

Page 308 Test yourself on prior knowledge

- 1 a) Sun: 270 N kg^{-1} ; Earth: 9.8 N kg^{-1} ; Mercury: 3.7 N kg^{-1} ; Ceres: 0.3 N kg^{-1} .

The Sun has the largest field strength, due its great mass; Ceres the lowest field strength due to its low mass. You know Earth's field strength, so you can deduce that of Mercury – which is a smaller planet than Earth.

- 2 Work done climbing Everest = $m g h$

$$= 120 \text{ kg} \times 9.8 \text{ N kg}^{-1} \times 8\,800 \text{ m}$$

$$= 10.3 \text{ MJ}$$

Work done climbing Olympus Mons = $m g h$

$$= 120 \text{ kg} \times 3.7 \text{ N kg}^{-1} \times 22\,000 \text{ m}$$

$$= 9.8 \text{ MJ}$$

- 3 a) $v^2 = 2 g h$

$$v = (2 g h)^{\frac{1}{2}}$$

$$= (2 \times 9.8 \text{ N kg}^{-1} \times 5 \text{ m})^{\frac{1}{2}}$$

$$= 9.9 \text{ m s}^{-1}$$

b) $g = \frac{v^2}{2h}$

$$= \frac{(100 \text{ m}^2 \text{s}^{-2})}{2 \times 31 \text{ m}}$$

$$= 1.6 \text{ N kg}^{-1}$$

c) $v = (2 g h)^{\frac{1}{2}}$

So, if h is doubled, v increases by $\sqrt{2}$ and $v_{2h} = 10 \times \sqrt{2} = 14 \text{ m s}^{-1}$

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1 a) $F = \frac{G m_1 m_2}{r^2}$

$$= \frac{(6.7 \times 10^{-11} \text{ Nm}^2 \text{kg}^{-2}) \times (4 \times 10^{25} \text{ kg}) \times 100 \text{ kg}}{(8.4 \times 10^6 \text{ m})^2}$$

$$= 3\,800 \text{ N}$$

b) $g = \frac{3800 \text{ N}}{100 \text{ kg}}$

$$38 \text{ N kg}^{-1}$$

2 a) $F = \frac{G m_1 m_2}{r^2}$

$$= \frac{(6.7 \times 10^{-11} \text{ Nm}^2 \text{kg}^{-2}) \times (80 \text{ kg}) \times (80 \text{ kg})}{(10 \text{ m})^2}$$

$$= 4 \times 10^{-9} \text{ N}$$

We have assumed (incorrectly) that the two people are spheres.

- b)** 4×10^{-9} N is a tiny force.
- c)** Electrostatic forces are far larger than gravitational forces.
- 3** The pull on Jupiter is 300 x larger due to its larger mass, but 5^2 times smaller due to the larger distance. So the pull on Jupiter is $300/25 = 12$ times larger.
- 4 a)** $I \propto \frac{1}{r^2}$
- So the intensity will be 3^2 or 9 times smaller.
- So $I = 0.022 \text{ W m}^{-2}$
- b)** $I = \frac{\text{Light power}}{4\pi r^2}$
- $\Rightarrow \text{Light Power} = 4\pi \times (1 \text{ m})^2 \times 0.2 \text{ W m}^{-2}$
- $= 2.5 \text{ W}$
- Since it is 20% efficient, the total power transformed by the bulb is $2.5 \text{ W} \times 5 = 12.5 \text{ W}$.
- 5** $F = \frac{Gm_1m_2}{r^2}$
- $$= \frac{(6.7 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}) \times (7 \times 10^{11} \times 2 \times 10^{30} \text{ kg}) \times (7 \times 10^{11} \times 2 \times 10^{30} \text{ kg})}{(2.4 \times 10^{22} \text{ m})^2}$$
- $= 2 \times 10^{29} \text{ N}$
- (The two galaxies are expected to collide to form one giant galaxy in about 4 billion years time.)

