Page 381 Test yourself on prior knowledge

1 $Q = I \times t = 25 \times 10^{-3}$ mA × 60 s = 1.5 C

2 $W = Q \times V$

$= 1.6 \times 10^{-19}$ C × 4 800 V

$= 7.7 \times 10^{-16}$ J or 4800 eV

3 $I = V/R$

$= 7.7$ V / 1.1×10^{3} Ω

$= 7.0 \times 10^{-3}$ A

4 $E = I^2 \times R \times t$

$= (1.2\text{ V})^2 \times 39\ \Omega \times 3\text{ min} \times 60\text{ s min}^{-1}$

$= 1.0 \times 10^{4}$ J

5 $\varepsilon = V_1 + V_2$

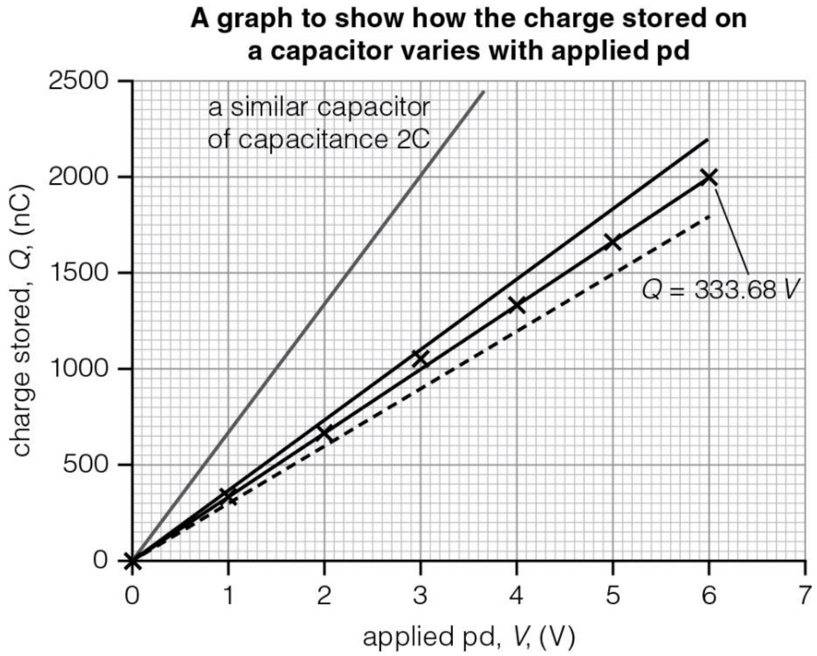
$\Longrightarrow V_2 = 12\text{ V} - 5.7\text{ V}$

$= 6.3$ V

Page 383 Activity

Measuring the capacitance of a capacitor

1 and 2 Your graph should be similar to this one:



The error in V should be ± 0.05 V.

The error in Q should be ± 10%.

3 Graph is linear and goes through (0,0), hence $Q \propto V$.

4 Q = CV so the gradient is equal to the capacitance. Excel gives a value of 333.21 for the gradient. Taking into account the unit prefixes for *Q* and *V*, the gradient is 333×10^{-9} F, or 330 nF (2 sf). The blue and red lines give maximum and minimum gradient lines, leading to 367 nF and 300 nF. Hence C = 330 ± 30 nF (2 sf).

5 See graph.

Page 383–884 Test yourself

1 The ability of a capacitor to store charge; the charge stored per unit potential difference, C = Q/V.

2

Units	C	A	F	V
Quantities	charge	current	capacitance	pd

3 Any three from: Potential difference, *V*; area of capacitor plates, *A*; separation of the capacitor plates, *d*; the insulating ability of the dielectric/insulating material between the plates (the permittivity of the material).

