Answers

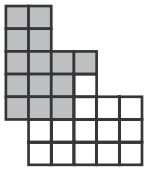
Chapter 1: Fractions, decimals and rounding

Starter 1 (page 1)

1 100 cm (1 metre)

2 $\frac{1}{8}$

3



4 One person cuts and the other one chooses.

Exercise 1.1 (page 6)

1 $\frac{1}{4}$ **2** $\frac{3}{7}$ **3** $\frac{4}{41}$
4 $\frac{1}{4}$ **5** $\frac{1}{5}$ **6** $\frac{3}{4}$
7 $\frac{3}{5}$ **8** $\frac{3}{5}$ **9** $\frac{4}{7}$
10 $\frac{3}{5}$ **11** $\frac{7}{13}$ **12** $\frac{7}{9}$
13 $\frac{1}{2}, \frac{11}{20}, \frac{3}{5}, \frac{5}{8}$ **14** $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}$ **15** $\frac{2}{3}, \frac{4}{5}, \frac{5}{6}, \frac{13}{15}$
16 $\frac{7}{9}$ **17** $\frac{9}{16}$ **18** $\frac{5}{6}$
19 $\frac{13}{24}$ **20** $\frac{3}{8}$ **21** **TK**
22 $\frac{37}{48}$ **23** $\frac{155}{252}$ **24** $5\frac{13}{40}$
25 $10\frac{67}{72}$ **26** $2\frac{15}{56}$ **27** $3\frac{5}{8}$
28 $2\frac{3}{5}$ **29** $2\frac{3}{4}$ **30** $8\frac{1}{2}$
31 $3\frac{3}{20}$

Exercise 1.2 (page 9)

1 $\frac{2}{5}$ **2** $\frac{5}{9}$ **3** $\frac{11}{20}$
4 $\frac{4}{9}$ **5** 15 **6** $\frac{1}{15}$
7 $\frac{5}{28}$ **8** 120 **9** $\frac{8}{15}$
10 $\frac{1}{2}$ **11** $\frac{15}{16}$ **12** $\frac{1}{18}$
13 $\frac{7}{16}$ **14** $\frac{1}{18}$ **15** $\frac{3}{4}$
16 1 **17** $\frac{5}{16}$ **18** 2
19 $\frac{9}{16}$ **20** $1\frac{1}{2}$ **21** **TK**
22 $20\frac{5}{8}$ **23** 6 **24** $31\frac{1}{2}$
25 12 **26** 100 **27** $\frac{1}{2}$

Exercise 1.3 (page 11)

1 $\frac{6}{25}$ **2** $\frac{18}{25}$ **3** $\frac{3}{10}$
4 $\frac{5}{8}$ **5** $\frac{91}{100}$ **6** $\frac{1}{40}$
7 $1\frac{47}{50}$ **8** $\frac{19}{50}$ **9** $2\frac{1}{8}$
10 $\frac{303}{1000}$ **11** $\frac{7}{9}$ **12** $\frac{29}{99}$
13 $1\frac{1}{3}$ **14** $\frac{1733}{3330}$ **15** $\frac{43}{99}$
16 $\frac{6}{11}$ **17** $\frac{107}{333}$ **18** $1\frac{113}{330}$
19 0.625 **20** $0.\dot{4}28\,57\dot{1}$ **21** $0.\,\dot{4}$
22 0.45 **23** 0.28
24 Wrong. $0.\,\dot{3}\dot{5} \times 2 = 0.707\,070\ldots$ which is not $0.\,\dot{7}$

Exercise 1.4 (page 14)

1 3.142 **2** 3.1416 **3** 16.2
4 0.24 **5** 14.1 **6** 14.8
7 6.2240 **8** 1.90 **9** 15.4
10 20 **11** 14.3 **12** 359 300
13 370 **14** 10 **15** 0.0021
16 11.0
17 **a)** 1000 **b)** 1000 **c)** 20
d) 6 **e)** 8 **f)** 20
g) 4 **h)** 5 **i)** 6
18 \$10 800 or \$12 000 **19** 34.5 cm **20** 21 500 mm^2
21 Not possible, last week must be at least 24 500, the week before not more than 24 450, so no overlap is possible.
22 No. The age is only given to the nearest million years so Charlie's calculation is not valid.

Exercise 1.5 (page 16)

1 24.058
2 24.01
3 5.089 204 26; 5.09
4 11
5 0.292 279 412; 0.292
6 4.929 503 018; 4.93
7 2.342 857 143; 2.34
8 1.161 290 323; 1.16
9 17.576
10 5.88
11 4.301 162 634; 4.30
12 1.16
13 2.150 326 797; 2.15
14 5.816 356 248; 5.82
15 1.666 666 667; 1.67

Exercise 1.6 (page 18)

1 **a)** 50 cm, 46 cm **b)** 156.25 cm^2, 132.25 cm^2
2 **a)** 34 cm **b)** 52.25 cm^2
3 **a)** 29.25 cm^2 **b)** 0.538
4 **a)** 60 mph **b)** 56.86 mph **c)** 63.27 mph
5 28.5 cm^2

Review Exercise 1 (page 18)

1 $\frac{13}{20}$ **2** $1\frac{4}{15}$ **3** $3\frac{11}{24}$
4 $4\frac{11}{20}$ **5** 5 **6** $23\frac{1}{3}$
7 $3\frac{3}{4}$ **8** 14 **9** $\frac{7}{12}, \frac{5}{8}, \frac{3}{4}, \frac{5}{6}$
10 $\frac{3}{5}, \frac{2}{3}, \frac{7}{10}, \frac{11}{15}$
11 **a)** 300 **b)** 100 **c)** 8
12 $\frac{5}{8}$, 0.65, $0.\dot{6}$, $\frac{3}{4}$
13 **a)** $\frac{7}{8}$ **b)** $\frac{5}{11}$
14 **a)** 0.067, 0.56, 0.6, 0.605, 0.65
b) $-10, -6, -4, 2, 5$
c) $\frac{2}{5}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}$
15 $\frac{5}{12}$ **16** $\frac{1}{5}$

17 For example, $\frac{1}{3}$ and $\frac{3}{8}$

18 855.4 kg

19 **a)** $\frac{31}{40}$ **b)** $2\frac{11}{12}$

20 **a)** 69.3 **b)** 6.93 **c)** 0.0693

21 **a)** 100.5 mm **b)** 101.5 mm

22 **a)** 119.31 **b)** 119 310 **c)** 1.23

23 $\frac{29}{99}$

24 41.2

25 **a)** 46.416 376 42 **b)** 46

26 **a)** 0.787 965 007 **b)** 0.79

27 **a)** 17.9867 **b)** $(1.6 + 3.8 \times 2.4) \times 4.2$

28 2.56

29 **a)** $\frac{13}{99}$

b) $\frac{254}{495}$

30 **a)** $\frac{4}{11}$

b) $2\frac{3}{22}$

31 $2\frac{1}{15}$

32 **a)** $\frac{3}{11}$

b) Let $X = 0.\dot{0}\dot{x}$

So $X = 0.0x0x0x...$

$100\,X = x.0x0x0x\ldots$

Subtracting gives:

$99X = x$

So $X = \dfrac{x}{99}$

33 1.865

34 18

35 0.0205

Internet Challenge 1 (page 23)

1 $\frac{1}{6}$ **2** $\frac{11}{20}$ **3** $\frac{1}{4} + \frac{1}{12}$ **4** $\frac{1}{2} + \frac{1}{3} + \frac{1}{12}$ **5** $\frac{1}{2} + \frac{1}{6} + \frac{1}{9}$

6 No, but the Erdos–Strauss conjecture claims that fractions of the form $\frac{4}{n}$ may be written as the sum of three Egyptian fractions. The conjecture has been checked for every individual case up to $n = 10^{14}$.

7 Emily will write $\frac{5}{8}$ as $\frac{1}{2} + \frac{1}{8}$.

For each of the first four sacks, divide the contents of the sack into two equal piles, giving eight piles of $\frac{1}{2}$ a sack each. For the fifth sack, divide its contents into eight equal piles, giving $\frac{1}{8}$ of a sack each. Give each chicken coop one of the $\frac{1}{2}$ sack piles and one of the $\frac{1}{8}$ sack piles. The idea is that it is easier to judge splitting a sack into equal piles ($\frac{1}{2}$ or $\frac{1}{8}$) than to divide it asymmetrically into $\frac{3}{8}$ and $\frac{5}{8}$.

Chapter 2: Ratios and percentages

Starter 2 (page 24)

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Exercise 2.1 (page 26)

1 2 : 5 **2** 2 : 3 **3** 7 : 11
4 3 : 13 **5** 4 : 5 : 7 **6** 3 : 4 : 6
7 3 : 4 : 7 **8** 2 : 5 **9** 5 : 7 : 8
10 £15, £20, £25 **11** 60, 100, 140
12 £50, £150, £250 **13** $24, $36, $84
14 24, 48, 60 **15** £50, £125, £175
16 £48, £60, £108 **17** **a)** 5 km **b)** 33.4 cm
18 **a)** £720 **b)** £3360
19 **a)** £960 **b)** £1600
20 150 g flour, $\frac{3}{8}$ tsp salt, 75 g suet, $\frac{1}{2}$ tsp herbs
21 **a)** 1 : 250 000 **b)** 1 : 12 500
c) 1 : 40 **d)** 1 : 80 000
22 **a)** €1627.91 **b)** $2.42

Exercise 2.2 (page 29)

1 $\frac{2}{5}$ **2** $\frac{9}{25}$ **3** $\frac{1}{20}$ **4** $\frac{3}{10}$
5 $\frac{33}{100}$ **6** $\frac{1}{3}$ **7** $\frac{1}{8}$ **8** $\frac{1}{200}$
9 75% **10** 40% **11** 30% **12** 85%
13 84% **14** 95% **15** $87\frac{1}{2}\%$ **16** $7\frac{1}{2}\%$
17 66%, $\frac{2}{3}$, 0.67, 69%, 0.7, $\frac{5}{7}$
18 **a)** 84% **b)** 155 **c)** 71%
19 70% and 71.1% so both scores are about the same
20 20%

Exercise 2.3 (page 30)

1 271.2 **2** 382.2 **3** 1380, 1490.4
4 £35.99 **5** 14%
6 **a)** £11 480 **b)** £5190
7 2 hours 21 minutes (or 141 minutes)
8 **a)** £39 520 **b)** £39 125 **c)** $1.04 \times 0.99 = 1.0296$
9 **a)** 35% **b)** 9%
10 **a)** $2393.58 **b)** £1388.28

Exercise 2.4 (page 32)

1 **a)** £280 **b)** £85 **2** **a)** £66 **b)** £650
3 £76 **4** **a)** £20.40 **b)** £15.30
5 900 milliseconds
6 **a)** £9200 **b)** £3405
7 **a)** 160 pounds **b)** 11 stone 6 pounds
8 2.86 million light years **9** 15 400
10 **a)** 1200 by 775 **b)** Area reduced by 36%

Exercise 2.5 (page 34)

1 £56.48 **2** £1604.06 **3** £750
4 24 **5** 806.69
6 **a)** $4\% \times 25 = 100\%$ (which is the wrong calculation)
b) $1.04^{18} = 2.026$ so 18 years is sufficient

Review Exercise 2 (page 34)

1 2 : 3 **2** 54° **3** 8 : 10 : 15
4 95% **5** £384 **6** 40%
7 **a)** £504 **b)** 80% **c)** £680
8 **a)** 12 grams **b)** 280 grams
9 £40
10 **a)** 360 **b)** 22%
11 £9720
12 **a)** 16.8 cm **b)** 25 cm
13 $68.77
14 £800
15 **a)** £239.76 **b)** £351
16 85
17 £5895.60
18 200 g flour, 150 g almonds, 225 g sugar, 150 g butter, 10 pears
19 £624.32
20 **a)** £35.70 **b)** £18.40
21 **a)** £6.37 **b)** £69.52
22 **a)** 4% **b)** £980
23 £5062.50
24 4 years

Internet Challenge 2 (page 37)