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- 6 a)** $g = \frac{GM}{r^2}$
- $$= \frac{(6.7 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}) \times (4.6 \times 10^{23} \text{ kg})}{(3.2 \times 10^6 \text{ m})^2}$$
- $= 3.0 \text{ N kg}^{-1}$
- b)** $R = 6\,400 \text{ km} + 3\,200 \text{ km}$
- $= 9\,600 \text{ km}$
- $g \propto \frac{1}{r^2}$
- since r^2 is 9 times larger, g is 9 times less.
- So $g = \frac{3}{9} \text{ N kg}^{-1}$
- $= 0.33 \text{ N kg}^{-1}$
- 7 a)** $\frac{r_1}{r_2} = \frac{8.4 \times 10^5 \text{ km}}{14 \text{ km}}$
- $= 6 \times 10^4$

$$\begin{aligned}\text{b) } g &= 400 \text{ N kg}^{-1} \times (6 \times 10^4)^2 \\ &= 1.4 \times 10^{12} \text{ N kg}^{-1}\end{aligned}$$

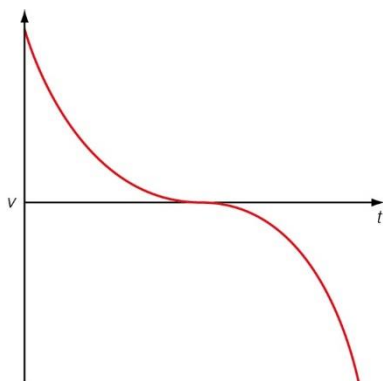
$$\begin{aligned}8 \quad g \text{ at sea level} &= \frac{GM}{r_1^2} \\ &= \frac{(6.7 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}) \times (6.0 \times 10^{24} \text{ kg})}{(6.4 \times 10^6 \text{ m})^2} \\ &= 9.81 \text{ N kg}^{-1}\end{aligned}$$

At the top of Mount Everest:

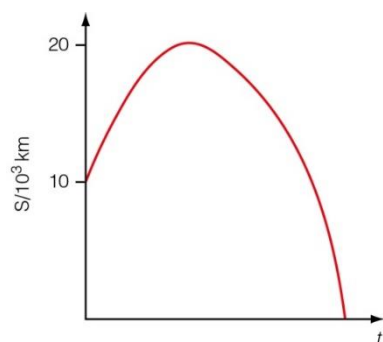
$$\begin{aligned}g &= \frac{(6.7 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}) \times (6.0 \times 10^{24} \text{ kg})}{(6.4088 \times 10^6 \text{ m})^2} \\ &= 9.79 \text{ N kg}^{-1}\end{aligned}$$

So, to two significant figures, we can take g to be about 9.8 N kg^{-1} in both places.

- 9 a) The gradient of the velocity time graph is the acceleration, which is equal to the gravitational field strength.



- b) The gradient of the displacement graph is the velocity of the spacecraft at any point.



- 10 G has units $\text{N m}^2 \text{ kg}^{-2}$

$$\text{and } 1 \text{ N} = 1 \text{ kg m s}^{-2}$$

$$\text{so } G \text{ has units: } \text{kg m s}^{-2} \text{ m}^2 \text{ kg}^{-2}$$

$$= \text{m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

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- 11 a) Gravitational potential difference is the gravitational potential energy difference per kg between two points.
- b) J kg^{-1}
- c) An equipotential is a line or surface where the gravitational potential is the same along that line or surface.

12 a) $\Delta E = m \Delta V$

$$= 5 \text{ kg} \times 100 \text{ J kg}^{-1}$$

$$= 500 \text{ J}$$

b) 0

c) $\Delta E = m \Delta V$

$$= 5 \text{ kg} \times 300 \text{ J kg}^{-1}$$

$$= 1500 \text{ J}$$

13 $\frac{\Delta v}{\Delta h} = \frac{400 \text{ J kg}^{-1}}{80 \text{ m}}$

$$= 5 \text{ N kg}^{-1}$$

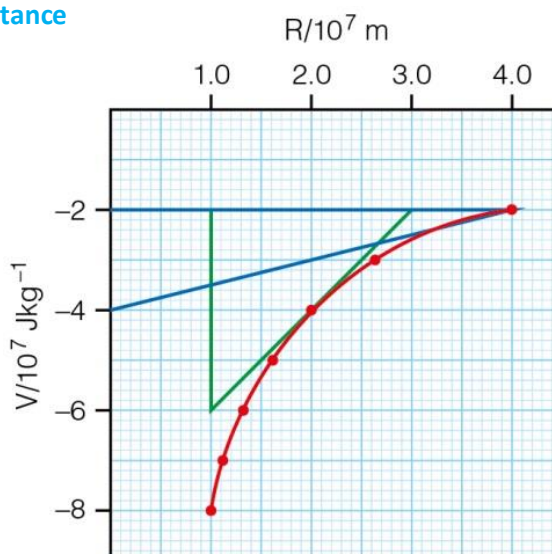
This is the same value as the gravitational field, g , but in the opposite direction. When dealing with units: note that $\text{J m}^{-1} = \text{N}$

