

Pages 577–578 Test yourself

1 Remember to convert all measurements to standard units in this question.

a) Volume = $0.02 \text{ m} \times 0.02 \text{ m} \times 0.02 \text{ m} = 8 \times 10^{-6} \text{ m}^3$

b) Percentage uncertainty will be the sum of the percentage uncertainties in each measurement.

$$\% \text{ uncertainty in measurement of one dimension} = \frac{2 \times 10^{-4}}{0.02} \times 100\% = 1\%$$

$$\therefore \% \text{ uncertainty in volume} = (1\% + 1\% + 1\%) = 3\%$$

c) density = $\frac{\text{mass}}{\text{volume}}$

$$= \frac{0.0582}{8 \times 10^{-6}}$$

$$= 7\,280 \text{ kg m}^{-3} \text{ (to 3 sf)}$$

d) % uncertainty in density = % uncertainty in mass + % uncertainty in volume

$$\text{uncertainty in mass} = \left(\frac{2 \times 10^{-4}}{0.0582} \right) \times 100\% = 0.3\%$$

$$\text{so \% uncertainty in density} = 3.3\%$$

$$\text{absolute uncertainty} = \frac{(3.3 \times 7\,280)}{100} = 240 \text{ kg m}^{-3}$$

The density is therefore written as $7\,280 \text{ kg m}^{-3} \pm 240 \text{ kg m}^{-3}$

2 Spring constant = $\frac{10 \text{ N}}{0.032 \text{ m}} = 313 \text{ N m}^{-1}$

$$\% \text{ uncertainty in force} = \left(\frac{0.1 \text{ N}}{10 \text{ N}} \right) \times 100\% = 1\%$$

$$\% \text{ uncertainty in extension} = \left(\frac{0.001 \text{ m}}{0.032 \text{ m}} \right) \times 100\% = 3.1\%$$

$$\text{total percentage uncertainty} = 1\% + 3.1\% = 4.1\%$$

So the spring constant is $313 \text{ N m}^{-1} \pm 4.1\%$

3 speed = $\frac{\text{distance}}{\text{time}}$

$$\text{speed} = \frac{100.00 \text{ m}}{9.63 \text{ s}} = 10.38 \text{ m s}^{-1}$$

$$\% \text{ uncertainty in distance} = \left(\frac{0.01 \text{ m}}{100.00 \text{ m}} \right) \times 100\% = 0.01\%$$

$$\% \text{ uncertainty in time} = \left(\frac{0.01 \text{ s}}{9.63 \text{ s}} \right) \times 100\% = 0.1\%$$

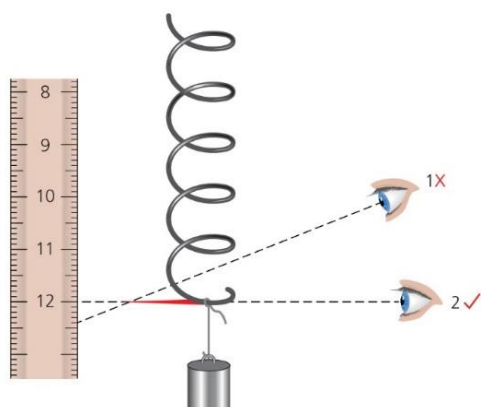
$$\text{So \% uncertainty in speed} = 0.11\%$$

$$\text{absolute uncertainty} = \frac{(0.11 \times 10.38)}{100} = 0.01 \text{ m s}^{-1}$$

$$\text{so average speed} = 10.38 \text{ m s}^{-1} \pm 0.01 \text{ m s}^{-1}$$

4 a) Timing 10 oscillations reduces the effect that the reaction time of the student has on the accuracy of the measurements. If only 1 oscillation were measured then her reaction time would be a significant percentage of the measurement.

- b) The student should ensure that she reduces the effect of parallax by measuring the height from eye level as shown in the figure.