1 The RPI is an average measure of change in the prices of goods and services bought by the vast majority of households in the UK.
2 It is compiled and published monthly.
3 Inflation varies so check on the internet for the most up-to-date value at www.statistics.gov.uk. At the time of publication the value is 1.1%.
4 The Bank of England sets a rate at which it lends money to other financial institutions; this in turn affects the rates building societies and so on charge their customers.
5 It is reviewed monthly.
6 4.5% (January 2006), updates available at www.bankofengland.co.uk.
7 1920 to 1923
8 Old definition: one trillion = 1 000 000 000 000 000 000
New definition: one trillion = 1 000 000 000 000
9 The former Yugoslavia.
10 Hyperinflation

Chapter 3: Powers and roots

Starter 3 (page 38)

Task 1 **a)** 17 **b)** 14 **c)** 45 **d)** 70 **e)** 92 **f)** 609

Task 2 **a)** XXI **b)** XXIV **c)** XXXIX **d)** CCXII **e)** CCCXIX **f)** XLVII

Task 3 Star Wars: 1977
Lion King: 1994

Exercise 3.1 (page 40)

1 25 **2** 8 **3** 49
4 27 **5** 81 **6** 64
7 144 **8** 1000 **9** 12
10 15 **11** 4 **12** 6
13 14 **14** 5 **15** 9
16 10 **17** 361 **18** 3.24
19 213.16 **20** -729 **21** 4330.747
22 -1.728 **23** 3.61 (3 s.f.) **24** 17.3 (3 s.f.)
25 2.88 (3 s.f.) **26** 3.68 (3 s.f.) **27** 1.58 (3 s.f.)
28 1.89 (3 s.f.) **29** 8.49 or -8.49 **30** 3.362

Exercise 3.2 (page 42)

1 81 **2** 1 000 000 **3** 6 **4** 10 000
5 729 **6** 32 **7** 2 **8** 1024
9 10 **10** 20 **11** 1728 **12** 2
13 249 000 **14** 9224 **15** 6.275 **16** 0.8145
17 1.817 **18** 1.86 **19** 1.445 **20** 3.46

Exercise 3.3 (page 43)

1 8 **2** 9 **3** 125 **4** 243
5 216 **6** 512 **7** 16 **8** 100 000
9 27 **10** 625 **11** 64 **12** 8

Exercise 3.4 (page 45)

1 $\frac{1}{9}$ **2** $\frac{1}{1000}$ **3** $\frac{1}{25}$ **4** $\frac{1}{4}$
5 $\frac{1}{81}$ **6** $\frac{1}{16}$ **7** $\frac{1}{32}$ **8** $\frac{1}{10}$
9 $\frac{1}{5}$ **10** $\frac{1}{400}$ **11** $\frac{5}{3}$ **12** $\frac{3}{4}$
13 $\frac{2}{5}$ **14** $\frac{25}{16}$ **15** $\frac{27}{8}$ **16** 2
17 $\frac{8}{3}$ **18** $\frac{9}{25}$ **19** $\frac{343}{1000}$ **20** $\frac{16}{9}$

Exercise 3.5 (page 47)

1 2^7 **2** 5^7 **3** 8^9
4 6^3 **5** 9 **6** 3
7 2^{12} **8** 3^6 **9** 3^2
10 2^4 **11** 3^2 **12** 6
13 32 **14** 3 **15** 1 000 000
16 16 **17** 64 **18** 243
19 16 **20** 49 **21** 27
22 1 **23** 1 000 000 **24** 1

Exercise 3.6 (page 49)

1 3.5×10^5 **2** 4×10^4
3 3.52×10^8 **4** 1.93×10^7
5 7.65×10^2 **6** 4.5×10^{-3}
7 8×10^{-1} **8** 2.03×10^{-3}
9 8.27×10^{-10} **10** 3.3×10^{-4}
11 7 400 000 **12** 21 500 000
13 105 000 **14** 2 000 000 000
15 8400 **16** 0.005
17 0.000 002 5 **18** 0.000 000 100 4
19 0.000 000 000 083 **20** 0.000 505

Exercise 3.7 (page 51)

1 7.04×10^8 **2** 2.04×10^6
3 6.95×10^7 **4** 2.2×10^5
5 6×10^{17} **6** 4.2×10^{14}
7 5×10^2 **8** 7.5×10^8
9 150 000 000 **10** 2.98×10^{-5}
11 5.76×10^2 **12** 8×10^7
13 2.25×10^{14} **14** 1.65×10^{10}
15 1.87×10^{11}
16 **a)** 2×10^n **b)** $6.4 \times 10^{6n+1}$
17 4 hours and 4 minutes

Exercise 3.8 (page 53)

1 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37
2 **a)** 4 **b)** 37
c) 13 and 31 **d)** 7×13
3 **a)** $2^4 \times 5$ **b)** $2 \times 3^2 \times 5$
c) $2 \times 3^2 \times 5^2$
4 **a)** $2^2 \times 3^2$ **b)** 3^4
c) $2^4 \times 3^2$
All use an even number of each factor
5 a = 3, b = 7

Exercise 3.9 (page 54)

1 **a)** 6 **b)** 15 **c)** 11
d) 9 **e)** 1 **f)** 26
2 **a)** $2^2 \times 5$, 2^5, 4
b) $2^2 \times 3^2$, $2^2 \times 3 \times 5$, 12
c) $2^4 \times 5$, $2^2 \times 3^2 \times 5$, 20
d) $2^3 \times 3^2$, $2^2 \times 3^3$, 36
e) $2^3 \times 3 \times 5$, $3 \times 5 \times 13$, 15
f) $2^4 \times 3^2$, $2^3 \times 32 \times 5$, 72
3 **a)** 6 **b)** 12 **c)** 24
d) 10 **e)** 6 **f)** 2

Exercise 3.10 (page 56)

1 60 **2** 208 **3** 90
4 200 **5** 144 **6** 60
7 154 **8** 150 **9** 180
10 220 **11** 144 **12** 84
13 180 **14** 210 **15** 108
16 165
17 **a)** $2^2 \times 3 \times 5$, $2^2 \times 3 \times 7$
b) 420
18 **a)** $2 \times 3 \times 11$, 32×11
b) 198
c) 33
19 **a)** 2×5, $2^2 \times 3^2$, $2^3 \times 7$
b) 2
c) 2520
20 **a)** $2^3 \times 5$, $2^4 \times 3$, $2^3 \times 3 \times 5^2$
b) 8
c) 1200
21 Every 20 days
22 In 120 days' time

Review Exercise 3 (page 57)

1 **a)** 125 **b)** 12 **c)** 225
2 **a)** $\frac{1}{100}$ **b)** $\frac{1}{64}$ **c)** 4
3 **a)** 6^5 **b)** 3^3 **c)** 4^6
4 **a)** $2 \times 5 \times 7$ **b)** $2^2 \times 31$
c) $2^5 \times 3$ **d)** $2^4 \times 3 \times 5$

5 **a)** 8 **b)** 168
6 **a)** 2 **b)** 220
7 **a)** $2^3 \times 3^2 \times 5$ **b)** 720
8 Lilian. The numbers have no factors in common other than 1, e.g. 15 and 4
9 7.6×10^{-2}, 15 300, 3.2×10^8, 1.4×10^9
10 30
11 **a)** $\frac{1}{9}$ **b)** 6 **c)** 9 **d)** $\frac{27}{8}$
12 **a)** 1 **b)** $\frac{1}{16}$ **c)** 64
13 **a)** 64 **b)** 3 **c)** 12
14 **a)** 8.4×10^7 **b)** 2.1×10^{-5}
15 **a)** 125
b) **(i)** 4.472 135 955
(ii) 4.47
16 8.01×10^{10}
17 **a)** 4.2×10^5
b) 2.4×10^{-6} grams
18 4.3×10^3 or -4.3×10^3
19 480
20 $c = 4$, $d = 23$
21 **a)** $2^3 \times 3^2$, $2^5 \times 3$ **b)** 24

Internet Challenge 3 (page 60)

1 2×10^{30} **2** 5×10^9
3 4×10^6 **4** 6×10^3
5 2.25×10^8 **6** 2.998×10^8
7 5.5×10^{-7} **8** 10^{-9}
9 2.8×10^6 **10** 10^{11}

Chapter 4: Working with algebra

Starter 4 (page 61)

1 0 3 if misread as 3×1
2 2 0 if misread as $6 - (5 + 1)$
3 25 10 if misread as 5×2
4 14 20 if misread as $(2 + 3) \times 4$
5 9 7 if misread as $(4 + 10) \div 2$
6 9 -9 if misread as $-(3)^2$ or if an incorrect key sequence is used on a calculator

Exercise 4.1 (page 63)

1 8 **2** 200 **3** -14 **4** 36
5 20 **6** 14 **7** -22 **8** 100
9 36 **10** 10 **11** 7 **12** 3
13 **a)** 3625 **b)** 2500
14 **a)** 13 **b)** 3.61
15 **a)** 260 **b)** 18.5

Exercise 4.2 (page 66)

1 k^3 **2** u^2 **3** x^5 **4** n^4
5 $2g^2$ **6** $5t^3$ **7** x^8 **8** y^7
9 z^9 **10** $10x^7$ **11** $24x^6$ **12** $2y^3$
13 $8y^6$ **14** $6z^5$ **15** $6x^9$ **16** $\frac{z^2}{2}$ or $\frac{1}{2}z^2$
17 1 **18** x^6 **19** x^8 **20** y^6
21 $9z^6$ **22** $16x^{10}$ **23** y^{30} **24** $16z^8$
25 $4x^8$ **26** $125x^6$ **27** $64x^3y^3$ **28** $36x^4y^2$
29 $16x^2$ **30** y^3 **31** $15x^5$ **32** $2y$
33 $27z^6$ **34** y **35** $40x^5$ **36** $3x^3$
37 $100x^{10}$ **38** x^6y^2 **39** x^9y^6 **40** $12x^5$
41 $\frac{4x}{y}$ **42** $\frac{6}{y}$

Exercise 4.3 (page 69)

1 $3x^2 + 2x$ **2** $20x^2 + 5x$ **3** $6x - 12x^2$
4 $21x - 3x^2$ **5** $2x^3 + 4x^2 - 2x$
6 $6x^3 - 15x^2 - 9x$
7 $7x + 20$ **8** $8x + 13$
9 $11x + 29$ **10** $8x + 25$
11 $22x + 16$ **12** $7x + 11$
13 $16x + 19$ **14** $7x + 4$
15 $17x + 1$ **16** $38x + 3$
17 $10x - x^2$ **18** $11x^2$
19 $21x - 9$ **20** $2x + 10$
21 $3x + 3$ **22** $31x - 50$
23 $175x - 90$ **24** $6x + 12$
25 $6x + 33$ **26** $4x + 12$
27 $3x - 9$ **28** $5x - 3$
29 $10x^2 - 19x$ **30** $12x - 24x^2$

Exercise 4.4 (page 72)

1 $3x^2 + 13x + 12$ **2** $4x^2 + 13x + 10$
3 $2x^2 + 9x + 4$ **4** $2x^2 + 9x - 5$
5 $2x^2 - 4x - 6$ **6** $4x^2 + 24x + 11$
7 $3x^2 + 10x + 8$ **8** $6x^2 - 35x - 6$
9 $4x^2 + 4x - 15$ **10** $4x^2 + 51x - 13$
11 $6x^2 + 13x + 6$ **12** $8x^2 + 18x - 5$
13 $14x^2 - 15x - 9$ **14** $x^2 - 7x + 12$
15 $6x^2 + 5x - 6$ **16** $2x^2 - 11x + 15$
17 $x^2 - 49$ **18** $4x^2 - 9$
19 $x^2 + 6x + 9$ **20** $x^2 - 2x + 1$
21 $9x^2 - 24x + 16$ **22** $x^3 + 6x^2 + 11x + 6$
23 $x^3 + 9x^2 + 26x + 24$ **24** $x^3 + 4x^2 - 7x - 10$
25 $x^3 - 11x^2 + 38x - 40$ **26** $x^3 - 3x - 2$
27 $x^3 - 6x^2 + 12x - 8$ **28** $2x^3 - x^2 - 13x - 6$
29 $6x^3 + 13x^2 + x - 2$ **30** $27x^3 + 54x^2 + 36x + 8$