Page 317 Activity

Equipotentials and variation of potential with distance

1

Potential/ 10^7 J kg^{-1}	$r/$ 10^7 m	$1/r$ $/10^{-7} \text{ m}^{-1}$
-8	1.00	1.00
-7	1.14	0.88
-6	1.33	0.75
-5	1.60	0.63
-4	2.00	0.50
-3	2.70	0.37
-2	4.00	0.25



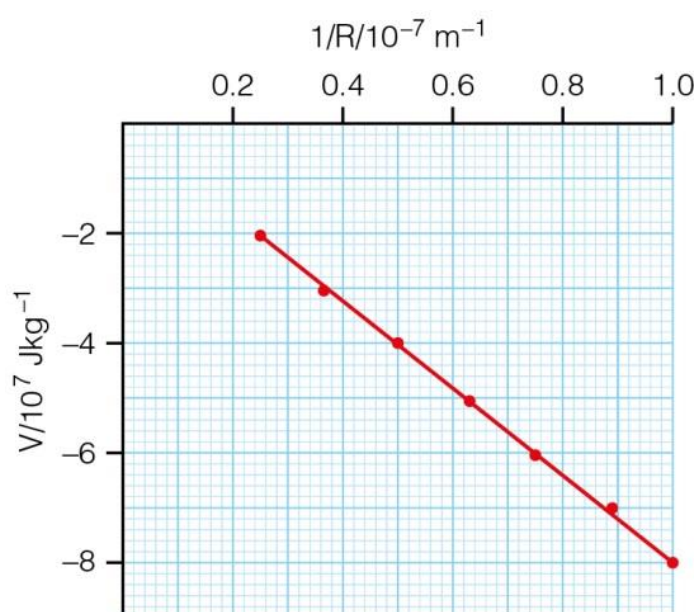
2 a) graph gradient: $g = -\frac{\Delta v}{\Delta r}$
 $= 2 \text{ N kg}^{-1}$

b) graph gradient: $g = -\frac{\Delta v}{\Delta r}$
 $= 0.5 \text{ N kg}^{-1}$

Note that as the distance is doubled the field reduces to a $\frac{1}{4}$ of its former value.

The line used to calculate the second value is much harder to draw accurately as there is less data around that point.

3 The graph should look similar to this:



$$V = -\frac{GM}{R}$$

So gradient $= -GM$

$$\text{gradient} = \frac{6 \times 10^7 \text{ J kg}^{-1}}{0.75 \times 10^{-1} \text{ m}^{-1}}$$

$$= 8.0 \times 10^{14} \text{ J m kg}^{-1}$$

Thus $GM = 8.0 \times 10^{14} \text{ J m kg}^{-1}$

$$M = \frac{8 \times 10^{14} \text{ J m kg}^{-1}}{6.7 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}}$$

$$= 1.2 \times 10^{25} \text{ kg}$$

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14 This definition ensures that all planets and stars have the same zero point for their potentials.

$$\begin{aligned}
 15 \quad V &= -\frac{GM}{R} \\
 &= -\frac{(6.7 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}) \times (1.9 \times 10^{27} \text{ kg})}{7.0 \times 10^7 \text{ m}} \\
 &= -1.8 \times 10^9 \text{ J kg}^{-1}
 \end{aligned}$$

Make sure you have the correct unit and sign.

$$16 \text{ a) i) } \Delta W = 0$$

$$\begin{aligned}
 \text{ii) } \Delta W &= m \Delta V \\
 &= 1200 \text{ kg} \times (6 \times 10^7 - 4 \times 10^7) \text{ J kg}^{-1} \\
 &= 2.4 \times 10^{10} \text{ J}
 \end{aligned}$$

$$\text{iii) } \Delta W = 0$$

$$\begin{aligned}
 \text{iv) } \Delta W &= m \Delta V \\
 &= 1200 \text{ kg} \times (4 \times 10^7 - 2 \times 10^7) \text{ J kg}^{-1} \\
 &= 2.4 \times 10^{10} \text{ J}
 \end{aligned}$$

b) The spacecraft stays in its circular orbit at a constant speed, because no work is done by the planet's field to change its direction. It is on an equipotential so its energy remains unchanged.

