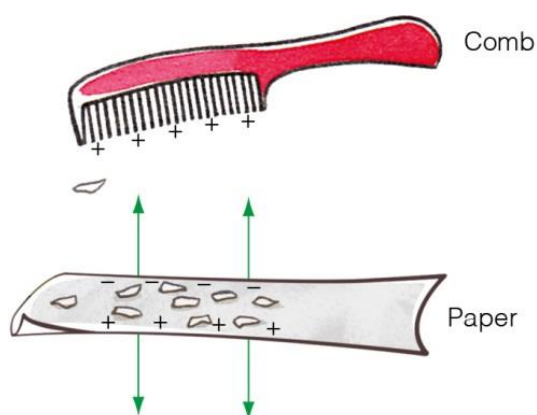


Page 359 Test yourself on prior knowledge

- 1 Magnesium also has 12 electrons in orbit round the nucleus. There is as much positive as negative charge, so the atom is neutral.
- 2 When an electron is removed from an atom a positive ion is formed. When an electron is added to an atom a negative ion is formed. Ions always occur in pairs, because charge is conserved. An electron which is removed from an atom will either remain as a free electron, or attach itself to another atom.
- 3 A charged particle experiences a force in an electric field.
Magnetic fields (in which moving charges and magnetic materials experience forces) and gravitational fields (which create forces on objects that have mass) are the most common examples of other fields.
Elementary particles experience forces in nuclear force fields.
- 4 Electrons in the paper can move. The positive charge on the comb attracts electrons towards the top of the paper, leaving the bottom positively charged. The top of the paper is attracted towards the comb and the bottom of the paper is repelled. However, because the top of the paper is closer to the comb, the attractive force is larger than the repulsive force, so there is a resultant upwards force.



Page 361 Activity

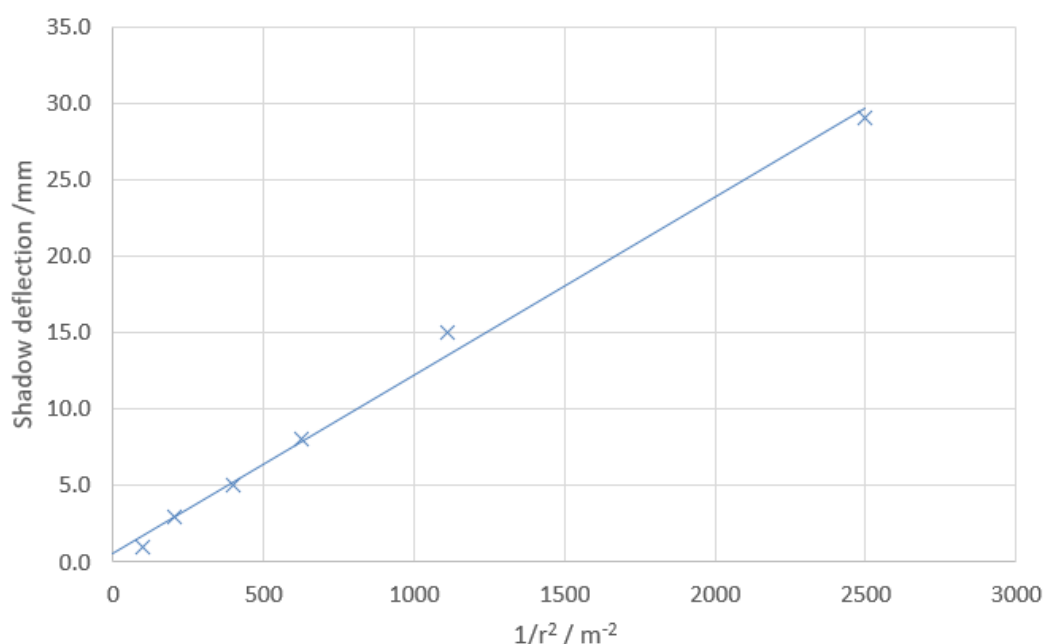
Testing Coulomb's law

1 (Graph overleaf)

Distance between centres of spheres' shadows / mm	100	70	50	40	30	20
Deflection of sphere B's shadow / mm	1	3	5	8	15	29
$1/r^2$ / m^{-2}	100	204	400	625	1111	2500

If Coulomb's law applies, a graph of s against $1/r^2$ should be a straight line through the origin (see answer to 2, below).

These results produce a straight line which passes very close to the origin. Allowing for a small systematic error in the measurement of s (0.6 mm using value of intercept given by Excel), they therefore support Coulomb's law.



