Page 285 Test yourself on prior knowledge

1 a) T $= \frac{1}{f}$

$= \frac{1}{2.6 \times 10^9\ \mathrm{s}^{-1}}$

$= 3.8 \times 10^{-10}$ s

b) f $= \frac{6}{30\ \mathrm{s}}$

= 0.2 Hz

2 a $= \frac{v_2 - v_1}{t}$

$= \frac{15\ \mathrm{ms}^{-1} - (-20\ \mathrm{ms}^{-1})}{120\ \mathrm{s}}$

= 0.29 m s^{-2} in a southerly direction

3 a) 0.01

b) 0

c) −1

Page 290 Test yourself

1 a) T = 3.2 s

f $= \frac{1}{3.2\ \mathrm{s}}$ = 0.31 s^{-1}

b) A = 12 cm

c) i) v_{max} = 2πfA

= 2π × 0.31 s^{-1} x 0.12 m

= 0.24 m s^{-1}

ii) v = ± 2πf$(A^2 – x^2)^{1/2}$

= ±2π x 0.31 $s^{-1}(0.12^2 – 0.04^2)^{1/2}$ m

= ± 3.96 s^{-1} × 0.113 m

= ± 0.22 m s^{-1}

d) a $= –(2\pi f)^2 x$

= – (2π x 0.31 $s^{-1})^2$ x 0.06 m

= – 0.23 m s^{-2} i.e. an acceleration to the left towards B

2 **a)** $v_{max} = 2\pi fA$

$= 2\pi \times 100\ s^{-1} \times 1.8 \times 10^{-3}\ m$

$= 1.1\ m\ s^{-1}$

b) $a_{max} = (2\pi f)^2 A$

$= (2\pi \times 100 s^{-1})^2 \times 1.8 \times 10^{-3}\ m$

$= 710\ m\ s^{-2}$

The maximum accelerations occur at the points of maximum displacement.

c) **i)** At the midpoint of oscillation.

ii) At both ends of the oscillation – 1.8 mm from the midpoint/equilibrium point.

3 **a)** $v_{max} = 2\pi fA$

$= \frac{2\pi}{2.8\ s} \times 0.9\ m$

$= 2.0\ m\ s^{-1}$

b) $x = 0.9\ m - 1.4\ m$

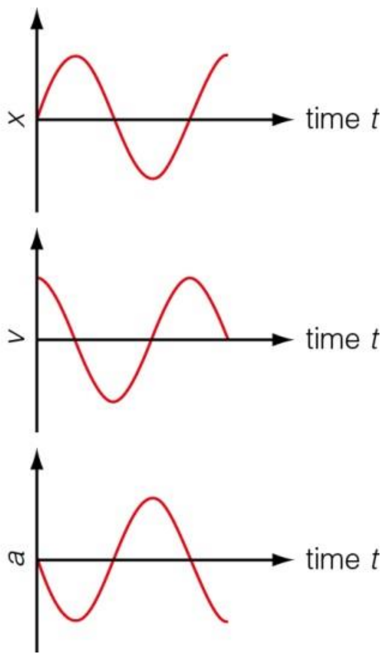
$= -0.5\ m$

So $a = -(2\pi f)^2 x$

$= (\frac{2\pi}{2.8\ s})^2 \times -0.5\ m$

$= 2.5\ m\ s^{-1}$ upwards (because the sign is positive)

