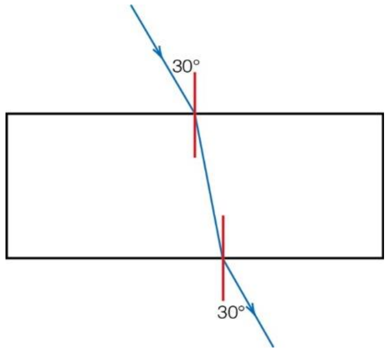
Page 501 Test yourself on prior knowledge

1



2 A laser is directed at a narrow slit with a width of between 10^{-6} m and 10^{-5} m. A white screen is placed on the other side of the slit. It is important that the slit is very narrow, as the wavelength of red laser light is about 6×10^{-7} m. If the slit is much larger than the suggestion above, diffraction will not easily be observed.

3 a) $d = 3 \times 10^8 \text{ m s}^{-1} \times (365 \times 24 \times 60 \times 60) \text{ s}$

$= 9.46 \times 10^{15}$ m

b) $d = 9.46 \times 10^{15} \text{ m} \times 2 \times 10^9$

$= 1.9 \times 10^{25}$ m

4 a) Assuming all galaxies have the same number of stars:

$n = 300 \times 10^9 \times 200 \times 10^9$

$= 6 \times 10^{22}$

b) Mass of hydrogen in the sun = $0.75 \times 2.0 \times 10^{30}$ kg = 1.5×10^{30} kg

Mass of hydrogen = mass of hydrogen atom x number of hydrogen atoms in the sun (n)

$$n = \frac{1.5 \times 10^{30} \text{ kg}}{1.67 \times 10^{-27} \text{ kg}}$$

$= 9 \times 10^{56}$ atoms of hydrogen

Number of hydrogen atoms in the universe, N,

= n × no of stars in the universe.

$N = 9 \times 10^{56} \times 6 \times 10^{22} = 5.4 \times 10^{79}$

$\approx 10^{80}$ hydrogen atoms

Here we have assumed all stars are roughly the same size and we have ignored any dark matter between stars.

Page 505 Activity

A simple model telescope

1 You should see an inverted, virtual, magnified image.

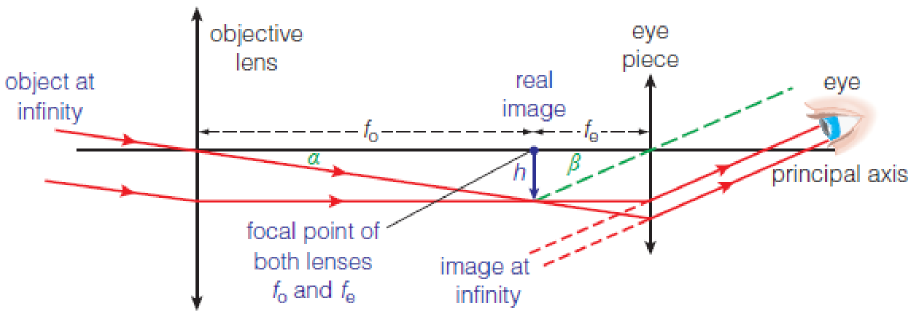
2 For an objective lens of focal length 50 cm and an eyepiece of focal length 10 cm:

$M = \frac{f_o}{f_e}$

$= \frac{50}{10}$

$= 5$

3



Page 505 Test yourself

1 **a)** $\tan\alpha = \frac{1.7\text{ m}}{10\text{ m}} = 0.17$

$\alpha = 0.168$ rad $= 0.17$ rad (2 sf)

b) $\tan\alpha = \frac{1.7\text{ m}}{120\text{ m}} = 0.0142$

$\alpha = 0.0142$ rad

c) The approximation is valid in (b) to 3 significant figures in (a) to two significant figures, which is appropriate since the height of the man is given to two significant figures.

2 **a)** A long focal length objective lens projects a larger real image.

b) A short focal length eyepiece magnifies the image from the objective lens more powerfully; the overall magnification is $\frac{\text{fo}}{\text{fe}}$.

3 $L = f_o + f_e$

$= 2.50\text{ m} + 0.04\text{ m}$

$= 2.54\text{ m}$

4 **a)** The long focal length projects a large image, so the magnification of the telescope is greater.

b) $M = \frac{10.5\text{ m}}{0.05\text{ m}}$

$= 210$

5 $M = \frac{\text{fo}}{\text{fe}} = \frac{2.40\text{ m}}{0.04\text{ m}}$

$= 60$

So $\theta = 0.05° \times 60$

$= 3°$

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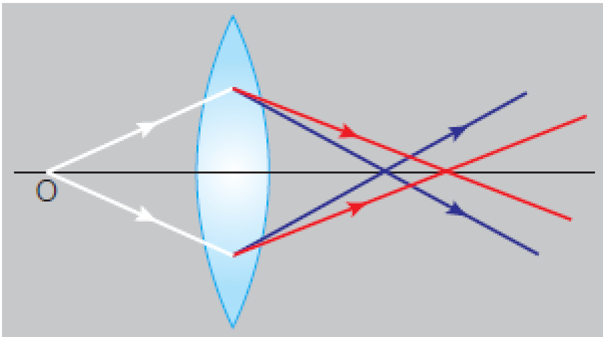
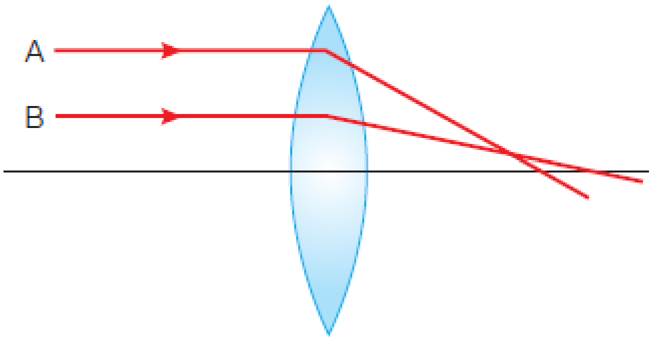
6 a) Chromatic aberration occurs because the refractive index of a material varies with wavelength so a lens has different focal lengths for light of different wavelengths. This results in a blurred image or/and the appearance of coloured fringes.



b) Spherical aberration causes blurring of images because light passing through different parts of the lens is brought to a focus at a slightly different position.



