Page 468 Test yourself on prior knowledge

1 a) In one half-life, half of a radioactive isotope decays to an isotope of another element.
b) A random process is unpredictable – a die has a probability of 1/6 of turning up a six but you cannot predict which throw will produce a six.
c) A radioactive isotope (or radioisotope) is an isotope which is unstable; such an isotope decays by radioactive emission.

2 6 pm to midnight is 6 h or 3 half-lives, so amount remaining will be $\frac{1}{2^3}$ the original amount

$\frac{1}{8} \times 1.6$ g = 0.2 g remains.

3 a) $^{208}_{84}\text{Po} \rightarrow {}^{204}_{82}\text{Pb} + {}^{4}_{2}\text{He}$

b) $^{138}_{56}\text{Ba} \rightarrow {}^{138}_{57}\text{La} + {}^{0}_{-1}\text{e} + \bar{\nu}_e$

4 There are many possible uses for you to research.

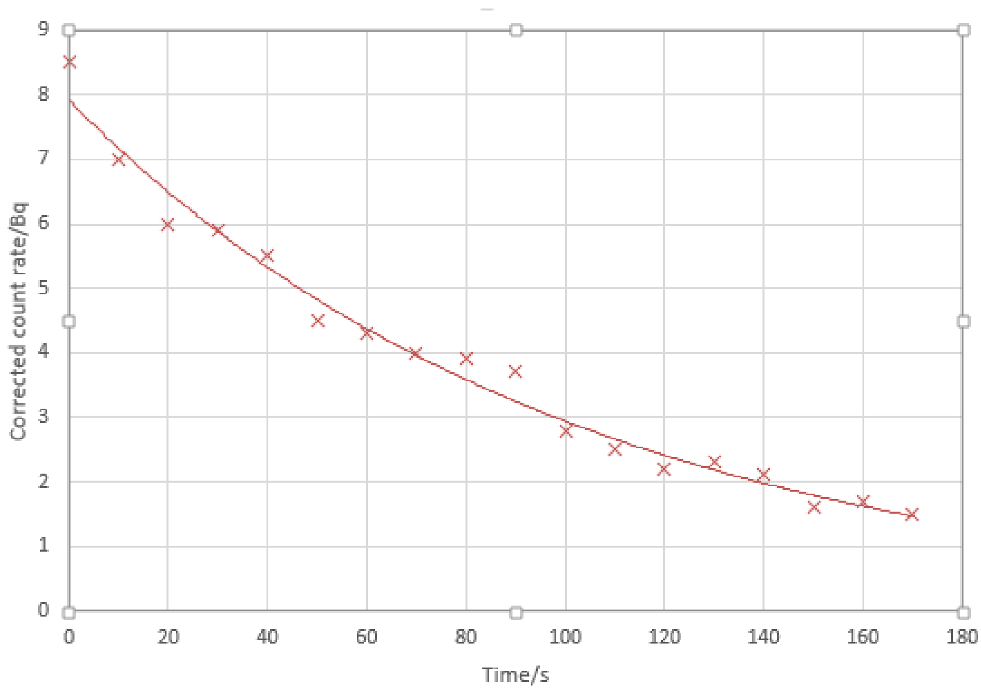
Page 468 Activity

Half-life of protactinium

1 Corrected count rate.

time/s	0	10	20	30	40	50	60	70	80
count rate/Bq	8.8	7.3	6.3	6.2	5.8	4.8	4.6	4.3	4.2
corrected count rate/Bq	8.5	7.0	6.0	5.9	5.5	4.5	4.3	4.0	3.9

time/s	90	100	110	120	130	140	150	160	170
count rate/Bq	4	3.1	2.8	2.5	2.6	2.4	1.9	2	1.8
corrected count rate/Bq	3.7	2.8	2.5	2.2	2.3	2.1	1.6	1.7	1.5