- c) 3.86 s to 3.96 s

- d) uncertainty in 10 oscillations = (largest reading – smallest reading) / 2 = ± 0.05 s

- e) Mean time for 10 oscillations = $\frac{3.91 \text{ s} + 3.86 \text{ s} + 3.87 \text{ s} + 3.92 \text{ s} + 3.96 \text{ s}}{5} = 3.90 \text{ s} \pm 0.005 \text{ s}$

Mean time for 1 oscillation = $0.390 \text{ s} \pm 0.005 \text{ s}$

- 5 a) i) A systematic error is an error which affects a set of measurements in the same way each time making them consistently too large (or too small).

ii) The student has subtracted the background count rate from his readings.

- b) Uncertainty is equal to half the range of the measurements

$$= (295 - 276)/2$$

$$= \pm 10 \text{ min}^{-1}$$

- c) The student should have repeated the measurement at 6.0 cm.

- 6 For a diffraction grating, $d \sin \theta = n\lambda$

$$d = (1 \times 10^{-3} / 750) \text{ m}$$

Calculating the wavelength using the student's measured value, $\theta = 23^\circ$:

$$\lambda = d \sin \theta$$

$$\text{so } \lambda = \frac{1 \times 10^{-3} \text{ m}}{750} \times \sin 23^\circ = 5.21 \times 10^{-7} \text{ m}$$

Calculating the wavelength for the minimum value of θ possible from the uncertainty, $\theta = 22^\circ$

$$\text{so } \lambda_{\min} = \frac{1 \times 10^{-3} \text{ m}}{750} \times \sin 22^\circ = 4.995 \times 10^{-7} \text{ m}$$

Calculating wavelength using $\theta = 24^\circ$

$$\lambda_{\max} = \frac{1 \times 10^{-3} \text{ m}}{750} \times \sin 24^\circ = 5.423 \times 10^{-7} \text{ m}$$

$$\text{so uncertainty} = \pm 0.214 \times 10^{-7} \text{ m}$$

$$\text{measured wavelength} = 5.21 \times 10^{-7} \text{ m} \pm 0.214 \times 10^{-7} \text{ m}$$

The stated wavelength of 510 nm = $5.10 \times 10^{-7} \text{ m}$ is within the range of uncertainty of the student's measurement.

7 a) 14.50 mm

b) To increase the accuracy of her measurement.

If the ball was not perfectly spherical, or if she didn't place the callipers at the widest point of the ball, then this would be noticeable from the repeat measurements.

Pages 579–580 Activities

Dropping a bouncy ball

1 Ball may not be dropped from measured height. Mitigate by ensuring ruler is fixed and vertical (plumb line), positioning head to ensure no parallax errors or even using another ruler/card at right angles to rule position base of ball.

Measuring maximum bounce height is very subjective. Could do trials from each height to get rough idea of where ball will get to to ensure observer is in a suitable position. If have a suitable camera, could film experiment and examine relevant frames to determine position.

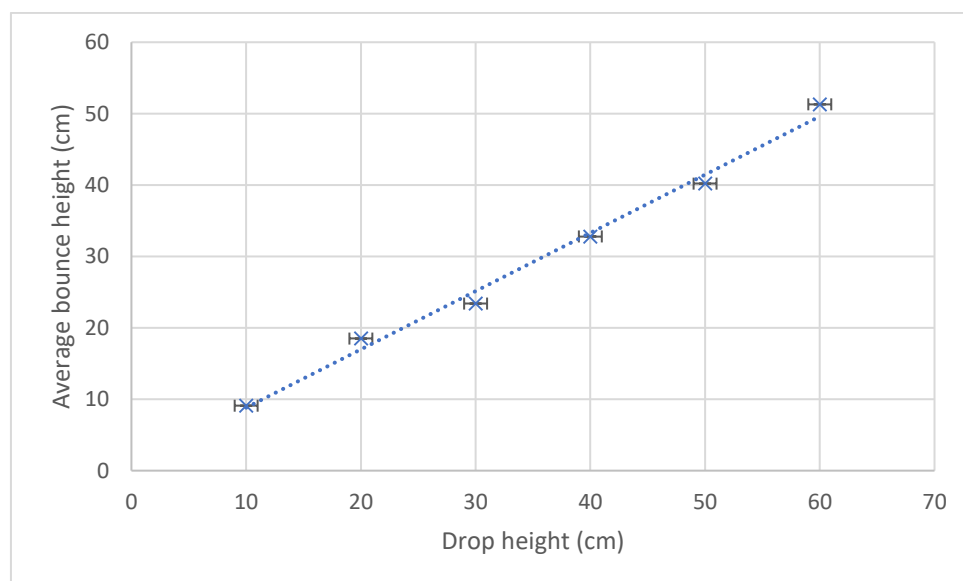
2 If a method is used that ensures good alignment, the uncertainty in drop height could be as little as ± 0.1 cm

