Pages 200–201 Test yourself on prior knowledge

1 In physics, work is done when a force acts over a distance. In the case of *Voyager*, there are effectively no forces acting on *Voyager*, so no work is done. The steady motion of the spacecraft is an example of Newton's first law.

2 Weight = mass × gravitational field strength

On Earth, weight = 8800 N

On Mars, weight = 3300 N

3 To calculate work done up the stairs we are only interested in the vertical height that the suitcase was lifted.

The force is the weight of the suitcase = 20 kg × 9.81 Nkg^{-1} =196 N.

W = Fs

W = 196 N (20 × 0.18 m)

W = 706 J

4 $F = \frac{W}{s}$

F = 12 000 J ÷ 100 m = 120 N

5 a) m = 0.02 kg, so weight of penguin is 0.2 N

W = Fs

Work done = 0.2 N × 0.2 m = 0.004 J

b) The change in gravitational potential energy as the penguins slide down the slides will be the same as the work done because work is calculated using the distance moved in the direction of the force. The force is the weight, which acts downwards, so the distance we use in our calculation would be the vertical distance the penguins descend as they slide down the slides.

Page 203 Test yourself

1 Both materials can have a different chemical composition (e.g. steel often contains carbon atoms) which will change the mass of the material. The difference in mass means that the density is also different.

2 a) Mass = 0.0321 kg

Volume = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{19.98\times10^{-3}}{2}\right)^3$ = 4.176×10^{-6} m^3

Density $= \frac{\text{mass}}{\text{volume}}$

$= \frac{0.0321\text{ kg}}{4.176\times10^{-6}\text{ m}^3}$

= 7687 kg m^{-3}

b) Percentage uncertainty $= \left(\frac{\text{instrument precision}}{\text{measurement}}\right) \times 100\%$

For the vernier calliper $= \left(\frac{0.02}{19.98}\right) \times 100\%$

$= 0.1\%$

3 a) Total mass of stars = 300 000 × 2 × 10^{30} = 6 × 10^{35} kg

1 light-year = 9.46 × 10^{15} m

Volume of a sphere = $\frac{4}{3}\pi r^3$

Volume of globular cluster = $\frac{4}{3}\pi\left(\frac{145\times9.46\times10^{15}}{2}\right)^3 = 1.35 \times 10^{54}$ m^3

Average density = $\frac{\text{total mass}}{\text{volume}}$

Average density = 4.4 × 10^{-19} kg m^{-3}

b) The average density of the globular cluster is very low. An individual star would have a much greater density than the value in (a). That suggests that there is a lot of empty space in the globular cluster which reduces the average density. If there is a lot of empty space it is likely that the stars are spread out throughout the cluster and so they are unlikely to collide.

4 Volume of window = area × depth

Volume = 1.5 × 2 × 3 × 10^{-3} = 9 × 10^{-3} m^3

Mass = volume × density

Mass = 9 × 10^{-3} m^3 × 2 200 kg m^{-3} = 20 kg (2sf)

5 Volume of the washer = (volume of disc) – (volume of hole) = $\pi r_d^2 t - \pi r_h^2 t$

Using cm as units for this question as this is what the density has been given in.

Volume of washer = π × 0.08 cm × [(0.85 cm/2)^2 – (0.4 cm/2)^2]= 0.035 cm^3

Mass = density × volume = 8.4 g cm^{-3} × 0.035 cm^3 = 0.29 g

6 Total volume = 0.1039 × 0.03

= 3.12 × 10^{-3} m^3

Mass = volume × density = 3.12 × 10^{-3} m^3 × 20 kg m^{-3}

Mass = 0.06 kg

7 First, estimate the volume of a typical body. The easiest way to do this is to assume that the body is a regular cuboid. taking typical measurements for a 17-year-old: height ≈ 1.65 m, width ≈ 0.5 m, depth ≈ 0.2 m. Thus, the volume = 1.65 m × 0.5 m × 0.2 m = 0.165 m^3

Assume a mass of 55 kg.