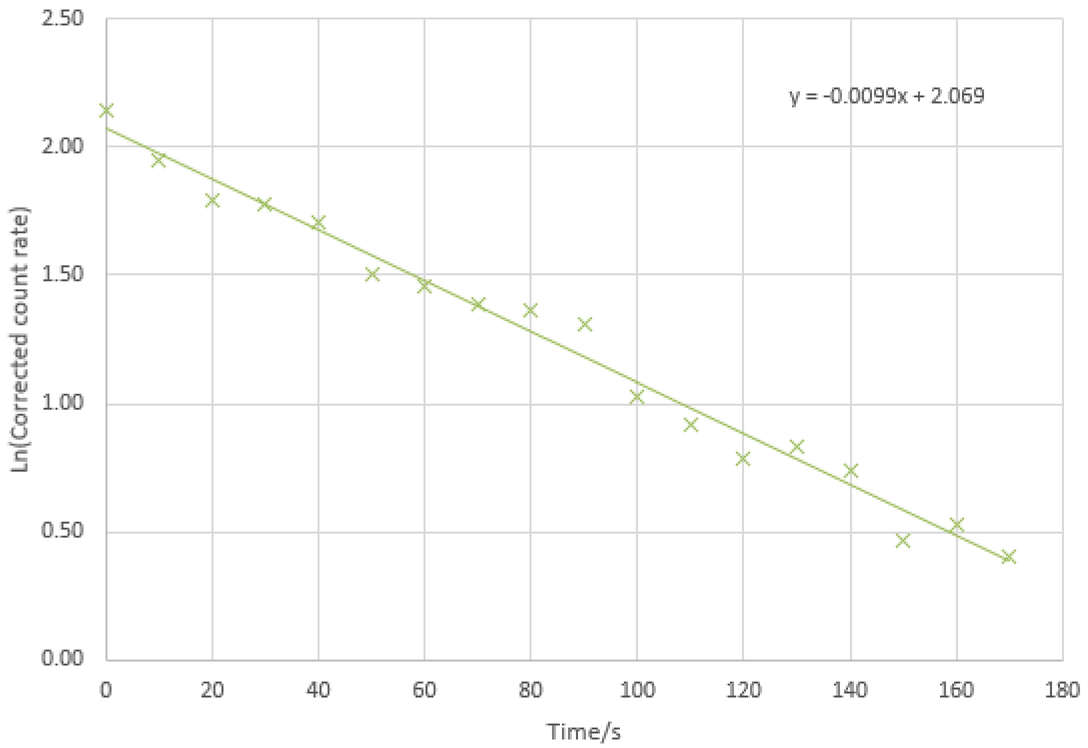
2 Draw the best curve through the points (this is quite hard to do).
Calculate some half-lives by measuring the time the count takes to drop from e.g. 7 Bq to 3.5 Bq, 6 Bq to 3 Bq and 5 Bq to 2.5 Bq. You will find slightly different answers for each case. But you can take an average that should come out at around 70 s.

3

time/s	0	10	20	30	40	50	60	70	80
ln A	2.14	1.95	1.79	1.77	1.70	1.50	1.46	1.39	1.36

time/s	90	100	110	120	130	140	150	160	170
ln A	1.31	1.03	0.92	0.79	0.83	0.74	0.47	0.53	0.41

4



Decay constant, $-\lambda$ = gradient of the graph

Use value given by Excel or calculate:

$$\lambda = \frac{2.06 - 0.40}{170\ \text{s}}$$

$$= \frac{1.66}{170}\ \text{s}^{-1}$$

$$= 0.0098\ \text{s}^{-1}$$

$$T = \frac{0.693}{0.0098}\ \text{s}$$

$$= 71\ \text{s}$$

Using a log graph helps you to average out the random errors. It is easier to draw (or calculate) the best straight line through the points. The answer obtained through this method is likely to be more accurate.