3

Drop height (cm)	10.0	20.0	30.0	40.0	50.0	60.0
Bounce 1 (cm)	9.0	18.0	23.2	32.0	40.0	50.5
Bounce 2 (cm)	9.5	17.5	23.0	33.5	40.5	52.0
Bounce 3 (cm)	8.8	20.0	24.0	32.8	40.0	51.5
Average bounce height (cm)	9.1	18.5	23.4	32.8	40.2	51.3
Uncertainty in bounce height (cm)	0.35	1.25	0.5	0.75	0.25	0.75

4 (no answer required)

5



6 Using the value of the gradient calculated by Excel, the efficiency is 82 %.
By drawing lines of steepest and shallowest fit, the uncertainty can be estimated at 3%.

Measuring wave speeds in shallow water

- 1 The independent variable is the depth of water.
- 2 Use the equation speed = distance / time. The distance the wavefront has travelled is 5 x length of tray, and the time is the time that she has measured.
- 3 You are given the uncertainty in the depth of water. To calculate the uncertainty in the mean wavespeed use half the range and this is then doubled to find the error in v^2 . E.g. for $d = 0.010$ m:

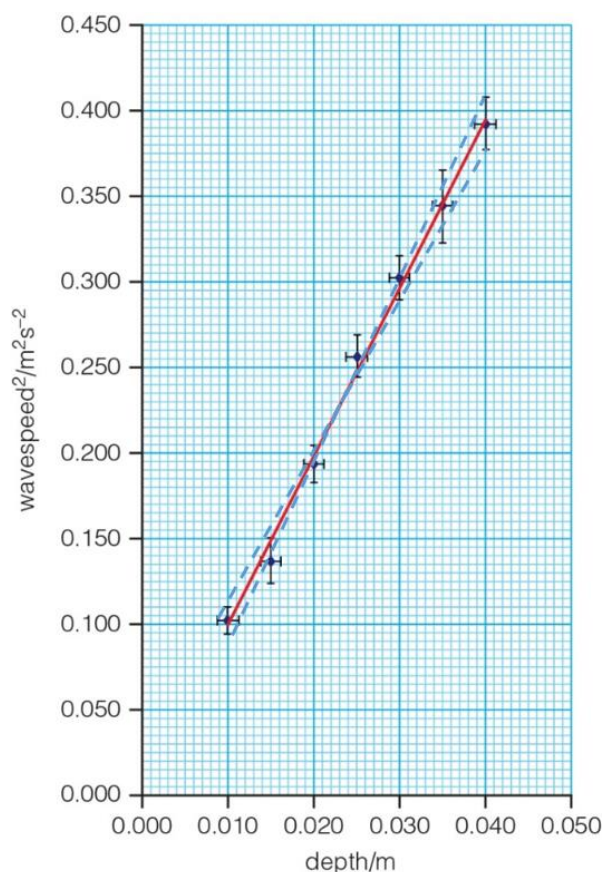
$$\text{Uncertainty in } v_{\text{mean}} = (0.33 - 0.31)/2 = 0.01 = \frac{0.01}{0.32} \times 100\% = 3.2\%$$

$$\text{Uncertainty in } v_{\text{mean}}^2 = 6.4\% = \frac{6.4}{100} \times 0.102 = 0.006$$

