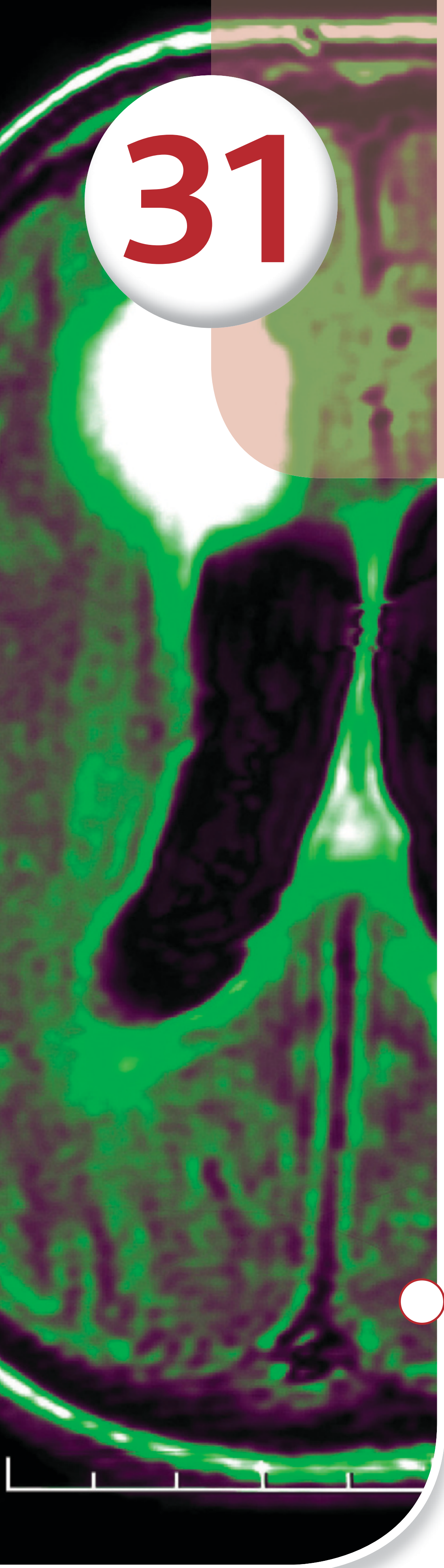


31 Medical Physics

PRIOR KNOWLEDGE

Before you start, make sure that you are confident in your knowledge and understanding of the following points:

- Light is a wave which shows wave properties including reflection and refraction.
- When light crosses from air into glass, the light slows down and is refracted towards the normal. It refracts away from the normal when it travels from glass into air.
- Sound is a wave which can be transmitted through media. The speed of sound depends on the medium.
- Sound is reflected off denser media. This is called an echo.
- Ultrasound is a high frequency sound wave above the frequency range of human hearing.
- X-rays are used in medicine as a diagnostic tool.
- Radioactive materials emitting beta and gamma rays are used in radiotherapy.

TEST YOURSELF ON PRIOR KNOWLEDGE

1 A ray of light is incident on one face of a parallel-sided block of glass, at an angle of 30° to the normal. Draw a sketch to show the path of the ray as it passes through the block.

2 An ultrasonic wave travelling in water at a speed of 1500 m s^{-1} has a frequency of 100 kHz. Calculate the wavelength of the ultrasound.

3 A boy shouts in a mountainous area and hears an echo reflected back to him off a cliff after a time of 2.2 s. Calculate the distance of the cliff away from the boy. Sound travels at 330 m s^{-1} in air.

4 Technetium-99 is a gamma-emitting radioisotope with a half-life of 6 hours. Explain why technetium-99 is used as a tracer in medical diagnosis.

5 Iridium-192 ($^{192}_{77}Ir$) decays by β^- emission to platinum (Pt).

a) Write a balanced nuclear equation to describe the decay.

b) Explain why iridium-192 can be used effectively as an implant to treat a small cancer in the body.

Lenses

Refraction in lenses

Many optical instruments use glass lenses to form an image. A lens forms an image by refracting light.

There are two types of lens:

- convex (or converging)
- concave (or diverging).

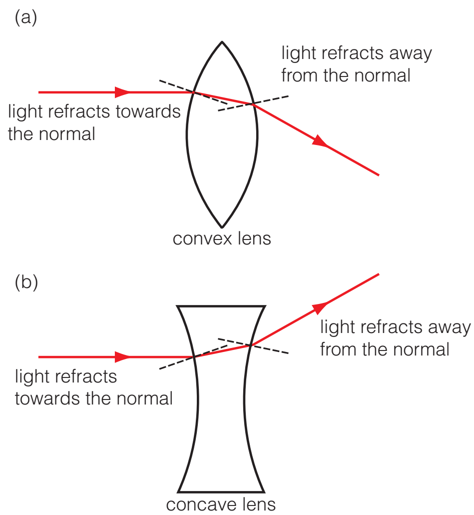


Figure 31.1

Figure 31.1 shows how each type of lens refracts light. The rules of refraction apply to both types of lens. When light enters the lens it bends towards the normal. When light leaves the lens it bends away from the normal.

Convex lenses

As a result of refractions, parallel rays of light entering a convex lens converge and meet at a point. This point is the principal focus of the lens (Figure 31.2).

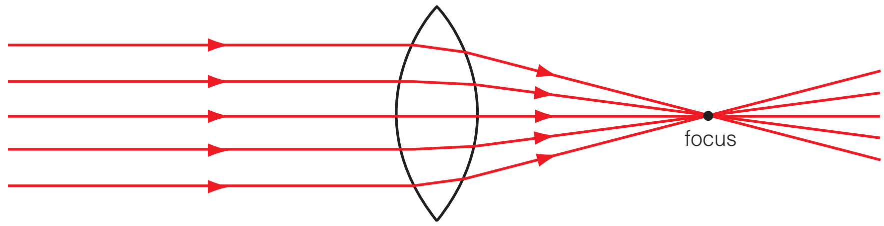


Figure 31.2

Light can pass through the lens in either direction. So parallel rays coming from the right of the lens converge at a focus that is an equal distance on the left.

A converging lens can form an image of a distant object onto a screen or a piece of paper. The image is sharpest when the piece of paper or screen is at the **principal focus** of the lens. The distance between the principal focus and the lens is called the **focal length** of the lens.

When a lens forms an image that we can see on a screen, it is a **real image**.

The **principal focus** of a lens is the point at which rays parallel to the principal axis of the lens are brought to a focus.

The **focal length** of a lens is the distance between the centre of the lens and the principal focus (or focal point).

A **real image** is formed when light rays converge to a point.

Figure 31.3

Ray diagrams

When an object is placed beyond the focal length, as shown in Figure 31.4, the lens forms a real image of the object.

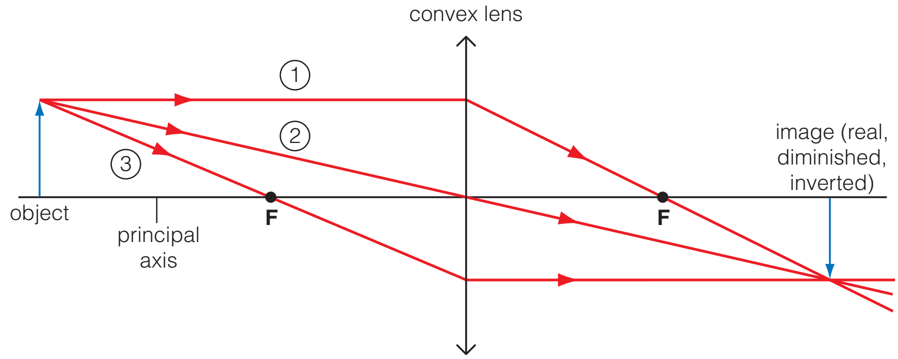


Figure 31.4

Rays come at all angles into the lens, but there are three rays which we can use to locate the image. These rays are chosen because we can predict their path through the lens.

Ray 1 A ray parallel to the principal axis (on the left) passes through the focal point on the right.

Ray 2 A ray which passes through the centre of the lens does not change direction.

Ray 3 A ray which passes through the focal point of the lens on the left-hand side emerges from the lens parallel to the principal axis on the right-hand side.

The place where these three rays meet on the right-hand side of the lens, locates the top of the image. The bottom of the image lies on the **principal axis**.

In this example, where the object is a long way from the lens, the image is:

- real
- inverted
- diminished in size.

A real image is formed when rays meet at a point, and the image can be seen by many people on a screen.

TIP

You should know the rules of refraction and be able to predict which way a light ray bends when it crosses a curved surface. Although you need to understand that light is refracted at both surfaces of a lens, we use a helpful approximation. We make it easier to draw ray diagrams by showing refraction to occur in one place. The refraction in Figure 31.2 is shown to occur at the centre of the lens. We use the symbol to represent a concave lens, and the symbol to represent a convex lens.

The **principal axis** of a lens is an imaginary line that passes through the centre of the lens and through the centres of curvature of the faces of the lens.

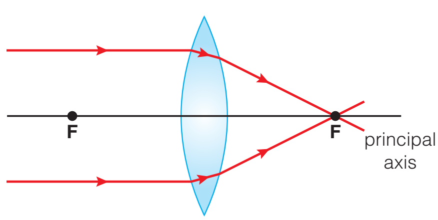


Figure 31.5

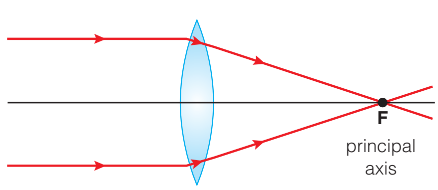


Figure 31.6

Focal length and lens shape

The focal length of a lens depends on the curvature of the lens surface. The lens in Figure 31.5 has a short focal length because its surfaces have large radii of curvature, and the light is refracted through relatively large angles. The lens in Figure 31.6 is a thinner lens, with less curved surfaces. Its focal length is longer than the lens in Figure 31.5.

A lens with a short focal length is said to be more powerful than a lens with a longer focal length. The power of the lens is defined as the reciprocal of its focal length measured in metres. The power is measured in dioptres, D.

For example, the power of a converging lens of focal length 40 cm is calculated as follows.

$$\text{Power } (P) = \frac{1}{0.4\,\text{m}}$$

$$= +2.5 \text{ dioptres or } +2.5 \text{ D}$$

Diverging lenses have a negative power, as they diverge light rather than converge it.

When two thin lenses are placed in contact with each other, the powers of the lenses add up to produce a combined power of the lenses.

$$P = P_1 + P_2$$

Locating an image by drawing or calculation

To find the position of an image you only need to draw two of the rays, not all three. So, from now on, we will just use rays 1 and 2.

Figure 31.7 shows the size and position of the image when the object is just outside the focal length of the lens. In this diagram, the following distances and heights are given these letters:

- the focal length of the lens, f
- the distance from the centre of the lens to the object, u
- the distance from the centre of the lens to the image, v
- the height of the object, h
- the height of the image, H

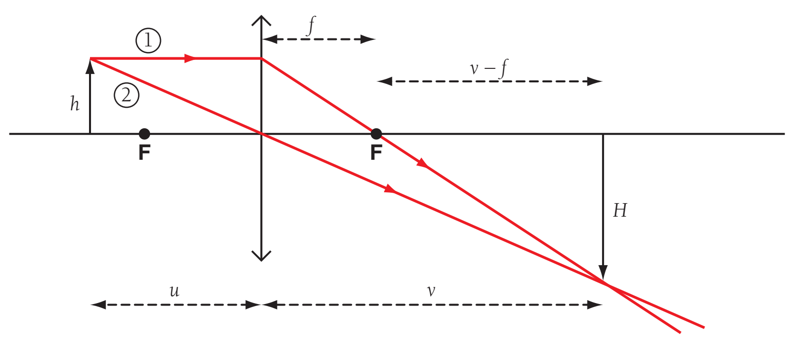


Figure 31.7

The magnification produced by a lens is given by the equation:

$$\text{magnification} = \frac{\text{image height}}{\text{object height}}$$

Magnification is a ratio of two heights and so has no units.

Using similar triangles we can see from Figure 31.7 that:

$$\frac{H}{h} = \frac{v}{u}$$

or the magnification,

$$m = \frac{v}{u}$$

EXAMPLE

In Figure 31.7 the height of the object is 14 mm and the height of the image is 28 mm.

$$\text{magnification} = \frac{\text{image height}}{\text{object height}}$$

$$= \frac{28}{14}$$

$$= 2.0$$

The position of the image, the position of the object and the focal length are linked by the following formula:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Here we use the real-is-positive convention.

- All distances are measured from the centre of the lens.
- Distances of real objects and real images are positive.
- Distances of virtual images (and virtual objects) are negative.
- The focal length of a converging lens is positive.
- The focal length of a diverging lens is negative.

EXAMPLE

Calculate the position of the image, when an object is placed 9.0 cm away from a lens of focal length 6.0 cm.

Answer

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

so

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

and

$$\frac{1}{v} = \frac{1}{6} - \frac{1}{9}$$

therefore

$\frac{1}{v} = \frac{1}{18}$ and v = 18 cm

and because v is positive, the image is real.

MATHS BOX

Here we derive the formula given above. You are not expected to be able to do this, but the derivation is provided for the interested mathematician.

From Figure 31.7, and using similar triangles, we can write the following equation:

$$\frac{H}{v - f} = \frac{h}{f}$$

so

$$\frac{H}{h} = \frac{v - f}{f} = \frac{v}{f} - 1$$

However, we know that:

$$\frac{H}{h} = \frac{v}{u}$$

Therefore:

$$\frac{v}{u} = \frac{v}{f} - 1$$

$$\frac{1}{u} = \frac{1}{f} - \frac{1}{v}$$

And finally:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

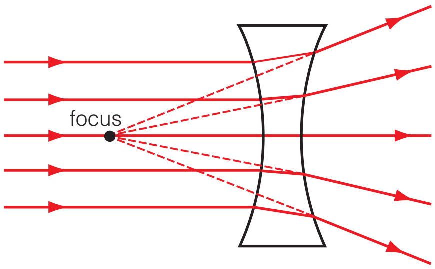


Figure 31.8

Concave lenses

As a result of refraction, parallel rays of light entering a concave (or diverging) lens spread out as they pass through the lens (Figure 31.8). The rays look as though they spread out or diverge from a single point. This point is the principal focus of the lens – but because the light only seems to diverge from this point, it is a virtual focus.

The power of a diverging lens is negative. For example, a diverging lens of focal length 67 cm is calculated as follows.

$$\text{Power} = -\frac{1}{0.67\ \text{m}} = -1.5\ \text{D}$$

Images and ray diagrams

Ray diagrams can be drawn for diverging lenses in the same was as for converging lenses. A ray of light incident on the centre of the lens does not change direction. A ray of light that is parallel to the principal axis is refracted so that it seems to have come from a virtual focus.

A virtual image is formed where the two rays seems to cross. It does not matter where the object is, inside or outside the focal length of the lens, the nature of the image is always the same:

- virtual
- upright
- diminished.

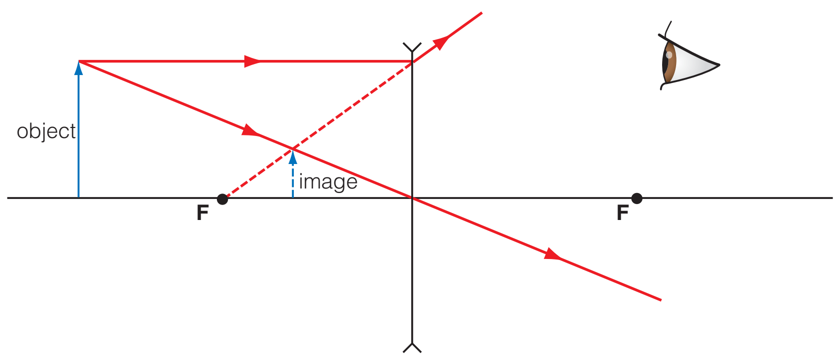


Figure 31.9

We can also calculate the position of a virtual image using the same formula as we did for the concave lens.

EXAMPLE

Calculate the position of the virtual image seen when an object is placed 9.0 cm away from a diverging lens of focal length 6.0 cm.

Because the focal length is virtual, it is negative.

Answer

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{9} + \frac{1}{v} = -\frac{1}{6}$$

$$\frac{1}{v} = -\frac{1}{6} - \frac{1}{9}$$

$$\frac{1}{v} = -\frac{5}{18}$$

And $v = -3.6$ cm

The minus sign tells us that the image is virtual and therefore on the same side of the lens as the object, as shown in Figure 31.9.

TEST YOURSELF

1 A diverging lens has a 'virtual' principal focus. What does this mean?

2 a) What is meant by (i) a real image (ii) a virtual image?
  b) Which type of lens always produces a virtual image?

3 a) Copy and complete the diagram in Figure 31.10 to show how the lens forms an image.

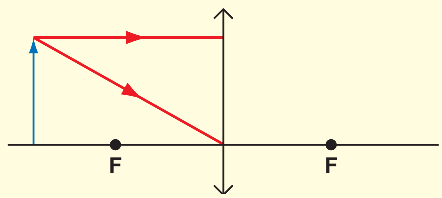


Figure 31.10

  b) State whether the image is
    i) real or virtual
    ii) magnified or diminished
    iii) upright or inverted.

4 A converging lens produces a magnification of × 1.5 when a real image is formed at a distance of 15 cm from the lens.
  a) Draw a scale diagram to show the formation of the image.
  b) Use your diagram to calculate the focal length of the lens.
  c) Check your answer to part (b) by doing a calculation.

5 a) A converging lens has a focal length of 20 cm. An object with a height of 2 cm is placed 25 cm from the lens. Calculate the position and height of the image. Is this a real or virtual image? Which way up is the image?
  b) A diverging lens has a focal length of 10 cm. An object is placed 30 cm from the lens. Calculate the position and apparent height of the image seen by an observer. Is this a real or a virtual image? Which way up is the image?

6 a) i) Calculate the power of a converging lens with a focal length of 250 cm.
    ii) Calculate the power of a diverging lens with a focal length of 50 cm.
  b) The two lenses are placed close together. What is the combined power of the lenses now?

The eye

The structure of a human eye is shown in Figure 31.11. The eye is filled with fluid to maintain its even spherical shape. There are two chambers of fluid on either side of the lens: the aqueous humour and vitreous humour. The iris controls the amount of light entering the pupil: in bright light the pupil is small and it is large in low light intensities. The lens and the cornea focus the light in exactly the same way as any other convex lens. A real, inverted, diminished image of an object is formed onto the retina. The retina is the light sensitive part of the eye. The light-sensitive receptors in the eye are known as rods (slender rod-like elements) and cones (narrow conical elements). These receptors have diameters of a few microns.

TIP
1 micron = 1 μm = 10^{-6} m

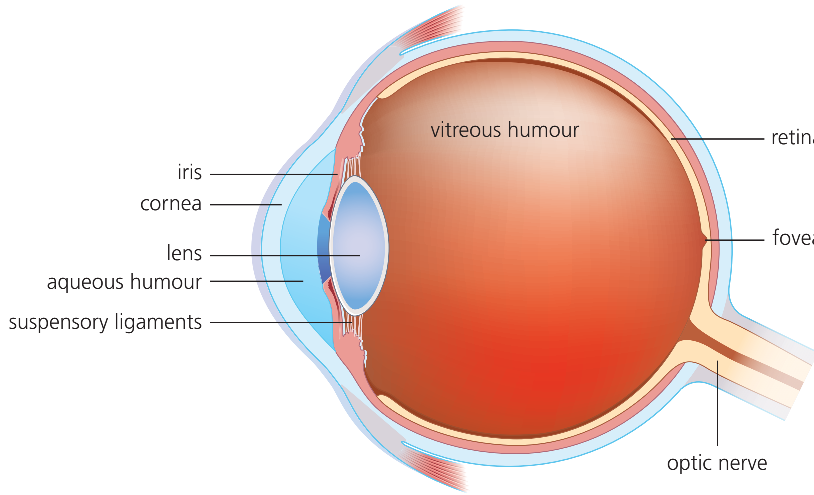


Figure 31.11

Although the image on the retina is upside down, our brains interpret this image so that we see things the right way up.

The eye as a photodetector

When the retina is illuminated with visible light it produces electrical signals, which enable the brain to visualise the image. The retina contains two types of light-sensitive cell – rods and cones, as shown in Figure 31.12. There are about twenty times more rods than cones. Rods are used in low-intensity light detection. They produce a simple perception of light; there is little detail and rods do not differentiate between colours – we see shades of grey. Cones are sensitive to different wavelengths (and hence colours of light – see Figure 31.13). Each cone is connected by one nerve fibre to the brain, so we see greater detail with cones and detailed colour. Cones are sensitive to high-intensity light, but do not function well in low-intensity light, where rods are more effective.

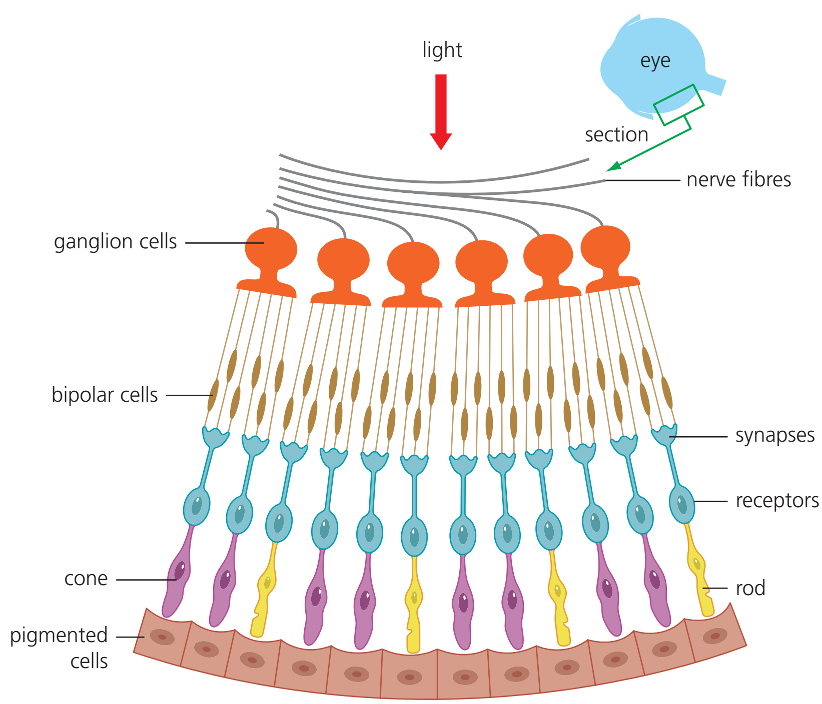


Figure 31.12 There are three different types of cone which respond primarily to blue, green and red light.

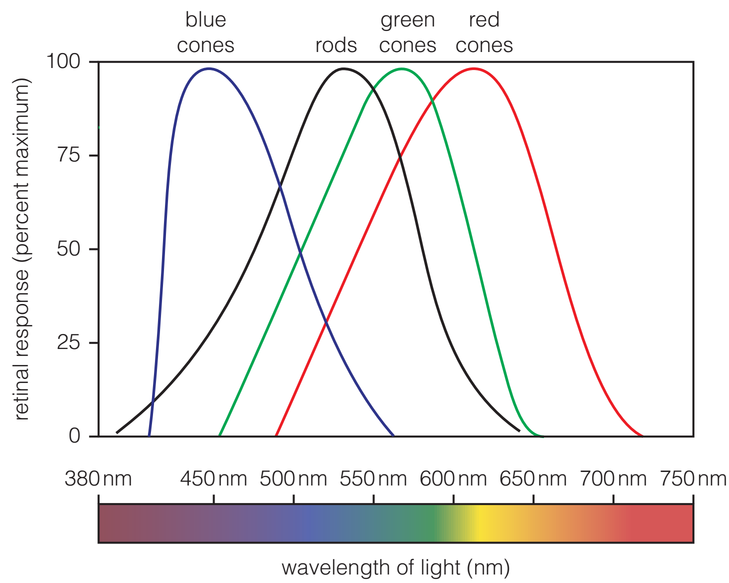


Figure 31.13 The rods respond to all wavelengths of light but do not allow us to distinguish colours.

Each rod and cone behaves like a small photoelectric cell, because the light incident on the cell generates an electrical signal, which passes to the brain.

The ratio of rods to cones is not the same in all parts of the retina. The fovea, the region of the retina in line with the pupil, consists entirely of thin closely packed cones. This means that the spatial resolution is high.

Further away from the fovea there are many fewer cones and many more rods. When the eye looks directly at an object, the image is formed in the fovea. Due to the high resolution of the closely packed cones, we can see in great detail. However, the rods also form a very important part of our vision. Rods enable us to detect movement in our peripheral vision. As rods are sensitive to low levels of light, they are responsible for our night vision.

TEST YOURSELF

7 a) Describe the differences between the two types of light-detecting cells in the retina.
b) Why do we see in greater detail when we look straight at an object?
c) Why do we see less detail in our peripheral vision?
d) Explain why rods are used in night vision, rather than cones.

8 a) Use Figure 31.13 to state the range of wavelengths that the eye can detect.
b) Explain which two types of cone are used to detect yellow light. Why is the eye particularly sensitive to yellow light?

Accommodation

You learnt earlier that the position of an image formed by a converging lens depends on the position of the object. The eye is of a fixed length (about 2.5 cm from the cornea to the retina) so the image position is fixed. When an object is in a different position, the eye lens changes shape so

Accommodation is the name given to the automatic mechanism whereby the eye lens changes shape to focus on objects at different distances from the eye.

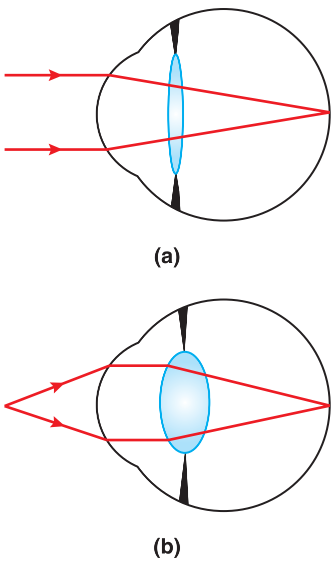


Figure 31.14

that we always get a sharp image on our retina. This automatic mechanism is known as **accommodation**. The shape of the eye lens is controlled by the ciliary muscles. Figure 31.14(a) shows how the eye acts as a refracting system when it is looking at a distant object and Figure 31.14(b) shows it when looking at a nearby object. Most of the refraction occurs at the air/cornea boundary, due to the large change in refractive index across this interface. The cornea acts as a lens with a power of about 41 D.

When the eye lens looks at a distant object, it has a relatively flat shape, providing an additional power of 18 D. So the eye has a total power of about 59 D.

When a young human eye looks at a nearby object, the eye lens can accommodate and provide about an extra 11 D. This provides a refracting system with a total power of about 70 D. The eye in Figure 31.14(b) provides a more powerful refracting system than the lens in Figure 31.14(a).

In a young healthy eye, the accommodation of the eye allows a person to see clearly between a near point of about 25 cm to objects in the far distance. As people get older, the eye lens becomes less flexible and its range of accommodation reduces. So it is common for people who are about 40 years old to have clear vision for distant objects but to focus less well on nearby objects.

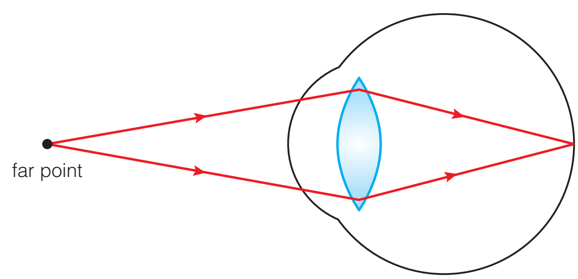
Defects in vision

Short sight (myopia)

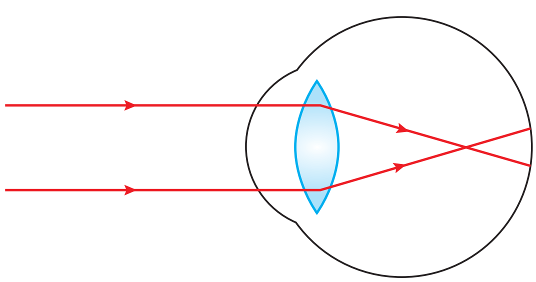
When an eye is short sighted, near objects can be seen clearly, as shown in Figure 31.15(a). But the eye lens is too powerful for the length of the eye and distant objects are focused in front of the retina, as shown in Figure 31.15(b). The far point of the eye is the furthest distance that the eye can focus on clearly.

Figure 31.15(c) shows how a diverging lens can be used to correct the eye. For example, if the far point of the eye is 2 m away, a diverging lens with this focal length must be put in front of the eye. Then parallel rays coming from a distant object appear to diverge from the far point of the eye.

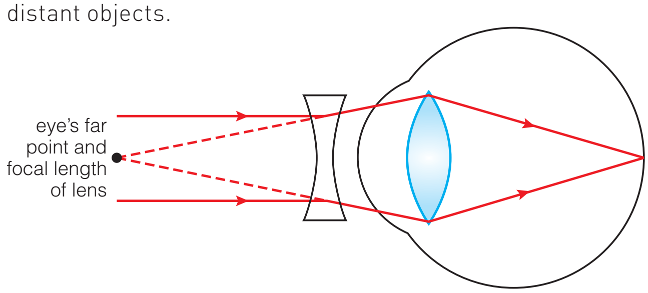
So a diverging lens allows a short-sighted person to view distant objects. If the person then wishes to read, he or she simply removes the glasses, because it is easy to focus on objects close to the eye.



