figure 33.31 Work done by torque.

As the wheel rotates the force remains constant in magnitude and stays tangential. If the force causes the wheel to rotate through an angle, θ then

Work done = force × distance PQ

$= F \times r\,\theta$

$=$ torque $\times\ \theta$

= KE gained

and the work done by the torque is equal to the increase in energy stored in the wheel.

EXAMPLE

1 Calculate the work done by a torque of 8 N m in rotating a wheel through 5 revolutions.

Answer

Work done = torque × θ

$= 8\ \text{N m} \times 5 \times 2\pi$

$= 251\ \text{J}$

2 Calculate the number of revolutions a wheel completes after a constant braking torque of 8 N m is applied. The moment of inertia of the wheel is 3 kg m^2 and it is rotating about its centre with an angular velocity of 20 rad s^{-1} when the braking torque is applied.

Answer

Work done by torque = decrease in KE

$$T\theta = \frac{1}{2} I \omega^2$$

$$8 \times \theta = \frac{1}{2} \times 3 \times 20^2$$

$$\theta = 75 \text{ rad}$$

$$\text{Number of complete revolutions} = \frac{75 \text{ rad}}{2\pi} = 11.9 \approx 11$$

Power

Power is the rate of doing work or rate of energy transfer:

$$\text{Power} = \frac{\text{work done}}{\text{time}} \text{ and as } W = T\theta$$

$$\text{power} = \text{torque} \times \frac{\theta}{\text{time}}$$

Angular velocity is defined as

$$\omega = \frac{\theta}{t}$$

and therefore

$$\text{power} = \text{torque} \times \text{angular velocity}$$

$$= T\,\omega$$

This can be used to calculate the power delivered by an applied torque or the work done per second by a frictional torque.

EXAMPLE

A circular saw blade accelerates uniformly from rest to 620 rad s^{-1} in 3 seconds. If the saw has a moment of inertia of 5.9×10^{-4} kg m^2, calculate the average power transferred to accelerate the blade. You may assume friction is negligible.

Answer

$$\text{Rotational } E_K \text{ gained} = \tfrac{1}{2} I \omega^2$$

$$= \tfrac{1}{2} \times 5.9 \times 10^{-4} \text{ kg m}^2 \times (620 \text{ rad s}^{-1})^2$$

$$= 113.4 \text{ J}$$

$$\text{Power} = \frac{E_K \text{ gained}}{\text{time}}$$

$$= \frac{113.4}{3}$$

$$= 37.8 \text{ W}$$

TEST YOURSELF

Angular kinetic energy, work and power

1 Flywheels have been tested as energy storage devices for small-scale power systems. Estimate the kinetic energy which could be stored in a 60 tonne flywheel with a diameter of 10 m when rotating at 120 rpm. The flywheel has a moment of inertia of $\frac{1}{2}Mr^2$.

2 A hurricane can be thought of as a rotating cylinder of air. If the rotation is assumed to be constant, calculate the maximum E_K stored in a hurricane of diameter 200 km, height 4 km and whose speed at the outer edge is 180 Km h^{-1}.

The density of air is 1.2 kg m^{-3} and the moment of inertia of a cylinder is $\frac{1}{2}Mr^2$.

3 Calculate the work done per second by a small aeroplane engine in turning the propeller with a speed of 2000 rpm, if the engine applies a torque of 800 N m.

4 A truck can be run for a short time on the energy stored in a flywheel. The flywheel has a mass of 500 kg, a radius of 1.2 m and a rotational speed of 628 π rad s^{-1}.

a) Calculate the kinetic energy of the flywheel.

b) If the truck uses an average power of 10 kW, calculate for how many minutes it can operate using only the energy stored in the flywheel.
The moment of inertia of a solid disc can be calculated using $I = \frac{1}{2}MR^2$.

Stretch and challenge

5 A wheel of mass 5 kg is pulled up a ramp which is angled at 25° to the horizontal. The 40 N force acts parallel to the ramp. If the wheel has a radius of 0.4 m and moment of inertia of 0.6 kg m^2, calculate the translational velocity the wheel has gained after travelling 10 m along the ramp.

6 Two identical solid balls of the same mass and radius are released from the top of two ramps, one ramp has friction and so enables the ball to roll, the other is frictionless and so the ball slides without rolling.

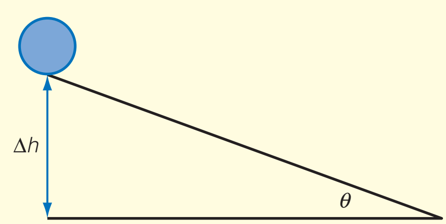


Figure 33.32

Both ramps make an angle of θ to the horizontal. If the height of the ramp is 1.5 m and it makes an angle of 10° to the horizontal, calculate which ball reaches the bottom first. You can assume the moment of inertia of a sphere is $\frac{2}{5}Mr^2$.

Angular momentum

Linear momentum is defined as the product of the mass and the velocity of a moving object. Objects which are rotating also possess angular momentum. We can calculate the **angular momentum** of a single particle moving in a circular path.

The angular momentum L of a particle about an axis is defined as the **moment** of momentum:

$$L = \text{momentum} \times r$$
$$= m\, v\, r$$
$$= mr^2\, \omega$$
$$= I\, \omega$$

This can be compared to linear momentum, $p = mv$, where mass is replaced by moment of inertia and velocity by angular velocity.

Angular momentum of a rotating body $= I\,\omega$

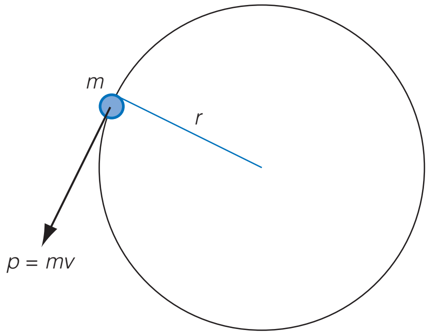


Figure 33.33 The angular momentum (L) of a particle moving in a circle.

Conservation of angular momentum

Just as with linear momentum, **angular momentum is also conserved**.

Newton's second law of motion for a body undergoing linear motion states that:

resultant force × time = change in momentum = $mv - mu$

For a rotating object we can write that:

resultant torque × time = change in angular momentum

Replacing mass in the linear version of Newton's second law with moment of inertia and velocity with angular velocity results in

$$\Sigma T \times t = I\,\omega_1 - I\,\omega_2$$
$$\Sigma T \times t = I\,\Delta\omega$$
$$T\Delta t = \Delta(I\omega)$$

This is the rotational version of Newton's second law of motion.

The **law of conservation of angular momentum** is stated as follows: 'The angular momentum about an axis is constant if no external torque acts about that axis'.

Angular impulse = change in angular momentum

TIP

In many examples the moment of inertia can also change. Due to the law of conservation of angular momentum this will lead to a change in the angular velocity of the rotating object without requiring an external torque to be applied and so if $\Sigma T = 0$, the law of conservation of angular momentum can be written as

$$I_1\,\omega_1 = I_2\,\omega_2.$$

EXAMPLE

A wheel has a moment of inertia of 3 kg m^2 about its centre. Calculate the steady braking torque required to decelerate it from an angular velocity of 18 rad s^{-1} to 10 rad s^{-1} in 7 seconds.

Answer

$$T \times t = I\,\omega_1 - I\,\omega_2$$
$$T \times 7 = 3\,(18 - 10)$$
$$T = \frac{24}{7}$$
$$T = 3.4 \text{ N m}$$

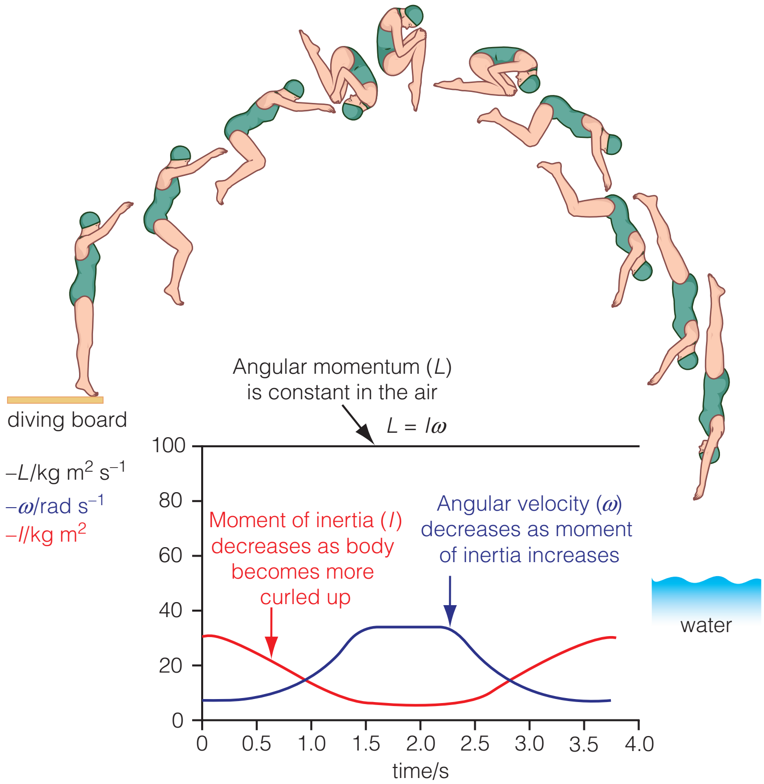


Figure 33.34 Conservation of angular momentum in a dive.

Applications of conservation of angular momentum

The angular velocity of a spinning ice skater or dancer increases if they pull in their arms as this reduces their moment of inertia and so increases their angular velocity. Collapsing stars also begin to spin faster because as their radius reduces, so does their moment of inertia. As $I = \Sigma mr^2$ a high diver will decrease their moment of inertia by curling up their body (decreasing r) leading to an increase in their angular velocity. They will therefore complete more somersaults before they hit the water.

Figure 33.34 shows the conservation of momentum in a dive. At any moment total $L = I\ \omega$ as there is no external torque acting on the diver. The diver therefore spins fastest when the distance to the axis of rotation is smallest.

EXAMPLE

1 A dancer spins about a vertical axis with her arms outstretched with a speed of 3 revolutions per second (rps). She then moves her arms to her side and reduces her moment of inertia to two-thirds of the value it was with arms outstretched. Calculate her new angular speed.

Answer

$$I_1\ \omega_1 = I_2\ \omega_2$$

$$I_1 \times 3 = \frac{2}{3} \times I_1 \times \omega_2$$

$$\omega_2 = 4.5 \text{ rev s}^{-1}$$

2 A student sits on a rotating stool holding two weights, each of mass 2.00 kg. When her arms are extended horizontally, the weights are 0.70 m from the axis of rotation and the student rotates with an angular speed of 1.5 rad s^{-1}.

While spinning, the student pulls the weights inward horizontally to a position 0.20 m from the axis of rotation.

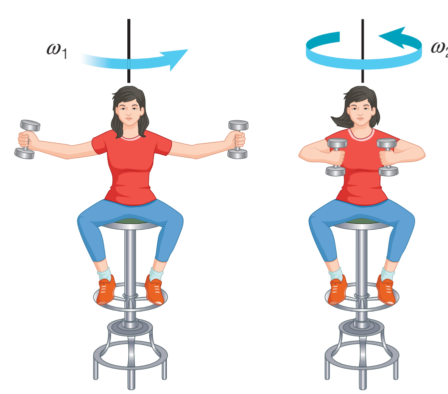


Figure 33.35

Calculate the angular speed of the student after the weights have been pulled inward.

The moment of inertia of the student plus stool is 3.00 kg m^2.

Answer

$I = I_{student} + I_{weight\ 1} + I_{weight\ 2}$

$I_1 = 3 + (2 \times 0.7^2 + 2 \times 0.7^2) = 4.96$ kg m^2

$I_2 = 3 + (2 \times 0.2^2 + 2 \times 0.2^2) = 3.16$ kg m^2

No external torque is applied and so from the law of conservation of angular momentum

$I_1\ \omega_1 = I_2\ \omega_2$

4.96 kg m^2 × 1.5 rad s^{-1} = 3.16 kg m^2 × ω_2

ω_2 = 2.35 rad s^{-1}

3 A potter's wheel is rotating freely at a speed of 30 rpm about its centre. It has a moment of inertia of 1.5 kg m^2. The potter drops a lump of clay of mass 0.8 kg onto the wheel 30 cm from its axis. Explain what happens to the speed of the wheel and calculate by what percentage the wheel's rotation changes.

Answer

The angular velocity of the wheel decreases because mass has been added and therefore the moment of inertia increases as $I = \Sigma Mr^2$. There is no external torque so for angular momentum to be conserved ($I_1\ \omega_1 = I_2\ \omega_2$) the angular velocity must decrease as I_2 has increased.

$\omega_1 = 30 \text{ rpm} \times \left(\frac{2\pi}{60}\right) \text{s}$

= 3.14 rad s^{-1}

By applying the law of conservation of momentum, the moment of inertia of the wheel is increased by $I = Mr^2$ after the clay is added so

$I_2 = I_1 + Mr^2$

where M = mass of clay added to the wheel and r = distance between the clay and the axis of rotation.

$I_2 = 1.5 + (0.8 \times 0.30^2) = 1.572$

$I_1\ \omega_1 = I_2\ \omega_2$

1.5 kg m^2 × 3.14 rad s^{-1} = 1.572 kg m^2 × ω_2

ω_2 = 3.00 rad s^{-1}

The wheel slows down by only a small amount as the additional moment of inertia of the clay is small compared with the large moment of inertia of the wheel.

% decrease in angular velocity of wheel = 4%

$\left(\frac{\omega_2}{\omega_1} = \frac{3.00 \text{ rad s}^{-1}}{3.14 \text{ rad s}^{-1}} \times 100 = 96\%\right)$

Simple clutches

A clutch is a mechanical device which enables the transfer of rotational power from the engine of a car to the wheels. In a manual transmission car, the clutch controls the connection between the rotating shaft coming from the engine and the shafts which turn the wheels. The car's engine generates power all the time, and has parts which are constantly rotating, but the wheels are not constantly spinning. To allow the car to both change speed and to come to a complete stop without turning off the engine, the connection between the wheels and the engine needs to be temporarily broken.

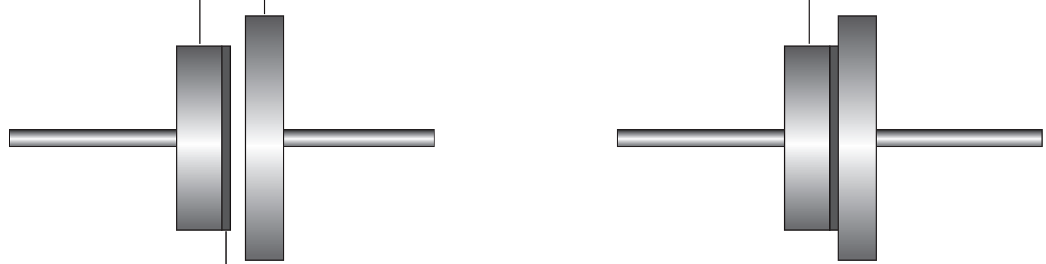


Figure 33.36 A simple (flat) clutch in a manual car. Most real clutches are in fact a conical shape to increase the amount of friction and reduce slipping. This ensures better power transmission.

A set of springs keep a pressure plate pushed up against the clutch plate. This pressure pushes the clutch plate up against the flywheel connecting the engine to the drive shaft which transfers motion to the wheels. When the clutch pedal is pushed down the pressure plate is pulled away from the clutch plate. This breaks the connection between the rotating engine and the wheels, meaning that the wheels continue to spin but under their own inertia, not through the power of the engine. This design allows the driver to disengage the wheels from the engine in order to change gear.

Conservation of angular momentum can be applied to the two plates.

Total angular momentum before engagement = $I_1 \omega_1 + I_2 \omega_2$

Total angular momentum after engagement = $(I_1 + I_2)\, \omega$

where ω is the common angular velocity of the two plates after they engage.

$$\omega = \frac{(I_1 \omega_1 + I_2 \omega_2)}{(I_1 + I_2)}$$

TEST YOURSELF

Angular momentum

1 A figure skater can reduce their rate of rotation from an initial rate of 1 revolution every 1.8 s to a final rate of 3 revolutions per second.

   a) Calculate their initial angular velocity in rad s^{-1}.

   b) If their initial moment of inertia was 5.1 kg m^2, use the law of conservation of angular momentum to calculate their final moment of inertia.

   c) Explain how they physically changed their moment of inertia.

2 A bullet strikes the outer edge of a cylinder as shown in Figure 33.37 and becomes embedded in the cylinder.

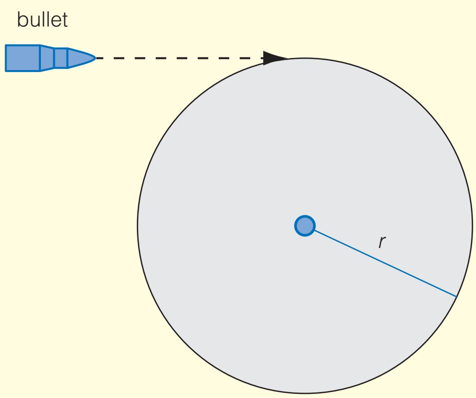


Figure 33.37 Bullet striking the outer edge of a cylinder.

The cylinder has a radius of 30 cm, a mass of 3 kg and is initially at rest.

Just before striking the cylinder the velocity of the 5 gram bullet is 960 m s^{-1}.

The moment of inertia of a cylinder is $\frac{1}{2}Mr^2$.

Use this data to calculate the angular velocity of the cylinder after the bullet becomes embedded.

3 At the end of the Sun's life it will collapse into a white dwarf star, losing half its current mass and with a radius of 1.5% it current value.
If the Sun's current rotation period is 30 days, what will be its rotation period as a white dwarf?
You may assume that the ejected mass does not carry any angular momentum with it.
The moment of inertia of a sphere is $\frac{2}{5}Mr^2$.

4 A person of mass 70 kg stands at the centre of a playground roundabout of radius 2 m which is rotating at 0.9 rad s^{-1}. They walk from the centre to the edge of the roundabout.
The moment of inertia of the roundabout is 950 kg m^2.

   a) Describe what happens to the speed of the roundabout.

   b) Calculate the change in angular velocity after the person completes their walk to the edge.

Rotational and linear motion compared

TEST YOURSELF

Rotational and linear motion compared

1 Complete the table below to compare linear and rotational motion.

Linear motion	Rotational motion
Velocity, v	Angular velocity, $\omega = \frac{v}{r}$
Momentum = mv	
Kinetic energy = $\frac{1}{2}mv^2$	
	Resultant torque, $T = I\alpha$
Change in momentum = $F \times t$	Change in angular momentum = $T \times t$
Work done = change in $E_K = F \times s$	
Power = $\frac{\text{work done}}{\text{time}}$	
Linear momentum is conserved if no external forces act	Angular momentum is conserved if no external torques act, $I_1\omega_1 = I_2\omega_2$

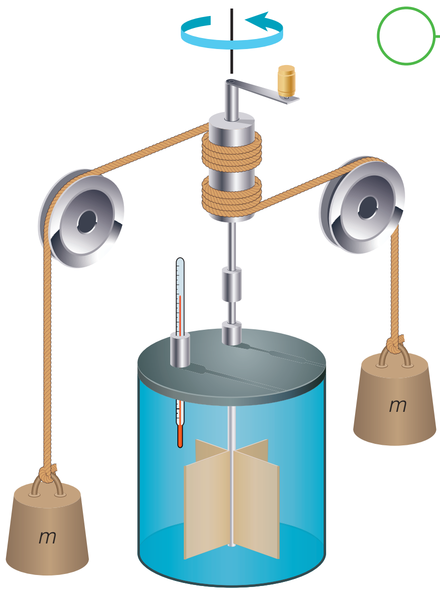


Figure 33.38 Joule's experiment on the mechanical equivalent of heat. As the mass falls and loses gravitational potential energy it causes the paddles to move through the water. Friction between the paddle wheels and the water causes the temperature of the water to increase slightly.

Thermodynamics and engines

The core thermal physics topic introduced you to thermal energy transfer, ideal gases and kinetic theory. This part of the option covers these in greater depth and is also concerned with how these ideas can be applied to making engines and heat pumps.

The first law of thermodynamics

James Joule (1818–1889) carried out many experiments, all of which showed that energy in any one form could be converted into any other form, with a loss of useful energy in the process. Many of his experiments involved doing mechanical work on a liquid or gas and then measuring the temperature increase.

Joule worked during a time of great industrial change and much of his work was concerned with improving the efficiency of steam engines. By carrying out very careful measurements he was able to show that the work done by an engine added to the thermal energy produced by friction and the thermal energy lost from a chimney and exhaust is equal to the thermal energy produced by the burnt fuel. This is led to the principle of conservation of energy.

You should remember from the A-level thermal physics topic that:

Heat = energy flow due to a temperature difference. Energy always transfers from a higher temperature body to a lower temperature body.

Internal energy = sum of the randomly distributed kinetic and potential energies of all its particles. Ideal gases are assumed to have zero potential energy as there are no forces of attraction or repulsion between the particles (apart from when they are colliding).

If a gas is warmed and expands then the heat (Q) given to it causes an increase in the internal energy (ΔU) and also causes work (W) to be done. The first law of thermodynamics enables the link to be made between energy transferred by mechanical means and the energy transferred due to a temperature difference.

TIP

We can express this principle mathematically as an equation:

$Q = \Delta U + W$

where

Q = amount of energy transferred to the system by heating

ΔU = increase in internal energy of the system

W = work done **by** the system.

This is known as the first law of thermodynamics and it enables us to calculate how the internal energy of a body changes as heat is transferred or work is done.

TIP

Sign conventions and the first law. This law historically related to steam engines where the main interest was in the heat input and the work output. If work is done on the system (e.g. to compress a gas) then W is negative. If heat is lost from the system then Q is also negative.

EXAMPLE

1 3000 J of energy is transferred to a system by heating and 1200 J of work is done on the system. Calculate the change in internal energy of the system.

Answer

$Q = \Delta U + W$

$3000\,\text{J} = \Delta U - 1200\,\text{J}$

$\Delta U = 4200\,\text{J}$

2 50 J of energy was transferred to a gas by heating it, causing the gas to expand and do 30 J of work. Calculate the change in internal energy of the system.

Answer

$Q = \Delta U + W$

$50\ \text{J} = \Delta U + 30\ \text{J}$

$\Delta U = 20\ \text{J}$

The internal energy of the gas increases by 20 J. This would be seen as an increase in the rms speed of the molecules and so the temperature of the gas would increase.

3 Calculate the work done when 130 J of energy is transferred to a gas by heating and its internal energy increases by 200 J.

Answer

$Q = \Delta U + W$

$130 = 200 + W$

$W = -70\ \text{J}$

The work done is negative which indicates that work is done on the gas, i.e. the gas is being compressed.

4 Use the first law of thermodynamics to explain why air cools when it expands rapidly.

Answer

$Q = \Delta U + W$

$Q = 0$ as heat has no time to transfer

$-\Delta U = W$

U (internal energy) is related to the temperature of the gas and so if U decreases, the temperature decreases.

The first law of thermodynamics and ideal gases

The work done when a gas expands can be calculated by considering a gas trapped inside a cylinder which is fitted with a movable piston.

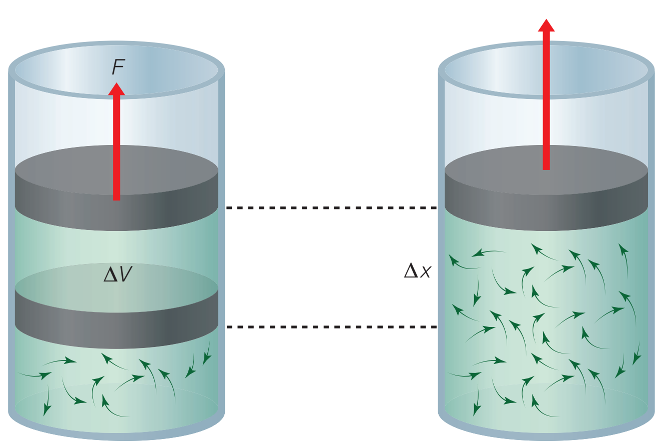


Figure 33.39 A trapped gas does work as it expands and pushes the piston.