Page 474 Test yourself

1 $\frac{I_0}{I_{20}} = \frac{1}{0.818} = 1.22$

$\frac{I_{20}}{I_{40}} = \frac{0.818}{0.670} = 1.22$

$\frac{I_{40}}{I_{60}} = \frac{0.670}{0.548} = 1.22$ etc.

The ratios are the same for each 20 s interval.

2 a) $\lambda = \frac{\ln 2}{T_{\frac{1}{2}}}$

$= \frac{0.693}{453 \times 24 \times 3600}\ s^{-1}$

$= 1.7 \times 10^{-8}\ s^{-1}$

b) i) 109 g of cadmium-109 contain 6×10^{23} atoms

1 g of cadmium-109 contains $6 \times 10^{23}/109$ atoms

80 µg of cadmium-109 contain $80 \times 10^{-6} \times 6 \times 10^{23}/109 = 4.4 \times 10^{17}$ atoms

ii) A $= \lambda N$

$= 1.7 \times 10^{-8}\ s^{-1} \times 4.4 \times 10^{17}$

$= 7.5 \times 10^{9}$ Bq

iii) $A = A_0 e^{-\lambda t}$

It is more convenient for us to work in days here using $\lambda = \frac{0.693}{453}\ d^{-1}$

$\lambda t = \frac{0.693}{453d} \times (2 \times 365)$ d

$= 1.12$

So after two years

A $= 7.5 \times 10^{9} \times e^{-1.12}$

$= 2.5 \times 10^{9}$ Bq

3 A $= A_0 e^{-\lambda t}$

$\ln\left(\frac{A}{A_0}\right) = e^{-\lambda t}$

ln (1/3) = $-\lambda t$

$-1.1 = -\lambda \times (45 \times 60)$ s

So $\lambda = 4.0 \times 10^{-4}\ s^{-1}$

4 a) The fraction of γ-rays absorbed $= \frac{0.4\ cm^2}{4\pi \times 5^2 cm^2}$

$= 1.3 \times 10^{-3}$

So $\frac{70}{\text{total count}} = 1.3 \times 10^{-3}$

Total count $= \frac{70}{1.3 \times 10^{-3}}$

$= 5.5 \times 10^{4}$ Bq

b) $A = \lambda N$

$5.5 \times 10^4 = \lambda \times 4 \times 10^{16}$

$\lambda \;= \frac{5.5\times10^4}{4\times10^{16}}$

$= 1.4 \times 10^{-22}\ s^{-1}$

$T_{\frac{1}{2}} = \frac{0.693}{\lambda} = \frac{0.693}{1.4\times10^{-12}s^{-1}}$

$= 5.0 \times 10^{11}$ s = 15 900 years

5 a) N atoms of P would have twice the activity of N atoms of Q so λ_p is twice λ_Q

Since $\lambda = \frac{0.693}{T_{\frac{1}{2}}}$ it follows that the half-life of P is half that of Q.

So half-life of P = 50 years.

b) i) 200 years is 2 half-lives of Q, so $\frac{N}{4}$ atoms of Q remain.

200 years is 4 half-lives of P, so $\frac{2N}{16} = \frac{N}{8}$ atoms of P remain.

ii) 50 years is 1 half-life of P, so $\frac{2N}{2}$ = N atoms of P remain.

In half a half-life $\frac{N}{\sqrt{2}}$ atoms of Q remain.

(After 100 years – 2 lots of 50 years – $\frac{N}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{N}{2}$ are left.)

Or $N_{50} = N_0 e - \left(\frac{\ln 2}{100} \times 100\right)$

$= N\ e^{-0.5\ \ln 2}$

$= N\ e^{-0.3465}$

$= 0.707\ N = \frac{N}{\sqrt{2}}$

6 You need to describe how you determine the background count.
Set up a rate meter to determine the count rate as a function of time.
Correct for background count.
Plot the activity against the time. Draw a smooth curve. Check the time for the activity to halve – repeat with different parts of the graph.
Or Plot ln (A) against t and measure the gradient which is – λ.