$$\begin{aligned}
 \text{c) } \frac{1}{2}mv_2^2 - mv_5^2 &= m\Delta V \\
 v_2^2 - v_5^2 &= 2\Delta V \\
 v_2^2 &= v_5^2 + 2\Delta V \\
 &= (5.2 \times 10^3 \text{ m s}^{-1})^2 + 2 \times 4 \times 10^7 \text{ J kg}^{-1} \\
 &= 2.7 \times 10^7 \text{ m}^2 \text{ s}^{-2} + 8 \times 10^7 \text{ m}^2 \text{ s}^{-2} \\
 v_2 &= 10300 \text{ m s}^{-1}
 \end{aligned}$$

17 Using the diagram to scale, the distance to the first equipotential above the planet is about 1400 km.

$$\begin{aligned}
 \text{So } g &= -\frac{\Delta V}{\Delta r} \\
 &= -\frac{(8-7) \times 10^7 \text{ J kg}^{-1}}{1.4 \times 10^6 \text{ m}} \\
 &= -7.1 \text{ N kg}^{-1}
 \end{aligned}$$

The minus sign means the field is (of course) pointing towards the surface.

OR

$$v = \frac{GM}{r}; g = \frac{GM}{r^2}$$

$$\text{so } g = -\frac{V}{r} \text{ at the surface}$$

$$= -\frac{8 \times 10^7 \text{ J kg}^{-1}}{10^7 \text{ m}}$$

$$= -8.0 \text{ N kg}^{-1}$$

As you can see both methods give about the same answer, but the answers are not exactly the same as the first method is only approximate, as the field reduces in size above the surface.

$$\mathbf{18 \ a)} \quad v^2 = \frac{GM}{r}$$

$$r = \frac{GM}{v^2}$$

$$= \frac{6.7 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \times (10^{31} \text{ kg})}{(3 \times 10^8 \text{ m s}^{-1})^2}$$

$$= 7400 \text{ m or } 7.4 \text{ km}$$

$$\mathbf{b) \ i)} \quad g = \frac{GM}{r^2}$$

$$= \frac{(6.7 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}) \times 10^{31} \text{ kg}}{(7400 \text{ m})^2}$$

$$= 1.2 \times 10^{13} \text{ N kg}^{-1}$$

$$\mathbf{ii)} \quad V = -\frac{GM}{r}$$

$$= -\frac{(6.7 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}) \times 10^{31} \text{ kg}}{7400 \text{ m}}$$

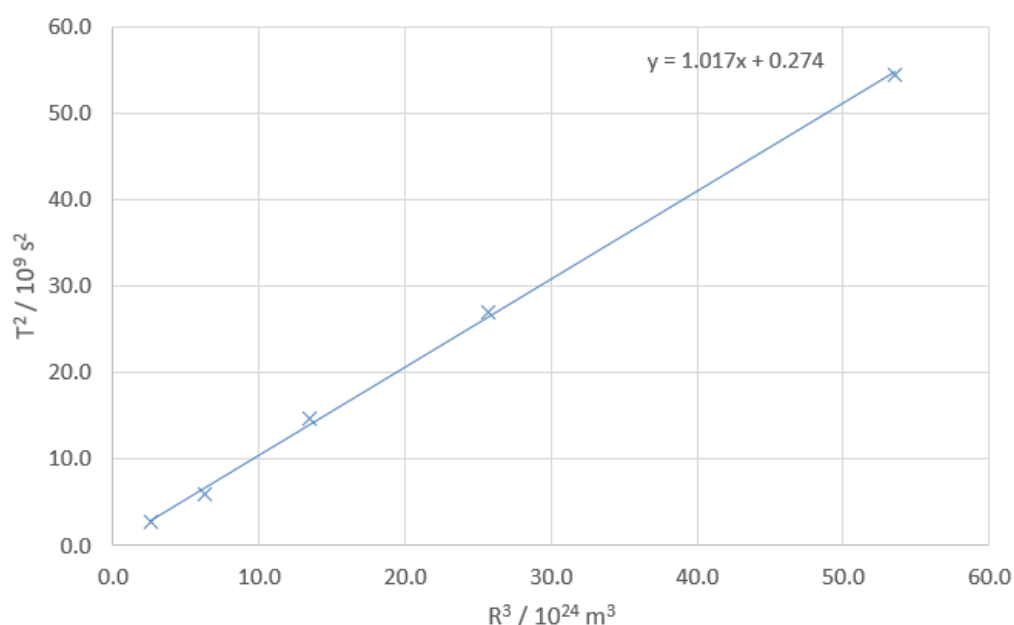
$$= -9 \times 10^{16} \text{ J kg}^{-1}$$

Page 322 Activity

The moons of Saturn

1 a)