Work done by expanding gas = pressure × volume change = $p\Delta V$

Integration can be used for larger changes or when the pressure is not constant.

$$W = \int_{V_1}^{V_2} p\Delta V$$

TIP

Remember to use the correct SI units. For work to be in joules, the pressure must be in Pa or N m^{-2} and the volume in m^3.

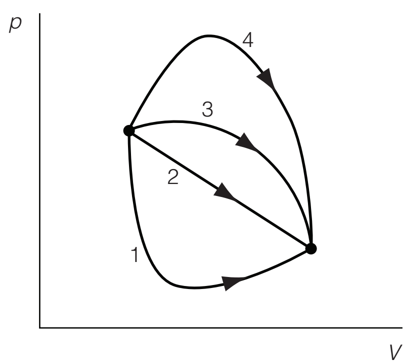


Figure 33.40

The molecules of gas are colliding with the piston and so exerting a force given by

Force = Pressure × Area

If the gas is allowed to expand and moves the piston a distance Δx, the **work done by the expanding gas** = force × distance = $F\,\Delta x$.

$$\text{Work done} = pA\,\Delta x$$
$$= p\Delta V$$

as volume change = $A\,\Delta x$.

EXAMPLE

1 Energy is transferred to a gas by heating, causing the volume to increase from 0.500 m^3 to 0.510 m^3. If the pressure remains constant at 100 000 Pa, calculate the work done by the gas.

Answer

$W = p\Delta V = 100\,000\ \text{Pa} \times 0.010\ \text{m}^3 = 1000\ \text{J}$

2 Calculate the work done when the volume of a gas decreases from 0.300 m^3 to 0.280 m^3 at a constant pressure of 100 000 Pa.

Answer

$W = 100\,000\ \text{Pa} \times (-0.020\ \text{m}^3) = -2000\ \text{J}$

The negative sign indicates that work is being done **on** the gas to compress it.

This work done on the gas would cause a temperature rise or would lead to energy being transferred (i.e. a change in Q).

3 Figure 33.40 shows four paths followed by an ideal gas on a p-V indicator diagram.

Rank the paths according to

a) the change in internal energy of the gas
b) the work done by the gas
c) heat transferred between the gas and the surroundings.

Answer

$Q = \Delta U + W$

a $\Delta U = Q - W$, this is constant for all processes and is independent of the path followed by the gas as it expands. So all four paths result in the same change in internal energy.
b W = area under curve 4, 3, 2, 1
c $Q = \Delta U + W$
As ΔU is the same for all paths, the path resulting in the most work being done also results in the most heat being transferred, in this case from the surroundings to the gas. 4, 3, 2, 1

TEST YOURSELF

First law and work done

1 A sample of gas is contained in a cylinder with a moveable piston of diameter 14 cm. The gas is heated, transferring 600 J of energy to the gas and causing it to expand. As it expands it pushes the piston up 25 cm.

Calculate

a) the work done by the gas

b) the increase in internal energy of the gas.

The external pressure is 110 kPa (atmospheric pressure).

2 Energy is transferred from a gas causing it to cool and its volume decreases from 5 m^3 to 3.5 m^3. Calculate the work done on the gas as it contracts. During this process the gas remains at a constant pressure of 100 000 Pa.

3 800 grams of water is heated until it reaches 100°C and becomes steam. During the change of state from liquid to gas the volume increases by 4 m^3.

a) Calculate the work done by the gas in expanding as it changes state.

b) Explain what happens to the rest of the heat input.

Atmospheric pressure is 101 000 Pa.

Latent heat of vaporisation of water = 2 260 000 J kg^{-1}.

4 An ideal gas is held in a container by a moveable piston and energy is supplied to the gas by heating. This causes it to expand at a constant pressure of 1.2×10^5 Pa.

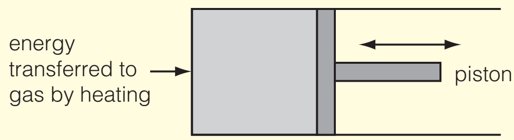


Figure 33.41

The initial volume of the container is 0.050 m^3 and after expansion the volume is 0.10 m^3. The total energy supplied to the gas by heating during the process is 8 kJ.

a) Determine the work done by the gas.

b) Calculate the change in internal energy of the gas.

Ideal gas equation

A transfer of energy or work being done can result in a change in the temperature, volume or pressure of a gas. The following equation applies to an **ideal gas**:

$$pV = nRT$$

where p = pressure (Pa), V = volume (m^3), n = number of moles, R = molar gas constant 8.31 J mol^{-1}, T = temperature (K).

An **ideal gas** is one which obeys the ideal gas equation at all temperatures and pressures. Its internal energy only depends on temperature, i.e. the KE of the particles. Real gases will condense into liquids as they are cooled but the ideal gas equation gives a very good description of the behaviour of most gases at typical temperatures and pressures.

The ideal gas equation for a fixed mass of gas can also be written as

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

since $\frac{pV}{T} = nR$ and nR is constant.

EXAMPLE

1 A cylinder contains 400 cm^3 of gas at a pressure of 5.0 MPa.

a) Calculate the volume of air which escapes when the cylinder is opened to the atmosphere (P = 101 kPa) whilst the temperature remains constant.

Answer

$$p_1V_1 = p_2V_2$$

$$V_2 = \frac{p_1V_1}{p_2}$$

$$= \frac{5 \times 10^6 \text{ Pa} \times 4 \times 10^{-4} \text{ m}^3}{101\,000 \text{ Pa}}$$

$$= 0.0198 \text{ m}^3 = 19\,801 \text{ cm}^3$$

b) Calculate the volume this gas would have at atmospheric pressure, V_2.

Answer

The volume of escaped air = $V_2 - V_1$

$$= 19\,801\text{ cm}^3 - 400\text{ cm}^3$$

$$= 19\,401\text{ cm}^3$$

2 Air is trapped in a rigid bottle which is unable to expand. Its pressure is 120 000 Pa when stored in a room of temperature 15°C. The lid of the bottle will pop off when the pressure increases to 140 000 Pa. Calculate the temperature at which this occurs.

Answer

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

$$T_2 = \frac{p_2 T_1}{P_1} = \frac{140\,000 \times 288}{120\,000} = 336\text{ K} = 63°\text{C}$$

TEST YOURSELF

Ideal gases

1 State what is meant by an ideal gas and explain why the internal energy of an ideal gas is kinetic energy only.

2 The atmospheric pressure is about 100 000 Pa and the temperature about 300 K. Estimate the number of moles of air in the room you are in now.

3 A quantity of 0.35 mol of air enters a diesel engine at a pressure of 1.03×10^5 Pa and a temperature of 30°C.

a) Calculate the volume occupied by the gas.

b) When the gas is compressed to one-twentieth of its original volume the pressure rises to 8.0×10^6 Pa. Calculate the temperature of the gas immediately after the compression.

c) State any assumptions you are making.

TIP

A change which is carried out at constant temperature ($\Delta U = 0$) is called an isothermal process.

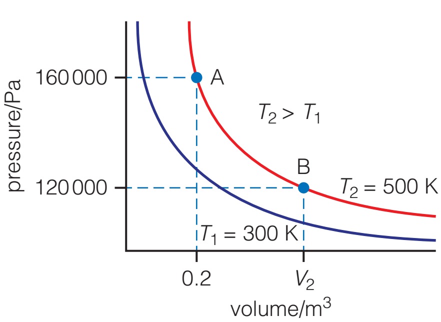


Figure 33.42 *p*-*V* diagram showing isothermal changes at two different temperatures. Each point on the curve shows the pressure and volume of the gas at that moment.

Non-flow processes

Isothermal changes

If the temperature remains constant we can plot a diagram to show how pressure depends on volume. The lines on these indicator diagrams show 'isotherms'. They are '$y = \frac{1}{x}$' curves in accordance with Boyle's Law which showed experimentally that pressure × volume = constant, that is pressure is inversely proportional to volume $\left(p \propto \frac{1}{V}\right)$.

$$T_2 > T_1$$

because $pV = nRT$

If $T_2 > T_1$

$$p_2 V_2 > p_1 V_1$$

EXAMPLE

a Use Figure 33.42 on page 38 to calculate the new volume V_2 as the gas expands from A to B.

Answer

Isothermal change so constant temperature.

$p_2V_2 = p_1V_1$

$120\,000 \times V_2 = 160\,000 \times 0.2$

$V_2 = 0.27\ \text{m}^3$

b Calculate the number of molecules of gas present.

Answer

Use ideal gas equation, $pV = nRT$, for any pair of points on the isothermal

$160\,000\ \text{Pa} \times 0.2\ \text{m}^3 = n \times 8.31 \times 500\ \text{K}$

$n = 7.7\ \text{mol}$

$N = N_A \times n$

$= 6.02 \times 10^{23} \times 7.7$

$= 4.64 \times 10^{24}$

Isobaric change

Isobaric means 'constant pressure'.

As the pressure remains constant the work done can be calculated using $W = p\Delta V$.

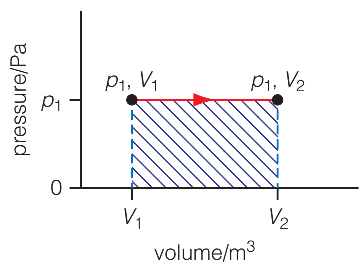


Figure 33.43 Isobaric change.

EXAMPLE

A gas expands at a constant pressure of 100 kPa from a volume of 10 m^3 to a volume of 35 m^3. Calculate the work done by the gas.

Answer

$W = p\Delta V$

$= 100\,000\ \text{Pa} \times (35\ \text{m}^3 - 10\ \text{m}^3)$

$= 2.5 \times 10^6\ \text{J}$

Isobaric changes are changes at constant pressure and are seen as horizontal lines on a p-V diagram.

Isovolumetric changes are changes at constant volume and are seen as vertical lines on a p-V diagram.

In an **isovolumetric change**, no work is done ($\Delta V = 0$, $W = 0$) and so $\Delta U = Q$.

An **adiabatic process (or adiabatic change)** is one in which no heat is allowed to flow into or out of the system, $Q = 0$.

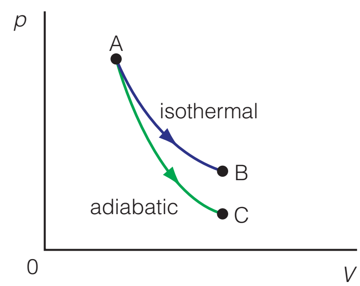


Figure 33.44 p-V diagram showing adiabatic and isothermal changes when a gas expands without any heat flowing into or out of the system.

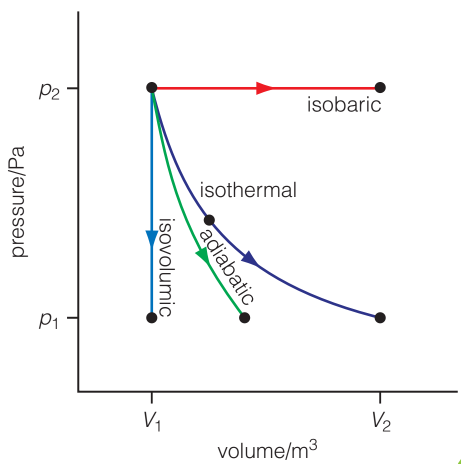


Figure 33.45 Comparing isothermal, isobaric, isovolumetric and adiabatic processes.

TIP

γ depends on the type of gas. For a monatomic gas $\gamma = 1.67$ and for a diatomic gas $\gamma = 1.40$. You will always be given these values in an examination.

TIP

The specification expects you to be familiar with isothermal, adiabatic, constant pressure and constant volume changes

EXAMPLE

A gas is cooled to produce a decrease in pressure. The volume is held constant at 10.0 m^3 and the pressure decreases from 1×10^5 N m^{-2} to 0.5×10^5 N m^{-2}. Calculate the work done on the gas.

Answer

$W = p\Delta V$

No work is done as $\Delta V = 0$.

Adiabatic changes

Adiabatic changes could happen in a system which is very well insulated or where a gas is expanding or being compressed very rapidly before there is time for heat to flow in or out. The rapid expansion of gases in a car internal combustion engine is an example of a process that is very nearly adiabatic.

If adiabatic changes are plotted on a p-V diagram the curve is steeper than for an isothermal change. This is because the adiabatic change does not allow any heat to flow into or out of the system ($Q = 0$) and so if the gas is expanding the internal energy of the gas must decrease ($-\Delta U = W$). In other words, the work done by the gas leads to a corresponding decrease in the internal energy (and therefore a temperature drop) and a corresponding drop in pressure.

For an adiabatic change, $pV^{\gamma} = \text{constant}$

that is, $p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$

You do not need to know the derivation of this.

EXAMPLE

A gas at an initial pressure of 150 kPa is expanded adiabatically until its volume is doubled. If the initial temperature is 280 K, calculate the final pressure and temperature of the gas. $\gamma = 1.40$.

Answer

$p_1 = 150\ 00$ Pa, $p_2 = ?$

$V_2 = 2\,V_1$

$$p_2 = p_1 \left(\frac{V_1^{\gamma}}{V_2^{\gamma}}\right)$$

$$= 150\ 000 \left(\frac{1}{2}\right)^{1.4}$$

$$= 56.8 \text{ kPa}$$

The ideal gas equation can be used to find the temperature.

$pV = nRT$, therefore $\frac{pV}{T} = nR$ and as no gas escapes we can use the ideal gas equation in the form

⇨

MATHS BOX

When dealing with indices, remember

$$x^n \times y^n = (xy)^n$$

so

$$\frac{(V_1)^\gamma}{(V_2)^\gamma} = \left(\frac{V_1}{V_2}\right)^\gamma$$

Try using this rule to tackle the example above.

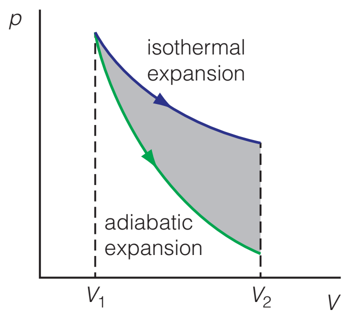


Figure 33.46 *p*-*V* diagram comparing isothermal expansion and adiabatic expansion. More work is done by the gas when it expands isothermally and this additional work is shown by the shaded area (shaded area = work done by gas expanding isothermally – work done by gas expanding adiabatically).

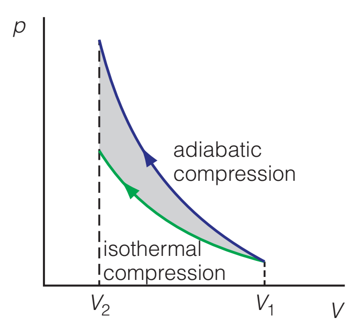


Figure 33.47 *p*-*V* diagram comparing isothermal compression and adiabatic compression. More work is done on the gas when it is compressed adiabatically and this additional work is shown by the shaded area (shaded area = work done on gas compressing adiabatically – work done on compressing gas isothermally).

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\frac{150\,000 \times V_1}{280} = \frac{56\,800 \times 2\,V_1}{T_2}$$

$$T_2 = \frac{113\,600 \times 2800}{150\,000}$$

$$= 212 \text{ K}$$

Kinetic theory in isothermal and adiabatic changes

In an **adiabatic** expansion the gas has done work to expand and the energy to do this work has come entirely from a decrease in the internal energy of the gas (as no energy can flow into the gas). The particles will now be moving less quickly, resulting in fewer collisions with the container walls per second and so the pressure will decrease more rapidly than in an isothermal change where energy can be taken in.

For an **isothermal** change, the temperature and hence E_K of the molecules remains constant and so the number of collisions with the container walls per second does not decrease as rapidly and so the pressure does not decrease as rapidly.

The work done in adiabatic and isothermal changes can be compared in Figures 33.46 and 33.47. The gas has the same initial temperature at the beginning of the processes.

What happens to the temperature of a gas when it is compressed or expands?

Here we will discuss changes which are **not** isothermal.

Consider a gas trapped in a container with a movable piston. As work is done to push the piston in, the gas molecules gain kinetic energy from it and rebound with a greater velocity than their incident velocity. The gain in their E_K is equal to the work done in pushing in the piston. Kinetic theory (see pages 349–352 of the Student's Book) shows that temperature is directly proportional to the average kinetic energy of the particles ($\frac{1}{2} m (c_{rms})^2 = \frac{3}{2} kT$) therefore the temperature rises.

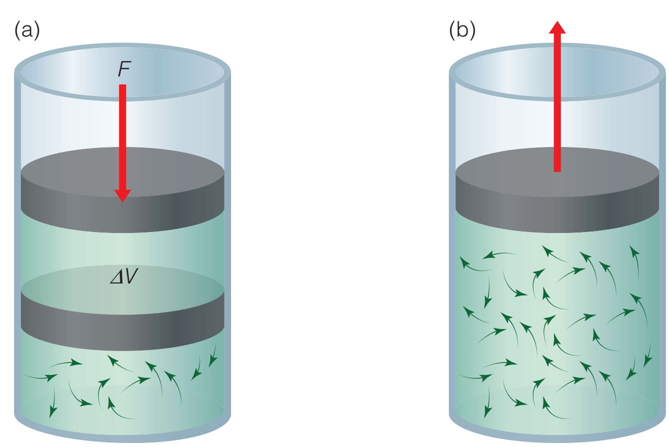


Figure 33.48 (a) The particles rebound off the piston with greater velocity and so their kinetic energy increases. (b) the particles rebound off the piston with less kinetic energy.

When the particles bombard the piston in order to push it outwards they rebound with less kinetic energy (and therefore less speed) as the collisions cannot be perfectly elastic. The E_K they lose is equal to the work done in pushing the piston outwards. If the E_K of the particles is decreasing then the temperature of the gas will decrease.

EXAMPLE

1 Explain how an isothermal compression can take place in practice because the compression by the piston increases the speed (and kinetic energy) of the molecules.

Answer

The particles must be able to transfer energy away from the gas and so the container walls should be thin and able to conduct heat well. The compression must also be slow in order to enable time for heat flow.

2 By considering the motion of the particles, explain how an isothermal expansion of a gas trapped in a container with a movable piston can take place in practice.

Answer

The container should enable heat to pass into it from the surroundings so that the E_K transferred by the particles by doing work to push out the piston can be replaced. The expansion should also take place slowly in order to allow time for heat to pass back into the gas from the surroundings in order to maintain a constant mean E_K and therefore temperature.

TEST YOURSELF

Non-flow processes

1 2 moles of gas are compressed isothermally from an initial volume of 0.060 m^3 to a final volume of 0.042 m^3. The initial pressure is 150 kPa. Calculate the final pressure of the gas.

2 A diatomic gas at an initial pressure of 120 kPa is expanded adiabatically until its volume is tripled. The initial temperature is 312 K. No gas escapes during the process.
 a) Calculate the final pressure of the gas.
 b) Calculate the final temperature of the gas.
 The gas is diatomic and so $\gamma = 1.40$.

3 A monotonic gas has an initial volume of 5 m^3 at a pressure of 200 kPa. Calculate the new pressure if it is
 a) compressed isothermally to a new volume of 0.5 m^3
 b) compressed adiabatically to a new volume of 0.5 m^3.
 c) Sketch a p-V diagram to illustrate these two processes.
 $\gamma = 1.67$

4 A fixed mass of gas undergoes two processes. It is first compressed isothermally from an initial pressure of 1.00×10^5 Pa and a temperature of 12°C until it reaches a quarter of its initial volume. It then expands adiabatically back to its original volume. Determine the final pressure and temperature.

5 Distinguish between an isothermal process and an adiabatic process as applied to an ideal gas.

An ideal gas is held in a container by a moveable piston and energy is supplied to the gas such that it expands at a constant pressure of 1.2×10^5 Pa.

State and explain whether this process is either isothermal or adiabatic or neither.

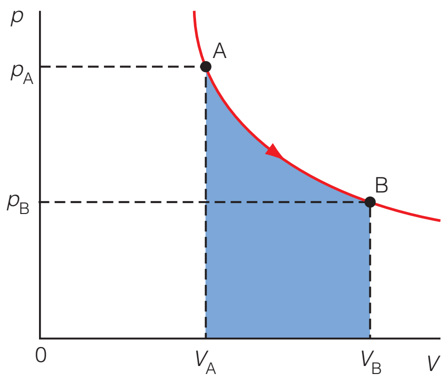


Figure 33.49 Work done by an ideal gas when it expands from A to B is equal to the shaded area under the *p*-*V* curve.

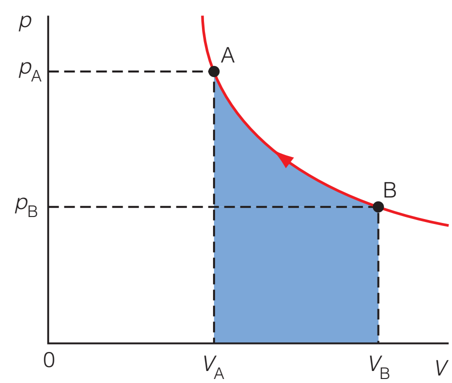


Figure 33.50 Work done on an ideal gas when it is compressed from B to A is equal to the shaded area under the *p*-*V* curve.

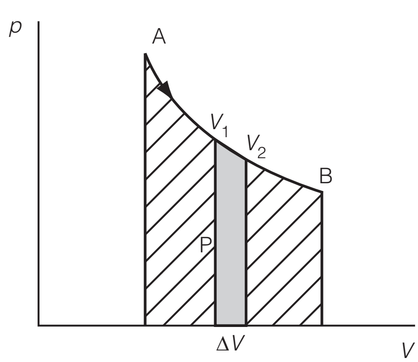


Figure 33.51 The area under a *p*-*V* curve is equal to work done.

Using pressure against volume curves

The area underneath the *p*-*V* curve represents the work done. If the gas is being compressed then the area represents the work done by an external force **on** the gas and if it is expanding then the area represents the work done **by** the gas.

Examples of *p*-*V* graphs

Moving from B to A the gas would be being compressed (its volume at A is smaller than at B) and so the area under the curve would be equal to the work done *on* the gas.

Figure 33.47 shows that the total work done when gas expands is equal to the sum of many small changes in volume × pressure. For the small change shown (V_1 to V_2) the pressure can be considered to be constant. By adding up many of these small changes the total work done can be determined.

EXAMPLE

1 Describe the changes a gas undergoes during the process A → B → C as shown in Figure 33.52.

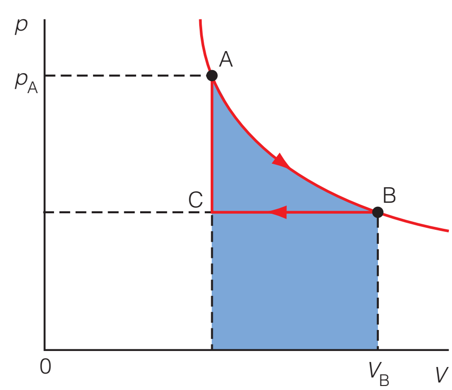


Figure 33.52

Answer

A → B: Isothermal change, pressure is inversely proportional to volume. The gas is expanding.

B → C: The gas is being compressed at constant pressure. This means the internal energy of the gas must be decreasing as the E_K of the molecules must be decreasing in order to maintain a constant pressure when the volume is decreased.

C → A: The pressure of the gas is increasing but the volume remains constant. The internal energy of the gas must be increasing as the E_K of the molecules must be increasing in order to increase the pressure of the gas at a constant volume.

The area ABC enclosed by the cycle is the net work done.

2 Use the *p*-*V* diagram shown in Figure 33.53 to estimate the work done by the gas as it expands from A to B.

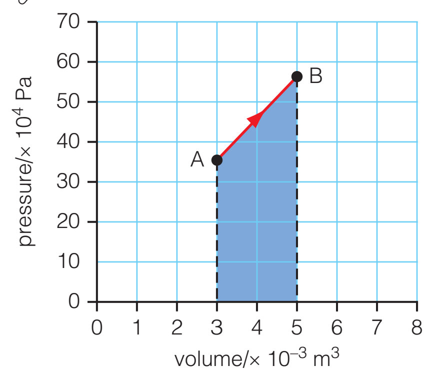


Figure 33.53

Answer

The area under the curve between A and B represents the work done.