7 They are cheaper.
It is easier to correct spherical aberration using a parabolic mirror.
Chromatic aberration is reduced as light does not pass through an objective lens.
It is possible to make larger diameter reflectors than refractors and so collect more light.

8 Compare the collecting power of each telescope:

$$\frac{A_2}{A_1} = \left(\frac{28}{12}\right)^2 = 5.4$$

So, to collect the same amount of light, the exposure time should be: $\frac{16}{5.4} = 3$ s

Page 510 Test yourself

9 Since the angle is small, the angular separation of the lamps is:

$$\theta = \frac{x}{y}$$

$$= \frac{1.5\text{ cm}}{600\text{ cm}}$$

$$= 0.0025 \text{ rad}$$

a) $\frac{\lambda}{D} = \frac{6.5\times10^{-7}\text{m}}{2.2\times10^{-4}\text{m}}$

$= 0.003$ rad

So the student sees just one source of light as she cannot resolve them.

b) $\frac{\lambda}{D} = \frac{5.4\times10^{-7}\text{m}}{2.2\times10^{-4}\text{m}}$

$= 0.025$ rad

She can just resolve the two sources.

c) $\frac{\lambda}{D} = \frac{4.7\times10^{-7}\text{m}}{2.2\times10^{-4}\text{m}}$

$= 0.0021$ rad

She can now see two sources of light clearly.

10 a) The 'effective diameter' of the telescope is reduced to 0.82 m due to atmospheric blurring.

So $\frac{\lambda}{D} \approx \frac{4\times10^{-7}\text{m}}{0.82}$

$= 4.9 \times 10^{-7}$ rad

So 4.9×10^{-7} rad $\approx \frac{x}{2.2\times10^{6}\text{ ly}}$

$x \approx 4.9 \times 10^{-7}$ rad $\times 2.2 \times 10^{6}$ ly

$= 1.1$ ly

b) $\frac{\lambda}{D} \approx \frac{4\times10^{-7}\text{m}}{2.4\text{ m}}$

$= 1.7 \times 10^{-7}$ rad

$1.7 \times 10^{-7} \approx \frac{x}{2.2\times10^{6}\text{ ly}}$

$x = 0.4$ ly

11 Angular separation of the lines is:

$\theta \approx \frac{x}{y}$

$= \frac{1\times10^{-3}\text{m}}{5\text{ m}}$

$= 2 \times 10^{-4}$ rad

$\frac{\lambda}{3\times10^{-3}\text{ m}} = 2 \times 10^{-4}$

$\lambda = 6 \times 10^{-7}$ m

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12 The minimum angle which a telescope can resolve is: $\theta = \frac{\lambda}{D}$

Because λ is very large for radio waves – perhaps as large as 3 m, D must be large to allow accurate resolution.

13 a) See Figure 27.17 (page 512) and Figure 27.20 (page 513).
X-rays need to strike a surface at a grazing angle to be focused. Light and other radiations can be reflected and focused using a Cassegrain arrangement of mirrors.

b) X-rays do not penetrate the earth's atmosphere. So the telescope is placed above the atmosphere.

14 a) The smallest angle a telescope can resolve is given by: $\theta = \frac{\lambda}{D}$

Herschel detects longer wavelengths than WISE, so a wider diameter improves Herschel's resolving power.

b) Collecting power $\propto D^2$

So $\frac{\text{Power (Herschel)}}{\text{Power (WISE)}} = \frac{350^2}{40^2}$

$= 77$

c) i) $\theta_H = \frac{\lambda}{D}$

$= \frac{50 \times 10^{-6}\ \text{m}}{3.5\ \text{m}}$

$= 1.4 \times 10^{-5}$ rad

$\theta_W = \frac{\lambda}{D}$

$= \frac{3 \times 10^{-6}\ \text{m}}{0.4\ \text{m}}$

$= 7.5 \times 10^{-6}$ rad

ii) $\theta_H = \frac{670 \times 10^{-6}\ \text{m}}{3.5\ \text{m}}$

$= 1.9 \times 10^{-4}$ rad

$\theta_W = \frac{25 \times 10^{-6}\ \text{m}}{0.4\ \text{m}}$

$= 6.3 \times 10^{-5}$ rad

Page 515 Test yourself

15 a) A star's apparent magnitude is a measure of its brightness as it appears in the sky. It is a logarithmic scale with lower numbers (including negative values) indicating brighter stars.

b) A is brighter

c) $(2.512)^6 = 250$

16 a) 2.512 (or 2.5)

b) $(2.512)^{3.1} = 17.4$

Page 517 Test yourself

17 a) i) α centauri 4.13×10^{16} m / 9.46×10^{15} m/light year = 4.4 light years

β centauri 3.31×10^{16} m / 9.46×10^{15} m/light year = 350 light years

ii) α centauri 1.3 pc

β centauri 107 pc

b) 1 Mpc = 3.26×10^6 ly = $3.26 \times 10^6 \times 9.46 \times 10^{15}$ m = 3.08×10^{22} m