Other values are then as shown in the table below

d/m	v_1/ms^{-1}	v_2/ms^{-1}	v_3/ms^{-1}	$v_{\text{mean}}/\text{ms}^{-1}$	Error $v_{\text{mean}}/\text{ms}^{-1}$	$v_{\text{mean}}^2/\text{m}^2\text{s}^{-2}$	Error $v_{\text{mean}}^2/\text{m}^2\text{s}^{-2}$
0.010 ± 0.001	0.33	0.32	0.31	0.32	0.01	0.102	0.006
0.015 ± 0.001	0.35	0.38	0.38	0.37	0.02	0.137	0.011
0.020 ± 0.001	0.44	0.45	0.43	0.44	0.01	0.194	0.009
0.025 ± 0.001	0.50	0.50	0.52	0.51	0.01	0.257	0.010
0.030 ± 0.001	0.56	0.54	0.55	0.55	0.01	0.303	0.011
0.035 ± 0.001	0.57	0.59	0.60	0.59	0.02	0.344	0.018
0.040 ± 0.001	0.62	0.62	0.64	0.63	0.01	0.393	0.013

Your graph should look similar to this:



- 4 Using $v = \sqrt{gd}$ to obtain $v^2 = gd$. Comparing to the equation of a straight-line, $y = mx + c$ shows that the gradient of the graph will be equal to acceleration due to gravity.

$$\text{gradient} = 9.89 \text{ m s}^{-2}$$

To obtain the uncertainty in the gradient we draw two additional lines on the graph which show the steepest and shallowest acceptable lines of fit (shown by the dotted lines on the graph). These lines should pass through the corners of the small box formed by the error bars.

You should obtain values similar to the following:

$$\text{Steepest gradient} = 11.1 \text{ m s}^{-2}$$

$$\text{Shallowest gradient} = 8.5 \text{ m s}^{-2}$$

$$\text{The error in the gradient} = (11.1 - 8.5) / 2 = 1.3 \text{ m s}^{-2}$$

$$\text{so } g = 9.89 \text{ m s}^{-2} \pm 1.3 \text{ m s}^{-2}$$

The usual value of g is within the uncertainty of the measurement.

- 5 The student could use a method of creating the initial wave which occurs automatically, rather than by dropping the tray. This would allow for the timing mechanism to be started more accurately.

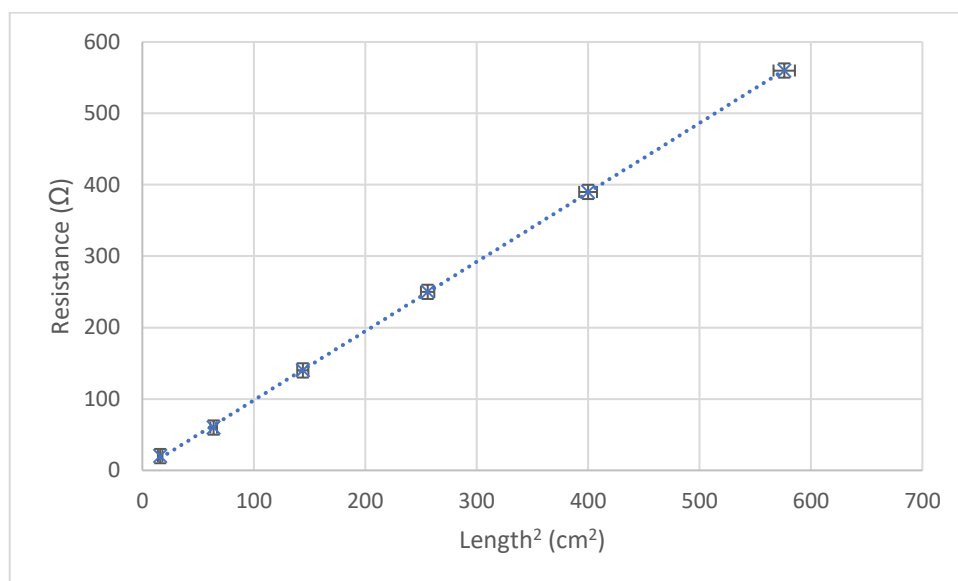
The student could use a video camera to record the motion of the wave, and then use video editing software (such as Tracker) to observe the motion of the wave front more clearly, and use the frame rate of the video to obtain a timing for the motion.