The area can be determined by counting squares. Remember to combine partial squares together to make whole squares.

Area = 7 whole squares + 1 + 1 = 9 squares

Area of 1 square = 10×10^4 Pa $\times 1 \times 10^{-3}$ m^3 = 100 joules

Work done = 9 × 100 = 900 J

TIP

If the area of a *p*-*V* graph is used to 'estimate' the work done then there will be some tolerance allowed by the examiner in the final answer.

MATHS BOX

Calculus can be used to calculate the work done for an isothermal expansion as work done by the gas is the area under the *p*-*V* graph as it expands from V_1 to V_2.

$$W = \int_{V_1}^{V_2} p \, dV$$

but $p V = n R T$

$$W = \int_{V_1}^{V_2} \frac{n R T}{V} dV$$

and as

$$\int_{x_1}^{x_2} \frac{dx}{x} = \ln x_2 - \ln x_1$$

$$W = n R T \int_{V_1}^{V_2} \frac{dV}{V}$$

$$W = nRT \ln \left(\frac{V_2}{V_1} \right)$$

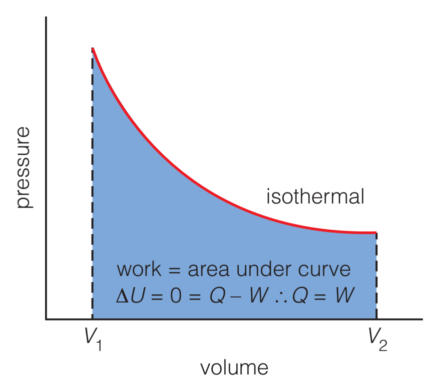


Figure 33.54

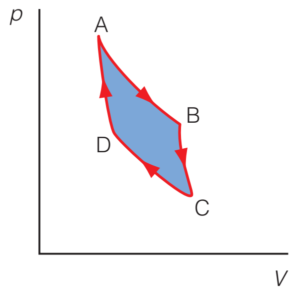


Figure 33.55 A typical cyclic process.

Cyclic process

Many processes involve expanding, heating, compressing and cooling one after the other where the gas returns to the initial pressure and volume after each cycle. Engines are examples of cyclic processes.

Figure 33.55 shows a typical cyclic process.

A–B: Isothermal expansion

B–C: Adiabatic expansion

C–D: Isothermal compression

D–A: Adiabatic compression

TIP

As the net work done = work done by gas when expanding – work done on gas when compressing,

net work done by the gas per cycle = area enclosed by loop = area ABCD

For a gas to do work there must be a supply of energy. This could be from heating or from the internal energy of the gas itself.

EXAMPLE

1 The *p-V* indicator diagram in Figure 33.56 shows various paths which can be followed by a gas. Which two curved paths (followed by connecting vertical constant volume expansions or compressions) should be followed for the net work done *by* the gas during a cycle to be at its maximum value?

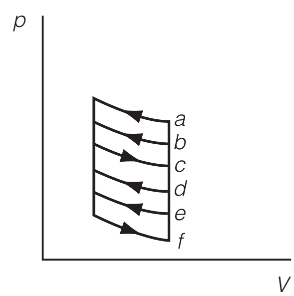


Figure 33.56

Answer

c–e

The work done by the gas is equal to the area enclosed by the loop. Work is done by the gas as it expands and on the gas as it is compressed. Path c is the highest path showing an expansion and path e is the lowest path showing a compression. The gas therefore moved clockwise around the cycle. a–f gives the greatest area but as path a is a compression this route would give the most work done *on* the gas.

2 Figure 33.57 shows a simplified cycle undergone by a gas. Its initial volume is 1.5 m^3, its initial pressure is 120 000 Pa and its initial temperature is 320 K. You may assume the gas behaves as an ideal gas.

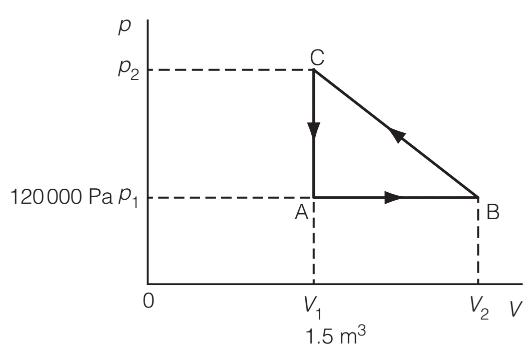


Figure 33.57

a) From A to B the gas is heated to 400 K and it expands. Calculate the new volume, V_2.

Answer

This is a change at constant pressure. For an ideal gas

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{1.5}{320} = \frac{V_2}{400}$$

$$V_2 = 1.9\ \text{m}^3$$

b) From B to C the gas is compressed isothermally. Calculate the new pressure at C.

Answer

There is no change in temperature so

$$p_1 V_1 = p_2 V_2$$

$$120\ 000 \times 1.9 = 1.5 \times p_2$$

$$p_2 = 152\ 000\ \text{Pa}$$

c) From C to A the gas cools and remains at a constant volume. Estimate the work done during the cycle and comment on your answer.

Answer

$$\text{Area enclosed by cycle} = \frac{(152\ 000 - 120\ 000) \times (1.9 - 1.5)}{2}$$

$$= \frac{(32\ 000 \times 0.4)}{2}$$

$$= 6400\ \text{J}$$

This is larger than the actual work done as the actual enclosed area is smaller than a triangle due to the change from B to C being isothermal (i.e. a $y = \frac{1}{x}$ curve).

TEST YOURSELF

Using *p–V* diagrams

1 Figure 33.58 shows a *p-V* indicator diagram for a fixed mass of gas in an engine cylinder completing a cycle of four processes.

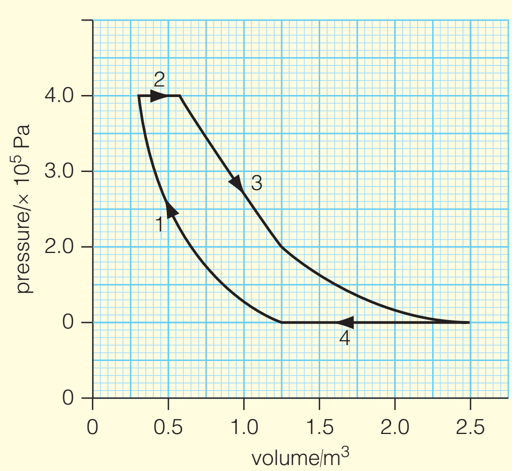


Figure 33.58

⇨

a) State and explain which of the four processes involve work being done by the gas.

b) Processes 1 and 3 are isothermal. If process 1 occurs at 310 K, determine the temperature at which process 3 occurs.

c) Use Figure 33.58 to calculate the work done per cycle.

d) If the engine has four cylinders and each cylinder completes 50 cycles per second, calculate the power generated by the engine.

2 Figure 33.59 shows part of a *p-V* indicator diagram for a fixed mass of gas passing through the following processes:

1–2 constant volume increase in pressure caused by a heat input of 260 J

2–3 isothermal expansion with work done by the gas equal to 250 J

3–4 constant volume cooling. The gas gives out 260 J of heat as it cools.

4–1 isothermal compression at a lower temperature than in stage 2–3. 180 J of work is done on the gas during this stage and the gas returns to its original pressure, temperature and volume.

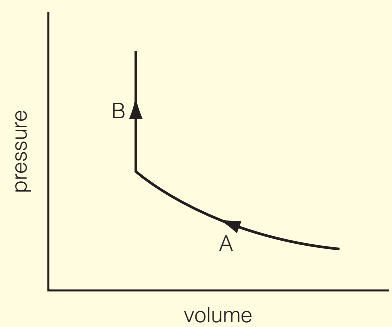


Figure 33.59

a) Complete the *p-V* indicator diagram showing the complete cycle.

b) Use the first law of thermodynamics to complete the following table.

Stage	*Q*	ΔU	*W*
1–2			0
2–3			
3–4			
4–1			
Whole cycle			

Engine cycles

Using engines to do work

It is easy to transfer energy into thermal energy by doing work – just try rubbing your hands together. Using thermal energy to perform useful work is a more complex task and the first practical device was only developed in the 1700s with the development of the steam engine. Steam engines were very inefficient – even the most efficient steam engines rarely achieved efficiencies of better than 10%. They also required a team of people to operate them as the fuel was burned in a separate boiler.

All engines work in a continuous cycle. A petrol engine heats up an air–fuel mixture inside a cylinder, causing the gas to expand, increase pressure, push a piston and do work. The air–fuel mixture then cools, the gas contracts and the cycle repeats. The work done by the gaseous mixture as it expands results in energy being transferred to the moving parts of the engine and so thermal energy has been transferred to kinetic energy. As the engine is working in a cycle it can be made to work continuously.

The main parts of an internal combustion engine

The main parts of an internal combustion engine are shown in Figure 33.60. The engine shown has one cylinder.

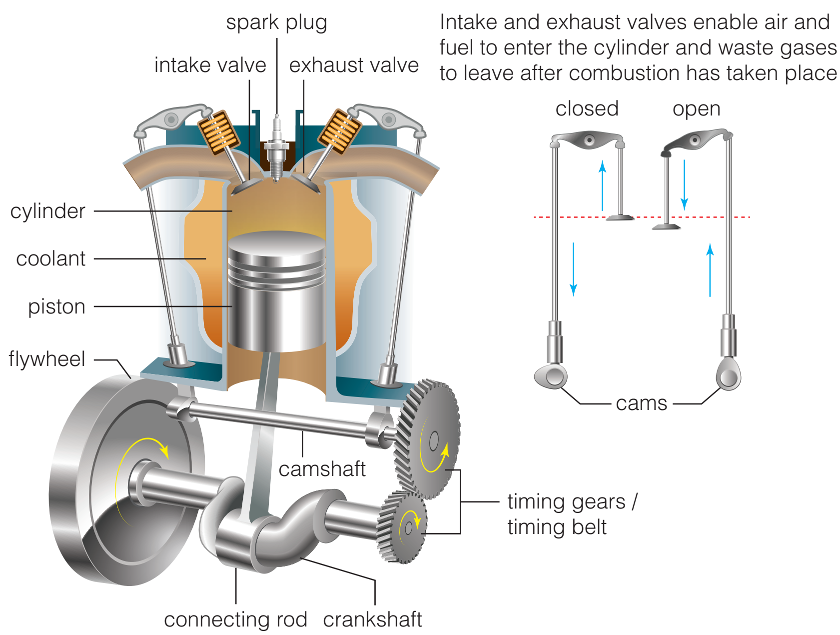


Figure 33.60 An internal combustion engine.

- The **valves** are located in the head of the cylinder. The intake valve opens to permit the intake of fuel and air. Then they must seal so that compression can take place. After the power stroke the exhaust valve must open to permit the exhaust gases to leave the combustion chamber.
- The **sparkplug** ignites the fuel, starting combustion and causing the air–fuel mixture to expand.
- The **piston** is forced down when the air–fuel mixture expands during the power stroke. The piston is connected by a connecting rod to the crankshaft.
- The **crankshaft** transfers the vertical oscillatory motion of the piston into rotary motion which can be used to drive the wheels.
- The **alternato**r charges the battery and powers other electrical devices whilst the engine is running.
- The **distributor** delivers a high pd at the correct moment to each spark plug to ignite the air–fuel mixture.

TIP

You do not need to know the parts of an internal combustion engine but you are expected to be able to describe and understand what happens to the air–fuel mixture in one of the cylinders during a complete 4-stroke cycle.

- The **timing belt** synchronises the rotating of the crankshaft so that the valves open at the correct moment in the intake and exhaust strokes.

The 4-stroke petrol internal combustion engine

Figure 33.61 shows the main parts of the cylinder of a 4-stroke petrol internal combustion engine typically used on motorbikes and cars.

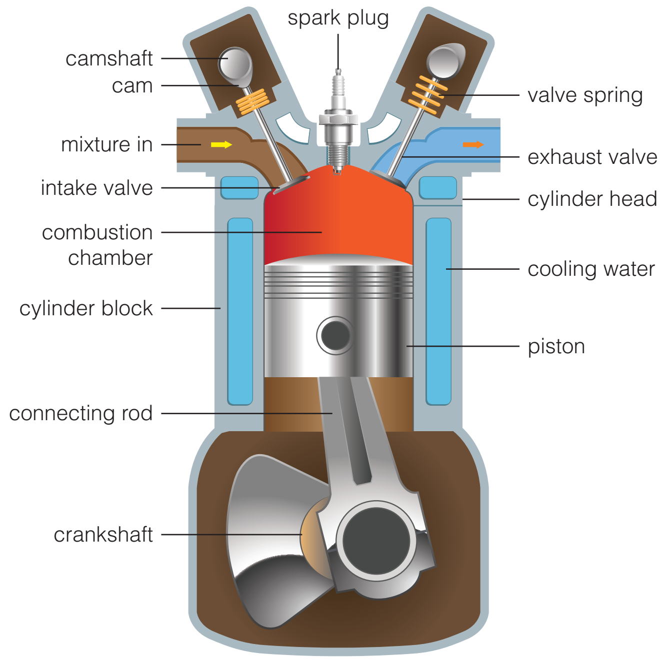


Figure 33.61 Inside a cylinder of a 4-stroke combustion engine.

As the piston is connected to a crankshaft the up and down motion of the piston can be transferred to the rotational motion required to drive the car or motorbike wheels.

Figure 33.62 shows how the piston moves during one cycle.

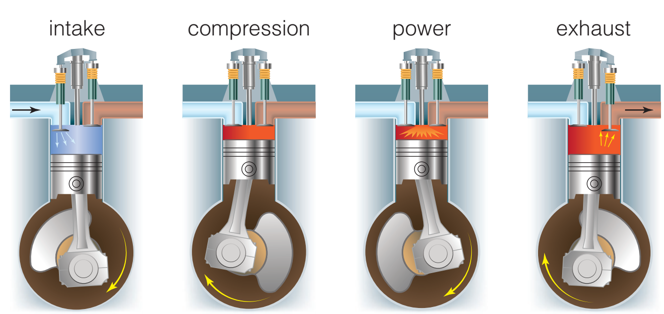
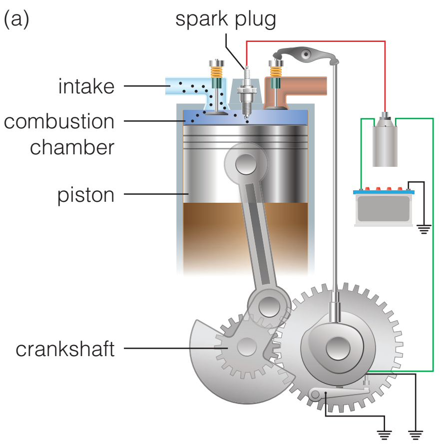
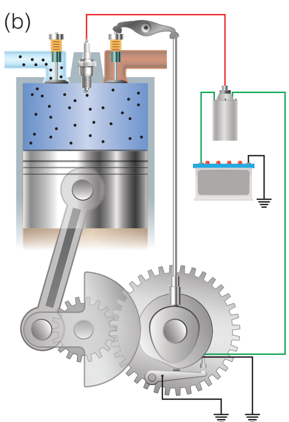


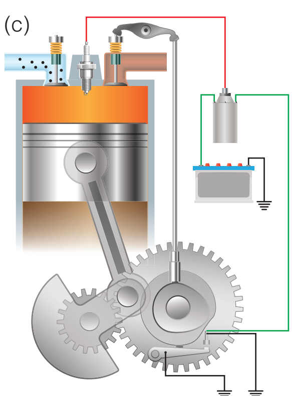
Figure 33.62



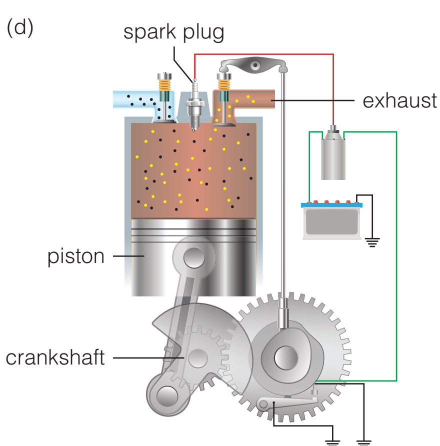
- Stroke 1: **Intake**. The piston moves down and air–fuel mixture is drawn into the chamber.



- Stroke 2: **Compression**. The air–fuel mixture in the cylinder is compressed adiabatically by the piston moving upwards and the spark plug ignites the mixture. This raises the temperature and pressure in the cylinder.



- Stroke 3: **Power**. The high temperature and pressure in the cylinder forces the piston to move downwards which causes the expanding gas to do work. The gas then cools rapidly and the pressure returns to the initial pressure.



- Stroke 4: **Exhaust**. Before the cycle can repeat, the exhaust gases are forced out of the chamber.

Figure 33.63 The 4-stroke combustion engine cycle: (a) intake – piston moving down, (b) compression – piston moving up, (c) power – piston moving down, (d) exhaust – piston moving up.

For a 4-stroke engine the crankshaft completes two whole rotations for each complete cycle as the piston moves down (intake) then up (compression), down (power) then up (exhaust).

The *p*-*V* indicator diagram for a 4-stroke petrol engine

Figure 33.64 shows the engine cycle on a *p*-*V* diagram.

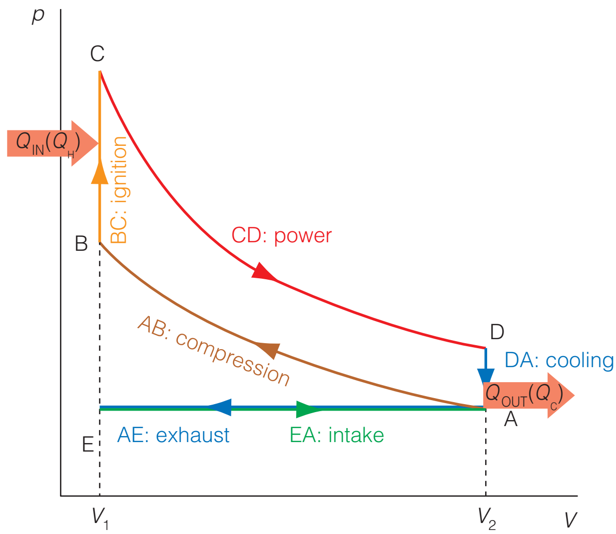


Figure 33.64 *p*-*V* diagram of the Otto cycle. This is named after Nikolaus Otto who designed the first petrol internal combustion engines in Germany in the 1860s.

The Otto cycle shown in Figure 33.64 is an ideal model of what happens in a real combustion engine. The diagram shows BC and DA happening at constant volume which therefore means thermal energy is transferred by heating. Q_{IN} (Q_H) is added when the air–fuel mixture ignites and it is removed during the exhaust cycle Q_{OUT} (Q_C).

The work done by each cylinder of the engine in one cycle will equal the area ABCD as the gas returns to the initial temperature at the end of each cycle. This can be shown using the first law of thermodynamics.

$$Q_{net} = \Delta U + W$$

as $\Delta U = 0$

$$Q_{net} = W = Q_H - Q_C$$

The diesel engine

The diesel engine was patented in Germany by Rudolf Diesel in 1892 and is now the world's most versatile engine. Diesel engines power a huge range of machines from container ships and trains to agricultural machinery, lorries and cars. A diesel engine does not use spark plugs to ignite the air–fuel mixture. In fact diesel fuel is much less volatile than petrol and is difficult to ignite and so not prone to fire. Early petrol engines were very dangerous as they leaked extremely flammable petrol vapour.

During the diesel induction stroke only air is pulled into the chamber. During the compression stroke the air is compressed until it reaches a temperature which is high enough to ignite diesel fuel. The fuel is then injected or sprayed into the chamber where it ignites. The piston is then forced downwards in the power stroke and then the exhaust gases are ejected from the chamber.

The p-V diagram for a diesel engine is shown in Figure 33.65.

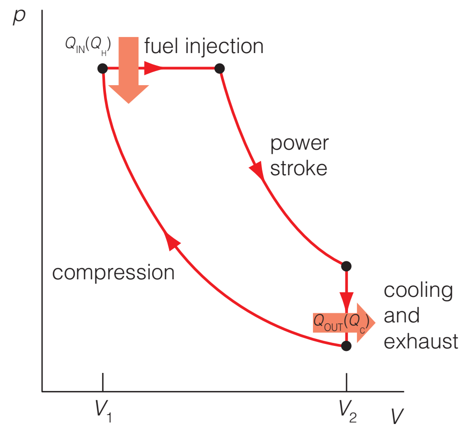


Figure 33.65 p-V diagram for a diesel engine.

There are two key differences between diesel and petrol combustion engines:

1 The fuel is injected after the air has been compressed to ignition temperature.
2 The compression stroke compresses the gas much more in a diesel engine than in a petrol engine.

What are the advantages and the disadvantages of a diesel engine compared with a petrol engine?

A diesel engine achieves better fuel efficiency as it achieves greater compression. The power stroke therefore results in a greater torque and so a diesel engine emits less CO_2 for the same power output. However as diesel engines operate at higher pressures and greater compression ratios they are heavier for the same power as the parts need to have a greater strength.

The high temperatures inside a diesel cylinder enable the nitrogen in the fuel to be oxidised forming nitrogen oxides which are harmful to human health. Diesel exhaust gases also contain particulate matter. This is solid particles formed by partial combustion of diesel fuel in the cooler parts of the cylinder. Most can be removed by particulate filters, but some will pass into the exhaust gases. Particulates are harmful as the small particles can penetrate deep into the lungs and cause respiratory problems. Nano-particles can even enter the bloodstream.

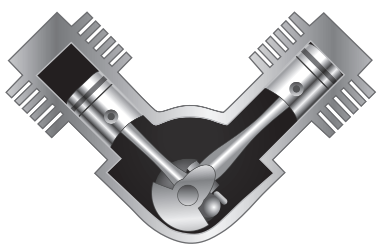


Figure 33.66 Cylinder configuration for a 2-cylinder V engine, typically used on a small motorbike.

Figure 33.67 A V8 engine. Here you can see the two banks of four cylinders at an angle to each other.

Some key developments in engine technology

V engines

A V engine, or Vee engine, is a common configuration for an internal combustion engine. The cylinders and pistons are aligned in two separate banks, so that they appear to be in a 'V' when viewed along the axis of the crankshaft. The Vee configuration generally reduces the overall engine length, height and weight compared to an equivalent inline configuration, enabling a more compact engine design. A V8 engine would have eight cylinders in this configuration, a V12 would have twelve.

Figure 33.68 A British Rail class 55 – 'Deltic' diesel locomotive.

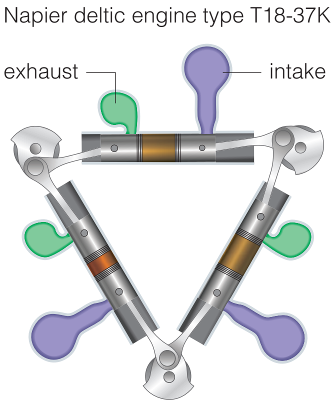


Figure 33.69 A deltic engine has opposing pistons which results in greater power output per stroke as higher compression is achieved.

Deltic engine

These used opposing pistons in a triangular arrangement and powered the diesel trains on the east coast main line between London, the North East and Scotland in the 1960s and 1970s. They were the most powerful diesel-powered trains of the time.

Pre-combustion chambers – Comet swirl chamber

The swirl, or turbulence chamber, is a disc-shaped antechamber designed to impart a circular motion to the air or fuel before it enters the cylinder. As peak vortex speed is reached the fuel injector is opened. As the piston reverses direction, a turbulent stream of burning fuel and superheated air exits the antechamber and rebounds off the piston, filling the main chamber. The turbulence of the flow causes the air and fuel to mix more thoroughly and so leads to more complete combustion.