4 Q = C V

= 4200×10^{-6} F × 6.0 V

= 0.025 C (to 2 sf)

5 C = Q/V

= 3.2×10^{-3} C / 6.0 V

= 0.00053 F = 530 μF (to 2 sf)

6 Gradient of graph = pd

$= \frac{0.25}{0.021} = 12$ V (to 2 sf)

7

Q	V	C
2.6 mC	6.0 V	440 μF
0.03 C	12.0 V	2.5 mF
30 μC	3 mV	10 000 μF
250 nC	5.0 V	5.0×10^{-8} F
1.1×10^{-6} C	9.0 V	120 nF

Page 384 Test yourself

8 C = $(\varepsilon_r \varepsilon_0 A)/d$

= $2.25 \times 8.854 \times 10^{-12}$ F m^{-1} × $(45 \times 10^{-2}$ m × 95×10^{-2} m) / 12.5×10^{-6} m

= 6.8×10^{-7} F

9 a) Q = CV

= 20×10^{-9} F × 6.0 V

= 1.2×10^{-7} C

b) $C = (\varepsilon_r \varepsilon_0 A)/d$

$\Longrightarrow \varepsilon_r = Cd / \varepsilon_0 A$

= $(20 \times 10^{-9}$ F × 5.0×10^{-6} m) / (8.854×10^{-12} F m^{-1} × 0.0016 m^2)

= 7.1

10 a) The maximum area will occur when the capacitance is biggest, 520 pF. The capacitance of one set of plates is therefore 520 pF / 5 = 104 pF. The area of the plates is therefore

A = $Cd / (\varepsilon_r \varepsilon_0)$

= 104×10^{-12} F × 0.5×10^{-3} m / 8.854×10^{-12} F m^{-1}

= 5.9×10^{-3} m^2

b) The minimum area will occur when the capacitance is smallest, 29 pF. The capacitance of one set of plates is therefore 29 pF / 5 = 5.8 pF. The area of the plates is therefore

A = $Cd / (\varepsilon_r \varepsilon_0)$

= 5.8×10^{-12} F × 0.5×10^{-3} / 8.854×10^{-12} F m^{-1}

= 3.3×10^{-4} m^2

Page 386 Activity

Measuring the relative permittivity of a dielectric material

1 $C = \frac{\varepsilon_r \varepsilon_0}{\mathrm{d}} A$ so the gradient of a graph of C against area of overlap will be $\frac{\varepsilon_r \varepsilon_0}{\mathrm{d}}$

The mean thickness of the material (average of values in Table 20.3) is 0.13 ± 0.01 mm

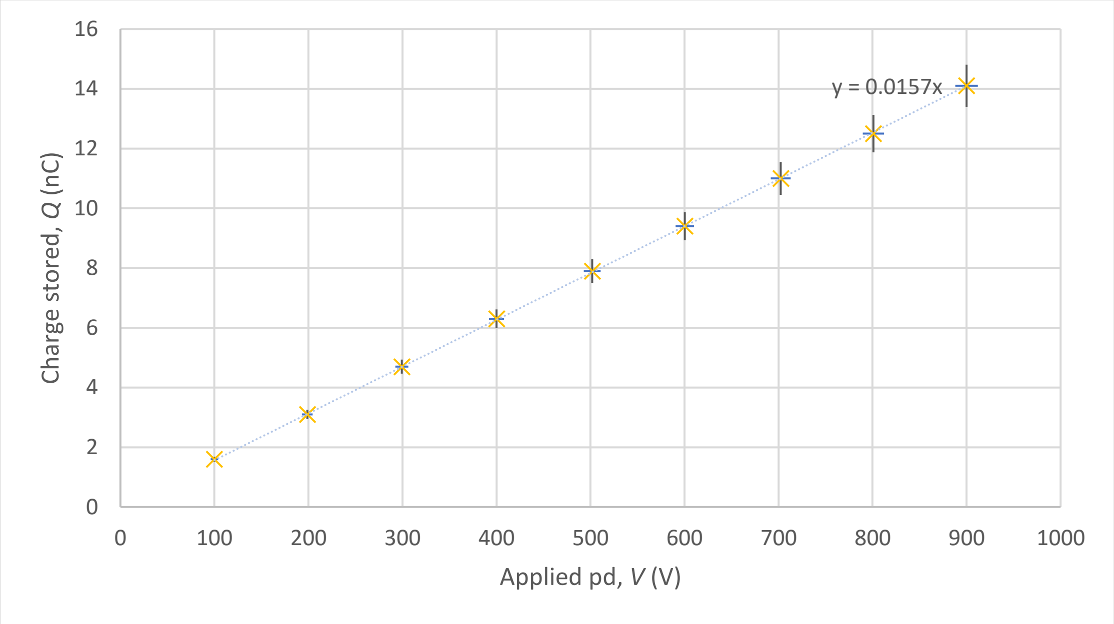
x dimension of plates in overlap/cm (±0.2cm)	*y* dimension of plates in overlap/cm (±0.2cm)	Capacitance, C/nF(±5%)	Area of plates in overlap/cm^2
30.0	30.0	14.1	900.0
28.3	28.3	12.5	800.9
26.5	26.5	11	702.3
24.5	24.5	9.4	600.3
22.4	22.4	7.9	501.8
20.0	20.0	6.3	400.0
17.3	17.3	4.7	299.3
14.1	14.1	3.1	198.8
10.0	10.0	1.6	100.0