4



Pages 292–293 Activities

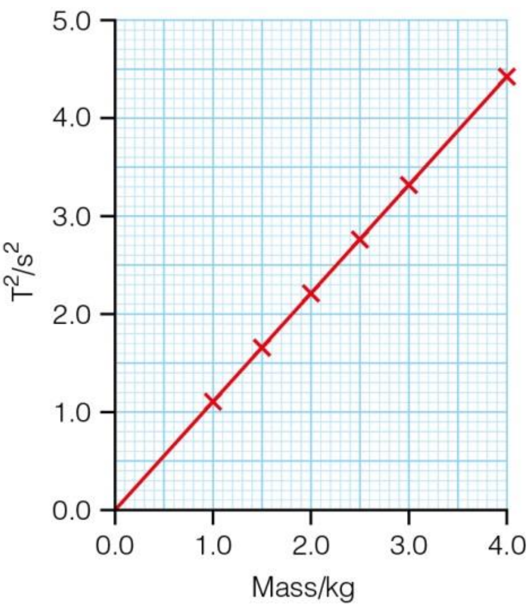
Oscillation of a tethered trolley

1 The restoring force is $2kx$, so $F = ma$ gives $a = \frac{2kx}{m}$

Since $a = \omega^2 x \Rightarrow \omega = \sqrt{\frac{2k}{m}}$

and $T = \frac{2\pi}{\omega} \Rightarrow T = 2\pi\sqrt{\frac{m}{2k}}$

2



a) $T = 2\pi\sqrt{\frac{m}{2k}}$

$T^2 = \frac{4\pi^2}{2k}m$

So graph is consistent since T^2 is proportional to m.

b) The gradient $= \frac{4.41\ s^{-1}}{4.0\ kg}$

$= 1.10\ s^2\ kg^{-1}$

The gradient should be:

$\frac{4\pi^2}{2k} = \frac{4\pi^2}{2 \times 17.8}\ s^2\ kg^{-1}$

$= 1.11\ s^2\ kg^{-1}$

so the two answers compare well.

Other systems that might show SHM

1 a) For the U-tube, when the water is displaced by x there is a restoring force due to the weight of the water 2ρAgx.

So ma = –2ρAgx

where ρ is the density of water, g the gravitational field strength, and A the cross-sectional area of the tube.

Then ρALa= –2ρAgx

Because the mass of the water in the tube is ρAL

and $a = \frac{-2g}{L}x$

and $T = 2\pi\sqrt{\frac{L}{2g}}$

So the period should be independent of the amplitude of the oscillations.

b) Check your measured answer with the theory.

2 a) When the tube is displaced downwards by x, there is an extra upthrust ρAgx equal to the weight of the water displaced. (Archimedes' principle); ρ is the density of water, A the cross-sectional area of the tube.

So F = ma = –ρAgx

But when the tube is in equilibrium, the weight of the water displaced equals the weight of the tube.

So mg = ρAgL

and m = ρAL

Therefore the equation describing the SHM is:

ρALa = – ρAgx

or $a = -\frac{g}{L}x$

so $T = 2\pi\sqrt{\frac{L}{g}}$

and the period should again be independent of the amplitude of the oscillations.

b) Check your measured answer with the theory.

Page 294 Test yourself

5 a) $T = 2\pi\sqrt{\frac{L}{g}}$

$= 2\pi\sqrt{\frac{2\ \text{m}}{9.81\ \text{m s}^{-1}}}$

$= 2.8$ s

b) 7.0 s

c) 360 s

6 a) i) $k = \frac{F}{x}$

$= \frac{9.5\ \text{kg} \times 9.81\ \text{N kg}^{-1}}{0.18\ \text{m}}$

$= 518\ \text{N m}^{-1}$ or $520\ \text{N m}^{-1}$ (2 sf)

ii) $T = 2\pi\sqrt{\frac{m}{k}}$

$= 2\pi\sqrt{\frac{9.5\ \text{kg}}{518\ \text{N m}^{-1}}}$

$= 0.85$ s

iii) $v_{max} = 2\pi fA$

$= \frac{2\pi}{T}A$

$= \frac{2\pi}{0.85\ \text{s}} \times 0.1$ m

$= 0.74\ \text{m s}^{-1}$

b) $T = 2\pi\sqrt{\frac{m}{k}}$

$m = (\frac{T}{2\pi})^2 k$

$= (\frac{1.0\ \text{s}}{2\pi})^2 \times 518\ \text{N m}^{-1}$

$= 13.1$ kg

OR

$\frac{T_2^2}{T_1^2} = \frac{m_2}{m_1}$

$m_2 = m_1\frac{T_2^2}{T_1^2}$

$= 9.5 \times (\frac{1}{0.85})^2 = 13.1$ kg

c) i) Weight is the pull of gravity on an object. This is a force and is measured in N.

ii) Mum was making the point that mass is a measure of how hard it is to set something in motion: 13.1 kg babies accelerate more slowly than 9.5 kg babies. Dad made a slip over the definition of weight and mass.