The swirl chamber was invented by Sir Harry Ricardo during the late 1920s and underwent numerous alterations during its long career. It was the introduction of a pre-combustion chamber that enabled the development of high-speed diesel engines which could power cars.

Inside the cylinder of a diesel engine

Sir Harry Ricardo (26 January 1885 – 18 May 1974) was one of the foremost engine designers and researchers of the internal combustion engine. He researched the physics of internal combustion and the design of combustion chambers.

Figure 33.70 The blue plaque outside 13 Bedford Square, London.

The following excerpt is from a lecture Harry Ricardo gave to the Royal Society of Arts on 23 November 1931.

I am going to take the rather unconventional course of asking you to accompany me, in imagination, inside the cylinder of a diesel engine. Let us imagine ourselves seated comfortably on the top of the piston, at or near the end of the compression stroke. We are in complete darkness, the atmosphere is a trifle oppressive, for the shade temperature is well over 500 Celsius – almost a dull red heat – and the density of the air is such that the contents of an average sitting-room would weigh about a ton; also it is very draughty, in fact, the draught is such that, in reality, we should *be blown off our perch and hurled about like autumn leaves in a gale. Suddenly, above our heads, a valve opens and a rainstorm of fuel begins to descend. I have called it a rainstorm, but the velocity of droplets approaches much more nearly that of rifle bullets than of raindrops.*

For a while nothing startling happens, the rain continues to fall, the darkness remains intense. Then suddenly, away to our right perhaps, a brilliant gleam of light appears, moving swiftly and purposefully; in an instant this is followed by a myriad others all around us, some large and some small, until on all sides of us the space is filled with a merry blaze of moving lights; from time to time the smaller lights wink and go out, while the larger ones develop fiery tails like comets; occasionally these strike the walls, but, being surrounded by an envelope of burning vapour, they merely bounce off like drops of water spilt on a red hot plate.

Right overhead all is darkness still, the rainstorm continues, and the heat is becoming intense; and now we shall notice that a change is taking place. Many of the smaller lights around us have gone out, but new ones are beginning to appear, more overhead, and to form themselves into definite streams shooting rapidly downwards or outwards from the direction of the injector nozzles.

Looking round again we see that the lights around are growing yellower; they no longer move in a definite direction, but appear to be drifting listlessly hither and thither; here and there they are crowding together in dense nebulae, and these are burning now with a sickly, smoky flame, half suffocated for want of oxygen. Now we are attracted by a dazzle, and looking up we see that what at first was cold rain falling through utter darkness, has given place to a cascade of fire as from a rocket. For a little while this continues, then ceases abruptly as the fuel valve closes.

Above and all around us are still some lingering fire balls, now trailing long tails of sparks and smoke and wandering aimlessly in search of the last dregs of oxygen which will consume them finally and set their souls at rest. If so, well and good; if not, some unromantic engineer outside will merely grumble that the exhaust is dirty and will set the fuel valve to close a trifle earlier.

So ends the scene, or rather my conception of the scene, and I will ask you to realise that what has taken me nearly five minutes to describe may all be enacted in one five-hundredth of a second or even less.

ACTIVITY – THE CONTROVERSY SURROUNDING DIESEL ENGINES

Four major cities move to ban diesel vehicles by 2025

The leaders of four major global cities say they will stop the use of all diesel-powered cars and trucks by the middle of the next decade.

The mayors of Paris, Mexico City, Madrid and Athens say they are implementing the ban to improve air quality.

They say they will give incentives for alternative vehicle use and promote walking and cycling.

The commitments were made in Mexico at a biennial meeting of city leaders.

The use of diesel in transport has come under increasing scrutiny in recent years, as concerns about its impact on air quality have grown. The World Health Organization (WHO) says that around three million deaths every year are linked to exposure to outdoor air pollution.

The term **horsepower** was invented by the engineer James Watt who lived from 1736 to 1819. At the time horses were used to pull up the coal from mines.

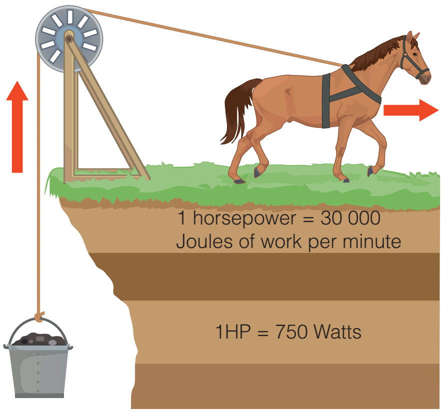


Figure 33.71 What is meant by 'horsepower'?

Watt made measurements and found that a typical coal pit pony could do about 30 000 J of work in a minute. He then increased that number by 50% (assuming larger horses could carry out more work) to make horsepower equivalent to 45 000 J of work in one minute. This converts to around $750\ \mathrm{J\,s^{-1}}$ ($\frac{45\,000\ \mathrm{J}}{60\ \mathrm{s}} = 750\ \mathrm{J\,s^{-1}}$).

Figure 33.72 Built in Finland the Wärtsilä, RT-flex is the world's largest and most powerful diesel engine.

The **indicated power** is the power developed inside the cylinder.
indicated power = area of p-V loop × number of cycles per second × number of cylinders

Input power = calorific value × fuel flow rate

$$\text{thermal efficiency} = \frac{\text{indicated engine power}}{\text{input power}}$$

Now, mayors from a number of major cities with well-known air quality problems have decided to use their authority to clamp down on the use of diesel.

In the UK, campaigners are calling for London's mayor to commit to phase out diesel vehicles from London by 2025.

Research the controversy surrounding diesel engines using a range of sources

a Explain why diesel engines are more polluting than petrol engines.
b Outline the different health problems caused by the exhaust gases from diesel engines.
c Explain why governments in the 1990s originally promoted diesel vehicles.
d Describe the technology which can be fitted to diesel vehicles to reduce their emissions and discuss why this is not more widely used.
e Describe some of the measures cities around the world are taking to reduce pollution from diesel vehicles.
f Evaluate the economic and social costs of this problem.

The most powerful engine in the world

Modern container ships are absolutely huge (more than 400 m long) and so require the largest engines ever built to power them.

- One of these engines could have a mass of 1800 tonnes and fill four floors of the ship.
- The volume of the 14 cylinders is more than 18 000 litres (compared to a couple of litres for a typical car).
- Power output is 75 000 **horsepower** (compared to 150 hp for a reasonable car).

The power output of an engine

The power developed by the pressure of the gas pushing against the piston is measured by recording and plotting the pressure and volume inside the piston through a complete cycle. Electronic sensors record the pressure and volume automatically. The area enclosed by the cycle can be calculated and the **indicated power** determined.

Modern vehicle engines have more than one cylinder and so the power of the engine is calculated by

indicated engine power = area of p-V loop × number of cycles per second × number of cylinders

TIP

Some exam questions quote the engine speed in rpm. The crankshaft completes two rotations for each 4-stroke cycle as the piston moves down (intake) then up (compression), down (power) then up again (exhaust). The number of cycles per second (around the p-V diagram) is therefore half the number of revolutions (of the crankshaft) per second.
The **input power** to the engine can be calculated if the calorific value (energy content per kg) of the fuel is known. This is the number of joules of energy released when one kilogram of fuel is burnt.

EXAMPLE

1 An engine requires 135 kW of power by burning fuel with a calorific value of 41 MJ kg^{-1}. Calculate the flow rate of fuel required.

Answer

Input power = calorific value × fuel flow rate

$$135\,000 = 41 \times 10^6 \times f$$

$$f = 3.3 \times 10^{-3}\ kg\ s^{-1}$$

2 If an engine burns fuel with a calorific value of 42.8 MJ kg^{-1} at a rate of 0.53 kg min^{-1}, calculate the input power.

Answer

Input power = calorific value × fuel flow rate

$$= 42.8 \times 10^6 \times \frac{0.53}{60} = 378\ kW$$

Engine efficiency and power

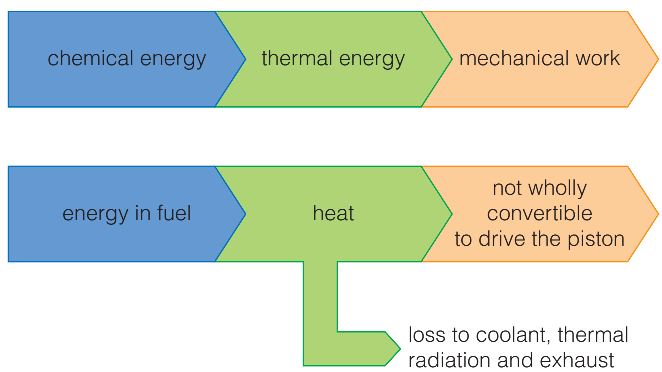


Figure 33.73 Energy transfers in an engine.

The actual efficiency of an engine is lower than the thermal efficiency due to the large amount of work done against friction between the moving parts, namely the transmission gears, driveshaft and bearings. Modern designs mean that frictional losses are only 6–9%.

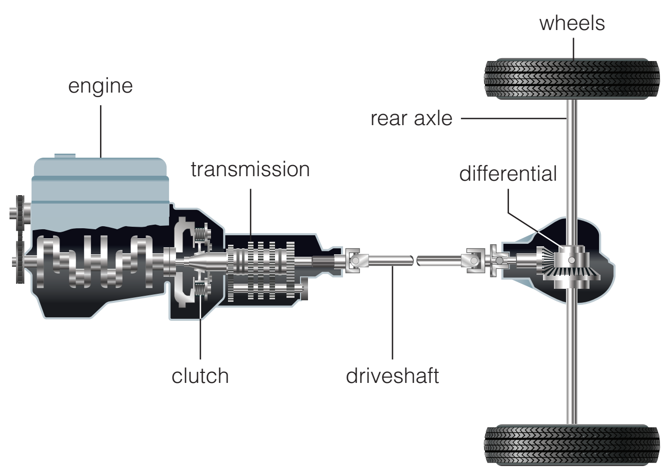


Figure 33.74 Drive shaft and transmission.

The driveshaft and transmission enable the motion from the piston crankshaft to be transmitted to the front wheels (and rear wheels if the vehicle has four-wheel drive).

The actual output power of an engine is known as its **brake power** or **output power**.

This is the **output power** delivered to the engine's crankshaft. **Brake power** is always less than indicated power due to energy losses due to friction in the gear box, differentials and other components.

$$\text{Brake power} = \text{torque} \times \text{angular speed of driveshaft}$$
$$= T\,\omega$$
$$P_{\text{OUT}} = T\,\omega$$

Among car manufacturers, the output power is often referred to as the brake power, or brake horsepower (bhp) but in A-level examinations you will be expected to work in watts (or kilowatts).

The indicated power (based on the area of the p-V indicator diagram) will be greater than the output power (brake power) due to the frictional losses in the engine and transmission.

The difference between the indicated power and the output power is the **frictional power**.

$$P_{\text{frictional}} = P_{\text{indicated}} - P_{\text{out}}$$

Since there are three key power indicators:

- input power
- indicated power
- output power

engine designers also consider three key indicators of efficiency:

1 **Thermal efficiency** = indicated power / input power. This has already been discussed and is related to the amount of energy released when the fuel burns and how this results in pressure and volume changes in the cylinder.

2 **Mechanical efficiency** = output power / indicated power. This relates to how much torque is produced at the driveshaft as the piston is driven up and down by the combusting fuel. Mechanical efficiency is typically 80–90%.

3 **Overall efficiency** = output power / input power from fuel.

Overall engine efficiency

Overall efficiency = mechanical efficiency × thermal efficiency

It is straightforward to show that the **overall efficiency** can be calculated by multiplying the thermal efficiency by the mechanical efficiency.

As

$$\text{thermal efficiency} = \frac{P_{\text{INDICATED}}}{P_{\text{INPUT}}}$$

and

$$\text{mechanical efficiency} = \frac{P_{\text{OUT}}}{P_{\text{INDICATED}}}$$

$$\text{thermal efficiency} \times \text{mechanical efficiency} = \frac{P_{\text{INDICATED}}}{P_{\text{INPUT}}} \times \frac{P_{\text{OUT}}}{P_{\text{INDICATED}}}$$

$$\text{overall efficiency} = \frac{P_{\text{OUT}}}{P_{\text{INPUT}}}$$

EXAMPLE

1 Measurements were made on a single-cylinder 4-stroke petrol engine producing the following data:
mean temperature of gases in cylinder during combustion stroke: 860°C
mean temperature of exhaust gases: 82°C
area enclosed by indicator diagram loop: 430 J
completes 1000 cycles per minute
power developed by engine at output shaft: 5.2 kW
calorific value of fuel: 47.6×10^6 J kg^{-1}
flow rate of fuel: 2.3×10^{-2} kg min^{-1}
Use this data to answer the following.

a) Calculate the indicated power of the engine.

Answer

$P_{INDICATED}$ = area of p-V diagram × number of cycles per second × number of cylinders

$$= 430\text{ J} \times \left(\frac{1000}{60}\right) \times 1$$

= 7167 W = 7.2 kW

b) Calculate the power dissipated in overcoming the frictional losses in the engine.

Answer

$$P_{FRICTION} = P_{INDICATED} - P_{OUT}$$

= 7167 − 5200 = 1967 W

c) Calculate the rate at which energy is supplied to the engine from the fuel.

Answer

P_{INPUT} = calorific value of fuel × flow rate

$$= 47.6 \times 10^6\text{ J kg}^{-1} \times \left(\frac{23 \times 10^{-2}}{60}\right)$$

$= 18\,247$ J s^{-1} = 18.2 kW

d) Calculate the overall efficiency of the engine.

Answer

$$\text{Overall efficiency} = \frac{P_{OUT}}{P_{INPUT}}$$

$$= \frac{5.2\text{kW}}{18.2\text{kW}}$$

= 0.29

2 An engine is used to pump water uphill. The engine is capable of pumping 10 tonnes of water through a height of 2.5 m during each 8 second cycle. The work done by the engine in one cycle is 340 kJ.

a) Calculate the indicated power of the engine.

Answer

$$\text{power} = \frac{\text{work done}}{\text{time}}$$

therefore

$$\text{indicated power} = \frac{\text{indicated work per cycle}}{\text{time to complete 1 cycle}}$$

$$= \frac{340\,000}{8} = 42\,500\text{ W}$$

b) Calculate the mechanical efficiency of the engine.

Answer

This engine is being used to pump water uphill so the output power is equal to the gravitational potential energy gained by the water each second.

$$P_{OUT} = \frac{(10\,000 \times 9.81 \times 2.5)}{8}$$

$$= 30\,656\text{ W}$$

$$\text{mechanical efficiency} = \frac{P_{OUT}}{P_{INDICATED}}$$

$$\text{mechanical efficiency} = \frac{30\,656}{42\,500} = 0.72 = 72\%$$

c) Explain why the efficiency in practice would be much lower than this.

Answer

In practice there are frictional losses in the moving parts of the engine and the pump. This will lead to energy being transferred to thermal energy.

Measuring brake power

Brake power can be measured by placing the engine on a test rig and measuring the engine torque when the drive shaft is turning at constant angular speed. A rope brake dynamometer is coupled to the engine drive shaft. The rope or belt provides a frictional force to oppose the turning of the drive shaft.

The Newton meter measures the tension in the rope/belt and the resultant force can be calculated. If the radius is known then the torque from the engine can be calculated.

Figure 33.75 A rope brake dynamometer.

How close do real engines get to an ideal engine?

In calculating the work done on the air–fuel mixture and analysing petrol and diesel *p*-*V* cycles we have assumed that:

- The petrol (or diesel)–air mixture behaves as an ideal gas.
- The heat energy (in the compression stroke) is taken in entirely at the single temperature T_H and rejected at the single temperature T_C.
- The processes that form the engine cycle are reversible.

These assumptions are not true of real engines as the high temperatures and pressures inside engines mean that the kinetic theory assumptions break down. Combustion inside the engine occurs at a range of temperatures and the air–fuel mixture will be taken in at a range of temperatures.

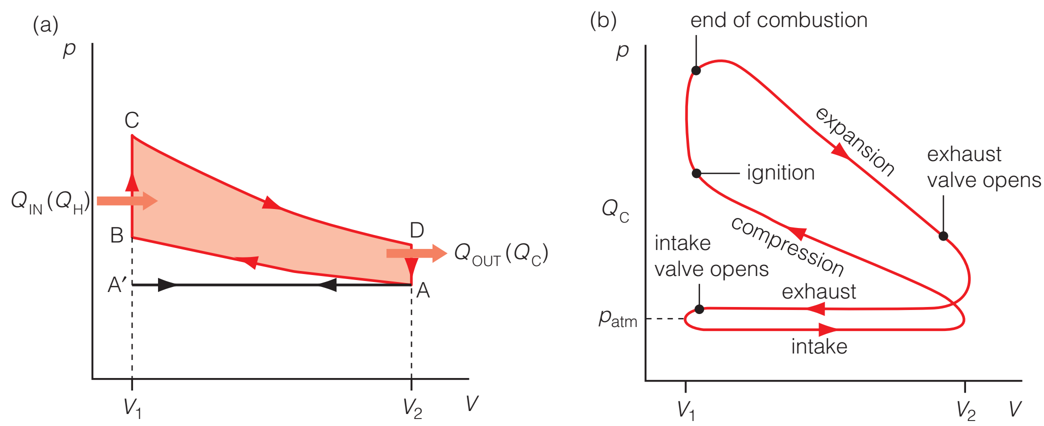


Figure 33.76 *p*-*V* diagram for a real 4-stroke petrol engine: (a) theoretical petrol engine cycle, (b) real petrol engine cycle.

The *p*-*V* diagram for a real engine will begin and end in slightly different places and will have rounded corners. The corners are rounded because combustion does not occur instantaneously. The intake and exhaust valves take time to open and so the sharp corners can never occur for a real engine.

You can see from Figure 33.76 that there are no isovolumetric changes in a real engine as the pistons are constantly moving. The expansion and compression strokes are only approximately adiabatic as some thermal energy inevitably is lost from the system and transferred to the surroundings.

TEST YOURSELF

Engine cycles, power output and efficiency

1 An engine burns fuel with a calorific value of 44.8 MJ kg^{-1} at a rate of 0.49 kg min^{-1}.

a) Calculate the input power.

b) What would happen to the indicated power if the cylinder completed each cycle at twice the speed?

2 Use *p*-*V* indicator diagrams to describe the key differences between diesel and petrol engine cycles.

3 The power output of a toy steam engine was measured as part of a school science project. The student fitted pressure and volume sensors to the single cylinder and plotted a *p*-*V* indicator diagram as the engine lifted some small weights. The engine completed 300 cycles per minute.

a) Use the *p*-*V* indicator diagram in Figure 33.77 to show that the indicated power was estimated to be 16 W.

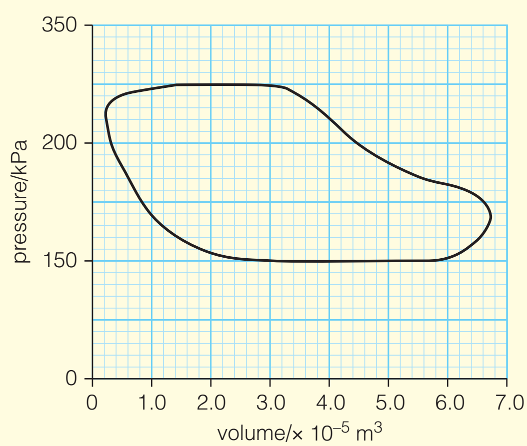


Figure 33.77

b) During lifting, a mass of 6 kg was lifted through a height of 1.8 m in 8.7 s. Estimate the mechanical efficiency of the engine.

c) Explain why the mechanical efficiency can never be equal to 1.

4 An engine requires 180 kW of power by burning fuel with a calorific value of 37 MJ kg^{-1}. Calculate the flow rate of fuel required.

5 The following data was measured for a 4-cylinder, 4-stroke engine:
Shaft speed = 2500 rev min^{-1}
Torque arm, R = 0.4 m
Net brake load $(T-W)$ = 200 N
Fuel consumption = 2 g s^{-1}
Calorific value of fuel = 42 MJ kg^{-1}
Indicated power = 6.54 kW per cylinder

Use this data to determine the following quantities for this engine:
a) brake power
b) mechanical efficiency
c) thermal efficiency
d) overall efficiency.

6 The thermal energy supplied by a kilogram of fuel is 51.0×10^6 J. Calculate the efficiency of a car which is producing mechanical energy at a rate of 42 000 J s^{-1} if it is using petrol at a rate of 7.8 kg per hour.

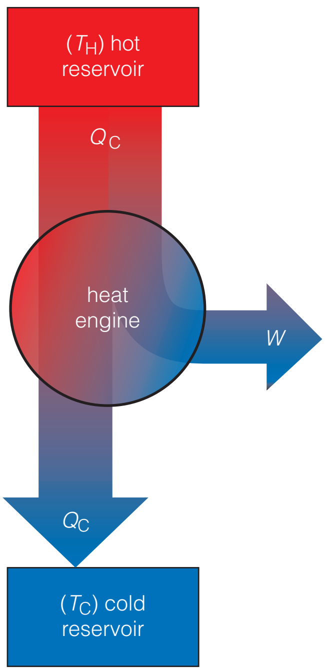


Figure 33.78 Schematic diagram of energy transfers in a heat engine.

The **efficiency** of any engine is defined as the ratio of work it does to the heat input:

$$\text{efficiency} = \frac{W}{Q_H}$$

This is similar to the broad definition of efficiency 'useful energy out / total energy in'.

$$W = Q_H - Q_C$$

Efficiency can be calculated as

$$\text{efficiency} = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H}$$

TIP

The efficiency can be expressed as a percentage by multiplying by 100, e.g. efficiency = 0.2 = 20%.

The second law of thermodynamics and engines

The basic idea of any engine is that mechanical energy can be obtained from thermal energy when heat flows from a high temperature to a lower temperature. This process enables some of the thermal energy to be transferred to work.

Figure 33.78 shows a schematic diagram of a heat engine.

The heat input (Q_H) partly transfers energy by doing work (W) and the remainder is exhausted from the system (Q_C). The engine begins each cycle with the same amount of internal energy, $\Delta U = 0$. Therefore by applying the law of conservation of energy (first law of thermodynamics), $W = Q_H - Q_C$.

EXAMPLE

A car engine has an efficiency of 20% and produces an average of 30 000 J of mechanical work per second. Calculate the amount of heat energy discharged as waste heat per second.

Answer

$$\text{efficiency} = \frac{W}{Q_H}$$

$$Q_H = \frac{30\,000}{0.2} = 150\,000 \text{ J s}^{-1}$$

$$W = Q_H - Q_C$$

$$Q_C = 150\,000 - 30\,000 = 120\,000 \text{ J s}^{-1}$$

The p-V diagram in Figure 33.79 shows the Otto cycle, and is the idealised cycle of a 4-stroke petrol engine as discussed on page 51.

As the area enclosed by the cycle (orange shaded area) is equal to the net work done W then it can be also seen from this diagram that $W = Q_H - Q_C$ according to conservation of energy.

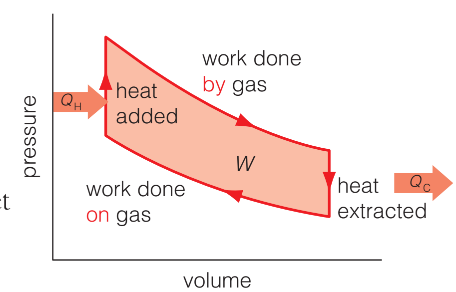


Figure 33.79 The Otto cycle.

An engine will be more **efficient** if Q_C can be as small as possible. If Q_C is reduced to zero then the engine would be 100% efficient. This would be a 'perfect' engine, where all of the heat input is used to do work.

The **second law of thermodynamics** states that an engine can never be 100% efficient and that during a complete cycle it is impossible for all of the heat supplied to be transferred into mechanical energy.

In other words, it is not possible to convert heat continuously into work without at the same time transferring some heat from a warmer to a colder body.