(a) A short-sighted eye sees clearly close to it.

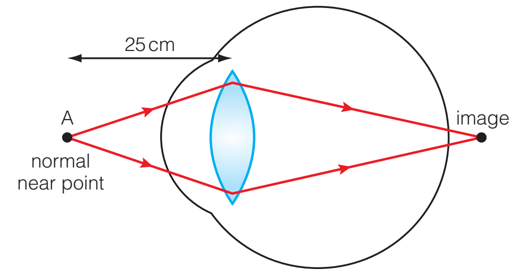


(b) A short-sighted eye is too powerful to focus on distant objects.

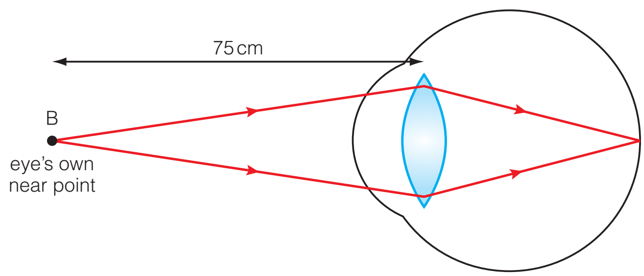


(c) A short-sighted eye is corrected with a diverging lens.

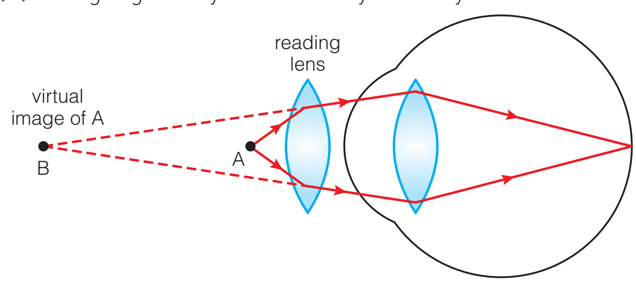
Figure 31.15 For simplicity, the refraction in the eye is shown to take place in the eye lens.



(a) A long-sighted eye is too weak to focus on near objects.



(b) A long-sighted eye sees clearly far away.



(c) A long-sighted eye is corrected with a converging lens.

Figure 31.16

Long sight (hypermetropia)

When people suffer from long sight, they can see things clearly at long distances, but their eyes cannot focus clearly on nearby objects. When this condition exists, people are likely to need reading glasses.

Figure 31.16(a) shows a long-sighted eye. The normal near point for a healthy eye is about 25 cm away from the eye. This eye is not powerful enough to focus an image on the retina and the image would come to a focus behind the retina. In this example, the eye's near point is 75 cm away from the eye, as shown in Figure 31.16(b). This is too far away for the person to read comfortably, so reading glasses are needed. Figure 31.16(c) shows how a converging lens helps the eye to focus an image on the retina when it looks at an object at the normal near point of vision, 25 cm away.

EXAMPLE

How do we calculate the power of the lens needed in Figure 31.16 (c)?

Answer

We assume that the lens is close to the eye so that the point A is 25 cm from the lens. [Note that Figure 31.16 (c) is not drawn to scale].

The image seen through the converging lens is at B, but this is a virtual image as the rays appear to come from B, rather than converging at B.

The focal length of the reading lens can be calculated from the formula:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

where u = 0.25 m and v = −0.75 m. Using the convention real-is-positive, v is a negative quantity as the image is virtual.

So

$$\frac{1}{0.25} - \frac{1}{0.75} = \frac{1}{f}$$

$$\frac{1}{f} = 4.00 - 1.33$$

$$= 2.67$$

So power $= \frac{1}{f} = 2.67\,\text{D}$

Note that if you work in metres $\frac{1}{f}$ gives the power in dioptres.

Some people have difficulty focusing both on distant objects and nearby objects. Then their eyesight can be corrected by using varifocal lenses. These glasses have weaker lenses at the top for distant vision and stronger lenses at the bottom for closer vision.

Astigmatism

So far, we have assumed that the surface of the cornea is a perfect sphere, like an orange or a football. However, it is very common for people to have eyes that have surfaces that are not spherical. So the curvature of the eye surface is more like that of an egg or a rugby ball, as shown in Figure 31.17.

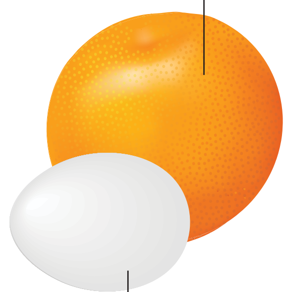


Figure 31.17 In astigmatism, the front surface of the eye has an irregular shape like an egg. Whereas the non-astigmatic eye is shaped like an orange.

In Figure 31.17, the egg-shaped 'eyeball' has a greater curvature in the vertical plane than the horizontal plane. This means that light is focused more strongly by the eye in the vertical plane than in the horizontal plane. So when opticians design our glasses, they take account of astigmatism by making the lenses cylindrical in shape to correct for astigmatism. Opticians define the direction of astigmatism relative to the eye using an angle, as shown in Figure 31.18.

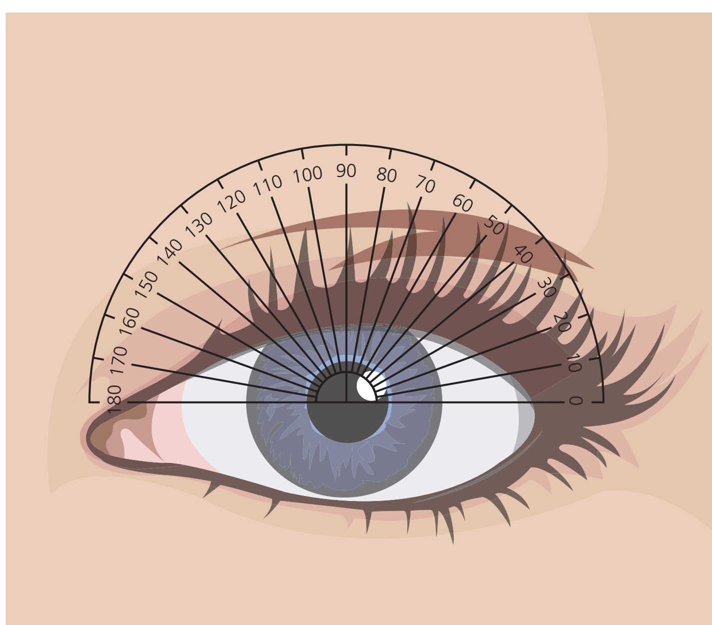


Figure 31.18

EXAMPLE

1 A patient has the following prescription. What does it mean?

Near	Sphere	Cylinder	Axis
Right	+2.00	−0.50	10°
Left	+2.25	−1.25	170°

Answer

First, the patient is long sighted as the correcting lenses for both eyes are converging with powers of 2.00 D and 2.25 D. The heading 'near' tells us that this prescription corrects for vision near to the eye.
The right eye lens has a power of 2.00 D along the 10° axis, and a power of 1.50 D at right angles to this along the 100° axis. The left eye lens has a power of 2.25 D along the 170° axis and 1.00 D at right angles to this along the 80° axis.

2 A patient has the following prescription. What does it mean?

Far	Sphere	Cylinder	Axis
Right	−1.25	0	
Left	−0.75	−0.25	40°

Answer

Here we can see that this prescription corrects for vision far from the eye. The negative powers of the lens show that the patient is short sighted. The right eye has no astigmatism, as the cylindrical measurement is zero. The left eye has a power of −0.75 D along the 40° axis and a power of −1.00 D along the 130° axis.

TIP

For an astigmatic eye:
- the power of the lens along the axis = sphere power
- the power of the lens perpendicular to the axis = sphere power + cylinder power

TEST YOURSELF

9 a) Make a sketch of a human eye and label the iris, pupil, cornea, lens, suspensory ligaments, retina and optic nerve.
b) Which two parts of the eye are responsible for refracting light and forming an image?

10 a) Explain what is meant by the term accommodation.
b) Draw diagrams to explain how an eye accommodates to focus on
i) an object far away
ii) an object close by.

11 An eye is short sighted. The far point of vision for the eye is 1.8 m.
a) Sketch a diagram to show how a lens can be used to enable the eye to focus on far away objects.
b) Calculate the power of the lens required for the eye to focus on far away objects.

12 An eye is long sighted and has a near point of vision of 30 cm.
a) Sketch a diagram to show how a lens can be used to help the eye to focus on a book 25 cm in front of it.
b) Calculate the power of the lens you have chosen. It is rare for reading glasses to be stronger than +4.0 D. Explain why.

13 a) Explain what is meant by the term astigmatism.
b) Two patients, X and Y, have the following prescriptions for their eyes. In each case, describe what is wrong with their eyes.

X	Near	Sphere	Cylinder	Axis
	Right	+0.5	0	
	Left	+0.75	0	

Y	Far	Sphere	Cylinder	Axis
	Right	−2.25	−1.0	80
	Left	−1.75	−0.5	100

c) What shape of lens is used to correct astigmatism?

Physics of the ear

The ear as a detection system

The ear consists of three regions: the outer ear, the middle ear and the inner ear.

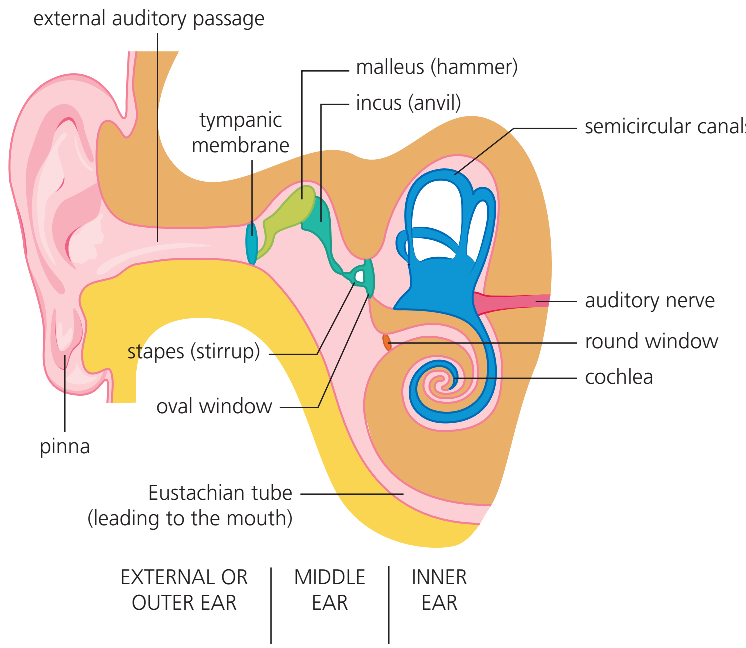


Figure 31.19

The outer ear begins with the ear flap (pinna) which is connected by the ear canal (external auditory passage) to the eardrum (the tympanic membrane).

The middle ear comprises a cavity which is connected by the Eustachian tube to the mouth. This allows the pressure in the ear to be adjusted – you can do this by swallowing; you will find you need to adjust your ear pressure when you go uphill in a car, for example.

The middle ear also has three small bone levers (ossicles) which connect the outer ear to the inner ear, where nerves can detect sound.

The inner ear has three semicircular canals, which help us with balance and detect changes in velocity. The inner ear also contains the cochlea, which is the organ responsible for transmitting the sensation of sound to our brains.

The cochlea is a helical, spiral shaped cavity, but its function can be best understood by reference to Figure 31.20, which is an 'uncoiled' representation of the cochlea. One end of the cochlea is connected to the oval window and the lower end of the cochlea is terminated by the round window. Sound waves are transmitted along the cochlea, where movement in the membrane causes small hairs in the cochlea to bend backwards and forwards. The distortion of the hair cells then initiates neural impulses which travel along the auditory nerve to the brain.

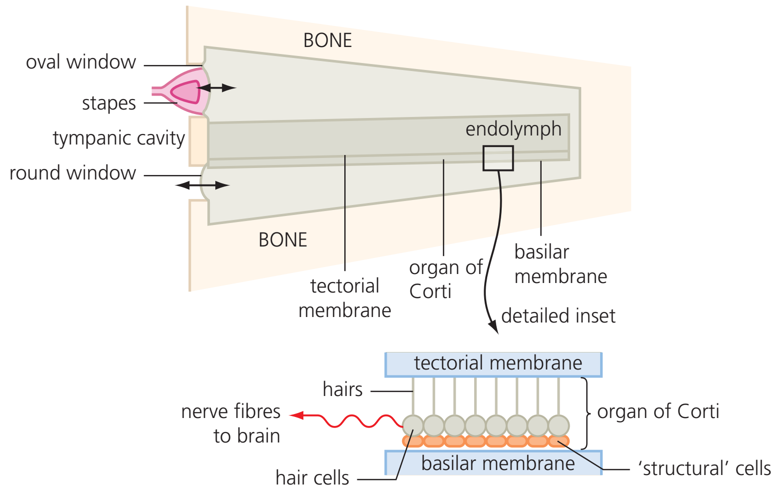


Figure 31.20

The mechanism of sound transmission

Each of the three parts of the ear plays a part in allowing us to hear.

The earflap is designed to collect sound and channel it down the ear canal. It also helps us to detect the direction of sounds. The eardrum (tympanic membrane) then vibrates as a result of the changes in pressure caused by the expansions and rarefactions in the sound waves. The eardrum is connected to the three ossicles which rock like levers to transfer energy to the oval window. Then, finally, the energy stimulates movement in the hair cells in the cochlea, to transmit a signal along the auditory nerve, which our brain recognises as sound.

Frequency range of sound

The range of audible frequency varies from person to person, but the average range is about 20 Hz to 20 000 Hz. However, as we get older we are less able to hear the higher frequencies and, for people past middle age, the upper limit of hearing is likely to be about 12 000 Hz.

The sensitivity of our ears depends on the frequency of the sound. Figure 31.21 shows that the ear is most sensitive to sounds of frequency about 3000 Hz. For us to hear lower or higher frequencies, the intensity of sound must be several orders of magnitude higher. For example, Figure 31.21 shows us that for us to hear sounds of frequency 100 Hz or 10 000 Hz, the lowest intensity of sound must be about 10^{-8} W m^{-2}.

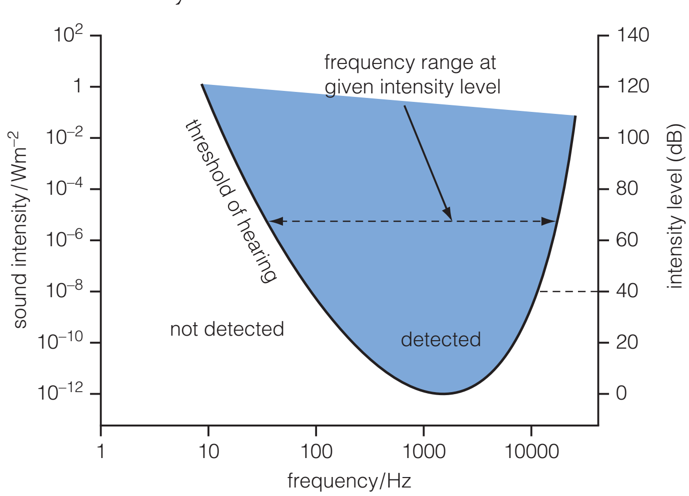


Figure 31.21

The ear's logarithmic response

The ear can detect a great range of intensities of sound. At the lower limit, the ear can detect intensities as low as 10^{-12} W m^{-2} at a frequency of 1 kHz, which is known as the threshold of hearing. At the upper limit, the ear can hear sounds as loud as 100 W m^{-2}, above which point we would be likely to suffer pain, and if we are exposed to such intensities for long periods, the ear is likely to be damaged and our hearing adversely affected.

Sound intensity is a direct measure of the power reaching an eardrum measured in W m^{-2}.

However, our detection and discernment of changes in intensity are not linear. We can more easily discern changes in intensity at low intensities of sound than at high intensities. For example, if the intensity of sound is increased from 1×10^{-6} W m^{-2} to 2×10^{-6} W m^{-2}, and then again from 2×10^{-6} W m^{-2} to 4×10^{-6} W m^{-2}, the loudness of the sound appears to change in equal steps. This is a logarithmic response to **sound intensity**, and it therefore makes sense for us to describe sound intensity using a logarithmic scale.

The definition of intensity and the decibel scale

The **intensity level of sound** is a measure relative to the threshold of hearing, defined as 0 dB. Intensity levels are measured in dB.

The **intensity level of sound** is defined by this equation:

$$\text{intensity level} = \log_{10}\left(\frac{I}{I_0}\right) \text{B}$$

where I_0 is the generally accepted threshold of hearing of 10^{-12} W m^{-2}.

In this scale the intensity is measured in bels (B).

However, since the bel is a large unit, representing intensity differences in the ratio 10 : 1, we use the decibel (dB) to describe sound intensities.

$$1\text{ B} = 10\text{ dB}$$

and $\text{intensity level} = 10\ \log_{10}\left(\frac{I}{I_0}\right) \text{dB}$

For example, a sound of intensity 10^{-11} W m^{-2} has an intensity level of:

$$\text{intensity level} = 10\ \log_{10}\left(\frac{10^{-11}}{10^{-12}}\right)$$

$$= 10 \text{ dB (or 1B)}$$

Note that we define intensity levels of sound relative to the threshold of hearing of 10^{-12} W m^{-2} at 1 kHz.

EXAMPLE

1 The sound intensity near to a road in a town centre is $3 \times 10^{-5}\ \mathrm{W\,m^{-2}}$. Calculate the intensity level in dB.

Answer

$$\text{Intensity level} = 10\ \log_{10}\left(\frac{3 \times 10^{-5}}{10^{-12}}\right)$$

$$= 75\ \text{dB}$$

2 Background noise from a loudspeaker produces an intensity level of 46 dB. Calculate the sound intensity of the music.

Answer

$$46 = 10\ \log_{10}\left(\frac{I}{10^{-12}}\right)$$

$$4.6 = \log_{10}\ \frac{I}{10^{-12}}$$

$$10^{4.6} = \frac{I}{10^{-12}}$$

So

$$I = 10^{-12} \times 10^{4.6}$$

$$= 10^{-12} \times 4 \times 10^{4}$$

$$= 4 \times 10^{-8}\ \mathrm{W\,m^{-2}}$$

The difference in intensity level between two sounds is given by:

$$10\left[\log_{10}\left(\frac{I_2}{I_0}\right) - \log_{10}\left(\frac{I_1}{I_0}\right)\right]\text{dB} = 10\ \log_{10}\frac{I_2}{I_1}\ \text{dB}$$

For example, the difference in intensity levels between a person shouting with sound intensity $8 \times 10^{-5}\ \mathrm{W\,m^{-2}}$ and someone speaking with sound intensity $2 \times 10^{-6}\ \mathrm{W\,m^{-2}}$ is:

$$10\ \log_{10}\left(\frac{8 \times 10^{-5}}{2 \times 10^{-6}}\right) = 10\ \log_{10} 40$$

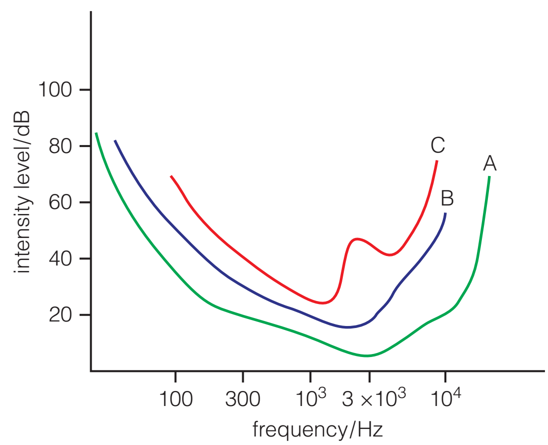
$$= 16\ \text{dB}$$

Equal loudness curves and the dBA scale

You read earlier that the ear is most sensitive to sounds in the region close to 3 kHz. The ear is less sensitive at frequencies above and below 3 kHz.

The decibel scale relates to the intensity of a sound calculated in $W\,m^{-2}$. However, what matters to us is not so much the sound intensity measured in $W\,m^{-2}$, but the loudness that we actually perceive. We can tolerate a low frequency noise of 100 Hz much more comfortably than the same sound intensity at 3 kHz.

The sensitivity of the ear varies from person to person. An equal loudness curve can be produced for a person by getting him/her to say what sounds appear to be of the same loudness. Figure 31.22 shows two typical responses for people of different ages, and also a response for someone with impaired hearing.



A Threshold of hearing for a 20 year old
B Threshold of hearing for a 55 year old
C Threshold of hearing for an ear damaged by excessive noise

Figure 31.22

Curve A shows the typical response for a young person with good hearing. Their threshold of hearing is at approximately the accepted value, $10^{-12}\,W\,m^{-2}$ at 1 kHz. You can see that for them just to hear a sound of frequency 100 Hz the intensity of the sound needs to be about 30 dB above the level to hear the same sound intensity at 3 kHz.

Loudness is a subjective quantity which depends on the sensitivity of an individual's ears.

Curve B shows an equal **loudness** curve for someone in middle age. They have suffered from a hearing loss of about 10 dB at lower frequencies, but the loss is more severe at high frequencies. So sounds need to be 10 dB to 20 dB louder for the middle-aged person to have the same loudness as have for the young person.

Curve C shows an equal loudness curve for someone whose hearing has been damaged by industrial noise. The ear is most sensitive to frequencies of about 3 kHz, so the loss can be greatest at this frequency.

The dBA scale

It is usual to quote the intensity of sounds on the dBA scale which takes account of the sensitivity of the human ear. These are called A-weighted sound levels. The dBA scale links the sound intensity at different frequencies to the relative loudness perceived by the human ear.

Figure 31.23 shows the A-weighted sound levels adjusted to dBA for the three different people highlighted in Figure 31.21.

These curves are the inverse shape of the equal loudness curves shown in Figure 31.22.

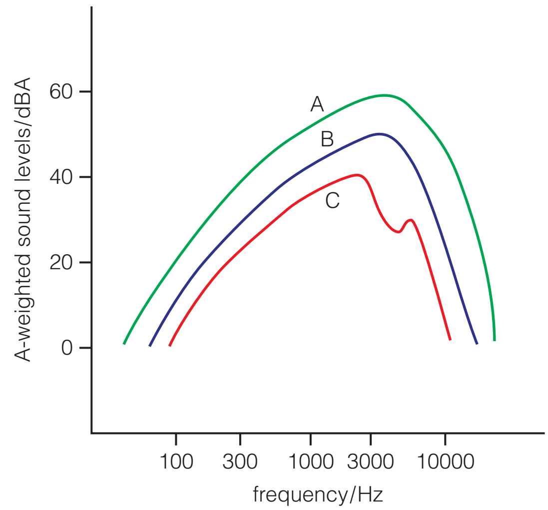


Figure 31.23 Three people A, B, C, are exposed to sounds of intensity level 60 dB across the audible range of frequencies. Due to the sensitivity of the ear, the perception of loudness changes with frequency. B and C have different corrected curves, because their hearing is not as good as A's.

- At 3000 Hz, A is exposed to an intensity level of 60 dB. When they are exposed to the same level at a frequency of 300 Hz, it sounds quieter to them. At 300 Hz, A's weighted sound level is 40 dBA, even though a microphone might record the intensity level of 60 dB. It is the subjective impact of the sound on the ear that gives rise to the loudness heard by each person.
- B suffers from some age-related hearing loss. In comparison with A, B has lost about 10 dB of hearing in the range close to 3000 Hz.
- C's hearing is worse that both A's and B's. C hears a narrower range of frequencies than the others. C's loss of hearing (at around 3 kHz) might have been caused by being exposed to excessive noises which have damaged the cochlea, or it might be as a result of an illness.

Table 31.1 shows some approximate intensity levels for various common sources of sound.

Table 31.1

Source of sound	Intensity level/dBA	Sound intensity/W m^{-2}
Silence (threshold of hearing)	0	10^{-12}
Whispering	20	10^{-10}
Normal speech	60	10^{-6}
A jet plane 1000 m overhead	100	10^{-2}
Thunder overhead	110	10^{-1}

TEST YOURSELF

14 Each of the following is part of the ear. Explain the function of each part.

a) malleus

b) tympanic membrane

c) oval window

d) cochlea

e) pinna

15 Give an account of how sound is transmitted from the outer ear to the inner ear.

16 **a)** Explain what is meant by the threshold of hearing.

b) What is the accepted value of the threshold of hearing?

17 **a)** A person speaks with an intensity of 7×10^{-6} W m^{-2}. Calculate the sound's intensity level in dB.

b) A car makes a sound with an intensity level of 65 dB. Calculate the intensity of the sound in W m^{-2}.

c) A musician plays two notes on a violin. A person hears two different sound intensities for the notes: 5×10^{-6} W m^{-2} and 8×10^{-7} W m^{-2}. Calculate the differences between the two intensity levels in decibels.

18 The three curves in Figure 31.23 show the A-weighted sound levels for three people.

a) **i)** Explain what is meant by an A-weighted sound level.

ii) Explain what is meant by the word 'loudness'.

iii) Two people listen to a sound, but do not agree on how loud it is. What do you know about loudness that would explain this?

b) Find the difference in the sensitivity of A's ear at frequencies of 100 Hz and 3000 Hz.

c) **i)** Use the graph to state the range of frequencies which C was able to hear in this test.

ii) State the hearing loss of C in dB, in comparison to A, at a frequency of 3 kHz. How many times more sensitive are A's ears than C's ears at this frequency?

19 **a)** Distinguish between the intensity, intensity level and loudness of a sound, and give the units to measure each quantity.

b) Express the following in decibels:

i) The increase in intensity level due to the power from a loudspeaker changing from 3 W to 12 W.

ii) The decrease in intensity level due to moving from 10 m to 20 m away from a source which is emitting sound in all directions.

[Hint: you need to assume that sound intensities obey an inverse square law $I \propto \frac{1}{r^2}$].

iii) The difference in intensity level between the noise from an express train of 4×10^{-3} W m^{-2} 10 m away and someone shouting from 5 m with a sound of 6×10^{-6} W m^{-2}.

Biological measurement

Simple ECG machines and the normal ECG waveform

The brain controls the action of muscles by sending a signal along nerves. For example, if we decide to pick something up, our brain sends a signal to our fingers. The muscle which is an exception to this rule is our heart, where the electrical stimulus for movement originates in the heart itself.

The heart muscle contains millions of cells which are more negatively charged on the inside and less positively charged on the outside. There is a potential difference of about 80 mV across the cell membranes; this is known as the resting membrane potential. Figure 31.24 shows how the potential varies with time across the cell membranes.

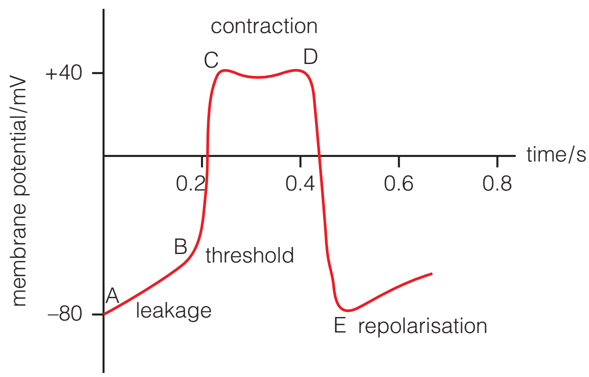


Figure 31.24

Over the period AB, charges migrate across the membrane, until the time B is reached (the threshold). At this point, charge migration becomes very rapid. Consequently, the cells redistribute the charges, so that the inside of the cells become more positively charged. This process is known as depolarisation. After the cells in each zone have depolarised, the charges rapidly move back to their original positions and the cell membrane is repolarised, as shown by part DE of the graph. This process repeats itself every second or so, and this is one beat of the heart.

Electrocardiography (ECG) measurements

During one heartbeat, different parts of the heart are stimulated at different times. For example, the atria are stimulated slightly before the ventricles. So cells in different places are depolarised at different times. Therefore there is a potential difference between cells which have depolarised and those which are about to depolarise. The body conducts electricity sufficiently well for this potential difference to be detected across different parts of the body.

The study of the waveform from the heart is known as electrocardiography. The machine that examines the waveform produces electrocardiograms which can be examined by cardiologists. Electrodes are connected to the body at various places. Good electrical contact is ensured by connecting the electrodes to the body using a conducting gel. Figure 31.25 shows a typical ECG which is produced when electrodes are placed in the right arm and in the left arm.

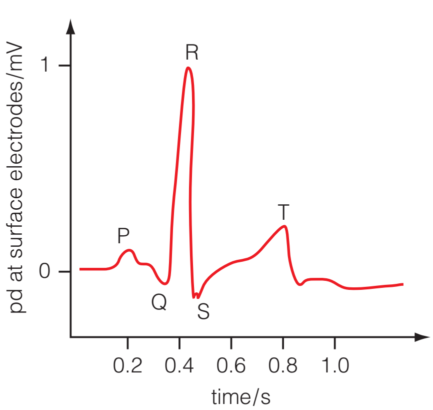


Figure 31.25

- The P-wave occurs during depolarisation of the atria which causes the contraction of the atria.
- The QRS pulse corresponds to the depolarisation and then the contraction of the ventricles.
- The T-wave occurs during the repolarisation of the ventricles which corresponds to the relaxation of the ventricles.