Pages 581–582 Test yourself

- 8 a) Rewriting the equation in the form $y = mx + c$ we get $R = \frac{\rho}{v} l^2$. So if a plot of R against l^2 is a straight-line graph with no y intercept, then the relationship is correct.
- b) i) To calculate the uncertainty in l^2 we need to double the percentage uncertainty in l and then calculate the absolute uncertainty. This is because we are multiplying the length by itself.

The data to be plotted is given in the table.

l^2 / cm^2	R / Ω
16 ± 2	20 ± 10
64 ± 3	60 ± 10
144 ± 5	140 ± 10
256 ± 6	250 ± 10
400 ± 8	390 ± 10
576 ± 9.6	560 ± 10



ii) Gradient = $0.973 \Omega \text{ cm}^{-2}$

iii) This will depend on the lines drawn. Gradients of these lines will be between $0.910 \Omega \text{ cm}^{-2}$ and $1.02 \Omega \text{ cm}^{-2}$. The uncertainty of the gradient will therefore be around $\pm 0.055 \Omega \text{ cm}^{-2}$.

c) gradient = $\frac{\rho}{V}$

So resistivity = gradient $\times V = 0.979 \Omega \text{ cm}^{-2} \times 26.8 \text{ cm}^3 = 26.2 \Omega \text{ cm}$

9 a) To reduce current flowing through the voltmeter.

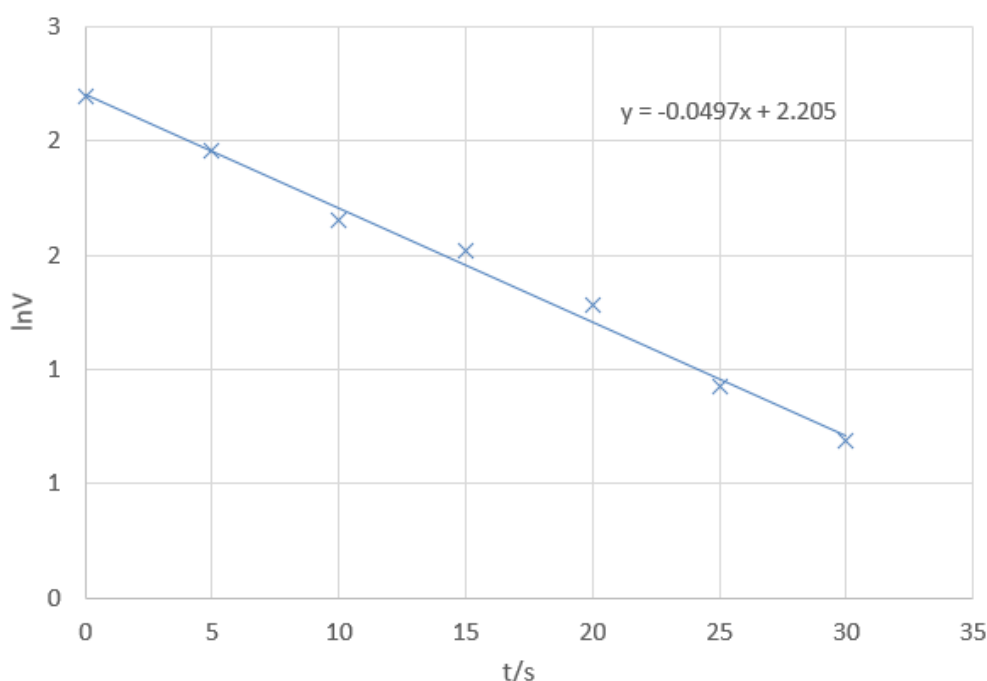
b) Rearrange the equation to have the form $y = mx + c$.