We can use a simple example to illustrate another application of the second law. If an object is dropped on the floor, gravitational potential energy is transferred into thermal energy in that object and the surroundings. The first law 'allows' the object to jump back upwards, because energy can be transferred from one store to another. But of course the object does not jump back. Thermal energy is random and disordered. The second law shows that energy does not flow by itself from a disordered state to an ordered state. It would not break the first law of thermodynamics if when holding a hot cup of coffee your hands got colder and the coffee got warmer so long as the total energy of the system did not change (i.e. the energy lost by the hands was gained by the coffee). However, this is never seen to happen as energy flow from hot to cold is a one-way process. We call these processes 'irreversible'. They are irreversible due to the fact that the entropy of a system always increases.

RESEARCH TASK

Find out as much as you can about entropy.

Why is this sometimes called the 'arrow of time'?

Can you define entropy:

1 in terms of temperature and energy
2 in terms of the number of ways of arranging the atoms and molecules in a system?

The Carnot cycle

In the early 1800s a French scientist, Sadi Carnot, was investigating how to increase the efficiency of engines. The Carnot cycle shows an ideal engine, where the expansion and compressions take place so slowly that the system returns back to its original state at the end of each cycle. The most efficient heat engine cycle is the Carnot cycle, consisting of two isothermal processes and two adiabatic processes.

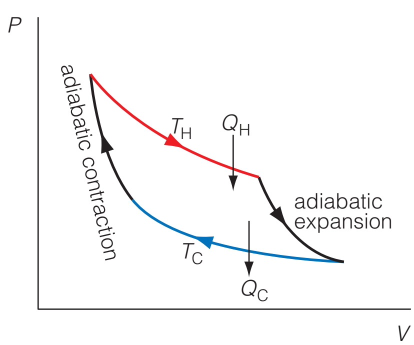


Figure 33.80 The Carnot cycle.

As this is the theoretically most efficient engine, heat should flow into and out of the engine without any losses. So, the gas should expand and contract isothermally when Q_H flows into the gas and Q_C flows out of it. The gas should stay at the temperature of the hot sink (T_H) and cold sink (T_C) during these expansions or contractions otherwise a temperature difference would cause a flow of energy.

The engine is operating between T_H and T_C and the most efficient way to move from one isothermal to the other is for there to be no heat transfer. This is why the other two processes to complete the cycle are adiabatic.

This is the most efficient heat engine cycle allowed by physical laws. Although such an engine could never be built in practice it enables us to calculate a limiting value on the fraction of the heat which can be used to do work.

Well-designed engines can reach 60% to 80% of the maximum theoretical **Carnot efficiency**.

Maximum theoretical efficiency (**Carnot efficiency**) $= \frac{T_H - T_C}{T_H}$

A typical car has a Carnot efficiency of around 55% but an actual efficiency of 25%.

The engine will be most efficient if there is a large temperature difference between the outside air and the gas inside the engine. Increasing T_H comes with engineering challenges as higher temperatures can cause materials to melt or change their properties. High-temperature gases will also be at high pressure and would require a stronger vessel, leading to a heavier engine.

EXAMPLE

An engine manufacturer claims that an engine's heat input per second is 10 kJ at 460 K and the heat output per second is 4.5 kJ at 300 K. Comment on the manufacturer's claims.

Answer

$$\text{Efficiency} = \frac{Q_H - Q_C}{Q_H} = \frac{(10\,000 - 4500)}{10\,000} = 0.55$$

$$\text{Maximum theoretical efficiency (Carnot efficiency)} = \frac{T_H - T_C}{T_H} = \frac{(460 - 300)}{460} = 0.35$$

The manufacturer's claims are false as they are claiming an efficiency greater than the maximum theoretical efficiency. Their engine would violate the second law of thermodynamics.

Combined heat and power (CHP) stations

Combined heat and power stations are sometimes called cogeneration plants. These plants produce thermal energy and mechanical or electrical energy by using one single source of fuel. In a conventional power plant, the fuel is burnt in a boiler, which in turn produces high-pressure steam. This high-pressure steam is used to drive a tribune, which in turn is connected to a turbine which drives a generator to produce electrical energy. The heat produced in this process is wasted and goes into the atmosphere and surroundings. In a CHP station the waste heat produced is also used in heating nearby buildings in addition to heating water to provide steam to the turbine and electrical generator.

As a CHP station makes use of the waste heat it is far more efficient than a conventional power station. The main difficulties with such a scheme are that few people live close enough to thermal power stations to enable them to make use of the waste heat.

TEST YOURSELF

Engine efficiency

1 Figure 33.81 shows the various changes of temperature, pressure and volume as a fixed mass of gas expands and contracts.

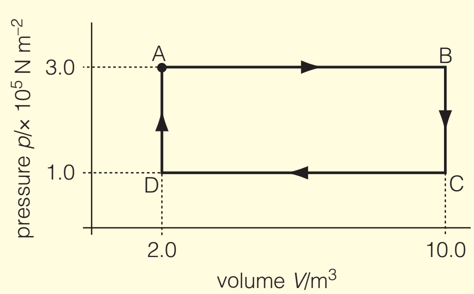


Figure 33.81

a) Use this p-V diagram to calculate the work done during one cycle.
b) If 2.0×10^8 J of thermal energy is ejected into the cold reservoir each cycle, calculate the efficiency of this engine.

2 Sketch the p-V changes that take place in a fixed mass of an ideal gas during one cycle of a Carnot engine. Indicate which change is occurring at each place in the cycle.

3 Explain why a petrol engine can never achieve maximum thermal or mechanical efficiency in practice. Discuss how the temperature of the surroundings affects the efficiency of an engine.

4 Early steam engines often had efficiencies of a few percent. Calculate the maximum possible efficiency for a steam engine using steam at a temperature of 100°C on a day when the temperature of the surroundings is 17°C and account for the large difference between theoretical and practical efficiencies.

Reversed heat engines

Refrigerators

Refrigerators, air-conditioning units and heat pumps are simply engines operating in reverse where work is done to transfer energy from a cold environment to a hot environment. According to the second law of thermodynamics it is impossible for heat to flow from a cooler body to a hotter one without putting in some energy or doing work. The work is usually done by an electric motor which compresses and pumps a fluid, known as the 'refrigerant'.

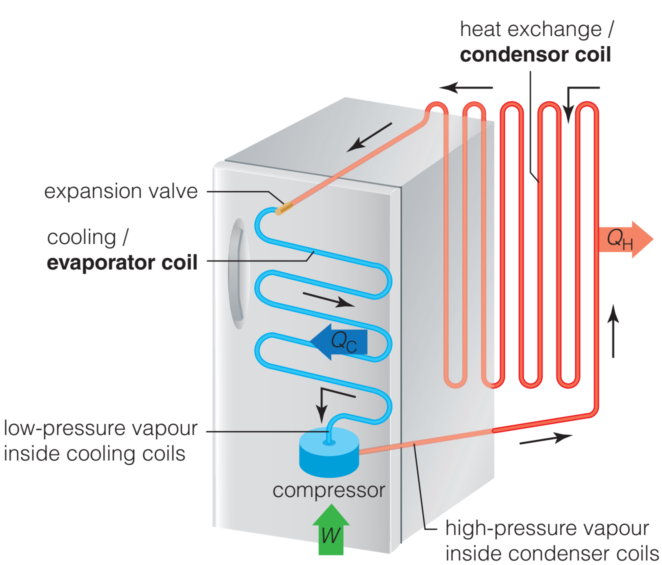


Figure 33.82 A typical refrigerator system.

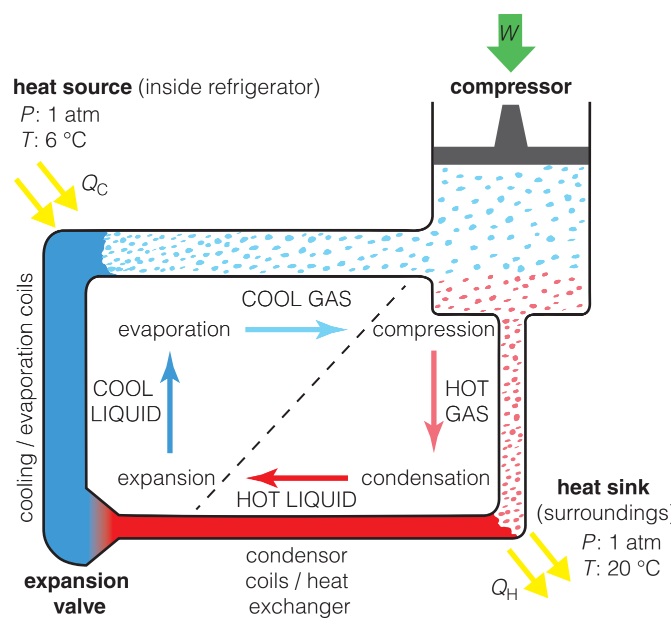


Figure 33.83 Schematic of a refrigeration system.

Thermal energy from inside the refrigerator is absorbed by the fluid inside the cooling coils as the food is at a higher temperature than the refrigerant. This energy causes the liquid refrigerant inside the coil to change state and evaporate at a constant temperature. The vapour then passes through the electric motor which compresses the gas and it condenses back into a liquid and passes through the condenser coil heat exchanger on the back of the refrigerator. The refrigerant is at a higher temperature than the surroundings and so heat energy (Q_H) passes into the surroundings. The gas cools to become a liquid before passing through the expansion valve and re-entering the cooling coils and the cycle begins again.

The refrigeration cycle

The refrigeration cycle can be shown on a *p-V* diagram. A refrigerator can be considered as a heat engine working in reverse and the arrows on the *p-V* diagram are in the opposite direction to those for a heat engine.

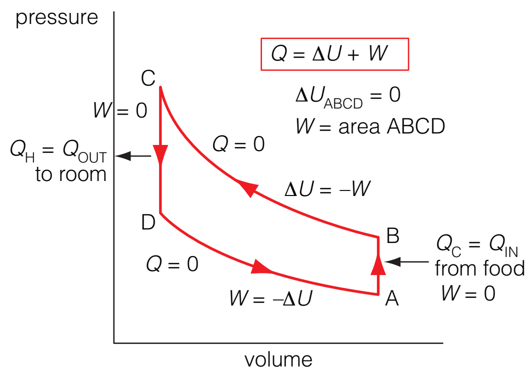


Figure 33.84 *p-V* indicator diagram for an ideal refrigerator.

- Stage A to B: Thermal energy taken in by the refrigerant from food inside the fridge.
- Stage B to C: The refrigerant is adiabatically compressed by the motor.
- Stage C to D: Thermal energy is transferred to the surroundings.
- Stage D to A: The refrigerant expands adiabatically.

EXAMPLE

Complete the table to show the work done, heat energy transferred and change in internal energy as the refrigerant completes one whole cycle. For each stage, explain what is happening to the refrigerant as it goes around the cycle.

Stage	Q	ΔU	W
AB	+58		0
BC		+53	
CD	–74	–74	0
DA			37

Answer

Stage AB: As $W = 0$, $Q = \Delta U = +58$. The refrigerant temperature is increasing as heat is being taken in but there is no volume change.

Stage BC: As this is an adiabatic compression, $Q = 0$ and so $\Delta U = -W$, $W = -53$. The temperature of the refrigerant is increasing as it is compressed.

Stage CD: $W = 0$ as the volume does not change. The temperature of the refrigerant falls as heat is lost to the surroundings.

Stage DA: As this is an adiabatic expansion, $Q = 0$ and so $W = -\Delta U$. The refrigerant temperature falls to the starting temperature.

The completed table therefore looks like this:

Stage	Q	ΔU	W
AB	+58	**+58**	0
BC	**0**	+53	**−53**
CD	−74	−74	0
DA	**0**	**−37**	37

Refrigerants and the environment

In the past, chlorofluorocarbons (CFCs) were popular refrigerants, but it was discovered that CFCs damaged the ozone layer with long-lasting effects. CFCs released at the surface of the Earth, through leaky refrigerators and air conditioners, can rise up to the ozone layer in the stratosphere. The ozone layer shields the Earth's surface from UV radiation and also helps to prevent water vapour escaping from the atmosphere. CFCs were banned in Europe in the early 1990s and now ozone-friendly refrigerants such as Freon or ammonia are used.

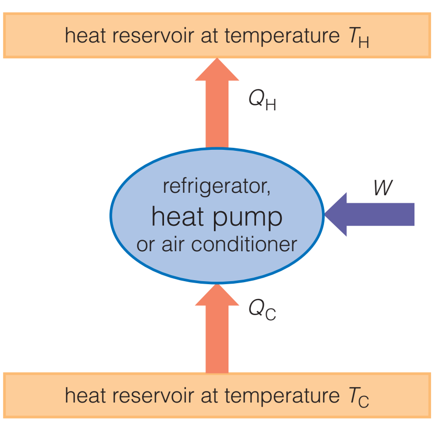


Figure 33.85 Schematic diagram of energy transfer for a refrigerator.

The **coefficient of performance** of a refrigerator

$$COP_{REF} = \frac{Q_C}{W} = \frac{Q_C}{Q_C - Q_H}$$

For an ideal refrigerator

$$COP_{IDEAL} = \frac{T_C}{T_H - T_C}$$

As the coefficient of performance is a ratio it does not have any units.

EXAMPLE

Explain why leaving the refrigerator door open on a hot day does not cool down the room.

Answer

The second law of thermodynamics shows that it is impossible for heat to flow from a cooler body to a hotter one without putting in some energy. Refrigerators use an electric motor to do work (W) to remove heat energy (Q) from the interior of the refrigerator and transfer it to the warmer surroundings. The heat energy removed is therefore Q but the energy returned to the surroundings is equal to $Q + W$ which means the room increases in temperature as more energy is returned than was removed.

The coefficient of performance

The **coefficient of performance** of a refrigerator is defined as the heat removed from inside the refrigerator where the food and drink is kept, divided by the work that is done by the compressor.

For every 1 unit of work done by the compressor, COP units of thermal energy (Q_C) are removed from the interior of the refrigerator.

TIP

You do not need to be able to derive the coefficient of performance equations.

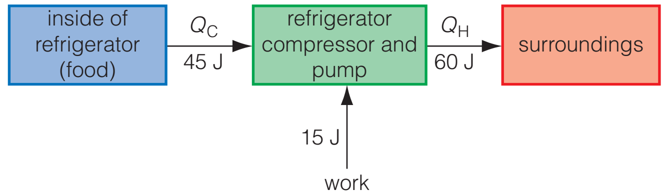


Figure 33.86 15 J of work are done by the compressor and pump to remove 45 J of thermal energy from the inside of the fridge. The COP_{REF} of this refrigerator is therefore 3 as 3 × 15 = 45.

EXAMPLE

1 A freezer has a COP_{REF} of 3.8 and uses 360 J of energy per second. How long would the freezer take to freeze 1.2 kg of water at 0°C into ice?

Answer

This requires a change of state from liquid to solid which occurs without a change in temperature. The latent heat of fusion of water is L_f = 330 000 J kg^{-1}.

So the energy needed to be removed in order to freeze 1.2 kg of water is

$$Q = mL = Q_C$$

$$= 1.2 \text{ kg} \times 330\,000 \text{ J kg}^{-1}$$

$$= 396\,000 \text{ J}$$

$$COP_{REF} = \frac{Q_c}{W}$$

$$W = \frac{396\,000}{3.8}$$

$$= 104\,211 \text{ J}$$

$$\text{Time} = \frac{\text{work done}}{\text{power}}$$

$$= \frac{1\,042\,115}{360 \text{ J s}^{-1}}$$

$$= 289.5 \text{ s}$$

In practice the freezer would take longer than this as not all of the electrical power in the motor goes to the refrigeration process. The electrical motor in itself is also inefficient.

2 A refrigerator with a 15 W compressor cools a 1 litre (mass of 1 kg) carton of orange juice from 25°C to 7°C in 26 minutes. Calculate the COP_{REF} of the refrigerator.

You may assume that the juice has a specific heat capacity of 4200 J kg^{-1} °C^{-1}.

Answer

Q_C = thermal energy removed from the juice

$$= mc\Delta\Phi$$

$$= 1 \text{ kg} \times 4200 \text{ J kg}^{-1}\,°\text{C}^{-1} \times 18\,°\text{C}$$

$$= 75\,600 \text{ J}$$

W = work done by compressor

$$= P \times t$$

$$= 15 \text{ W} \times 26 \text{ min} \times 60 \text{ s}$$

$$= 23\,400 \text{ J}$$

$$COP_{REF} = \frac{Q_c}{W}$$

$$= \frac{75\,600}{23\,400}$$

$$= 3.2$$

In practice this would take longer as the juice is likely to be in contact with other objects in the fridge and so will also absorb energy from them.

3 Calculate the maximum coefficient of performance of a refrigerator which uses a refrigerant which vaporises at a temperature of −18°C and condenses at 35°C. Explain why this maximum COP can never be achieved in practice.

Answer

$$COP_{IDEAL} = \frac{T_C}{T_H - T_C}$$

$$= \frac{255}{(308 - 255)}$$

$$= 4.8$$

In practice the ideal coefficient of performance can never be achieved as in each cycle energy will also be lost due to the flow of the refrigerant and friction.

Air-conditioning systems

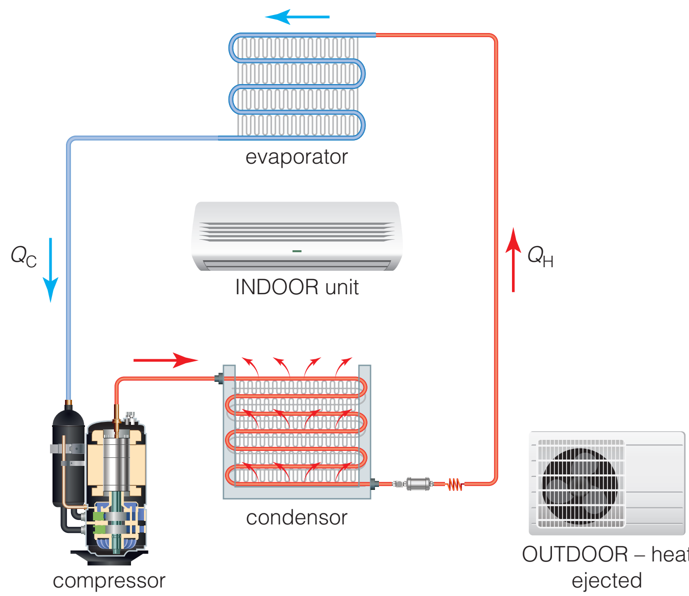


Figure 33.87 An air-conditioning system.

An air-conditioning system works in a similar way to a refrigerator but is constructed differently. However, the key purpose is still to transfer energy from inside to the higher-temperature surroundings outside the building.

Heat pumps

A heat pump is able to heat a building in winter by removing energy from the cooler surroundings and transferring it to the warmer building. As heat naturally flows from a high temperature to a low temperature a heat pump requires energy to be input in order for it to do this work.

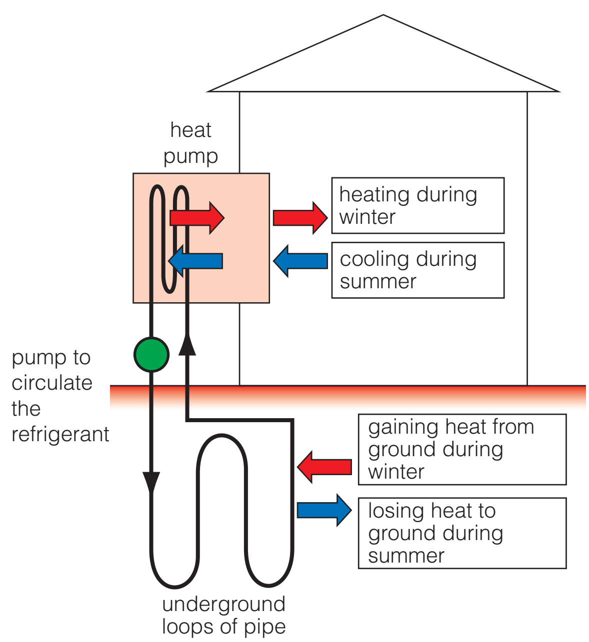


Figure 33.88 A heat pump for use in a cold climate does work to pump heat from cooler surroundings to the warmer inside of a building.

The **coefficient of performance** of a heat pump is defined as

$$COP_{HP} = \frac{\text{heat transferred into building, } Q_H}{\text{work done}}$$

$$= \frac{Q_H}{W}$$

$$= \frac{Q_H}{Q_H - Q_C}$$

A COP of 3 means that if 1 kWh of work is put in, we get 3 kWh of heat transferred into the building.

The maximum or ideal **COP of a heat pump**

$$COP_{IDEAL} = \frac{T_H}{T_H - T_C}$$

The heat pump will be more effective if the temperature difference between the outside and surroundings is smallest. A huge advantage of a heat pump is that it can be reversed to operate as an air-conditioning unit.

Provided the surroundings are above absolute zero they will contain some energy. Some of this energy can be transferred inside a building. A heat pump can be thought of as a refrigerator in reverse with the evaporation coils being placed outside the building and the heat exchanger (condenser coils) inside.

Heat pumps are extremely efficient because they move heat rather than generating heat by burning fuels.

EXAMPLE

1 A heat pump has a coefficient of performance of 3.8 and is rated to do work at a rate 2.5 kW.

a) How much heat could it add to a building per second?

Answer

$$COP_{HP} = \frac{Q_H}{\text{work done}}$$

$$Q_H = COP_{HP} \times \text{work done}$$
$$= 3.8 \times 2500 \text{ J s}^{-1}$$
$$= 9500 \text{ J s}^{-1}$$

b) Calculate the coefficient of performance of the heat pump when it is used as an air-conditioning unit in a cold climate summer.

Answer

$Q_C + W = Q_H$

therefore

$$Q_C = 9500 \text{ J s}^{-1} - 2500 \text{ J s}^{-1}$$
$$= 7000 \text{ J s}^{-1}$$

$$COP_{REF} = \frac{Q_C}{W}$$
$$= \frac{7000 \text{ J s}^{-1}}{2500}$$
$$= 2.8$$

2 A domestic heat pump is designed to keep the rooms at a constant temperature of 20°C. Calculate the change in the coefficient of performance on a day when the average outside temperature is 15°C compared to a cold winter's day when the average outside temperature is 1°C.

Answer

$$COP_{IDEAL} = \frac{T_H}{T_H - T_C}$$

When outside temperature is 15°C = 288 K

$$COP_{IDEAL} = \frac{293}{(293 - 288)}$$
$$= 58.6$$

When outside temperature is 1°C = 274 K

$$COP_{IDEAL} = \frac{293}{(293 - 274)}$$
$$= 15.4$$

From a cold day to a hot day the COP increases by

$$\frac{58.6}{15.4} = 3.8$$

That is

$$COP_{\text{warm day}} = 3.8 \times COP_{\text{cold day}}$$

3 Explain why electric heaters can never have an efficiency greater than 1 (100%) but heat pumps can have a COP_{HP} of 8 or higher.

Answer

The maximum efficiency of a 1 kW heater is 1 kW (i.e. 100%).

However a heat pump transfers energy from the cold source (outside) into the building and so transfers more energy than the work done by the pump.

Disadvantages of heat pumps

With such high coefficients of performance heat pumps may seem almost too good to be true. However there are some problems with them. The example above showed that they work better in the summer than in the winter which is the opposite of what is needed from a heating system. There are also large costs and difficulties involved with installing the large network of pipes underground needed to absorb energy from the surroundings.

TEST YOURSELF

Reversed heat engines

1 A particular refrigerator that uses an electric motor has a COP equal to 7. Show that for every unit of energy used by the electric motor, 8 units of energy will be ejected from the refrigerator to the surroundings.

2 Figure 33.89 shows the idealised relation between the pressure and volume of the refrigerant in a refrigerator as it is taken around one complete cycle.

