Page 274 Test yourself on prior knowledge

1 a) Speed is a scalar and is measured in m s^{-1}. Velocity is measured in m s^{-1}, but is a vector so we must also define a direction.

b) Acceleration is a vector, because acceleration is in a particular direction. A car speeds up by accelerating forwards but decelerates by accelerating backwards.

2 a) $a = \frac{v_2 - v_1}{t}$

$= \frac{4\text{ m s}^{-1} - 3\text{ m s}^{-1}}{10\text{ s}}$

$= 0.1$ m s^{-2} (to the right)

b) $a = \frac{v_2 - v_1}{t}$

$= \frac{-4\text{ m s}^{-1} - 3\text{ m s}^{-1}}{10\text{ s}}$

$= -0.7$ m s^{-1} (i.e. to the left)

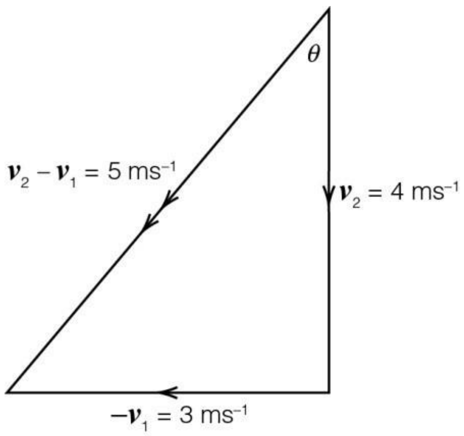
c) Now we must use a vector diagram, as shown, to calculate the acceleration.

$a = \frac{\mathrm{v}_2 - \mathrm{v}_1}{\mathrm{t}}$

$v_2 - v_1 = \sqrt{4^2 + 3^2}$ m s^{-1} = 5 m s^{-1}

$a = \frac{5\text{ m s}^{-1}}{10\text{ s}}$

$= 0.5$ m s^{-1}



tan θ = 3/4

θ = 37°

3 F = m a

a) F = 0.2 N (to the right)

b) F = 1.4 N (to the left)

c) F = 1.0 N (along the direction shown in the diagram)

4 The vehicle could speed up, slow down or change direction.

5 a) c $= 2\pi r$

$= 2\pi \times 10$ m

$= 20\pi$

A quarter of the circumference = 5 π m or 16 m.

b) s = $\sqrt{(20\text{ m})^2 + (20\text{ m})^2}$ = 14.1 m on a bearing 135° (or travelling southeast)

Page 276 Test yourself

1 a) ω $= \frac{2\pi}{T}$

$= \frac{2\pi}{24 \times 3600\text{ s}}$

$= 7.3 \times 10^{-5}$ rad s^{-1}

b) v $= \omega r$

$= 7.3 \times 10^{-5}$ rad s^{-1} $\times 6.4 \times 10^{6}$ m

$= 470$ m s^{-1}

c) r = 6 400 km cos 60°

= 3 200 km

v $= \omega r$

$= 7.3 \times 10^{-5}$ rad s^{-1} $\times 3.2 \times 10^{6}$ m

$= 230$ m s^{-1}

2 a) s $= v \times t$

t $= \frac{s}{v}$

$= \frac{2\pi \times 85\text{ m}}{3 \times 10^{8}\text{ m s}^{-1}}$

= 1.8 μs

b) f $= \frac{1}{T}$

$= 5.6 \times 10^{5}$ s^{-1}

c) ω $= 2\pi f$

$= 2\pi \times 5.6 \times 10^{5}$ s^{-1}

$= 3.5 \times 10^{6}$ rad s^{-1}

3 a) ω $= \frac{2\pi}{T}$

$= \frac{2\pi}{220 \times 10^{6} \times 3.16 \times 10^{7}\text{s}}$

$= 9.0 \times 10^{-16}$ rad s^{-1}

b) $v = \omega r$

$= 9 \times 10^{-16}\ \text{rad s}^{-1} \times (30\,000 \times 9.47 \times 10^{15}\ \text{m})$

$= 2.6 \times 10^{5}\ \text{m s}^{-1}$ or $260\ \text{km s}^{-1}$

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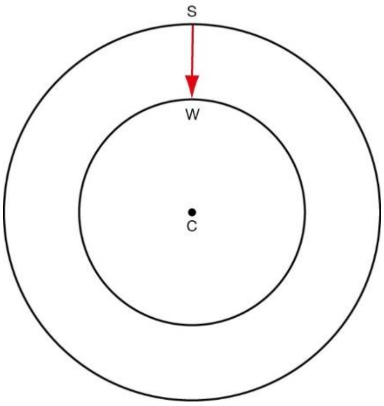
4 Velocity is a vector quantity. Acceleration is defined as (change of velocity)/time. So even if the speed stays constant, a body's velocity changes when it changes direction. So a change of direction results in a change of velocity and therefore acceleration.