Excel calculates the gradient of the line to be 0.0157 nF cm^{-2} (see graph on next page), providing the graph passes through the origin.

Converting this to F m^{-2} gives 1.57×10^{-7} Fm^{-2}

As the gradient is equal to $\frac{\varepsilon_r \varepsilon_0}{d}$

$\varepsilon_r = \frac{1.57 \times 10^{-7}\text{Fm}^{-2} \times 0.13 \times 10^{-3}}{8.854 \times 10^{-12}} = 2.3$



The maximum and minimum values of the gradient are dictated by the error bars. The maximum gradient is 1.67×10^{-7} Fm^{-2}, and the minimum is 1.47×10^{-7} Fm^{-2}. This gives an uncertainty in the gradient of $\pm 0.10 \times 10^{-7}$ Fm^{-2}.

The percentage uncertainty in the gradient is therefore (0.10/1.57) × 100 = 6% and the percentage uncertainty in the thickness of the bag is (0.01/0.13) × 100 = 7.7%, giving a total uncertainty of 13.7% in ε_r.

Therefore, the value of ε_r given by the experiment is 2.3 ± 0.3

Pages 388–389 Test yourself

11 A dielectric material is an insulating material whose molecules polarise inside an electric field.

12 Polar water molecules align themselves with the electric field. As the field alternates, the water molecules constantly rotate and realign. The rotating water molecules transfer kinetic energy to their surroundings heating them up.

13 $E = \frac{1}{2} CV^2$

$= 0.5 \times 3300 \times 10^{-6}\text{ F} \times (9.0\text{ V})^2$

$= 0.13$ J (2 sf)

14 $C = (2 \times E)/V^2$

$= (2 \times 0.25\text{ J})/(24\text{ V})^2$

$= 870$ µF (2 sf)

15 $Q = \sqrt{2 \times E \times C}$

$= \sqrt{2 \times 0.12\,J \ \times 220 \times 10^{-6} F}$

$= 7.3 \times 10^{-3}$ C (2 sf)

16 a) Y

b) X

c) Y

17 a) C = gradient of graph = 0.025/12 = 0.0021 F

b) At 8.0 V, Q = 0.017 C

so E = ½ QV = 0.5 × 0.017 C × 8.0 V = 0.068 J

Page 393 Test yourself

18 a) E = ½ CV^2

= 0.5 × 2.0 × 10^{-9} F × $(1.5\ V)^2$

= 2.3 × 10^{-9} J;

b) time constant = RC

= 3.9 × 10^{6} Ω × 2.0 × 10^{-9} F

= 7.8 × 10^{-3} s

c) $t_{1/2}$ = 0.693 RC = 5.4 × 10^{-3} s

so 2 × $t_{1/2}$ = 10.8 × 10^{-3} s

19 a) Q = VC

= 12 V × 2800 × 10^{-6} F

= 0.034 J

b) E = ½ CV^2

= 0.5 × 2800 × 10^{-6} F × $(12\ V)^2$

= 0.20 J

c) I = V/R

= 12 V /8.0 × 10^{3} Ω

= 1.5 × 10^{-3} A

d) The capacitor starts to discharge – with the current flowing in the opposite direction to the charging current.
The current varies following $I = I_0 e^{-t/RC}$ – exponential decay – so current starts to decrease.