Exercise 4.5 (page 73)

1 $x(x + 6)$ **2** $2x(x + 3)$
3 $2x(x + 3y)$ **4** $y(y - 10)$
5 $2y(y - 5)$ **6** $3x(2 + 3x)$
7 $4(3y^2 + 2)$ **8** $4y(3y + 2)$
9 $g(f + 3g)$ **10** $3y(3y + 4)$
11 $x^4(5x - 4)$ **12** $6x^2(2 - x)$
13 $7a(2a + 3b)$ **14** $5y(x + 2)$
15 $2(7 + 5y)$ **16** $3xy(5 - 3x)$
17 $4y^2(2 - 5y)$ **18** $4y(3y - 2)$
19 $6(1 + 3x^2)$ **20** $3pq(4q^2 - 4q + 5)$

Exercise 4.6 (page 75)

1 $(x + 1)(x + 7)$ **2** $(x + 2)(x + 7)$
3 $(x + 3)(x + 2)$ **4** $(x + 5)(x + 6)$
5 $(x + 8)(x + 2)$ **6** $(x - 1)(x - 3)$
7 $(x - 2)(x - 5)$ **8** $(x - 6)(x - 5)$
9 $(x - 2)(x - 1)$ **10** $(x - 3)(x - 4)$
11 $(x + 4)(x - 1)$ **12** $(x + 3)(x - 2)$
13 $(x + 2)(x + 3)$ **14** $(x + 1)(x - 5)$
15 $(x + 3)(x - 4)$ **16** $(x - 2)(x - 6)$
17 $(x + 8)(x + 4)$ **18** $(x - 9)(x + 8)$
19 $(x + 3)(x + 4)$ **20** $(x + 4)(x - 11)$

Exercise 4.7 (page 75)

1 $(2x + 1)(x + 1)$ **2** $(2x + 3)(x + 1)$
3 $(2x + 1)(x + 2)$ **4** $(3x - 2)(x - 1)$
5 $(3x + 1)(x - 1)$ **6** $(5x - 1)(x + 1)$
7 $(2x + 1)(x - 1)$ **8** $(5x + 1)(x - 2)$
9 $(3x - 2)(x - 2)$ **10** $(2x - 1)(x + 6)$
11 $(2x - 3)(x - 3)$ **12** $(2x + 1)(3x - 1)$
13 $(3x + 5)(2x - 5)$ **14** $(6x + 1)(2x + 1)$
15 $(3x + 2)(5x + 3)$ **16** $(2x - 1)(2x - 1)$
17 $(6x - 1)(x - 2)$ **18** $(2x + 7)(x + 1)$
19 $(2x + 3)(2x + 3)$ **20** $(2x + 3)(x - 3)$

Exercise 4.8A (page 76)

1 $(x + 1)(x - 1)$ **2** $(y + 11)(y - 11)$
3 $(x + 9)(x - 9)$ **4** $(y + 20)(y - 20)$
5 $3(x + 5)(x - 5)$ **6** $2(x + 3)(x - 3)$
7 $7(y + 3)(y - 3)$ **8** $10(x + 2)(x - 2)$
9 $3(x + 3)(x - 3)$ **10** $4(y + 5)(y - 5)$

Exercise 4.8B (page 77)

1 $(x + 1)(x + 5)$ **2** $x(x + 8)$
3 $(y + 4)(y + 11)$ **4** $(x + 6)(x + 5)$
5 $x(x + 7)$ **6** $(y - 2)(y + 5)$
7 $(4x - 1)(x - 2)$ **8** $(y - 6)(y + 5)$
9 $(x - 1)(x - 2)$ **10** $(x - 3)(x - 5)$
11 $(y + 4)(y - 4)$ **12** $5y(x - 2y)$
13 $(2x - 1)(2x - 3)$ **14** $7(y + 10)(y - 10)$
15 $(x + 6)(x - 4)$ **16** $(2y + 5)(y - 2)$
17 $4z(z - 1)$ **18** $(2x + 1)(x + 1)$
19 $3(x + 2)(x - 2)$ **20** $(2x - 1)(x + 3)$

Exercise 4.9 (page 78)

1 $P = 3x$ **2** $T = 30n$
3 $T = 60x + 5y$ or $T = 5(12x + y)$
4 $P = 500 + 10m$
5 $w = \frac{A}{l}$
6 **a)** $n + 11$ **b)** $T = 2n + 11$
7 **a)** $15x$ **b)** $T = 15x + 25y$
8 **a)** £15 **b)** $C = 5 + 2n$
9 **a)** 18 MB **b)** $S = 128 - 0.3n$ **c)** 426
10 **a)** $(a - 2)(b - 2)$ **b)** $V = (a - 2)(b - 2)$

Exercise 4.10 (page 80)

1 $r = \frac{A}{\pi l}$ **2** $u = v - at$
3 $a = \frac{v - u}{t}$ **4** $h = \frac{3V}{\pi r^2}$
5 $m = \frac{E}{c^2}$ **6** $x = \frac{y - 3}{4}$
7 $x = 5(y - 3)$ or $x = 5y - 15$
8 $x = 5y - 3$
9 $h = \frac{2A}{b}$ **10** $c = \sqrt{\frac{E}{m}}$
11 $y = \frac{A - x^2}{4x}$ **12** $R = \frac{P}{I^2}$
13 $x = \frac{y}{m} + a$ **14** $a = \frac{v^2 - u^2}{2s}$
15 $r = \sqrt{\frac{A}{4\pi}}$ **16** $x = \sqrt{y + 9}$
17 $y = \sqrt{x^2 - z^2}$ **18** $b = \frac{V}{ac}$
19 $r = \sqrt[3]{\frac{3V}{4\pi}}$ **20** $u = \sqrt{v^2 - 2as}$

Review Exercise 4 (page 81)

1 **a)** 24 **b)** 32 **c)** 31 **d)** −40
2 **a)** 18 **b)** −8 **c)** 36 **d)** 10
3 **a)** −1 **b)** 13 **c)** 10
4 x^7 **5** $3x^6$
6 $12x^5$ **7** $5y^3$
8 $4z$ **9** $3xy^3$
10 x^6 **11** $25x^2y^4$
12 $9x^2$ **13** $30x^6$
14 $10x^3$ **15** $2y^2$
16 $7x + 16$ **17** $5y + 13$
18 $8z - 7$ **19** $13x + 5$
20 $10x - 14$ **21** $2x + 8$
22 $30x - 2$ **23** $2x - 16$
24 $10x$ **25** 12
26 $4x - x^2$ **27** $10x^2 - 18x$
28 $x^2 + 6x + 5$ **29** $y^2 + 12y + 35$
30 $2z^2 + 9z + 4$ **31** $x^2 - x - 20$
32 $2x^2 + 7x - 15$ **33** $2x^2 - 3x + 1$
34 $3x^2 - 7x - 6$ **35** $4x^2 - 9$
36 $x^2 - 16$ **37** $5x^2 - 5$
38 $x^3 - 6x^2 + 11x - 6$ **39** $x^3 + 4x^2 - 11x - 30$
40 $2x^3 + 3x^2 - 14x - 15$ **41** $2x^3 + 7x^2 + 4x - 4$
42 $2x(12x + 5)$ **43** $(x + 7)(x + 3)$
44 $(y + 1)(y + 1)$ or $(y + 1)^2$
45 $(z + 8)(z - 8)$ **46** $(2y - 1)(y + 5)$
47 $(2x - 1)(x - 4)$ **48** $2x(6x - 5)$
49 $(2y + 3)(y + 2)$ **50** $4(x + 3)(x - 3)$
51 **a)i)** $x^2 + 2x + 1$ **ii)** $x^2 + 4x + 4$
b) $x^4 + 6x^3 + 13x^2 + 12x + 4$
52 **a)** 2 **b)** b^{-3} **c)** $\frac{4}{c^2}$
53 $T = 26x + 19y$
54 **a)** $T = 5x + 3y$ **b)** $x = y$
55 **a)** $5n + 2(10 - n)$ **b)** $3n + 20$
56 $r = \frac{C}{2\pi}$ **57** $a = \frac{2(s - ut)}{t^2}$
58 $r = \sqrt{\frac{A}{\pi}}$ **59** $l = \frac{gT^2}{4\pi^2}$ or $l = g\left(\frac{T}{2\pi}\right)^2$
60 **a)** $12x$ **b)** $12x + 10y$
61 $w = ph + b$
62 $C = 20 - 4n$
63 **a)** $8p - 3q$ **b)** x^5 **c)** $2(2x + 3)$
d) $x^2 + x - 6$ **e)** $2x^8$
64 **a)** y^7 **b)** $8x + 17$
c) **(i)** $2(2a + 3)$ **(ii)** $3p(2p - 3q)$
65 **a)** Bryani, because $4 \times 3^2 = 4 \times 9 = 36$
b) 64
66 **a)** $12a^5b^3$ **b)** $\frac{125p^9}{q^3}$ **c)** $\frac{4t^3}{u}$
67 **a)** $x^2 + 2x - 15$ **b)** $3a(2a - 3b)$
68 **a)** $x^2 + 2xy + y^2$ **b)** 25
69 **a)** p^9 **b)** $6q^8$
70 $x = \sqrt{5y - 4}$
71 **a)** k^3
b) **(i)** $7x - 1$ **(ii)** $x^2 + 5xy + 6y^2$
c) $(p + q)(p + q + 5)$
d) m^8
e) $6r^3t^6$

Internet Challenge 4 (page 85)

T	E	Q	U	A	T	I	O	N	N	D	P	O	R	I
V	X	S	P	G	F	E	N	Y	M	Z	S	E	R	Y
B	P	O	W	E	R	F	T	L	A	C	I	D	A	R
Q	R	X	O	Y	U	O	M	K	P	P	M	S	T	L
U	E	S	N	E	O	A	N	T	P	N	P	C	I	H
O	S	L	D	R	U	S	A	O	I	S	L	D	O	S
T	S	A	O	D	E	R	L	U	N	F	I	V	N	J
I	I	N	D	E	X	Y	E	A	G	I	F	A	A	Y
E	O	J	E	L	N	C	D	U	H	C	Y	R	L	T
N	N	D	N	O	I	T	C	N	U	F	E	I	Q	I
T	E	R	M	G	E	O	W	L	A	L	H	A	F	T
A	R	I	L	T	C	U	D	O	R	P	N	B	A	N
A	A	D	J	H	Y	T	E	D	W	T	X	L	B	E
L	Y	T	S	F	A	C	T	O	R	I	S	E	L	D
C	I	T	A	R	D	A	U	Q	F	V	H	I	Y	L

Chapter 5: Algebraic equations

Starter 5 (page 86)

Missing numbers clockwise from top:

2 14, 17 **3** 16, 7, 3 **4** 7, 10, 6

Exercise 5.1 (page 87)

1 Expressions: A, C
2 Equations: B, D, E, F, G, H, I, J (although D and H are actually identities)
3 Formulae: B, J
4 Identities: D, H

Exercise 5.2 (page 89)

1 5 **2** 7
3 9 **4** 30
5 12 **6** 2
7 4 **8** -9
9 $2\frac{1}{2}$ **10** $\frac{6}{5}$
11 16 **12** -3
13 $\frac{4}{7}$ **14** $\frac{8}{3}$
15 6 **16** 17
17 $\frac{1}{14}$ **18** 0
19 $\frac{5}{4}$ or $-\frac{5}{4}$ **20** 12 or -12

Exercise 5.3 (page 91)

1 3 **2** 1
3 4 **4** -2
5 3 **6** 5
7 2 **8** -2
9 0 **10** 11
11 $\frac{3}{4}$ **12** $\frac{7}{5}$
13 $\frac{9}{2}$ **14** $\frac{5}{8}$
15 $-\frac{1}{2}$ **16** -2
17 $-\frac{1}{2}$ **18** $\frac{5}{3}$
19 -5 **20** $\frac{7}{3}$

Exercise 5.4 (page 92)