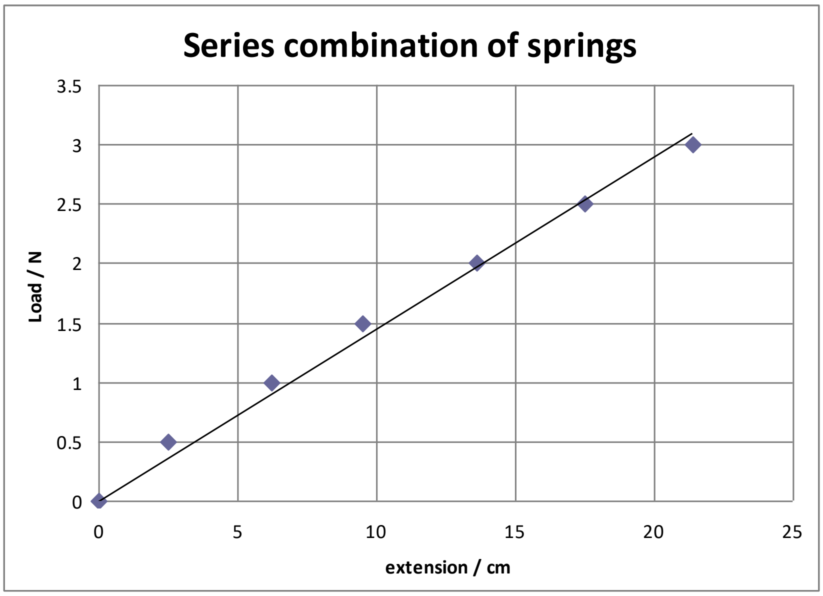
$\rho \approx \frac{55\text{ kg}}{0.165\ m^3}$ = 333 kg m^{-3}

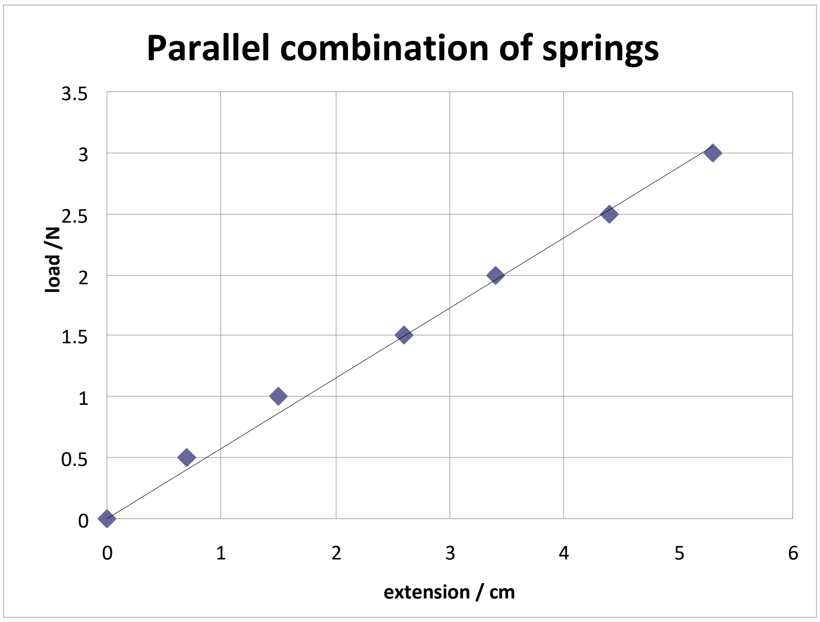
This is of the same order of magnitude as the often quoted value of 953 kg m^{-3}. A more accurate estimate could be obtained by assuming that the body is formed from a number of cuboids (for the torso) and cylinders (for arms and legs). You can then calculate the volume of these separately to obtain a better estimate of the volume. To obtain a very accurate measurement, the volume can be calculated by submerging the body completely in water and measuring the volume of water displaced.

Page 205 Activity

Extension of springs

1





2 Effective spring constant for springs in series = 0.14 N cm^{-1}

Effective spring constant for springs in parallel = 0.58 N cm^{-1}

3 $\frac{0.58}{4}$ = 0.145

The student's prediction is correct.

4 0.14 N cm^{-1}. Two springs in parallel doubles the spring constant, but the two further springs in series halve the spring constant. So this combination has the same constant as a single spring.