- 2 If the electrostatic force acting on the sphere is F then, from the diagram:

Resolving vertically: $mg = T \cos \theta$

Resolving horizontally: $F = T \sin \theta$

Dividing gives: $\frac{F}{mg} = \frac{T \sin \theta}{T \cos \theta} = \tan \theta$

For small angles: $\tan \theta \approx \sin \theta = \frac{s}{l}$

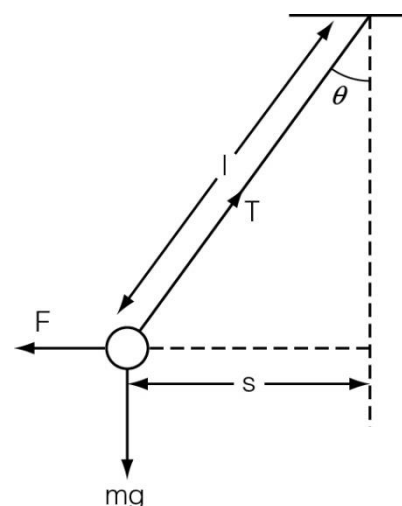
so $F = mg \tan \theta = mg \frac{s}{l}$

that is, the horizontal displacement of sphere B is proportional to the force.

From Coulomb's Law: $F = \frac{Q_1 Q_2}{4\pi \epsilon_0 r^2}$

Equating these expressions for force gives: $\frac{Q_1 Q_2}{4\pi \epsilon_0 r^2} = mg \frac{s}{l}$

and $s = \frac{Q_1 Q_2 l}{4\pi \epsilon_0 mg} \left(\frac{1}{r^2} \right)$



- 3 Possible sources of error include:

- There will be a systematic error if the centres of A and B are not on the same horizontal line.
- If the shadows are not sharp, it may be difficult to determine r and s precisely. Ambient light levels should therefore be low.
- Since the spheres are light, draughts may introduce errors.
- If the lamp is poorly placed (too close, or nearer one sphere than the other), the shadows cast on the screen will not be directly behind one or both spheres and this will also lead to errors in r and s .

- The force experienced varies with charge, so if the spheres are not charged to the same levels each time, results from each run cannot be compared. Charging using a supply reduces this error.
 - The spheres will gradually discharge to the atmosphere. Keeping sphere A connected to the supply, as shown in the diagram, means this is only an issue for sphere B. Charging using an EHT supply reduces the proportional change in charge; and the effect of losses can be reduced by recharging B between runs, making measurements quickly and, if possible, working in a dry environment.
- 4 Use a camera, making sure it is set at right angles to the graph paper, and make measurements from the resulting images.

Page 362 Test yourself

$$\begin{aligned}
 1 \text{ a) } F &= \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2} \\
 &= \frac{(2 \times 10^{-9} \text{ C}) \times (5 \times 10^{-9} \text{ C})}{4\pi \times (8.85 \times 10^{-12} \text{ Fm}^{-1}) \times (0.24 \text{ m})^2} \\
 &= 1.56 \mu\text{N}
 \end{aligned}$$

b) Calculate using same method or proportionality and answer to part (a):

$$F = 1.56 \mu\text{N} \times \left(\frac{240}{r}\right)^2 \text{ with } r \text{ in mm}$$

i) $6.2 \mu\text{N}$

ii) $14 \mu\text{N}$

iii) $25 \mu\text{N}$

iv) $39 \mu\text{N}$

$$\begin{aligned}
 2 \text{ } F &= \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2} \\
 &= \frac{(1.6 \times 10^{-19} \text{ C})^2}{4\pi \times (8.85 \times 10^{-12} \text{ Fm}^{-1}) \times (5 \times 10^{-11} \text{ m})^2} \\
 &= 9 \times 10^{-8} \text{ N}
 \end{aligned}$$

The electron exerts the same force on the proton (Newton's third law).

$$\begin{aligned}
 3 \text{ } F &= \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2} \\
 &= \frac{(92 \times 1.6 \times 10^{-19} \text{ C}) \times (2 \times 1.6 \times 10^{-19} \text{ C})}{4\pi \times (8.85 \times 10^{-12} \text{ Fm}^{-1}) \times (8 \times 10^{-15} \text{ m})^2} \\
 &= 660 \text{ N}
 \end{aligned}$$

This is about the weight of an adult – this shows us how strong the electrostatic force is.

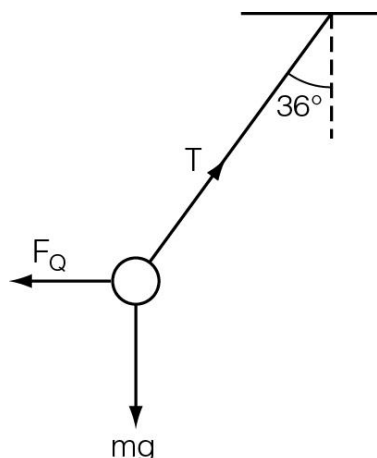
$$4 \quad F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2} = \frac{3Q \times Q}{4\pi\epsilon_0 r^2}$$

$$\text{So } Q^2 = \frac{4}{3}\pi \times (8.85 \times 10^{-12} \text{ F m}^{-1}) \times (0.1 \text{ m})^2 \times (80 \times 10^{-3} \text{ N})$$

$$Q = 1.7 \times 10^{-7} \text{ C}$$

$$\text{So the large charge is } 3Q = 5.2 \times 10^{-7} \text{ C}$$

5 a)