7 a) $v = \frac{d}{t}$

$= \frac{0.06\text{ m}}{0.04\text{ s}}$

$= 1.5\text{ m s}^{-1}$

b) $v_{max} = 2\pi fA$

$= \frac{2\pi}{T}A$

$= \frac{2\pi}{0.4} \times 0.1\text{ m s}^{-1}$

$= 1.57\text{ m s}^{-1}$

c) $T = 2\pi\sqrt{\frac{m}{k}}$

$T^2 = 4\pi^2\frac{m}{k}$

$k = \frac{4\pi^2 m}{T^2}$

$= \frac{4\pi^2 \times 0.5\text{ kg}}{(0.4)^2}$

$= 120\text{ N m}^{-1}$

Pages 296–297 Test yourself

8 a) About 1%.

b) $T = 2\pi\sqrt{\frac{m}{k}}$

$\sqrt{\frac{k}{m}} = \frac{2\pi}{T} = 2\pi f$

So, $k = (2\pi f)^2 m$

$= (2\pi \times 10^{13}\text{ s}^{-1})^2 \times 10^{-25}$

$= 400\text{ N m}^{-1}$ (1 sf)

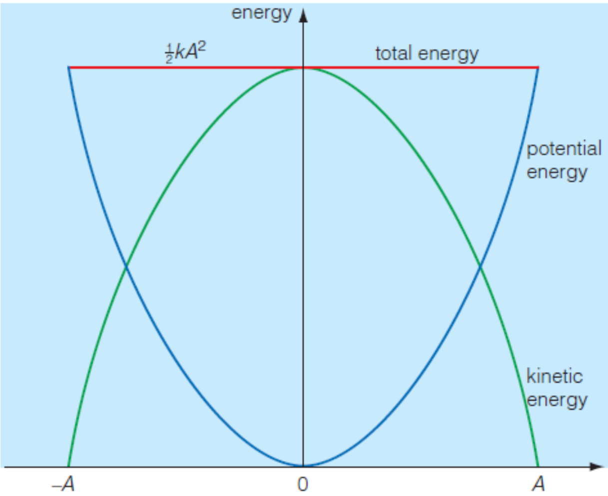
c) i) $E = \frac{1}{2}kA^2$

$= \frac{1}{2} \times 400\text{ N m}^{-1} \times (2 \times 10^{-12}\text{ m})^2$

$= 8 \times 10^{-22}\text{ J}$

ii) 8×10^{-22} J = $\frac{8\times10^{-22}\text{J}}{1.6\times10^{-19}\text{C}}$ = 5×10^{-3} eV