5 The Moon is accelerating towards the Earth all the time because the Earth exerts a gravitational force on it. So, in a way, the Moon *is* falling towards the Earth. However, the Moon has a large sideways velocity, so the Earth's pull keeps changing the direction of the Moon's motion.

6 **a)** The *only* force acting on the satellite is its weight, W, as shown in the diagram.



$W = m g$

$= 560\ \text{kg} \times 8.2\ \text{N kg}^{-1}$

$= 4\,600\ \text{N}$ (2 sig figs)

b) $a = 8.2\ \text{m s}^{-2}$

The pull of gravity provides this acceleration.

c) **i)** $a = \frac{v^2}{r}$

$\Rightarrow v^2 = a r$

$= 8.2\ \text{m s}^{-2} \times 7 \times 10^{6}\ \text{m}$

$\Rightarrow v = 7\,600\ \text{m s}^{-1}$ (2 sf)

ii) $T = \frac{2\pi}{\omega} = \frac{2\pi r}{v}$

$\Rightarrow T = \frac{2\pi \times 7 \times 10^{6}\,\text{m}}{7600\ \text{m s}^{-1}}$

$= 5800\ \text{s}$ (2sf)

7 a) i) The ball is displaced backwards because a component of the tension in the string provides the force to pull the ball forwards. This unbalanced force causes the ball to accelerate.

ii) F = m a

= 0.15 kg × 2 m s^{-2}

= 0.3 N

iii) The string hangs at an angle so that the vertical component of the tension supports the weight of the ball, *mg*, and the horizontal component provides the forwards force *ma*.

So $\tan\theta = \frac{a}{g}$

$= \frac{2.0}{9.8}$

$\theta = 11.5°$

b) i) The ball has a resultant force of 0.3 N acting on it towards the centre of the circle.

ii) The ball is accelerating at 2 m s^{-2} towards the centre of the circle.

iii) The ball is accelerating sideways. This causes a change of direction, not a change of speed.

iv) $a = \frac{v^2}{r}$

$r = \frac{v^2}{a}$

$= \frac{(55\ \mathrm{m\ s^{-1}})^2}{2\ \mathrm{m\ s^{-2}}}$

= 1500 m

c) i) F = m a

= 40 × 10^3 kg × 2 m s^{-2}

= 80 kN

This force is provided by the track.

ii) $a = \frac{v^2}{r}$

If r is small and v stays the same, the centripetal acceleration required to make the train turn the bend gets larger. This force is provided by the track. By Newton's Third Law the train exerts an equal and opposite force on the track. If the train goes too quickly the force on the track can be sufficient to break the rail.

Page 279–280 Activity

Investigating centripetal forces

1

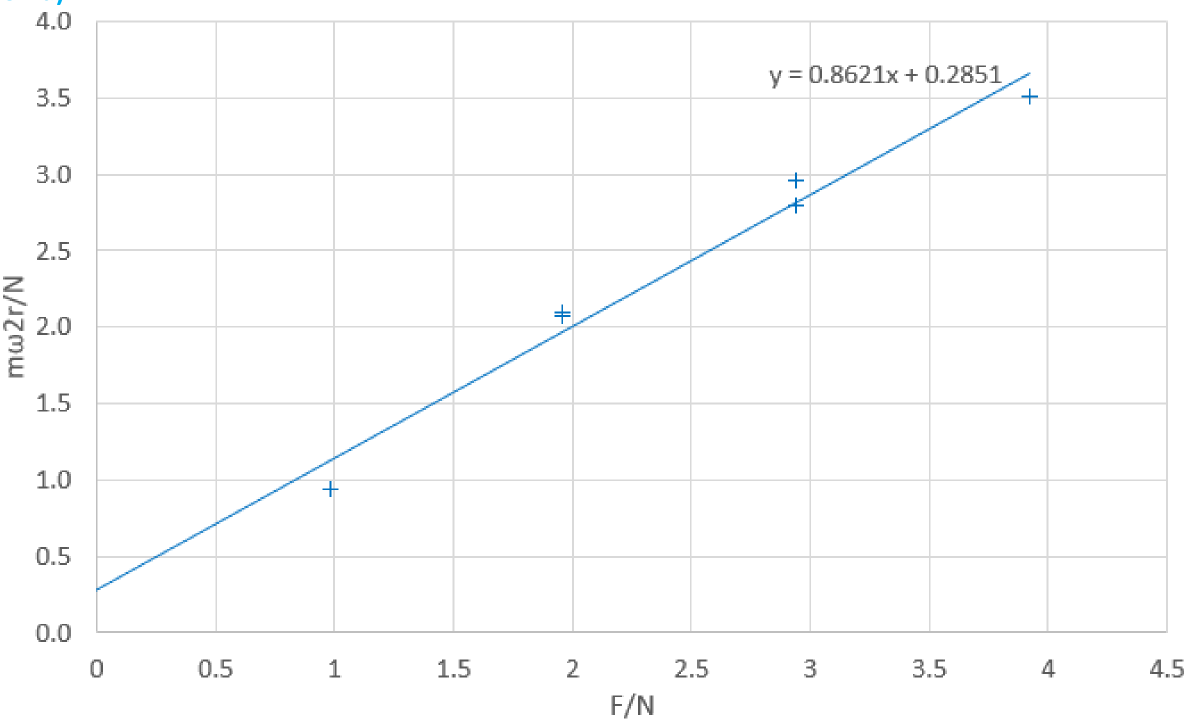
M/kg	F/N	10T/s	ω/rad s^{-1}	r/m	m $\omega^2 r$/N
0.1	0.98	18.8	3.3	0.94	0.94
0.2	1.96	11.5	5.5	0.78	2.12
0.2	1.96	9.8	6.4	0.56	2.06
0.3	2.94	8.8	7.1	0.61	2.77
0.3	2.94	7.9	8.0	0.52	3.05
0.4	3.92	5.6	11.2	0.31	3.50