e) time constant = RC = 8.0 × 10^{3} Ω × 2800 × 10^{-6} F = 22.4 s

f) i) stays the same

ii) stays the same

iii) doubles

iv) halves

20 a) Current is the rate of flow of charge, so charge is the integral of current over time – the area under the graph of current against time.

b) Q = Area under graph = 1.26×10^{-3} C

c) C = Q/V

= 1.26×10^{-3} C / 3.0 V

= 4.2×10^{-4} F

d) E = ½ CV^2

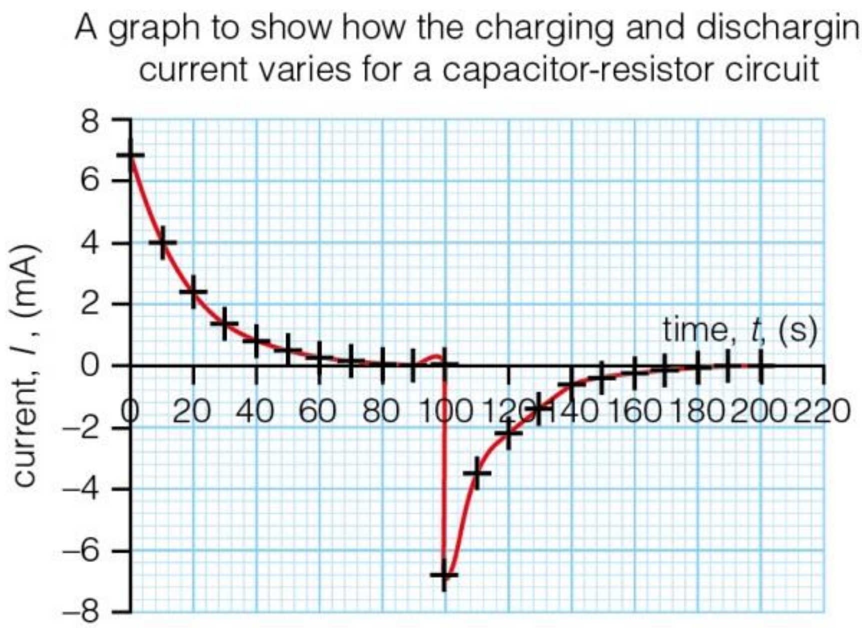
= $0.5 \times 4.2 \times 10^{-4}$ F × $(3\ V)^2$

= 1.9×10^{-3} J

Pages 395–396 Required practical 9

Investigating the charging and discharging of *capacitors*

1 Your graph should be similar to this:



2 Total charge = area under charging or discharging portion of graph.
Use a 'counting squares' technique to determine this value.
Q_0 = 0.126 C

3 Using the charging portion of the graph, where the maximum current is 6.81 mA, the half-life, $t_{1/2}$, occurs when I = (6.81 mA / 2) = 3.41 mA.
Using the graph, this takes approximately 13 s.
Then, using $t_{1/2}$ = 0.693 RC,