1 -3 **2** 7 **3** 2
4 6 **5** 10 **6** 4
7 4 **8** 3.5 **9** -4
10 4 **11** $\frac{1}{2}$ **12** 7
13 $2(x + 12) = 4x$, leading to $x = 12$
14 $n + 4 = 2(n - 5)$, leading to $n = 14$
15 **a)** $7x + 5 = 5(x + 7)$
b) 15

Exercise 5.5 (page 94)

1 5 **2** $2\frac{1}{2}$ **3** 4 **4** 2
5 5 **6** 7 **7** -3 **8** 3
9 -1 **10** 5

Review Exercise 5 (page 94)

1 **a)** formula **b)** expression
c) equation **d)** identity

2 6 **3** -2 **4** 7
5 $\frac{1}{3}$ **6** $\frac{7}{4}$ **7** $\frac{8}{5}$ or $-\frac{8}{5}$
8 4 **9** 9 or -9 **10** 20
11 4 **12** 4 **13** 2
14 -2 **15** $\frac{7}{4}$ **16** -1
17 5 **18** -2 **19** 0
20 $\frac{5}{3}$ **21** 4 **22** -1
23 9 **24** 0 **25** 6
26 2 **27** 2 **28** 5
29 6 **30** $10\frac{2}{3}$
31 Glenn should have written -8 instead of $+8$ in the second line.
He would then get a final answer of 4.
32 Seyi is right.
33 **a)** $p = 3$ **b)** $r = -11$
34 $\frac{6}{5}$
35 **a)** 8 **b)** $6\frac{1}{2}$ **c)** $\frac{5}{8}$
36 **a)** 3 **b)** -2 **c)** $4\frac{1}{5}$
37 23
38 **a)** $3\frac{1}{2}$ **b)** 7
39 **a)** $x + 2$ **b)** $4x + 14$ **c)** $1\frac{1}{2}$

Internet Challenge 5 (page 97)

1 German
2 Braunschweig, 30 April 1777
3 77 years
4 Construction of the heptadecagon.
5 He added them in pairs: $1 + 100$, $2 + 99$, … gives $101 \times 50 = 5050$.
6 Göttingen
7 Discovery of Ceres, the first known asteroid.
8 True (discovered by Gauss).
9 A polynomial of degree n will have exactly n solutions. For example, the equation $x^3 + 4x^2 + x - 6 = 0$ is of degree 3 (it has an x^3 term) and has 3 solutions. Note that some of the solutions may be duplicates, and some may only exist if you use complex numbers (which were also developed by Gauss).
10 Demagnetise it.
11 de Moivre; Normal distribution
12 Numbers containing two parts, a real part and a complex part based on the (imaginary) square root of minus 1.
13 When told his wife was dying.
14 The prince of mathematicians.
15 1855, Göttingen
16 Heptadecagon; no
17 A heptadecagon has 17 sides.

Chapter 6: Graphs of straight lines

Starter 6 (page 98)

1 Hint: The two squares that remain do not have to be the same size. Try removing two of the matches that meet in the centre of the original pattern.

2 Hint: Move the horizontal match half of its length to the right (or left).

3 Hint: Begin by removing the two upper left matches; place one of them to complete the fourth side of a square that now forms the face of the new fish.

4 Hint: Drag the right hand match slightly out, so a small square is formed where the matches meet.

Exercise 6.1 (page 100)

1 A (1, 2), B (−1, −2), C (−2, 3), D (3, −2), E (−3, 0)

2 **a)** M **b)** (0, 1) **c)** J
d) K **e)** (6, −1) **f)** N
g) D

3 (1, 3)

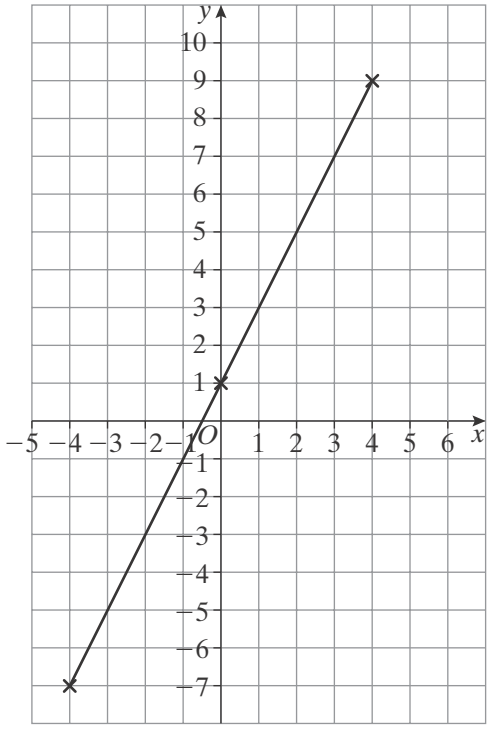
4 (1½, −3)

Exercise 6.2 (page 103)

1

x	−4	0	4
y	−7	1	9

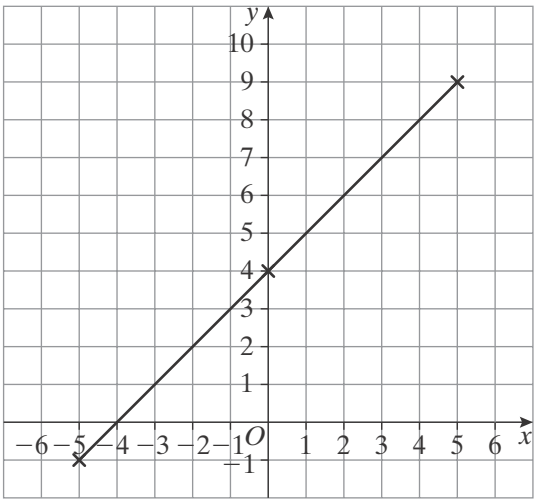
$y = 2x + 1$



2

x	−5	0	5
y	−1	4	9

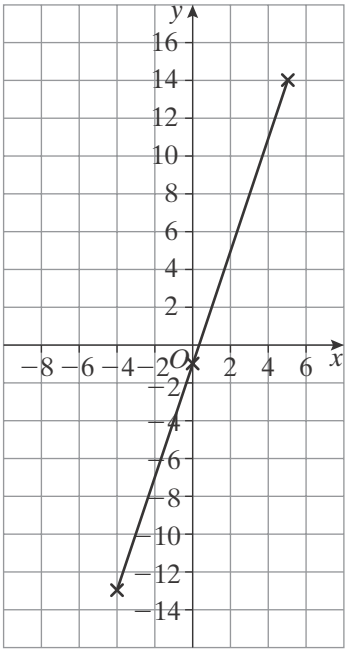
$y = x + 4$



3

x	−4	0	5
y	−13	−1	14

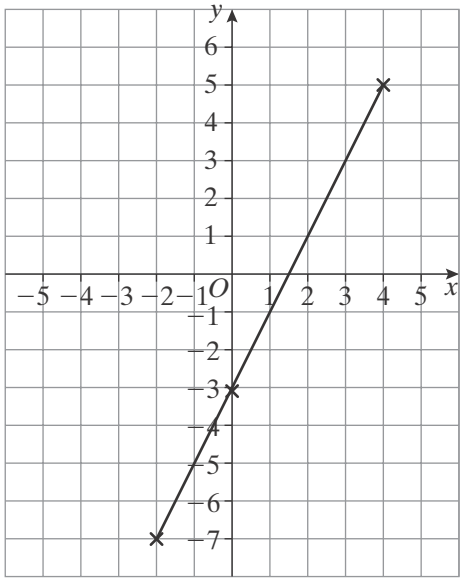
$y = 3x - 1$



4

x	−2	0	1
y	−7	−3	5

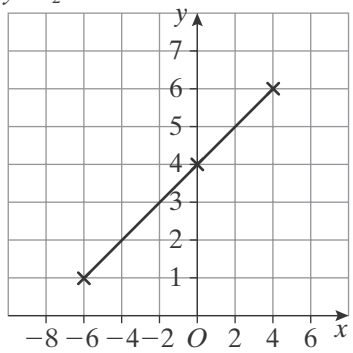
$y = 2x - 3$



5

x	-6	0	4
y	1	4	6

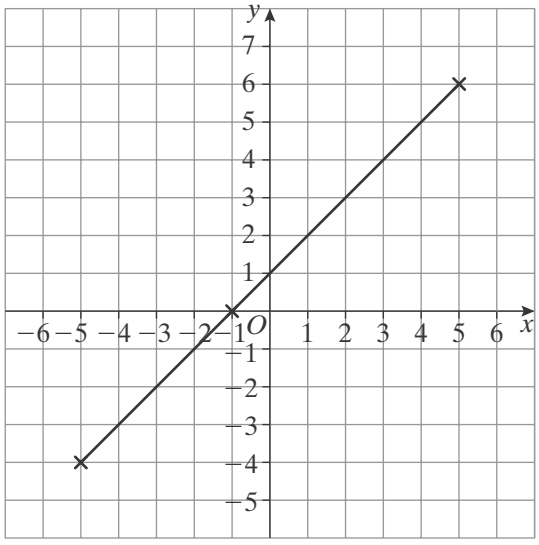
$y = \frac{1}{2}x + 4$



6

x	-5	0	5
y	-4	1	6

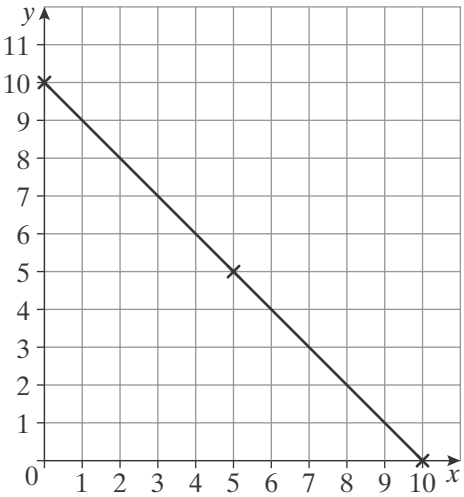
$y = x + 1$



7

x	0	5	10
y	10	5	0

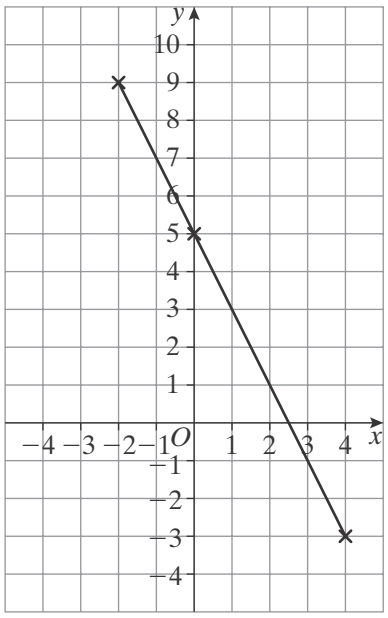
$x + y = 10$



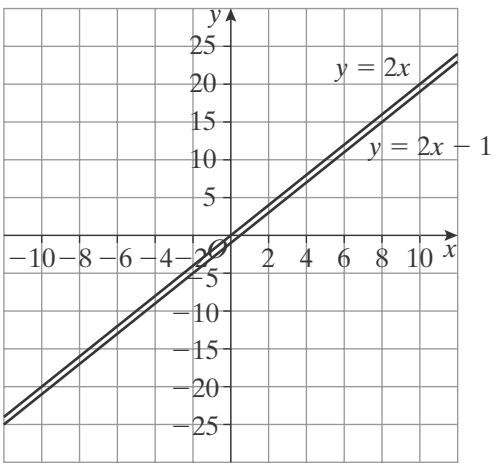
8

x	-2	0	4
y	9	5	-3

$2x + y = 5$

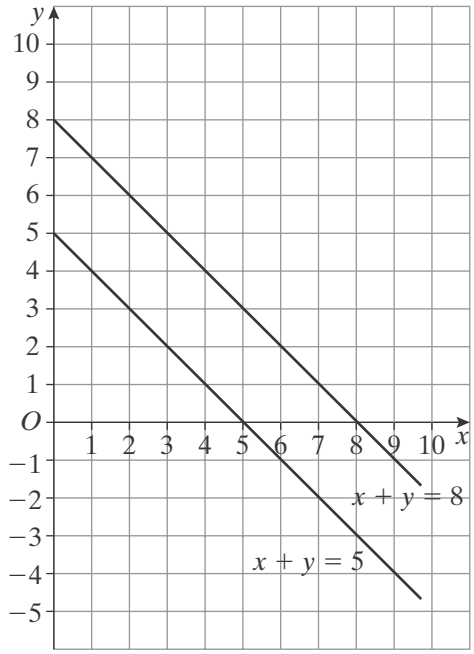


9



The graphs of the two lines are parallel.