Moon	Orbital radius/ 10^3 km	Orbital period/ days	$R^3 /$ 10^{24} m^3	$T^2 /$ 10^9 s^2
Atlas	137	0.6	2.6	2.7
Mimas	185	0.9	6.3	6.0
Methone	194		7.3	
Enceladus	238	1.4	13.5	14.6
Tethys	295	1.9	25.7	26.9
Dione	377	2.7	53.6	54.4



From the graph, T^2 for Methone is $7.7 \times 10^9 \text{ s}^2$

Methone's orbital period, $T = 87700 \text{ s} = 24.4 \text{ hours} = 1 \text{ day}$

b) The gradient of the graph is:

$$(5.44 \times 10^{10} - 2.7 \times 10^9) / (5.36 \times 10^{25} - 2.6 \times 10^{24}) = 1.01 \times 10^{-15} \text{ s}^2 \text{ m}^{-3}$$

$$\text{But the gradient} = \frac{4\pi^2}{GM}$$

$$\text{So } 1.01 \times 10^{-15} \text{ s}^2 \text{ m}^{-3} = \frac{4\pi^2}{(6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}) \times M}$$

$$M = \frac{4\pi^2}{(6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}) \times (1.01 \times 10^{-15} \text{ s}^2 \text{ m}^{-3})}$$

$$\approx 5.9 \times 10^{26} \text{ kg}$$

2 Dione and the Moon

$$\text{Since } T^2 = \left(\frac{4\pi^2}{GM} \right) r^3$$

$$\Rightarrow \text{if } r \text{ is constant, } T^2 \propto \frac{1}{M}$$

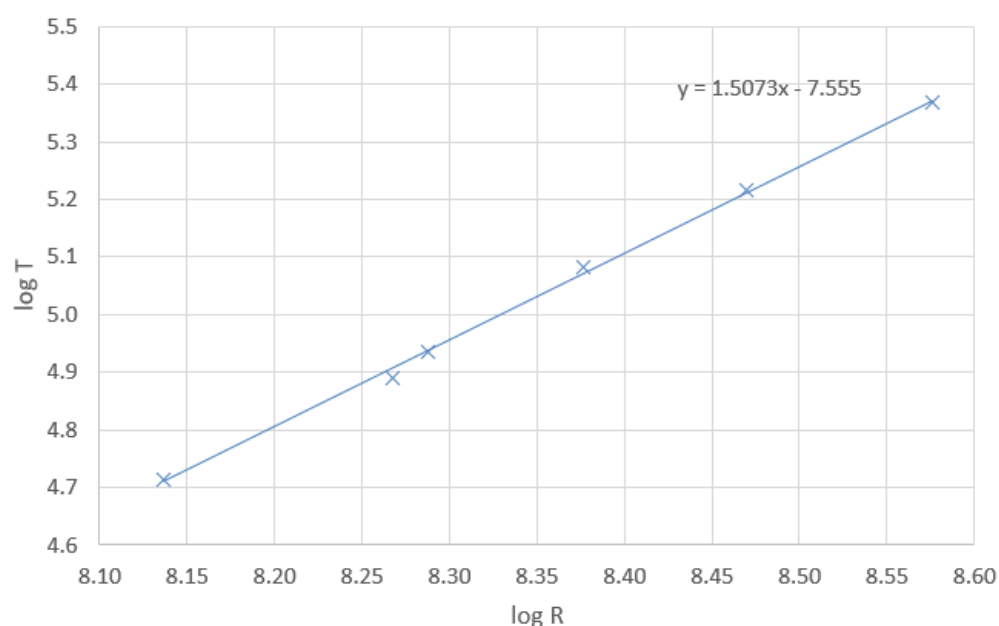
$$\text{So } \frac{M_S}{M_E} = \frac{(T_{\text{Moon}})^2}{(T_{\text{Dione}})^2}$$

$$= \left(\frac{27}{2.7} \right)^2$$

$$= 100$$

3

Moon	Orbital radius/ 10 ³ km	Orbital period/ days	log ₁₀ R	log ₁₀ T
Atlas	137	0.6	8.14	4.71
Mimas	185	0.9	8.27	4.89
Methone	194	1.0	8.29	4.94
Enceladus	238	1.4	8.38	5.08
Tethys	295	1.9	8.47	5.22
Dione	377	2.7	8.58	5.37



Excel gives the gradient of this graph as 1.5 (2sf) and thus it confirms that $T^2 \propto R^3$

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19 a) i) Surveillance, weather maps, etc.

ii) Communications.