$$b) \quad F = mg \tan \theta$$

$$= 10^{-3} \text{ kg} \times 9.8 \text{ N kg}^{-1} \times \tan 36$$

$$= 0.007 \text{ N}$$

$$6 \quad a) \quad F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}$$

$$= \frac{(79 \times 1.6 \times 10^{-19} \text{ C}) \times (1.6 \times 10^{-19} \text{ C})}{4\pi \times (8.85 \times 10^{-12} \text{ F m}^{-1}) \times (7 \times 10^{-13} \text{ m})^2}$$

$$= 0.037 \text{ N}$$

$$b) \quad F = \frac{mv^2}{r}$$

$$v^2 = \frac{Fr}{m}$$

$$= \frac{0.037 \text{ N} \times 7 \times 10^{-13} \text{ m}}{9.1 \times 10^{-31} \text{ kg}}$$

$$v = 1.7 \times 10^8 \text{ m s}^{-1} \text{ which is 56\% of the speed of light.}$$

Pages 366– 367 Test yourself

$$7 \quad V = \text{J C}^{-1}$$

$$J = \text{N m}$$

$$\text{So } V = \text{N m C}^{-1}$$

$$\text{Therefore } V \text{ m}^{-1} = \text{N m C}^{-1} \text{ m}^{-1} = \text{N C}^{-1}$$

$$\begin{aligned}
 8 \text{ a) } E &= \frac{V}{d} \\
 &= \frac{1500 \text{ V}}{0.075 \text{ m}} \\
 &= 2 \times 10^4 \text{ V m}^{-1}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } F &= EQ \\
 Q &= \frac{F}{E} \\
 &= \frac{1.5 \times 10^{-7} \text{ N}}{2 \times 10^4 \text{ V m}^{-1}} \\
 &= 7.5 \times 10^{-12} \text{ C}
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } W &= Fd \\
 &= 1.5 \times 10^{-7} \text{ N} \times 0.075 \text{ m} \\
 &= 1.1 \times 10^{-8} \text{ J}
 \end{aligned}$$

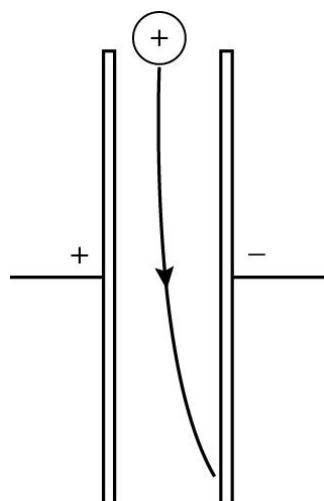
9 a) The electrons are deflected more as there is a larger upwards force on them.

b) The electrons are deflected less as they spend less time in the electric field.

10 a) At this speed, the weight is balanced by drag.

$$\begin{aligned}
 \text{b) } t &= \frac{d}{v} \\
 &= \frac{0.1 \text{ m}}{1 \text{ ms}^{-1}} \\
 &= 0.1 \text{ s}
 \end{aligned}$$

c) It will curve to the right. (If there is little drag to the right, this will be a parabola.)



$$\begin{aligned}
 \text{d) i) } E &= \frac{V}{d} \\
 &= \frac{3000 \text{ V}}{0.05 \text{ m}} \\
 &= 6.0 \times 10^4 \text{ N}
 \end{aligned}$$

$$\text{ii) } F = EQ$$

$$= 6.0 \times 10^4 \text{ V m}^{-1} \times 1.2 \times 10^{-8} \text{ C}$$

$$= 7.2 \times 10^{-4} \text{ N}$$

$$\text{e) } a = \frac{F}{m}$$

$$= \frac{7.2 \times 10^{-4} \text{ N}}{2 \times 10^{-4} \text{ kg}}$$

$$= 3.6 \text{ m s}^{-2}$$

$$\text{f) } s = \frac{1}{2} a t^2$$

$$= \frac{1}{2} \times 3.6 \text{ m s}^{-2} \times (0.1 \text{ s})^2$$

$$= 0.018 \text{ m} = 1.8 \text{ cm}$$

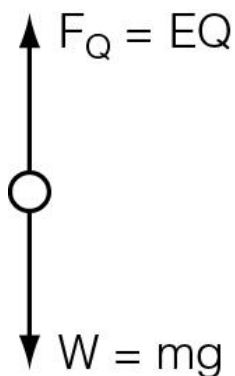
If there is significant drag the deflection will be less.