Virgo cluster 5.0×10^{23} m / 3.08×10^{22} m/Mpc = 16 Mpc

Corona Borealis 1.1×10^{25} m / 3.08×10^{22} m/Mpc = 360 Mpc (2 sf)

Page 518 Test yourself

18 a) $m - M = 5\log\left(\frac{d}{10}\right)$

$M = m - 5\log\left(\frac{d}{10}\right)$

$= 4.8 - 5\log\left(\frac{1800}{10}\right)$

$= 4.8 - 5 \times 2.25$

$= -6.5$

b) $M = m - 5\log\left(\frac{d}{10}\right)$

$M = -26.7 - 5\log((4.8 \times 10^{-6})/10)$

$= -26.7 - 5 \times (-6.3)$

$= 4.9$

19 $m = M + 5\log\left(\frac{d}{10}\right)$

$= -5.0 + 5\log\left(\frac{70}{10}\right)$

$= -5.0 + 5 \times 0.85$

$= -0.77$

20 a) $M = m - 5\log\left(\frac{d}{10}\right)$

Capella's distance = 42 ly / 3.26 ly pc^{-1} = 12.9 pc

M(Capella) $= 0.1 - 5\log\left(\frac{12.9}{10}\right)$

$= 0.1 - 5 \times 0.1$

$= -0.5$

Vega's distance = 25 ly / 3.26 ly pc^{-1} = 7.7 pc

M(Vega) $= 0 - 5\log\left(\frac{7.7}{10}\right)$

$= -5 \times -0.12$

$= 0.6$

b) Capella is one magnitude brighter than Vega so it emits about 2.512 ≈ 2times as much light.

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21 Temperature and radius (or surface area).

22 a) $P = \sigma A T^4$

$= 5.7 \times 10^{-8}\ \text{W m}^{-2}\ \text{K}^{-4} \times 4\pi \times (3.2 \times 10^{10}\ \text{m})^2 \times (6\,015\ \text{K})^4$

$= 9.6 \times 10^{29}\ \text{W}$

b) $T^4 = \frac{P}{\sigma A} = \frac{3.6\times10^{31}\ \text{W}}{5.7\times10^{-8}\ \text{Wm}^{-2}\text{K}^{-4}\times4\pi(1.1\times10^{10}\ \text{m})^2}$

$T^4 = 4.16 \times 10^{17}\ \text{K}^4$

$T = 25\,000\ \text{K}$

c) $A = \frac{P}{\sigma T^4}$

$= \frac{4\times10^{25}\ \text{W}}{(5.7\times10^{-8}\text{Wm}^{-2}\text{K}^4)\times(3900\ \text{K})^4}$

$= 3.0 \times 10^{18}\ \text{m}^2$

So $4\pi r^2 = 3.0 \times 10^{18}\ \text{m}^2$

$r = 4.9 \times 10^8$ m or 4.9×10^5 km

23 $P_{sun} = \sigma A T^4$

$= 4\pi \sigma r^2 T^4$

$P_{star} = 4\pi \sigma (4r)^2(3T)^4$

$= 4\pi \sigma r^2 T^4 \times 16 \times 81$

$= P_{sun} \times 1296$

24 a) $4\pi r^2 = \frac{P}{\sigma T^4} = \frac{1.4\times10^{24}\ \text{W}}{(5.7\times10^{-8}\text{Wm}^{-2}\text{K}^4)\times(3100\ \text{K})^4}$

$= 2.65 \times 10^{17}\ \text{m}^2$

$r = 1.5 \times 10^8$ m or 1.5×10^5 km

b) $\lambda_{max} T = 2.9 \times 10^{-3}$ m K

$\lambda_{max} = \frac{2.9\times10^{-3}\ \text{m K}}{3100\ \text{K}}$

$= 935$ nm

This is in the infrared.

c) The star is dull anyway – about 300 times less power emitted than the sun. Also a lot of light is emitted as infrared radiation (which we cannot see).

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25 This is the lowest energy state of the atom, with the electrons in their lowest energy levels.

26 When light passes through the cooler gases in the outer atmosphere of a star, atoms may absorb light of discrete wavelengths corresponding to electron transitions between energy levels in the atoms of the atmosphere. When light is absorbed in this way, the intensity of these wavelengths is reduced, so black lines appear across the continuous spectrum.

27 A series of lines in an absorption spectrum produced by electron transitions from the n = 2 energy levels to higher levels.

28 a) $E = \frac{hc}{E}$

$$\lambda = \frac{hc}{E}$$

$$= \frac{6.6\times10^{-34}\,\text{J s}\times3.0\times10^{8}\,\text{ms}^{-1}}{(13.6-3.4)\times1.6\times10^{-19}\,\text{J}}$$

= 120 nm

This is in the ultraviolet part of the spectrum.

b) $E = \frac{hc}{\lambda}$

$$= \frac{6.6\times10^{-34}\,\text{J s}\times3.0\times10^{8}\,\text{ms}^{-1}}{(3.4-0.8)\times1.6\times10^{-19}\,\text{J}}$$

= 480 nm

This is visible light.

29 The absorption lines may be produced by different elements and even those produced by the same element may arise from different electron transitions.
Cooler stars emit radiation with longer wavelengths/less energy than hotter stars and the photons may not have sufficient energy to promote electrons between levels in e.g. hydrogen, so the most prominent absorption lines are produced by e.g. metallic atoms or ions.
The electrons in the atoms of a hotter star's atmosphere may already be above the ground state, so the spectrum may contain absorption lines corresponding to transitions from this level (such as the Balmer series for hydrogen).