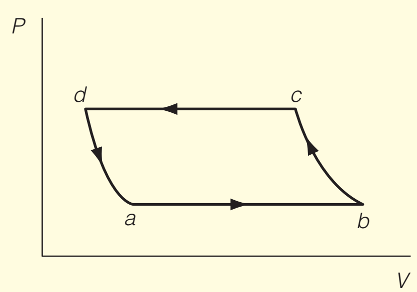


Figure 33.89

a) Label each stage of the cycle.

b) State and explain where in the cycle energy is absorbed from the cold reservoir and where it is ejected to the surroundings.

3 A freezer takes water at 18°C and turns it into ice at −15°C. The coefficient of performance of the freezer is 4.8.

Specific heat capacity of water = 4200 J kg^{-1} $°C^{-1}$
Specific heat capacity of ice = 2100 J kg^{-1} $°C^{-1}$
Latent heat of fusion = 330 J kg^{-1}

a) Calculate the power input to the freezer if it is required to make 8 kg of ice every hour.

b) Calculate the rate at which energy is delivered to the surroundings of the freezer.

4 The left-hand box in Figure 33.90 shows a combined heat and power scheme.

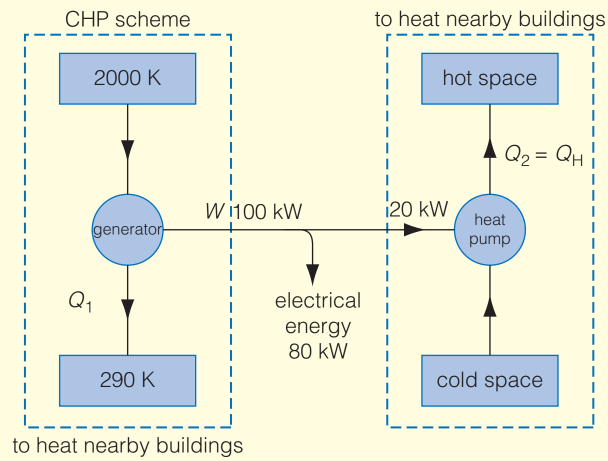


Figure 33.90

This provides electrical energy W from a gas turbine driven generator and heat Q_1 for nearby buildings. You should consider the generator to be operating as an engine.

Some of the electrical energy is used to drive a heat pump with Q_2 also used to heat buildings.

a) Calculate the maximum theoretical efficiency of the generator.

b) The output power of the generator is 100 kW. Calculate the input power to the generator.

c) Calculate the flow rate of gas into the engine if the gas fuel burned in the power station has a calorific value of 47 M J kg^{-1} and state what you are assuming when making this calculation.

d) The coefficient of performance of the heat pump is 2.9 and the power supplied by the electrical generator to the heat pump is 20 kW. Calculate the total power available for heating.

5 a) Explain why the coefficient of performance for a heat pump is greater than when the same heat pump is working as a refrigerator.

b) A student claims that 'A heat pump delivers more energy than is supplied to it'. Discuss whether or not this statement contradicts the first and second laws of thermodynamics.

Exam practice questions

Thermodynamics

1 A quantity of gas is enclosed in a metal cylinder fitted with a piston. The very many gas molecules present are represented in Figure 33.91 by the molecules shown. The cylinder walls are thermally conducting.

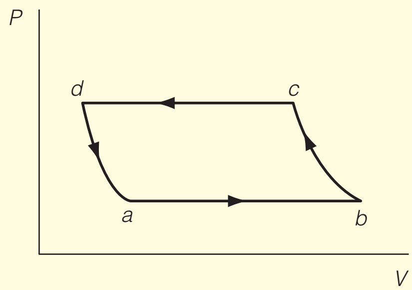


Figure 33.91 The motion of molecules in a gas is random.

a) The piston is moved outwards slowly so that the gas temperature is the same after the increase in volume. Explain why moving the piston slowly ensures the temperature of the gas does not increase as it expands. *(3)*

b) Explain whether work is done as the piston moves outwards and if so, is the work done on the gas or by the gas? *(2)*

c) Explain whether there is a flow of heat as the piston moves outwards. If so, does it flow into or out of the cylinder? *(2)*

d) Discuss whether heat can be completely converted to work in this expansion and the implications for using this arrangement to continuously convert heat into work. *(4)*

2 The graph below shows the variation with volume V of the pressure p for two isothermal changes of two ideal gases X and Y. The gases have the same number of moles.

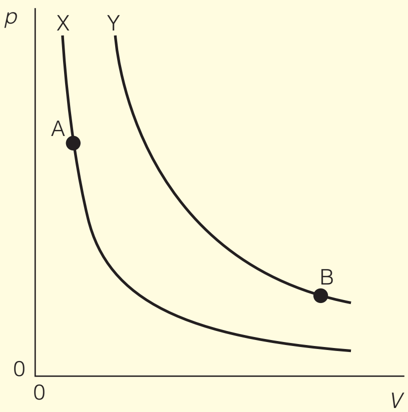


Figure 33.92

a) State what is meant by an isothermal change. *(1)*

b) Explain whether gas X in the state shown by point A (p_A,V_A) is at a higher or lower temperature than gas Y in the state shown by point B (p_B,V_B). *(2)*

c) Gas Y is compressed adiabatically from state (p_B,V_B) until it reaches the pressure p_A. Explain what happens to the temperature of gas Y during this process. *(3)*

d) Copy Figure 33.92 and shade an area to represent the work done when gas X compressed isothermally from V_B to V_A. *(2)*

3 A 2-cylinder lawnmower petrol engine was tested. Each cylinder produced the following results:

flow rate of fuel	2.5×10^{-2} kg min^{-1}
calorific value of fuel	52 MJ kg^{-1}
indicated power	4.8 kW
rotational speed of output shaft	1100 rev min^{-1}
torque measured at output shaft	50 Nm

Use this data to:

a) Calculate the rate at which energy is supplied to the engine. *(2)*

b) Calculate the overall efficiency of the engine. *(3)*

c) Estimate the power dissipated in overcoming frictional losses. *(1)*

4 a) State and explain, with reference to the first law of thermodynamics, two ways in which the internal energy of a gas can be decreased. *(4)*

b) A volume of 25 m^3 of exhaust gas leaves an engine during 1 cycle. The exhaust gas contracts to one third of its volume as it is cooled by the surrounding atmosphere at a pressure of 1.0×10^5 Pa. If 5.2 MJ of heat is transferred to the atmosphere during cooling, calculate the change in internal energy of the exhaust gas. *(3)*

c) Sketch a *p-V* indicator diagram to illustrate this process. *(3)*

5 An air rifle uses a spring held in compression to propel the pellet as shown in Figure 33.93.

When the trigger is pulled the spring is released. This pushes a piston rapidly to the left along the cylinder, compressing the air behind the pellet. A force is therefore exerted on the pellet which accelerates it along the barrel.

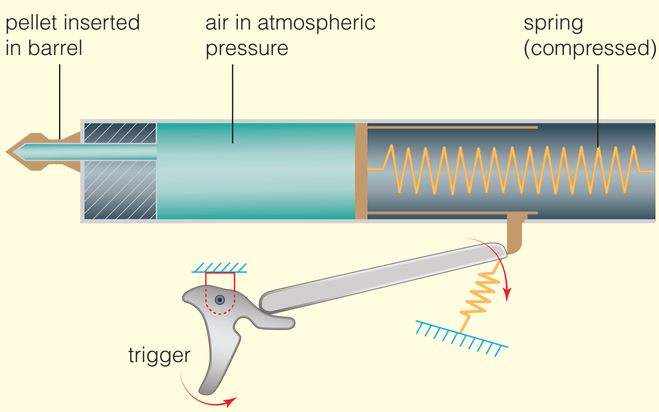


Figure 33.93 An air rifle mechanism which is ready to fire.

a) If the initial length of the cylinder is 8.5 cm, with an internal diameter of 1.5 cm, calculate the number of moles of air inside the cylinder. The air is at a temperature of 288 K and is at atmospheric pressure (101 kPa). *(2)*

b) The piston compresses the air adiabatically. Explain why the compression can be considered an adiabatic compression and calculate the pressure of the air when the piston has moved 5 cm. *(3)*

c) Estimate the temperature in the cylinder when the piston has travelled 5 cm along the cylinder. *(2)*

6 The graph in Figure 33.94 of pressure p against volume V shows the expansion of 2.0 moles of a monatomic ideal gas from state A to state B.

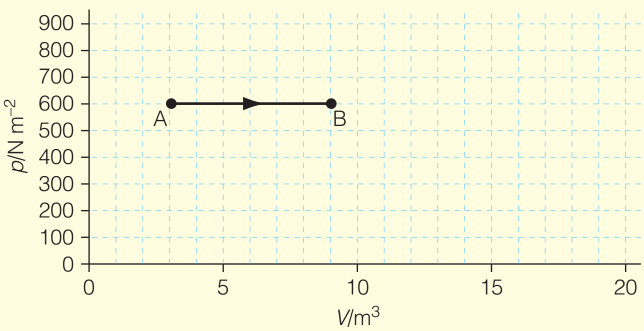


Figure 33.94

a) Calculate the work done as the gas expands. *(2)*

b) Calculate the change in internal energy of the gas as it expands. *(3)*

c) Calculate the heat added to or removed from the gas during this process. *(2)*

d) The pressure is then reduced to 200 N m^{-2} without changing the volume as the gas is taken from state B to state C. Copy Figure 33.94 and label state C and draw a line to represent this process. *(2)*

e) The gas is then compressed isothermally back to state A. Draw a line to represent this process and explain whether heat is added to or removed from the gas during this compression. *(3)*

7 An inventor claims to have invented four engines, each of which operates between a hot reservoir at 400 K and a cooler reservoir at 300 K.

Data on each engine is shown in the table.

Discuss the claims made for each engine in terms of the first and second laws of thermodynamics. *(6)*

Engine	Q_H	Q_C	W
1	200	–175	40
2	500	–200	400
3	600	–200	400
4	100	–90	10

8 a) The compressor motor in a refrigerator has a power of 250 W. If the freezing compartment is at 270 K and the outside air is at 300 K, calculate the maximum amount of energy that can be extracted from the freezing compartment in 20.0 minutes. *(4)*

b) State what you are assuming in answering part **(a)** and calculate the amount of energy transferred to the surroundings each second. *(2)*

9 An inventor claims to have constructed an engine that has an efficiency of 70% when operating between the boiling and freezing points of water.

a) Discuss whether this claim is possible. *(3)*

b) Explain why a real engine can never attain the maximum possible efficiency. *(2)*

10 An ideal gas initially at 300 K is compressed at a constant pressure of 20 N m^{-2} from a volume of 3.0 m^3 to a volume of 1.8 m^3. In this process, 75 J is lost as heat.

a) Calculate the change in internal energy of the gas. *(3)*

b) Calculate the final temperature of the gas. *(2)*

Rotational dynamics

1 A flywheel battery can be used to replace chemical batteries to provide a short-term electrical power supply. A motor is used to drive the flywheel up to speed. The mass of the flywheel is 200 kg and its moment of inertia is 32 kg m^2.

a) Explain how the energy can be recovered from the flywheel. *(1)*

b) If the flywheel has a radius of 0.40 m and can be rotated at a maximum angular speed of 48 000 rev min^{-1}, calculate the rotational kinetic energy stored when it is rotating at half its maximum speed. *(1)*

c) The manufacturer claims that losses due to friction when the flywheel is running at its maximum speed is 1.7 W and the mean power loss over the range of speeds from rest to its maximum speed is 0.8 W. Calculate the frictional torque acting on the rotor when spinning at its maximum speed and the time taken to come to rest from its maximum speed under the action of the frictional torque alone. *(4)*

d) When supplying electrical power the flywheel battery can supply a constant load of 3 kW for 24 hours. Calculate the flywheel's angular speed in rpm at the end of this period if it starts at its maximum angular speed. *(4)*

e) Explain why a flywheel has a maximum safe angular speed of rotation. *(1)*

f) Two possible designs for flywheels are shown in Figure 33.95. Both spin at the same angular speed and have the same mass.

Explain which design should be used for the flywheel battery discussed above. *(2)*

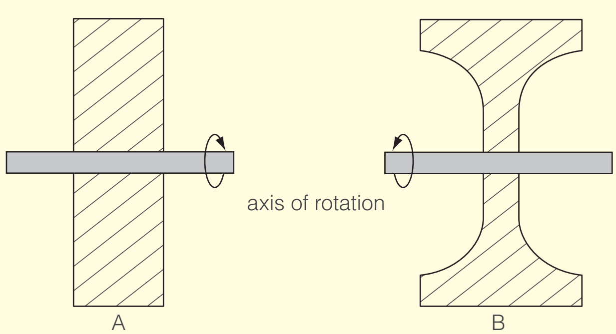


Figure 33.95

2 The RAF use human centrifuges to simulate the large 'g' forces experienced by pilots. A rotating arm is driven by an electric motor and the pilot sits in a capsule at the end of the rotating centrifuge arm as shown in Figure 33.96.

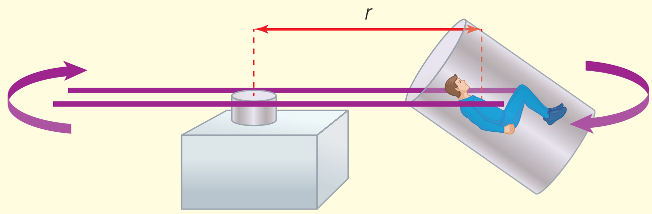


Figure 33.96

a) The motor driving the centrifuge is able to work at a maximum power of 200 kW for 5.2 s. During this time the angular speed increases from 19.0 rpm to 82.0 rpm. Use this data to estimate the moment of inertia of the rotating system. *(5)*

b) State what you are assuming in your calculation in part **(a)**. *(1)*

3 An early design of four-stroke engine stores kinetic energy in a flywheel attached to the crankshaft. The engine is started from rest and produces a torque which accelerates the flywheel to a speed of 105 rev min^{-1}.

The flywheel has a moment of inertia of 280 kg m^2 and takes 9.5 s to accelerate from rest to 105 rev min^{-1}.

a) Calculate the average accelerating torque acting on the flywheel. *(2)*

b) Calculate the average useful power output of the engine during the period when it is accelerating. *(2)*

When the engine is running at 105 rev min^{-1} the fuel supply is switched off and the flywheel continues to rotate for a further 36 turns before coming to a rest.

c) Estimate the average retarding torque acting on the flywheel. *(3)*

4 A KERS solid disc flywheel system is fitted to a delivery van in order to avoid 'wasting' kinetic energy when it stops to make a delivery. The stored rotational energy is recovered when the van re-starts.

When the van comes to rest from a speed of 50 km h^{-1}, 50% of its translational kinetic energy is transferred to the flywheel.

Use the following data to calculate the radius of the flywheel.

maximum safe angular speed of flywheel = 400 rad s^{-1}

mass of loaded van = 3000 kg

thickness of flywheel = 0.15 m

density of steel alloy from which flywheel is constructed = 8.0 kg m^{-3}

moment of inertia of a solid disk = $\frac{1}{2} Mr^2$ *(5)*

5 A grinding wheel is used to sharpen a knife. The heavy wheel rotates and the knife blade is pushed against the edge of the wheel with a tangential force of 2.5 N. An electric motor accelerates the grinding wheel uniformly up to its operating speed of 600 rpm.

The radius of the wheel is 0.20 m and it has a moment of inertia of 0.85 kg m^2.

a) Calculate the angular acceleration of the wheel caused by the torque from the knife. *(2)*

b) Explain what happens to the rotational kinetic energy lost by the grinding wheel. *(1)*

c) Calculate the amount of power supplied by the motor in order to maintain the grinding wheel at a speed of 600 rpm during sharpening. *(3)*

6 A light turntable is mounted on low-friction bearings. Describe an experimental method to determine the rotational inertia of the turntable.

You should include:

- A list of the equipment used, what measurements you will take and how they will be taken
- How the measurements will be used to calculate the moment of inertia
- Which measurements contribute the most uncertainty and how these uncertainties will be minimised. *(6)*

Figure 33.97

7 A student performed an experiment to measure the moment of inertia of a heavy pulley.

Masses are attached by a string to the pulley and the string is wrapped around the edge of the pulley several times. The masses are released from rest and the time taken for them to fall a distance, h, to the floor is recorded. The experiment is repeated three times but only data for the first run is shown below.

Radius of pulley = 0.125 m

Mass of falling masses = 2.5 kg

h = 2 m

Time to fall distance h = 1.41 s

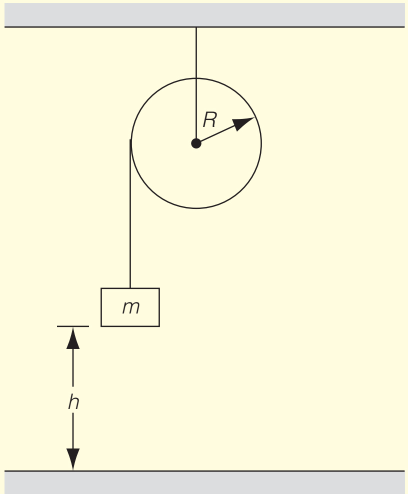


Figure 33.98 Moment of inertia of a flywheel.

a) Use the data to calculate the linear acceleration of the falling masses. *(1)*

b) The diameter of the pulley was measured to be 0.25 m. Calculate the angular acceleration. *(2)*

c) Determine the moment of inertia of the pulley. *(4)*

8 Low-friction motors are able to change their speed and direction of rotation rapidly.

a) Explain in terms of work done and torque why a low moment of inertia is desirable when the speed and direction of rotation must be changed quickly and describe the features of such a design of rotor. *(4)*

b) One such motor reverses direction from an angular speed of 170 rad s^{-1} to 90 rad s^{-1} in the opposite direction. If this is completed in 75 ms, calculate the angular acceleration and state what you are assuming. *(2)*

c) The moment of inertia of the motor's rotor is 3.8×10^{-5} kg m^2. Calculate the torque required to achieve the acceleration you calculated in part **(b)**. *(1)*

9 Figure 33.99 shows data for the torque measured at the crankshaft as a 4-stroke petrol engine completes one cycle.

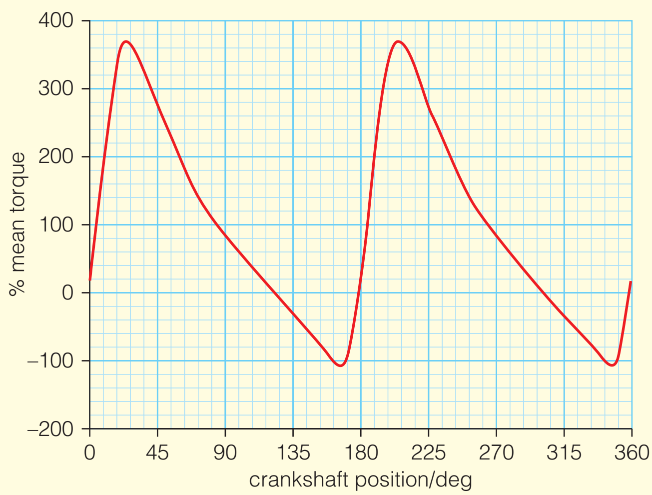


Figure 33.99 Instantaneous torque even-fire 4-cylinder engine.

a) Explain why the torque varies through one cycle. You should sketch a diagram to illustrate your answer. *(3)*

b) Explain how the graph could be used to calculate the work done by the engine in one revolution of the crankshaft. *(2)*

c) Describe a modification which could be made to 'smooth' out the motion of the crankshaft and deliver a more even power. *(3)*

10 A firework wheel consists of two rockets attached to a beam which is pivoted at the centre. Thrust from the exhaust gases of the rockets provides a torque.

The moment of inertia of the beam = 0.09 kg m^2.

a) Each rocket has a mass of 0.38 kg and is attached a distance of 0.4 m either side of the pivot. Calculate the moment of inertia of the firework wheel about the pivot. *(2)*

b) Both rockets are lit simultaneously and each produces a thrust of 2.8 N. Calculate the time taken for the fire rocket to complete three whole rotations, starting from rest. *(4)*

c) Explain why, after a short time, the firework wheel begins to rotate at a constant angular speed until the fuel runs out. *(2)*

Answers to Test yourself on prior knowledge questions

1 For small angles the chord length and arc length are approximately the same.

$s = r\,\theta$

$\theta = \frac{s}{r}$

$= \frac{0.05}{100}$

$= 5 \times 10^{-4}$ rad

$= 5 \times 10^{-4} \times \left(\frac{360}{2\pi}\right) = 0.029°$

2 $\omega = 2.5 \times 10^{-6}$ rad s^{-1}

3 The bubbles of CO_2 expand as their temperature increases in accordance with Charles' Law.

$\frac{V_1}{T_1} = \frac{V_2}{T_2}$

4 $P_1V_1 = P_1V_1$

$2.8 \times 2 = 1.9 \times V_2$

$V_2 = 0.34$ cm^2

Answers to Test yourself on rotational motion questions

1 (a) $v = \frac{d}{t}$ so the outer child travels further in the same time interval and so has the greater linear velocity.

(b) The angular velocity is the same for both as $\omega = \frac{\theta}{t}$ and both take the same time to complete one rotation.

2 (a) $\alpha = (\omega_2 - \omega_1)/t$:

$\omega_2 = \alpha t + \omega_1$

$= 0.05 \times 12$

$= 0.6$ rad s^{-1}

(b) $v = r\,\omega$

$= 2 \times 0.6$

$= 1.2$ ms^{-1}

3 (a) $\omega_2 = 2\pi f$

$= 2\pi \times \left(\frac{30\,000}{60}\right)$

$= 3142$ rad s^{-1}

$\alpha = \frac{(\omega_2 - \omega_1)}{t}$

$= \frac{(3142 - 0)}{20}$

$= 157$ rad s^{-2}

(b) $\theta = \frac{(\omega_2 - \omega_1)}{2\alpha}$

$= \frac{3142^2}{(2 \times 157)}$

$= 31\,440$ rad

$= \frac{31440}{2\pi} = 5003$ revolutions whilst accelerating

It then rotates at a constant angular velocity of 3142 rad s^{-1} for another 40 seconds.

$\theta = \omega t = 3142 \text{ rad s}^{-1} \times 40 = 125\,680 \text{ rad}$

Number of revolutions completed at constant angular velocity $= \frac{125\,680}{2\pi} = 20\,002.6$

Total number of revolutions = 5003 + 31 440 = 25 005 complete revolutions.

4 (a) Gradient of graph is greatest at t_A

(b) and least at t_B

5 (a) $\omega_2 = \omega_1 + \alpha t$

$= 0 + 0.21 \times 10 = 2.1 \text{ rad s}^{-1}$

(b) $\theta = \omega_1 + \frac{1}{2}\alpha t^2$

$= 0 + \frac{1}{2} \times 0.21 \times 10^2$

$= 10.5$ rad

6 (a) $\alpha = \frac{(\omega_2 - \omega_1)}{t}$ but $\omega = \frac{2\pi}{T}$ so $\alpha = \frac{\frac{2\pi}{T_2} - \frac{2\pi}{T_1}}{t}$

$$\alpha = \frac{\frac{2\pi}{0.033s + 1.26 \times 10^{-5}s} - \frac{2\pi}{0.033s}}{60 \times 60 \times 24 \times 365.25}$$

$= -2.31 \times 10^{-9} \text{ rad s}^{-2}$

(b) $t = \frac{(\omega_2 - \omega_1)}{\alpha}$

$$= \frac{0 - \left(\frac{2\pi}{0.033}\right)}{-2.31 \times 10^{-9} \text{ rad s}^{-2}}$$

$= 8.24 \times 10^{10}$ s = 2612 years

= 2600 years

Answers to Torque and moment of inertia questions

1 (a) $I = mr^2$

$I = 2 \text{ kg } (3\text{m})^2$

$I = 18 \text{ kg m}^2$

(b) $I = 3 \times (\frac{1}{3}MR^2)$

$= 30 \text{ kg} \times 7^2$

$= 1470 \text{ kg m}^2$

2 (a) $\alpha = \frac{T}{I}$

$$= \frac{(0.25 \times 2 \times 9.81)}{(\frac{1}{3} \times 2 \times 0.5^2)}$$

$= \frac{4.905}{0.167} = 29.4 \text{ rad s}^{-2}$

(b) $a = \alpha r$

$= 29.4 \times 0.5$

$= 14.7 \text{ ms}^{-2}$

This is greater than g and so an object balanced on the end of the shelf support would be left behind once the rod was allowed to fall freely.