$$\ln V = \frac{t}{RC} + \ln V_0$$

So plot a graph of $\ln V$ against t .

The gradient will be equivalent to $-\frac{1}{RC}$ and the y-intercept will be equal to $\ln V_0$.

t/s	V/V	lnV
0	9.00	2.20
5	7.07	1.96
10	5.21	1.65
15	4.56	1.52
20	3.61	1.28
25	2.52	0.92
30	1.99	0.69
35	1.53	0.43



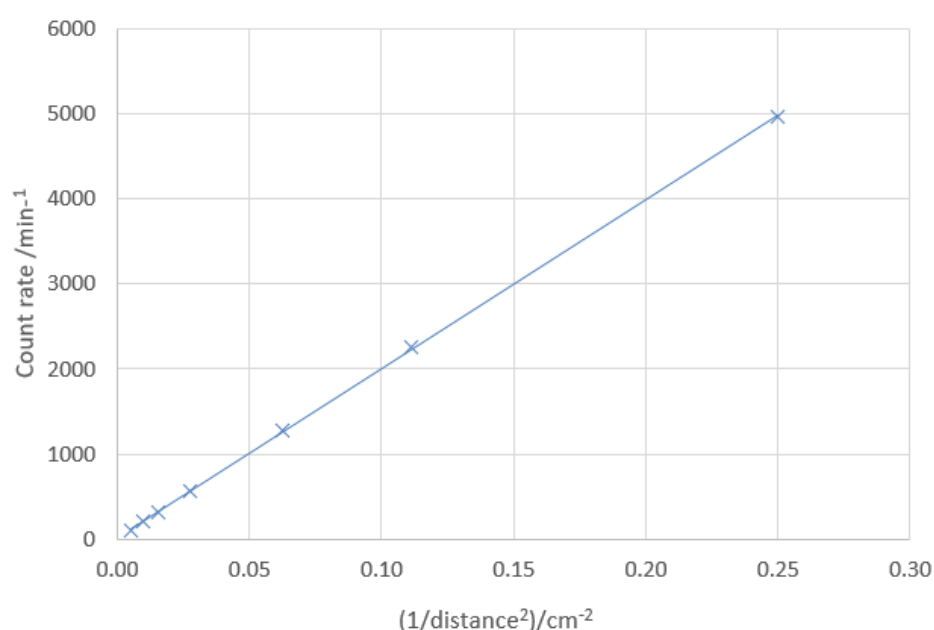
Gradient = -0.05 s^{-1} and $R = 220 \text{ k}\Omega$

$$\text{so } C = \frac{-1}{-0.05 \text{ s}^{-1} \times 220 \times 10^3 \Omega} = 91 \mu\text{F}$$

c) Percentage difference = $\frac{100 - 91}{100} \times 100\% = 9\%$

- 10 If the student is correct, a graph of count rate against $\left(\frac{1}{\text{distance}}\right)^2$ will be a straight line and it will pass through the origin if the count rate has been corrected. If the count rate has not been corrected, then there will be a systematic error as a result of background radiation. This will shift the line away from the origin, but it will still be a straight line.

Distance/cm	Count rate/min ⁻¹	(1/Distance ²)/cm ⁻²
2.0	4963	0.250
3.0	2254	0.111
4.0	1278	0.063
6.0	565	0.028
8.0	308	0.016
10.0	204	0.010
14.0	107	0.005
18.0	62	0.003



The graph is a straight line and Excel gives an intercept of 14. This suggests a background radiation level of 14 counts min⁻¹ which is a reasonable level, and the results therefore support the student's hypothesis.