Page 524 Test yourself

30 a) A main sequence star is one in which radiation is produced by the thermonuclear fusion of hydrogen nuclei into helium nuclei.

b) A red giant is a large, low-temperature, luminous star in which helium nuclei are fused fuse into more massive nuclei such as beryllium, carbon and oxygen.

c) A white dwarf is an extremely dense, hot star powered by the gravitational potential energy released as it contracts rather than by nuclear fusion.

31 Stars on the main sequence with high luminosities are massive and very bright, perhaps 10^6 times brighter than the sun, so nuclear fuel is used at a higher rate – 10^6 times faster than the sun in the example given. If a star has a mass of 100 times the sun it will live about $100/10^6$ or about 10^{-4} times as long: a few million years.

32 a) L = σ A T_4

$$A = \frac{L}{\sigma T_4}$$

$$= \frac{7.6\times10^{23}\text{W}}{5.7\times10^{-8}\text{Wm}^{-2}\text{K}^{-4}\times(25200\text{ K})^4}$$

= 3.3 × 10^{13} m^2

b) $4\pi r^2$ = 3.3 × 10^{13} m^2

r = 1.6 × 10^6 m or 1 600 km

33 $\rho = \frac{M}{V}$

$$= \frac{2\times10^{30}\text{ kg}}{\frac{4}{3}\pi(1.6\times10^{6}\text{m})^3}$$

= 1 × 10^{11} kg m^{-3}

This is very large indeed; the density of water is 10^3 kg m^{-3}.

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34 a) The nuclei must have large kinetic energies, which are sufficient to overcome the strong repulsive forces. The kinetic energy must be enough to transfer to the electrostatic potential energy which the protons have in close contact.

b) There are greater charges on the nuclei so the electrostatic potential energy is greater when the nuclei are in contact. So the initial kinetic energy, and therefore the temperature, must be greater.

c) The reasoning is the same as in (b). The electrostatic potential energy of two silicon nuclei close together is greater than that of a silicon and a helium nucleus.

Page 530 Test yourself

35 $\rho = \frac{M}{V}$

$$= \frac{4\times10^{30}\text{ kg}}{\frac{4}{3}\pi(14\times10^{3}\text{m})^3}$$

= 3 × 10^{17} kg m^{-3}

This is very large indeed; the density of water is 10^3 kg m^{-3}.

36 $R_s = \frac{2GM}{c^2}$

$$= \frac{2\times6.7\times10^{-11}\text{Nm}^2\text{kg}^{-2}\times100\times10^{6}\times2\times10^{30}\text{ kg}}{(3\times10^{8}\text{ms}^{-1})^2}$$

= 3 x 10^{11} m or 3 × 10^8 km

37 a) A collapsed star made of neutrons. It has a very high density.

b) A highly condensed state of matter that has an escape velocity higher than the speed of light.

c) A star or supernova of known brightness that can be used to calculate galactic distances.

d) A brief intense emission of gamma rays from a collapsing supergiant star.

e) The explosion of a massive star when it has run out of nuclear fuel.

38 a) A Type 1a supernova produces a nickel isotope which decays to cobalt that in turn decays to iron. Both these decay processes have relatively short half-lives (6 and 77 days respectively) so the light curve changes in a characteristic way over the first few hundred days.

b) Type 1a supernovae all have approximately the same absolute magnitude because they always occur in stars of about 1.4 solar masses.

39 $M = m - 5\log\left(\frac{d}{10}\right)$

$-19 = 11 - 5\log\left(\frac{d}{10}\right)$

$-30 = -5\log\left(\frac{d}{10}\right)$

$6 = \log\left(\frac{d}{10}\right)$

$10^6 = \left(\frac{d}{10}\right)$

So $d = 10 \times 10^6$ pc

$= 10$ Mpc

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40 a) The wavelength of the light changes because sometimes the star is moving towards us and sometimes away from us. When the wavelength is 655.9 nm the star is moving towards us; when the wavelength is 656.7 nm it is moving away from us.

b) $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$

$v = \frac{0.4 \text{ nm}}{656.3 \text{ nm}} \times 3 \times 10^8 \text{ m s}^{-1}$

$= 1.8 \times 10^5 \text{ m s}^{-1}$

41 $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$

$v = \frac{\Delta\lambda}{\lambda} \times 3 \times 10^8 \text{ m s}^{-1}$

$\frac{541 \text{ nm} - 486 \text{ nm}}{486 \text{ nm}} \times 3 \times 10^8 \text{ m s}^{-1}$

$= 0.11 \times 3 \times 10^8 \text{ m s}^{-1}$

$= 3.4 \times 10^7 \text{ m s}^{-1}$

42 This is a poor defence – he was going dangerously fast!

$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$

$\frac{\Delta\lambda}{\lambda} = \frac{650 \text{ nm} - 530 \text{ nm}}{650 \text{ nm}} = 0.18$

So, had he been able to see the colour change in this way, he would have been travelling at 0.18 of the speed of light.

Page 535 Test yourself

43 The three pieces of evidence are outlined as in the text are:

- Hubble's Law
- The cosmic microwave radiation.
- The percentage of hydrogen and helium in the early universe, as evidenced by the oldest objects in the universe. (75% H; 25% He)

44 a) v = H d

= 67.8 km s^{-1} Mpc^{-1} × 200 Mpc

= 13 600 km s^{-1}

b) v = H d

$d = \frac{v}{H}$

$= \frac{120\ 000 \text{ km s}^{-1}}{67.8 \text{ km s}^{-1} \text{ Mpc}^{-1}}$

= 1770 Mpc

45 a) $\frac{v}{c} = 0.22$

So v = 0.22 c

= 0.22 × 3 × 10^5 km s^{-1}

= 66 000 km s^{-1}

b) v = H d

So $d = \frac{v}{H}$

$= \frac{66\ 000 \text{ km s}^{-1}}{20.7 \text{ km s}^{-1} \text{ Mly}^{-1}}$

= 3200 Mly = 3.2 billion ly

Page 536 Test yourself

46 A quasar is a quasi-stellar object – a point-like, star-like object. But it is very bright.

47 High luminosity, high red shift, small size, strong radio source.

48 a) $M = m - 5 \log\left(\frac{d}{10}\right)$

$= 13 - 5 \log\left(\frac{10^9}{10}\right)$

$= 13 - 5 \log 10^8$

= 13 – 40

= –27

b) A magnitude of –27 is 5 magnitudes brighter than –22, which is equivalent to a factor of 100 in luminosity.