3 (a) $I = \Sigma Mr^2$

$= 10 \times 1^2 + 10 \times 1^2 = 20 \text{ kg m}^2$

(b) $(10 \times 0.5^2) + (10 \times 1.5^2)$

$= 2.5 + 22.25 = 24.75 \text{ kg m}^2$

4 Net torque = (20 × 0.45) – 1.5 = 7.5 N m

Apply Newton's 2nd law of motion for rotation: $\Sigma T = I\alpha$

$$7.5 \text{ N m} = \frac{I(\omega_2 - \omega_1)}{t}$$

$$7.5 = I \times \left(\frac{40}{5}\right)$$

$$I = 0.94 \text{ kg m}^2$$

5 $I_{\text{earth}} = \frac{2}{5} MR^2$

$$= \frac{2}{5}(5.98 \times 10^{24} \times (6.4 \times 10^6)^2)$$

$$= 9.7 \times 10^{37} \text{ kg m}^2$$

$$\omega_1 = \frac{2\pi}{(24 \times 60 \times 60)} = 0.73 \text{ rad s}^{-1}$$

$$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{(0 - 0.73)}{(12 \times 60 \times 60)} = -1.68 \times 10^{-9} \text{ rad s}^{-2}$$

Apply Newton's 2nd law of motion for rotation: $\Sigma T = I\alpha$

$$T = 9.7 \times 10^{37} \text{ kg m}^2 \times 1.68 \times 10^{-9} \text{ rad s}^{-2}$$

$$= 1.63 \times 10^{29} \text{ N m}$$

$$F = \frac{T}{R}$$

$$= \frac{1.63 \times 10^{29}}{6.4 \times 10^6}$$

$$= 2.6 \times 10^{22} \text{ N}$$

The force would need to be exerted tangentially to the radius. If the angle was less than 90° a greater force would be required.

Answers to Angular kinetic energy, work and power questions

1 $\omega = \frac{120 \times 2\pi}{60} = 12.57 \text{ rad s}^{-1}$

$$I = \frac{1}{2} \times 60\,000 \times 5^2$$

$$= 750\,000 \text{ kg m}^2$$

$$E_K = \frac{1}{2} I\omega^2$$

$$= 0.5 \times 750\,000 \times 12.57^2$$

$$= 59 \text{ MJ } (3.6 \text{ MJ} = 1 \text{ kWh})$$

$$= 16.4 \text{ kWh}$$

2 Mass = density × volume

$= 1.2 \times \pi \times 100\,000^2 \times 4000$

$= 1.51 \times 10^{14}$ kg

$v = \dfrac{180\,000}{3600} = 50 \text{ ms}^{-1}$

$\omega = \dfrac{v}{r}$

$= \dfrac{50}{100\,000}$

$= 5 \times 10^{-4} \text{ rad s}^{-1}$

$I = \dfrac{1}{2}Mr^2$

$= 0.5 \times 1.51 \times 10^{14} \times 100\,000^2$

$= 7.55 \times 10^{23} \text{ kg m}^2$

$E_K = \dfrac{1}{2} I\, \omega^2$

$= 0.5 \times 7.55 \times 10^{23} \times (5 \times 10^{-4})^2$

$= 9.44 \times 10^{16}$ J

Much less energy will be stored than this as speed of rotation is not constant.

3 $W = T\theta$

$t = 1$ second, $f = \dfrac{2000}{60} = 33.3$ rps, $\omega = \dfrac{\theta}{t}$ so $\theta = 33.3 \times 2\pi$

$W = 800 \times 33.3 \times 2\pi$

$= 167\,552 \text{ J s}^{-1}$

$= 168$ kW

4 (a) $E_K = \dfrac{1}{2} I\, \omega^2$

$I = \dfrac{1}{2}MR^2$

$= 0.5 \times 500 \text{ kg} \times 1.2^2$

$= 360 \text{ kg m}^2$

$E_K = 0.5 \times 360 \times 628^2$

$= 71 \times 10^6$ J

(b) Time $= \dfrac{\text{energy}}{\text{power}}$

$= \dfrac{71 \times 10^6}{10\,000}$

$= 7100$ s $= 118$ minutes

5 Work done = GPE gained + translation KE gained + angular KE gained

$\sin 25° = \dfrac{\Delta h}{10}$ therefore $\Delta h = 4.23$ m

$40 \times 10 = (5 \times 9.81 \times 4.23) + (\frac{1}{2} \times 5 \times v^2) + (\frac{1}{2} \times 0.6 \times \omega^2)$

$\omega = \dfrac{v}{r}$

therefore $400 = 207.5 + 2.5\, v^2 + \left(\dfrac{0.3\, v^2}{0.4^2}\right)$

$192.5 = 4.38\, v^2$

$v = 6.6 \text{ ms}^{-1}$

6 **Sliding**

Apply conservation of energy: $mg\Delta h = \dfrac{1}{2} m\, v^2$

therefore $v_{\text{sliding}} = \sqrt{2gh} = 5.42 \text{ m s}^{-1}$

Rolling

GPE lost = rotational KE gained + translational KE gained

$$mg\Delta h = \frac{1}{2} I\, \omega^2 + \frac{1}{2} m\, v^2$$
$$mg\Delta h = \frac{1}{2} \times \frac{2}{5} \times mr^2\, \omega^2 + \frac{1}{2} m\, v^2$$
but $\omega = \frac{v}{r}$ and so
$$mg\Delta h = \frac{1}{2} \times \frac{2}{5} \times mr^2 \left(\frac{v}{r}\right)^2 + \frac{1}{2} m\, v^2$$
which simplifies to
$$g\Delta h = \left(\frac{7}{10}\right) v^2$$
$$v = \sqrt{\frac{10\ g\ \Delta h}{7}} = \sqrt{\frac{10 \times 9.81 \times 1.5}{7}} = 4.58\ \text{ms}^{-1}$$
Therefore the ball which was only sliding reaches the bottom first as it has greater final velocity.

Answers to Angular momentum questions

1 (a) $\omega_1 = \frac{1}{1.8} = 0.56$ rev s^{-1}, $I_1 = 5.1$ kg m^2

(b) $I_1\, \omega_1 = I_2\, \omega_2$

$5.1 \times 0.56 = I_2 \times 3$

$I_2 = 0.95$ kg m^2

(c) They reduce their moment of inertia by bringing their arms in and so moving more mass closer to the axis of rotation which reduces I as $I = Mr^2$.

2 The system has no angular momentum before.

After, $L = I\, \omega$ where $I = I_{bullet} + I_{cylinder}$

$$I = \frac{1}{2} m_{cylinder}\, (r_{cylinder})^2 + m_{bullet}\, (r_{cylinder})^2$$
$$L = (\frac{1}{2}\, m_{cylinder} + m_{bullet})\, (r_{cylinder})^2\, \omega$$
$$m_{bullet}\, v\, \mathrm{r}_{cylinder} = (\frac{1}{2}\, m_{cylinder} + m_{bullet})\, (r_{cylinder})^2\, \omega$$
$$\omega = \frac{m_{bullet} v}{(\frac{1}{2}\, m_{cylinder} + m_{bullet})\, (r_{cylinder})}$$
$$\omega = \frac{0.005 \times 960}{(0.5 \times 3 + 0.005)\ 0.3} = \frac{4.8}{0.45} = 10.6 \text{ rad s}^{-1} = 1.69 \text{ rps}$$

3 $I_1\, \omega_1 = I_2\, \omega_2$

$$\frac{2}{5} M_0\, (r_0)^2\, \omega_1 = \frac{2}{5} \times 0.5\, M_0 \times (0.015\, r_0)^2\, \omega_2$$
$$\omega_1 = \frac{1}{2} \times 0.015^2\, \omega_2$$
$$\omega_2 = \frac{\omega_1}{1.125 \times 10^{-4}}$$
$$= 1861.7 \text{ rad day}^{-1}$$
$$T_2 = \frac{2\pi}{\omega_2}$$
$$= 3.4 \times 10^{-3} \text{ days}$$

This is rather fast but we have assumed that the ejected matter has not carried any angular momentum with it.

4 (a) The angular velocity decreases as the moment of inertia increases as the person moves their mass further from the axis of rotation.

(b) I = moment of inertia of roundabout + moment of inertia of person
$I = 950 + 70 \times 2^2 = 1230$ kg m^2
$950 \times 0.9 = 1230 \times \omega_2$
$\omega_2 = 0.7$ rad s^{-1}

Answers to Rotational and linear motion compared questions

1

Linear motion	Rotational motion
Velocity, v	Angular velocity, $\omega = \frac{v}{r}$
Momentum = mv	Angular momentum = $I\,\omega$
Kinetic energy = $\frac{1}{2} m v^2$	Rotational kinetic energy = $\frac{1}{2} I\,\omega^2$
Resultant force, $F = ma$	Resultant torque, $T = I\,\alpha$
Change in momentum = $F \times t$	Change in angular momentum = $T \times t$
Work done = change in KE = $F \times s$	Work done = change in KE = $T \times \theta$
Power = $\frac{\text{work done}}{\text{time}}$	$P = T\omega$
Linear momentum is conserved if no external forces act	Angular momentum is conserved if no external torques act, $I_1\,\omega_1 = I_2\,\omega_2$

Answers to First law and work done questions

1 (a) Area of piston $= \pi \left(\frac{d}{2}\right)^2 = 0.015$ m^2

$W = p\Delta V = 110\,000 \times 0.25 \times 0.015 = 412.5$ J

(b) $Q = \Delta U + W$
$\Delta U = Q - W$
$= 600 - 412.5$
$= 187.5$ J $= 188$ J

2 $W = p\Delta V = p(V_2 - V_1)$
$= 100\,000 \times (3.5 - 5)$
$= -150\,000$ J

The work done is negative as work is being done on the gas to compress it.

As $\Delta U = 0$ (in order for the pressure to remain constant), the 150 000 J is transferred from the gas ($Q = -150\,000$ J).

3 (a) $W = p\Delta V$
$= 101\,000 \times 4$
$= 404\,000$ J
$Q = mL_v = 0.8 \times 2.26 \times 10^6$ J
$= 1.808 \times 10^6$ J
$Q = \Delta U + W$ therefore $\Delta U = Q - W = 2.71 \times 10^6 - 404\,000$
$= 2.31 \times 10^6$ J

(b) The change in state occurs at a constant temperature and so the KE of the particles remains constant. Therefore the increase in internal energy (Q) increases the separation of the particles.

4 (a) $W = p\Delta V$

$= 1.2 \times 10^5 \times 0.05$

$= 6000$ J

(b) $Q = \Delta U + W$

$\Delta U = Q - W$

$= 8000 - 6000 = 2000$ J

Answers to Ideal gases questions

1 No intermolecular forces and so there are no forces between molecules so no potential energy. Internal energy = random kinetic energy of particles + potential energy.

2 Room temp = 300 K and a typical room could be 3 m high × 8 m × 5 m, volume = 120 m^3

$$n = \frac{pV}{RT}$$

$$= \frac{100000 \text{ Pa} \times 120 \text{m}^3}{8.31 \text{ J mol}^{-1} K^{-1} \times 300\text{K}} = 4813 = 4800 \text{ mol}$$

3 (a) $pV = nRT$, $V_1 = \frac{nRT}{p} = \frac{0.35 \times 8.31 \times 303}{1.03 \times 10^5}$

$= 8.56 \times 10^{-3}$ m^3

(b) $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$

$$T_2 = \frac{p_2 V_2 T_1}{p_1 V_1}$$

$$= \frac{8 \times 10^6 \times \left(\frac{8.56 \times 10^{-3}}{20}\right) \times 303}{1.03 \times 10^5 \times 8.56 \times 10^{-3}}$$

$= 1177$ K $= 904$ °C

(c) The gas behaves as an ideal gas at all temperatures and pressures.

Answers to Non-flow processes questions

1 $p_2 V_2 = p_1 V_1$

$$p_2 = \frac{150 \text{ kPa} \times 0.06}{0.042} = 214.3 \text{ kPa}$$

2 (a) $P_1 = 120$ kPa, $P_2 = ?$

$T_1 = 312$ K, $T_2 = ?$

$V_2 = 3\ V_1$

$p_1 (V_1)^\gamma = p_2 (V_2)^\gamma$

$$p_2 = p_1 \left(\frac{V_1}{V_2}\right)^\gamma$$

$$= 120 \text{ kPa} \left(\frac{1}{4}\right)^{1.4}$$

$= 25.8$ kPa

(b) $pV = nRT$, therefore $\frac{pV}{T} = nR$ and as no gas escapes we can use the ideal gas equation in the form

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\frac{120\,000 \times V_1}{312} = \frac{25\,800 \times 3\,V_1}{T_2}$$

$T_2 = 201.2\ K$

3 (a) $p_1(V_1)^{\gamma} = p_2(V_2)^{\gamma}$

$p_2 V_2 = p_1 V_1$

$p_2 = 200\ \text{kPa} \times \frac{5\ \text{m}^3}{0.5\ \text{m}^3}$

$= 2000\ \text{kPa}$

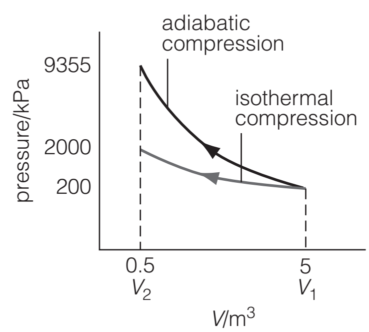
(b) $p_1 (V_1)^{\gamma} = p_2 (V_2)^{\gamma}$

$$p_2 = p_1 \left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$= 200\ \text{kPa} \left(\frac{5}{0.5}\right)^{1.67}$$

$= 9355\ \text{kPa}$

(c)



4 $V_2 = \frac{V_1}{4}$

$p_2 = \frac{p_1 V_1}{V_2} = 1.00 \times 10^5 \times \frac{V_1}{0.25\ V_1} = 4 \times 10^5$ Pa = pressure of compressed gas

For the adiabatic change, $T_1 = 12$ °C = 285 K as the first stage was isothermal.

$p_1(V_1)^{\gamma} = p_2(V_2)^{\gamma}$

$p_2 = p_1 \left(\frac{V_1}{V_2}\right)^{\gamma}$ so $p_2 = 4 \times 10^5 \times \left(\frac{1}{4}\right)^{1.4} = 0.57 \times 10^5$ Pa

To calculate the final temperature after the expansion use $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$ as we now know the initial and final pressures and volumes.

$$T_2 = \frac{p_2 V_2 T_1}{p_1 V_1} = \frac{0.57 \times 10^5\ \text{Pa} \times V_2 \times 285}{4 \times 10^5\ \text{Pa} \times 0.25\ V_2} = 162\ \text{K}$$

$= -110$ °C

5 Isothermal: takes place at constant temperature; adiabatic: no energy exchange between gas and surroundings. The expansion is not adiabatic as heat is transferred to the gas. If it expands at constant pressure it cannot be isothermal as the rate of collisions between the particles and the container walls would decrease as the volume increases causing the pressure to decrease. Temperature is a measure of the average KE of the particles and KE must increase in order to maintain the same rate of collisions between particles and container walls.

Answers to Using *p*-*V* diagrams questions

1 (a) Processes 2 and 3 show work being done by the gas as the gas is expanding.

(b) Use $pV = nRT$ to calculate the number of moles of gas.

$$n = \frac{pV}{RT}$$

Choose two points from the process 1 isotherm

$$n = \frac{2.5 \times 10^5 \text{ Pa} \times 0.5 \text{ m}^3}{8.31 \times 310}$$

$= 48.5$ mol

Choose two points from the process 3 isotherm

$$T = \frac{pV}{nR}$$

$$= \frac{1.25 \times 10^5 \text{ Pa} \times 1.25 \text{ m}^3}{48.5 \times 8.31}$$

$= 388$ K

(c) Area enclosed by cycle ≈ 355 small squares

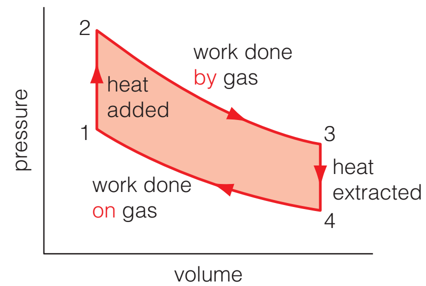
Each small square area $= 0.1 \times 10^5 \text{ Pa} \times 0.05 \text{ m}^3 = 500$ J

$W = 500 \times 355 = 177\,500$ J

(d) $P = 177\,500 \times 4 \times 50 = 35.5$ MW

This is a very powerful engine (a typical car engine has a power of ≈ 80 kW).

2 (a)



(b)

Stage	*Q*	Δ*U*	*W*
1–2	+260	+260	0
2–3	+250	0	+250
3–4	−260	−260	0
4–1	−180	0	−180
Whole cycle	70	0	70

Answers to Engine cycles, power output and efficiency questions

1 (a) Input power = calorific value × fuel flow rate

$$= 44.8 \times 10^6 \times \frac{0.49}{60}$$

$= 366$ kW

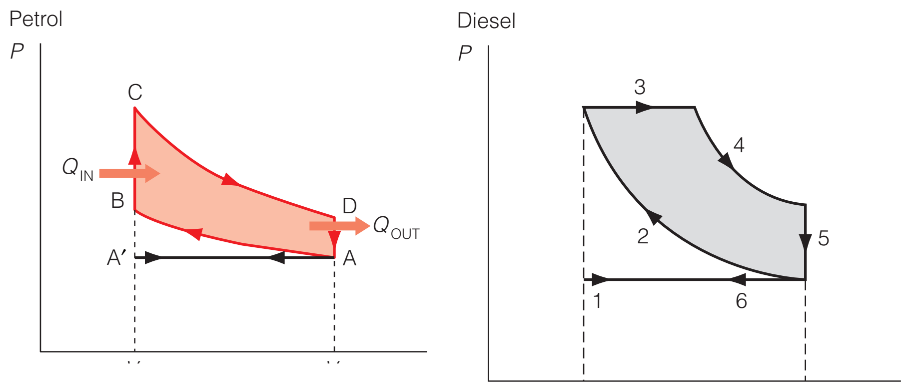
(b) The indicated power would double if the cycle was completed at twice the speed as the same amount of work would be done in half the time.

2 Correct p-V diagrams for both cycles labelled.

Petrol A′ to A – induction stroke, petrol vapour and air mixture at about 50°C is drawn into each cylinder. For diesel, only air is taken in during the induction stroke. (1)

Petrol B to C – A spark plug ignites the gas mixture at B. Diesel cycle – air is compressed during (2) and at (3). Diesel is sprayed into the cylinder where it ignites due to the high temperature of the air.

Diesel (3 & 4) is power stroke. Petrol (C–D) is the power stroke.



3 (a) Area enclosed by cycle ≈ 330 small squares (allow +/– 30 small squares)

Area of 1 small square = $0.2 \times 10^{-5} \times 5 \times 10^{3} = 0.01$ J

Total area = $330 \times 0.01 = 3.3$ J per cycle

Engine completes 300 cycles per minute therefore completes 5 cycles per second.

So $P_{indicated} = 3.3 \times 5 = 16.5$ W

(b) $P_{OUT} = \frac{mg\Delta h}{t}$

$= \frac{(6 \times 9.81 \times 1.8)}{8.7}$

$= 12.18$ W

Mechanical efficiency $= \frac{P_{OUT}}{P_{INDICATED}}$

$= \frac{12.18}{16.5}$

$= 0.74$

(c) Frictional heating in the moving parts of the engine

4 Input power = calorific value × fuel flow rate

$120\,000 = 37 \times 10^{6} \times f$

$f = 0.0032$ kg s^{-1} = 3.2 g s^{-1}

5 (a) Brake power $= T\omega = 200 \times 0.4 \times \frac{(2500 \times 2\pi)}{60} = 20.9$ kW

(b) Mechanical efficiency $= \frac{\text{brake power}}{\text{indicated power}}$

$= \frac{20.9}{(4 \times 6.54)} \times 100 = 79.9\%$

(c) Input power = calorific value × fuel flow rate
$= 0.002 \times 42\,000 = 84$ kW

Thermal efficiency $= \dfrac{\text{indicated power}}{\text{input power}} = \dfrac{(4 \times 6.54)}{84 \times 100} = 31.1\%$

(d) Overall efficiency $= \dfrac{\text{brake power}}{\text{input power}} = \dfrac{20.9}{84} \times 100 = 24.9\%$

6 P_{INPUT} = calorific value of fuel (J kg^{-1}) × flow rate (kg s^{-1})

$$= 51 \times 10^6 \times \frac{7.8\,\text{kg}}{36000}$$
$$= 110\,500\ \text{J s}^{-1}$$

$$\text{efficiency} = \frac{P_{\text{OUT}}}{P_{\text{INPUT}}}$$
$$= \frac{42\,000\ \text{J s}^{-1}}{110\,500\ \text{J s}^{-1}}$$
$$= 0.38$$

Answers to Engine efficiency questions

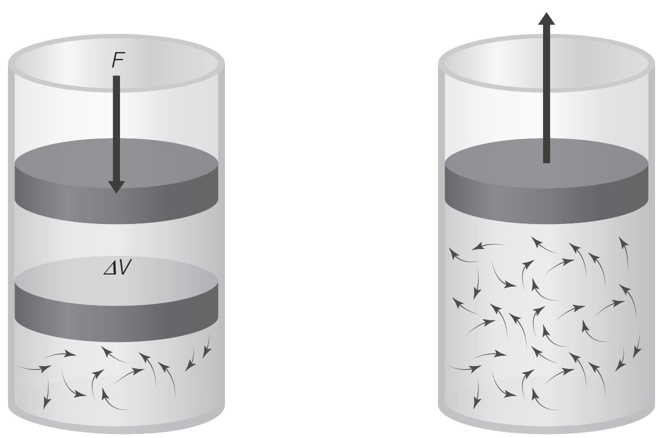
1 (a) Work done = enclosed area of cycle = area of rectangle
$= 2 \times 10^5 \times 8 = 16 \times 10^5$ J

(b) Efficiency $= \dfrac{\text{work done}}{\text{thermal energy input}} = \dfrac{W}{Q_H}$

Thermal energy input, $Q_H = Q_C + W = 2.1 \times 10^6 + 16 \times 10^5 = 3.7 \times 10^6$ J

Efficiency $= \dfrac{16 \times 10^5\ \text{J}}{3.7 \times 10^6\ \text{J}} \times 100 = 43\%$

2



3 Maximum efficiency $= \dfrac{T_H - T_C}{T_H}$

This equation implies that if the temperatures of the hot and cold sinks are similar then the efficiency is essentially zero ($\frac{0}{T_H} = 0$). If the temperatures are $T_H = \infty$ and $T_C = 0$ then the efficiency is maximised. The efficiency is therefore maximum when there is the greatest temperature difference between the hot and cold sink, i.e. when the engine is operating at a higher temperature.

An engine can never reach maximum efficiency in practice. It has many moving parts that produce **friction** leading to thermal losses. Some work is also done to move the air-fuel mixture into and out of the combustion cylinder. The combustion of the fuel depends on the amount of oxygen present and not all of the fuel entering the cylinder will combust and so the input power will be reduced. When the fuel is combusted, a lot of the heat (perhaps as much as 50%) will be lost from the engine without carrying out any work (e.g. passes through the cylinder walls or used in the cooling system).

4 Carnot efficiency $= 1 - \frac{T_C}{T_H}$

$$= 1 - \frac{290}{373}$$

$$= 22\%$$

Large losses occur due to friction in the moving parts of the engine. A lot of steam is also lost and the steam cools, transferring energy to the surroundings as it passes around the engine.