Page 476 Test yourself

7 a) Cobalt-60. Gamma rays can penetrate the plastic bag and syringe, then kill any bacteria. Very high intensity radiation is used, with operators hidden away behind protective screens.

b) Technetium-99. Gamma rays can escape from the body and be detected. Again radiographers need to keep away (as they do this all the time). The 6 h half-life means that there is high intensity radiation (which can be detected) for a short time. The overall dose to the patient is as low as possible.

c) Iridium-192. Beta particles are more heavily ionising than gamma, so they are better at killing the cancerous cells. The short half-life of 74 days means a small amount of iridium can deliver a high activity. But the wire is only inserted for a short time.

d) Bismuth-213. The short half-life means that the treatment can be administered in a few hours in the hospital. Alpha particles are most effective inside the body as they are highly ionising. Alpha particles are safe for the radiographer, as they will not get out of the patient's body.

8 a) In 20 minutes, we assume there has been negligible decay of chromium-51.
Volume of blood × 15 700 Bq = 10 ml × 7.4 x 10^6 Bq

Volume $= \frac{10 \text{ ml} \times 7.4 \times 10^6}{1.57 \times 10^4}$

$= 4.7\ \ell$

b) i) $A = A_0 e^{-\lambda t}$

$= 15.7 \text{ kBq} \times e^{-\left(\frac{0.693}{27.7} \times 2\right)}$

$= 15.7$ kBq × 0.951

$= 14.9$ kBq

ii) The lower-than-predicted count rate suggests that blood has been lost, but that new blood has been made to replace the lost blood. This way the total amount of the radioactive tracer in the blood has been reduced.

9 a) i) Beta particles are more heavily ionising than gamma rays, and their range in the body relatively short. So the radiation dose is localised and will have an effect on the cells near to where it is injected.

ii) Beta particles cannot penetrate the body, so they do not reach the surgeon.

b) 0.02 mg of ruthenium-106 contain $\frac{0.02 \times 10^{-3}}{106} \times 6 \times 10^{23}$ atoms $= 1.1 \times 10^{17}$ atoms.

$A = \lambda N$

$= \frac{0.693}{367 \times 24 \times 3600} \text{ s}^{-1} \times 1.1 \times 10^{17}$

$= 2.5 \times 10^9 \text{ s}^{-1}$

So total of beta particles: $2 \times 10^{12} = 2.5 \times 10^9 \text{ s}^{-1} \times t$

$t = \frac{2 \times 10^{12}}{2.5 \times 10^9 \text{ s}^{-1}}$

$= 800$ s or 13 m 20 s

10 a) $A = A_0 e^{-\lambda t}$

$66 = 90 \times e^{-\left(\frac{0.693}{5730} t\right)}$

$\ln\left(\frac{66}{90}\right) = -\frac{0.693}{5730} t$

$-0.310 = -\frac{0.693}{5730} t$

$t = 2560$ years

b) After 60 000 years over 10 half-lives have elapsed.
So $A = A_0 \times \left(\frac{1}{2}\right)^{10}$
90 × 0.001 = 0.09 counts per hour
The errors become large and dating unreliable, as this count rate is much smaller than the background count.

11 a) 5.5 MeV $= 5.5 \times 10^{6} \times 1.6 \times 10^{-19}$ J

$= 8.8 \times 10^{-13}$ J

b) i) Power from the radioactive decay × 0.35 = 470 W

$P = \frac{470}{0.35}$ W

$= 1340$ W

$P = \text{Activity} \times 8.8 \times 10^{-13}$ J

$A = \frac{P}{8.8\times10^{-13}}$

$= \frac{1340}{8.8\times10^{-13}}$

$= 1.5 \times 10^{15}\ s^{-1}$

ii) $A = \lambda N$

$= \frac{0.693}{87.7\times365\times24\times3600}\ s^{-1} \times N$

$= 2.5 \times 10^{-10}\ N$

So $N = \frac{1.5\times10^{15}}{2.5\times10^{-10}}$

$= 6.0 \times 10^{24}$ atoms

238 g of plutonium-238 contain 6×10^{23} atoms

So $\frac{6.0\times10^{24}\text{atoms}}{6.0\times10^{23}\text{atoms}} = \frac{\text{mass}}{238\text{ g}}$