2 The main errors are: measurement of T and r. T is significant as the calculation of ω^2 depends on T^2. Careful repetition of data is important. Or many data sets can be taken, then a graph plotted.

The effect of friction can be reduced by using reasonably heavy weights and a massive bung.

A significant error will be produced if the bung does not move in a horizontal circle. This is more likely to happen when the bung moves slowly – so keep it moving quickly (but then there might be a larger error in T).

3 a)



b) Since $F = m\omega^2 r$, you should get a straight line with gradient 1.

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8 **a)** F $= \frac{mv^2}{r}$ and, from text, $\frac{v^2}{r} = 55\ \mathrm{m\ s^{-2}}$

F $= 0.1\ \mathrm{kg} \times 55\ \mathrm{m\ s^{-2}}$

$= 5.5\ \mathrm{N}$

b) **i)** $R + m\,g = \frac{mv^2}{r}$

$\Rightarrow R = \frac{mv^2}{r} - mg$

R $= 5.5\ \mathrm{N} - 0.98\ \mathrm{N}$

$= 4.5\ \mathrm{N}$ (downwards)

ii) $R - m\,g = \frac{mv^2}{r}$

$\Rightarrow R = \frac{mv^2}{r} + mg$

$= 6.5\ \mathrm{N}$ (upwards)

iii) R $= \frac{mv^2}{r}$

$= 5.5\ \mathrm{N}$ (sideways)

c) The water falls out when $g \le \frac{v^2}{r}$

So minimum $v^2 = g\,r$

$= 9.8\ \mathrm{m\ s^{-2}} \times 0.95\ \mathrm{m}$

$= 9.3\ (\mathrm{m\ s^{-1}})^2$

V $= 3.0\ \mathrm{m\ s^{-1}}$

9 $F = \frac{mv^2}{r}$

$\Rightarrow v^2 = \frac{Fr}{m}$

a) $v^2 = \frac{15500\ \mathrm{N} \times 30\ \mathrm{m}}{620\ \mathrm{kg}}$

$= 750\ (\mathrm{m\ s^{-1}})^2$

v $= 27\ \mathrm{m\ s^{-1}}$

b) $v^2 = \frac{15500\ \mathrm{N} \times 120\ \mathrm{m}}{620\ \mathrm{kg}}$

$= 3000\ (\mathrm{m\ s^{-1}})^2$

v $= 55\ \mathrm{m\ s^{-1}}$

You might have seen that v would be twice as large as part (a) because v^2 is proportional to r.