5 To calculate percentage error use

$$\frac{\text{uncertainty}}{\text{measurement}} \times 100\%$$

extension (cm)	percentage error (%)
2.5	4.0
6.2	1.6
9.5	1.1
13.6	0.7
17.5	0.6
21.4	0.5

Page 206 Test yourself

8 The linear section of the graph will have a steeper gradient. This is because doubling spring constant means that it will take twice the force to achieve the same extension.

9 $F = k\Delta l$, k = 1 N cm^{-1}

When F = 10, Δl = 10 cm

Assuming the scale is 10 cm long, then the extension is equal to the length of the scale, so this spring would be suitable and provide the best possible use of the scale.

10 Δl = 0.7 m

Force = mg

= 55 × 10

= 550 N

k = $F/\Delta l$

= 790 N m^{-1}

11 Each spring will extend by Δl when the force, F, is applied, giving a total extension of $3\Delta l$. For the series combination the spring constant, $k_e = \frac{\text{force}}{\text{total extension}}$

$k_e = \frac{F}{3\Delta l}$

substituting in for $F = k\Delta l$ (where k is the spring constant of each individual spring) we get

$k_e = \frac{k\Delta l}{3\Delta l} = \frac{k}{3}$

12 a) Each spring in series will stretch by Δl. Each spring in parallel will stretch by $\frac{\Delta l}{2}$. The total extension will be (3+½)Δ

For the combination the effective spring constant $k_e = \frac{\text{force}}{\text{total extension}}$ substituting in for F = kΔl

$k_e = k\frac{\Delta l}{(3+½)\Delta l} = \frac{2}{7}k$

b) The two springs in parallel stretch by $\frac{\Delta l}{2}$, and the three springs in parallel stretch by $\frac{\Delta l}{3}$. So the total extension is $\frac{5\Delta l}{6}$.

Since $k_e = F/(\frac{5\Delta l}{6})$

$k_e = k\Delta l/(\frac{5\Delta l}{6})$

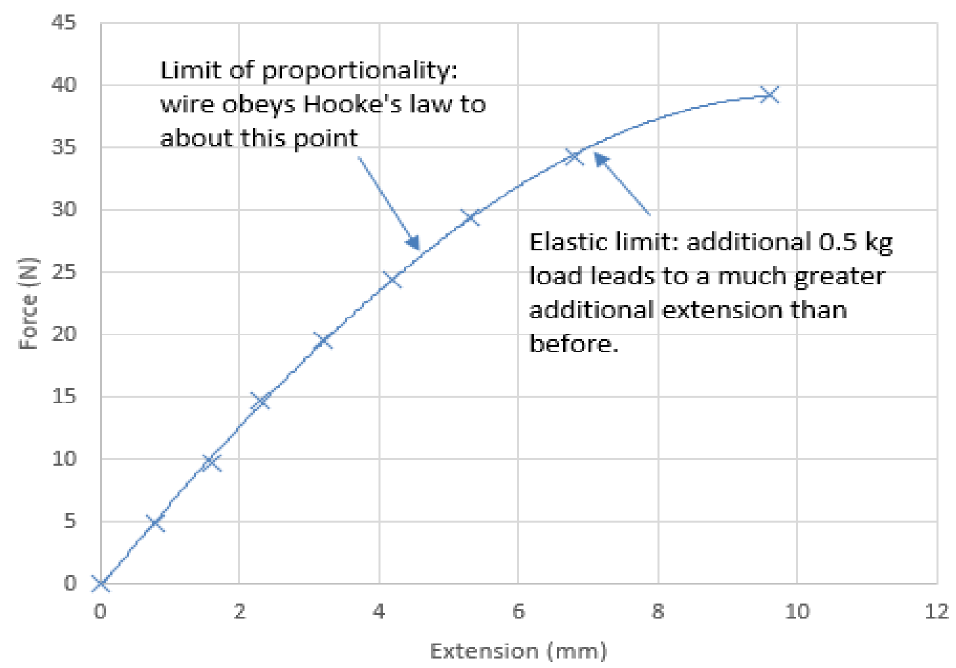
$k_e = (6/5)k$

Page 206–207 Activity

Investigating the properties of copper wire

1 Using sample results:

Mass (kg)	0	0.5	1	1.5	2	2.5	3	3.5	4
Extension (mm)	0	0.8	1.6	2.3	3.2	4.2	5.3	6.8	9.6
Force (N)	0	4.9	9.8	15	20	25	29	34	39



There is no indication in the sample results of when the wire breaks, but it may be at a load of 4.5 kg.