10



The graphs of the two lines are parallel.

Exercise 6.3 (page 106)

1 $m = 3, c = 1$ **2** $m = 1, c = 2$

3 $m = -2, c = 6$ **4** $m = \frac{1}{2}, c = 1$

5 $m = \frac{3}{4}, c = 2$ **6** $m = -\frac{2}{3}, c = 6$

7 $m = \frac{1}{2}, c = 1$ **8** $m = -\frac{1}{2}, c = 4$

Exercise 6.4 (page 109)

1 $y = 3x + 1$ **2** $y = x + 2$

3 $y = -2x + 6$ **4** $y = \frac{1}{2}x + 1$

5 $y = \frac{3}{4}x + 2$ **6** $y = -\frac{2}{3}x + 6$

7 $y = \frac{1}{2}x + 1$ **8** $y = -\frac{1}{2}x + 4$

9 **a)** P (2, 2), Q (8, 5)
b) $m = \frac{1}{2}, c = 1$
c) $y = \frac{1}{2}x + 1$

10 **a)** $m = -\frac{1}{2}, c = 5$ **b)** $y = -\frac{1}{2}x + 5$

Exercise 6.5 (page 112)

1 **a)** $y = x - 6$ **b)** $y = -2x - 5$
c) $y = \frac{1}{2}x + 1$ **d)** $y = \frac{1}{2}x + 2\frac{1}{2}$
Lines c) and d) are parallel.

2 **a)** $a = 5$
b) $b = -5$, so the line has equation $y = 5x - 5$

3 **a)** $m = 4$, $c = 3$, so the line has equation $y = 4x + 3$
b) $p = 15$

4 **a)** $y = 3x + 2$ **b)** $y = 3x - 5$

5 -2

6 $y = 4 - x$

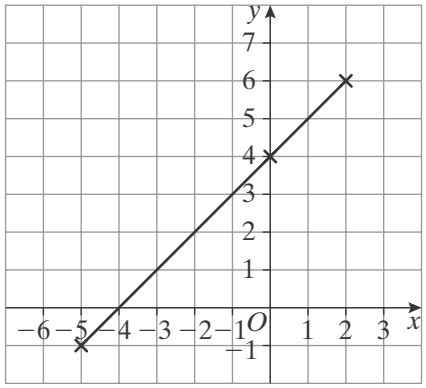
7 $x + 3y + x + 6 = 0$

Review Exercise 6 (page 113)

1

x	-5	0	2
y	-1	4	6

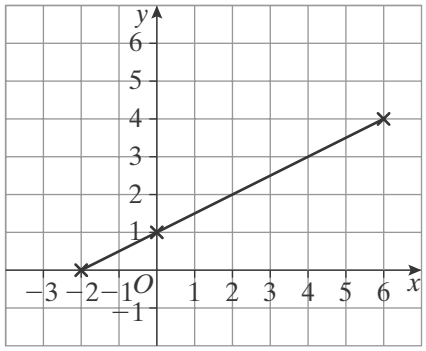
$y = x + 4$



2

x	-2	0	6
y	0	1	4

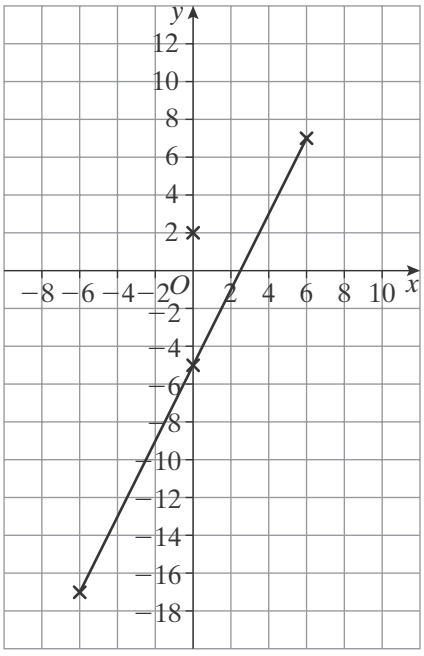
$y = \frac{1}{2}x + 1$



3

x	-6	0	6
y	-17	-5	7

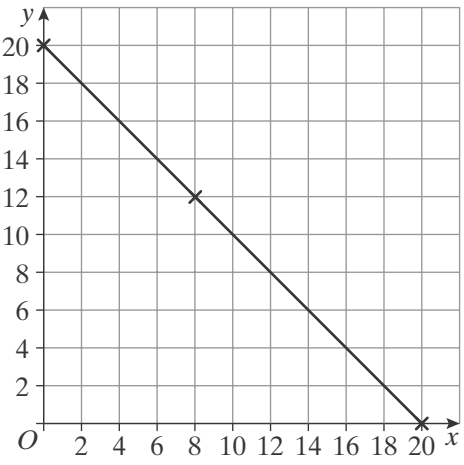
$y = 2x - 5$



4

x	0	8	20
y	20	12	0

$x + y = 20$



5 **a)** $m = \frac{1}{2}, c = 4 \quad y = \frac{1}{2}x + 4$
b) $m = -1, c = 7 \quad y = -x + 7$

6 $y = 4x + 7$

7 $y = 3x + 2$

8 Gradients are $\frac{2}{3}$ and $-\frac{3}{2}$; $-\frac{3}{2} \times \frac{2}{3} = -1$; product of gradients of perpendicular lines is -1

9 **a)** 2
b) $2y + x - 12 = 0$

10 $y = -2x + 5$

11 **a)** 8
b) Any line of the form $y = \frac{1}{2}x + k$
c) $x = 2y - 2$

12 **a)** $y = -\frac{1}{2}x + 3$
b) Gradient is $-\frac{1}{2}$
c) $x + 2y = 14$ or $y = -\frac{1}{2}x + 7$

13 $y = 2x + 6$

Internet Challenge 6 (page 116)

1 Parallelogram
2 Trapezium
3 A prism whose faces are all parallelograms.
4 Blondie
5 Border between USA and Canada.
6 When any decision arises, all the possible outcomes occur, each in a separate 'parallel universe' hidden from the others.
7 'If a straight line crossing two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if extended indefinitely, meet on that side on which are the angles less than two right angles.'
There are many other statements which are logically equivalent to the parallel postulate, including:
'Through a point not on a given line, exactly one line can be drawn in the plane parallel to the given line.'
8 On a (rather old) computer.
9 On a ski slope.
10 True
11 Yes (consider railway tracks going round a bend).
12 An electrician
13 **a)** All of them.
b) Bristol, Dr Richard Gregory

Chapter 7: Simultaneous equations

Starter 7 (page 117)

Cherry = 5, lemon = 3, apple = 12, orange = 8, grapes = 7, banana = 4

Exercise 7.1 (page 119)

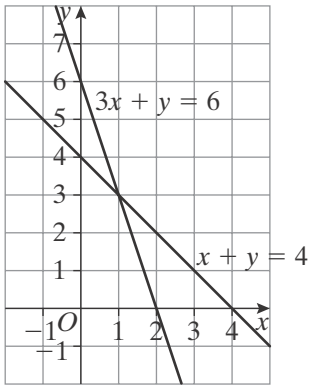
1 $x = 4, y = 1$
2 $x = 3, y = 3$
3 $x = -1, y = 6$
4 $x = -1, y = 5$
5 $x = -4, y = 1$
6 $x = 2, y = 3$
7 $x = 10, y = 1$
8 $x = 3, y = -1$
9 $x = 7, y = -2$
10 $x = 4, y = 0$

Exercise 7.2 (page 122)

1 $x = 4, y = 3$
2 $x = 1, y = -1$
3 $x = 2, y = 1$
4 $x = 2, y = -1$
5 $x = 5, y = 2$
6 $x = 1, y = 1$
7 $x = 5, y = -2$
8 $x = 0, y = -2$
9 $x = 1, y = 4$
10 $x = 2, y = 3$
11 $x = 1, y = -2$
12 $x = 2, y = 1$
13 $x = 0, y = -1$
14 $x = 10, y = -2$
15 $x = -1, y = -10$
16 $x = 2, y = -2$
17 $x = 3, y = 1$
18 $x = 4, y = -1$
19 $x = 2, y = 6$
20 $x = -1, y = -3$
21 $x = 4, y = \frac{1}{2}$
22 $x = 6, y = 1\frac{1}{2}$
23 $x = 3, y = -3$
24 $x = 5, y = 0$
25 $x = 2, y = -7$
26 $x = 3, y = 1\frac{1}{2}$
27 $x = \frac{1}{2}, y = -\frac{1}{2}$
28 $x = 4, y = -1\frac{1}{4}$
29 $x = 7, y = -6$
30 $x = 1\frac{1}{2}, y = -2\frac{1}{2}$
31 $x = 5, y = -3$
32 $x = -2, y = -5$

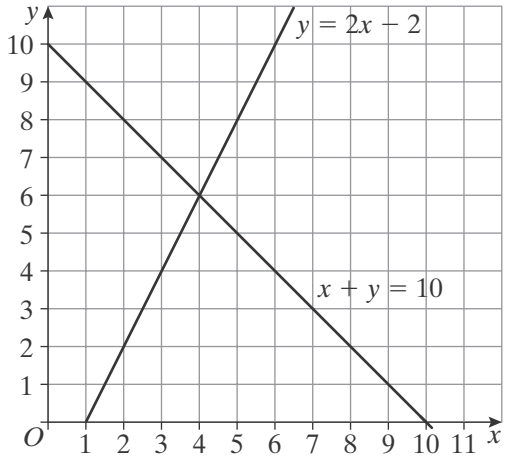
Exercise 7.3 (page 124)