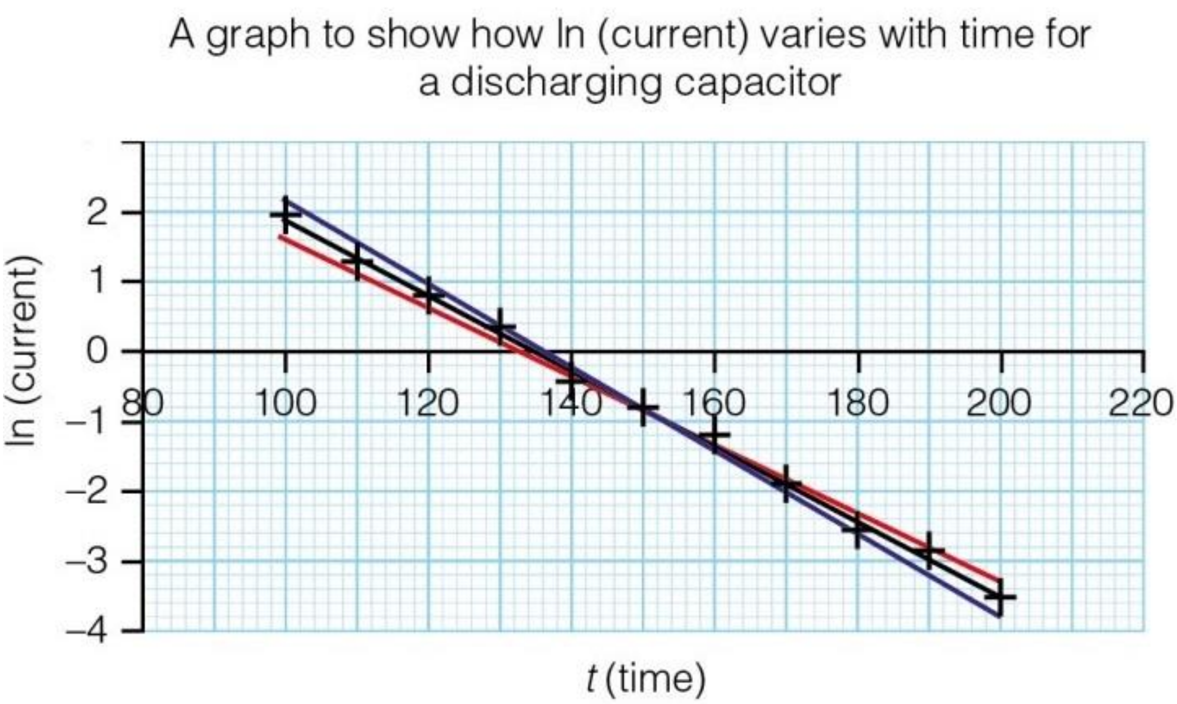
$$C = \frac{t_{\frac{1}{2}}}{0.693R} = \frac{13\ \text{s}}{0.693 \times 880\ \Omega}$$

= 0.021 F

4

Status	time, t, (s)	current, I, (mA)	ln(I)
Discharge	100	-6.81	1.92
	110	-3.50	1.25
	120	-2.20	0.79
	130	-1.40	0.34
	140	-0.65	-0.43
	150	-0.46	-0.78
	160	-0.30	-1.20
	170	-0.15	-1.90
	180	-0.08	-2.53
	190	-0.06	-2.81
	200	-0.03	-3.51

5 Your graph should be similar to this:



6 The gradient is (-1/RC) – the negative reciprocal of the time constant of the circuit (see pages 392–393)

Excel has calculated the gradient of the graph to be -0.0532

so RC = $\frac{-1}{-0.0532} = 18.8$ s

And, as R = 880 Ω,

C = $\frac{18.8\ \text{s}}{880\ \Omega} = 0.021$ F

The blue and red lines drawn on the graph represent maximum and minimum gradient lines that could be drawn through the scatter of the points. Measuring the gradients of each of these lines lead to a maximum value of C = 0.023 F and a minimum value of C = 0.019 F.

So this method gives C = (0.021 $\pm$ 0.002) F

7 The values calculated from both techniques are the same and the uncertainty is only about 10%.

Pages 397–401 Practice questions

1 C

2 D

3 D

4 C

5 A

6 B

7 D

8 B

9 B

10 A

11 D

12 a) i) I = V/R = 12.0 V/50 × 10^3 Ω = 2.4 × 10^{-4} A [1]

ii) Half-life = 14.5 s [1]

time constant, RC = half-life / 0.693 [1]

= $\frac{14.5}{0.693} = 21$ s value [1] unit [1]

OR

when t = RC, V = V_0e^{-1} [1]

V_0e^{-1} = 12.0 V × e^{-1} = 4.41 V [1]

$\Rightarrow$ t = 21 s (from graph) value [1] unit [1]