ECG electrode sites

The ECG electrodes may be placed over the heart, on the torso or on the limbs at points where the major arteries run close to the surface.

There are standard electrode sites, which allow electrocardiographers to observe the heart's electrical activity; these are listed in the table. The standard sites are connected in pairs to the two terminals of an ECG amplifier to give 12 possible connection systems (known as 'leads').

right arm	and left arm	lead 1
right arm	and left leg	lead 2
left arm	and left leg	lead 3
right arm	and left arm and leg	aVR
left arm	and left leg and right arm	aVL
left leg	and right and left arm	aVF
one of six chest sites	both arms and left leg	V_1, V_2, V_3, V_4, V_5, V_6

- aVR stands for augmented vector right
- aVL stands for augmented vector left
- aVF stands for augmented vector foot

Examination of electrocardiograms allows specialists to note various heart complaints, such as irregularity in rhythm or parts of the body where blood flow is restricted.

TEST YOURSELF

20 **a)** What is meant by the term ECG?
b) Explain how an ECG can allow cardiologists to investigate whether a patient's heart is healthy.

21 **a)** State how good electrical contact is made between the electrodes of an ECG and the body.
b) Name four parts of the body where electrodes are placed for an ECG.

22 Sketch a graph of potential difference, obtained at the electrodes, against time for the waveform obtained from a single beat of the heart in a healthy person. Give approximate scales on the axes and label your diagram with the chief features of interest in cardiology.

Non-ionising imaging

Ultrasound imaging

Piezoelectric devices

Ultrasound is a sound wave above the upper end of the human hearing range of 20 kHz. Ultrasound is produced using piezoelectric crystals such as quartz or the synthetic ceramic, lead zirconate titanate. When a potential difference is applied across a piezoelectric crystal it compresses or extends along the direction of the p.d. Therefore, if a high frequency p.d. is applied across the crystal, it vibrates backwards and forwards at that frequency, so that ultrasonic waves are produced. See Figure 31.26.

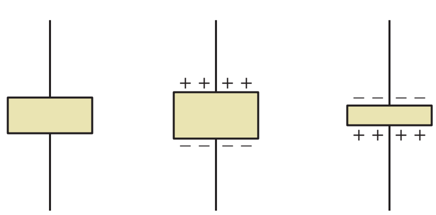


Figure 31.26 A p.d. applied across a piezoelectric crystal causes it to change shape.

An ultrasonic transducer is a device which converts the electrostatic energy supplied to a crystal into the energy transferred by the ultrasonic waves. The same transducer can be used as a receiver to detect ultrasound when the ultrasound impacts onto the piezoelectric crystal, causing it to distort. The oscillating crystal now produces an a.c. signal which can be detected and amplified.

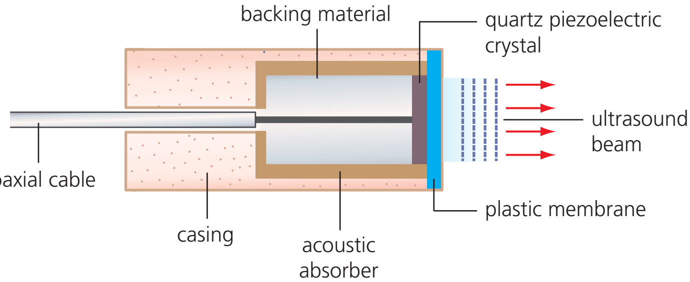


Figure 31.27

Ultrasound as a diagnostic tool

Attenuation is the gradual loss in intensity as sound (or any other wave) passes through a medium.

Typical ultrasonic frequencies range between 1 MHz and 18 MHz. The choice of frequency is a balance between the spatial resolution of the image and the imaging depth. Higher frequency (small wavelength) sounds produce a narrow beam because they diffract less than lower frequency (long wavelength) sounds. The smaller wavelength sounds are also capable of reflecting or scattering off smaller structures. Higher frequency sounds are absorbed more readily by tissues. Therefore, the lower frequency sounds can penetrate more deeply into body tissues. When energy is absorbed, the intensity of the transmitted sound decreases; this is called **attenuation**. Ultrasound can be used to build up images of the inside of our bodies because ultrasound waves reflect differently off the various organs or bones inside the body.

Reflection of ultrasound

The reflection of ultrasound (or sound) between the interface of two mediums depends significantly on the acoustic impedances of the two materials.

The acoustic impedance of a material is defined by the following equations.

Acoustic impedance = density × speed of sound

$$Z = \rho \times c$$

Where Z is the acoustic impedance in kg m^{-2} s^{-1}

ρ is the density in kg m^{-3}

c is the speed of sound in m s^{-1}

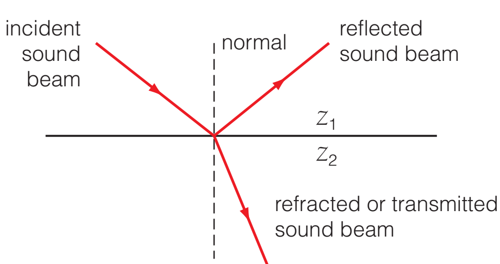


Figure 31.28

Figure 31.28 shows a beam of ultrasound incident on the boundary between two media: 1 and 2. The acoustic impedance of medium 1 is Z_1, and the acoustic impedance of medium 2 is Z_2.

It can be shown that the ratio of the reflected intensity I_r, to the incident energy I_i is given by:

$$\frac{I_r}{I_i} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$

This equation assumes that the energy is incident along the normal (at right angles) to the surface.

The ratio $\frac{I_r}{I_i}$ is called the intensity reflection coefficient.

Table 31.2 below shows the velocity of sound in various body tissues and air, together with the associated densities and acoustic impedances.

Table 31.2

Material	Velocity of sound/m s^{-1}	Density/kg m^{-3}	Acoustic impedance/ kg m^{-2} s^{-1} × 10^6
Air	330	1.3	4.30×10^{-4}
Oil	1500	950	1.43
Water	1500	1000	1.50
Bone	4100	1900	7.79
Brain	1540	1030	1.59
Muscle	1580	1080	1.71
Liver	1590	1040	1.65
Fat	1450	950	1.38
Blood	1570	1060	1.66

EXAMPLE

1 A beam of ultrasound passes through the muscle tissue in a patient's stomach and is incident on the bladder full of water. Calculate the fraction of the ultrasound reflected off the water in the bladder.

Answer

Z_2 (water) = 1.50×10^6 kg m^{-2} s^{-1}

Z_1 (muscle) = 1.71×10^6 kg m^{-2} s^{-1}

$$\frac{I_r}{I_i} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$

$$= \frac{(1.50 - 1.71)^2}{(1.71 + 1.50)^2}$$

$$= 4.28 \times 10^{-3}$$

Note that since all impedances are multiplied by 10^6, this factor can be cancelled out before doing the calculation.

So only a small amount of energy is reflected. However, this is sufficient for a small echo to be produced, and for the echo or reflection to be noticeable. In this example, the tissues are acoustically well matched, and the reflection is small. When $Z_1 \gg Z_2$ or $Z_2 \gg Z_1$ the materials are poorly matched acoustically and there is a large reflection.

2 Ultrasound travels from air into muscle tissue. What fraction of the energy is reflected now?

Answer

Z_2 (muscle) = 1.71×10^6 kg m^{-2} s^{-1}

Z_1 (air) = $4.3 \times 10^{-4} \times 10^6$ kg m^{-2} s^{-1}

= 4.3×10^2 kg m^{-2} s^{-1}

$$\frac{I_r}{I_i} = \frac{\left(1.71 \times 10^6 - 4.3 \times 10^2\right)^2}{\left(1.71 \times 10^6 + 4.3 \times 10^2\right)^2}$$

= 0.999

In this case nearly all the energy is reflected.

This second example explains why, when an ultrasonic transducer is placed on the body, it must have a layer of gel between it and the body. The gel ensures a good acoustic match; if any air gets between the transducer and the body, a lot of energy is reflected.

TEST YOURSELF

23 An ultrasonic scanner uses a frequency of 10 MHz.
- **a)** State the advantages of using a higher frequency of ultrasound.
- **b)** State the disadvantages of using a higher frequency ultrasound.

24
- **a)** What is meant by a piezoelectric crystal?
- **b)** Explain how a piezoelectric crystal can be used to generate high frequency ultrasound.

25 Oil has a density of 950 kg m^{-3} and sound travels through it at a speed of 1500 m s^{-1}.
- **a)** Calculate the acoustic impedance of oil.
- **b)** A beam of ultrasound in oil is incident on an oil/water boundary. Water has an acoustic impedance of 1.50×10^6 kg m^{-2} s^{-1}. Calculate the fraction of energy
  - **i)** reflected at the surface
  - **ii)** transmitted into the surface.

26
- **a)** Rock and air are acoustically poorly matched. Explain what this means.
- **b)** Use the data in Table 31.2 and the data below to show why sound is reflected very efficiently off a granite cliff. Speed of sound in granite is 6000 m s^{-1}; density of granite is 2700 kg m^{-3}.

The A-scan

The A-scan system is a range-measuring system. The system operates by emitting short pulses of ultrasound which are reflected off the interface between the tissues in the body. The time measuring instrument – usually a cathode ray oscilloscope (CRO), must be synchronised with the transmitter and receiver.

- The transmitter is a piezoelectric transducer that emits short bursts of ultrasound of duration a few microseconds.
- The timer is the oscilloscope timebase, where the spot moves at a constant speed across the screen.
- Ultrasound is reflected off the various interfaces in the body. The reflections produce potential differences at the receiving transducer probes. These are amplified at the receiver and then applied to the Y-plates of the CRO. These are then displayed. Each vertical line corresponds to a reflection off the various body surfaces. See Figure 31.29.

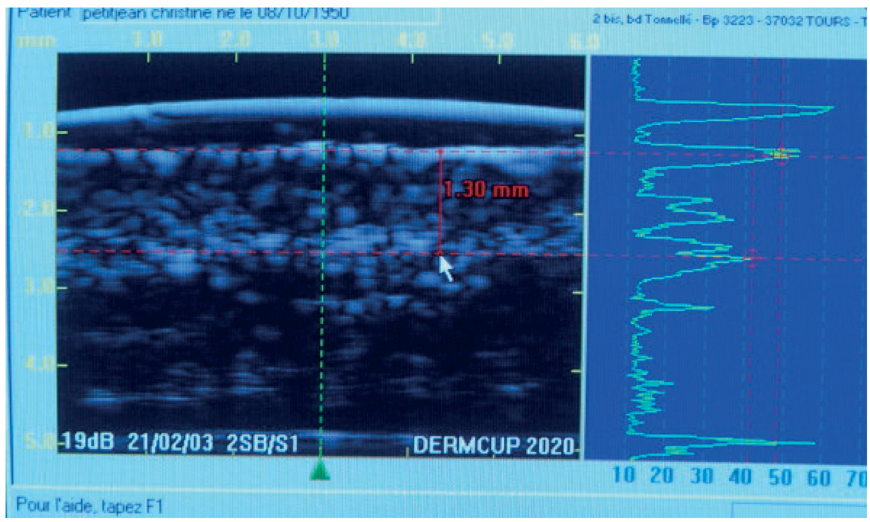


Figure 31.29 The receiver detects reflections of ultrasound off successive body surfaces.

The B-scan

The B-scan is used when it is necessary to build up a more complicated image, for example for looking at a foetus inside a mother's womb. The B-scan echoes are now used to control the brightness of the image at a particular place. The transducer is moved to different positions so that echoes can reveal a section through the body. A system of potentiometers enables voltages to record the position of the transmitter and receiver for each echo recorded. The information is stored by a computer, which can then build up the image which is seen on a screen.

The B-scan requires experience and skill from the operator. The transmitter must be positioned in various sites, and also rocked at different angles. The rocking is necessary because the transmission and reflection system requires the waves to be normal to the surface that reflects the ultrasound.

Figure 31.30 In a B-scan, an image is built up of a structure by recording many reflections from many different directions.

Use of ultrasound scanning

Ultrasound scanning is widely used in medicine. A few uses are listed below.

- Echocardiography is an essential tool in cardiology to diagnose the dilation of parts of the heart and function of heart ventricles and valves.
- In emergency medicine, ultrasound can be used to locate the position of a trauma inside the body.
- Organs such as the pancreas, liver, gall bladder and kidneys may be examined.
- In the neck, the thyroid, lymph nodes and saliva glands may be examined.
- In the field of obstetrics, ultrasound is widely used during pregnancy. Scans can: determine the date of the pregnancy; determine the viability of the foetus; check on the location of the foetus and thus ensure safety during birth; check for physical abnormalities; check on movement and heartbeat; and determine the gender of the baby.
- Ultrasound can also be used to examine tendons, muscles, nerves and ligaments. Sometimes ultrasound is used as an alternative to X-rays for patients up to the age of 12.

Advantages and disadvantages of ultrasound

- It is non-ionising, has no known side effects and, for nearly all patients, produces no discomfort. This makes ultrasound particularly useful for examining unborn babies and infants, for whom ionising radiation is particularly dangerous.
- It shows the structures of organs.
- It images muscle, soft tissue and bone surfaces very well. It shows up clearly the boundary between solid and fluid-filled spaces.
- The resolution of ultrasound is high – but not as clear as X-rays or MR scanners.

Some of the disadvantages of ultrasound are listed below.

- Ultrasound does not penetrate bone.
- Ultrasound performs poorly when there is gas between the transmitter and organ to be examined due to the extreme differences in acoustic impedances. For example, lung imaging is not possible.
- The depth of ultrasound penetration can be limited. This can be a particular problem in obese patients. Greater penetration can be obtained by using lower frequency ultrasound, but then the resolution is poorer.
- The method depends on the operator. A high level of skill and experience is required to produce good quality images.

TEST YOURSELF

27 a) Why is ultrasound used in medicine as a diagnostic tool? State two advantages of using ultrasound, and two disadvantages.

b) Give three examples of how ultrasound is used as a diagnostic tool.

28 Explain the difference between the uses of A and B-scans in ultrasonic diagnosis.

29 When structures near the surface of the body are examined – for example muscles and tendons – ultrasound of frequency 7–18 MHz is used. Deeper structures such as the liver and kidney are imaged at lower frequency 1–6 MHz. Explain why different frequencies are used for these investigations.

30 Figure 31.31 shows the intensity of reflections when ultrasound is reflected off different layers of the bodies.
Discuss qualitatively why the intensity of the reflections is different at each surface.

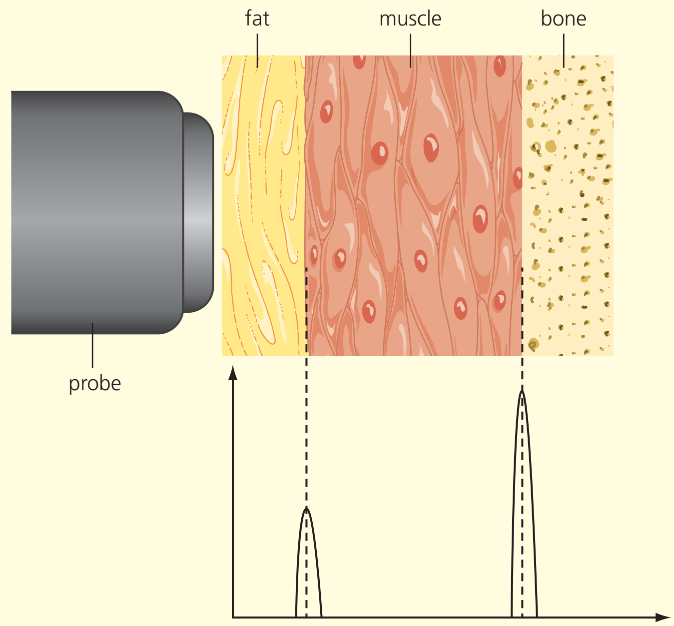


Figure 31.31

31 a) Why is a coupling medium necessary between a source of ultrasound and a body during ultrasonic investigation of the body?

b) The velocities of sound in air, oil and in body tissue (on average) are 0.33 km s^{-1}, 1.50 km s^{-1} and 1.53 km s^{-1}. The densities of the three media are respectively: 1.3 kg m^{-3}, 950 kg m^{-3}, 1065 kg m^{-3}. Estimate the fraction of the sound intensity reflected at each of the following interfaces.

i) an air – tissue interface

ii) an oil – tissue interface

iii) an air – oil interface

Comment on your results.

32 When ultrasound passes through the body, some energy is absorbed. This is called attenuation. The graph in Figure 31.32 shows the attenuation of the intensity of ultrasound 2 MHz with depth. In this investigation, a child's kidney is to be examined.

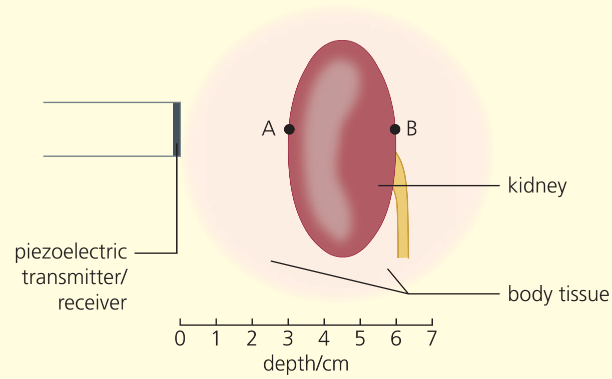


Figure 31.32 (a)

The acoustic impedance of the kidney is 1.65×10^6 kg m^{-2} s^{-1} and that of the surrounding body tissue is 1.40×10^6 kg m^{-2} s^{-1}. Use this information and the information in the graph to calculate the ratio of the intensity of the reflections from A and B, by the time the pulses come back to the receiver. (Remember that attenuation happens in both directions and assume that attenuation occurs at the same rate in the kidney and the body tissue.)

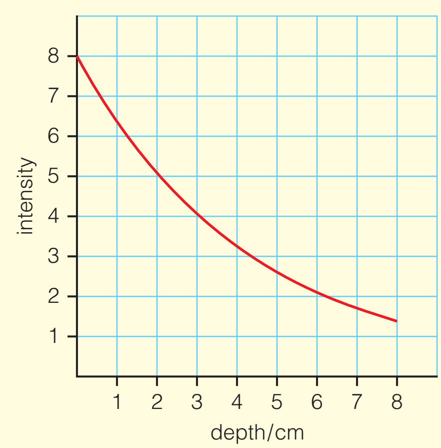


Figure 31.32(b)

Figure 31.33

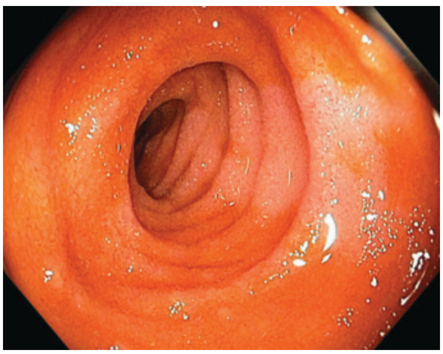


Figure 31.34

Fibre optics and endoscopy

Figure 31.33 shows a surgeon using an endoscope as he operates on a patient, and Figure 31.34 shows what he sees through the endoscope.

An endoscope is a fibre optic device which depends on light being transmitted, without loss, along an optical fibre. When light strikes the inside of the optical fibre at a shallow angle the light undergoes total internal reflection. (See page 86 of the Student's Book.)

The fibre (or the core) which transmits the light is surrounded by a cladding which has a slightly lower refractive index than the fibre. The cladding serves two purposes.

- Firstly, the cladding protects the inner fibre.
- Secondly, by having a refractive index just smaller than the inner fibre, the critical angle reaches a large value. This reduces the number of reflections along a length of fibre.

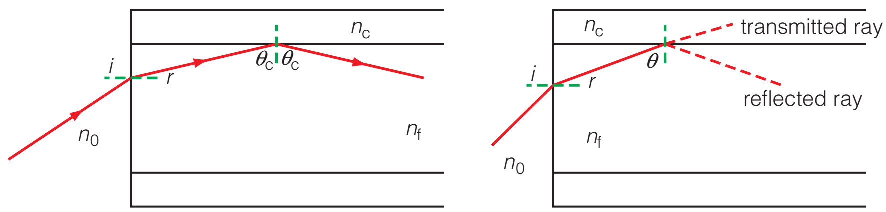


Figure 31.35(a) and (b) A ray is reflected at the critical angle inside a fibre.

Typical values for the refractive indices are:

- refractive index of inner fibre 1.52 n_f
- refractive index of cladding 1.48 n_c
- refractive index of air 1.00 n_0

EXAMPLE

1 Calculate the critical angle inside the fibre in Figure 31.35.

Answer

The critical angle is given by the formula (see page 86 of the Student's Book):

$$\sin\theta_c = \frac{n_c}{n_f}$$

$$= \frac{1.48}{1.52}$$

$$= 0.974$$

$$\theta_c = \sin^{-1} 0.974$$

$$= 77° \text{ (2 s. f.)}$$

Figure 31.35 shows what happens if a light ray enters the fibre at a larger angle of incidence. The angle of refraction r becomes larger and therefore the angle of incidence on the fibre, θ, becomes smaller, and below the critical angle. [The angle θ is equal to $(90° - r)$.]

2 Calculate the greatest angle of incidence, i_{max}, which ensures that the light undergoes total internal reflection inside the fibre.

Answer

In Figure 31.35(a)

$$\theta_c = 77°$$

Therefore

$$r = 90° - 77°$$

$$= 13°$$

$$\frac{\sin i_{max}}{\sin r_{max}} = \frac{n_f}{n_0}$$

So

$$\sin i_{max} = \frac{1.52}{1.00} \times \sin 13°$$

$$= 0.34$$

$$i = \sin^{-1} 0.34$$

$$= 20°$$

Thus from the above example, we can see that if such a fibre is used in an endoscope, the instrument has a field of view of 20°, on either side of the normal. This is adequate for most investigative or surgical procedures.

This analysis assumes that the fibre is straight. An endoscope is flexible, so the fibres can become curved to allow passage down a patient's throat (for example). When a fibre is curved, the angles of incidence vary, and losses can occur if the angle falls below the critical angle. In practice, a radius of curvature as small as about 20 times the fibre diameter can be tolerated without significant losses of the light intensity.

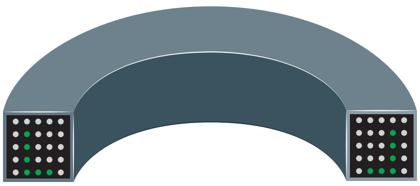


Figure 31.36 A bundle of fibres is placed in order. The ends of the fibres are cut flat and polished. The image falls on a number of fibres and is transmitted to the other end where it can be viewed.

Coherent and incoherent bundles

A coherent bundle of fibres is the name given to fibres that are placed together in an orderly fashion. In this bundle, a fibre remains in the same position relative to its neighbouring fibres. Each fibre transmits a small portion of an image, so that a coherent bundle allows the eye to see a clear image at the other end of the endoscope. (See Figure 31.36.) The image can be viewed using an eyepiece at the end of the endoscope.

Typically, fibres of diameter 10 μm are used in coherent bundles. The small diameter allows greater resolution of the image, as each fibre images a small part of the object under examination. If fibres of diameter smaller than about 5 μm are used, diffraction starts to be significant, so there is a loss of clarity in the image.

Incoherent (or non-coherent) bundles of fibres are also used in endoscopes. Here the fibres are arranged at random. Incoherent bundles are not used for image formation, but only to transmit light. The fibres in an incoherent bundle are relatively large compared with the fibres in a coherent bundle, having diameters of 50 μm–100 μm.

Fibre-optic endoscope

Figure 31.37 shows the basic principles behind an endoscope used for investigation or operations inside a patient's body.

The long flexible shaft of the endoscope is usually made of steel, which is sheathed in a protective PVC coating.

The following lie within the shaft.

- An aperture to pass and control instruments.
- A channel for air or water.
- A bundle of incoherent optical fibres to transmit light.
- A bundle of coherent optical fibres, with a lens, to enable the surgeon to see an image, at the far end of the endoscope, through the eyepiece.

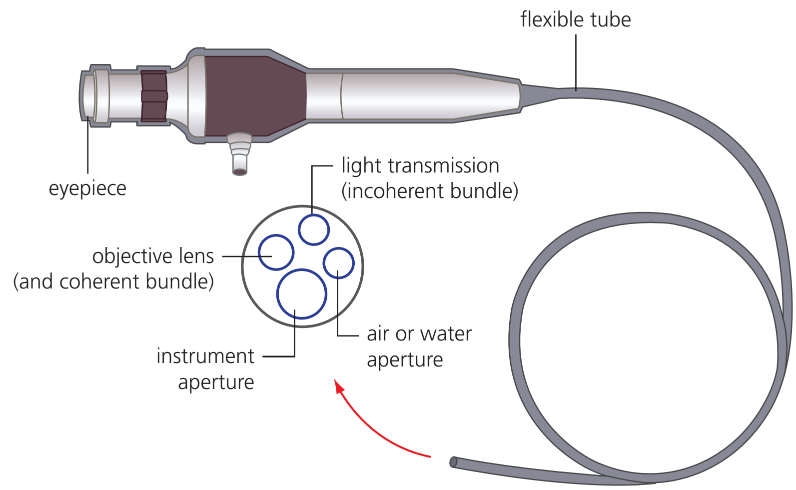


Figure 31.37 An endoscope used for surgery or investigation.

TEST YOURSELF

33 **a)** Explain what is meant by the term total internal reflection.
b) How is internal reflection put to use in an endoscope?

34 **a)** Explain the difference between a coherent and an incoherent bundle of optical fibres.
b) A coherent bundle of fibres has fibres with a diameter of 5 μm.
i) What is the advantage of having fibres with a small diameter?
ii) What is the disadvantage of having fibres with a diameter smaller than 5 μm?

35 **a)** A fibre has a refractive index of 1.50. It is surrounded with a cladding of refractive index 1.45. Calculate the critical angle for the fibre.
b) Use your answer for part (a) to show that the greatest angle of incidence, i_{max}, for a ray incident at the end of the fibre is 23°. Assume the light enters the end of the fibre from air.
c) Explain how an endoscope could be constructed with a wider field of view.

Magnetic resonance (MR) and scanner

The magnetic resonance scanner is a very high-resolution diagnostic device that provides detailed information about any part of a patient's body. Magnetic resonance imaging provides more detailed information than X-rays or ultrasound, and has the advantage (over X-rays) that it is non-ionising and therefore safe to use. However, magnetic resonance imaging is used sparingly as it is expensive.

Nuclear magnetic resonance

Nuclear spin

Spin is the name given to the quantum mechanical state of a proton or neutron associated with the nuclear magnetic field.

Figure 31.38 Protons have two possible spin states.

Magnetic field refers to the *B*-field which is also known as the magnetic flux density. The unit is the tesla, T. (See page 405 in the Student's Book.)

Protons and neutrons have a quantum mechanical property called **spin**, which provides each particle with a **magnetic field** like a miniature bar magnet. This is referred to as a magnetic dipole moment, or just magnetic moment. Each particle has two possible magnetic states, which are sometimes referred to 'spin up' and 'spin down' states. In many nuclei where there are even numbers of protons and neutrons (for example $^{16}_{8}O$ with 8 protons and 8 neutrons) the protons and the neutrons pair off in pairs of protons and pairs of neutrons, so that the magnetic fields of the spin up and spin down protons and the spin up and spin down neutrons cancel out. However, nuclei with odd numbers of protons and neutrons can have a magnetic field associated with them.

Magnetic resonance imaging depends on examining protons (hydrogen nuclei) which have these two possible spin or magnetic field states.

In the absence of a magnetic field the two spin states of a proton have the same energy and equal numbers of protons occupy the two possible states. However, when a magnetic field is applied, there is a difference in energy between the two spin or magnetic states. The spinning protons precess about the direction of the applied field, in the same way that a spinning top precesses under the influence of gravity.

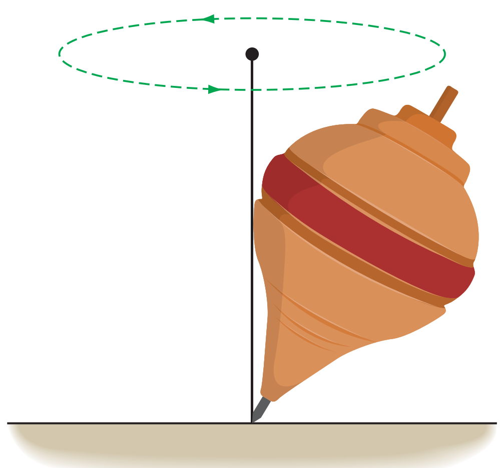


Figure 31.39 A spinning top precesses around the direction of the gravitational field.

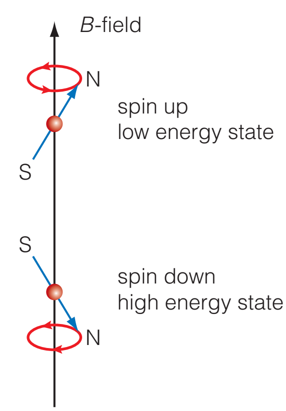


Figure 31.40 The spinning protons precess around the direction of the magnetic field.

Figure 31.40 shows how the magnetic moments of two protons precess around the direction of the magnetic field. The spin up proton has its magnetic moment in the same direction as the external field, and this now lies in a lower energy level than the spin down proton. So when a magnetic field is applied, there is a difference between the two spin states and most nuclei therefore lie in the lower energy state.