49 a) i) $V = -\frac{GM}{r}$

$= \frac{-(6.7\times10^{-11}\text{Nm}^{-2}\text{kg}^{-2})\times10^{39}\text{kg}}{3\times10^{12}\text{m}}$

$= -2 \times 10^{16}$ J kg^{-1} (1 sf)

ii) $\Delta PE = m\Delta V$

$= 2 \times 10^{16}$ J kg^{-1} $\times 2 \times 10^{30}$ kg

$= 4 \times 10^{46}$ J

b) i) Potential energy transferred to electromagnetic wave energy is:

4×10^{46} J $\times 20 \times 0.3 = 2.4 \times 10^{47}$ J

$L = \frac{2.4\times10^{47}\text{J}}{(365\times24\times3600)\text{s}}$

$= 8 \times 10^{39}$ W

ii) The quasar is $\frac{8\times10^{39}\text{W}}{4\times10^{26}\text{W}} = 2 \times 10^{13}$ times more luminous than the sun.

Page 538 Test yourself

50 You should discuss each of (see the text):

- Direct imaging
- Doppler shift detection
- Transit method

51 $\frac{\text{change in luminosity}}{\text{luminosity}} = \frac{\text{Area of a planet's disc}}{\text{Area of a star's disc}}$

$\frac{4}{100} = \frac{A_p}{A_s} = \frac{\pi r_p{}^2}{\pi r_s{}^2}$

So $\frac{r_p}{r_s} = \left(\frac{4}{100}\right)^{\frac{1}{2}}$

$= \frac{2}{10} = 0.2$

52 Our solar system has four gas giants. They are not as big as the planets in the HR 8799 system, but the orbit of Jupiter has a smaller diameter than that of HR 8799e. There may, therefore, be rocky planets in closer orbits, like our planets, Mercury, Venus, Earth, Mars.

Page 539–544 Test yourself

1 D

2 B

3 C

4 C

5 A

6 C

7 C

8 A

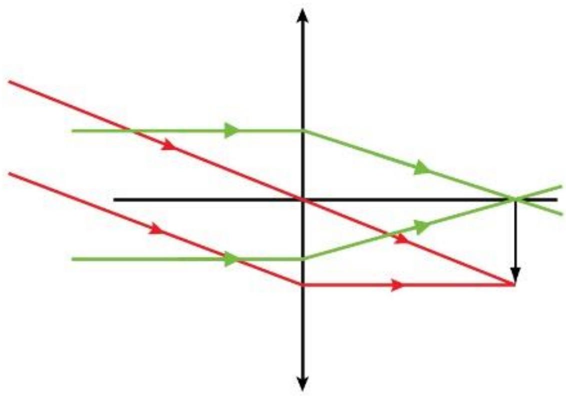
9 D

10 B

11 a) Incident rays from top of object (red) focused correctly [1]

Incident rays from bottom of object (green) focused correctly [1]

(a)

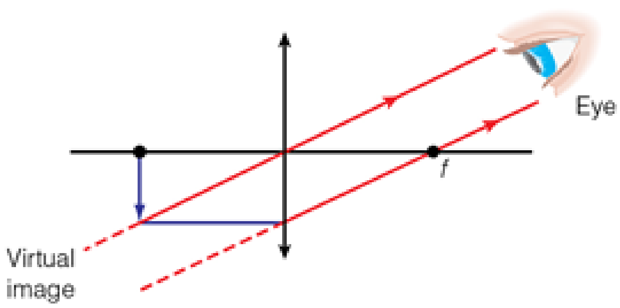


b) Undeviated ray through C [1]

Ray from top of real image parallel to principal axis refracted through f on other side [1]

Position of eye [1]

(b)



c) $f_o + f_e = 228$ cm and $\frac{f_o}{f_e} = 75$ [1]

So $75 f_e + f_e = 228$ cm

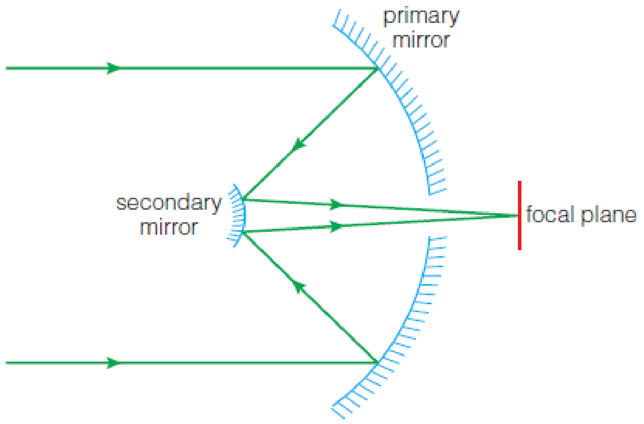
$76 f_e = 228$ cm

$f_e = 3$ cm and f_o 225 cm [1]

d) Red light travels slightly faster through glass than blue light. [1]

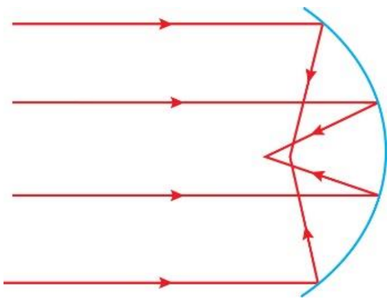
So red and blue light refract/bend by different amounts and come to a focus at different places [1].