These satellites are often in *geosynchronous orbits*.

b) i) $g = \frac{GM}{r^2}$

Since G and M are constant, $g_1 r_1^2 = g_2 r_2^2$

$$\frac{g}{9.8} = \frac{6400^2}{(6400+300)^2}$$

$$g = 8.9 \text{ N kg}^{-1}$$

ii) $g = \omega^2 r$

$$\text{So } \omega^2 = \frac{g}{r}$$

$$\left(\frac{2\pi}{T}\right)^2 = \frac{g}{r}$$

$$T^2 = \frac{4\pi^2 r}{g}$$

$$= \frac{4\pi^2 \times 6.7 \times 10^6 \text{ m}}{8.94 \text{ ms}^{-2}}$$

$$T = 5436 \text{ s} \approx 1.5 \text{ h}$$

20 a) Planets have nearly circular orbits. Comet orbits are often highly elongated ellipses.

b) The comet has low potential energy and high kinetic energy near the Sun.

As the comet moves away from the Sun, its potential energy increases and kinetic energy decreases.

21 a) $\omega = \frac{2\pi}{T}$

$$= \frac{2\pi}{120 \times 60 \text{ s}}$$

$$= 8.7 \times 10^{-4} \text{ s}^{-1}$$

b) $\frac{GM}{r^2} = \omega^2 r$

$$\text{so } r^3 = \frac{GM}{\omega^2}$$

$$= \frac{4 \times 10^{14} \text{ N kg}^{-1} \text{ m}^2}{(8.7 \times 10^{-4} \text{ s}^{-1})^2}$$

$$r = 8.0 \times 10^6 \text{ m}$$

22 a) i) $\omega = \frac{2\pi}{T}$

$$= \frac{2\pi}{365 \times 24 \times 3600 \text{ s}}$$

$$= 2 \times 10^{-7} \text{ s}^{-1}$$

ii) $v = \omega r$

$$= 2 \times 10^{-7} \text{ s}^{-1} \times 1.5 \times 10^{11} \text{ m}$$

$$= 3.0 \times 10^4 \text{ m s}^{-1} \text{ or } 30 \text{ km s}^{-1}$$

b) i) $F = m \omega^2 r$

$$= 6.0 \times 10^{24} \text{ kg} \times (2 \times 10^{-7} \text{ s}^{-1})^2 \times 1.5 \times 10^{11} \text{ m}$$

$$= 3.6 \times 10^{22} \text{ N}$$

ii) $F = \frac{GMm}{r^2}$

$$= \frac{(6.7 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}) \times (2 \times 10^{30} \text{ kg}) \times (6 \times 10^{24} \text{ kg})}{(1.5 \times 10^{11} \text{ m})^2}$$

$$= 3.6 \times 10^{22} \text{ N}$$

c) Since the gravitational field strength at a point is the force acting on unit mass, the simplest way to calculate this is to divide the previous answer by the mass of the Earth:

$$g = \frac{3.6 \times 10^{22} \text{ N}}{6 \times 10^{24} \text{ kg}}$$

$$= 0.006 \text{ N kg}^{-1}$$

More formally:

$$g = \frac{GM}{r^2}$$

$$= \frac{(6.7 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}) \times (2 \times 10^{30} \text{ kg})}{(1.5 \times 10^{11} \text{ m})^2}$$

$$= 0.0060 \text{ N kg}^{-1}$$

Pages 324–327 Practice questions

1 C

2 C

3 D

4 A

5 D

6 B

7 D

8 C

9 B

10 B

11 a) $\frac{g^2}{G}$

Units:

$$\begin{aligned}\frac{(\text{N kg}^{-1})^2}{\text{N m}^2 \text{ kg}^{-2}} &= \frac{\text{N}^2 \text{ kg}^{-2}}{\text{N m}^2 \text{ kg}^{-2}} \\ &= \text{N m}^{-2} \quad [1] \\ &= \text{kg m s}^{-2} \text{ m}^{-2} \\ &= \text{kg m}^{-1} \text{ s}^{-2} \quad [1]\end{aligned}$$

b) $g = \frac{GM}{r^2}$

$$= \frac{G \times \frac{4}{3} \pi \rho r^3}{r^2}$$

$$g = \frac{4}{3} \pi G \rho r \quad (\text{this equation should be known}) \quad [1]$$

$$\text{So } r = \frac{g}{\frac{4}{3} \pi G \rho}$$

$$\begin{aligned}\frac{r_{\text{Earth}}}{r_{\text{Moon}}} &= \frac{g_E}{\rho_E} \times \frac{\rho_M}{g_M} = \frac{g_E}{g_M} \times \frac{\rho_M}{\rho_E} \quad [1] \\ &= 6 \times 0.6 \\ &= 3.6 \quad [1]\end{aligned}$$