OR

line showing gradient of graph at t = 0 intercepts x-axis at t = RC [1]

appropriate line drawn [1]

$\Rightarrow$ t = 21 s (from graph) value [1] unit [1]

iii) Time constant RC = 21 s

so C = 21 s / 50 × 10^3 Ω [1]

= 4.2 × 10^{-4} F [1]

iv) Q = $Q_0e^{-t/RC}$ = $V_0Ce^{-t/RC}$ [1]

= 12.0 V × 4.2 × 10^{-4} F × $e^{-30\,s/21\,s}$ [1]

= 1.2 × 10^{-3} C [1]

b) i) E = ½ CV^2 [1]

i.e. E $\propto$ V^2 so, if V halves, then E $\Rightarrow$ E/4 [1]

ii) Time constant = RC; it is independent of V so does not change [1]

13 a) Another person should be able to read your handwriting and your spelling, punctuation and grammar should be sufficiently accurate for the meaning to be clear, this will include spelling the scientific keywords correctly and using capital letters, full stops and commas where appropriate. You can gain a maximum of 6 marks.

Your answer will be assessed as a whole using a 'best fit' to one of the three levels below using the following criteria:

High Level (good to excellent) 5 or 6 marks:

- Clear organisation of your answer.
- Logical and coherent (in the correct order).
- Appropriate specialist vocabulary used.
- The question is answered fully.
- Comprehensive and logical description of the sequence of releasing the ball and taking measurements of initial and final voltages.
- Correct distance measurement identified.
- Description of how to use the measurements to calculate the time and acceleration.
- The drop time should be found from capacitor discharge, using known C and R values.
- Repeated readings – 6 marks, otherwise 5 marks if left out of answer.

Intermediate Level (modest to adequate) 3 or 4 marks:

- Less well organised answer.
- Not fully coherent – may be out of sequence.
- Less use of specialist vocabulary, or specialist vocabulary may be used incorrectly.
- There is a comprehensive and logical description of the sequence of releasing the ball and taking measurements of the initial and final voltages.
- Some use of the *suvat* equations to calculate the acceleration, although the answer may not involve the measurement of the height that the ball falls through.

Low Level (poor to limited) 1 or 2 marks:

- Poorly organised answer.
- May not include relevant information and may be very incoherent.
- Your answer includes little correct use of specialist vocabulary.
- May not actually answer the question.
- The answer may include measurement of the initial and final voltages, but may not include any other measurements.
- You only included a few details of how to calculate the acceleration from the voltage measurements.

A good answer will include the following points in a relevant sequence:

Measurements

- initial pd across C (V_0) using the voltmeter.
- the height s that the ball falls between the switches
- final pd across C (V) using the voltmeter
- repeated measurements and an average

Analysis

- time t is found from $V = V_0e^{-t/RC}$, giving $t = RC \ln(V_0/V)$
- from $s = ut + ½ t^2$ with $u = 0$, acceleration $g = 2s/t^2$
- repeat and find average g from several results

b) $RC = 440 \times 10^{-6}$ F $\times 10 \times 10^3\ \Omega = 4.4$ s [1]

$V = V_0e^{-t/RC} \Rightarrow t = -RC \ln(V/V_0)$ [1]

$t = -4.4$ s $\times \ln(5.4$ V / 6.0 V$) = 0.46$ s [1]

c) $s = ut + ½at^2$, $u = 0$, $a = g \Rightarrow g = 2s/t^2$ [1]

$g = 2 \times 1.0$ m / $(0.46$ s$)^2 = 9.5$ m s^{-2} [1]

14 a) $E = ½ CV^2$ [1]

$E = 0.5 \times 12\,800 \times 10^{-6}$ F $\times (3.6$ V$)^2 = 0.083$ J [1]

b) $E = VIt = 3.6$ V $\times 0.84 \times 10^{-3}$ A $\times 3 \times 3600$ s $= 32.7$ J [1]

32.7 J / 0.083 J = 394 ≈ 400 [1]

c) Any two from the following:

- The capacitor(s) that stored enough energy for reasonable operating time would be to big (to fit in the tablet computer). [1]
- The capacitor(s) would need recharging too often, or capacitor could only supply energy to the tablet for a short time. [1]
- The capacitor voltage [or current or charge] would fall continuously whilst in use. [1]

15 a) charge (stored) [1]

per unit potential difference [1]

OR

C = Q/V where Q = charge (stored by one plate) [1]

V = pd (across plates) [1]

b) C = Q/V [1]

Q = 2.88×10^{-3} C (from graph) [1]

C = 2.88×10^{-3} C / 9.0 V = 3.2×10^{-4} F [1]

c) 63% of 2.88 mC = 1.8 mC [1]

and from graph, RC = 7 s [1]

d) R = 7 s / 3.2×10^{-4} F [1]

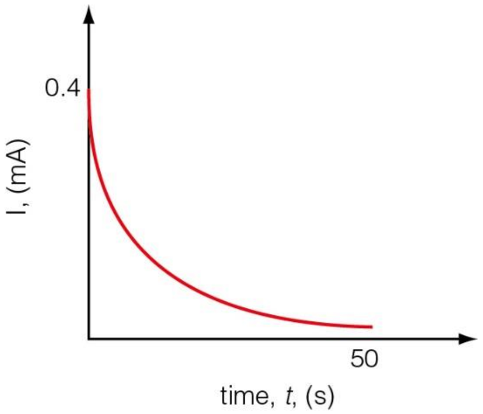
= 21 875 Ω (20 kΩ to 1 sf) [1]

e) The current [1]

f) I = V/R = 9.0 V / 21875 Ω = 4.1×10^{-4} A = 0.41 mA [1]

OR
calculation of gradient of graph at t = 0 s giving an answer in the range 0.3– 0.5 mA [1]

g) [1 mark for exponential decay shape; 1 mark for start at I = 0.41 mA]



16 a) i) RC = 48×10^3 Ω × 640×10^{-6} F = 30.72 s (31 s to 2 sf) [1]

ii) I = V/R = 12 V / 48×10^3 Ω = 2.5×10^{-4} A [1]

iii) Q_0 = VC = 12 V × 640×10^{-6} F = 7.68×10^{-3} C (7.7 mC to 2 sf) [1]

iv) E = ½ CV^2 = 0.5 × 640×10^{-6} F × $(12\text{ V})^2$ = 0.046 J [1]

b) i) Q = $Q_0 e^{-t/RC}$ [1]

= 7.68×10^{-3} C × $e^{-40/30.72}$

= 2.09×10^{-3} C ≈ 2.1 mC (2 sf) [1]

ii) V = Q/C = 2.09×10^{-3} C / 640×10^{-6} F = 3.3 V (2 sf) [1]

iii) E = ½ CV^2 = 0.5 × 640×10^{-6} F × $(3.3\text{ V})^2$ = 3.5×10^{-3} J [1]

17 a) Maximum current = 25 μA [1]

R = V/I = 3.0 V / 25×10^{-6} A = 120 000 Ω [1]

b) $t_{1/2}$ = 53 s [1]

$t_{1/2}$ = 0.693 RC ⇒ RC = 53 s / 0.693 = 76 s [1] (2 sf)

c) C = RC / R = 76 s / 120 000 Ω = 633 × 10^{-6} F = 630 μF [1] (2 sf)

Page 402 Stretch and challenge

18 a) $E_1 = ½ C_1V_1^2$

$V_1 = 1 \times 10^5$ V

$C_1 = \frac{\varepsilon_0 A_1}{d_1}$

where V_1 is the p.d. beteween the cloud and the ground, A_1 is the area of the thundercloud and d_1 is the distance from the cloud to the ground

$$\Rightarrow E_1 = \frac{1}{2} \times \frac{\varepsilon_0 A_1}{d_1} \times V_1{}^2$$

$$= 0.5 \times \frac{8.85 \times 10^{-12}\ \mathrm{F\,m} \times 25 \times 10^6\ \mathrm{m}^2 \times (1 \times 10^5)^2\ \mathrm{V}^2}{750\ \mathrm{m}} = 1475\ \mathrm{J}$$

(accept any value from 1.47 × 10^3 to 1.48 × 10^3 J)

b) i) Energy increases as work has to be done to further separate the charge on the cloud against the attraction from opposite charge on the ground; the charge on the cloud is constant.

ii) $E_2 = ½ C_2V_2^2$ and $V_2 = \frac{Q_1}{C_2}$

where Q_1 is the charge on the cloud, which remains constant, C_2 is the new capacitance and V_2 is the new potential.