11 ΔV = area under the graph

$$= \frac{1}{2} (2\,000 \text{ V m}^{-1} + 1\,000 \text{ V m}^{-1}) \times 0.4 \text{ m}$$

$$= 600 \text{ V}$$

12 a) EQ upwards is balanced by mg downwards.



$$\text{b) } mg = \frac{vQ}{d} \quad (1)$$

$$\text{So } Q = \frac{mgd}{v}$$

$$= \frac{1.7 \times 10^{-15} \text{ kg} \times 9.8 \text{ N kg}^{-1} \times 0.006 \text{ m}}{312 \text{ V}}$$

$$= 3.2 \times 10^{-19} \text{ C}$$

c) Using same method with new value of V gives $Q = 4.8 \times 10^{-19} \text{ C}$

d) The smallest charge the drop can carry is $1.6 \times 10^{-19} \text{ C}$ (the charge on an electron).

$$\text{Rearranging (1) gives } v = \frac{mgd}{Q}$$

$$= \frac{1.7 \times 10^{-15} \text{ kg} \times 9.8 \text{ N kg}^{-1} \times 0.006 \text{ m}}{1.6 \times 10^{-19} \text{ kg}}$$

$$= 624 \text{ V}$$

- e) This experiment is usually attributed to Robert A. Millikan, although there is some controversy surrounding the contribution of his colleague Harvey Fletcher (and also around his selection of results for publication).

Pages 368–369 Test yourself

13 a) i) $E = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}$

Combining constants into a single term and measuring in mm $\Rightarrow 300 = \frac{k}{100^2}$

$\Rightarrow k = 300 \times 100^2$

and field strength at 50 mm, $E_{50} = \frac{300 \times 100^2}{50^2}$

$= 1200 \text{ N C}^{-1}$

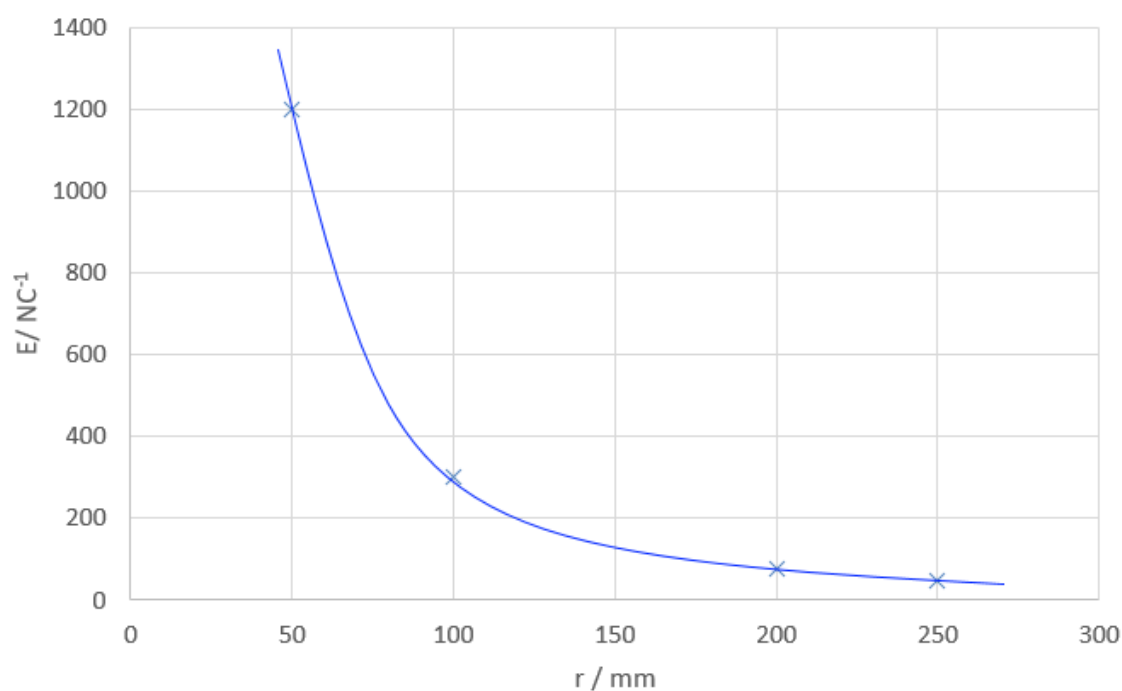
In the same way:

ii) $E_{200} = 75 \text{ N C}^{-1}$

iii) $E_{250} = 48 \text{ N C}^{-1}$

In each case:

- b) Your graph will show an inverse square law relationship.



$$14 \quad E = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}$$

Combining constants into a single term and taking the distance OA as one unit $\Rightarrow 3600 = \frac{k}{1^2}$

so field strength at a distance r from O, $E_r = \frac{3600}{r^2}$

This leads to the values shown in the table on the next page for the field strength.