9



10 a) i) $\frac{1}{2}mv^2 = 0.08$ J

$v^2 = \frac{2\times0.08\text{ J}}{0.1\text{ kg}}$

$v = 1.3$ m s^{-1}

ii) $\frac{1}{2}mv^2 = 0.08$ J – 0.02 J = 0.06 J

So, $v^2 = \frac{0.12\text{ J}}{0.1\text{ kg}}$

$v = 1.1$ m s^{-1}

b) i) $v_{max} = 2\pi fA$

$2\pi f = \frac{1.3\text{ m s}^{-1}}{\text{A}}$

$= \frac{1.3\text{ m s}^{-1}}{0.2\text{ m}}$

$= 6.5$ s^{-1}

$\frac{2\pi}{\text{T}} = 6.5$ s^{-1}

T = 0.97 s ≈ 1 s

ii) $T = 2\pi\sqrt{\frac{l}{g}}$

$\frac{l}{g} = (\frac{T}{2\pi})^2$

$l = (\frac{T}{2\pi})^2 g$

$= (\frac{0.97}{2\pi})^2 \times 9.81$ m s^{-1} = 0.23 m

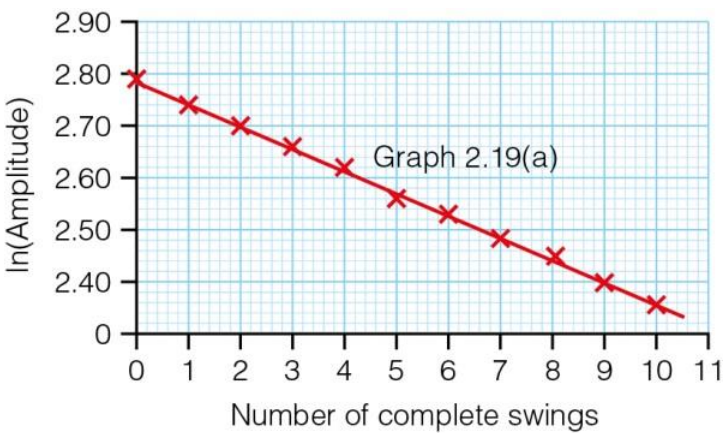
Page 299 Activity

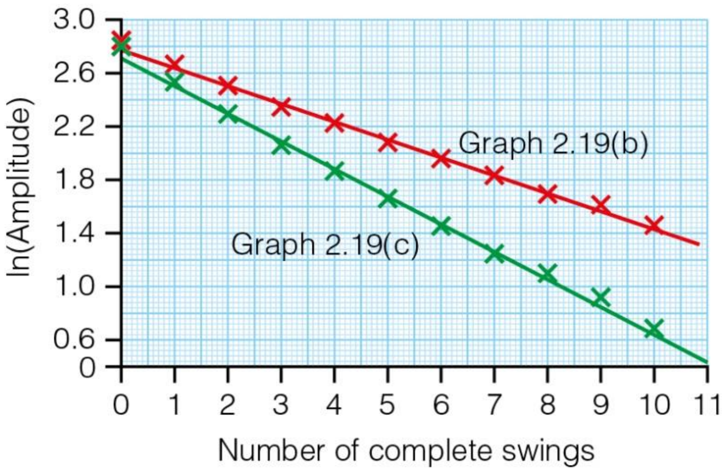
A damped oscillator

1

Number of swings	Amplitude (degrees) graph (a)	ln (A) graph (a)	Amplitude (degrees) graph (b)	ln (A) graph (b)	Amplitude (degrees) graph (c)	ln (A) graph (c)
0	16.3	2.79	17.2	2.85	16.5	2.80
1	15.5	2.74	14.5	2.67	12.7	2.54
2	14.9	2.70	12.3	2.51	10.0	2.30
3	14.3	2.66	10.5	2.35	7.9	2.07
4	13.7	2.62	9.3	2.23	6.5	1.87
5	13.0	2.56	8.0	2.08	5.3	1.67
6	12.5	2.53	7.0	1.95	4.3	1.46
7	12.0	2.48	6.3	1.84	3.5	1.25
8	11.6	2.45	5.5	1.7	3.0	1.10
9	11.0	2.40	5.0	1.61	2.5	0.92
10	10.6	2.36	4.3	1.46	2.0	0.69

2 a)





b) All three oscillations are close to obeying an exponential decay since all three plots show a linear relationship. Further investigation may be necessary to prove this.

c) In each case, $-\lambda$ = gradient of the graph.

Using values of the gradient from Excel plots:

Graph (a) $\lambda = 0.043$ swings^{-1}

$T_{\frac{1}{2}} = \frac{0.693}{0.043}$ swings

= 16.1 swings ≈ 16 swings

Graph (b) $\lambda = 0.132$ swings^{-1}

$T_{\frac{1}{2}} = \frac{0.693}{0.132}$ swings

= 5.25 swings ≈ 5 swings

Graph (c) $\lambda = 0.208$ swings^{-1}

$T_{\frac{1}{2}} = \frac{0.693}{0.208}$ swings

= 3.3 swings ≈ 3 swings

Page 303 Test yourself

11 a) The fifth from the left is the same length as the driver pendulum. This will swing with large amplitude.

The other six pendulums will swing with much smaller amplitudes.

You could add something about phase. The two on the right (those longer than the driver) swing in phase with the driver. The fifth from the left moves 90° out of phase with the driver. The four (shorter) pendulums on the left move out of phase with the driver.

b) The motion is damped, so the pendulums move with smaller amplitude. This is most noticeable with the resonant pendulum (fifth from left).