Mass $= 10 \times 238$ g

$= 2.4$ kg

c) The power available is proportional to the activity of the isotope so

$P = P_0\, e^{-\lambda t}$

$320 = 470\, e^{-\lambda t}$

$0.68 = e^{-\lambda t}$

$\ln(0.68) = -\lambda\, t$

$-0.38 = -\frac{0.693}{87.7}\, t$

$t = 48.6$ years

So Voyager's instruments will stop working in about 2025 or 2026.

d) The half-life of americium-241 is about five times that of plutonium-238, so the activity of a given number of atoms is about five times less. Americium-241 and plutonium-238 are of similar mass, so about 5 times as much Americium would be needed to produce the same power. Presumably, NASA decided that by 2025 the mission would be complete. So the plutonium option was probably cheaper and allowed a smaller nuclear generator.

Page 479 Test yourself

12 a) When an atom or nucleus is in a metastable state, it exists for an extended time in a state other than the system's state of least energy.

b) In an atom, electrons that orbit the nucleus in the lowest level are in the 'K shell'. These electrons are very tightly bound to the nucleus and actually spend some time inside the nucleus itself. The nucleus can capture such an electron, so that a proton is turned into a neutron.

13 a) $^{256}_{103}\mathrm{Lr} \rightarrow {}^{256}_{101}\mathrm{Md} + {}^{4}_{2}\mathrm{He}$

b) $^{248}_{100}\mathrm{Fm} \rightarrow {}^{244}_{98}\mathrm{Cf} + {}^{4}_{2}\mathrm{He}$

c) $^{210}_{85}\mathrm{At} \rightarrow {}^{210}_{84}\mathrm{Po} + {}^{0}_{1}\mathrm{e} + \nu$

d) $^{196}_{77}\mathrm{Ir} \rightarrow {}^{196}_{78}\mathrm{Pt} + {}^{0}_{-1}\mathrm{e} + \bar{\nu}$

e) $^{165}_{68}\mathrm{Er} + {}^{0}_{-1}\mathrm{e} \rightarrow {}^{165}_{67}\mathrm{Ho} + \nu$

14 a) Heavy nuclei have more neutrons to counterbalance the large electrostatic repulsion of the protons. With more neutrons there is a great attractive nuclear force.

b) Nuclei which are deficient in protons tend to decay by emission of β^-; nuclei which are rich in protons tend to decay by emitting β^+.

15 $^{237}_{93}\mathrm{Np} \rightarrow {}^{209}_{83}\mathrm{Bi} + 7{}^{4}_{2}\mathrm{He} + \mathrm{n}{}^{0}_{-1}\mathrm{e} + \mathrm{n}\bar{\nu}$

$93 = 83 + (7 \times 2) - n$

$n = 97 - 93 = 4$

Pages 480–484 Practice questions

1 B

2 C

3 C

4 D

5 C

6 A

7 D

8 C

9 B

10 B

11 a) $^{238}_{92}U \rightarrow {}^{206}_{82}Pb + 8\,^{4}_{2}He + x\,^{0}_{-1}e + n\bar{\nu}$