Point	Distance from O / arbitrary units	Field strength / N C^{-1}
B	2	900
C	3	400
D	4	225
E	$\sqrt{1^2 + 1^2}$	1800
F	$\sqrt{2^2 + 1^2}$	720
G	$\sqrt{3^2 + 1^2}$	360
H	$\sqrt{2^2 + 2^2}$	450
I	$\sqrt{4^2 + 2^2}$	180
J	$\sqrt{3^2 + 3^2}$	200
K	$\sqrt{4^2 + 3^2}$	144

$$15 \text{ a) } E_x = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$= \frac{5 \times 10^{-9} \text{ C}}{(4\pi \times 8.85 \times 10^{-12} \text{ Fm}^{-1}) \times (0.3 \text{ m})^2}$$

$$= 500 \text{ N C}^{-1} \text{ (to the right)}$$

$$E_z = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$= \frac{10^{-8} \text{ C}}{(4\pi \times 8.85 \times 10^{-12} \text{ Fm}^{-1}) \times (0.2 \text{ m})^2}$$

$$= 2250 \text{ N C}^{-1} \text{ (to the right)}$$

So the total field strength $= E_x + E_z = 2750 \text{ N C}^{-1}$

b) i) Now the field is $-2250 \text{ N C}^{-1} + 500 \text{ N C}^{-1} = -1750 \text{ N C}^{-1}$ (i.e. to the left)

$$\text{ii) } F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}$$

$$= \frac{5 \times 10^{-9} \times 10^{-8} \text{ C}}{(4\pi \times 8.85 \times 10^{-12} \text{ Fm}^{-1}) \times (0.5 \text{ m})^2}$$

$$= 1.8 \times 10^{-6} \text{ N}$$

The force is repulsive.

Pages 374–375 Test yourself

$$\begin{aligned}
 16 \text{ a) i) } V &= \frac{Q}{4\pi\epsilon_0 r} \\
 &= \frac{1.5 \times 10^{-7} \text{ C}}{(4\pi \times 8.85 \times 10^{-12} \text{ Fm}^{-1}) \times 0.25 \text{ m}} \\
 &= 5400 \text{ V}
 \end{aligned}$$

ii) V is inversely proportional to r so

$$\begin{aligned}
 V &= 5400 \text{ V} \times \frac{0.25}{0.75} \\
 &= 1800 \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } \Delta W &= Q \Delta V \\
 &= 2.0 \times 10^{-8} \text{ C} \times (5400 - 1800) \text{ V} \\
 &= 7.2 \times 10^{-5} \text{ J}
 \end{aligned}$$

$$17 \Delta W = Q \Delta V$$

$$\begin{aligned}
 \text{a) } \Delta W &= 2 \times 10^{-7} \text{ C} \times 0 \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } \Delta W &= 2 \times 10^{-7} \text{ C} \times (500 - 300) \text{ V} \\
 &= 4 \times 10^{-5} \text{ J}
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } \Delta W &= 2 \times 10^{-7} \text{ C} \times (800 - 300) \text{ V} \\
 &= 10^{-4} \text{ J}
 \end{aligned}$$

$$18 \text{ a) i) } 25 \text{ V} + 25 \text{ V} - 25 \text{ V} + 25 \text{ V} = 50 \text{ V}$$

$$\text{ii) } 25 \text{ V} - 25 \text{ V} + 25 \text{ V} - 25 \text{ V} = 0$$

$$\text{iii) } 2(25 \text{ V}) - 3(25 \text{ V}) + 25 \text{ V} - 25 \text{ V} = -25 \text{ V}$$

b) The second: A balances C's field; B balances D's field

$$19 \text{ V} \propto \frac{1}{r} \text{ Taking the distance AB as 1 unit gives } V = \frac{120 \text{ V}}{r} \text{ and thus:}$$

$$\text{a) i) At C: } r = 2 \text{ units, } V = 60 \text{ V}$$

$$\text{ii) At D: } r = 3 \text{ units, } V = 40 \text{ V}$$

$$\text{iii) At E: } r = 4 \text{ units, } V = 30 \text{ V}$$

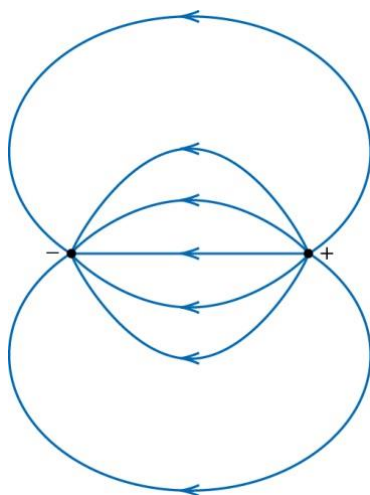
$$\text{iv) At F: } r = (4^2 + 3^2)^{\frac{1}{2}} = 5 \text{ units, } V = 24 \text{ V}$$

$$\text{b) i) EB} = 3 \text{ units, } V = 120 \text{ V} + 40 \text{ V} = 160 \text{ V}$$

$$\text{ii) EC} = 2 \text{ units, } V = 60 \text{ V} + 60 \text{ V} = 120 \text{ V}$$

$$\text{iii) ED} = 1 \text{ unit, } V = 40 \text{ V} + 120 \text{ V} = 160 \text{ V}$$