12 a) An oscillator undergoes high amplitude oscillations (resonance) when the driving frequency is the same as the natural frequency.

b) As the book says, a microwave takes advantage of the resonance of water molecules. The frequency of the microwaves is matched to the natural frequency of oscillation of water molecules. So, when something is cooked in the microwave oven, water molecules absorb energy from the microwaves. The water molecules start to vibrate. This energy is then dissipated as random vibrational energy among all the molecules in the food. Random vibrational energy is thermal energy.

c) Again, as the book says, even a large structure such as a chimney or a bridge can be set oscillating by eddies of wind. And, if the wind causes vortices of just the right frequency, large oscillations can build up.

Pages 304–307 Practice questions

1 C

2 B

3 B

4 C

5 A

6 B

7 C

8 D

9 A

10 C

11 a) The restoring force is proportional to and in the opposite direction to the displacement. [1]

b) F = kx

$k = \frac{F}{x}$

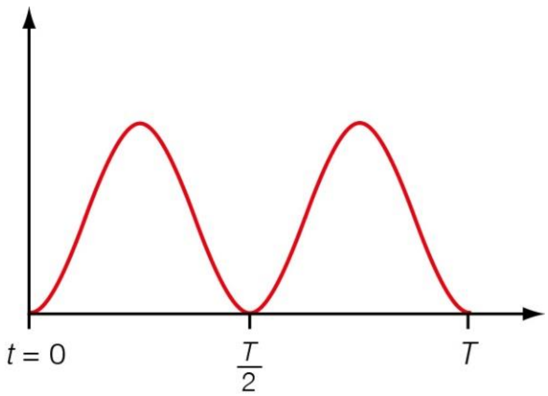
$= \frac{28\ \text{N}}{0.05\ \text{m}}$

$= 560\ \text{N m}^{-1}$ [1]

c) $T = 2\pi\sqrt{\frac{65\ \text{kg}}{560\ \text{N m}^{-1}}}$ [1]

$= 2.1\ \text{s}$ [1]

d) One mark for smooth curves, always positive and one mark for position of minima. [2]



12 a) $T = 2\pi\sqrt{\frac{l}{g}}$ [1]

$$\left(\frac{T}{2\pi}\right)^2 = \frac{l}{g}$$

$$l = \left(\frac{T}{2\pi}\right)^2 g$$

$l = \left(\frac{2.79}{2\pi}\right)^2 \times 9.81$ [1]

= 1.9 m [1]

b) $mgh = \frac{1}{2}mv^2$ [1]

$h = \frac{v^2}{2g}$

$= \frac{(3.5\ \text{m s}^{-1})^2}{(2 \times 9.81\ \text{m s}^{-2})}$

= 0.62 m [1]

c) i) $F = \frac{mv^2}{r}$

$= \frac{42\ \text{kg} \times (3.5\ \text{m s}^{-1})^2}{1.9\ \text{m}}$ [1]

= 270 N [1]

ii) F = 270 N + 42 kg × 9.81 N kg^{-1}

= 680 N [1]

13 a) F = −kx [1]

When the trolley comes into contact with the block the spring is compressed. The force acts to the right, which is in the opposite direction to the displacement of the trolley. The equation describes SHM. [1]

b) $a = \frac{\Delta v}{\Delta t}$ i.e. the acceleration is the gradient of the v–t graph. [1]
The acceleration is greatest when the v–t graph is steepest. [1]
Since the gradient is negative the acceleration is negative. [1]

c) The graphs show the trolley is in contact with the block for half a period of the SHM

$T = 2\pi\sqrt{\frac{m}{k}}$ [1]

$= 2\pi\sqrt{\frac{0.7\ \text{kg}}{25\ \text{N m}^{-1}}}$

= 1.05 s [1]

$t = \frac{T}{2} = 0.53$ s [1]

d) i) $v_{max} = 2\pi fA$

$= \frac{2\pi A}{T}$

So $A = \frac{v_{max}T}{2\pi}$ [1]

$= \frac{0.1\ \mathrm{m\ s^{-1}} \times 1.05\ \mathrm{s}}{2\pi}$

= 0.017 m [1]

OR

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$x = \sqrt{\frac{m}{k}}v_{max}$ [1]

$= \sqrt{\frac{0.7\ \mathrm{kg}}{25\ \mathrm{N\ m^{-1}}}} \times 0.1\ \mathrm{m\ s^{-1}}$