12 a) [1 mark for correct rays, 1 mark for correct telescope construction.]



b) i) The reflector reduces chromatic aberration because light does not pass through a thick objective lens. Instead, it is reflected off a metal mirror. [1]

ii)



Rays from different parts of the mirror do not focus at the same point. [1]

iii) Mirrors are made in a parabolic shape. [1]

c) $\theta \approx \frac{\lambda}{D}$ [1]

$= \frac{670 \times 10^{-9}\ \text{m}}{0.3\ \text{m}}$

$= 2.2 \times 10^{-6}$ rad [1]

d) $\theta = \frac{4800\ \text{km}}{1400 \times 10^{6}\ \text{km}}$

$= 3.4 \times 10^{-6}$ rad [1]

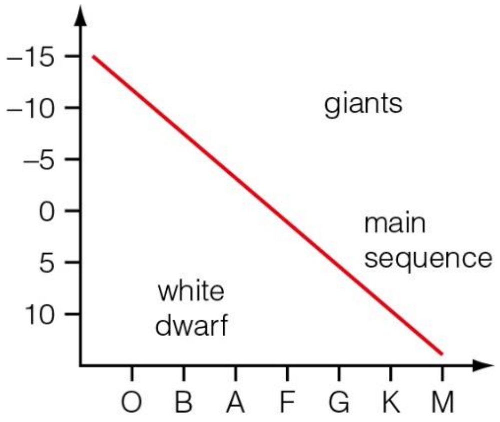
So $\frac{\lambda}{D} \approx 3.4 \times 10^{-6}$ rad

Choosing the red end of the spectrum λ = 700 nm,

$D = \frac{700 \times 10^{-9}}{3.4 \times 10^{-6}}$ m [1]

$= 0.2$ m [1]

13 a) main sequence sloping as shown below [1]; dwarfs and giants correctly placed [1]; appropriate figures added to magnitude scale [1]



b) i) $\lambda_{max}T = 2.9 \times 10^{-3}$ mK [1]

$T = \frac{2.9 \times 10^{-3}\ \text{mK}}{2.7 \times 10^{-7}\ \text{m}}$

= 10 700 K [1]

ii) $P = \sigma A T^4$

$P_s = \sigma\, 4\pi\, r_s^2\, (5700)^4$

$P_A = 100\, P_s = \sigma\, 4\pi\, r_A^2\, (10\,700)^4$ [1]

So $\frac{P_A}{P_s} = 100 = \frac{r_A^2\,(10\,700)^4}{r_S^2\,(5800)^4}$ [1]

So $\left(\frac{r_A}{r_s}\right)^2 = 100 \times \frac{(5800)^4}{(10\,700)^4}$

$\frac{r_A}{r_s} = 10 \times \left(\frac{5\,800}{10\,700}\right)^2$

= 2.9

So $r_A = 2.9 \times 6.96 \times 10^8$ m $\approx 2.1 \times 10^9$ m [1]

c) Light of all wavelengths is emitted from Alioth's black body surface producing a continuous spectrum [1]

There is some hot hydrogen in the star's atmosphere. [1]

The hydrogen is so hot that electrons are in the excited level n = 2. [1]

Light of particle wavelengths have photons of the right energy to take the electrons from n = 2 to higher (n = 3 or n = 4) levels. [1]

This light is absorbed then re-emitted in all directions, so reducing the intensity, [1]

so these wavelengths look black against the continuous spectrum. [1]

14 a) The boundary of the region inside of which light cannot escape. [1]

b) i) $r = \frac{2GM}{c^2}$ [1]

$= \frac{2 \times 6.7 \times 10^{-11} \times \text{N kg}^{-2}\,\text{m}^{-2} \times 8 \times 10^7 \times 2.0 \times 10^{30}\ \text{kg}}{(3 \times 10^8\ \text{m s}^{-1})^2}$

$= 2.4 \times 10^{11}$ m [1]

ii) $\rho = \frac{m}{V}$ [1]

$$= \frac{8 \times 10^{7} \times 2.0 \times 10^{30}\ \mathrm{kg}}{\frac{4}{3}\pi\,(2.4 \times 10^{11}\ \mathrm{m})^3}$$

$= 2800\ \mathrm{kg\ m^{-3}}$ [1]

(The density inside a large black hole is less than the density in a small black hole.)

15 a) i) Apparent magnitude is an arbitrary scale against which stars have been judged and defined as 1st, 2nd, 3rd magnitude stars, etc. [1]

ii) Absolute magnitude is the visual brightness that a star would have at a distance of 10 parsecs. [1]