So $E_2 = \frac{1}{2} C_2 \left(\frac{Q_1}{C_2}\right)^2 = \frac{1}{2}\frac{Q_1{}^2}{C_2}$

But $Q_1 = \frac{V_1}{C_1}$

Substituting for this and expressing capacitances in terms of the system dimensions gives

$$E_2 = \frac{1}{2}\left(\frac{\varepsilon_0 A_1 V_1}{d_1}\right)^2 \frac{d_2}{\varepsilon_0 A_1} = \frac{1}{2}\varepsilon_0 A_1 V_1{}^2 \frac{d_2}{d_1{}^2}$$

$$= \frac{1}{2} \times 8.85 \times 10^{-12}\ \mathrm{F\,m^{-1}} \times 25 \times 10^6\ \mathrm{m}^2 \times (1 \times 10^5)^2 \times \frac{1250\ \mathrm{m}}{(750\ \mathrm{m})^2}$$

= 2458 J

Alternatively: $E_2 = \frac{1}{2}\left(\frac{\varepsilon_0 A_1 V_1{}^2}{d_1}\right)\left(\frac{d_2}{d_1}\right) = E_1 \times \frac{1250}{750}$ = 2458 J

Increase in energy, ΔE = 2458 – 1475 = 983 J

19 a) $Q_1 = Q_2$ (otherwise a current will flow between the capacitors)

b) $V_1 = Q_1/C_1$

$V_2 = Q_1/C_2$

c) $\boldsymbol{\varepsilon} = \frac{Q_1}{C}$ and $\boldsymbol{\varepsilon} = V_1 + V_2$

So $\frac{Q_1}{C} = Q_1\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$

$\Longrightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

d) $E = \frac{1}{2} C \boldsymbol{\varepsilon}^2 = \frac{\boldsymbol{\varepsilon}^2}{2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$

Substituting for $\boldsymbol{\varepsilon}$

$E = \frac{1}{2} C \left(\frac{Q_1}{C}\right)^2$

$= \frac{1}{2}\frac{Q_1^2}{C}$

20 a) Initially the two capacitors have different pds so when they are joined a current flows. When a current flows, energy is dissipated as heat.

Total charge stored = $2 \times 240 \times 10^{-6}$ C

Total capacitance = $C_1 + C_2 = 2.0 \times 10^{-6} + 4.0 \times 10^{-6} = 6.0 \times 10^{-6}$ F

$V = Q/C = 240 \times 10^{-6}$ C / 6.0×10^{-6} F = 40 V

b) $E_1 = \frac{Q_1^2}{2 \times C_1} = \frac{(120 \times 10^{-6}\ \mathrm{C})^2}{2 \times 2.0 \times 10^{-6}\mathrm{F}} = 3.6 \times 10^{-3}$ J

$E_2 = \frac{Q_2^2}{2 \times C_2} = \frac{(120 \times 10^{-6}\ \mathrm{C})^2}{2 \times 4.0 \times 10^{-6}\mathrm{F}} = 1.8 \times 10^{-3}$ J

$E_{together} = \frac{Q_{together}^2}{2 \times C_{together}} = \frac{(240 \times 10^{-6}\ \mathrm{C})^2}{2 \times 6.0 \times 10^{-6}\ \mathrm{F}} = 4.8 \times 10^{-3}$ J

$\Delta E = 3.6 \times 10^{-3}\ \mathrm{J} + 1.8 \times 10^{-3}\ \mathrm{J} - 4.8 \times 10^{-3}\ \mathrm{J} = 6 \times 10^{-4}$ J

The change in pd causes work to be done on the charge.