- 11** For each paperclip chain the length and the period need to be measured.

Link together a number (e.g. 20) of paperclips.

Use a ruler to measure the length of the chain and record the data.

Suspend the chain from a fixed point such as clamp stand. Displace the chain to one side and note the position to which it is displaced.

Release the chain and record the time taken for 10 oscillations. The period is calculated by dividing this time by 10.

The timing could be repeated three times.

Repeat the measurements for at least 5 further lengths of chain.

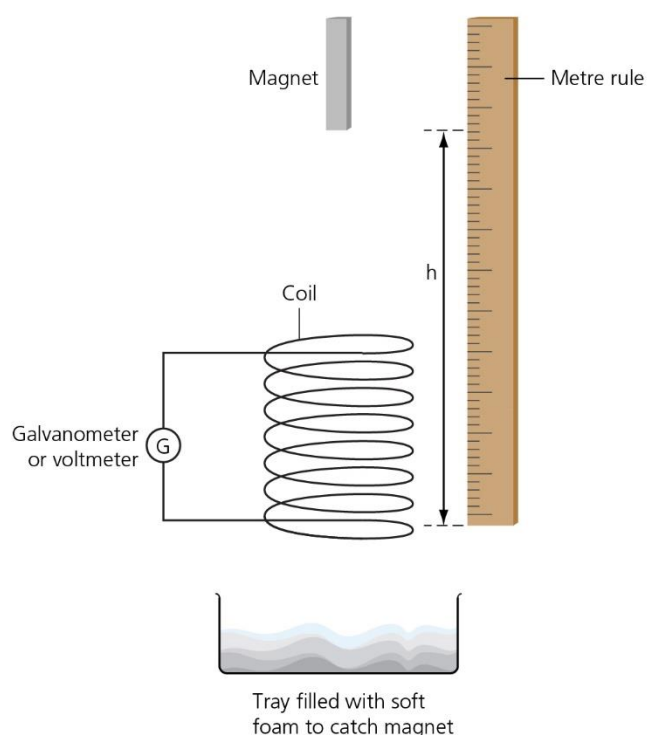
Chain must be released from same position each time, and in the same way. It should swing in the same plane. A marker should be used to indicate this position.

Using 10 oscillations reduces the effect of human error / reaction time on the measurement of period. Place a marker behind the equilibrium position and using that position to time the oscillations will further reduce error.

A graph of periodic time against $\sqrt{\text{length}}$ is plotted. A straight line graph through the origin would indicate that the relationship is correct.

Page 583 Practice questions

1 [5 key points indicated by individual marks below, supported by diagram and context = 6]

**Outline of procedure:**

Method to alter speed of magnet: change the height that the magnet is dropped from. Can either measure speed using timing device, or calculate speed (see below). [1]

Method to measure induced emf: galvanometer, voltmeter, CRO or voltage sensor linked to datalogger. [1]

Tray of soft foam/paper placed underneath the coil to catch the magnet as it falls.

Could also use a non-magnetic guide tube to ensure that magnet falls through the same path each time.

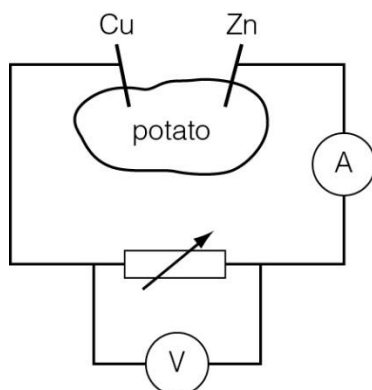
- Measure height from magnet to bottom of coil (h shown clearly in the diagram). [1]
- Drop magnet and record the induced emf as the magnet emerges from the bottom of the coil.
- Repeat 3 times, checking for possible anomalies each time.
- Change height between magnet and bottom of coil and repeat measurements.