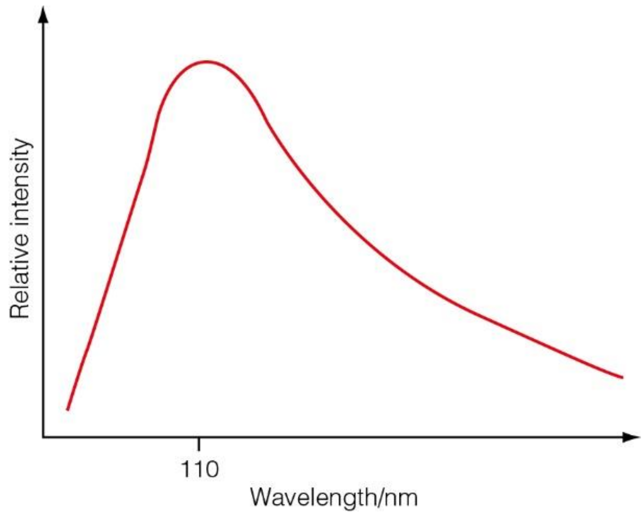
b) Betelgeuse – they both have about the same absolute magnitude, but the apparent magnitude of Betelgeuse is greater by 1.3 magnitudes so it must be closer. [1]

c) i) $\lambda_{max}T = 2.9 \times 10^{-3}$ mK [1]

$$\lambda = \frac{2.9 \times 10^{-3}\ \mathrm{mK}}{26\,200}$$

$= 110$ nm [1]

ii) steep climb to maximum [1]; shallow concave fall [1]; peak value marked as 110 nm [1]



d) i) O

ii) Helium

iii) Betelgeuse is too cold to excite electrons to the n = 2 state in hydrogen.

e) Alnilam has a higher luminosity than Betelgeuse because it is at a higher temperature and luminosity is given by $P = \sigma A T^4$ (However, Betelgeuse has a greater area – about 1300 times greater – which reduces the difference.) [1]

Although Alnilam has greater luminosity, its spectrum peaks in the ultra-violet so much of its radiation is in the wavelengths below visible light, reducing its visual magnitude to a level similar to that of Betelgeuse: a greater fraction of Betelgeuse's radiation is in the visible wavelengths. [1]

16 Less chromatic aberration. [1]

Less spherical aberration as mirrors are easier to make than lenses. [1]

Mirrors can be made larger than lenses [1]

which means more light (collecting power ∝ D^2) [1]

and better resolution since $\theta \propto \frac{\lambda}{D}$ [1]

Reflectors can be used for wavelengths outside the visible. [1]

17 Any two points for each of three regions of the spectrum: [6]

Radio waves

- D must be large for resolution because $\theta \approx \frac{\lambda}{D}$ and λ is long.
- Radio telescopes must be on earth as they are huge.
- Radio waves penetrate the atmosphere.

Ultraviolet

- Ultraviolet telescopes are best in space due to atmospheric absorption.
- Since λ is small, resolution is better than for telescopes operating in the visible
- even for relatively small D.

X –ray

- X-ray telescopes are best in space due to atmospheric absorption.
- X-rays are focused by grazing reflections off mirrors.

18 a) $\frac{v}{c} = 0.367$ [1]

So v $= 0.367 \times 3 \times 10^8$ m s^{-1}

$= 1.10 \times 10^8$ m s^{-1} or 1.1×10^5 km s^{-1} [1]

v = Hd [1]

So d $= \frac{v}{H}$

$= \frac{1.1 \times 10^5 \text{ km s}^{-1}}{65 \text{ km s}^{-1} \text{ Mpc}^{-1}}$

= 1700 Mpc [1]

b) i) They are unusually strong radio sources.

ii) Quasars are supermassive black holes in the centre of young galaxies. [1]
They are swallowing perhaps 100 solar masses a year. [1]
The gravitational potential energy of infalling stars is transferred to electromagnetic radiation. [1]

19 a) i) As a galaxy recedes the light is stretched – a Doppler shift to longer (red) wavelengths. [1]

ii) v $= 0.048$ c [1]

$= 0.048 \times 3 \times 10^8$ m s^{-1}

$= 1.4 \times 10^7$ m s^{-1} = 14 400 km s^{-1} [1]

iii) v = Hd

$d = \frac{v}{H}$

$= \frac{14\,400 \text{ km s}^{-1}}{65 \text{ km s}^{-1} \text{ Mpc}^{-1}}$

= 220 Mpc [1]

b) i) Standard absolute magnitude of about −19. [1]

The light curve is always the same shape due to the decay of radioisotopes Co, Ni. [1]

ii) $M = m - 5\log\left(\frac{d}{10}\right)$

$-19.2 = 17.3 - 5\log\left(\frac{d}{10}\right)$ [1]

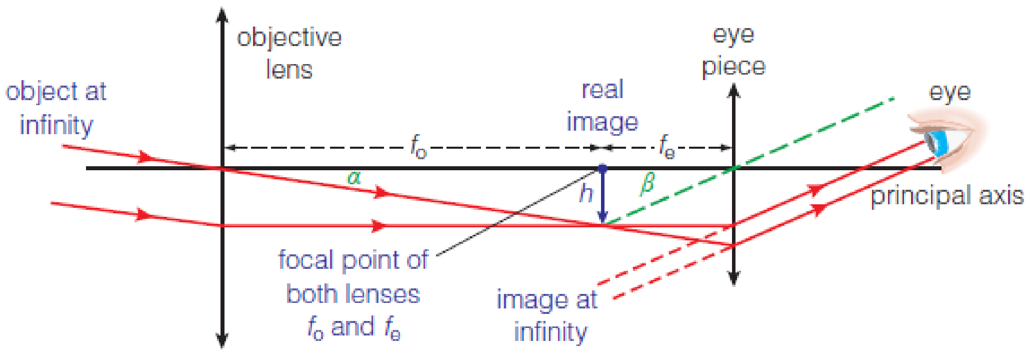
$36.5 = 5\log\left(\frac{d}{10}\right)$

$7.3 = \log\left(\frac{d}{10}\right)$

$\frac{d}{10} = 10^{7.3} = 200 \times 10^6$ [1]

d = 200 Mpc [1]