2 While the wire obeys Hooke's Law, each 0.5 kg added produces an additional extension of 0.8 mm

$$k = \frac{F}{\Delta l} = \frac{0.5\,\text{kg} \times 9.8\ \text{Nkg}^{-1}}{0.8 \times 10^{-3}\text{m}} = 6125\ \text{Nm}^{-1}$$

3 You can think of a long wire as several shorter wires in series. As the calculations in Q11 & Q12 show, a longer wire will, therefore, have a lower spring constant. This means a given force will produce a greater extension.

4 The position of the edge of the tape on the rule could be read to 0.5 mm and the error in the extension is therefore ±1 mm (as it will be calculated from two such readings). This is likely to be a much more significant error than any in the values of the masses used (manufacturers quote a figure that is usually <1%), so the error in the spring constant will be of the same order of magnitude (unless likely gradients of the linear portion of the graph demonstrate even greater uncertainty).

Page 211 Test yourself

13 Elastic strain energy = ½ FΔl

= ½ × 35 N × 0.8 × 10^{-3} m

= 0.014 J

14 Elastic strain energy = ½ FΔl

= ½ (30 kg × 9.81 N kg^{-1}) × (2.05 m – 2 m)

= 7.4 J

15 Elastic strain energy = ½ FΔl

= ½ × 27.5 N × (0.785 m – 0.750 m)

= 0.48 J

16 Energy required to stretch the springs will be equal to the elastic strain energy stored at this extension = ½ $k\Delta l^2$

= ½ × 2500 N m^{-1} × $(0.4\ \text{m})^2$

= 200 J

OR

work done = ½ FΔl

= ½ × 1000 N m × 0.4 m

= 200 J

Therefore, the person who is exercising must to provide at least 200 J of energy to the equipment to stretch the springs. However, there may be energy loss to heat, so this is a minimum value.

17 Elastic strain energy = ½ $k\Delta l^2$

= ½ × 1250 N m^{-1} × $(0.03\ \text{m})^2$

= 0.56 J

Pages 213–214 Required practical

Determination of the Young modulus by a simple method

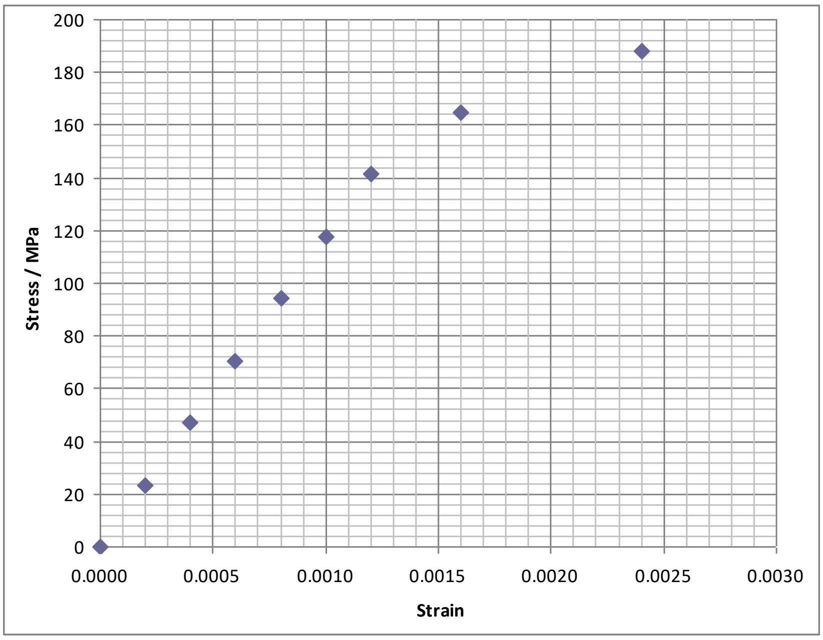
1 The wire might not be uniform, so taking three measurements allows for a mean value to be calculated.