$92 = 82 + (8 \times 2) - x$

$x = 98 - 92$

$x = 6$

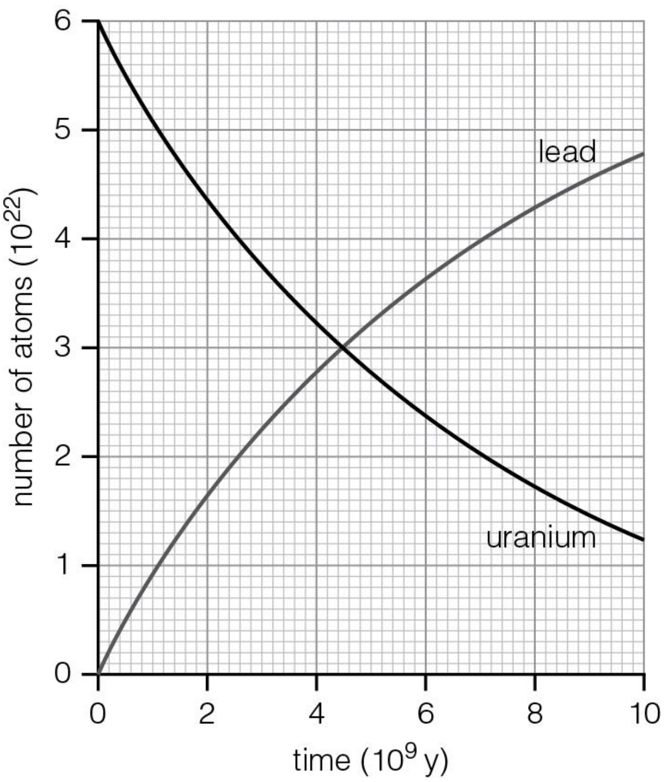
b) i) $A = \lambda N$

$= \frac{0.693}{4.5 \times 10^9 \times 365 \times 24 \times 3600} s^{-1} \times 6 \times 10^{22}$ [1]

$= 2.9 \times 10^5$ Bq [1]

ii) Uranium decreases on curve, lead increases on curve – both labelled [1]

Values add to 6×10^{22} and intersect 4.5×10^9 years, 3×10^{22} atoms [1]



iii) ¾ of the material is Uranium and ¼ lead; so

$\frac{3}{4} = e^{-\lambda t}$ [1]

$\ln\left(\frac{3}{4}\right) = -\lambda t$

$= -\frac{0.693}{T_{\frac{1}{2}}} \times t$ [1]

$t = \frac{\ln\left(\frac{3}{4}\right)}{-0.693} \times T_{\frac{1}{2}}$

$= \frac{-0.288}{-0.693} \times 4.5 \times 10^9$ years

$= 1.87 \times 10^9$ years [1]

12 a) $^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}e + \bar{\nu}$ [1]

b) i) The decay constant is the probability of one nucleus decaying per unit time; or the fraction of atoms present decaying per unit time.

ii) $T_{\frac{1}{2}} = \frac{0.693}{\lambda}$ [1]

$= \frac{0.693}{3.8 \times 10^{-12}\ \text{s}^{-1}}$ [1]

$= 1.8 \times 10^{11}$ s [1]

= 5780 years[1]

iii) A = λN [1]

$= 3.8 \times 10^{-12}\ \text{s}^{-1} \times 2 \times 10^{22} \times 10^{-12}$ [1]

= 0.076 Bq [1]

c) $A = A_0 e^{-\lambda t}$ [1]

$\ln\left(\frac{0.051}{0.076}\right) = -\frac{0.693}{5780\ \text{years}} \times t$ [1]

$t = \frac{0.399}{0.693} \times 5780$ years

= 3330 years [1]

13 a) ${}^{131}_{53}\text{I} \rightarrow {}^{131}_{54}\text{Xe} + {}^{0}_{-1}\text{e} + \overline{\nu}$ [1]

b) β-particles are ionising. [1]
Ionisation damages the cells by creating acids which can denature DNA. [1]

c) γ-rays are highly penetrating and will pass through the body. [1]
Most of the β-particles will be stopped by the body. [1]

d) 32 days is 4 half-lives. [1]
$\left(\frac{1}{2}\right)^4 = \frac{1}{16}$ of the iodine will be left, so $\frac{15}{16}$ has decayed. [1]

e) $A = A_0 e^{-\lambda t}$ [1]