1



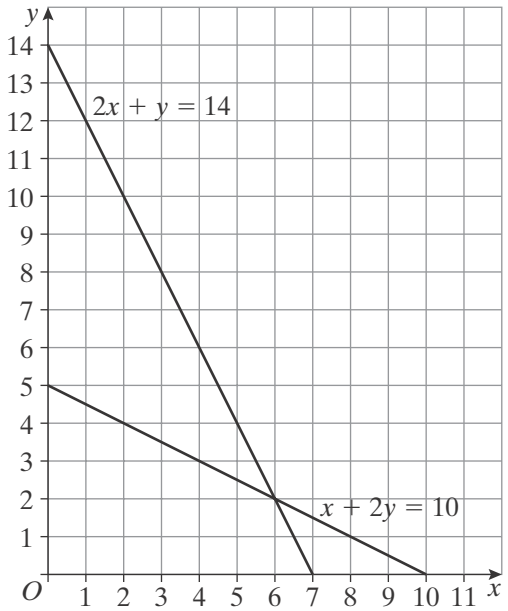
$x = 1, y = 3$

2



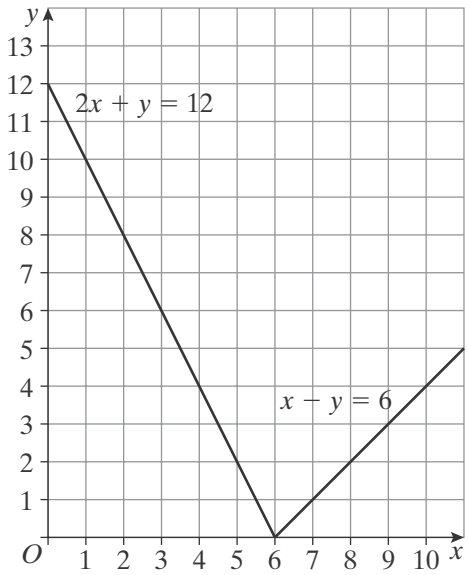
$x = 4, y = 6$

3



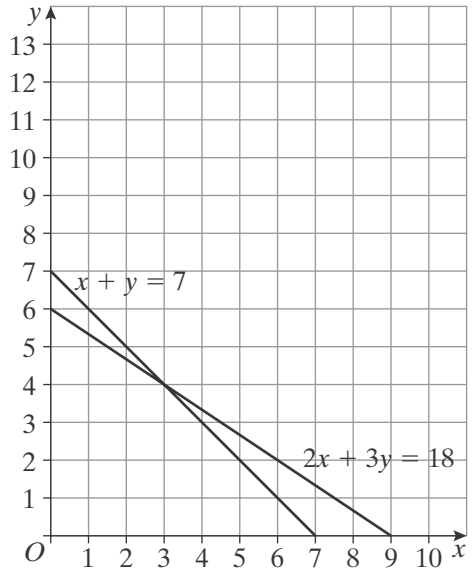
$x = 6, y = 2$

4



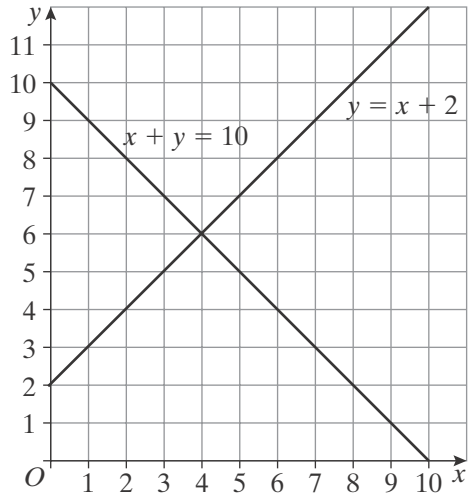
$x = 6, y = 0$

5



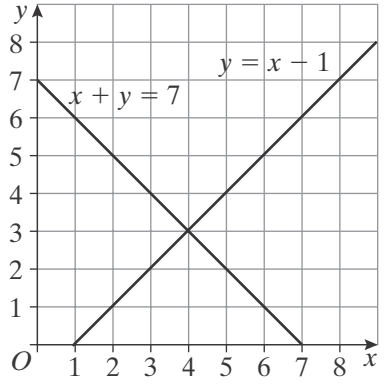
$x = 3, y = 4$

6



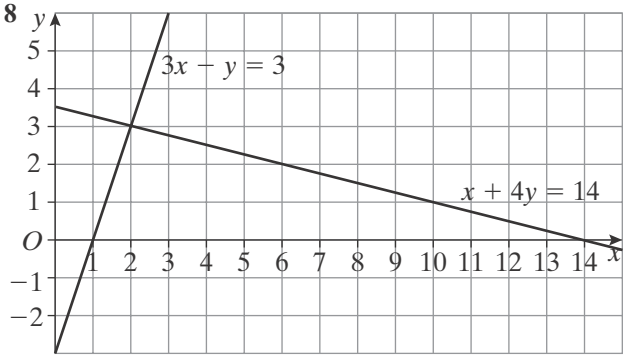
$x = 4, y = 6$

7



$x = 4, y = 3$

8



$x = 2, y = 3$

Exercise 7.4 (page 125)

1 a) $10x + 3y = 104$ $4x + y = 38$
b) Shirt £5, jacket £18

2 a) $3x + 4y = 180$ $5x + 2y = 230$
b) 40 passengers

3 a) If there are x A-level books and y IGCSE books then $x + y = 160$ and $10x + 15y = 1800$
b) 120 A-level books and 40 IGCSE books

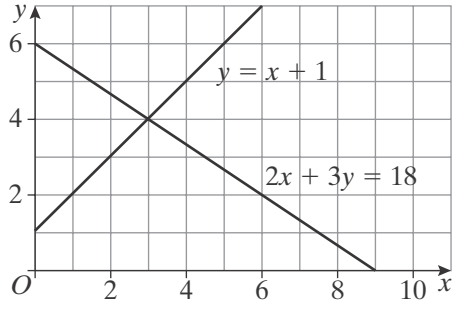
4 a) If there are x 2-litre cans and y 5-litre cans then $x + y = 500$ and $2x + 5y = 1420$
b) 360 2-litre cans and 140 5-litre cans

5 Tomato plants 35 pence each, peppers 45 pence each

Review Exercise 7 (page 126)

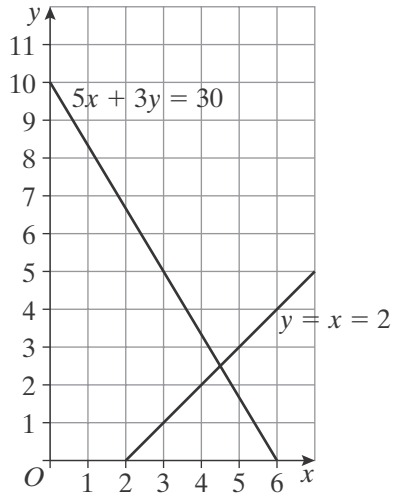
1 $x = 4, y = 3$ **2** $x = 16, y = 0$
3 $x = -1, y = 5$ **4** $x = 1, y = -1$
5 $x = 4, y = 1$ **6** $x = 2, y = -1$
7 $x = 3, y = -2$ **8** $x = -1, y = -2$
9 $x = 5, y = \frac{1}{2}$ **10** $x = 2, y = -\frac{1}{2}$
11 $x = \frac{1}{2}, y = -4$ **12** $x = 2, y = -1\frac{1}{2}$

13



$x = 3, y = 4$

14



$x = 4.5, y = 2.5$

15 a) $5x + y = 207, 2x + 3y = 166$
b) Cola 35p, orange 32p

16 a) $3c + 2s = 19, 4c + 5s = 30$
b) $c = 5, s = 2$
c) 42 minutes

17 a) $100x + 50y = 400, 150x + 100y = 650$
b) $x = 3, y = 2$
c) £10

18 $x = 2.5, y = -2$

19 $x = 4, y = -1$

Internet Challenge 7 (page 128)

1

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

2 A four by four magic square

3 Enter the number 1 in the middle of the bottom row. Work up through 2, 3, 4, …, moving right and down one cell each time; if this takes you outside the grid then move up or left (or both) by a number of squares equal to the dimension of the grid. If you reach a cell that is full then move up one cell.

a) b) Check students' magic squares.

4 a)

52	61	4	13	20	29	36	45
14	3	62	51	46	35	30	19
53	60	5	12	21	28	37	44
11	6	59	54	43	38	27	22
55	58	7	10	23	26	39	42
9	8	57	56	41	40	25	24
50	63	2	15	18	31	34	47
16	1	64	49	48	33	32	17

5 a) The Loh-Shu

b) It dates from 2800 BC, and so is nearly 4000 years old.

Chapter 8: Inequalities

Starter 8 (page 129)

The treasure is buried in the Swamp, at (7, 9).

Exercise 8.1 (page 131)

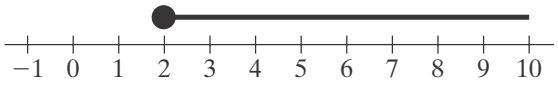
1 4, 5, 6, 7, …
2 3, 4, 5, 6, …
3 …, −1, 0, 1, 2, 3
4 …, −1, 0, 1, 2
5 1, 2, 3, 4, 5, 6
6 0, 1, 2, 3, 4, 5, 6
7 2
8 1
9 3, 4, 5
10 1, 2
11 1, 2, 3, 4, 5
12 1, 2
13 …, −1, 0, 1, 2, 3, 4, 5
14 4, 5, 6, 7, …
15 3, 4, 5
16 1, 2, 3, 4
17 1, 2, 3, 4
18 6, 7, 8, 9, 10
19 97, 98
20 −2, −1, 0, 1, 2

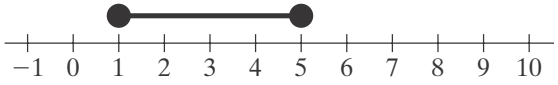
Exercise 8.2 (page 132)

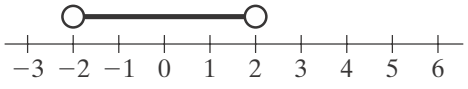
1 $x \geqslant 8$
2 $x > 5$
3 $x < -3$
4 $x < 2$
5 $x < 6$
6 $x \leqslant 3$
7 $x \geqslant 5$
8 $4 < x$
9 $2 \leqslant x$
10 $x > -1$
11 $x \geqslant 4$
12 $x < 4\frac{1}{2}$
13 $2 > x$
14 $x \leqslant -4$
15 $x \leqslant 0$
16 $-5 < x$
17 $x \geqslant 36$
18 $x < 0$
19 $x \geqslant -1$
20 $x < 3\frac{1}{2}$
21 $2 < x \leqslant 4$
22 $1 < x < 2$
23 $6 \leqslant x \leqslant 9$
24 $-1 \leqslant x < 3$
25 $x \leqslant 14$
26 $x > 5$
27 $x < 27$
28 $x \geqslant 11$

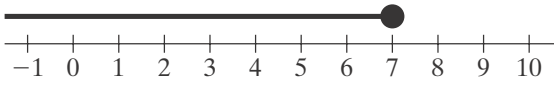
Exercise 8.3 (page 133)

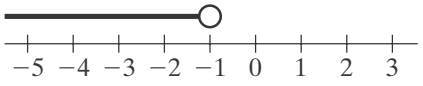
1

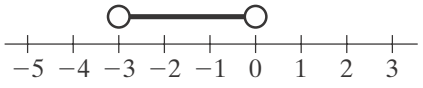

2


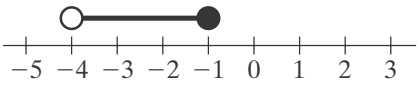
3


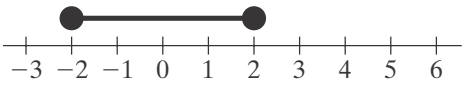
4


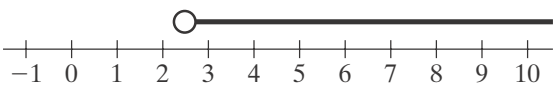
5


6


7


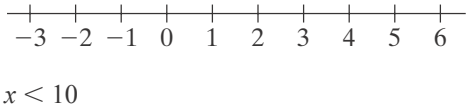
8


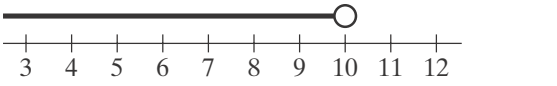
9


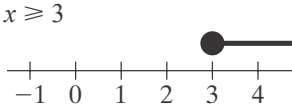
10


11 $x < 14$

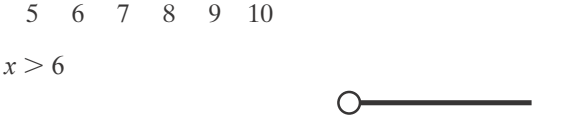

12 $x \leqslant 5$

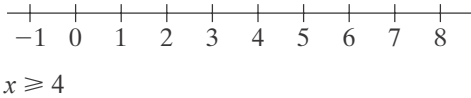

13 $x < 10$


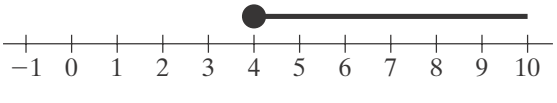
14 $x \geqslant 3$


15 $5 < x < 8.5$


16 $x > 6$


17 $x \geqslant 4$


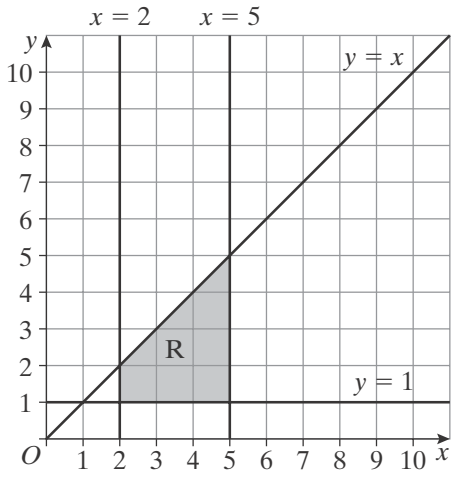
18 $x < 3$


19 $x > 3$


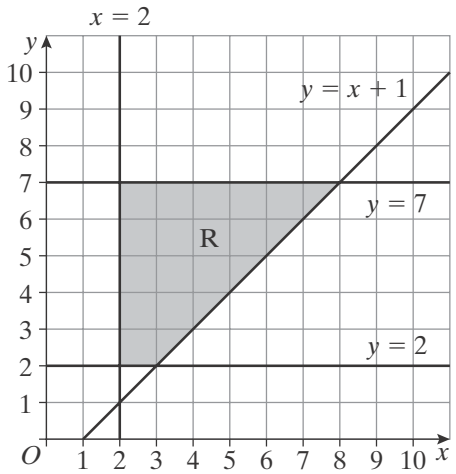
20 $x \leqslant -1$


Exercise 8.4 (page 136)