- iv) $EF = 3$ units, $V = 24 \text{ V} + 40 \text{ V} = 64 \text{ V}$
- c) i) $120 \text{ V} - 40 \text{ V} = 80 \text{ V}$
- ii) $60 \text{ V} - 60 \text{ V} = 0$
- iii) $40 \text{ V} - 120 \text{ V} = -80 \text{ V}$
- iv) $24 \text{ V} - 40 \text{ V} = -16 \text{ V}$
- 20 a) i) True: the field is at right angles to the equipotential, and it points along the path from the positive to the negative ion.
- ii) True: the potential gradient is larger.
- b) i) 0
- ii) $\Delta W = Q \Delta V$
 $= e \times (5.0 \text{ V} - (-2.5 \text{ V})) = 7.5 \text{ eV}$
- c)



Page 376 Test yourself

- 21 a) $F = \frac{GM_1M_2}{r^2}$
 $= \frac{(6.7 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}) \times (1.67 \times 10^{-27})^2}{(10^{-10} \text{ m})^2}$
 $= 1.9 \times 10^{-44} \text{ N}$
- b) $F = \frac{Q_1Q_2}{4\pi\epsilon_0 r^2}$
 $= \frac{(1.6 \times 10^{-19} \text{ C})^2}{(4\pi \times 8.85 \times 10^{-12} \text{ Fm}^{-1}) \times (10^{-10} \text{ m})^2}$
 $= 2.3 \times 10^{-8} \text{ N}$
- c) $\frac{F_E}{F_g} = \frac{2 \times 10^{-8} \text{ N}}{2 \times 10^{-44} \text{ N}} \approx 10^{36}$

- d) It will remain 10^{36} as both forces follow inverse square laws.
- 22 The strong force is short range – the nucleus is only a few femtometres (10^{-15} m) across.
The strong force must be stronger than the electric force, so that the electric force does not cause the nucleus to disintegrate.
- 23 A metal box will exclude electric fields from the inside because all points on a conducting surface will have the same potential.

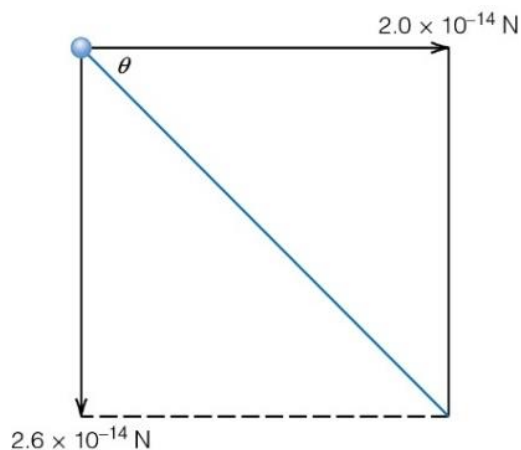
Pages 377–380 Practice questions

- 1 B
- 2 C
- 3 D
- 4 B
- 5 B
- 6 A
- 7 C
- 8 D
- 9 C
- 10 C
- 11 a) i) To the left. [1]
 ii) The acceleration remains constant. [1]
 The field is constant, so the force is constant. Since $F = ma$, the acceleration is constant. [1]
- b) i) The acceleration decreases as the electron moves to the left and the field decreases. [1]
 ii) The proton accelerates to the right [1]
 the acceleration is less as $a = \frac{F}{m}$ and mass is bigger [1]
 the acceleration increases as the proton moves to the right where the field is stronger. [1]
- 12 a) $F = EQ$ [1]
 $= \frac{V}{d} \times Q$ [1]
 $= \frac{300 \text{ V}}{0.06 \text{ m}} \times 4.0 \times 10^{-18} \text{ C}$
 $= 2.0 \times 10^{-14} \text{ N}$ [1]

b) $F = mg$

$$= 2.6 \times 10^{-15} \text{ kg} \times 9.81 \text{ N kg}^{-1}$$

$$= 2.6 \times 10^{-14} \text{ N} \quad [1] + [1] \text{ for diagram}$$



c) Resultant force acting on the sphere is

$$F = (2.0^2 + 2.6^2)^{\frac{1}{2}} \times 10^{-14} \text{ N}$$

$$= 3.3 \times 10^{-14} \text{ N} \quad [1]$$

$$a = \frac{F}{m} \quad [1]$$

$$= \frac{3.3 \times 10^{-14} \text{ N}}{2.6 \times 10^{-15} \text{ kg}}$$

$$= 13 \text{ m s}^{-2} \quad [1]$$

at a direction θ to the horizontal where

$$\tan \theta = \frac{2.6}{2.0}$$

$$\theta = 52^\circ \quad [1]$$

d) The gravitational force is doubled because the mass is doubled; and the electric force is doubled because the charge is doubled. [1]

Since these forces act at right angles, the resultant force is also doubled, but the acceleration stays the same as $a = F/m$. [1]