2 radius of wire = 0.52 mm ÷2 = 0.26×10^{-3} m
Area of wire = $\pi r^2 = \pi(0.26 \times 10^{-3}\ \text{m})^2 = 2.12 \times 10^{-7}\ \text{m}^2$

Stress $= \dfrac{\text{mass} \times 9.81\ \text{N kg}^{-1}}{2.12 \times 10^{-7}\ \text{m}^2}$ Strain $= \dfrac{\text{extension}}{2.5\ \text{m}}$

Stress/MPa	Strain
0	0.0000
23	0.0002
46	0.0004
69	0.0006
93	0.0008
116	0.0010
139	0.0012
162	0.0016
185	0.0024

3 The graph will look similar to this.



To calculate Young Modulus, you need to find the gradient of the straight line section of the graph (up to stress of approximately 140 MPa). The gradient of the graph should be approximately 116 GPa.

4 The wire will not return to its original length. The graph starts to curve at higher values of stress. This shows that the copper wire has passed the limit of proportionality, and has exceeded its elastic limit: it will no longer obey Hooke's law and has undergone plastic deformation.

Page 216 Test yourself

18 First convert all measurements to standard units.

a) Stress = $\frac{F}{A}$

Area = $\pi r^2 = \pi \times (2 \times 10^{-4}\ m)^2 = 1.25 \times 10^{-7}\ m^2$

Stress = $\frac{32N}{1.25 \times 10^{-7} m^2}$

= $2.5 \times 10^8\ N\ m^{-2}$

b) Strain = $\frac{\Delta l}{l} = \frac{3 \times 10^{-3}}{1.5}$

Strain = 2×10^{-3}

c) Young modulus = $\frac{stress}{strain}$

= $2.5 \times 10^8\ N\ m^{-2}/2 \times 10^{-3}$

= 1.25×10^{11} Pa

= 125 GPa

19 E = $\frac{Fl}{A\Delta l}$ and rearrange for Δl

$\Delta l = \frac{Fl}{EA}$

Area = $\pi r^2 = \pi \times (3 \times 10^{-2}\ m)^2 = 2.8 \times 10^{-3}\ m^2$

$\Delta l = \frac{(23 \times 10^3\ N \times 0.25\ m)}{(200 \times 10^9\ N\ m^{-2} \times 2.8 \times 10^{-3}\ m^2)}$

= 1.0×10^{-5} m

20 Maximum strain is 500 MPa

Strain = $\frac{F}{A}$

Area of the bar is $2.8 \times 10^{-3}\ m^2$

So maximum force will be $500 \times 10^6\ N\ m^{-2} \times 2.8 \times 10^{-3}\ m^2 = 1.4 \times 10^6$ N

21 strain = $\frac{\Delta l}{l}$

0.001 = $\frac{\Delta l}{0.5m}$

$\Delta l = 5 \times 10^{-4}$ m

22 The equations for tensile situations can also be used for compressive forces. In that case, strain is compression/original length.

$E = \frac{Fl}{A\Delta l}$

$F = \frac{EA\Delta l}{l}$

$= \frac{(2 \times 10^{11}\ \text{N m}^{-2})(0.025\ \text{m}^2)(3 \times 10^{-4}\ \text{m})}{2\text{m}}$

$F = 7.5 \times 10^5$ N

23 Assume person has a mass of 55kg and that the bones in the lower leg are 3.0cm in diameter. The weight will be distributed across both legs so one leg will support half the weight. Assume that $g=10$ N kg^{-1}.

Force on each bone = 27.5 kg × 10 N kg^{-1} = 275 N

Area = π × $(1.5 \times 10^{-2}\ \text{m})^2 = 7.1 \times 10^{-4}\ \text{m}^2$

Stress = $\frac{275\text{N}}{7.1 \times 10^{-4}\text{m}2}$ = 390 kPa

This is likely to be less that the ultimate compressive stress because the stresses on the bone will be much larger when running or jumping. Bones must therefore be able to withstand greater forces.