Magnetic resonance

Figure 31.41 shows how the energy difference between the two magnetic states depends on the magnetic field that is applied to the protons. The diagram shows an energy difference E_1 when the applied magnetic field is B_1, or E_2 when the field is B_2.

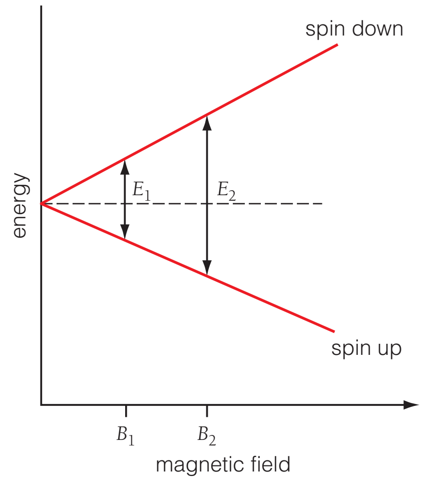


Figure 31.41

Nuclear magnetic resonance occurs when a nucleus absorbs a photon of exactly the energy required to flip its spin from a lower energy state to a higher energy stage.

When the field is B_1, if a proton is irradiated with a photon of energy exactly E_1, the proton's spin can be flipped from the spin up to the spin down state. So the proton's spin moves to the higher energy level. This is called **nuclear magnetic resonance**.

However, the nuclei do not stay in their excited state for long, and they soon return to their lower every state – by emitting a photon of exactly the same energy E_1. The time taken to return to the lower energy state is called the relaxation time. In a magnetic field of about 10 T, the photon has a frequency of about 420 MHz, which is in the radiofrequency range. This is a very low energy photon, which does not cause any damage to body tissues.

Magnetic resonance scanning

Figure 31.42 shows a simplified version of a magnetic resonance scanner that could be used in a laboratory.

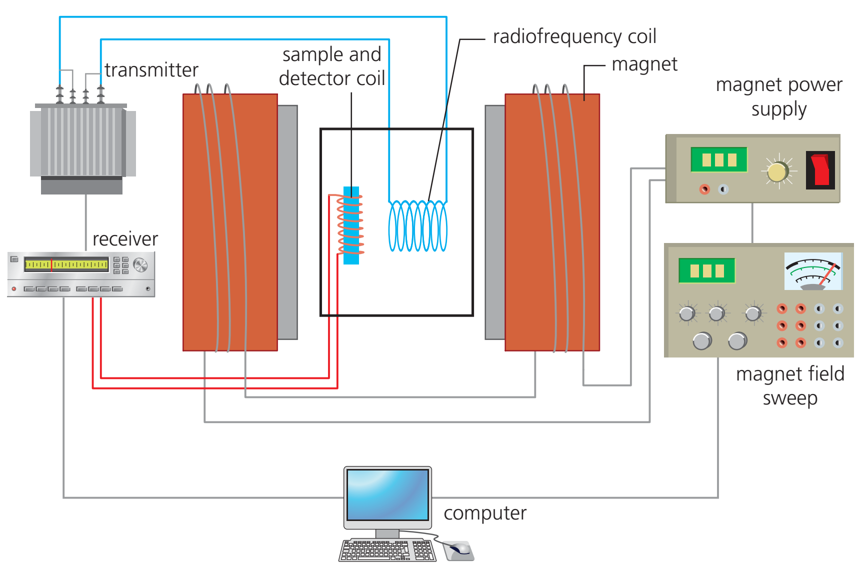


Figure 31.42 A magnetic resonance scanner.

- The magnetic field causes the energy levels of the proton's two magnetic spin states to separate – Figure 31.41. Magnetic fields are now usually generated by superconducting magnets which can produce flux densities as high as 20 T.
- The radiofrequency coil emits a range of frequencies which, if absorbed by a proton, can flip it into a higher energy state.
- When the proton returns to its lower energy level (this is known as spin relaxation), a radiofrequency photon is emitted, which is detected by the detector coil.
- The magnetic field sweep varies the strength of the magnetic field over the sample to be examined.

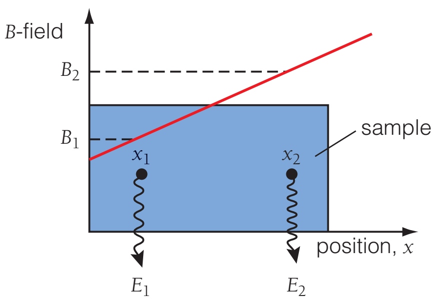


Figure 31.43 The two protons in positions x_1 and x_2 are located by a computer analysing the emissions. The computer knows the magnetic field at B_1 and B_2, and by detecting the energies E_1 and E_2, it can match the protons to those two positions, x_1 and x_2.

Since the frequency of the detected signal is proportional to the applied magnetic field, changing the strength of the field produces a different detected frequency. By placing a magnetic field gradient across a sample, the position of the proton which is emitting the radio frequency signal can be located.

Magnetic shielding

As well as detecting the position of a proton, magnetic resonance can also detect what type of chemical compound the proton is attached to. The electrons in a compound shield the nucleus (the proton) from the external magnetic field to an extent that depends on the electron configuration. Thus the frequency emitted by a relaxing proton depends not only on the strength of the external field, but on what chemical the proton is attached to. The relaxation time of the proton also depends on the material in which it is placed. The position and type of tissue is identified from the field strength, frequency of a photon, and the relaxation time.

Magnetic resonance can probe deep into our bodies to locate protons and the chemical they are attached to. Thus computers can build up a visual picture of the body, picking out variations of tissue types.

Hospital MR scanners

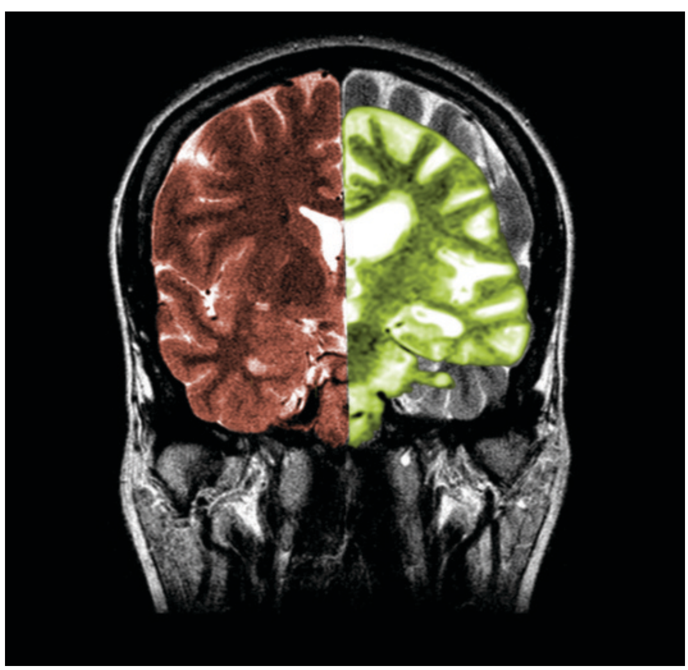


Figure 31.44 This magnetic resonance scan shows a composite image, with a normal coronal (frontal) cross-sectional MRI image of the brain (in brown), and then a superimposed coronal MRI image of a brain with advanced Alzheimer's disease (in green).

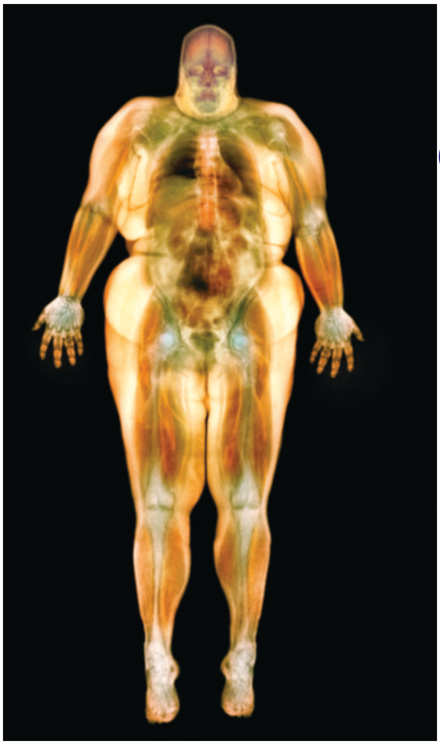


Figure 31.45 The photograph shows a patient who is having a whole body scan.

In hospitals, MR scanners can be used to build up detailed images of a patient's body. The procedure can take up to an hour as the magnetic fields are used to scan small parts of the body in turn.

TEST YOURSELF

36 How many spin states does a proton have?

37 a) Explain what is meant by the word 'precession'.

b) Draw a diagram to describe the behaviour of a proton in a magnetic field. Explain why the proton's magnetic moment has two possible energy levels in a magnetic field.

c) What is meant by the term 'spin flip'?

38 a) Explain what is meant by the term 'magnetic resonance'.

b) In a magnetic resonance experiment, a proton absorbs and then re-emits a photon of frequency 630 MHz.
Calculate the energy of the photon in (i) J (ii) eV.

c) A molecule usually requires an energy of about 1 eV or more to ionise it. Explain why the photon described in part (b) is of no danger to humans.

39 The energy gap between the spin up and spin down levels in Figure 31.41 can be calculated using the formula:

$$E = \frac{ehB}{4\pi m_p}$$

where B is the magnetic flux density, e the electronic charge, h is the Planck constant and m_p the mass of the proton (look up these quantities).

a) Calculate the frequency emitting in magnetic resonance when B is 10 T.

b) State the frequency when B = 20 T.

40 Explain why magnetic resonance scanners vary the strength of the magnetic field.

41 Why does the frequency of radiation absorbed by a proton in a magnetic field depend on which compound the proton is attached to?

X-ray imaging

The physics of diagnostic X-rays

X-rays are widely used in hospitals for diagnosing injuries or illness. Different wavelengths of X-rays are used for different purposes: for example the image of the broken bone shown in Figure 31.46 was taken using X-rays of wavelength about 4×10^{-11} m.

Figure 31.46 X-ray images can easily diagnose a broken bone.

Production of X-rays

An X-ray is produced when a highly energetic electron transfers some (or all) of its kinetic energy to a photon. You have met this idea before in Chapter 3 of the Student's Book: we explained how electrons in a light bulb transfer energy to photons to produce a continuous spectrum.

To produce X-rays, electrons with an energy of 20 kV to 150 kV are fired at a metal target, such as tungsten. Much of each electron's kinetic energy is transferred in many low energy collisions and only succeeds in heating up the metal target. However, if an electron passes close to a nucleus, or hits a nucleus, it can lose a large amount of kinetic energy by emitting an X-ray photon.

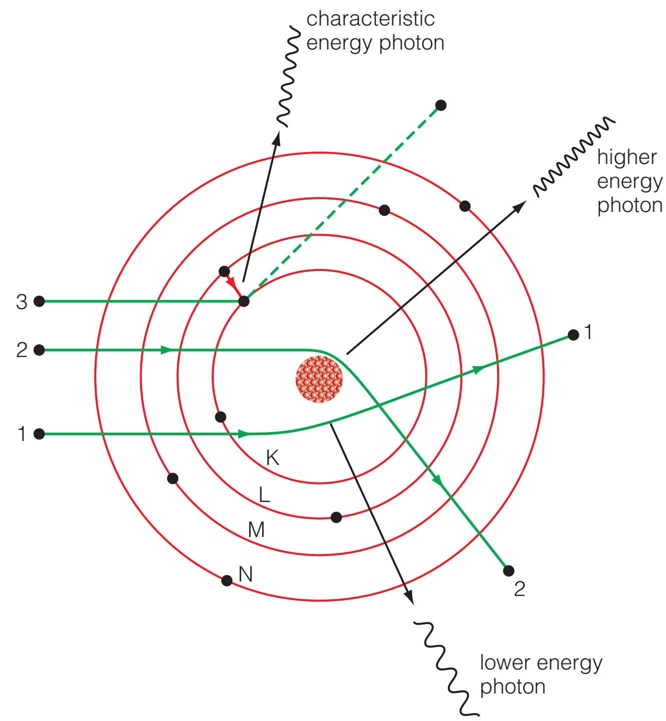


Figure 31.47

In Figure 31.47:

- electron 1 is deflected and slowed by the nucleus and emits an X-ray photon.
- electron 2 also emits an X-ray photon but with greater energy – and therefore smaller wavelength.

Electrons can transfer any amount of energy by the process described above. So if, for example, a beam of electrons has an energy of 100 keV, then photons of any energy up to 100 keV may be emitted.

So there is a continuous spectrum of X-rays produced. This is sometimes called *bremsstrahlung*, which in German means **braking radiation**.

Braking radiation is the name given to X-rays produced when an electron slows down. This radiation can have any energy up to the maximum kinetic energy of the electron.

However, each metal target also produces its own characteristic radiation, which is unique to each element.

In Figure 31.47, electron 3 strikes and ejects an electron in the atom's lowest energy level (known as the K level). With a vacancy in the atom's lowest energy level, another electron can fall from a higher level, L, M, N, etc. to fill that vacancy. When an electron transfers to the lowest level in an atom, an X-ray is emitted. However, this X-ray is always of the same wavelength, as the energy levels in atoms are fixed. If an electron is removed from the next level (L), X-rays are also emitted in some heavier atoms.

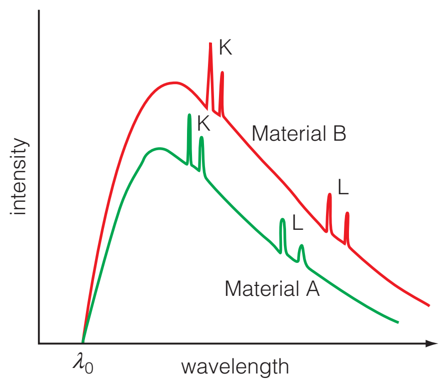


Figure 31.48

Figure 31.48 shows the typical spectra one would expect to see when electrons of the same energy are fired at two metals A and B. Each material has characteristic peaks of X-ray emission associated with the removal of electrons from the K and L shells. However, these peaks are at different wavelengths, because the energy levels are different for the two materials. Both materials produce a continuous spectrum of X-rays due to the deceleration of the electrons. And, because the incident electrons have the same energy, each material produces the same maximum energy of X-ray photon, or minimum wavelength of X-ray.

EXAMPLE

Electrons of energy 80 keV are incident on a metal surface. What is the minimum wavelength of X-ray that can be produced?

Answer

$$eV = hf$$

$$= \frac{hc}{\lambda_{\text{min}}}$$

So

$$\lambda_{min} = \frac{hc}{eV}$$

$$= \frac{6.63 \times 10^{-34}\ \text{J s} \times 3.00 \times 10^{8}\ \text{m s}^{-1}}{1.60 \times 10^{-19}\ \text{C} \times 80 \times 10^{3}\ \text{V}}$$

$$= 1.50 \times 10^{-11}\ \text{m}$$

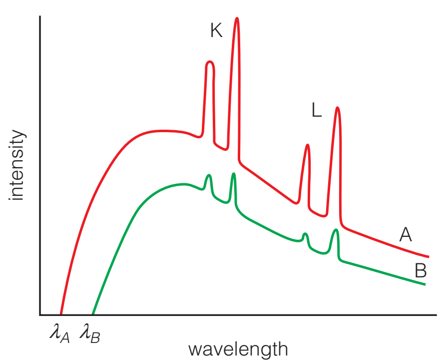


Figure 31.49

Figure 31.49 shows typical X-ray spectra produced when electrons of different energies are fired at the same metal.

- For both energies the characteristic peaks associated with K and L transitions occur at the same wavelengths.
- Electrons of high energy produce greater intensities of X-rays, and the higher energy electrons produce a minimum wavelength which is smaller than the minimum wavelength produced by the lower energy electrons.

Rotating-anode X-ray tube

Figure 31.50 shows the principle behind a rotating anode X-ray tube.

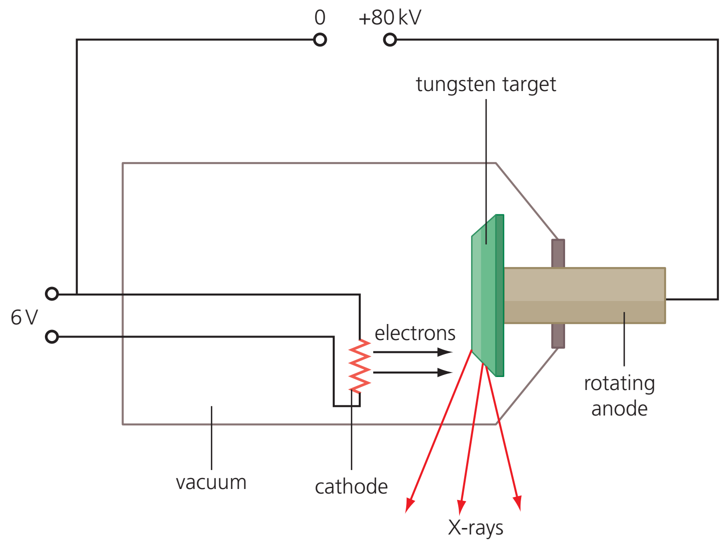


Figure 31.50

A heated cathode produces electrons by thermionic emission. A large potential difference accelerates these electrons so that they strike the bevelled edge of the anode. As a result of the electron collisions with the tungsten target in the anode, X-rays are emitted. However, most of the kinetic energy carried by the electrons is dissipated due to low energy collisions in the target which transfer thermal energy to the target. By having a rotating anode, the thermal energy produced by the electron collisions is spread more evenly. Thus the target heats up less and does not melt.

The intensity of the X-rays produced by the tube can be controlled by adjusting the accelerating potential difference, or by adjusting the temperature of the cathode. A hotter cathode produces more electrons per second, so the current carried by the electrons may be increased. A larger accelerating potential difference between anode and cathode increases the kinetic energy of the electrons and therefore the intensity of the X-rays.

TEST YOURSELF

42 What is meant by these two terms:
- **i)** a continuous X-ray spectrum;
- **ii)** a characteristic X-ray spectrum?

43 **a)** Explain why electrons striking a metal target produce a continuous spectrum of X-rays.
- **b)** What determines the shortest wavelength of X-rays produced by an electron beam?
- **c)** Explain why the 'characteristic spectrum' of a source of X-rays, depends on the metal target.

44 Describe and explain the operation of a rotating-anode X-ray tube.

45 In an X-ray tube electrons are accelerated through a p.d. of 40 kV.

a) Calculate the shortest wavelength of X-rays which the tube produces.

b) State the shortest wavelength of X-rays which are produced by the tube when the electrons are accelerated through a p.d. of 120 kV.

46 Figure 31.51 shows three electron energy levels in a metal. Calculate the wavelengths of three characteristic X-rays produced by this element.

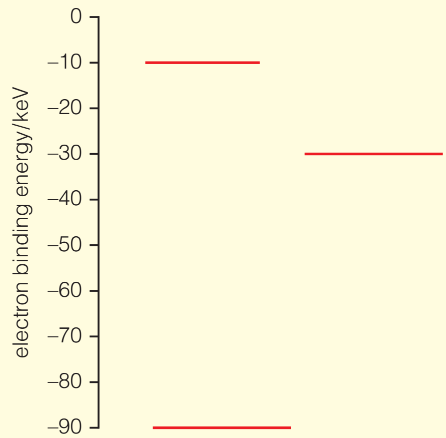


Figure 31.51

Radiographic image sharpness and contrast

One common use of X-rays is to investigate whether a bone has been broken. The principle behind this is quite straightforward. The intensity of X-rays reduces (or **attenuates**) as it passes through a material. However, the degree of attenuation is much greater when X-rays enter bone, whereas the attenuation is much less when X-rays pass through flesh. Therefore, when an ‘X-ray’ is taken of an arm, the bones cast a deeper shadow than the flesh. In this way we can ‘look’ inside the arm and inspect the bone.

It is very difficult to focus X-rays, so an image is taken by directing X-rays at the patient and placing a flat detector (or film) behind. It is important to get as sharp an image as possible so that the bone can be seen clearly. Producing a sharp X-ray image is similar to using a light source to produce a sharp shadow. In Figure 31.52 you can see that a point source of light produces a sharp shadow, whereas an extended source produces a fuzzy shadow. The dark area in the centre is called the umbra – the region where no light reaches from the source. Round the edge of the shadow is the penumbra where light reaches from part of the source.

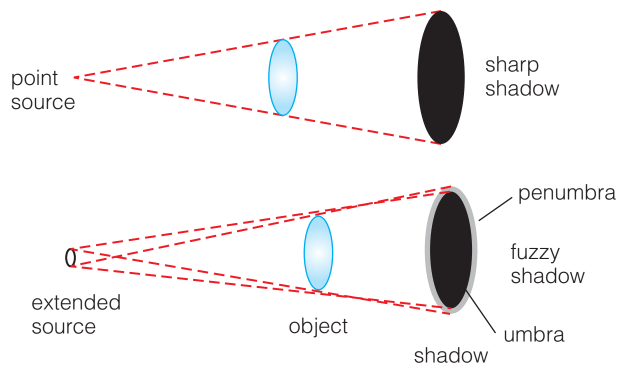


Figure 31.52

When an X-ray image is taken of a bone, it is important that the penumbra is small, so that the greatest possible clarity is achieved. However, in taking X-ray images, there is always a compromise between image brightness and image clarity.

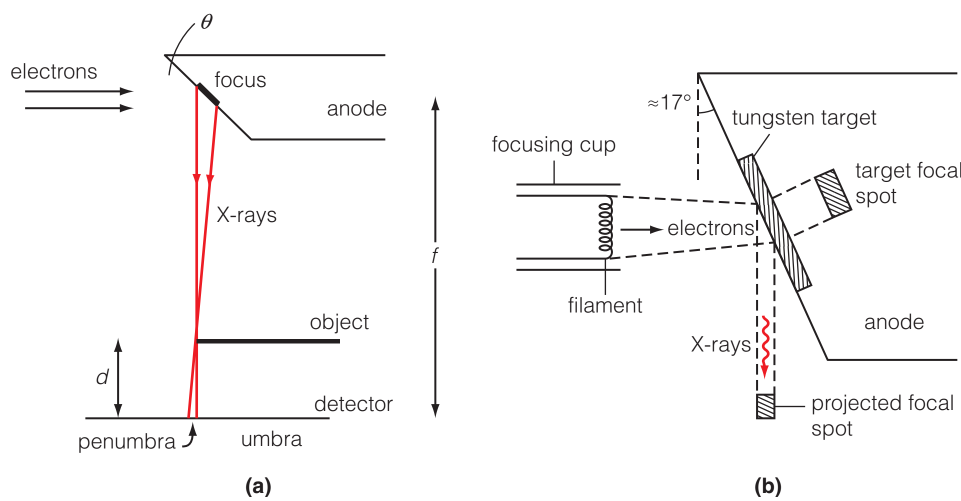


Figure 31.53 Electrons are incident on the focal spot in the tungsten target. By inclining the target at a sharp angle (usually 17°) the X-rays appear to come from a smaller focal spot, or apparent focus.

Figure 31.53(a) illustrates how the penumbra of the X-ray image can be reduced by choosing a favourable geometrical arrangement. A sharp beam of X-rays is produced when the anode is shaped so that the angle θ is about 17^{o}. Then, for taking an X-ray of a limb, the distance between the anode and screen, f, is about 1 m, and the distance between the patient and detector is a few cm. In practice, the focus has a diameter of a few mm, so there is always some blurring of the image. This is called geometric unsharpness.

Length of exposure and clarity

In order for a clear image to be produced, the X-ray detector (or film) must receive sufficient exposure for a clear contrast to be seen between different parts of the body.

- It is relatively easy to produce clear X-rays of bones or teeth, because the optical density of bones and teeth is high. This means that these materials absorb or attenuate the X-ray beam much more effectively than the surrounding body tissue. Also, a foot, for example, can be immobilised: the patient can sit still for a few seconds. The advantage of using a longer exposure is that the intensity of the X-rays and the electron beam, which is incident on the anode, can be reduced. Then a smaller focus on the anode can be used, because the heating of the anode due to the electron beam is less. The advantage of the smaller focus is that the X-rays emerge from a smaller source, and the sharpness of the image is increased.
- Taking an X-ray image of the stomach is more difficult. The contrast between the optical density of the body tissues is lower, and the stomach has involuntary movements. Such movement produces further blurring of the image which is known as movement unsharpness. It is not possible to sharpen the image by using an intensive beam of X-rays on a small focus, as the anode would melt.

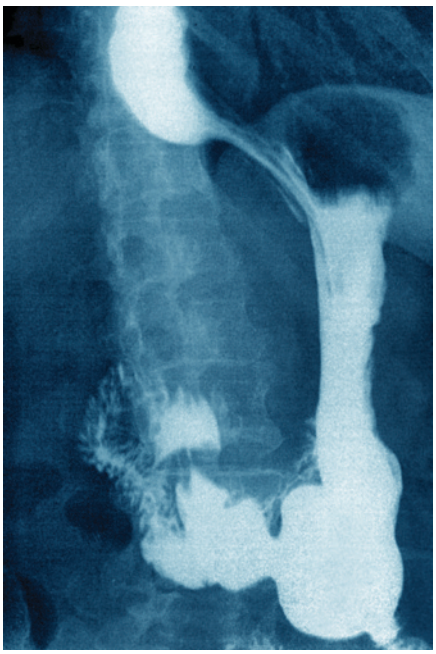


Figure 31.54 Barium sulfate is **opaque** to X-rays. Thus the barium meal allows a good contrast to be seen between the inside of the stomach and the surrounding body tissues.

Contrast enhancement

One way for an X-ray image to be sharpened is to use a contrasting medium. A barium meal is a diagnostic test used to detect abnormalities of the oesophagus, stomach, intestine and bowel. A patient swallows the contrasting medium, barium sulfate, which is **opaque** to X-rays. The barium sulfate coats the lining of the digestive tract, allowing accurate X-ray imaging. The images of the abdomen may be viewed in real time, as the barium meal passes through the digestive system.

An object which is **opaque** to X-rays does not allow X-rays to pass through it.

TEST YOURSELF

47 In the context of X-ray imaging, explain the terms geometric and movement unsharpness.

48 **a)** What is meant by the focus of the anode in an X-ray tube?
b) Explain why a small focus produces a sharper X-ray image. Illustrate your answer with a diagram.
c) Discuss the factors that determine the size of the focus.

49 **a)** Write a paragraph to explain what adjustments a radiographer would make to the size of the focus in an X-ray tube, and the time of exposure, to take an image of:
i) a bone in a hand
ii) the stomach.
b) What is meant by the contrast of an X-ray image? How can the contrast between body tissues be improved?

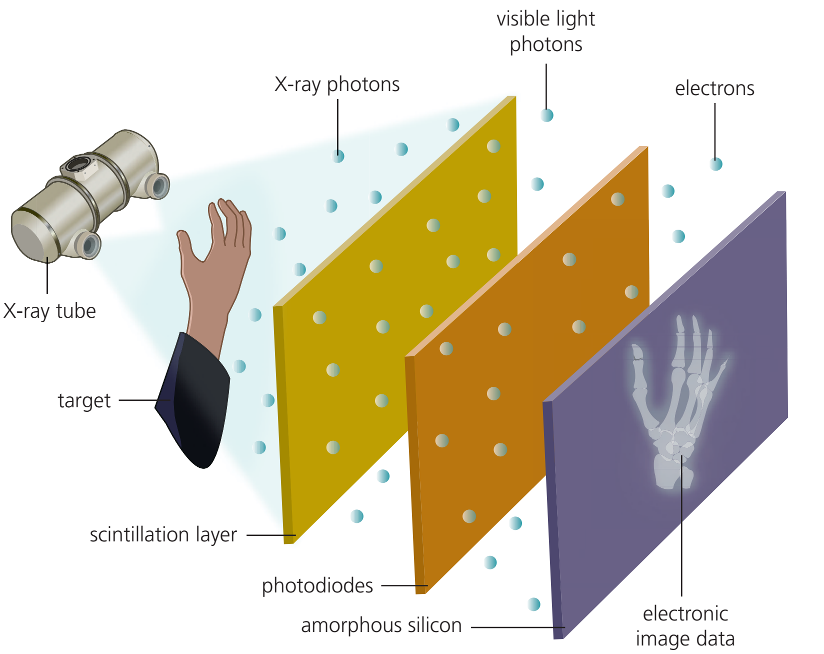


Figure 31.55 This diagram shows the main steps in producing an electronic image using X-rays. The photodiodes produce electrical signals in wires which are then processed to form an image.

Image detection

Figure 31.55 shows the simple geometry of taking an X-ray image of a patient: an X-ray tube is placed about 1 m away from a patient, and X-rays form an image on a flat detector, which is placed a few centimetres behind the object. Until about 2005, the detector used in hospitals or by dentists was a photographic film. Such a film produced very good images of the body, but the main disadvantage of a film was the time spent on developing it. A patient would have to wait for up to an hour in an accident and emergency department while the film was developed. And if a dentist took an X-ray of a tooth, the patient might have made a second appointment to return at a later date. Now, with the help of new technology, X-ray images are stored electronically within seconds.

Now X-ray flat panel (FTP) detectors include an X-ray scintillator, photodiode pixels and an electronic scanner which allows the image to be displayed in real time on a screen, or to be stored on a disc or in the computer memory. Figures 31.55 and 31.56 show the principle.