$A_0 = Ae^{\lambda t}$

$\lambda = \frac{0.693}{8}$

$= 0.087\ \text{days}^{-1}$ [1]

$\lambda t = 0.087 \times 2$

= 0.173

$A_0 = 900\ \text{kBq} \times e^{0.173}$

= 900 kBq × 1.189

= 1070 kBq [1]

14 a) ${}^{48}_{24}\text{Cr} + {}^{0}_{-1}\text{e} \rightarrow {}^{48}_{23}\text{V} + \nu$ [1]

b) i) γ-rays can penetrate the cylinder wall. [1]

ii) The short half-life produces a high activity for a small mass. [1]
The trial only lasts a few days, so there is no need for a long half-life. With a short half-life there is no nuclear waste over a long period. [1]

iii) $A = A_0e^{-\lambda t}$ [1]

$A = 450\ e^{-\lambda t}$

$\lambda t = \frac{0.693}{22} \times 40$

$= 1.26$ [1]

$A = 450 \times e^{-1.26}$

$= 128$ counts per minute [1]

c) Fraction left = $\frac{115}{128}$ [1]

$= 0.9$

So 0.1 has been worn away. [1]

15 a) ${}^{244}_{96}\text{Cm} \rightarrow {}^{240}_{94}\text{Pu} + {}^{4}_{2}\text{He}$ [1]

b) i) $N = \frac{6 \times 10^{23}}{244} \times 20$ [1]

$= 4.9 \times 10^{22}$ [1]

ii) $A = \lambda N$

$= \frac{0.693}{18 \times 365 \times 24 \times 3600} \times 4.9 \times 10^{22}$ [1]

$= 6.0 \times 10^{13}$ Bq [1]

iii) Energy of α-particle $= 5.8 \times 10^{6} \times 1.6 \times 10^{-19}$ J

$= 9.3 \times 10^{-13}$ J [1]

$P = 9.3 \times 10^{-13}\ \text{J} \times 6.0 \times 10^{13}\ \text{s}^{-1}$ [1]

$= 56$ W [1]

iv) $P \propto A$, and 36 years = 2 half lives, so

$P = \frac{1}{4} \times 56$ W

$= 14$ W [1]

16 a) i) Two isotopes of the same element have the same number of protons but different numbers of neutrons. [1]

ii) In one half-life, half the of the nuclei in a sample of a radioisotope will decay. [1]

b) $\lambda = \frac{0.693}{0.8\ \text{s}}$

$= 0.87\ \text{s}^{-1}$ [1]

c) Proton-rich isotopes decay by β^+. [1]
Isotopes which are rich in neutrons decay by β^-. [1]

d) i) ${}^{35}_{18}\text{Ar} \rightarrow {}^{35}_{17}\text{Cl} + {}^{0}_{1}\text{e} + \nu$ [1]

ii) ${}^{37}_{18}\text{Ar} + {}^{0}_{-1}\text{e} \rightarrow {}^{37}_{17}\text{Cl} + \nu$ [1]

iii) ${}^{41}_{18}\text{Ar} \rightarrow {}^{41}_{19}\text{K} + {}^{0}_{-1}\text{e} + \bar{\nu}$ [1]

17 We need to use A = λN and $\lambda = \frac{0.693}{T_{\frac{1}{2}}}$ [1]

so that $T_{\frac{1}{2}} = \frac{0.693\ N}{A}$ [1]

We can calculate the number of atoms in a sample by knowing its mass. [1]

N = $\frac{6 \times 10^{23} \times \text{mass}}{238}$ [1]

Activity can be measured over a period of time using a GM tube [1]

And total activity of the sample calculated knowing the fraction of emissions entering the GM tube. [1]