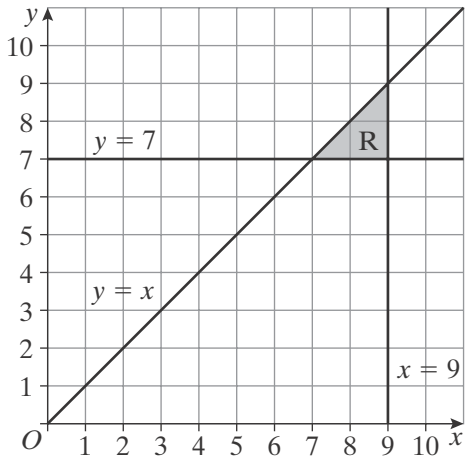
1



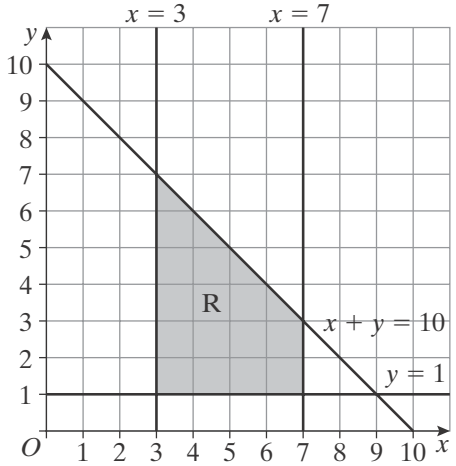
2



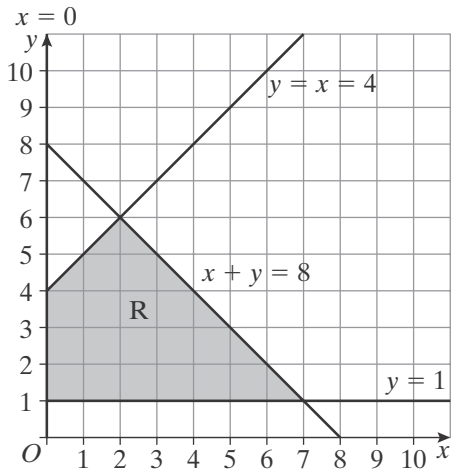
3



4



5

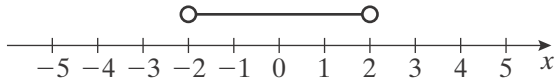


6 **a)** L_1: $y = 5$; L_2: $x = 7$; L_3: $y = x + 3$

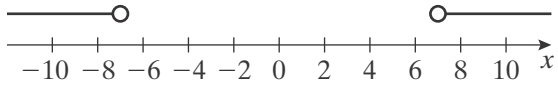
b) L_1: $y > 5$; L_2: $x < 7$; L_3: $y < x + 3$

Exercise 8.5 (page 138)

1 $-5 < x < 5$
2 $-9 \leqslant x \leqslant 9$
3 $y > 4$ or $y < -4$
4 $x \geqslant 2$ or $x \leqslant -2$
5 $-8 \leqslant x \leqslant 8$
6 $-7 < x < 7$
7 $y \geqslant 12$ or $y \leqslant -12$
8 $-3.5 < x < 3.5$
9 $-2 < x < 2$



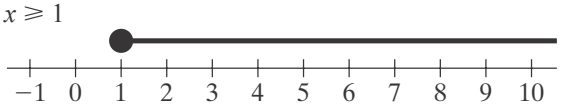
10 $x > 7$ or $x < -7$



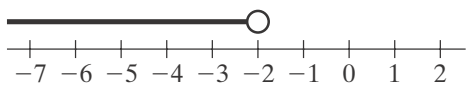
11 $x \geqslant 5$ or $x \leqslant -5$
12 $-3 \leqslant x \leqslant 3$

Review Exercise 8 (page 138)

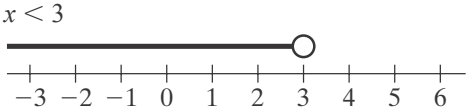
1 0, 1, 2, 3
2 1, 2, 3, 4, 5, 6, 7, 8
3 4, 5, 6, 7, 8
4 3, 4, 5, 6, 7, 8, 9
5 −1, 0, 1, 2, 3, 4, 5, 6
6 5, 6, 7
7 1, 2
8 2, 3, 4, 5
9 4, 5, 6
10 2
11 $x \geqslant 1$



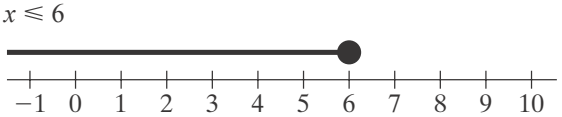
12 $x < -2$



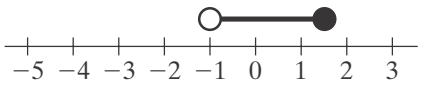
13 $x < 3$



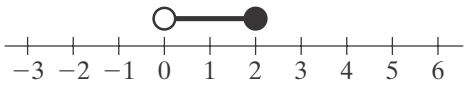
14 $x \leqslant 6$



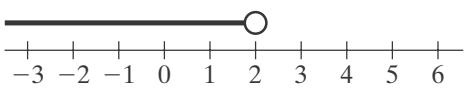
15 $-1 \leqslant x < 1.5$



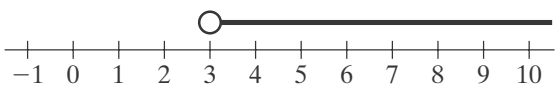
16 $0 < x \leqslant 2$



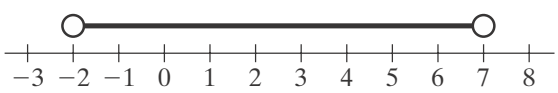
17 $x < 2$



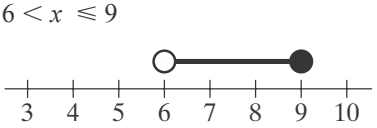
18 $x > 3$



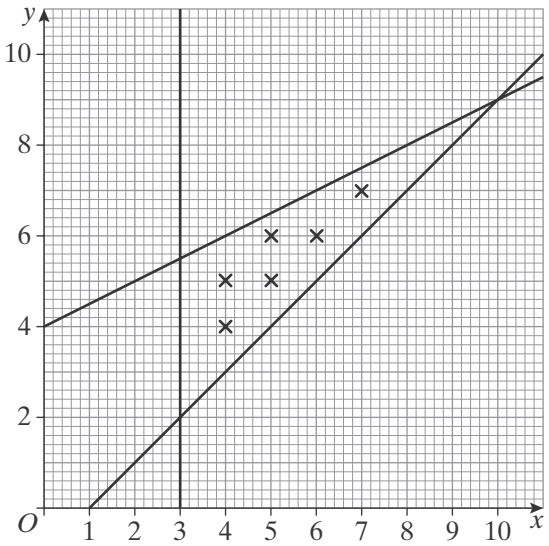
19 $-2 \leqslant x < 7$



20 $6 < x \leqslant 9$

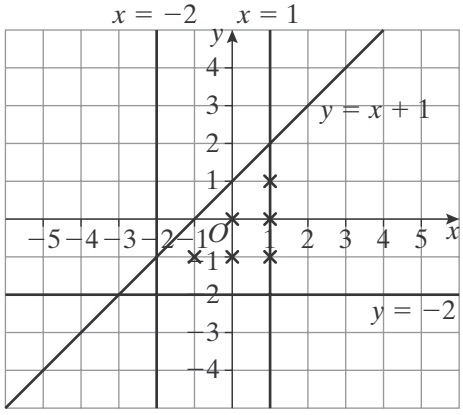


21



22 a) $-1, 0, 1$

b)



23 4, 5, 6

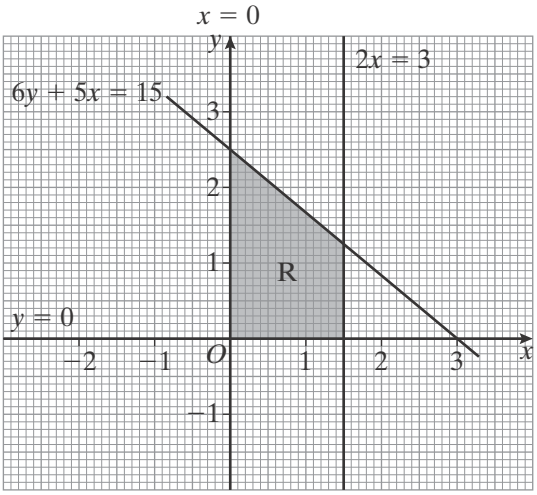
24 a) $y \geqslant -0.5$

b) 0

25 a) $y = -\frac{5}{6}x + 2\frac{1}{2}$

b) $k = 20$

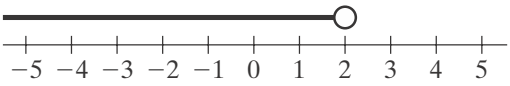
c) (i)



(ii) (1, 1)

26 a) (i) $x < 2$

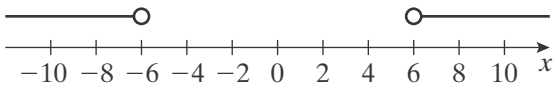
(ii)



b) $-2, -1, 0, 1$

27 a) $x > 6$ or $x < -6$

b)

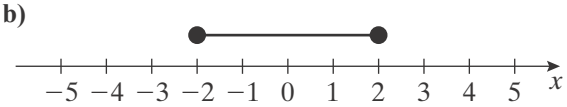


28 $x \leqslant 10$ or $x \leqslant -10$

29 $-3 < y < 3$

30 a) $-2 \leqslant x \leqslant 2$

b)



Internet Challenge 8 (page 142)

ϕ	The Golden Ratio.
$\sum$	The eighteenth letter of the Greek alphabet, denotes 'the sum of'.
$\div$	This 17th century symbol was formerly used in Europe to indicate subtraction.
∞	A sculpture of this symbol, by Marta Pan, stands on the A6 roadside in France.
i	The (not real) square root of minus one.
$<$	First used in Harriot's *Artis Analyticae Praxis* in 1631.
0	This originated from Hindu mathematics, where it was known as *sunya*.
$=$	Invented by Robert Recorde in 1557.
θ	The eighth letter of the Greek alphabet, used to denote an unknown angle.
$\surd$	This 16th century symbol may be a corrupted abbreviation for *radix*.

Chapter 9: Sequences and series

Starter 9 (page 143)

Task 1: Add 1 each time
Task 2: Add 1, then 2, then 3 (triangular numbers, starting at 0)
Task 3: Seems to double each time
Task 4: Pattern 5 has 5 points, 10 lines, 16 regions
Pattern 6 has 6 points, 15 lines 31 regions
The first two rules seem to work, the third does not.