13 a) $F = k \times d$

$$= 0.12 \text{ N m}^{-1} \times 9 \times 10^{-3} \text{ m} \quad [1]$$

$$= 1.1 \times 10^{-3} \text{ N} \quad [1]$$

b) $E = \frac{V}{d} \quad [1]$

$$= \frac{500 \text{ V}}{0.2 \text{ m}}$$

$$= 2500 \text{ V m}^{-1} \quad [1]$$

c) $F = EQ$ [1]

$$Q = \frac{F}{E}$$

$$= \frac{1.1 \times 10^{-3} \text{ N}}{2500 \text{ V m}^{-1}}$$

$$= 4.4 \times 10^{-7} \text{ C} \text{ [1]}$$

d) Force acting on the sphere is proportional to displacement and in the opposite direction. [1]

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$$= 2\pi \sqrt{\frac{15 \times 10^{-3} \text{ kg}}{0.12 \text{ N m}^{-1}}} \text{ [1]}$$

$$= 2.2 \text{ s} \text{ [1]}$$

14 a) $E = \frac{V}{d}$

$$= \frac{600 \text{ V}}{0.12 \text{ m}}$$

$$= 5000 \text{ V m}^{-1} \text{ [1]}$$

b) $\tan \theta = \frac{F}{mg}$ [1]

$$F = 2.4 \times 10^{-3} \text{ kg} \times 9.81 \text{ N kg}^{-1} \times \tan 10^\circ \text{ [1]}$$

$$= 4.15 \times 10^{-3} \text{ N} \text{ [1]}$$

c) $Q = \frac{F}{E}$

$$= \frac{4.15 \times 10^{-3} \text{ N}}{5000 \text{ V}} \text{ [1]}$$

$$= 8.3 \times 10^{-7} = 8 \times 10^{-7} \text{ C} \text{ [1]}$$

15 a) 0 [1]

b) i) $V = \frac{Q}{4\pi\epsilon_0 r}$ [1]

$$= \frac{1.6 \times 10^{-19} \text{ C}}{(4\pi \times 8.85 \times 10^{-12} \text{ F m}^{-1}) \times (1.5 \times 10^{-10} \text{ m})} \text{ [1]}$$

$$= 9.6 \text{ V} \text{ [1]}$$

ii) $2 \times 9.6 \text{ V} = 19.2 \text{ V} \text{ [1]}$

c) Electrical potential energy of an electron at C = -19.2 eV [1]

$$\text{Total energy} = -19.2 \text{ eV} + 3.7 \text{ eV}$$

$$= -15.5 \text{ eV} \text{ [1]}$$

d) The ionisation energy is 15.5 eV, because this is the energy required to take the electron to infinity (with zero KE). [1]

16 a) $E_A = \frac{Q}{4\pi\epsilon_0 r^2}$ [1]

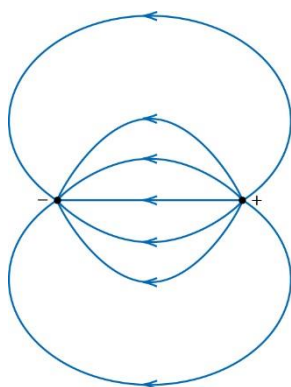
$$= \frac{6 \times 10^{-9} \text{ C}}{(4\pi \times 8.85 \times 10^{-12} \text{ F m}^{-1}) \times (0.5 \text{ m})^2}$$

$$= 216 \text{ N C}^{-1} \text{ [1]}$$

E_B has the same magnitude as the charges and distances are the same in both cases.

At C, both E_A and E_B are in the same direction, so total field, due to both = 432 N C^{-1} . [1]

b) i) Correct shape [1], arrows in correct direction [1]



ii) The direction of the field at D is parallel to AB and directed to the left. [1]

17 a) $E = \frac{\Delta V}{\Delta x}$ [1]

The potential gradient is greater at A than it is at C (the equipotential are closer together). So the field is stronger. [1]

b) i) 0 [1]

ii) $W = Q\Delta V$ [1]

$$= 2 \times 10^{-9} \text{ C} \times 100 \text{ V}$$

$$= 2 \times 10^{-7} \text{ J [1]}$$

18 a) $E = \frac{\Delta V}{\Delta x}$ [1]

The potential gradient is greater near the tree because the equipotentials are closer together. So the field is greater. [1]

b) i) $W = VQ$

$$= E \times d \times Q \text{ [1]}$$

$$= 46 \times 10^6 \text{ V m}^{-1} \times 0.5 \times 10^{-6} \text{ m} \times e \text{ [1]}$$