24 Stress = $\frac{F}{A}$

Assuming constant density, the real bridge has $(20)^3$ times the mass, and thus the force on the pillars also increases by $(20)^3$; the real pillars have $(20)^2$ the cross-sectional area. So the ratio of the stresses is 20.

25 If a cat is scaled up by a factor of 10, mass goes up by a factor of 1000 and the area of legs by 100. This means that the stress goes up by 10. So large animals need proportionately wider legs.

Pages 217–220 Test yourself

1 A

2 C

3 C

4 C

5 D

6 C

7 A

8 B

9 A

10 D

11 C

12 Ultimate tensile stress is the stress at which the sample breaks.

Stress = Force /area

= 840 N/ (1.3×10^{-5} m^2)

= 6.46×10^7 Pa [1]

13 a) Density = mass / volume

Volume = 6.3×10^{-3} m^3

Density = 2060 kg m^{-3} [1]

b) Stress = Force / area

Maximum stress will occur for smallest area

Stress = (13 kg × 9.81 N kg^{-1}) / (0.3 m × 0.035 m) [1]

= 1.2×10^4 Pa [1]

14 a) Strain = Extension / original length

= (100 cm – 20 cm) / 20 cm

= 4 [1]

b) Stress = Force / cross-sectional area

Cross-sectional area = 30 N/ 14×10^6 m^2

= 2.1×10^{-6} m^2 [1]

15 a) The deformation is elastic because the wire returns to its original length when unloaded. [1]

b) [1 mark each for method to measure length, extension and one safety point. 1 additional mark for one other point.]

Original length is measured with a ruler.

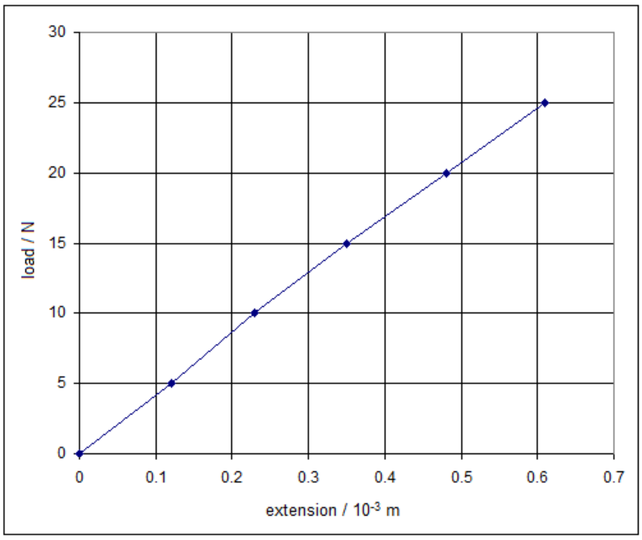
Extension may be measured by marker attached to wire and a scale.

Extension may also be measured using a vernier scale fixed to a reference wire hung next to the sample wire.

Safety goggles must be worn when measuring extension in case of the wire breaking.

Cushioning should be placed under the weights to prevent damage to floor or feet in case of the wire breaking.

c) [1 mark for correct graph]



Gradient = 4.16×10^4 N m^{-1} [1]

Spring constant = 4.16×10^4 N m^{-1} [1]

d) Diameter of the wire to calculate the cross-sectional area and thus stress. [1]

16 a) ρ = m/V

Volume = $30 \times \pi \left(\frac{1}{2} \times 20 \times 10^{-3}\right)^2 = 9.4 \times 10^{-3}$ m^3

Mass = 9550 kgm^{-3} × 9.4×10^{-3} m^3 = 90 kg [1]

b) Assume that the lift contains five passengers, and that the lift is supported only by the cable.

Stress = F / A

= (500 + 450 + 90) kg × 9.81 N $m^{-1}/\pi(0.01\ m)^2$ [1]

= 3.2×10^7 Pa [1]

This is approximately 30MPa. [1]

17 a) Any four points:

Elastic band suspended from fixed point fix a ruler close, and parallel, to the band.

Unstretched length measured. Ensure that band is straight to reduce measurement error. (small pointer attached to bottom of band could be used to aid reading)

Mass hanger added and new length measured.

Masses continue to be added in regular increments and length measured each time.