= 0.017 m [1]

ii) $a = \left(\frac{2\pi}{T}\right)^2 A$ [1]

$= \left(\frac{2\pi}{1.05\ \mathrm{s}}\right)^2 \times 0.017\ \mathrm{m}$

$= 0.6\ \mathrm{m\ s^{-2}}$ [1]

e) i) The maximum acceleration doubles. [1]

ii) There is no change to the time. [1]

f) The time of contact increases [1]

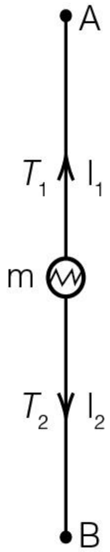
$T = 2\pi\sqrt{\frac{m}{k}}$ or we can explain that it takes longer for the force to slow the mass down. [1]

14 See Activity on page 300 – one mark for each point below

- Set up a mass on a spring and attach it to a variable signal generator.
- Measure the natural frequency.
- Adjust the driving frequency.
- Measure the amplitude of the oscillations.
- Plot amplitude against driving frequency.
- The maximum amplitude should occur at the natural frequency of oscillation.

Page 307 Stretch and challenge

15 a)



distance AB = $3l = l_1 + l_2$

$F = \frac{ke}{l}$

so $T_1 = \frac{k}{l}(l_1 - l)$

and $T_2 = \frac{k}{l}(l_2 - l) = \frac{k}{l}(3l - l_1 - l) = \frac{k}{l}(2l - l_1)$

At equilibrium: $T_1 - T_2 = mg$

Substituting for T_1 and T_2 gives

$\frac{k}{l}(l_1 - l) - \frac{k}{l}(2l - l_1) = mg$

$\Rightarrow kl_1 - kl - 2kl + kl_1 = mgl$

$2kl_1 = mgl + 3kl$

So, at equilibrium, the distance between A and the particle is

$$l_1 = \left(\frac{mg}{2k} + \frac{3}{2}\right)l$$

b) When m is pulled down a small distance z:

T_1 increases by $\frac{kz}{l}$

T_2 decreases by $\frac{kz}{l}$

So the resultant force = $\frac{2kz}{l}$ upwards

Therefore ma = $-\frac{2kz}{l}$

$a = -\frac{2kz}{ml}$

This is the SHM equation with:

$\omega^2 = (2\pi f)^2 = \frac{2k}{ml}$

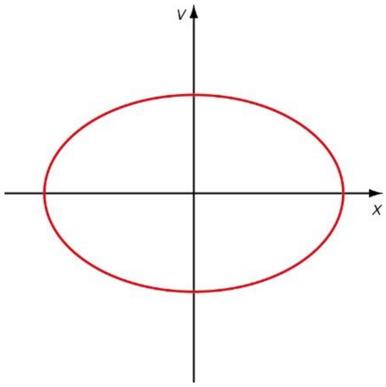
Therefore, the time period, $T = 2\pi\left(\frac{ml}{2k}\right)^{1/2}$

and $v_{max} = 2\pi f\, z_0$

$= z_0\left(\frac{2k}{ml}\right)^{1/2}$

c) The equation for velocity is $v = \pm\, 2\pi f\, (z_o{}^2 - z^2)^{1/2}$

$v^2 = (2\pi f)^2\, (z_o{}^2 - z^2)$ is the equation of an ellipse, as shown below

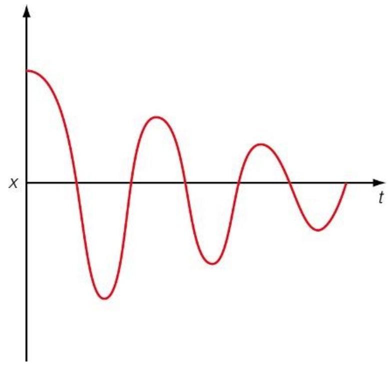


d) i) The original solution: $l_1 = \frac{mgl}{2k} + \frac{3l}{2}$

assumes that $\frac{mgl}{2k} < \frac{l}{2}$ so that the lower elastic remains in tension.

If z_0 is large l_2 will go slack and there will be uneven time periods on either side of the equilibrium position.

ii) There is damped SHM:



Decreasing amplitude, but constant time period.