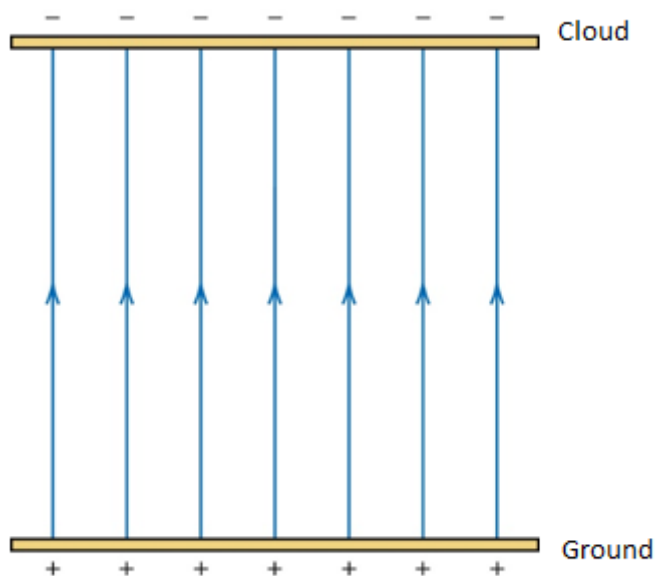
$$= 23 \text{ eV [1]}$$

- ii) In lower pressure the distance travelled between collisions, d , is bigger. [1]

Thus, the electron gains more energy between collisions for a given field, so the field to produce ionisation can be less. [1]

(Referring to the equation used for the last part of the question: $W = E \times d \times Q$
 Q and W are constant, so if d increases, E decreases.)

- c) i) Diagram [1 mark]



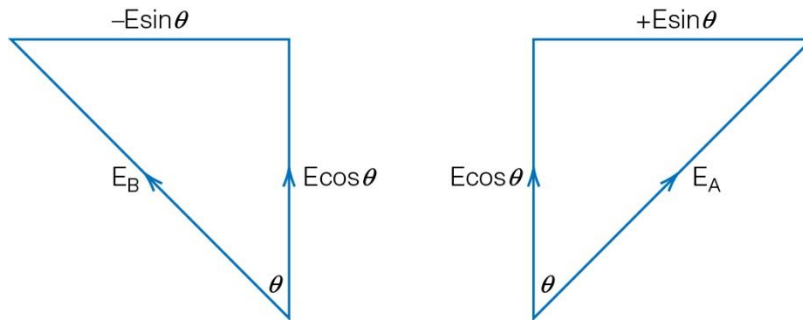
$$\begin{aligned} \text{ii) } E &= \frac{V}{d} \\ &= \frac{7 \times 10^8 \text{ V}}{300 \text{ m}} [1] \\ &= 2.3 \times 10^6 \text{ V m}^{-1} [1] \end{aligned}$$

$$\begin{aligned} \text{iii) } I &= \frac{Q}{t} [1] \\ &= \frac{4.5 \text{ C}}{0.024 \text{ s}} \\ &= 190 \text{ A} [1] \end{aligned}$$

$$\begin{aligned} \text{iv) } W &= VQ \\ &= 7 \times 10^8 \text{ V} \times 4.5 \text{ C} [1] \\ &= 3 \times 10^9 \text{ J} = 3 \text{ GJ} [1] \end{aligned}$$

Page 380 Stretch and challenge

19 a) The fields at C, due to the charges at A and B are as shown.



As the charges are equal: $E_A = E_B = E$

The vector sum of the fields at C is $2 E \cos \theta$ along the line OC, where E is given by:

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$\text{So } E_C = \frac{2q \cos \theta}{4\pi\epsilon_0 r^2}$$

$$\text{but } \cos \theta = \frac{x}{r}$$

$$\text{So } E_C = \frac{q x}{2\pi\epsilon_0 r^3}$$

$$\text{Since } r = (x^2 + a^2)^{\frac{1}{2}}$$

$$E_C = \frac{qx}{2\pi\epsilon_0 (x^2 + a^2)^{\frac{3}{2}}}$$

b) For a maximum we require $\frac{dE_C}{dx} = 0$

Using the quotient rule for differentiation:

$$\text{i.e. if } f(x) = \frac{u}{v} \text{ then } f'(x) = \frac{v u' - u v'}{v^2}$$

$$\frac{dE_C}{dx} = \frac{q}{2\pi\epsilon_0} \left[\frac{(x^2 + a^2)^{\frac{3}{2}} - x \left(\frac{3}{2} (x^2 + a^2)^{\frac{1}{2}} \times 2x \right)}{(x^2 + a^2)^3} \right]$$

When $\frac{dE_C}{dx} = 0$, the numerator of this is zero

$$\Rightarrow (x^2 + a^2)^{\frac{3}{2}} = 3 x^2 (x^2 + a^2)^{\frac{1}{2}}$$

$$x^2 + a^2 = 3 x^2$$

$$2 x^2 = a^2$$

$$x = \pm \frac{a}{\sqrt{2}}$$

20 KE is transferred to electrical potential energy.

At O the particle has zero kinetic energy and maximum electrical potential energy

The electrical field at O is given by $E = \frac{2q}{4\pi\epsilon_0 a}$

so the electrical potential energy of the particle is $W = \frac{2q^2}{4\pi\epsilon_0 a}$

$$\text{So } \frac{1}{2}mv^2 = \frac{q^2}{2\pi\epsilon_0 a}$$

$$v^2 = \frac{2q^2}{2\pi\epsilon_0 am}$$

$$v = \frac{q}{(\pi\epsilon_0 ma)^{1/2}}$$