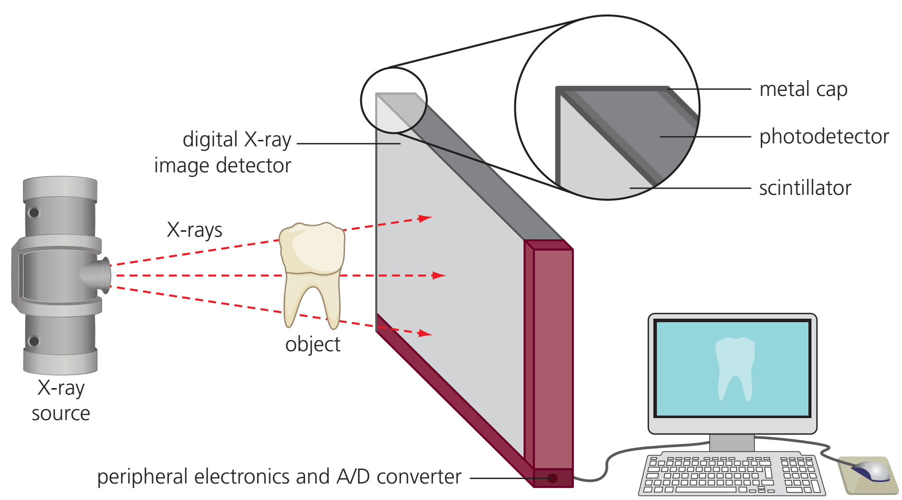


Figure 31.56

Scintillation is a flash of visible light produced in a transparent material by the passage of a particle (an electron, alpha particle, ion or high-energy photon).

- An X-ray tube directs X-rays towards an object (the hand in Figure 31.56). An image of the bones and flesh in the hand is formed on the **scintillation** layer behind.
- The high-energy X-ray photons are absorbed by the scintillation layer behind the hand. As a result of the absorption of energy, electrons in the scintillator are excited into higher energy levels. These electrons then fall back into lower levels emitting visible photons of light. So when the X-ray photon is absorbed, the material produces a flash of light: this is called scintillation.
- Visible photons which are emitted by the scintillation layer are then absorbed by photodiodes. The photodiodes generate a p.d., which is processed and transmitted to an image panel or stored on a computer.

Figure 31.57 X-ray detectors can be made very small. Here a small detector is placed inside a patient's mouth and irradiated by X-rays from outside. Within seconds the patient and the dentist can view the image on a screen and discuss what to do next.

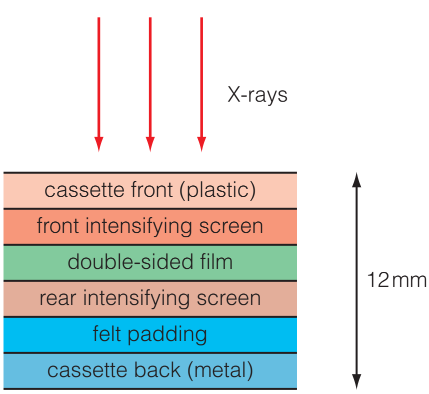


Figure 31.58

Photographic detection

When a patient is exposed to X-rays, it is important to reduce the time of exposure in order to minimise the risk of damage being done by ionising radiation, and also to reduce the time a patient must remain still.

When a photographic film is used to form an X-ray image, intensifying screens are used to shorten the exposure time. In the simple intensifying cassette shown in Figure 31.58, two fluorescent screens are placed in close contact with two faces of a double-sided light-sensitive X-ray film. X-rays penetrate the film and the two fluorescent screens. The X-rays produce an image on the film directly, but 95% of the activation of the film derives from the light emitted on either side by the fluorescent screens. Thus the exposure time can be reduced by a factor of about 20.

The image intensifier

The image intensifier described below was first used in hospitals in 1948. It is still in use in some parts of the world, but it has now been replaced by flat panel detectors in UK hospitals.

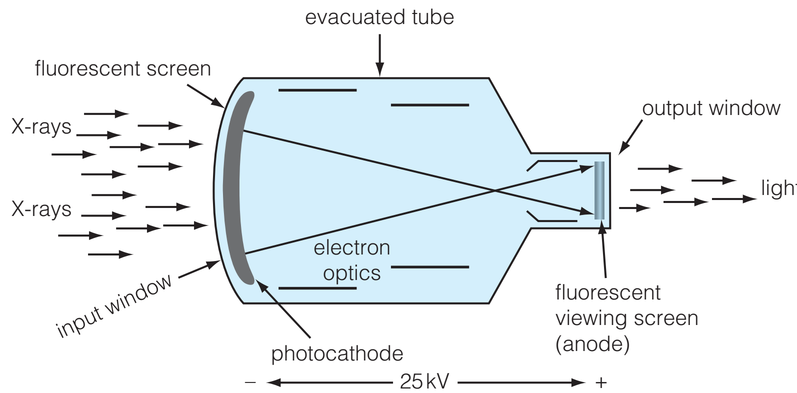


Figure 31.59

The function of the image intensifier is illustrated in Figure 31.59. X-rays from the patient arrive at the input window and they strike a fluorescent screen. Light from the screen is blocked by a very thin layer of aluminium, but electrons are able to pass through this layer. The pattern of electrons which appears on the inside of the photocathode replicates the pattern of X-rays on the input window. These electrons are accelerated by a potential difference of 25 kV so that they strike the output fluorescent screen, which acts as the anode of this electron gun. A system of electric plates acts as an electron lens, so that an image is formed of the electron pattern on the photocathode. The image on the anode is much brighter because the electrons are concentrated into a smaller area; the electrons are very energetic because they have been accelerated through a p.d. of 25 kV. The image can be increased in brightness 5000 times by this method.

The image intensifier allows a surgeon to view the body in real time, using a much lower dose of X-rays – thus minimising the danger of ionising radiation to the patient. For example, a surgeon can use the image intensifier as they pass a catheter into the body along a vein to carry out minimally invasive heart surgery.

TEST YOURSELF

50 a) Write a paragraph to explain how a flat panel detector is used to produce a digital X-ray image.
b) What advantages are there in digital images over photographic images?

51 a) Explain how intensifying screens are used to enhance X-ray photographic images.
b) In the text, it is stated that an intensifying screen reduces exposure times by about a factor of 20. Explain where this number comes from.
c) Why is it important to reduce exposure times when X-ray images are taken?

52 a) Describe how an image intensifier works to produce a bright image of X-rays.
b) For what sort of procedures do doctors use an image intensifier?

Absorption of X-rays

In common with all types of electromagnetic waves, X-rays are able to travel an infinite distance in space. However, the intensity of a beam of X-rays is attenuated as it passes through matter. As X-rays pass through a medium, they interact with electrons or the nucleus of an atom. When a photon interacts with matter, it can lose energy by exciting electrons or the nucleus. An X-ray photon is more likely to interact with matter the further it travels through a particular material.

The intensity of a beam of X-rays decreases exponentially with the distance it travels through a material. The intensity of a beam of radiation can be calculated using the equation:

$$I = I_0 e^{-\mu x}$$

where I is the transmitted intensity after passing through a length x of material.

I_0 is the incident intensity

μ is the linear attenuation coefficient in m^{-1}.

The intensity of a beam of radiation can be measured in W m^{-2}. However, for most purposes it is sufficient to compare the relative intensities using the ratio I/I_0.

The value of μ depends on the energy of the incident photons; μ also depends on the density of the material.

A denser material either has atoms with more massive nuclei, or more nuclei per unit volume. An X-ray photon is more likely to interact with a nucleus if there are more of them along its path.

The attenuation of X-rays is also described using the mass attenuation coefficient:

$$\mu_m = \frac{\mu}{\rho}$$

where μ_m is the mass attenuation coefficient $m^2\,kg^{-1}$

μ is the linear attenuation coefficient in m^{-1}

ρ is the density of the material in kg m^{-3}

The difference in density between body tissues is important, as it allows X-rays to distinguish between fat and a kidney, for example. A kidney has a higher density than fat, so X-rays are attenuated more as they pass through the kidney than they are as they pass through fat.

The exponential decay of the intensity of X-rays as they pass through a material means that it is impossible to reduce the intensity of X-rays to zero, no matter how thick the material is. However, we can describe the effectiveness of the absorption of X-rays using the half-value thickness $x_{\frac{1}{2}}$, which is the thickness of a material that reduces the intensity of an X-ray beam to half its initial value.

Half-value thickness and linear attenuation coefficient

The half-value thickness, $x_{\frac{1}{2}}$, and linear attenuation coefficient μ, may be linked as follows.

$$I = I_0 e^{-\mu x}$$

Since $I = \dfrac{I_0}{2}$ when $x = x_{\frac{1}{2}}$

we get:

$$\frac{I_0}{2} = I_0 e^{-\mu x_{\frac{1}{2}}}$$

$$\frac{1}{2} = e^{-\mu x_{\frac{1}{2}}}$$

and

$$\ln\frac{1}{2} = -\mu x_{\frac{1}{2}}$$

or

$$\ln 2 = \mu x_{\frac{1}{2}}$$

Therefore

$$x_{\frac{1}{2}} = \frac{\ln 2}{\mu}$$

Calculations using attenuation coefficients

Table 31.3 shows linear attenuation coefficients for various body tissues for three different X-ray photon energies.

Table 31.3

Photon energy/keV	Bone μ/m^{-1}	Muscle μ/m^{-1}	Fat μ/m^{-1}	Water μ/m^{-1}
50	74.2	24.0	20.5	22.7
80	39.0	19.3	16.7	18.7
100	32.6	17.9	15.5	17.1

The densities of these materials are shown below.

Bone	$1750\ \mathrm{kg\ m^{-3}}$
Muscle	$1060\ \mathrm{kg\ m^{-3}}$
Fat	$960\ \mathrm{kg\ m^{-3}}$
Water	$1000\ \mathrm{kg\ m^{-3}}$

Note that the lower the photon energy, the higher the linear attenuation coefficient; a lower energy photon is more easily stopped than a higher energy photon.

EXAMPLE

1 Calculate the mass attenuation coefficient for 80 keV X-rays in bone.

Answer

$$\mu_m = \frac{\mu}{\rho}$$

$$= \frac{39.0\ \mathrm{m}^{-1}}{1750\ \mathrm{kg\ m}^3}$$

$$= 0.0223\ \mathrm{m}^2\ \mathrm{kg}^{-1}$$

2 Calculate the half-value thickness of water for 100 keV X-rays.

Answer

$$x_{\frac{1}{2}} = \frac{\ln 2}{\mu}$$

$$= \frac{0.693}{17.1}$$

$$= 0.041\ \mathrm{m}\ \text{or}\ 4.1\ \mathrm{cm}$$

3 Calculate the intensity reduction when a beam of 100 keV X-rays passes through 6 cm of muscle.

Answer

$$I = I_0 e^{-\mu x}$$

$$\frac{I}{I_0} = e^{-\mu x}$$

$$= e^{-17.9 \times 0.06}$$

$$= e^{-1.074}$$

$$= 0.34$$

So the intensity of X-rays is reduced to about a third of the initial value.

TEST YOURSELF

53 a) Explain the term linear attenuation coefficient, μ.

b) Does μ increase or decrease as the energy of X-rays increases for a particular material?

54 a) Calculate the mass attenuation coefficient of muscle for 80 keV X-rays.

b) Calculate the half-value thickness of bone for 100 keV X-rays.

c) Calculate the reduction in intensity of a beam of 50 keV X-rays passing through 5 cm of muscle.

d) Calculate the reduction in intensity of 50 keV X-rays passing through 5 cm of fat. Explain why it is possible for a radiographer to distinguish between fat and muscle.

e) Calculate the reduction in intensity of 50 keV X-rays passing through 5 cm of bone. Explain why bones are shown clearly in X-ray images.

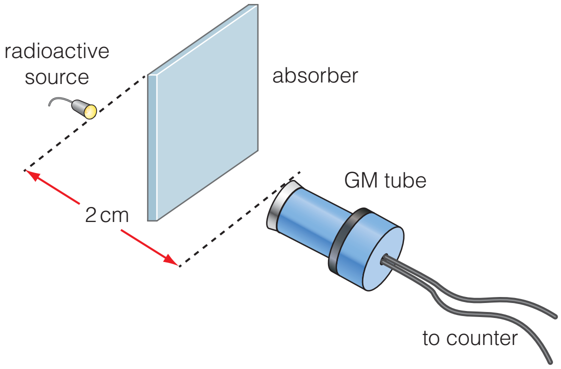


Figure 31.60

Optional experiment

Figure 31.60 shows an experimental arrangement that you can use to test whether the intensity of gamma radiation decreases exponentially as the thickness of lead absorbers increased.

You can do this for yourself in the laboratory or use the data provided.

The Geiger–Müller tube is used to record the activity of a cobalt-60 source, which emits gamma rays and beta particles. The activity is recorded over a 10 second period for different thicknesses of lead absorber.

The following results were recorded.

Table 31.4

Thickness of lead absorber/mm	2	3	5	8	10	15	20
Recorded activity over a 10 second period	2260	2130	1880	1560	1380	1010	740

Experimental note: now caesium-137 sources are also available. which emit just gamma rays.

If the intensity of the gamma rays reduces exponentially with thickness the following relationship holds:

$$I = I_0 e^{-\mu x}$$

or $$\frac{I}{I_0} = e^{-\mu x}$$

Therefore $\ln\left(\frac{I}{I_0}\right) = -\mu x$

and $\ln I - \ln I_0 = -\mu x$

or $\ln I = \ln I_0 - \mu x$

So if we plot a graph of: ln (count) against the thickness (in m) the gradient of the graph is $-\mu$.

ACTIVITY

1 Plot a suitable graph to determine μ.
2 Calculate μ_m for lead. The density of lead is 11 300 kg m^{-3}.
3 Calculate the half-value thickness of lead for cobalt-60 gamma rays.
4 Explain why the experimenter did not record a value for the count with no absorber present. [Hint: what other radiation is emitted by cobalt-60?]

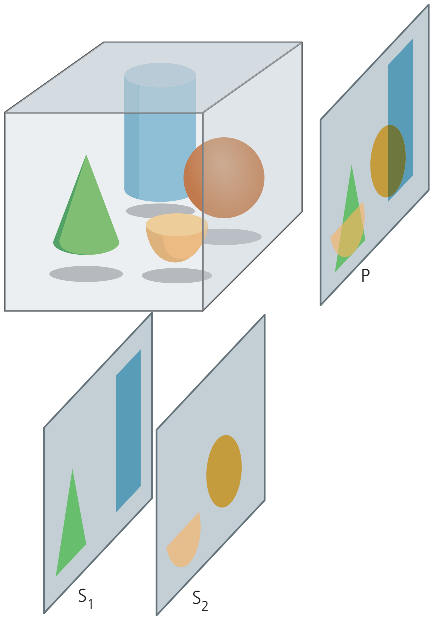


Figure 31.61 The computer builds up many images at different depths.

CT scanner

A CT scan is the shorthand name we give to X-ray computerised tomography (X-ray CT) or computerised axial tomography (CAT scan). The word tomography refers to imaging by sections using any type of penetrating wave.

In a CT, scan radiographers make a sectional image through the body by moving an X-ray tube and detectors in opposite directions during the exposure. This method of investigation allows a thin slice of the body to be brought into sharp focus with other planes appearing blurred. Throughout a CT scan, a narrow beam of **monochromatic** X-rays is directed at different planes of the body, and an array of detectors surrounding the patient builds up images of sections of the organ under investigation. The information from the detectors goes to a computer which turns electrical signals into the detailed image we see. Figure 31.61 shows the principle of a CT scan.

With the usual X-ray technique an image similar to Figure 31.46 is produced: a two-dimensional image of a three-dimensional object. A CT scan allows slices (S_1 and S_2) to be produced, so that a clear focus is seen for each section of the body.

Figure 31.62 shows a CT scan through a patient's head. Many slices of the head are seen allowing doctors to make a detailed diagnosis of any illness.

Monochromatic means of one colour, but in the context of X-rays it means of one wavelength.

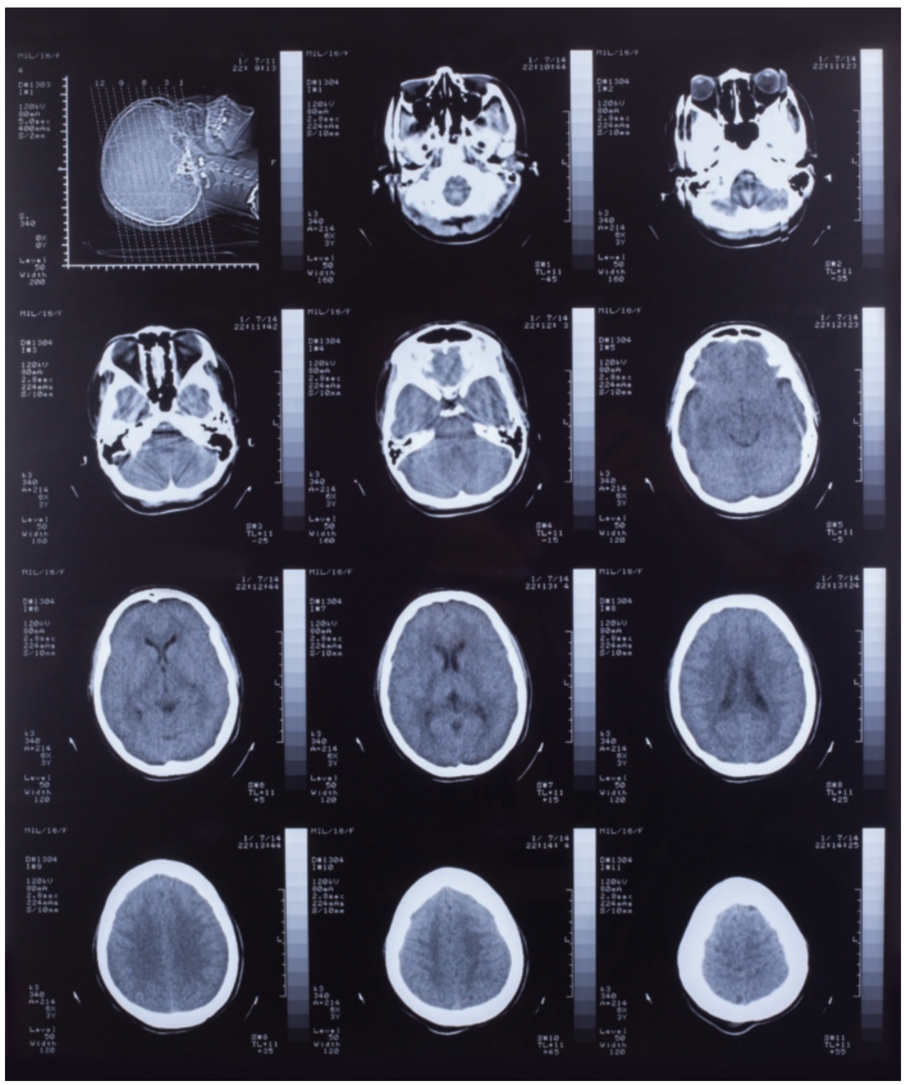


Figure 31.62 Computerised tomography of the human brain, from the base of the skull to the top.

Advantages of CT scans

There are three advantages that CT has over conventional two-dimensional radiography.

- CT completely eliminates superimposition of images of structures outside the area of interest.
- CT scans produce high resolution and high contrast images, so that differences between tissues that differ in density by 1% can be detected. This is very important for the diagnosis of cancer.
- A CT scan allows images to be formed from any of the three planes of the body – see Figure 31.63.

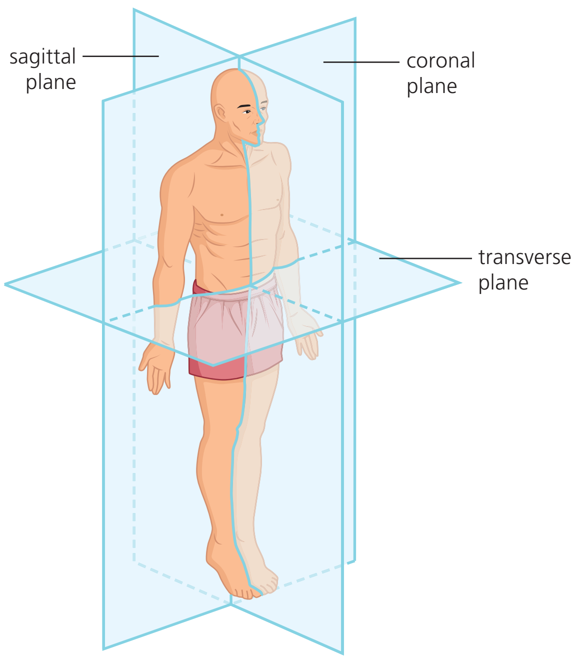


Figure 31.63

Disadvantages of CT scans

There are two main disadvantages: cost and safety.

- CT scanners are costly to build and take between 15 to 60 minutes for a scan to be completed – so they are costly on manpower too.
- A CT scan exposes a patient to a relatively high radiation dose which can damage body cells (see below). Such cell damage can lead to cancer. The radiation dose received from a CT scan can be equivalent to 100 to 1000 times the dose for a conventional chest X-ray. Although the dose from a CT scan is small, it does carry a risk of inducing a cancer, so it must only be used sparingly. Doctors must weigh up carefully the potential benefit to a patient against the risk. For example, a doctor would only decide to expose a 1-year-old child to a CT scan if a serious life-shortening illness is under investigation. Exposure to radiation might cause illness many years later. By contrast, the risk posed by a CT scan to a patient aged 65 or over is relatively small. If a patient of that age is suffering from cancer already, a CT scan can provide important information about how to treat it.

Radiation damage

You do not need to know any detail about radiation doses, but this brief section is included to provide some additional background information. Radiation can be ionising or non-ionising. Ionising radiation can cause damage to our bodies. Common types of ionising radiation include alpha and beta particles, other heavy ions, neutrons, gamma rays and X-rays. Damage can be caused in two ways.

- Direct damage is caused by a particle colliding directly with a cell in the body. An alpha particle, for example, behaves like a miniature bullet and destroys body tissue.
- Indirect damage is caused by ionisation. Radiation produces ions by removing electrons from atoms and molecules. Such ions can produce strong acids in our bodies such as H_3O^+. These acids can destroy cells in our bodies or cause mutations to our genes.

Both types of damage can lead to cancer.

Measuring the dose

The heavier type of radiation, such as alpha particles, cause the greatest damage. The alpha particle typically has a lot of energy and because it travels relatively slowly, it transfers its energy over a short distance. Thus it causes intense localised damage. X-rays and gamma rays penetrate further into flesh, and spread their energy over a greater distance.

When we assess the dangers of radiation, we take into account now much we have been exposed to. We measure radiation dose in sieverts, Sv. Doses are often quoted in thousandths of sieverts – millisieverts, mSv.

The dose takes into account the quality of the radiation. For example, an alpha particle delivers 20 times the dose of a gamma ray with the same energy. Table 31.5 provides some examples of radiation doses which are used in medicine, including a comparison with the annual background dose we are exposed to in the UK.

Table 31.5

Dose/mSv	Source of dose	Equivalence with UK background radiation	Effect of dose
0.0005	dental X-ray	18 hours	low risk
0.02	chest X-ray	3 days	low risk
2.0	head CT	300 days	low risk
2.4	yearly background radiation	1 year	low risk
10.0	chest, abdomen and pelvis CT	4.2 years	low risk
1000	highly targeted dose of radiotherapy	420 years	high risk, but balanced by a short-term cure of cancer.

TEST YOURSELF

55 (G) a) State what is meant by the phrase CT scan.
b) Give a brief outline of how a CT scan is done.
c) i) What are the advantages of a CT scan over a conventional X-ray?
ii) What are the disadvantages of a CT scan over a conventional X-ray?

56 (G) a) What is meant by the term radiation dose?
b) With reference to Table 31.5, compare the risks posed by: a dental X-ray; a CT scan; targeted radiation therapy.

Radionuclide imaging and therapy

Imaging techniques

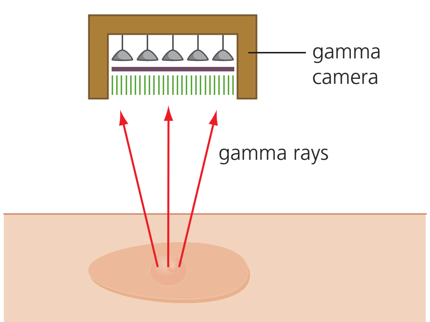


Figure 31.64

Radionuclines are commonly used in medicine as a diagnostic tool. The principle of radionuclide imaging is shown in Figure 31.64.

A radionuclide is absorbed by a particular organ that is to be investigated. Then gamma rays which are emitted by the radionuclide allow medical physicists to form an image of the organ using a gamma camera (see Figure 31.71 and page 57). A range of gamma-emitting **radioisotopes** is used, depending on the investigation which is to be undertaken. However, the following broad principles apply to **tracers**.

- The isotope used must have a chemical affinity for the organ or cells to be investigated. Unless the isotope is naturally absorbed in the right part of the body, no investigation can take place.
- The isotope should emit low energy gamma rays. This radiation can pass through the body and be detected outside by the gamma camera,

A **radioisotope** is an isotope that is radioactive.

A **tracer** is a radioisotope that is absorbed by the body and therefore allows its radioactive emissions to help form an image of an organ.

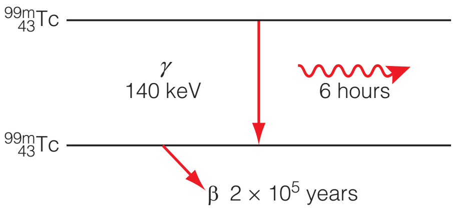


Figure 31.65 The decay of $^{99m}_{43}Tc$

whereas beta or alpha radiation would not escape from the body. Gamma radiation is ionising and so can cause damage to our body; by using low energy gamma rays, such radiation damage is kept to a minimum.

- The isotope should have a short half-life: of a few hours or a few days. The advantage of the short half-life is that the total radiation dose absorbed by the patient is kept to a minimum. An isotope with a short half-life produces a high activity for a short time. The high activity allows the radiation to be detected and thus form an image of an organ under investigation, but the isotope soon decays once the investigation has been completed.

Technetium–99m

Technetium-99m, $^{99m}_{43}Tc$, is a metastable state of technetium-99. A metastable state is an excited state of a nucleus. The nucleus decays to its ground state by emitting a gamma ray; this is a similar process to an excited electron returning to its ground state by emitting a photon of visible or ultraviolet light.

Technetium-99m is used, world-wide, in tens of millions of diagnostic procedures annually, making it the most commonly used medical radioisotope. Technetium-99m is well suited to this role as it emits low energy gamma rays (140 keV), which are of similar wavelength and energy to X-rays used by conventional diagnostic equipment. The isotope has a half-life of 6 h, which means that the patient is exposed to a low radiation dose. The typical dose received by a patient undergoing an investigative procedure with technetium-99m is about 10 mSv, which is about 5 times the annual dose due to background radiation in the UK.

A **radiopharmaceutical** is a drug used to diagnose a disease that relies on the radioactive emissions of the drug for the purposes of image formation.

There are about 30 **radiopharmaceuticals** based on $^{99m}_{43}Tc$. Uses of these compounds include investigation of the brain, thyroid, lungs, liver, gall bladder, kidneys, skeleton, blood and tumours. For example, one radiopharmaceutical allows blood flow round the brain to be investigated and another labels white blood cells, so that the site of an infection can be determined.

Iodine-131

Iodine-131 ($^{131}_{53}I$) decays with a half-life of 8.0 days with both beta-minus and gamma ray emissions. The decay takes place in two steps: first to metastable xenon-131, and then to stable xenon-131 by the emission of a gamma ray.

$$^{131}_{53}\mathrm{I} \rightarrow {}^{131m}_{54}\mathrm{Xe} + {}^{0}_{-1}\mathrm{e} + \bar{\nu}_e$$

$$^{131m}_{54}\mathrm{Xe} \rightarrow {}^{131}_{54}\mathrm{Xe} + \gamma$$

The maximum energy of the electrons emitted in the beta decay process is about 600 keV, and the gamma ray emitted has an energy of 360 keV.

Due to the emission of the high energy beta particle, iodine-131 is not a suitable isotope to be used first for diagnostic purposes. The beta particles have a tissue penetration of between 0.6 mm and 2.0 mm, and therefore cause significant damage to any tissue that they strike. If an isotope of iodine

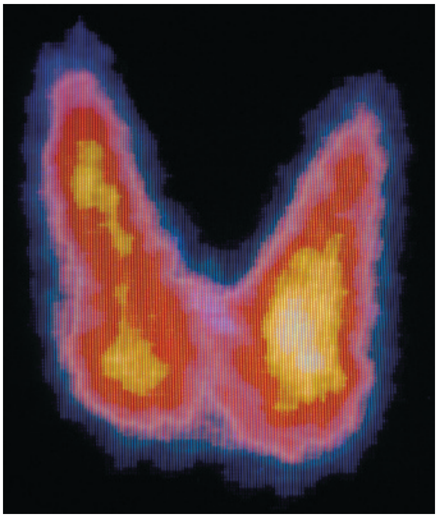


Figure 31.66 This scan of the thyroid taken with a gamma camera reveals an abnormality in the gland.

is needed for diagnostic purposes only, a more suitable isotope is iodine-123, which emits gamma rays of energy 160 keV with a half-life of 13.0 hours.