Page 484 Stretch and challenge

18 a) $A_p = \frac{0.693}{300} N$

$A_Q = \frac{0.693 \times 2\ N}{150}$

$\frac{A_P}{A_Q} = \frac{N}{300} \times \frac{150}{2N} = \frac{1}{4}$

So $A_p = \frac{1}{4} A_Q$

b) If A_0 is the initial activity of sample *Q*

$A_p = \frac{1}{4} A_0 e^{-\frac{0.693 \times t}{300}}$

$A_Q = A_0 e^{-\frac{0.693 \times t}{150}}$

At time t, $A_P = A_Q$

So $\frac{1}{4} e^{-\frac{0.693 \times t}{300}} = e^{-\frac{0.693 \times t}{150}}$

$\ln\frac{1}{4} - \frac{0.693 \times t}{300} = -\frac{0.693 \times t}{150}$

$\ln\frac{1}{4} = -\frac{0.693 \times t}{300}$

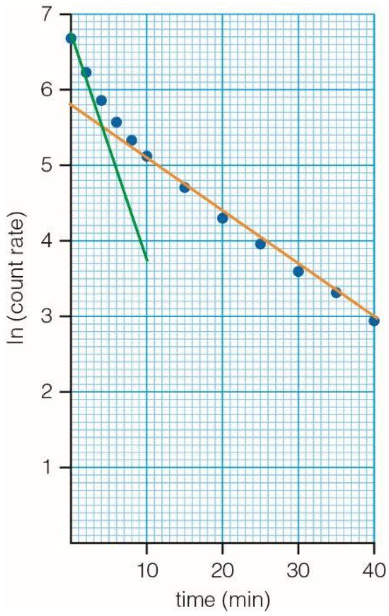
$-1.386 = -\frac{0.693 \times t}{300}$

t = 600 days

Or you might see that after 600 days P has had 2 half-lives and Q has had 4 half-lives. So P's activity reduces to ¼ of its original and Q's activity to 1/16 of its original. So they are now equal.

19

Count rate s^{-1}	800	511	352	261	205	166	109	75	53	37	27	19
ln (count)	6.68	6.24	5.86	5.56	5.32	5.11	4.69	4.31	3.97	3.61	28	2.94
Time/min	0	2	4	6	8	10	15	20	25	30	35	40



Orange line is the best-fit for second source, green line is best-fit for first source

Considering the orange line:

gradient = $-\lambda_2 = \frac{3-5.8}{40-0} = 0.07$

$\lambda_2 = 0.07\ \text{min}^{-1}$

$T_{\frac{1}{2}} = \frac{0.693}{0.07}$

$= 10.0$ min

By about 20 min the count from the first source has become negligible.
So we can now work out the contribution to the total count from each source, using the idea that $T_{\frac{1}{2}}$ (second source) is 10 minutes and that the activity of this source at 20 minutes is 75 counts per second (values at 0 minutes and 10 minutes can be calculated by inspection and the others by using $A = A_0e^{-\lambda t}$).

Count rate/s^{-1}	800	511	352	261	205	166	109	75	53	37	27	19
Source 1 count rate/s^{-1}	500	250	125	63	31	16	3	0	0	0	0	0
Source 2 count rate/ s^{-1}	300	261	227	198	172	150	106	75	53	37	27	19
Time/min	0	2	4	6	8	10	15	20	25	30	35	40

By inspection we can see that the half-life of source 1 is 2 min. This can also be worked out from the graph: $\lambda = 0.3\ \text{min}^{-1}$ so $T_{\frac{1}{2}} = 2$ min.

20 $\lambda = \lambda_1 + \lambda_2$

$$\frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2}$$

$$\frac{1}{T} = \frac{1}{24} + \frac{1}{36}$$

$$= \frac{3}{72} + \frac{2}{72}$$

$$= \frac{5}{72}$$

$$T_{\frac{1}{2}} = \frac{72}{5} = 14.4 \text{ h}$$

So it takes 28.8 h for the activity to drop to a quarter of the initial value (100 Bq to 25 Bq)