To measure unloading, the masses are removed – again in regular increments – and the length measured once each mass has been removed.

b) This will increase the likelihood of incorrect readings as random errors will be less obvious. [1]

c) No. Hooke's law states that force is proportional to extension and will give a straight line graph [1].

The elastic band does not have a straight line force-extension graph so doesn't show Hooke's law behaviour [1].

d) The amount of energy dissipated is the difference in area between the loading and unloading graph. The student could count the number of squares between the loading and unloading lines.

Each small square on the graph is equivalent to 2×10^{-3} J. She should then multiply the number of small squares by this value to obtain an estimate of the energy dissipated. [2]

e) The elastic band would become warm. [1]

18 a) Strain is a ratio of extension to original length. A ratio does not have units. [1]

b) The copper obeys Hooke's law: the extension is proportional to the force applied. [1]

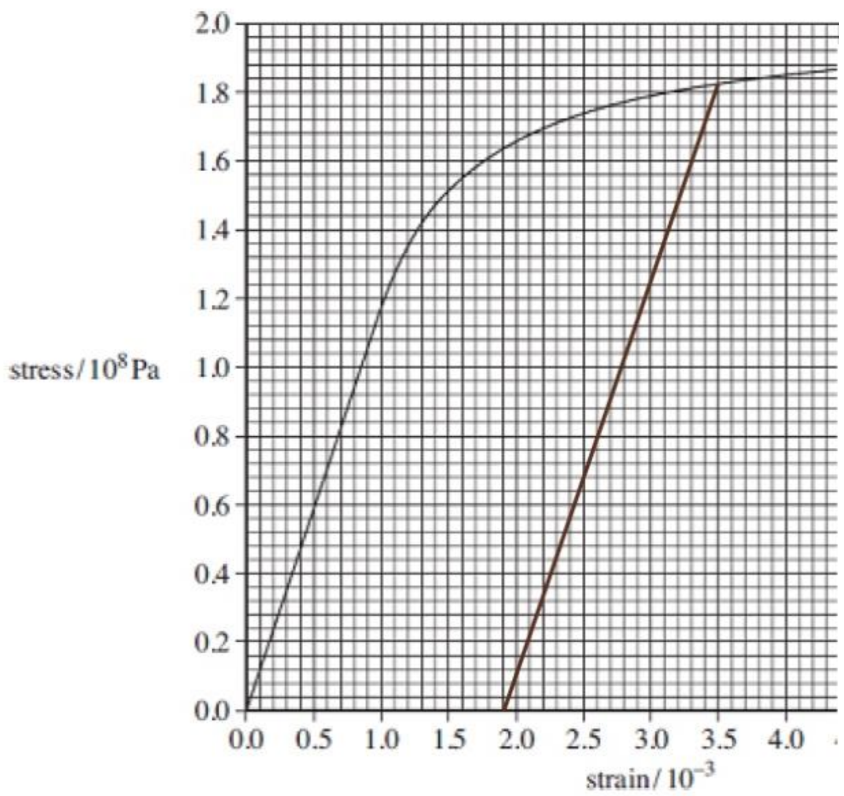
c) 190×10^6 Pa (190 MPa or 1.9×10^8 Pa) [1]

d) Young modulus = stress / strain

At strain of 1.0×10^{-3} stress is 118×10^6 Pa [1]

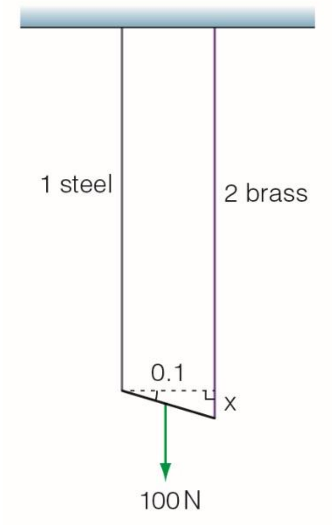
Young modulus = 1.18×10^{11} Pa [1]

e) At strain of 3.5×10^{-3} wire has passed its elastic limit and is exhibiting plastic behaviour. When the load is removed the wire does not return to its original length and has been permanently stretched as shown in the diagram. [1 mark explanation, 1 mark sketch]



Page 220 Stretch and challenge

19 a) The diagram shows the extension of the wires when the 100N force is applied.