Iodine is readily absorbed by the thyroid gland. Thus iodine-131 has two main uses.

- In the case of an overactive thyroid gland (hyperthyroidism) a dose of radioactive iodine destroys part of the thyroid gland, so that the thyroid gland then functions at a normal level.
- In the case of thyroid cancer, it is usual for the thyroid gland to be removed. Then a large dose of iodine-131 is used to destroy any remaining thyroid tissue in the area. The radioactive iodine also circulates throughout the body and can be picked up by any thyroid cells which have moved round the body.

Although iodine-131 is primarily used to treat conditions, the fact that the isotope emits gamma rays allows it also to act as a tracer. By viewing the emitted gamma rays, it is possible to monitor how effective the treatment has been.

Indium-111

Indium-111 is a radioisotope which decays to the stable nucleus cadmium-111 by electron capture (see page 478 of the Student's Book) with a half-life of 68 hours.

$$^{111}_{49}\text{In} + ^{0}_{-1}\text{e} \rightarrow ^{111}_{48}\text{Cd} + \nu + \gamma$$

In the process of decay, the nucleus emits gamma rays of energy 170 keV and 250 keV.

Indium-111 is a very expensive radioisotope, which is used to label antibodies, polypeptides and white blood cells. This allows the diagnosis of blood disorders and rare cancers. The short half-life of 67 hours and the emission of only gamma rays makes indium-111 an ideal choice for a tracer.

The molybdenum–technetium generator

One of the problems with using radioisotopes with short half-lives is that the material cannot readily be stored or transported. The most commonly used tracer, technetium-99m, has a half-life of only 6 hours. Thus if you stored a sample in a hospital ready for use the next day, about 94% of it would have decayed. The solution is to use a generator from which the isotope can be extracted.

Molybdenum-99 ($^{99}_{42}\text{Mo}$) is the parent nucleus of technetium-99. The isotope $^{99}_{42}\text{Mo}$ is produced in nuclear reactors and it has a half-life of 66 hours, which makes it far more convenient to transport to hospitals.

A hospital is supplied with a molybdenum–technetium generator. The isotope $^{99}_{42}\text{Mo}$ then decays to $^{99m}_{42}\text{Tc}$. The two elements are separated by a chemical process, which allows a supply of technetium-99m to be produced. Since molybdenum-99 has a half-life of 66 h, it means that a weekly supply of the radioisotope allows the hospital to produce technetium-99m on a daily basis ready for use.

Positron emission tomography (PET)

Positron emission tomography (PET) is an expensive, but high resolution, imaging process which allows highly detailed images of the body to be produced. PET is used for monitoring the treatment of cancers such as lymphoma and lung cancer. PET scans are also most effective at monitoring brain activity and blood flow to the brain.

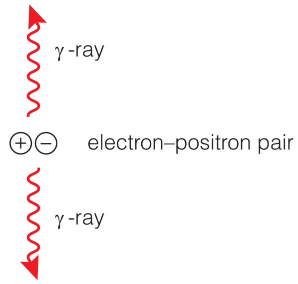


Figure 31.67

PET requires a radioisotope which decays by positron emission to be absorbed by the body. The most commonly used tracer is fluorodeoxyglucose, which contains the isotope fluorine-18. ${}^{18}_{9}F$ decays by positron decay with a half-life of 110 minutes.

$${}^{18}_{9}F \rightarrow {}^{18}_{8}O + {}^{0}_{1}e + \nu$$

As soon as a positron has slowed down, it encounters an electron, when the electron–positron pair annihilate to produce a pair of γ-rays with energy 512 keV each. Due to the conservation of momentum, these two γ-rays move off in opposite directions (Figure 31.67).

The production of pairs of gamma photons allows an image of the body to be constructed using computers. Figure 31.68 shows the operation of a PET scanner. Here the patient's brain is being examined. The head is surrounded by a three-dimensional array of detectors, which detect pairs of gamma rays. The computer and detectors use the information recorded to determine the origin of a pair of gamma rays.

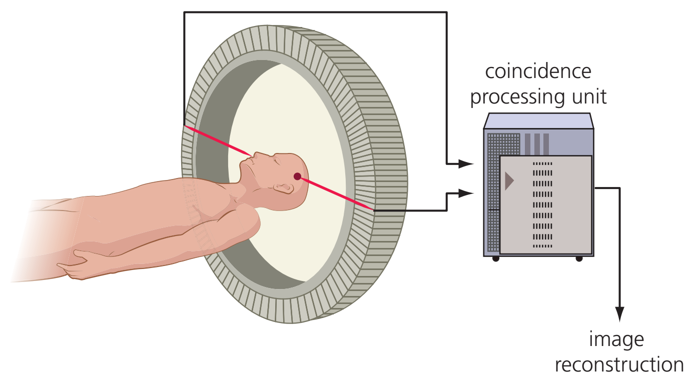


Figure 31.68 The annihilation of a position–electron pair produces a pair of γ-rays (shown in red).

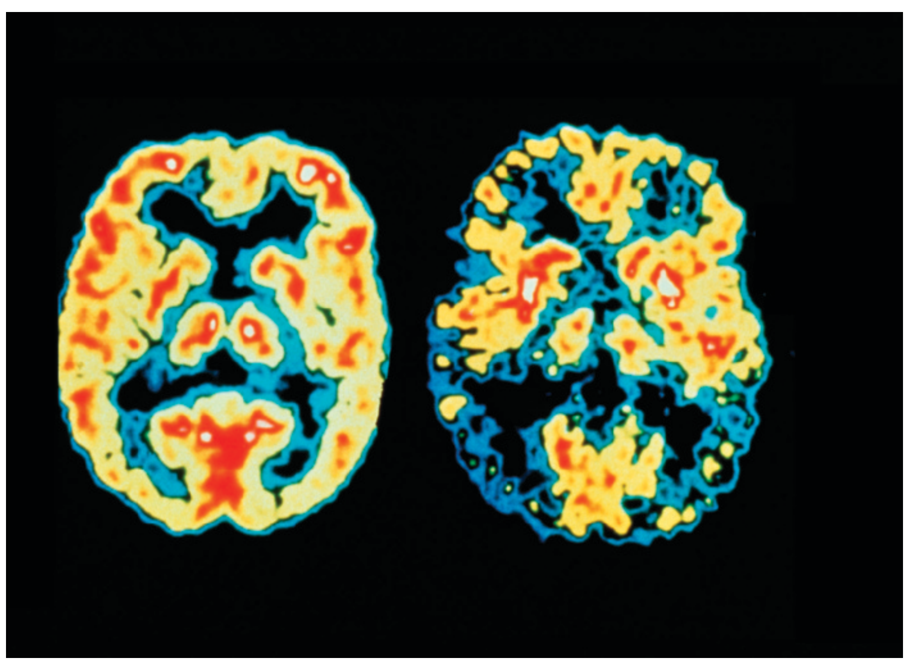


Figure 31.69 In this PET scan of a human brain the red colour indicates areas of higher brain activity.

TEST YOURSELF

57 Give three properties which are essential for a radioactive tracer to be used safely and effectively.

58 **a)** Explain why technetium-99m is used commonly as a radioactive tracer.

b) Explain why iodine-123 is used in preference to iodine-131 as a radioactive tracer.

c) For what purpose is the isotope-131 used? Explain why.

59 What is a molybdenum–technetium generator? Explain how the generator works, and why it is essential for hospitals to have one delivered every week.

60 Explain the principle behind a PET scan.

61 Technetium-99m has a half-life of 6 hours. A sample of technetium-99m of mass 2 g is stored in a hospital over the weekend. How much is left of the sample after 48 hours?

62 In the text it is stated that an electron–positron pair produce two gamma rays each of 512 keV.

a) Why must two gamma rays be produced when an electron–positron pair annihilate?

b) An electron and a positron each have a mass of 9.1×10^{-31} kg. Confirm by calculation:

i) the energy of each gamma ray emitted

ii) the wavelength of each gamma ray.

Half-life

Physical half-life

As a physicist, you are already familiar with the concept of physical half-life. For example, strontium-90 has a half-life of 29 years; this means that the amount of the isotope in a sample will halve in 29 years, and there will be a quarter of the original amount of the isotope left after 58 years.

Biological half-life

The biological half-life of a substance is the time it takes for a substance (a drug for example) to lose half of its physiological activity. The activity of a drug decays exponentially in the same way as the activity of a radioactive sample decays. The body cleanses a drug out of its system through the action of the kidneys and liver, and through excretion. Morphine has a biological half-life of 2–3 hours, which is why it is an effective anaesthetic – note that a biological half-life will vary from person to person. So a patient can receive an anaesthetic and recover completely from its effects within a day.

Effective half-life

When a radioisotope is used in the body as a tracer, the amount left in the body after a period of time depends both on the physical half-life of the tracer, T_P, and the biological half-life of the tracer, T_B. To calculate the amount of a radioisotope left in the body after a particular time, you use the effective half-life of the substance, T_E.

The three half-lives are connected by the equation:

$$\frac{1}{T_E} = \frac{1}{T_P} + \frac{1}{T_B}$$

EXAMPLE

The biological half-life of technetium-99m is 1 day, and the physical half-life of technetium-99m is 6 hours.

Calculate the effective half-life of the isotope.

Answer

$$\frac{1}{T_E} = \frac{1}{T_P} + \frac{1}{T_B}$$

$$= \frac{1}{6} + \frac{1}{24}$$

$$= \frac{4}{24} + \frac{1}{24}$$

$$= \frac{5}{24}$$

$$T_E = \frac{24}{5}$$

= 4.8 hours

So the activity of a sample of ${}^{99m}_{43}Tc$ halves every 4.8 hours.

MATHS BOX

When a radioisotope is in the body, there is a biological activity and a physical activity. So the total activity is:

$$A_E = A_P + A_B$$

or

$$\lambda_E N = \lambda_P N + \lambda_B N$$

but

$$\lambda = \frac{0.693}{T}$$

So

$$\frac{0.693\ N}{T_E} = \frac{0.693\ N}{T_P} + \frac{0.693\ N}{T_B}$$

and

$$\frac{1}{T_E} = \frac{1}{T_P} + \frac{1}{T_B}$$

TEST YOURSELF

63 Define each of the terms:
- **i)** Physical half-life
- **ii)** Biological half-life
- **iii)** Effective half-life.

64 Explain why a patient, who has been treated with a radioactive tracer, is kept in isolation for a period of time. What factors determine the length of the isolation?

65 Cobalt-60 has a physical half-life of 5.3 years and a biological half-life of 10 days. Explain why a patient who has consumed some cobalt-60 will show a low activity after a few weeks.

66 Strontium-90 has a biological half-life of 50 years and a physical half-life of 29 years. Calculate its effective half-life.

67 **a)** Zinc-65 has a biological half-life of 933 days and a physical half-life of 244 days. Calculate the effective half-life of this isotope.
b) A person consumes a dose of zinc-65, which produces an activity, A, in his body. Calculate the activity in the body, in terms of A, after 1 year.

The gamma camera

The principle of a gamma camera is illustrated in Figure 31.70.

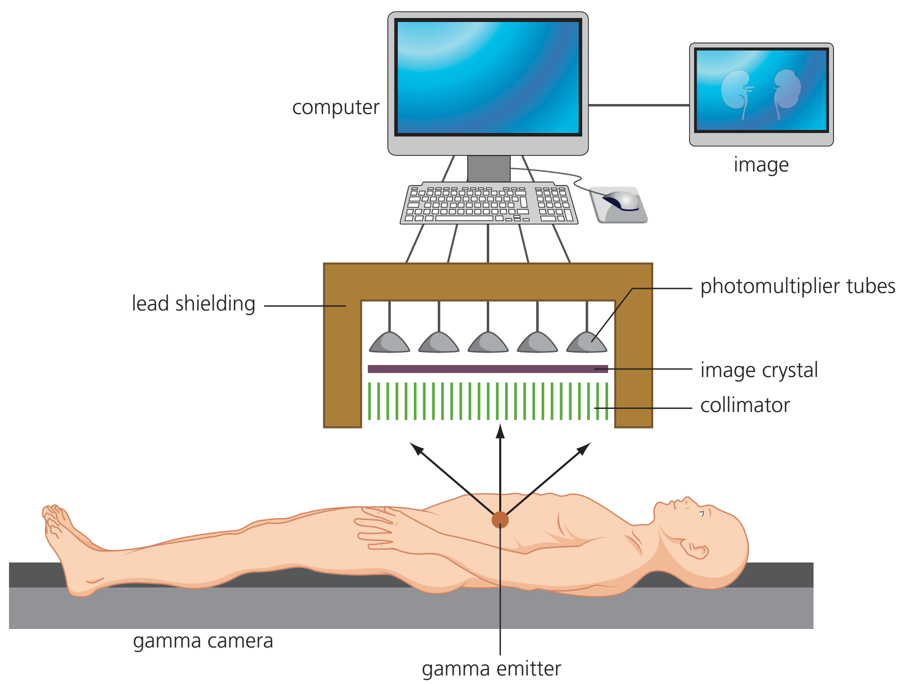


Figure 31.70

Once a patient has consumed a radiopharmaceutical, they are positioned so that the organ to be investigated is under the gamma camera. In Figure 31.70, the gamma camera is taking an image of the patient's kidneys. The gamma rays are emitted in all directions from the patient's body, but the camera only allows gamma rays to enter it from one direction. At the front

A **collimator** is a device which ensures that a beam of particles travels in a narrow beam in one direction.

TEST YOURSELF

68 a) Explain the function of the collimator in a gamma camera.

b) Why must the collimator be made out of a heavy metal?

69 Draw a diagram to explain how a gamma camera works. Explain the function of each part of the camera.

of the camera is a **collimator**; this is a thick plate of lead riddled with a large number of very thin parallel channels. The gamma rays which pass through the collimator are those whose direction is perpendicular to the surface of the lead plate and the scintillating crystal. The lead collimator reduces gamma rays travelling at an oblique angle to an insignificant level.

When the gamma photons strike the scintillating crystal, photons of visible light are emitted on the upper side of it. Above the crystal, an array of small photomultipliers (or photodiodes) converts photons of light into electrical signals. From these signals, the computer can determine the energy of the incoming gamma photons and the position of the impacts on the crystal. Now the computer can build up an image of the gamma ray emissions from the patient's kidneys, which provides useful diagnostic information.

Due to the difficulties of focusing gamma rays, the gamma camera has relatively poor resolution – it is able to resolve areas about 6 mm apart in an organ. But the advantage of gamma ray diagnosis is that we can gather information about the function of the inside of an organ.

Radiotherapy

When a patient is diagnosed with cancer, the oncologist (a doctor who treats cancer) will decide on a course of treatment. Sometimes a cancer can be cured by surgery or by chemotherapy (treatment using anti-cancer drugs), and cancers may also be treated using radiotherapy, in addition to the two other treatments.

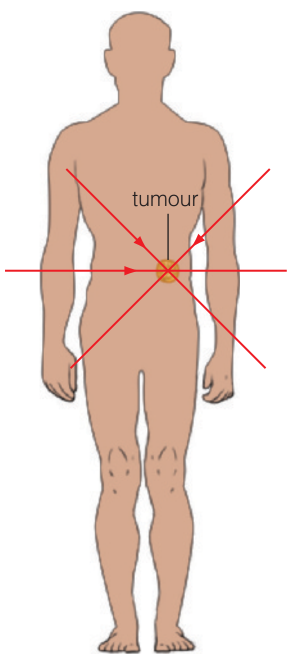


Figure 31.71

Use of high energy X-rays

One type of radiotherapy is an external treatment using high energy X-rays. In this treatment, X-rays are directed towards the tumour from outside the body. Figure 31.71 illustrates how X-rays are directed from different directions towards a deep tumour. Since X-rays can damage healthy cells as well as cancerous cells, it is important to direct the X-ray beam from different directions in order to reduce the damage to the surrounding healthy tissue.

X-ray energies

The type of X-rays used to treat cancer depends on the nature and position of the tumour. [Refer to pages 35–37 for information about X-ray production and energies.]

- Superficial X-rays 50 keV to 200 keV
  These X-rays do not penetrate very far before they are absorbed, reaching a depth of about 2 mm. These X-rays are used to treat diseases of the skin. This type of treatment is the least likely to cause side effects, as the X-rays can be focused on a small area of tissue.
- Orthovoltage X-rays 200 keV to 500 keV
  These X-rays can penetrate to a depth of 4–6 cm, so they can be used to treat problems close to the skin.
- Megavoltage X-rays 1 MeV to 25 MeV
  Megavoltage X-rays penetrate deep into the body and are used to treat cancers in the bladder, bowel, prostate, lung or brain. This treatment brings a risk to healthy tissue, but this risk has to be weighed against the

likely benefit of slowing or stopping entirely the growth of the tumour. The risk can also be reduced by directing the beam from different directions, as shown in Figure 31.71. The risk to the body is also reduced by using X-ray filters. When X-rays are produced, they have a wide range of energies and wavelengths (Figure 31.48), so when megavoltage X-rays are used, the less energetic X-rays are removed from the beam by placing a sheet of aluminium or lead in front of the beam. The filtering allows only high energy X-rays to reach the patient, which then penetrate to the tumour. Lower energy X-rays, which would be absorbed by the skin and shallow body tissues, are prevented from reaching the patient by filtering.

Use of radioactive implants

Some cancers are best treated with a radioactive implant, which is placed inside the body next to the cancerous tumour. This type of treatment is also called brachytherapy, from the Greek word *brachys*, meaning 'short-distance'.

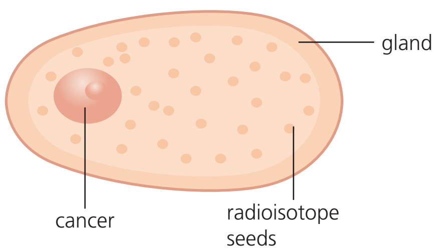


Figure 31.72 Seeds like these are placed near to the cancerous tumour.

This type of treatment has considerable advantages over external therapy. A radioactive source can be placed in a small wire implant, which is inserted inside the body next to the cancer. Then the radiation dose is applied only to the tissue next to the implant, which thus reduces the risk of radiation damage to healthy body tissue.

The radioisotopes used for short-range internal therapy are placed in very small seeds, which are then implanted in the body tissue. Some seeds are placed in the body temporarily, so that a large dose is administered over a short period of time. On the other hand, some cancers – prostate cancer, for example – are treated with small seeds which are left permanently in the body; the radioisotope used has a short half-life so that the dose is administered over a short period of time.

The radiation emitted in internal therapy must be short range, and must be able to pass through the metal casing of the seed.

- Alpha radiation is unsuitable for two reasons: firstly, it will not pass through the metal case; secondly, even if the alpha particles could get through the seed casing they would lose their energy in a very small distance, causing intense localised damage.
- Beta radiation is commonly used. Iridium-92 is a β^- emitter with a half-life of 72 days, and ruthenium-106 is a β^- emitter with a half-life of 367 days. A beta emitter produces electrons with a range of energies, which allows body tissues to be irradiated at a distance of a few millimetres away from the source.
- Gamma radiation is also used, but the gamma rays emitted must be of low energy so that they do not penetrate far from the source. A suitable gamma emitter is iodine-125, which has a half-life of 59 days and emits gamma rays with an energy of 29 keV.

TEST YOURSELF

70 a) Discuss the advantages and disadvantages of internal and external radiotherapy.

b) What precautions are taken to reduce the risk posed by

i) external radiotherapy

ii) internal radiotherapy?

71 What factors determine the choice of X-ray energy for the treatment of tumours by external radiotherapy?

72 Explain what is meant by an X-ray filter. Why does an X-ray filter help to make radiotherapy safer for patients?

73 a) Iridium-72 is a beta-emitter with a half-life of 72 days. The maximum energy carried by a beta particle emitted by an iridium source is 0.67 MeV. Explain why iridium is a suitable isotope for internal radiotherapy.

b) Calculate the period of time over which an iridium implant delivers 90% of its radiation dose.

A comparison of imaging techniques

Throughout this option of medical physics you have studied various imaging techniques. In the text, comments have been made about the advantages and disadvantages of each technique. Table 31.6 includes a summary. Questions in the A-level exam will be limited to consideration of imaging resolution, convenience and safety issues.

Table 31.6

Method	Resolution	Advantages	Disadvantages
Gamma rays from an ingested source	6 mm	Shows images of the inside of organs and their function. Relatively easy to administer	Low resolution. Presents some radiation danger.
X-rays from outside the body	0.5 mm	Cheap and easy to use.	Limited contrast between some types of tissues. Some organs cannot be seen. Some radiation danger.
CT scan	0.5 mm	High resolution. Can distinguish clearly between types of tissue.	Expensive. Relatively high radiation dose equivalent to a few years' background radiation dose.
MRI	1 mm	No radiation danger. Images are of high quality. Can distinguish between types of tissue.	Expensive. Some patients find it claustrophobic.
Ultrasound	2 mm	No radiation dangers	Some organs cannot be reached by ultrasound. Low resolution, and less clear images.
PET scans	1 mm	Allows a study of the function of organs. Excellent images of the brain.	Some organs cannot be imaged. Expensive.

Exam practice questions

1 a) A person with a defective eye wears spectacles to look at an object 25 cm away from his eye. The power of the correcting lens in the spectacles is +2.50 D.

i) Calculate the distance of the image seen by the eye through the correcting lens. *(3)*

ii) State the defect of vision that is being corrected. *(1)*

b) Draw a labelled diagram to show how the lens in part (a), together with the eye lens, forms an image of the object 25 cm away from the eye. Your diagram does not have to be to scale. *(3)*

2 a) Copy and complete the ray diagram in Figure 31.73 to show the formation of the image of a real object by a diverging lens. *(3)*

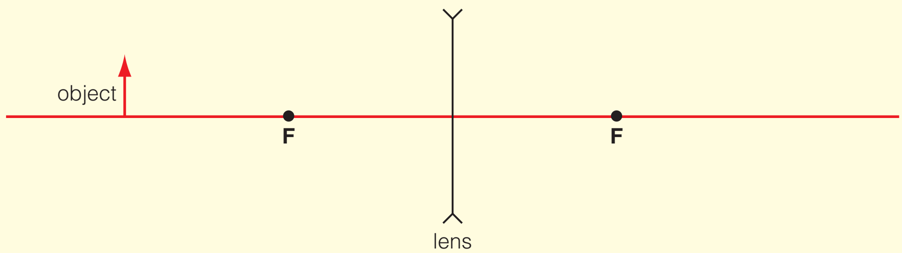


Figure 31.73

b) A lens of focal length –0.45 m is used to correct a defect of vision in an eye.

i) Name this defect of vision. *(1)*

ii) Calculate the power of the correcting lens. *(1)*

c) The defective eye has an unaided near point at 0.18 m from the eye.

Calculate the aided near point with the correcting lens, expressing your answer to an appropriate number of significant figures. *(3)*

3 A patient's prescription for her eyes is shown in the table below. Explain what the patient's eye defects are. *(4)*

	Sphere	Cylinder	Axis
Right	+1.75	0	
Left	+1.50	–0.25	50°

4 a) Describe how the vibrations of a sound wave are received by the outer ear and transmitted to the inner ear. *(3)*

b) An intensity meter, set to the dB scale, measures the intensity level of a sound as 57 dB. Calculate the intensity of sound at the meter, giving the appropriate unit. *(3)*

c) The scale on the intensity meter is now set to the dBA scale. The new reading is found to be 52 dBA. Explain this change. *(2)*

5 a) Define the threshold of hearing I_0. *(2)*

b) A person speaks with an intensity of 8×10^{-5} W m^{-2}. Calculate the intensity level of the sound in dB.

c) Give two reasons why we use a logarithmic scale to define the intensity level of a sound. *(2)*

d) A hearing test was used to obtain threshold hearing audiograms for several people. The audiogram in Figure 31.74 was obtained for a person with normal hearing.

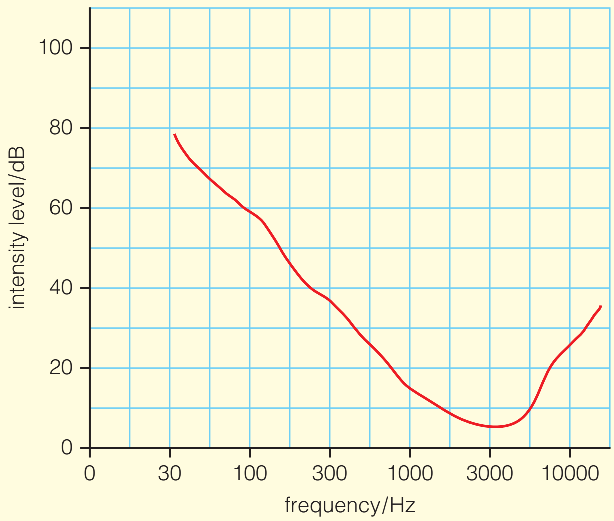


Figure 31.74

Copy the graph and on the same axes sketch curves to show:

i) a curve labelled X to show a person suffering from hearing loss due to excessive noise *(2)*

ii) a curve labelled Y to show a person suffering from hearing loss due to old age. *(2)*

6 a) Figure 31.75 shows an ultrasound transducer used in an A-scan.

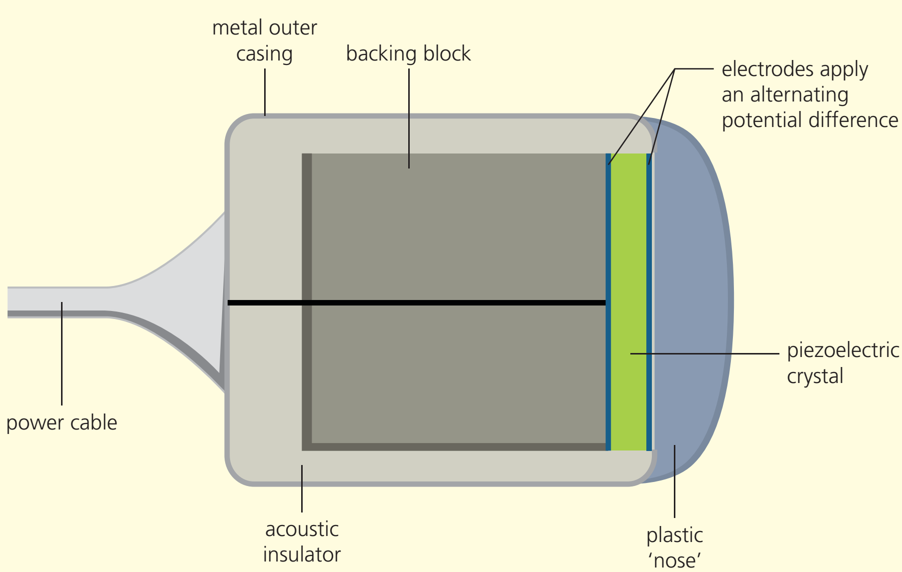


Figure 31.75

Explain, with reference to Figure 31.75, the process by which the transducer produces a pulse of ultrasound. *(4)*

b) Table 31.7 shows the acoustic impedances of some media.

Table 31.7

Medium	**Acoustic impedance/ kg m^{-2} s^{-1} × 10^{6}**
Air	4.3×10^{-4}
Muscle	1.7
Liver	1.6

i) Calculate the percentage of an incident beam of ultrasound which would be transmitted from muscle into the liver. *(2)*

ii) When obtaining the ultrasound image of an unborn foetus, a coupling gel is used. Explain why a coupling gel is needed and state the desirable properties of such a gel. *(2)*

iii) State and explain one advantage and one disadvantage of ultrasound imaging. *(2)*

7 Oil is used as a coupling gel between an ultrasonic transducer and body tissue. Use the information in Table 31.8 to calculate the percentage of energy from the ultrasonic transducer that is transmitted into the body. *(6)*

Table 31.8

Material	**Density/kg m^{-3}**	**Ultrasound velocity/m s^{-1}**
Oil	950	1500
Body tissue	1065	1530

8 Figure 31.76 shows the cross-section through an optical fibre. The refractive index of the core is 1.52 and the refractive index of the cladding is 1.45.

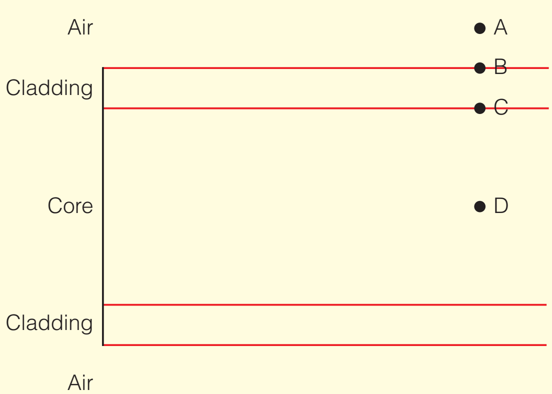


Figure 31.76

a) Sketch a graph to show how the refractive index of the fibre changes along the line ABCD. *(3)*

b) In an endoscope, there are two types of optical fibre: coherent and in coherent. Explain the purpose of each type of optical fibre. *(2)*

9 a) State the changes to a normal eye when the eye changes from focusing on a distant object to focusing on a near object, both being viewed in bright light. *(2)*

b) State two differences between the image seen by an eye when it looks at a coloured object in bright white light and the image seen of the same object in very dim light. *(2)*

10 When protons are placed in a strong magnetic field, there is an energy difference between the two possible spin states for the protons, as shown in Figure 31.77. A proton in the higher energy state can move to the lower state by emitting a radiofrequency photon. Information from radiofrequency photons is used to build up an image in an MRI scan.

a) What does the term MRI stand for? *(1)*

b) Why is there an energy difference between the two spin states of a proton in a magnetic field? *(1)*

c) A photon of frequency 750 MHz is emitted when a proton 'relaxes' from the higher state to the lower state.