12 a) Work done = $m\Delta V$ [1]

$$\text{Since } V_A = -16 \text{ MJ kg}^{-1} \text{ then } V_B = -8 \text{ MJ kg}^{-1} \quad [1]$$

$$\begin{aligned}\text{So, work done} &= 120 \text{ kg} \times (16 - 8) \text{ MJ kg}^{-1} \\ &= 960 \text{ MJ} \quad [1]\end{aligned}$$

b) $g \propto \frac{1}{r^2}$ [1]

$$\begin{aligned}\text{So } g &= \frac{1}{4} \times 4 \text{ N kg}^{-1} \\ &= 1 \text{ N kg}^{-1} \quad [1]\end{aligned}$$

13 $g = \frac{4}{3} \pi G \rho r$ (see question 11)

$$g \propto \rho r \quad [1]$$

$$\begin{aligned}\text{So } g \text{ at Betelgeuse surface} &= 270 \text{ N kg}^{-1} \times 0.01 \times 1\,000 \quad [1] \\ &= 2700 \text{ N kg}^{-1} \quad [1]\end{aligned}$$

14 a) Work done = $m\Delta V$ [1]

$$\begin{aligned}&= 600 \text{ kg} \times 8 \times 10^7 \text{ J kg}^{-1} \quad [1] \\ &= 4.8 \times 10^{10} \text{ J} \quad [1]\end{aligned}$$

$$\text{b) } g = \frac{GM}{r^2} [1]$$

$$\text{At P: } g_1 = g_2 [1]$$

$$\text{So } \frac{GM_A}{(3r)^2} = \frac{GM_B}{r^2}$$

Planet A is 9 times more massive than planet B [1]

$$\text{15 a) } \omega = \frac{2\pi}{T} [1]$$

$$= \frac{2\pi}{42 \times 3600} \text{ s}^{-1} [1]$$

$$= 4.2 \times 10^{-5} \text{ s}^{-1} [1]$$

$$\text{b) } \frac{GM}{r^2} = \omega^2 r [1]$$

$$M = \frac{\omega^2 r^3}{G} [1]$$

$$= \frac{(4.1 \times 10^{-5} \text{ s}^{-1})^2 \times (4.2 \times 10^8 \text{ m})^3}{(6.7 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})} [1]$$

$$= 1.9 \times 10^{27} \text{ kg} [1]$$

$$\text{c) } T_{\text{Ganymede}} = (168 \times 3600) \text{ s} \Rightarrow \omega_{\text{Ganymede}} = 1.0 \times 10^{-5} \text{ s}^{-1} [1]$$

$$r^3 = \frac{GM}{\omega^2} [1]$$

$$= \frac{(6.7 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}) \times (1.9 \times 10^{27} \text{ kg})}{(1.0 \times 10^{-5} \text{ s}^{-1})^2} [1]$$

$$r = 1.06 \times 10^9 \text{ m} [1]$$

OR

$$\text{Since } r = kT^{2/3} [1]$$

$$\frac{r_G}{r_I} = \left(\frac{T_G}{T_I} \right)^{\frac{2}{3}} [1]$$

$$r = 4.2 \times 10^8 \text{ m} \times \left(\frac{168}{42} \right)^{\frac{2}{3}} [1]$$

$$= 1.06 \times 10^9 \text{ m} [1]$$

$$\text{16 a) Force} = \frac{2 \times 2}{4^2} F [1]$$

$$= \frac{1}{4} F [1]$$

$$\text{b) } F = \frac{Gm_1m_2}{r^2} [1]$$

$$m_2 = \frac{Fr^2}{Gm_1}$$

$$m_2 = \frac{(3.7 \times 10^{-9} \text{ N}) \times (0.2 \text{ m})^2}{(6.7 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}) \times 3.0 \text{ kg}} [1]$$

$$= 0.74 \text{ kg} [1]$$

$$\text{c) } g = \frac{\Delta V}{\Delta x} [1]$$

$$= \frac{800 \text{ J kg}^{-1}}{20 \text{ m}} [1]$$

$$= 40 \text{ N kg}^{-1} [1]$$

17 a) We are used to feeling a reaction force equal to our weight, which gives us the sensation of weight. So, when there is no reaction force, we feel weightless. In this case:

$$mg = m\omega^2 r [1]$$

$$\text{So } g = \left(\frac{2\pi}{T}\right)^2 r$$

$$T^2 = (2\pi)^2 \frac{r}{g}$$

$$T = 2\pi \sqrt{\frac{r}{g}}$$

$$= 2\pi \left(\frac{6.4 \times 10^6 \text{ m}}{9.8 \text{ m s}^{-2}}\right)^{\frac{1}{2}} [1]$$

$$= 5080 \text{ s}$$

$$\approx 1.4 \text{ h} [1]$$

$$\text{b) i) } \frac{GM}{r^2} = \frac{v^2}{r} [1]$$

$$v^2 = \frac{GM}{r}$$

$$v = \left(\frac{GM}{r}\right)^{\frac{1}{2}} [1]$$

$$\text{ii) } \frac{2\pi r}{T} = \left(\frac{GM}{r}\right)^{\frac{1}{2}} [1]$$

$$T = 2\pi r \times \left(\frac{r}{GM}\right)^{\frac{1}{2}}$$

$$= 2\pi \left(\frac{r^3}{GM}\right)^{\frac{1}{2}} [1]$$

18 a) If M is the mass of the Moon:

$$\frac{G \times 81 M}{r_1^2} = \frac{GM}{r_2^2} \quad [1]$$

$$\text{So } \frac{r_1}{r_2} = \sqrt{81} = 9 \quad [1]$$

b) i) $\Delta E = m\Delta V \quad [1]$

$$= 2 \times 10^4 \text{ kg} \times (62.8 - 1.3) \text{ MJ kg}^{-1} \quad [1]$$

$$= 1.2 \times 10^{12} \text{ J} \quad [1]$$

ii) The Moon's gravity now pulls the craft to a place of lower potential. [1]

c)

- The Earth's gravitational field is much larger than the Moon's, and the gravitational potential at or near the surface is much lower. So more work is done to take the craft to N from the Earth than it is from the Moon.
- On the return journey there is only a small potential change required to take the craft from the Moon's surface to N.
- On the return once the craft is at N, Earth's gravity pulls it back. Also on the return journey the spacecraft carries much less fuel, so its mass is less, so less work is done to lift it.

Pages 327–328 Stretch and challenge

19 a) i) $V = -\frac{GM}{R}$

ii) $\frac{1}{2}mc^2 = \frac{1}{2}c^2$ (as $m = 1 \text{ kg}$)

iii) $\frac{GM}{r} = \frac{1}{2}c^2$

$$r = \frac{2GM}{c^2}$$

$$= \frac{(2 \times 6.7 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}) \times (2 \times 10^{30} \times 5 \times 10^9 \text{ kg})}{(3 \times 10^8 \text{ m s}^{-1})^2}$$

$$= 1.5 \times 10^{13} \text{ m}$$

This is very big – about 0.002 light years.

b) i) $\frac{GM}{r^2} = \omega^2 r$

$$\omega^2 = \left(\frac{4\pi^2}{T^2} \right) = \frac{GM}{r^3}$$

$$\Rightarrow T^2 = \left(\frac{4\pi^2}{GM} \right) r^3$$

$$= \frac{4\pi^2 \times (3 \times 10^5 \times 9.5 \times 10^{15} \text{ m})^3}{(6.7 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}) \times (6 \times 10^{12} \times 2 \times 10^{30} \text{ kg})}$$

$$T = 3.4 \times 10^{16} \text{ s} \approx 1000 \text{ million years}$$

ii) Slow-moving stars feed the black hole.

20 a) $V = \frac{GM}{r}$ or $Vr = GM$

So $V_J.r_J = V_S.r_S$

$$\frac{r_S}{r_J} = \frac{-172}{-93}$$

$$= 1.85$$

b) The Sun's gravitational pull accelerates it.

Or, the potential is lower, so it transfers GPE to KE.

c) i) $\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = m\Delta V$

ii) $v_B^2 - v_A^2 = 2\Delta V$

$$v_B^2 = (3 \times 10^4 \text{ m s}^{-1})^2 + (2 \times 800 \times 10^6)(\text{m s}^{-1})^2$$

$$v_B = 5 \times 10^4 \text{ m s}^{-1}$$

iii) $v_C = v_B = 5 \times 10^4 \text{ m s}^{-1}$

$$v_D = v_A = 3 \times 10^4 \text{ m s}^{-1}$$