Page 282–283 Practice questions

1 C

2 B

3 A

4 A

5 B

6 D

7 C

8 C

9 D

10 A

11 a) Velocity is a vector quantity, so when something changes its direction, the velocity changes [1]

Since $a = \frac{\Delta v}{\Delta t}$, the astronaut accelerates. [1]

b) $a = \frac{v^2}{r}$ [1]

$$v = \frac{2\pi r}{T} = \frac{2\pi \times 7.0\text{ m}}{(24.3\text{s}/10)} = 18\text{ ms}^{-1}(2\text{s.f.})\ \ [1]$$

$\Longrightarrow a = \frac{(18\text{ m s}^{-1})^2}{7\text{ m}}$

$= 46\text{ m s}^{-2}$ or about 4.6 g [1]

12 a) GPE is transferred to KE, so

$mgh = \frac{1}{2}mv^2$ [1]

$\Longrightarrow v^2 = 2gh$

$= 2 \times 9.8\text{ N kg}^{-1} \times 4.0\text{ m}$

$= 78.4\ (\text{ms}^{-1})^2$ [1]

$v = 8.9\text{ m s}^{-1}$ [1]

b) $T = mg + \frac{mv^2}{r}$ [1]

$= 2100\text{ kg} \times 9.8\text{ N kg}^{-1} + \frac{2100\text{ kg} \times (8.9\text{ m s}^{-1})^2}{8.0\text{ m}}$ [1]

$T = 20.6\text{ kN} + 20.8\text{ kN}$ [1]

$= 41\text{ kN}$ (2s.f.) [1]

13 a) $\omega = 2\pi f$

$f = \frac{960}{60}$ = 16 rotations per second [1]

$\omega = 2\pi \times 16 \text{ rad s}^{-1}$

$= 100 \text{ rad s}^{-1}$ [1]

b) $v = \omega r$

i) $v = 101 \text{ rad s}^{-1} \times (0.2 + 0.7) \text{ m}$

$= 90 \text{ m s}^{-1}$ [1]

ii) $v = 101 \text{ rad s}^{-1} \times 0.2 \text{ m} = 20 \text{ m s}^{-1}$ [1]

c) A large force is required to keep it in its circular path so the propeller must be strong. [1]

The force can be calculated from $F = m\omega^2 r$ and the force is reduced if m is small; low density materials reduce m. [1]

d) Near B, because the force at B must keep more of the blade rotating than at A. At A the force acting through the blade only has to keep the tip rotating. [1]
Also the cross-sectional area is smaller at B. [1]

e) $F = m\omega^2 r$ [1]

$= 3.5 \text{ kg} \times (101 \text{ rad s}^{-1})^2 \times 0.6 \text{ m}$ [1]

$= 21 \text{ kN}$ (2sf) [1]

Page 284 Stretch and challenge

14 a) i) $R = mg$

$= 80 \text{ kg} \times 9.8 \text{ m s}^{-2}$

$= 780 \text{ N}$

He is not accelerating upwards, so he feels his normal weight.

ii) $R = mg + ma$

$= 80 \text{ kg} \times 9.8 \text{ m s}^{-2} + 80 \text{ kg} \times 1.5 \text{ m s}^{-2}$

$= 900 \text{ N}$

He is accelerating upwards, so the lift floor provides an extra force on him.

iii) $R = mg - ma$

$= 80 \text{ kg} \times 9.8 \text{ m s}^{-2} - 80 \text{ kg} \times 1.5 \text{ m s}^{-2}$

$= 660 \text{ N}$

He is accelerating downwards, so the lift floor provides less force on him.

b) i) $\frac{1}{2}mv^2 = mgh$

so $v^2 = 2gh$

$v^2 = 2 \times 9.8\ \text{N kg}^{-1} \times 20\ \text{m}$

$v = 20\ \text{m s}^{-1}$

ii) The height difference between A and C = 20 m – (2 × 7 m) = 6 m

$v^2 = 2 \times 9.8\ \text{N kg}^{-1} \times 6\ \text{m}$

$v = 11\ \text{m s}^{-1}$

c) i) $a = \frac{v^2}{r}$

$= \frac{(20\ \text{m s}^{-1})^2}{7\ \text{m}}$

$= 57\ \text{m s}^{-2}$

ii) $a = \frac{(11\ \text{m s}^{-1})^2}{7\ \text{m}}$

$= 17\ \text{m s}^{-2}$

d) i) $R = mg + ma$

$= 70\ \text{kg} \times 9.8\ \text{m s}^{-2} + 70\ \text{kg} \times 57\ \text{m s}^{-2}$

$= 4700\ \text{N}$

This means the passengers feel about 7 times their usual weight, rather more than most astronauts' experience.

ii) $R + mg = ma$

$R = ma - mg$

$= 70\ \text{kg} \times 17\ \text{m s}^{-2} - 70\ \text{kg} \times 9.8\ \text{m s}^{-2}$

$= 500\ \text{N}$

While the vehicle will stay on the track, the whole ride looks terrifying. The rapid changes of direction and speed are likely to do serious damage to passengers' necks. Health and safety stop this one.

e) $g = \omega^2 r$

$\omega^2 = \frac{g}{r}$

$= \frac{9.8\ \text{m s}^{-2}}{60\ \text{m}}$

$\omega = 0.40\ \text{rad s}^{-1}$