$\tan(\theta) = \text{opp} / \text{adj}$

$\tan(1) = x / 0.1$

Difference in extensions = 1.7×10^{-3} m

b) Each wire will be subject to half of the applied force.

Young modulus = Stress / strain

$\Delta l = F\, l / E\, A$

Area = $\pi r^2 = 5.0 \times 10^{-7}\ m^2$

$\Delta l = (50\ N \times 2.0\ m) / (2 \times 10^{11}\ N\ m^{-2})\ (5 \times 10^{-7}\ m^2)$

$\Delta l = 1.0 \times 10^{-3}$ m

c) Difference in extension of steel and brass wire is 1.7×10^{-3} m. So brass wire will have extended this amount + the amount that the steel wire extended.

Total extension of brass wire = 2.7×10^{-3}m

For brass wire, strain = 2.7×10^{-3} m / 2.0 m = 0.00135

Area of brass wire = $\pi r^2 = 3.6 \times 10^{-7}\ m^2$

Strain on brass wire = 50 N / $3.6 \times 10^{-7}\ m^2 = 1.38 \times 10^{8}$ Pa

Young modulus = stress / strain

Young modulus brass= $1.0 \times 10^{11}\ Nm^{-2}$

d) Elastic strain energy $= \frac{1}{2} F \Delta l$

$= \frac{1}{2} \times 50 \times 1.0 \times 10^{-3}$

$= 0.025$ J

20 The initial energy of the ball will be equal to the energy stored in the spring. The final energy of the ball will be KE + GPE. These two will be equal as there are no energy loses in the system.

Elastic strain energy = ½ k x^2

Gravitational potential energy = mgh

Using trigonometry to calculate the gain in height of the ball.

sin(8.5) = h / 0.05

h = 0.05sin(8.5)

GPE = 0.1 × 9.8 × 0.05sin8.5

GPE = 7.24×10^{-3}J

½ kx^2 = ½ mv^2 + GPE

k = 2(½ mv^2 + GPE)/x^2

$$k = 2\times\frac{\left(\frac{1}{2}\times0.100\times0.68^2\right)+7.24\times10^{-3}}{0.05^2}$$

k = 24.3 Nm^{-1}

21 The Young modulus of spider silk is twenty times lower than that of steel.

Young modulus = stress / strain

For a particular value of stress (e.g. the weight of a fly), the strain on the silk will be 20 times greater than that of the steel. (This means that the extension of the silk will be much more than that of a steel rope.)

The energy which can be stored (or absorbed) by a unit volume of each rope = ½ (stress × strain).

If we consider two ropes of the same dimensions, then spider silk is able to absorb 20 times more energy from e.g. a moving object.

(However, this only applies if the maximum force is such that the breaking strength is not exceeded as the spider silk is not as strong as steel – the breaking strength of steel is twice that of the silk.)

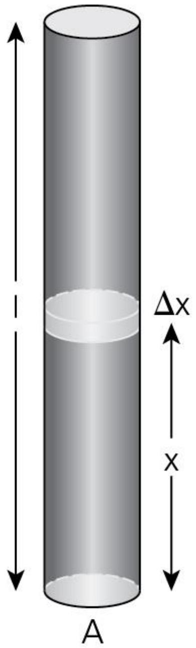
22 The stress due to a length x is:

$= \frac{F}{A}$

$= \frac{\rho A x g}{A}$

$= \rho x g$

If the extension of a section Δx due to the weight of the bar beneath it is Δe,



Young modulus = stress / strain

$= \frac{\rho x g}{\Delta e / \Delta x}$

$= \frac{\rho x g \Delta x}{\Delta e}$

Therefore $\Delta e = \frac{px g \Delta x}{E}$

So total extension $\frac{\rho g}{E} \int_0^l x dx$

$= \frac{\rho g l^2}{2E}$

Alternatively, we could say:

The average stress is $\frac{\rho g l}{2}$

So

$\frac{\Delta e}{l} = \frac{\rho g l}{2E}$

$\Delta e = \frac{\rho g l^2}{2E}$