Calculate the energy of this photon. *(2)*

d) Explain why a magnetic field gradient is applied to a patient during an MRI scan. *(2)*

e) Explain the advantages and disadvantages of an MRI scan as a diagnostic tool. *(2)*

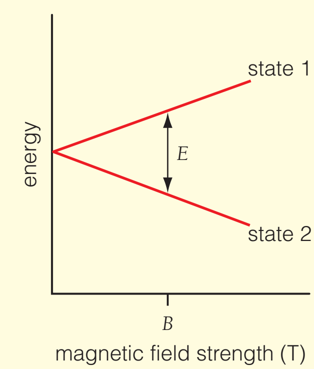


Figure 31.77

11 Figure 31.78 shows the X-ray spectrum produced by a beam of electrons which strike a tungsten target.

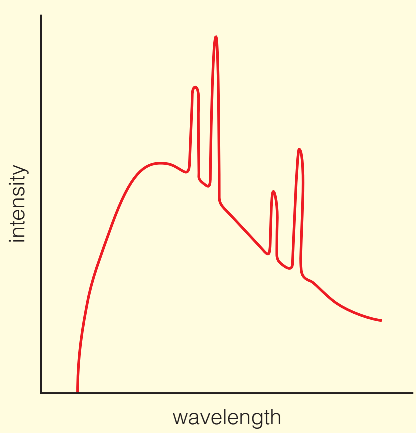


Figure 31.78

a) i) Explain why the spectrum has some sharp peaks in intensity. *(2)*

ii) Explain why there is also a continuous spectrum with a well-defined minimum wavelength. (2)

b) Copy the graph and sketch a second graph of a spectrum which is produced by an electron beam in which the electrons have slightly less energy than in part (a). *(3)*

c) An electron is accelerated through a p.d. of 80 kV.

i) Calculate the kinetic energy of the electron, expressing your answer in joules. *(2)*

ii) Assuming all of this kinetic energy is transferred into the energy of an X-ray photon, calculate the wavelength of the photon. *(2)*

12 A diagnostic X-ray tube produces a beam of X-rays. The beam passes through an aperture, the size of which is controlled by two pairs of lead sheets that move at right angles to each other. The beam then passes through an aluminium filter.

a) i) Explain the purpose of the lead sheets. *(1)*

ii) Explain the purpose of the aluminium filter. *(2)*

b) A monochromatic beam of X-rays passes through an aluminium sheet of thickness 5.0 mm. The intensity of the beam is reduced by 20%.

i) Calculate the mass attenuation coefficient for these X-rays. State an appropriate unit for your answer. The density of aluminium is $2700\,kg\,m^{-3}$. *(5)*

ii) Calculate the intensity reduction of the beam when it passes through an aluminium sheet of thickness 10 mm. (2)

13 a) A CT scan is carried out using X-rays. Give an account of how a CT scan is carried out. *(3)*

b) Explain the main advantages and disadvantages of a CT scan in comparison with a conventional X-ray imaging process. *(2)*

14 a) Describe the operation of a gamma camera. *(3)*

b) Explain what is meant by the term radiopharmaceutical. *(1)*

c) A doctor is considering a choice of two isotopes for use as a tracer to examine the thyroid of a patient. Explain which isotope would be better, using the data in Table 31.9. *(4)*

Table 31.9

Isotope	Half-life	Radiation emitted	Energy of radiation
$^{123}_{53}I$	13 hours	γ	160 keV
$^{131}_{53}I$	8 days	γ β	300 keV up to 600 keV

15 Technetium-99 is a tracer widely used in diagnostic medicine. This isotope has a physical half-life of 6 h and a biological half-life of 24 h.

a) Explain the terms biological and physical half-life. *(2)*

b) A patient consumes a dose of technetium-99. The doctor decides to keep the patient in isolation until 90% of the technetium has been removed from the body. How long will the patient spend in isolation? *(5)*

16 A cancerous tumour can be treated by using radiotherapy. Sometimes treatment can be applied externally and sometimes treatment is carried out using a radioactive implant.

a) Explain how X-rays are used to treat

i) a tumour near the skin

ii) a tumour deep inside the body. *(6)*

Include in your explanation a discussion about the energy of the X-rays used and how radiation damage is minimised.

b) Explain how internal radiotherapy is carried out. Include in your explanation a discussion of the choice of radioisotope for this purpose. *(6)*

17 a) State the differences between the two types of light detecting cells in the retina. *(2)*

b) Explain why we see objects with the sharpest clarity when we look straight ahead. *(2)*

c) Explain why we are more likely to see something at night time using our peripheral vision, rather than looking straight ahead. *(2)*

Photo credits

Answers

Activity (page 48)

1

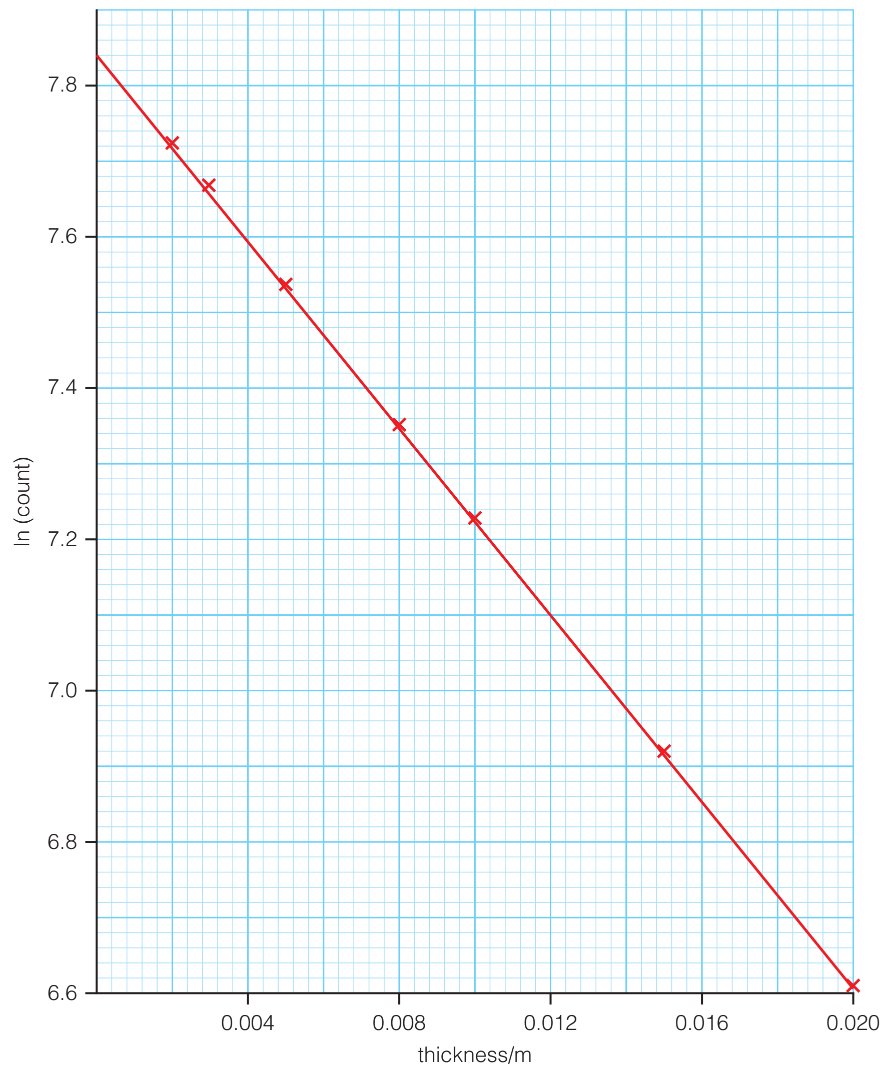


Figure A31.1

$$\mu = -\text{gradient}$$

$$= \frac{7.84 - 6.61}{20 \times 10^{-3}}$$

$$= 62\,\text{m}^{-1}$$

2 $\mu_m = \dfrac{\mu}{\rho}$

$$= \frac{62\ \text{m}^{-1}}{11\ 300\ \text{kg m}^{-3}}$$

$$= 5.5 \times 10^{-3}\ \text{m}^2\ \text{kg}^{-2}$$

3 $x_{\frac{1}{2}} = \dfrac{\ln 2}{\mu}$

$$= \frac{0.693}{62\ \text{m}^{-1}}$$

$$= 0.011\,\text{m or } 1.1\,\text{cm}$$

4 Cobalt-60 also emits β-particles, which would increase the count when no lead is present.

Answers to Test yourself on prior knowledge

1

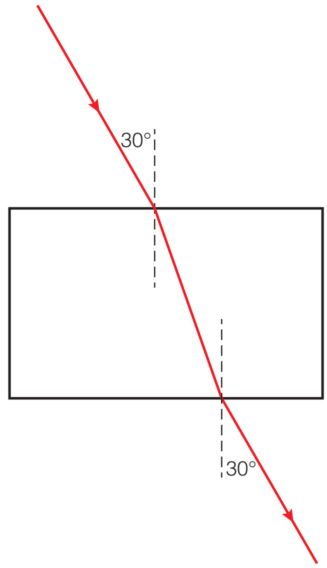


Figure A31.2

2 $\lambda = \dfrac{v}{f}$

$= \dfrac{1500}{10^5}$

$= 1.5 \times 10^{-2}$ m

3 $d = vt$

$= 330 \times 2.2$

$= 726$ m

So the cliff is $\dfrac{726}{2} = 363$ m away.

4 The gamma rays can pass through the body and be detected outside. Gamma rays are less ionising, per metre travelled, than alpha or beta particles. The short half-life minimises the dose to the patient.

5 a) $^{192}_{77}\mathrm{Ir} \rightarrow {}^{192}_{78}\mathrm{Pt} + {}^{0}_{-1}\mathrm{e}$

b) Beta radiation travels only a short distance through the body, and will cause some localised ionisation which can destroy cancer cells.

Answers to Test yourself questions

1 Rays parallel to the principal axis of a diverging lens appear to have diverged from a point – this is the virtual focus.

2 a) i) A real image can be projected on to a screen and can be seen by many people at once. Light rays converge at the image position.

ii) A virtual image only appears to be in a position. The light rays appear to come from the image position, due to the refraction by a lens. This type of image cannot be projected onto a screen.

b) A diverging lens always produces a virtual image; a converging lens can produce either a virtual or real image depending on whether the distance of the object from the lens is less than or greater than the focal length.

3 a)

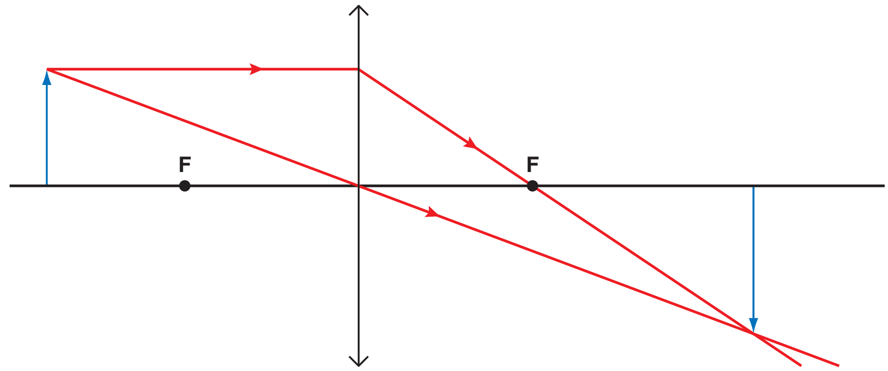


Figure A31.3

b) The image is:

i) real

ii) inverted

iii) magnified.

4 a) and b)

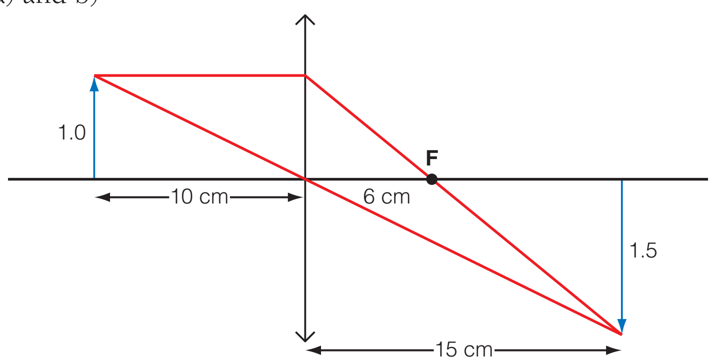


Figure A31.4

c) $m = \frac{v}{u}$ $v = 15$ cm so

$$1.5 = \frac{15}{u}$$

Therefore $u = 10$ cm.

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$= \frac{1}{15} + \frac{1}{10}$$

$$= \frac{10 + 15}{150}$$

$$\frac{1}{f} = \frac{25}{150}$$

$$f = 6 \text{ cm}$$

5 a) $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{20} = \frac{1}{25} + \frac{1}{v}$

$\frac{1}{v} = \frac{1}{20} - \frac{1}{25}$

$= \frac{25-20}{500}$

$= \frac{5}{500}$

$v = 100$ cm

$m = \frac{v}{u}$

$= \frac{100}{20}$

$= 5$

So the image is 10 cm high, inverted and real.

b) $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$-\frac{1}{10} = \frac{1}{v} + \frac{1}{30}$ (the lens has a virtual focus, so f is negative)

$\frac{1}{v} = -\frac{1}{10} - \frac{1}{30}$

$= -\left(\frac{30+10}{300}\right)$

$\frac{1}{v} = -\frac{40}{300}$

$v = -7.5$ cm

The minus sign tells us the image is virtual. The image is the right way up and diminished by a factor of 4 $\left(m = \frac{7.5}{30} = \frac{1}{4}\right)$.

6 a) i) Power $= \frac{1}{2.5}$

$= +0.4$ D

ii) Power $= -\frac{1}{0.5}$

$= -2.0$ D

b) -2.0 D $+ 0.4$ D $= +1.6$ D

7 a)
- Rods are sensitive to low level light intensity – they are responsible for night vision and allow us to see movement in our peripheral vision.
- Cones are sensitive to different wavelengths of light and allow us to see colour.

b) Cones are very closely packed in the fovea. When we look directly at an object the image is formed on the fovea, where the high resolution provided by the closely packed cones allows us to see in detail.

c) There are only rods to detect our peripheral vision. Rods are not so closely packed.

d) Rods are sensitive to low level light, but cones do not work well in low light intensity.

8 a) 390–720 nm

b) The eye is particularly sensitive to yellow light, as both the red and green cones detect it; blue light and red light are each only detected by one type of cone – the blue and red, respectively.

9 a) Refer to Figure 31.11 on page 8.

b) Cornea and lens.

10 a) Accommodation is the mechanism by which the eye lens changes shape to allow the eye to focus on near and far objects. To see close by, the lens becomes fatter and more curved; the lens is thinner and less curved for distant vision.

b) Refer to Figure 31.14 on page 10.

11 a) Refer to Figure 31.15(c) on page 10.

b) The object is at infinity, $u = \infty$.

The virtual image is at the far point $v = -1.8$ m.

(Remember that using the real-is-positive convention, a virtual image distance has a negative sign.)

Using $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$:

$$\frac{1}{f} = -\frac{1}{1.8}$$

So $f = -1.8$ m

i.e. we use a diverging lens of focal length 1.8 m or power

$$-\frac{1}{1.8} = -0.56 \text{ D.}$$

12 a) Refer to Figure 31.16(c) on page 11.

b) The object distance is 25 cm, so $u = 0.25$ m.

The virtual image distance is 30 cm, so $v = -0.3$ m. (Remember that a virtual distance has a negative sign.)

Using $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$:

$$\frac{1}{f} = \frac{1}{0.25} - \frac{1}{0.3}$$

$$= 4 - 3.33$$

$$= 0.67$$

So the power is 0.67 D (because power $= \frac{1}{f}$).

When v is very large, $\frac{1}{v}$ tends to zero, so the power of the lens tends to 4 D.

13 a) Astigmatism occurs when an eye has different curvatures in different planes. So an eye might focus more strongly in the vertical plane than it does in the horizontal plane.

b) X does not have astigmatism, but is long sighted. The right eye's near vision is corrected by a lens of power +0.5 D, and the left eye needs a lens of +0.75 D.
Y has astigmatism and is short sighted. The far vision of the right eye is corrected by a cylindrical lens of power −2.25 D along the 80° axis and a lens of power −3.25 D along the 170° axis. The left eye is corrected by a cylindrical lens of power −1.75 D along the 100° axis and a lens of power −2.25 D along the 10° axis.

c) Cylindrical lens.

14 a) The malleus is a bone in the middle ear that transmits vibrations from the outer to the inner ear.

b) The tympanic membrane (or eardrum) vibrates when sound waves reach it.

c) The oval window is a membrane that connects the bones in the middle ear to the inner ear (which transmits signals to the brain).

d) The cochlea is the organ that turns mechanical vibrations into electrical impulses that are transmitted to the brain.

e) The pinna is the visible part of the outer ear (earflap) that helps collect sound, and enables the direction of sound to be judged.

15 The pinna collects sound and channels it into the outer ear. The eardrum vibrates and the bones of the middle ear transmit the energy through to the oval window in the inner ear. The oscillations of the oval window stimulate hairs in the cochlea, which transmit electrical impulses along the auditory nerve into the brain. The hairs in the cochlea resonate when a sound of their natural frequency reaches the oval window.

16 a) The threshold of hearing is the lowest intensity of sound that can be heard by the human ear.

b) This differs from person to person, but it is defined as an intensity of 10^{-12} W m^{-2} at 1 kHz.

17 a) $$\text{Intensity level} = 10\log_{10}\left(\frac{I}{I_0}\right)\text{dB}$$

$$= 10\log_{10}\left(\frac{7\times10^{-6}}{10^{-12}}\right)\text{dB}$$

$$= 68.5\text{ dB}$$

b) $$\text{Intensity level} = 10\log_{10}\left(\frac{I}{I_0}\right)\text{dB}$$

$$65 = 10\log_{10}\left(\frac{I}{10^{-12}}\right)$$

$$6.5 = \log_{10}\left(\frac{I}{10^{-12}}\right)$$

$$10^{6.5} = \frac{I}{10^{-12}}$$

$$3.16\times10^{6} = \frac{I}{10^{-12}}$$

$$I = 3.16\times10^{-6}\text{ W m}^{-2}$$

c) Difference in intensity $= 10\log_{10}\left(\frac{I_2}{I_1}\right)$

$$= 10\log_{10}\left(\frac{5\times10^{-6}}{8\times10^{-7}}\right)$$

$$= 10\log_{10}\left(\frac{50}{8}\right)$$

$$= 10\times0.796$$

$$= 8.0\text{ dB}$$

18 a) i) An A-weighted sound level takes account of the ear's sensitivity. The ear is most sensitive to sounds of about 3 kHz frequency. Thus sounds with the same intensity in W m^{-2} sound louder to us at 3 kHz than at 10 kHz or 500 Hz.

ii) Loudness is a measure of the way in which a person perceives the amplitude of a sound. It is therefore subjective.

iii) Our perception of sound is subjective. One person may say the television is too loud, another person may complain that they cannot hear it.

b) At 3000 Hz the A-weighted sound for A is 60 dBA, and it is 20 dBA at 100 Hz. So the difference is 40 dBA

c) i) C could hear sounds between 100 Hz and 10 kHz.

ii) C has a hearing loss of about 60 dBA – 25 dBA or 35 dBA.

$$35\text{ dBA} = 10\log_{10}\left(\frac{I_2}{I_1}\right)$$

$$\frac{I_2}{I_1} = 10^{3.5}$$

$$= 3200$$

So A's ears are 3200 times more sensitive than C's ears at 3 kHz.

19 a) Intensity is measured in W m^{-2} so measures power reaching the ear per m^2.

Intensity level is measured in dB. This is a logarithmic scale that compares sounds with the threshold of hearing (10^{-12} W m^{-2}).

Loudness is subjective, but can be measured in dBA – a scale that allows relative loudness to be measured for an individual.

b) i) Assuming that the intensity is measured at the same distance from the loudspeaker:

the increase in intensity level $= 10\log_{10}\left(\frac{I_2}{I_1}\right)$ dB

$$= 10\log_{10}\left(\frac{12}{3}\right)\text{dB}$$

$$= 6.0\text{ dB}$$

ii) Intensity $\propto \frac{1}{r^2}$ $\quad \frac{I_2}{I_1} = \frac{\frac{1}{20^2}}{\frac{1}{10^2}} = \frac{1}{4}$

Change in intensity level $10\log_{10}\left(\frac{1}{4}\right) = -10\log_{10}4$

So the intensity falls by a factor of 4 and the intensity level falls by $10\log_{10}4$ or 6 dB (see part i)).

iii) Difference in intensity level $= 10\log_{10}\left(\frac{I_2}{I_1}\right)$

$$= 10\log_{10}\left(\frac{4\times10^{-3}}{6\times10^{-6}}\right)$$

$$= 10\log_{10}(667)$$

$$= 28\text{ dB}$$

20 a) Electrocardiography

b) When the heart beats it produces a potential difference across various parts of the chest. A healthy heart produces a particular signal as a function of time. By looking at this waveform, a cardiologist (heart specialist) can see if the heart is healthy.

21 a) Contact is made using conducting gel.

b) Contact is made using the four limbs, or one of six places on the chest.

22 See Figure 31.24 on page 21.

23 a) The advantage of using high-frequency ultrasound is that the resolution is better because the wavelength is shorter. There is more scattering and reflection by very small structures in the body when using high-frequency ultrasound, so these are more easily observed.

b) High-frequency ultrasound is absorbed more readily by the body than low frequency. So the strength of the reflected signal is less than at low frequencies i.e. there is more attenuation.

24 a) A piezoelectric crystal is one that changes shape when a p.d. is applied across it.

b) A high-frequency alternating p.d. sets the crystal vibrating (at its resonant frequency). This vibration produces the high-frequency ultrasound.

25 a) $Z = \rho\times c$

$$= 950\text{ kg m}^{-3}\times 1500\text{ m s}^{-1}$$

$$= 1.425\times10^{6}\text{ kg m}^{-2}\text{ s}^{-1}$$

b) i) $\frac{I_r}{I_i} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$

$$= \frac{(1.5\times10^{6} - 1.425\times10^{6})^2}{(1.5\times10^{6} + 1.425\times10^{6})^2}$$

$$= 0.000\,66$$

ii) Fraction transmitted $= 1 - 0.000\,66$

$$= 0.999\,34 \text{ or } 1.0 \text{ to 2 s.f.}$$

26 a) This means that the acoustic impedances of the two materials are very different. Thus there is little sound transmitted from air into rock.

b) Acoustic impedance of granite $= \rho\times c$

$$= 2700\text{ kg m}^{-3}\times 6000\text{ m s}^{-1}$$

$$= 16.2\times10^{6}\text{ kg m}^{-2}\text{ s}^{-1}$$

$$\frac{I_r}{I_i} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$

$$= \frac{\left(16.2 \times 10^6 - 4.30 \times 10^2\right)^2}{\left(16.2 \times 10^6 + 4.30 \times 10^2\right)^2}$$

$$\approx 1.0$$

27 a) Ultrasound is reflected off interfaces between layers of body tissues, and so is able to produce an image of the body. Ultrasound also has limited resolution.

Advantages: non-ionising, it images muscles and bones well.

Disadvantages: ultrasound can be limited in depth of penetration – it does not penetrate bone. It requires a skilled operator.

b) See text on page 26.

28 The A-scan is a depth finder and works on timing the reflection off a surface.

The B-scan relies on reflections from several directions to build up a three-dimensional image, for example of a foetus.

29 Lower frequency ultrasound, 1–6 MHz, penetrates deeper into the body, so that the insides of organs may be examined. Higher frequency ultrasound does not go so far into tissues, but provides better resolution, for example enabling tendons to be seen more clearly.

30 The strength of the reflections depends on the ratio of the acoustic impedances of each surface. There is a bigger difference in impedance between bone and muscle than there is between fat and muscle.

31 a) The coupling medium ensures that the acoustic impedance of the materials match closely – then there is little reflection when the ultrasound enters the body (as there would be from an air–body interface).

b) The data is recorded in the table, with velocity of sound expressed in m s^{-1}. Acoustic impedance for each material is calculated from $Z = \rho \times c$.

Material	Velocity of sound, v (m s^{-1})	Density, ρ (kg m^{-3})	Acoustic impedance, Z (kg m^{-2} s^{-1})
Air	330	1.3	430
Oil	1500	950	1.425×10^6
Body tissue	1530	1065	1.629×10^6

$$\frac{I_r}{I_i} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$

i) Air–tissue: $\frac{I_r}{I_i} = 1.0$

ii) Oil–tissue: $\frac{I_r}{I_i} = 0.0045$

iii) Air–oil: $\frac{I_r}{I_i} = 1.0$

32 This looks complicated but the equation turns out to be more to do with attenuation than reflection.

At each surface, A and B, only a small fraction of energy is reflected.

$$\frac{I_r}{I_i} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$

$$= \frac{(1.65 \times 10^6 - 1.40 \times 10^6)^2}{(1.65 \times 10^6 + 1.40 \times 10^6)^2}$$

$$= 0.0067$$

Thus without attenuation 1 – 0.0067 = 0.9933 of the energy would reach B, then 0.0067 of that is reflected. We can make it easier by saying that 0.9933 ≈ 1. So the same amount of energy is reflected at both A and B.

The difference in intensity is therefore due to the additional attenuation of the signal because it has travelled 6 cm further through body tissue (it has travelled through the kidney and back again). From the graph, after travelling 6 cm the intensity of the signal is reduced to about $\frac{1}{4}$.

So $\frac{I_A}{I_B} = 4$

33 a) Total internal reflection occurs when light at a large angle of incidence is reflected on the inside of a material, which has a refractive index higher than the material on the other side of the reflecting surface. Above a critical angle all the light is reflected. Refer to Figure 31.35 on page 29.

b) An endoscope transmits light (without loss of intensity) from its viewing end to an eyepiece, so that a patient may be examined internally.

34 a) A coherent bundle of fibres keeps all the fibres in the same pattern, so that a clear image of an object can be formed.

An incoherent bundle is used only for transmitting light, so the pattern of fibres does not matter.

b) i) A small radius allows the fibre to be bent more easily, which is useful when manoeuvring the endoscope.

ii) If the diameter of the fibres is smaller than 5 μm, then diffraction becomes significant, with an ensuing loss of clarity.

35 a) $\sin\theta_c = \frac{n_c}{n_f}$

$$= \frac{1.45}{1.50}$$

$$= 0.967$$

$$\theta_c = 75.2°$$

b) Refer to Figure 31.35 on page 29.

$$\theta_c = 75.2°$$

So $r = 90° - 75.2°$

$$= 14.8°$$

$n_f = 1.50$ and $n_0 = 1.00$

So for light entering the fibre, $\dfrac{\sin i_{max}}{\sin r_{max}} = \dfrac{n_f}{n_o} = 1.5$

$$\sin i_{max} = 1.5 \sin 14.8°$$

$$i_{max} = 23°$$

c) A higher refractive index in the core will provide a wider field of view, because rays will be refracted more along the direction of the fibre (towards the normal).

36 2

37 a) Precession occurs when the axis of rotation of a spinning body circles around a fixed axis, as shown by a spinning top.

b) Refer to Figure 31.40 on page 32.

The proton has two energy states:

1 when its magnetic moment lies parallel to the magnetic field

2 when the moment lies antiparallel to the field.

c) The spin (magnetic moment) of a proton can be in the direction of the applied magnetic field or in the opposite direction. The direction of precession changes when the spin flips (Figure 31.40 on page 32).

38 a) Nuclear magnetic resonance occurs when a proton absorbs a photon of exactly the energy required to flip its spin from a lower energy state (spin up) to a higher energy state (spin down).

b) i) $E = hf$

$$= 6.63 \times 10^{-34}\ \text{J s} \times 6.3 \times 10^{8}\ \text{s}^{-1}$$

$$= 4.2 \times 10^{-25}\ \text{J}$$

ii) $E = \dfrac{4.2 \times 10^{-25}\ \text{J}}{1.60 \times 10^{-19}\ \text{J eV}^{-1}}$

$$= 2.6 \times 10^{-6}\ \text{eV}$$

c) The photon energy above will not cause any ionisation – ionising radiation is dangerous to us, because DNA can be damaged by ions.

39 a) $\Delta E = \dfrac{ehB}{4\pi m_p} = hf$

So $f = \dfrac{eB}{4\pi m_p}$

$$= \frac{1.60 \times 10^{-19}\ \text{C} \times 10\ \text{T}}{4\pi \times 1.67 \times 10^{-27}\ \text{kg}}$$

$$= 76.2\ \text{MHz}$$

b) $f = 153$ MHz

40 The frequency of the emitted radiofrequency photon depends on the strength of the magnetic field. The computer maps the magnetic field by position, and can thus match an emitted frequency to a particular position in the patient.

41 The electrons in a compound can shield or alter the external magnetic field. So the nuclear magnetic resonant frequency depends on the external field strength and the compound to which the proton is attached. This means the scan can detect different types of body tissues, which is vital for detecting cancerous tissues.