20 a) [1 mark for lenses; 1 mark for rays converging at real image; 1 mark for virtual image at infinity]



b) $f_o + f_e = 2.05$ m and $\frac{f_o}{f_e} = 40$

So $40 f_e + f_e = 2.05$ m

$41 f_e = 2.05$ m

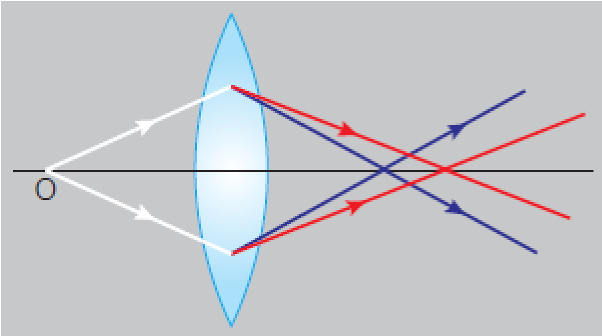
$f_e = 0.05$ m = 5 cm [1]

$f_o = (2.05 \text{ m} - 0.05 \text{ m}) = 2$ m = 200 cm [1]

c) $\theta = 40 \times \frac{7 \times 10^4}{7.8 \times 10^8}$ [1]

= 0.0036 rad [1]

d) Diagram shows different colours of light have different focal points [1] but if blue to a focus inside red [2]



21 a) A main sequence star is converting hydrogen to helium by thermonuclear fusion. [1]
A red giant is a large star which is burning helium (or heavier elements) in its core. [1]

b) $M = m - 5\log\left(\frac{d}{10}\right)$

$-2.5 = 3.1 - 5\log\left(\frac{d}{10}\right)$ [1]

$-5.6 = -5\log\left(\frac{d}{10}\right)$

$\log\left(\frac{d}{10}\right) = \frac{5.6}{5}$

$= 1.12$

$\frac{d}{10} = 10^{1.12}$ [1]

So $d = 10 \times 13.2$

$= 132$ pc [1]

c) $P = \sigma A T^4$

$\frac{P_A}{P_B} = \frac{\sigma\, 4\pi\, r_A^2\, T_A^4}{\sigma\, 4\pi\, r_B^2\, T_B^4}$ [1]

$= \left(\frac{r_A}{r_B}\right)^2 \times \left(\frac{T_A}{T_B}\right)^4$

$= \left(\frac{50000}{2000}\right)^2 \times \left(\frac{4300}{12900}\right)^4$

$= 25^2 \times \left(\frac{1}{3}\right)^4$ [1]

$= 7.7$ [1]

d) The difference between the magnitudes is $-0.3 - (-2.5) = 2.2$

So Albireo A is $(2.51)^{2.2}$ times brighter than Albireo B, which is 7.6. [2]

22 a) About 0.01 Mly [1]

Any sensible answer is acceptable here as you might be able to resolve a smaller distance than this.

b) $\theta = \frac{0.01\ \text{Mly}}{600\ \text{Mly}}$ [1]

$\theta = 1.7 \times 10^{-5}$ rad [1]

c) $\theta = \frac{\lambda}{D}$ [1]

$D = \frac{0.15\ \text{m}}{1.7 \times 10^{-5}}$

= 8800 m or about 9 km [1]

Page 545 Stretch and challenge

23 a) $P = \sigma A T^4 = \sigma 4\pi r^2 T^2$

Define the sun as having radius = 1, T = 1

The luminosity of the red giant is:

$P = \sigma\, 4\pi \times 200^2 \times 0.5^4$

So the ratio

$\frac{P}{P_S} = \frac{\sigma\, 4\pi\, (200)^2 \times (0.5)^4}{\sigma\, 4\pi\, (1)^2 \times (1)^4}$

$P = 2500\ P_s$

b) A difference of 1 magnitude = 2.51

So $(2.51)^n = 2500$

So $\log(2.51)^n = \log(2500)$

$n \log(2.51) = \log(2500)$

$n = \frac{\log(2500)}{\log(2.51)}$

$= \frac{3.4}{0.4}$

= 8.5

Therefore the red giant has an absolute magnitude of

4.6 − 8.5 = −3.9

24 $\lambda_0 = \lambda_s \dfrac{\left(1+\frac{v}{c}\right)}{(1-\frac{v^2}{c^2})^{\frac{1}{2}}}$

$\dfrac{\lambda_o}{\lambda_s} = \dfrac{\left(1+\frac{v}{c}\right)}{(1-\frac{v^2}{c^2})^{\frac{1}{2}}}$

$\lambda_o = \lambda_s + \Delta\lambda$

$\dfrac{\lambda_o}{\lambda_s} = 1 + \dfrac{\Delta\lambda}{\lambda_s}$

So $z = \dfrac{\Delta\lambda}{\lambda_s} = \dfrac{\lambda_o}{\lambda_s} - 1 = \dfrac{\left(1+\frac{v}{c}\right)}{(1-\frac{v^2}{c^2})^{\frac{1}{2}}} - 1$

$1 + \dfrac{\Delta\lambda}{\lambda_s} = 1 + 7 = 8$

$8 = \dfrac{\left(1+\frac{v}{c}\right)}{(1-\frac{v^2}{c^2})^{\frac{1}{2}}}$

So $64 = \dfrac{\left(1+\frac{v}{c}\right)^2}{\left(1-\frac{v^2}{c^2}\right)}$

$= \dfrac{\left(1+\frac{v}{c}\right)\left(1+\frac{v}{c}\right)}{\left(1+\frac{v}{c}\right)\left(1-\frac{v}{c}\right)}$

$= \dfrac{1+\frac{v}{c}}{1-\frac{v}{c}}$

So $64\left(1 - \frac{v}{c}\right) = \left(1 + \frac{v}{c}\right)$

$64 - 64\left(\frac{v}{c}\right) = 1 + \left(\frac{v}{c}\right)$

$63 = 65\frac{v}{c}$

So $v = \frac{63}{65}c$