Answers to Reversed heat engines questions

1 $\text{COP} = \frac{Q_C}{W}$

but $Q_C = Q_H + W$ therefore $\text{COP} = \frac{Q_H - W}{W}$ so $\text{COP} = \frac{Q_H}{W} - 1$

$Q_H = (7 + 1)\ W = 8W$

If $W = 1$ joule then $Q_H = 8$ joules, i.e. for every 1 unit of work done 8 joules of heat are ejected

2 (a) a–b and c–d are isothermal, b–c and d–a are adiabatic

(b) Heat (Q_C) is absorbed from the cold sink during a–b. Heat is absorbed at this stage because the substance is undergoing an isothermal expansion and therefore to maintain a constant temperature energy must be absorbed.

Heat (Q_H) is ejected to the surroundings during c–d. As the refrigerant is being compressed isothermally it must lose heat to maintain a constant temperature.

3 (a) Energy required to cool 8 kg of water down to 0°C so it can freeze

$Q = mc\Delta\theta = 8 \times 2400 \times 18 = 604\,800$ J

Energy required to freeze 8 kg of water

$Q = mL = 8 \times 330\,000 = 2.64 \times 10^6$ J

Energy needed to cool ice from 0 °C to −12°C

$Q = mc\Delta\theta = 8 \times 2100 \times 12 = 201\,600$ J

Total energy removed from water per hour $= 3.45 \times 10^6$ J $= Q_C$

$W = \frac{Q}{\text{COP}} = \frac{3.45 \times 10^6\ \text{J}}{4.8} = 718\,\text{kJ}$

$P = \frac{W}{t} = \frac{718\,000}{3600} = 199.2$ watts $= 200$ watts

(b) $Q_H = Q_C + W = 3.45 \times 10^6 + 718\,000 = 4.168 \times 10^6$

Q_H per second $= \frac{Q_H}{t} = \frac{4.168 \times 10^6}{3600} = 1158$ watts

4 (a) Max theoretical efficiency = $\frac{T_H - T_C}{T_H} = \frac{2000 - 290}{2000} = 0.86$

(b) Input power = $\frac{\text{output power}}{\text{efficiency}} = \frac{100\,\text{kW}}{0.86} = 117$ kW

(c) Fuel flow rate = $\frac{\text{power}}{\text{calorific value}} = \frac{117\,000\,\text{W}}{46 \times 10^6\,\text{J kg}^{-1}}$
$= 2.5 \times 10^{-3}$ kg s^{-1}

Assume the engine operates at maximum theoretical efficiency and the generator is 100% efficient.

(d) Q_1 = 117 kW – 100 kW = 17 kW

COP = $\frac{Q_H}{W}$ therefore $Q_2 = W \times$ COP = 20 kW × 2.9 = 58 kW

so total energy available for heating each second = 17 kW + 58 kW = 75 kW

5 (a) $Q_H = W + Q_C$

So $Q_H > Q_C$

By definition, $COP_{HP} = \frac{Q_H}{W}$ and $COP_{REF} = \frac{Q_c}{W}$

As $Q_H > Q_C$, $COP_{HP} > COP_{REF}$

(b) Heat pump does deliver more energy than is input as work but this comes from a decrease in the energy of the cooler surroundings.

Conservation of energy is obeyed as $W + Q_c = Q_H$, i.e. work + energy from cold outdoor surroundings = energy transferred into building.

The second law is also obeyed as it operates between a hot and cold source/sink.

Answers to Exam practice questions

Thermodynamics

1 (a) If the piston is moved quickly, KE will be transferred to the molecules and they will be moving faster after they collide with the piston. *(1)*

Temperature is proportional to KE ($\frac{1}{2}mc^2 = \frac{3}{2}kT$) *(1)*

so if the average KE of the molecules increases, the temperature of the gas will increase. *(1)*

(b) Work is done BY *(1)* the gas as it is expanding. *(1)*

(c) Heat flows into the cylinder. *(1)*

As the piston moves out the gas would cool, so heat flows in through the conducting walls until the temperature is the same as the surroundings, *(1)* which was the state before the expansion.

(d) Heat is completely converted into work during the expansion. *(1)*

A cyclical process is required for a continuously operating machine. *(1)*

As the gas must be returned to its initial state, some heat would be given out during this returning stage *(1)* or the second law of thermodynamics would be contravened. *(1)*

2 (a) Change in which the temperature remains constant *(1)*

(b) State X is at a lower temperature *(1)*; because $p_A, V_A < p_B, V_B$ and pV is proportional to T *(1)* ($pV = nRT$).

(c) Work is done on the gas *(1)* as it is being compressed and so ΔW is negative. *(1)*

$Q = \Delta U - W$ but as Q is zero, $\Delta U = W$ so temperature increases as internal energy increases. *(1)*

(d) Area under curve X *(1)* should be shaded between volume A and volume B. *(1)*

3 (a) Input power = fuel flow rate × calorific value *(1)*

$P_{in} = \left(\frac{2.5 \times 10^{-2}}{60}\right) \times 52 \times 10^6 = 21.7 \times 10^3$ W = 22 kW *(1)*

(b) Efficiency = $\frac{\text{power output}}{\text{power input}}$

$\omega = 2\pi f = 2\pi \times \frac{1100}{60} = 115.2$ rad s^{-1} *(1)*

$P_{out} = T\omega = 50 \times 115.2 = 5759.6$ W *(1)*

Efficiency = $\frac{5759.6}{21.7 \times 10^3} = 0.27$ (or 27%) *(1)*

(c) Frictional losses = indicated power – power output

6500 – 5760 = 740 W

4 (a) $Q = \Delta U + W$ so $\Delta U = Q - W$

Remove heat (Q) from the gas *(1)*

so Q is negative, W zero, therefore ΔU must be negative so internal energy (U) decreases *(1)*

Gas does work (i.e. expands) *(1)*

W is positive, Q zero, then ΔU must be negative so internal energy (U) decreases *(1)*

(b) $W = p\Delta V = 1 \times 10^5 \times \left(\frac{25}{3}\right) = 8.3 \times 10^5$ J *(1)*

As gas is contracting W is negative

Q transferred to surroundings so Q is negative *(1)*

$Q = \Delta U + W$

$-5.2 \times 10^6 = \Delta U - 8.3 \times 10^5$

$\Delta U = -4.37 \times 106$ J *(1)*

(c) p on y-axis and V on x-axis with suitable scale/key features *(1)*

Horizontal line going from 25 m^3, 1.0×10^5 Pa to 8.3 m^3, 1.0×10^5 Pa arrow pointing to left *(1)*

5 (a) V = area × length = $\pi \times \left(\frac{0.015}{2}\right)^2 \times 0.085 = 1.5 \times 10^{-5}$ m^3 *(1)*

$\frac{pV}{RT} = n = \left(\frac{101\,000 \times 1.5 \times 10^{-5}}{8.31 \times 288}\right) = 6.3 \times 10^{-4}$ mol *(1)*

(b) $V_1 = 1.5 \times 10^{-5}\ m^3$

$$V_2 = \pi \times \left(\frac{0.015}{2}\right)^2 \times 0.035 = 6.2 \times 10^{-6}\ m^3 \quad (1)$$

$\gamma = 1.4$ as air is mainly a diatomic gas

$$p_2 = p_1 \left(\frac{V_1}{V_2}\right)^{\gamma} = 101\ 000 \left(\frac{1.5 \times 10^{-5}}{6.2 \times 10^{-6}}\right)^{1.4} \quad (1)$$

$p_2 = 348$ kPa *(1)*

(c) $$T_2 = \frac{p_2 V_2}{nR} = \left(\frac{348\ 000 \times 6.2 \times 10^{-6}}{6.3 \times 10^{-4} \times 8.31}\right) \quad (1)$$

$= 412$ K *(1)*

6 (a) $W = p\Delta V$ *(1)*

$= 6\ m^3 \times 600\ N\ m^{-2} = 3600$ J *(1)*

(b) $pV = nRT$

$$T_A = \frac{pV}{nR} = \left(\frac{600 \times 3}{2 \times 8.31}\right) = 108.3\,K \quad (1)$$

$$T_B = \frac{PV}{nR} = \left(\frac{600 \times 9}{2 \times 8.31}\right) = 324.9\,K \quad (1)$$

Internal energy is all KE as the gas is ideal.

$$\Delta U = \frac{3}{2} nRT$$

$$\Delta U = \Delta U_B - \Delta U_A$$

$$= \frac{3}{2} \times 2 \times 8.31\ (324.9 - 108.3)$$

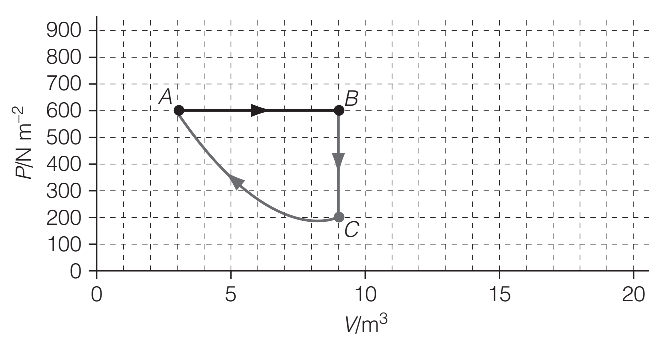
$= 5399.8$ J *(1)*

(c) $Q = \Delta U + W$ *(1)*

$= 5399.8 + 3600$

$= 9000$ J *(1)*

(d)



(1 mark per line)

(e) $Q = \Delta U + W$ *(1)*

$\Delta U = 0$ as C–A is an isothermal process

therefore $Q = W$

Work is negative as work is done **on** the gas as it is compressed, Q must also be negative and so heat is lost. *(1)*

Engine	Q_H	Q_C	W
1	200	−175	40
2	500	−200	400
3	600	−200	400
4	100	−90	10

7 First law of thermodynamics $Q = \Delta U + W$ (work is positive if done by the gas expanding)
For an ideal engine cycle $\Delta U = 0$ and so $Q = W$, statement of first law *(1)*
Engines 1 and 2 break the first law as $Q \neq W$
Engines 3 and 4 do not break the first law as $Q = W$ *(1)*
(*both correct for 1 mark*)
Carnot (maximum possible efficiency) = $\frac{T_H - T_C}{T_H}$, statement of second law and calculation

$= \frac{(400 - 300)}{400} = 0.25$ *(1)*

Efficiency of 1st engine = $\frac{W}{Q_H}$

$= \frac{40}{200} = 0.2$

Efficiency of 4th engine = $\frac{10}{100} = 0.1$ *(1)*

These both comply with the second law of thermodynamics as their efficiency is lower than the maximum possible efficiency.

Efficiency of 2nd engine = $\frac{400}{500} = 0.8$

Efficiency of 3rd engine = $\frac{400}{600} = 0.67$ *(1)*

Engines 2 and 3 break the 2nd law of thermodynamics as their efficiency is greater than the Carnot efficiency. *(1)*

8 (a) $COP_{REF} = \frac{Q_C}{W} = \frac{Q_C}{(Q_C - Q_H)}$

For an ideal refrigerator, $\mathbf{COP}_{IDEAL} = \frac{T_C}{T_H - T_C}$ *(1)*

$COP = \frac{270}{30} = 9$ *(1)*

$Q_C = COP \times W$
$= 9 \times 250$
$= 2250\,J\,s^{-1}$ *(1)*

In 20 mins, $Q = 2250 \times 60 \times 20$
$= 2.6$ MJ *(1)*

(b) Assume it is an ideal refrigerator *(1)*, 2250 + 250 = 2500 J transferred to the surroundings each second *(1)*

9 (a) The efficiency of a real engine must be less than the efficiency of a Carnot engine when operating between the same temperatures. *(1)*

Ideal efficiency = $\frac{T_C}{T_H}$

$= \frac{(373 - 273)}{373}$

$= 0.27$ *(1)*

The claimed efficiency is almost 3 times the maximum efficiency possible so the inventor's engine would violate the second law of thermodynamics. *(1)*

(b) In a real engine the processes are not reversible *(1)* and there are losses due to turbulent flow of the gas and friction in the moving parts. *(1)*

10 (a) $W = p\Delta V$

$= 20 \times (1.8 - 3.0)$ *(1)*

$= -24\,J$ *(1)*

$Q = \Delta U + W$

$-75 = \Delta U - 24$

$\Delta U = -51\,J$ *(1)*

(b) $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$T_2 = \left(\frac{V_2}{V_1}\right) T_1$ *(1)*

$= \left(\frac{1.8}{3}\right) \times 300$

$= 180\ K$ *(1)*

Rotational dynamics

1 (a) To recover the energy the motor used to spin up the flywheel/charge, the flywheel is operated in reverse as a generator. *(1)*

(b) $E_K = \frac{1}{2} I\,\omega^2$

$= 0.5 \times 32 \times \left(\frac{2\pi \times 24\,000}{60}\right)^2$

$= 101 \times 10^6\,J$ *(1)*

(c) $\omega_{max} = 48\,000 \times \frac{2\pi}{60}$

$= 5026.5\ rad\ s^{-1}$ *(1)*

$T = \frac{P}{\omega}$

$= \frac{1.7}{5026.5}$

$= 3.38 \times 10^{-4}\ Nm$ *(1)*

$E_{K\,max} = \frac{1}{2} I\,(\omega_{max})^2$

$= 0.5 \times 32 \times (5026.5\ rad\ s^{-1})^2$

$= 404.3 \times 10^6\,J$ *(1)*

$\text{Time} = \frac{\text{energy}}{\text{power}}$

$= \frac{404.3 \times 10^6\,J}{0.8\ W}$

$= 505 \times 10^6\ s$ *(1)*

(d) Energy supplied = Pt

$= 3000 \times 24 \times 3600$

$= 2.59 \times 10^8\ J$ *(1)*

E_K lost $= \frac{1}{2} I\,(\omega_1)^2 - \frac{1}{2} I\,(\omega_2)^2$

$2.59 \times 10^8 = 4.04 \times 10^8 - (0.5 \times 32 \times (\omega_2)^2)$ *(1)*

$\omega_2 = 3010\ rad\ s^{-1}$

$= \frac{3010}{2\pi} \times 60$

$= 28\,747 = 29\,000\,rpm$ *(1) (1) (appropriate sig. figs)*

(e) The forces and stresses in the wheel increase as the speed increases which can cause the material to break apart. *(1)*

(f) Design B would have a greater moment of inertia as more of the mass is concentrated further away from the axis of rotation. *(1)*

This would enable more kinetic energy to be stored for the same angular velocity as KE lost = $\frac{1}{2} I \omega_2$ *(1)*

2 (a) $\omega_1 = 19.0 \text{ rpm} = \frac{19.0}{60} \times 2\pi = 1.98 \text{ rad s}^{-1}$ *(1)*

$\omega_2 = \frac{82.0}{60} \times 2\pi = 8.59 \text{ rad s}^{-1}$ *(1)*

$E = Pt = 200\,000 \times 5.2 = 1.04 \times 10^6 \text{ J}$ *(1)*

KE gained $= \frac{1}{2} I (\omega_2)^2 - \frac{1}{2} I (\omega_1)^2$

$1.04 \times 10^6 = \frac{1}{2} I (8.59^2 - 1.98^2)$ *(1)*

$I = 29.8 \times 10^3 \text{ kg m}^2$ *(1)*

(b) The power output of the motor is constant/no electrical energy is wasted due to friction in the bearings. *(1)*

3 (a) Use $\omega_2 = \omega_1 + \alpha t$

Convert 105 rev min^{-1} to rad s^{-1}

$(\frac{105}{60}) \times 2\pi = 10.996 \text{ rad s}^{-1}$

$\alpha = \frac{10.996}{9.5} = 1.16 \text{ rad s}^{-2}$ *(1)*

$T = I\alpha$

$= 280 \times 1.16 = 324 \text{ N m}$ *(1)*

(b) $E_K = \frac{1}{2} I \omega^2$

$= \frac{1}{2} \times 280 \times 10.996^2$

$= 16\,928 \text{ J}$ *(1)*

$P = \frac{E_K}{t}$

$= \frac{16\,928}{9.5}$

$= 1781 \text{ W} = 1.7 \text{ kW}$ *(1)*

(c) $\theta = 36 \times 2\pi = 226.2 \text{ rad}$ *(1)*

Use = $\omega_2^2 = \omega_1^2 + 2\alpha\theta$

$\alpha = \frac{10.996^2}{2 \times 226.2}$

$= -0.27 \text{ rad s}^{-2}$ *(1)*

$T = I\alpha$

$= 280 \times -0.27$

$= -74.8 \text{ N m}$ *(1)*

4 $50 \text{ km h}^{-1} = \frac{50\,000}{3600}$

$= 13.8 \text{ m s}^{-1}$

Translational E_K of van $= \frac{1}{2} m v^2$

$= \frac{1}{2} \times 3000 \times 13.8^2$

$= 285\ 660\,J$ *(1)*

50% of this is stored in flywheel = 142 830 J

Rotational $E_K = \frac{1}{2} I \omega^2$ *(1)*

$I = \frac{2 E_K{}^2}{\omega^2}$

$I = \frac{285\,660}{400^2}$

$= 1.79\ kg\ m^2$ *(1)*

$M = \rho V$

Solid disk so $V = \pi r^2 t$

$M = \rho \pi r^2 t$

$I = \frac{M r^2}{2}$

$I = \frac{\rho \pi r^2 t\, r^2}{2}$ *(1)*

$r = \sqrt[4]{\frac{2I}{\rho \pi t}}$

$= \sqrt[4]{\frac{2 \times 1.79}{8 \times 10^3 \times 3.14 \times 0.15}}$

$= 0.18\ m$ *(1)*

5 (a) $T = F r$

$= 2.5 \times 0.2$

$= 5.0\ N\,m$ *(1)*

$T = I \alpha$

$\alpha = \frac{0.5}{0.85}$

$= 0.59\ rad\ s^{-2}$ *(1)*

(b) Some thermal energy is produced and also some sound. *(1)*

(c) Initial rotational $E_K = \frac{1}{2} I \omega^2$

600 rpm = 62.8 rad s^{-1}

$E_K = \frac{1}{2} \times 0.85 \times 62.8^2$

$= 1677.8\,J = 1.7\,kJ$ *(1)*

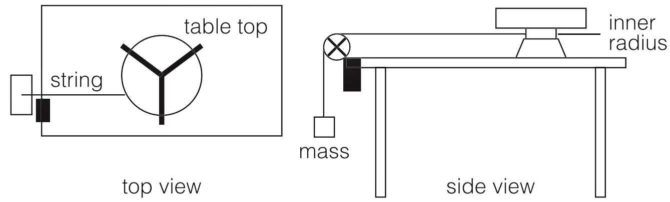
After 1 second of knife being held against wheel

Rotational $E_K = \frac{1}{2} \times 0.85\ (62.8 - 0.59)^2 = 1644.8\ J$ *(1)*

$\Delta E_K = 1677.8 - 1644.8 = 33\ J$ *(1)*

So power supplied must be 33 W.

6 A list of the equipment used, what measurements you will take and how they will be taken:



- hanging masses can be attached to the centre of the turntable and suspended off the side of the table with a pulley
- string wrapped around the central axle of the turntable which unravels as the masses fall and the turntable begins to spin
- these will provide a torque to the turntable
- time how long it takes the masses to fall a certain distance
- use 6 different falling masses (m), repeat 3 times and average
- measure the diameter of the axle and divide by 2 to get the radius (R) of the axle

(2 marks available)

How the measurements will be used to calculate the moment of inertia:

- use the linear uniform acceleration equations to calculate, a ($s = ut + \frac{1}{2}at^2$, where $u = 0$)
- use $a = \alpha R$ to calculate the turntable's angular acceleration
- calculate the tension in the string ($mg - ma = F$, $T = FR$)
- use $T = RT$ to calculate the torque
- plot a graph of T against α. The gradient = I and the y-intercept should be 0

(2 marks available)

Which measurements contribute the most uncertainty and how these uncertainties will be minimised:

- the distance should be relatively large in order to increase the time taken as human reaction time is of order 0.2 s, the time should ideally be longer than 4 s in order to keep the % uncertainty below 5%
- markers should also be used for the start and stop points and the eye carefully lined up level with these in order to avoid parallax
- an angular position sensor could be fitted and used with a data logger to record the rotation of the turntable

(2 marks available)

7 (a) Use $s = u\,t + \frac{1}{2}a\,t^2$, $a = \frac{2s}{t^2}$

$a = \frac{(2 \times 2)}{1.41^2}$

$a = 2.01\,\text{m}\,\text{s}^{-2}$ *(1)*

(b) $\alpha = \frac{a}{r}$

$= \frac{2.01}{0.125}$ *(1)*

$= 16.1 \text{ rad s}^{-2}$ *(1)*

(c) To calculate torque you need to calculate tension

Resolve vertically and apply Newton's second law to falling masses

$mg - T = ma$ *(1)*

$T = mg - ma$

Tension = $(2.5 \times 9.81) - (2.5 \times 2.01)$

$= 19.5$ N *(1)*

Torque = Fr

$= 19.5 \times 0.125$

$= 2.44$ N m *(1)*

Torque = $I\alpha$

$I = \frac{T}{\alpha}$

$= \frac{2.44}{16.1}$

$= 0.15 \text{ kg m}^2$ *(1)*

8 (a) $T = I\alpha$, α large so large torques needed unless I small *(1)*

Kinetic energy = $\frac{1}{2}I\omega^2$ so small I results in less stored energy so less work needs to be done to change the motion *(1)*

As $I = mr^2$ *(1)*

Low mass, small diameter gives a low moment of inertia *(1)*

(b) $\alpha = \frac{\omega_2 - \omega_1}{t}$

$= \frac{170 - (-90)}{75 \times 10^{-3}}$ *(1)*

$= 3466.7 \text{ rad s}^{-2}$ *(1)*

(c) $T = I\alpha$

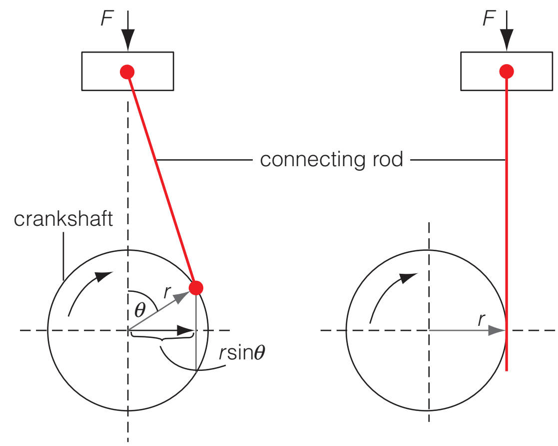
$= 3.8 \times 10^{-5} \times 3466.7$

$= 0.13$ N m *(1)*

9 (a) As the crankshaft rotates, the angle made by the connecting rod changes. *(1)*

$T = Fr$ where r is the perpendicular distance between the line of action of F and the pivot.

The change in angle therefore affects the torque as the crankshaft rotates ($T = Fr \cos \theta$). *(1)*



(b) $W = T\,\theta$ so the area under the graph is the work done. *(1)*

The angular displacement needs to be converted from degrees to radians. *(1)*

(c) A flywheel is added to the crankshaft *(1)*

which results in less fluctuation in rotational speed. *(1)*

It has a high moment of inertia and so keeps the crankshaft rotating even during the exhaust stroke $\Delta I\omega = T\,t$ *(1)*

10 (a) $I = mR^2$

$0.38 \times 0.4^{4}_{2} = 0.0608$ kg m^2

Two rockets so $I = 2 \times 0.0608 = 0.1216$

Total $I = I$ of rockets $+ I$ of beam $= 0.1216 + 0.09 = 0.21$ kg m^2

(b) $T = F\,r$

$T = (2.8 \times 0.38) \times 2 = 2.13$ N m

$\alpha = \frac{T}{I} = \frac{2.13}{0.21}$

$= 10.13$ rad s^{-2}

$\theta = 2\pi \times 3 = 18.8$ rad

Use $\theta = \omega_1\,t + \frac{1}{2}\,\alpha\,t^2$

$t = \left(2\frac{\theta}{\alpha}\right)^{\frac{1}{2}}$

$t = 1.36$ s

(c) Frictional couple due to air resistance increases as the angular speed increases. *(1)*

When frictional couple = driving torque there is no resultant torque and so no acceleration. *(1)*

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