42 i) A continuous X-ray spectrum is emitted by high-energy electrons, which slow down when they strike a target. Any energy (up to a maximum energy) may be transferred by the electron, which gives a continuous range of wavelengths, down to a minimum.

ii) A characteristic spectrum is seen which depends on the target. Here high energy electrons remove a bound electron in a low energy level. Then an electron from a higher level falls to the lower state – emitting a photon of a characteristic wavelength: $\Delta E = \frac{hc}{\lambda}$.

43 a) When an electron decelerates it loses some kinetic energy. The kinetic energy lost by the electron is transferred to the energy of a photon according to:

$$\Delta E = \frac{hc}{\lambda}$$

so any wavelength can be seen depending on how much energy is transferred.

b) The minimum wavelength (λ_{min}) corresponds to all of the kinetic energy of the electron being transferred.

c) The characteristic lines in a spectrum correspond to energy transitions in a particular element. Since electron energy levels differ from element to element, each metal target produces its own X-ray spectrum (Figure 31.48 on page 37).

44 Refer to Figure 31.50 on page 38.

- An electron gun directs high-energy electrons towards the rotating anode.
- Electrons collide with the target in the rotating anode and slow down, which results in the emission of X-rays of all wavelengths (down to a minimum value) and characteristic X-rays at particular wavelengths depending on the target.
- The anode rotates so that the target does not overheat, as electrons transfer their kinetic energy into the thermal store of the metal target.

45 a) $eV = \frac{hc}{\lambda_{min}}$

$$\lambda_{min} = \frac{hc}{eV}$$

$$= \frac{6.63 \times 10^{-34}\ \text{J s} \times 3.00 \times 10^{8}\ \text{m s}^{-1}}{1.60 \times 10^{-19}\ \text{C} \times 40 \times 10^{3}\ \text{V}}$$

$$= 3.1 \times 10^{-11}\ \text{m}$$

b) $\lambda_{min} = 1.0 \times 10^{-11}$ m

46 The possible energy values of the photons and their wavelengths are:

$E_1 = 20$ keV $\quad \lambda_1 = 6.2 \times 10^{-11}$ m

$E_2 = 60$ keV $\quad \lambda_2 = 2.1 \times 10^{-11}$ m

$E_3 = 80$ keV $\quad \lambda_3 = 1.6 \times 10^{-11}$ m

(Use $\lambda = \frac{hc}{eV}$ for each of the energy differences between energy levels, E_1, E_2 and E_3.)

47 Geometric unsharpness or blurring is caused by the source having an extending size – see Figure 31.52 on page 40. This is inevitable as a point source of X-rays would be too hot, and melt the target.

Blurring can also be caused by the movement of the body, e.g. the stomach will always have some movement in it.

48 a) The focus of an anode is the part of the target from which X-rays are emitted.

b) See Figure 31.52 on page 40.

c) The smaller the focus, the sharper the image, but a small focus also gets very hot. So there is a compromise between the target overheating and the sharpness of the image.

49 a) i) Bone in the hand – the hand can be immobilised. So a small focus can be used, with a low intensity beam, with a longer exposure. This produces a very clear image of the hand.

ii) Stomach – this will be more blurred. The stomach cannot be immobilised, so the X-ray must be taken with a short exposure, but then the focus will be larger (with geometric blurring).

b) Contrast is a clear difference observable in the image, where one tissue type appears noticeably darker than another.

There is a sharp contrast between flesh and bone, because the bone is opaque to X-rays. However, there is little contrast between the stomach and surrounding tissues, as they transmit/absorb X-rays equally. The contrast for the stomach can be enhanced by swallowing a barium 'meal', for example.

50 a) Refer to Figure 31.55 on page 42 (a copy of which would enhance your answer).

An X-ray flat panel detector includes an X-ray scintillator, photodiode pixels and an electronic scanner. X-rays are directed towards a patient and an image formed on the scintillation layer. As a result of the absorption of X-rays, electrons in the scintillation layer emit visible photons. Photodiodes absorb the visible light, which then release electrons that stimulate a pixel. Now the image can be stored digitally.

b) Digital images have the advantage that they are produced in real time, which thus allows quick diagnoses. The images can easily be stored and transmitted.

51 a) In an image intensifier, two fluorescent screens are placed on either side of a double-sided X-ray film. The X-rays expose the film, but the fluorescent screens emit light that also exposes the photographic film.

b) The text states that 95% of the image is produced by light emitted by the intensifying screens. Thus only $\frac{1}{20}$ of the light needed to produce the image comes from the X-rays. This gives a reduction in exposure time by a factor of 20.

c) Reducing the exposure time reduces the ionising dose of radiation received by a patient.

Also, the patient does not have to be stationary for so long.

52 a) In an image intensifier, X-rays are incident on a fluorescent screen and form an image. Where X-rays strike the screen, electrons are emitted. These pass through a thin layer of aluminium foil, forming an 'electron' image on the inside of a cathode. These electrons are now accelerated through a large p.d. to be focused on an output window. The image is intensified by:

- focusing the electrons on to a smaller window
- giving the electrons a large amount of energy which causes bright fluorescence on the output window.

b) Image intensifiers are used when a doctor needs to see inside the body to carry out a procedure such as passing a catheter along a vein.

53 a) The linear attenuation coefficient gives a measure of the attenuation for a particular material for a specified energy of X-rays. The half-value thickness for a material is $x_{\frac{1}{2}} = \frac{\ln 2}{\mu}$; for X-rays travelling a distance $x_{\frac{1}{2}}$ through the material, the intensity of the X-rays is reduced to a half.

b) μ decreases as the energy of the X-rays increases.

54 a) $\mu_m = \frac{\mu}{\rho}$

$$= \frac{19.3\,\mathrm{m^{-1}}}{1060\ \mathrm{kg\,m^{-3}}}$$

$$= 0.0182\ \mathrm{m^2\,kg^{-1}}$$

b) $x_{\frac{1}{2}} = \frac{\ln 2}{\mu}$

$$= \frac{0.693}{32.6}$$

$$= 0.021\,\mathrm{m}$$

c) $I = I_0 e^{-\mu x}$

$$\frac{I}{I_0} = e^{-24.0 \times 0.05}$$

$$= 0.30$$

d) $\frac{I}{I_0} = e^{-20.5 \times 0.05}$

$$= 0.36$$

Although the reduction in intensity of X-rays is similar for muscle and fat, there is enough difference for the two tissues to be distinguished.

e) $\frac{I}{I_0} = e^{-74.2 \times 0.05}$

$$= 0.024$$

There is a pronounced difference in intensity between images of bone and other tissues.

55 a) A CT scan is an X-ray computerised tomography scan, which enables slices of the body to be brought into a sharp focus.

b) In a CT scan, the body is surrounded by X-ray detectors that allow a computer to build up a sharp image of a thin section through the body. By moving the X-ray detectors, the computer builds up many images of sections of the body (or an organ) at different depths.

c) i) Advantages:

- a clear focus of one thin slice can be produced
- very high-resolution images
- images can be taken from any direction.

ii) Disadvantages:
- CT scans are time consuming and expensive
- patients are exposed to a relatively high radiation dose.

56 a) Radiation dose is a measure of the damage (and hence risk) caused to the body by ionising radiations. The dose depends on the total energy transferred to the body and the type of radiation. Alpha radiation is more dangerous than the same energy of gamma radiation, due to the localised damage caused by an alpha particle.

b) Dental X-ray – very low risk

CT scan – low risk

Targeted radiation therapy – high risk, but balanced by the chance of curing cancer.

57
- A tracer needs to be a gamma emitter so that the radiation can be detected outside the body.
- Low-energy gamma rays reduce the possible ionisation damage.
- A tracer with a short half-life reduces the total dose given to the patient but produces a high gamma intensity over a short period of time.

58 a) It has the properties described in the answer to question 57: low-energy gamma rays are emitted, and the half-life is short (6 hours).

b) Iodine-123 emits low-energy gamma rays only and has a short half-life (13 h). Iodine-131 emits high-energy gamma rays and beta particles and has a longer half-life (8 days).

c) Iodine-131 is used to reduce overactive thyroids or for treating thyroid cancer. This isotope emits both gamma rays and beta particles. The beta particles produce ionisation over a short range, and thus they are able to destroy cancerous cells.

59 The molybdenum–technetium generator produces technetium from molybdenum. Technetium is widely used as a tracer, because it has a half-life of 6 hours. But that means that the supply runs out quickly. The advantage of molybdenum-99 is that it has a half-life of 66 hours. Thus the molybdenum–technetium generator supplies technetium in a hospital for a week.

60 PET stands for positron emission tomography. In PET scanning the body absorbs a glucose that contains the isotope fluorine-18, which decays by positron emission. When the emitted positron meets an electron in the body tissue, both particles are annihilated. A pair of gamma rays is produced from the annihilation of the electron–positron pair, with the gamma rays moving in opposite directions. An array of detectors is used to detect the gamma rays and therefore their point of emission. Different body tissues absorb the glucose in different concentrations, and thus PET provides information about the health of body tissues.

61 48 hours is 8 half-lives.

Thus the amount left is $2\,\text{g} \times \left(\frac{1}{2}\right)^8 = 0.0078\ \text{g}$

62 a) Momentum is conserved, so the gamma rays have equal and opposite momentum.

b) i) $E = mc^2$

$= 9.11 \times 10^{-31}\ \text{kg} \times (3.00 \times 10^8)^2(\text{m s}^{-1})^2$

$= 8.20 \times 10^{-14}\ \text{J}$

ii) $E = \dfrac{hc}{\lambda}$

$\lambda = \dfrac{hc}{E}$

$= \dfrac{6.63 \times 10^{-34}\ \text{Js} \times 3.00 \times 10^8\ \text{ms}^{-1}}{8.20 \times 10^{-14}\ \text{J}}$

$= 2.43 \times 10^{-12}\ \text{m}$

63 i) Physical half-life is the half-life of a radioisotope (T_P).

ii) Biological half-life is the time taken for the physiological activity of a drug to halve (T_B); this happens because the body excretes the drug.

iii) Effective half-life of a radioisotope in the body depends on both physical and biological half-lives:

$$\frac{1}{T_E} = \frac{1}{T_P} + \frac{1}{T_B}$$

64 The patient is radioactive and can irradiate other people. The time in isolation depends on the type of radiation and the physical and biological half-lives of the tracer.

65 Although the physical half-life of cobalt-60 is over 5 years, the biological half-life is short. The rate at which the body excretes the drug gives it a biological half-life of 10 days, so that in a few weeks there is very little left in the body.

66 $\dfrac{1}{T_E} = \dfrac{1}{T_P} + \dfrac{1}{T_B}$

$= \dfrac{1}{29} + \dfrac{1}{50}$

$T_E = 18$ years

67 a) $\dfrac{1}{T_E} = \dfrac{1}{T_P} + \dfrac{1}{T_B}$

$= \dfrac{1}{244} + \dfrac{1}{933}$

$T_E = 193$ days

b) 1 year is approximately 2 half-lives, so after 1 year the activity will be $\dfrac{A}{4}$.

68 a) The collimator ensures that the gamma rays enter the gamma camera from one direction only – vertically upwards in Figure 31.71 on page 58.

b) Gamma rays will pass through most metals. A metal such as lead is used to reduce the intensity of gamma radiation reaching the camera from directions other than vertically upwards.

69 See Figure 31.71 on page 58.

70 a) See pages 58–59.

Internal therapy has the advantage of allowing a source of radiation to be placed close to the cancer. If this cannot be achieved, external therapy can be used – with the risk of damage to surrounding healthy cells.

External therapy is the obvious choice for skin cancers. Low-energy X-rays are directed to surface tumours, without the risk of damage to deeper tissues.

b) i) For treatment close to the skin, low energy X-rays are used so that the rays do not penetrate beyond the area for treatment. For deeper treatment, more energetic X-rays are needed. A filter is used to remove lower energy X-rays, which could cause damage near to skin.

ii) The radiation must not penetrate far from the implant site. Thus beta rays or low energy gamma rays are used.

71 Low-energy X-rays are used for tumours close to the skin as they do not penetrate deep inside the body. High-energy X-rays are used to treat tumours further inside the body, because they are attenuated less by body tissues.

72 An X-ray filter can be used to filter out low- and medium-energy X-rays, allowing only high-energy X-rays into the body. These then get deep into the body and can treat the tumour. This improves safety as the low-energy X-rays do not affect surface tissues.

73 a) The beta particles do not travel far inside the body, so all their energy is transferred locally to the cancerous tissue. A half-life of 72 days means that the isotope is active for a few months. It should be removed after treatment.

b) $N = N_0 e^{-\lambda t}$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$0.9 = e^{-\left(\frac{0.693}{T_P}\right)t}$$

$$\ln(0.9) = -\left(\frac{0.693}{72}\right)t$$

$$t = \frac{-0.105 \times 72}{-0.693}$$

$$= 11 \text{ days}$$

ANSWER

Answers to Exam practice questions

1 a) i) Focal length of the lens: $f = \frac{1}{2.5} = 0.40\,\text{m}$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$= \frac{1}{0.40} - \frac{1}{0.25}$$

$$= 2.5 - 4.0$$

$$= -1.5$$

$$v = -0.67\ \text{m}$$

The negative sign shows that the image is virtual 0.67 m in front of the lens.

ii) The defect is long sightedness.

b)

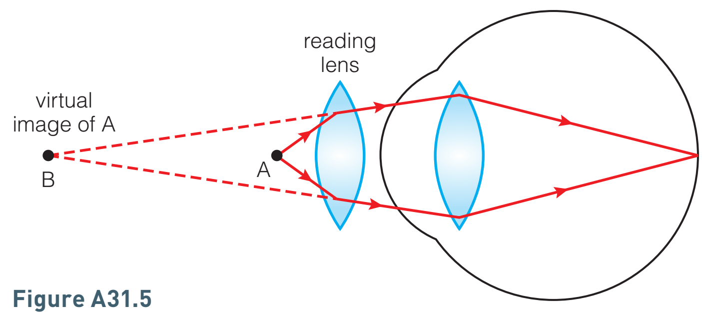


Figure A31.5

2 a)

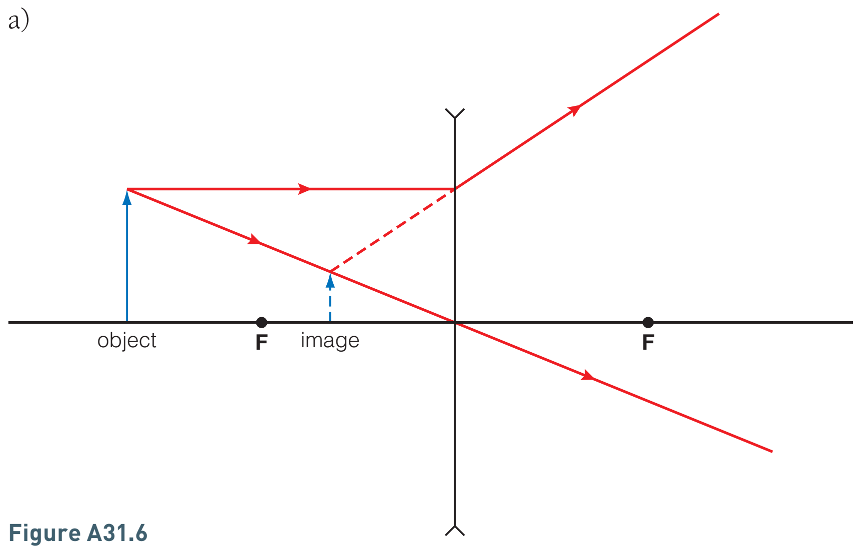


Figure A31.6

b) i) Myopia – short sightedness.

ii) Power $= \frac{1}{-0.45\text{m}}$

$= -2.2\ \text{D}$

c) The formation of the image is the same principle as shown in Figure A31.6.

The image is formed at the unaided near point and the object is at the aided near point.

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$

The image is virtual and the lens has a negative power, so the equation is:

$$-\frac{1}{0.45}=\frac{1}{u}-\frac{1}{0.18}$$

$$\frac{1}{u}=\frac{1}{0.18}-\frac{1}{0.45}$$

$$= 5.55 - 2.22$$

$$= 3.33$$

$$u = 0.3 \text{ m}$$

3 The right eye is long sighted and needs a correcting lens of power 1.75 D. It has no astigmatism.

The left eye is long sighted and has astigmatism. A correcting lens with power of 1.50 D along the 50° axis and 1.25 D at right angles (140°) to that axis is required.

4 a) Vibrations in the air set the eardrum vibrating. The vibrations are transmitted through the middle ear by three bones to the oval window, which is connected to the inner ear. Hairs in the cochlea are stimulated by movements of the oval window, and electrical impulses are sent down the auditory nerve to the brain. A vibration in the oval window will cause a hair to vibrate at its resonant frequency.

b) Intensity level $= 10\log\left(\frac{I}{I_0}\right)\text{dB}$

$$57 = 10\log\left(\frac{I}{10^{-12}}\right)$$

$$10^{5.7} = \frac{I}{10^{-12}}$$

$$I = 10^{5.7} \times 10^{-12}\ \text{W m}^{-2}$$

$$= 5.0 \times 10^{-7}\ \text{W m}^{-2}$$

c) The dBA scale takes account of the sensitivity of the human ear to different frequencies. Thus it gives a measure of how loud a sound will appear.

5 a) The threshold of hearing is the lowest intensity of sound that a healthy ear can detect. This is taken to be 1×10^{-12} W m^{-2} at a frequency of 1 kHz.

b) Intensity level $= 10\log_{10}\left(\frac{8\times10^{-5}}{10^{-12}}\right)$

$$= 10\log_{10}\left(8\times10^{7}\right)$$

$$= 70 \log_{10} 8$$

$$= 63 \text{ dB}$$

ANSWER

c) The ear can detect sounds over a great range of intensities from 10^{-12} W m^{-2} to 100 W m^{-2}. So a logarithmic scale is helpful to deal with such a range of numbers. But the ear's perception of loudness is logarithmic: if the intensity is doubled and then doubled again, we perceive that the loudness has gone up in equal steps.

d)

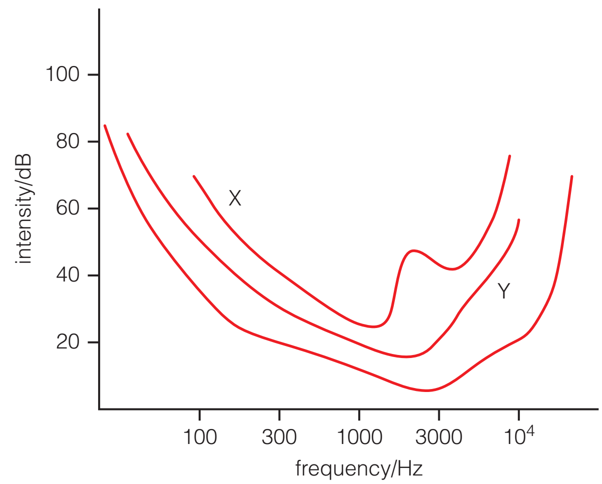


Figure A31.7

6 a) When a p.d. is applied across a piezoelectric crystal it changes shape. When a high-frequency p.d. is applied across a piezoelectric crystal it vibrates. (The vibrations have a large amplitude close to the crystal's resonant frequency.) At very high frequencies ultrasound waves are emitted from the crystal.

b) i) The fraction of ultrasound reflected at a surface is given by the ratio

$$\frac{I_r}{I_i} = \frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}$$

$$= \frac{(1.7\times10^6 - 1.6\times10^6)^2}{(1.7\times10^6 + 1.6\times10^6)^2}$$

$$= 0.0009$$

Thus the fraction transmitted is: 1 – 0.0009 = 0.9991.

ii) Without the coupling gel there would be a layer of air between the transmitter and body. The poor match of acoustic impedance would lead to a very large fraction of the energy being reflected at the body surface. The gel needs to have an acoustic impedance close to that of the body (and not have air bubbles in it).

iii) Advantage: Ultrasound is safe as it is non-ionising.

Disadvantage: Ultrasound is attenuated by body tissues and it is not possible to get high-resolution images deep inside the body.

7 $Z_{oil} = \rho \times c$

$= 950\ \mathrm{kg\,m^{-3}} \times 1500\ \mathrm{m\,s^{-1}}$

$= 1.425 \times 10^6\ \mathrm{kg\,m^{-2}\,s^{-1}}$

$$Z_{\text{tissue}} = \rho \times c$$
$$= 1065\ \text{kg}\,\text{m}^{-3} \times 1530\,\text{m}\,\text{s}^{-1}$$
$$= 1.629 \times 10^{6}\ \text{kg}\,\text{m}^{-2}\,\text{s}^{-1}$$

$$\frac{I_r}{I_i} = \frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}$$

$$= \frac{(1.629\times10^{6} - 1.425\times10^{6})^2}{(1.629\times10^{6} + 1.425\times10^{6})^2}$$

$$= 0.0045$$

So the fraction transmitted is: 1 – 0.0045 = 0.996.

8 a)

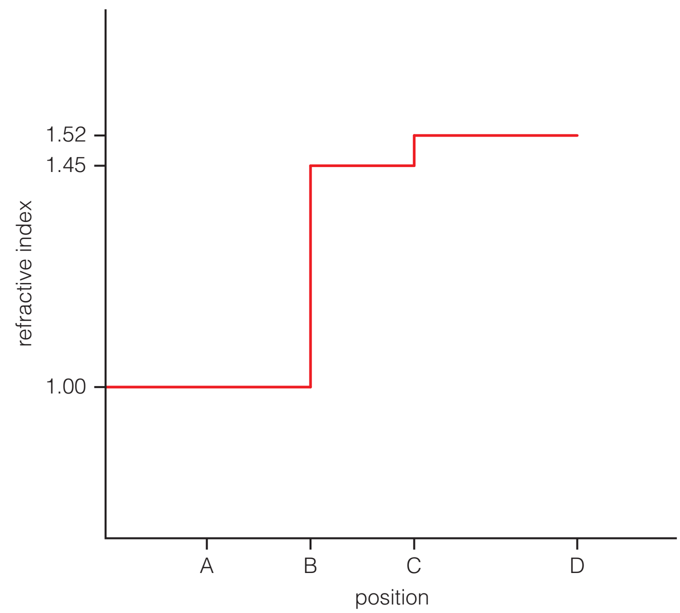


Figure A31.8

(The refractive index of air is 1.00.)

b) The coherent fibres keep the same arrangement along their length, so that a clear image may be formed. Incoherent fibres carry light just for illumination purposes.

9 a) When focused on a distant object, the eye lens is relaxed and flat – so the lens is at its least powerful. When looking at a nearby object, the muscles pull the lens into a fatter, more curved lens. Now the lens is at its most powerful, so that the diverging light rays may be focused on the retina.

b) In bright white light the cones are stimulated and the pupil is small. A clear well-resolved image is seen in colour.

In dull light the cones do not work well, so the rods are stimulated. The rods are less densely packed than cones, so the image is less well resolved and the image is seen in black and white only.

10 a) Magnetic resonance imaging.

b) In one state, the proton magnetic field is parallel to the external field; in the other state the proton field is antiparallel to the external field.

c) $E = hf$

$= 6.63 \times 10^{-34}\ \text{J s} \times 750 \times 10^{6}\ \text{s}^{-1}$

$= 5.0 \times 10^{-25}\ \text{J}$

d) The energy difference between the two proton states depends on the strength of the magnetic *B*-field. When an excited proton relaxes it emits a particular frequency of radiation, which can be linked to the *B*-field strength and therefore a position in the body. This gives information about the body at that point.

e) The advantage of an MRI scan is that it has excellent resolution and can diagnose small differences between cells and thus show cancers. The disadvantages of an MRI scan are that it is time consuming and expensive.

11 a) i) An incident electron can remove an electron in a low-lying energy level (E_1). Then an electron from a higher level (E_2) falls to that lower state. A photon of energy $E_2 - E_1$ is emitted, which has a characteristic wavelength: $E_2 - E_1 = \frac{hc}{\lambda}$.

ii) When an electron slows down it emits a photon. Since the energy change can be of any value, there is a continuous spectrum. But there is a maximum energy that can be lost (the initial kinetic energy of the electron), and therefore a minimum wavelength given by the relationship $E_{max} = \frac{hc}{\lambda_{min}}$.

b)

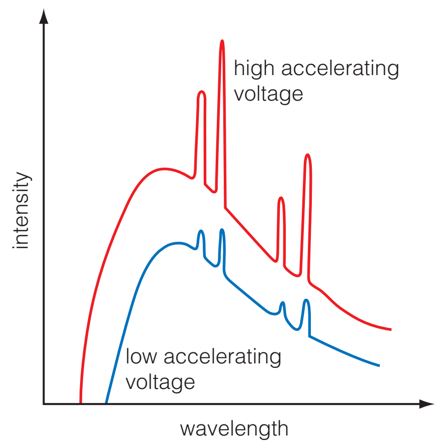


Figure A31.9

c) i) $E_K = eV$

$= 1.60 \times 10^{-19}\ \text{C} \times 80 \times 10^{3}\ \text{V}$

$= 1.28 \times 10^{-14}\ \text{J}$

ii) $E = \frac{hc}{\lambda}$

$\lambda = \frac{hc}{E}$

$$= \frac{6.63\times10^{-34}\ \text{Js}\times3.00\times10^{8}\ \text{ms}^{-1}}{1.28\times10^{-14}\ \text{J}}$$

$= 1.6 \times 10^{-11}\ \text{m}$

12 a) i) The lead is there to reduce significantly the spread of X-rays, so that X-rays pass through the aluminium in a narrow beam.

ii) The aluminium filter reduces the intensity of low-energy X-rays.

b) i) $I = I_0 e^{-\mu x}$

$0.8 = e^{-\mu \times 0.005}$

$\ln 0.8 = -\mu \times 0.005$

$-0.223 = -\mu \times 0.005$

$$\mu = \frac{0.223}{0.005}\,\mathrm{m}^{-1}$$

$$= 44.6\ \mathrm{m}^{-1}$$

$$\mu_\mathrm{m} = \frac{\mu}{\rho}$$

$$= \frac{44.6\ \mathrm{m}^{-1}}{2700\ \mathrm{kg\ m}^{-3}}$$

$$= 0.017\,\mathrm{m}^2\ \mathrm{kg}^{-1}$$

ii) The transmitted intensity will be $0.8 \times 0.8 = 0.64$ of the original intensity, so the intensity reduction is 0.36 or 36%.

13 a) A CT scan allows an X-ray image to be formed of a thin slice or plane of the body. The body is surrounded by an array of detectors that allow X-rays to be detected from all directions. A computer builds up images of 'slices' of the body.

b) The main advantages of the CT scan are the detailed images of each part of an organ, and that an image can be taken in any plane. The disadvantages are that a CT scan is time consuming, costly and exposes patients to relatively high doses of radiation.

14 a) A patient consumes a gamma-emitting radiopharmaceutical, so that gamma rays are emitted from inside the body. A collimator then ensures that gamma rays can only enter the camera from one direction. An image crystal and photomultipliers enable a computer to build up a digital image of the body.

b) A radiopharmaceutical is a compound that can be absorbed by the body which carries a radioisotope that emits gamma (or other) radiation.

c) ${}^{123}_{53}\mathrm{I}$ is the better isotope. This isotope has a short half-life, so it produces a high activity for a short period. Thus the organ is investigated and the radioisotope then decays. The gamma rays emitted by iodine-123 have lower energy than those from iodine-131 so are less damaging; the latter also emits dangerous beta particles.

15 a) Biological half-life measures the time taken for the body to excrete a drug until its physiological activity has reduced by a half.

Physical half-life measures the time taken for half of the nuclei of a radioisotope to decay.

b) $\dfrac{1}{T_E} = \dfrac{1}{T_P} + \dfrac{1}{T_B}$

$= \dfrac{1}{6} + \dfrac{1}{24}$

$T_E = 4.8$ hours

$N = N_0 e^{-\left(\frac{0.693}{T_E}\right)t}$

$0.1 = e^{-\left(\frac{0.693}{T_E}\right)t}$

$\ln 0.1 = -\dfrac{0.693t}{T_E}$

$-2.303 = -\dfrac{0.693t}{4.8}$

$t = 15.9$ hours

16 a) i) Low-energy X-rays can be used to treat a skin cancer. The low-energy X-rays can reach the cancer, as it is external, but as the X-rays are of low energy they do not penetrate deep into the body, so the risk of secondary damage is reduced.

ii) To treat a tumour deep inside the body, X-rays can be directed at the tumour from an external source. The rest of the body is protected by:

- directing the X-rays from different directions
- using filters to cut out low-energy X-rays (which would have damaged tissues close to the surface). Then only the more energetic X-rays reach the tumour deep inside the body.

b) Internal radiotherapy is when a radioactive implant is placed into the tumour itself. The radioisotope used should:

- have a short half-life so that it has high activity
- emit alpha or beta particles, which are short-ranged and highly ionising, so that only the tumour is damaged.

Sometimes the implant can be removed after the dose has been delivered, but some implants are left in the body.

17 a) There are three types of cone, which can detect red, green and blue light. Rods are sensitive to dim light and they allow us to sense movement and see in black and white.

b) When we look straight ahead, light falls on the fovea, which has a high density of cones, so we can resolve detail.

c) Rods lie at the edge of the eye and respond to low levels of light, so we use these for night vision. Since rods dominate the peripheral vision we tend to pick up movement at night out of the 'corner' of the eye. Cones are not sensitive to low light intensities, so our forward vision is less effective at night.