Use of data

The speed can be calculated using the measured height in equation $v = \sqrt{2gh}$ or $v = 2h/t$ [1]

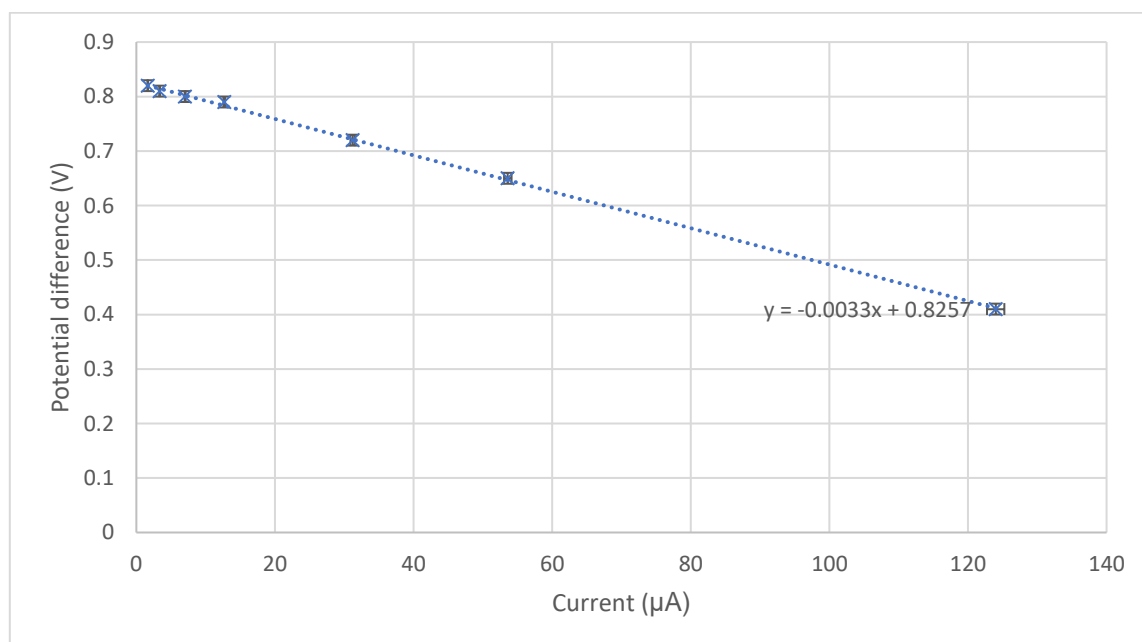
Plot a graph of E against v . If the graph is a straight-line going through the origin then the relationship is valid. [1]

- 2 a) Circuit shows potato as cell [1]; ammeter and voltmeter correctly positioned [1]



- b) Points correctly plotted [1]; error bars shown (for size of those in I, see table) [1]

$I/\mu\text{A}$	$V \pm 0.01/\text{V}$
1.64 ± 0.02	0.82
3.36 ± 0.03	0.81
7.04 ± 0.07	0.80
12.68 ± 0.12	0.79
31.20 ± 0.31	0.72
53.6 ± 0.54	0.65
124.00 ± 1.24	0.41



- c) Comparing $V = \mathcal{E} - Ir$ to $y = mx + c \Rightarrow$ the gradient of this graph is $-r$ and the intercept is \mathcal{E} . [1]
From graph above, $-r = -0.0033 \text{ V}/\mu\text{A}$

So $r = 3300 \Omega$ [1]

Highest and lowest values of r can be calculated from the gradients of alternative lines of best fit drawn through error bars. [1]

Error in final answer is half the range given by these values. [1]

- d) From graph above, $\mathcal{E} = 0.83 \text{ V}$ [1]

Highest and lowest values of \mathcal{E} are given by the intercepts of alternative lines of best fit drawn through error bars. [1]

Error in final answer is half the range given by these values. [1]