Exercise 9.1 (page 145)

1 70, 80; add 10; $10n$
2 17, 19; add 2; $2n + 3$
3 63, 65; add 2; $2n + 49$
4 28, 32; add 4; $4n$
5 728, 2186; powers of 3 take 1; $3^n - 1$
6 0.000 01, 0.000 001; divide by 10 each time; $\frac{1}{10n}$
7 280, 360; triangular numbers $\times$ 10; $10 \times \frac{n(n+1)}{2} = 5n(n+1)$
8 98, 128; double square numbers; $2n^2$
9 **a)** 13
b) $2n - 1$
10 **a)** n^2
b) 900

Exercise 9.2 (page 147)

1 5, 7, 9, 11, 13
2 1, 4, 10, 22
3 **a)** 7, 15, 23, 31, 39 **b)** 159
4 **a)** 2, $3\frac{1}{2}$, 5, $6\frac{1}{2}$, 8, $9\frac{1}{2}$ **b)** 35
5 **a)** Start at 12, go up 3 each time **b)** 39
6 **a)** 99, 98, 97, 96, 95 **b)** 50
7 **a)** 3, 9, 27, 81, 243 **b)** Powers of 3
8 **a)** 10, 17, 24 **b)** 73
c) 150th term
9 **a)** 1, 3, 6, 10 **b)** 465
c) Either n or $n + 1$ is even
d) The triangular numbers
10 $6n + 7 = 2770$ gives $n = 460.5$ which is not a whole number, so must be wrong.

Exercise 9.3 (page 150)

1 **a)** 57 **b)** $5n + 7$
2 **a)** -6 **b)** $-8n + 66$
3 $3n + 5$
4 $5n - 3$
5 $-n + 11$
6 $5n - 1$
7 $3n + 18$
8 $-2n + 14$
9 **a)** 19 **b)** $3n + 1$
c) 3 because 3 sticks are added to form each new square.
1 because 1 stick is needed at the start.
10 **a)** 7 **b)** 5 **c)** $7n - 2$

Exercise 9.4 (page 153)

1 **a)** 670 **b)** 1010
c) -890 **d)** -470
2 **a)** $2n + 5$ **b)** 45
c) 2800
3 500 500
4 15 150
5 **a)** d = 4 **b)** 25
c) 1275
6 1090
7 15 350
8 **a)** $a = 97$, $d = -2$ **b)** -200
c) 2401

Review Exercise 9 (page 153)

1 66, 77, 88; $11n$ **2** 64, 128, 256
3 14, 17, 20; $3n - 1$ **4** 36, 49, 64
5 5, 4, 3; $-n + 11$ **6** 85, 79, 72
7 7, 6, 3, -6
8 **a)** 2, 5, 9, 14, 20 **b)** No
9 **a)** 26 **b)** 61
c) $5n + 1$
10 **a)** 15, 9 **b)** $4n - 3$
11 **a)** 21, 25 **b)** $4n - 3$
12 $5n + 1$
13 **a)** **i)** $4n + 5$ **ii)** 940
b) **i)** $9n - 5$ **ii)** 1790
c) **i)** $94 - 4n$ **ii)** 1040
d) **i)** $59 - 9n$ **ii)** -710
14 **a)** $4n + 11$
b) 91
c) 5650
15 6375
16 **a)** 4 **b)** 125 **c)** 31 375
17 $a = 60$ and $S100 = 3525$

Internet Challenge 9 (page 156)

3 The ratios get increasingly close to 1.6180
4 $1 - \phi$ and $\frac{1}{\phi}$ are equal.
5 Various parts of the Parthenon are rectangles with sides in the Golden Ratio.
6 Leonardo da Vinci
7 Seurat
8 Born 1170, died 1250
9 Yes, for example Binet's formula

$$\frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}}$$

10 Nautilus

Chapter 10: Travel and other graphs

Starter 10 (page 157)

1 235 **2** 2333
3 11.25 seconds **4** 600 metres

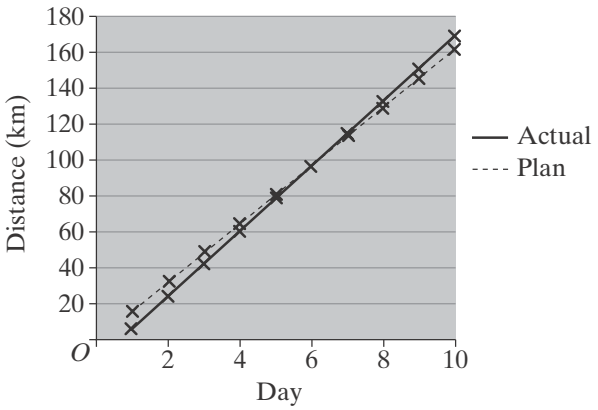
Exercise 10.1 (page 159)

1 a) 20 minutes
b) 12 km/h
c) No

2 a)

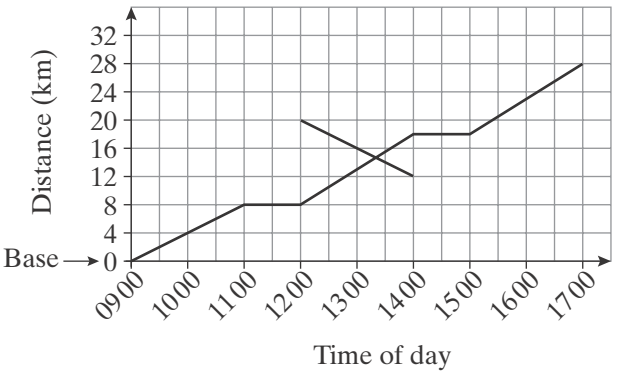
Day number n	3	4
Total distance travelled (D km)	42	60

b), c) Tom is back on schedule by end of Day 6.



d) $D = 18n - 12$
e) End of Day 14; original schedule end of Day 15.

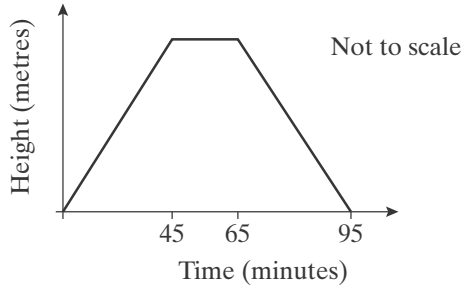
3 a), b)



c) About 1320

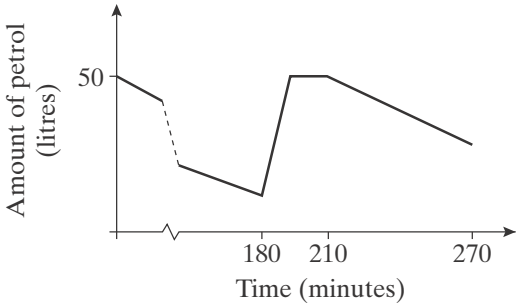
Exercise 10.2 (page 163)

1 a) 450 metres (above his start point)
b) 30 minutes
c)



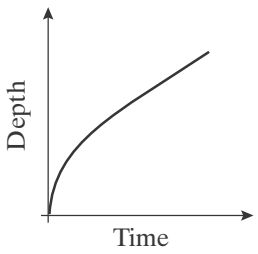
2 B. The initial rate at which the water level rises is fast, since the cross section is small. The rate of increase in depth decreases as the cross section of the bowl gets wider.

3 a) $50 - 3 \times 12 = 14$ litres
b) Not to scale:

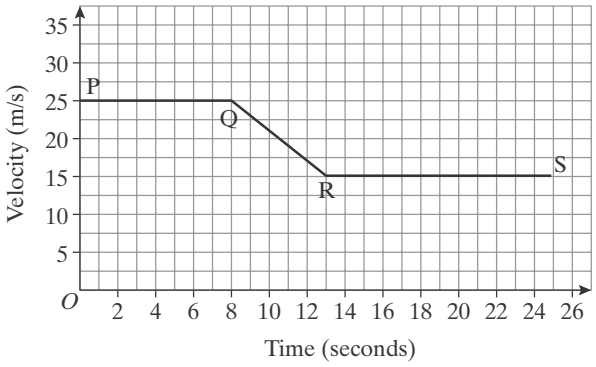


c) 380 litres

4



5

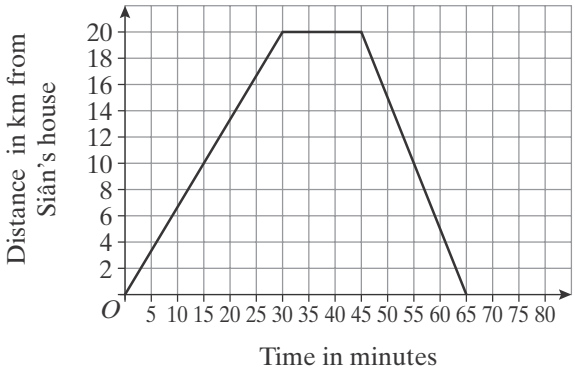


b) 200 m
c) 180 m

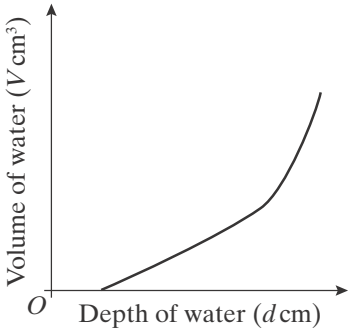
Review Exercise 10 (page 164)

1 a) 0905 **b)** 7 km
c) 10 minutes **d)** 21 km/h

2 a) 40 km/h
b)



3



4

Container	Graph
A	R
B	S
C	Q
D	P

5 **a)** 1305 (or 1306) **b)** 1000 and 1100
c) 50 km/h

6 **a)** 6 m/s **b)** 10.7 m/s **c)** 15 metres

Internet Challenge 10 (page 167)

The tea clipper Cutty Sark	32.19 km/h
Challenger 2 tank	59.55 km/h
Disney's Space Mountain roller coaster (Paris)	69.2 km/h
Intercity 225 train	225.31 km/h
Porsche 911 GT3 RS car	305.78 km/h
Boeing 747-400 passenger jet aircraft	1013.89 km/h
Speed of sound (in air)	1223.1 km/h
Eurofighter Typhoon jet aircraft	2124.33 km/h
Orbiting Space Shuttle	18 324.45 km/h
Apollo 11 spacecraft	39 428.93 km/h

Chapter 11: Working with shape and space

Starter 11 (page 168)

$a = 40°$, $b = 140°$, $c = 40°$, $d = 55°$, $e = 60°$, $f = 36°$, $g = 65°$, $h = 30°$, $i = 60°$, $j = 60°$, $k = 120°$, $l = 120°$, $m = 36°$, $n = 18°$, $o = 54°$, $p = 63°$, $q = 58°$, $r = 45°$, $s = 66°$, $t = 15°$, $u = 45°$, $v = 30°$, $w = 140°$, $x = 58°$, $y = 14°$, $z = 166°$

Exercise 11.1 (page 171)

1 $a = 68°$, $b = 68°$, $c = 68°$
2 $d = 131°$, $e = 49°$
3 $f = 61°$, $g = 61°$, $h = 119°$
4 $i = 71°$, $j = 65°$, $k = 44°$
5 $l = 13°$, $m = 13°$, $n = 77°$
6 $o = 52°$, $p = 52°$, $q = 90°$
7 $r = 55°$
8 $s = 132°$, $t = 48°$

Exercise 11.2 (page 176)

1 $a = 25°$
2 $x = 64°$
3 $x = 59°$, largest angle is 71°
4 $8y - 4 = 180$ leading to $y = 23$
Angles are then 23°, 69°, 88°
5 **a)** $3x + 90 = 180$ **b)** $x = 30°$
c) 38°, 52°, 90°
6 **a)** $16c + 4 = 180$ **b)** $c = 11°$
c) 48°, 48°, 84° **d)** Isosceles
7 85° **8** 88° **9** 106°
10 **a)** $10y + 50 = 360$
b) $y = 31°$ so angles are 82°, 98°, 103°, 77°
c) They are parallel
11 $3k + 3 = 180$ so $k = 59$ giving angles of 90°, 115°, 90°, 65°
12 **a)** $11x + 8 = 360$ **b)** $x = 32°$
c) 48°, 62°, 105°, 145°

Exercise 11.3 (page 180)

1 **a)** 1080° **b)** 3240°
2 142°
3 **a)** 60° **b)** 24°
4 **a)** 24 sides
b) 360 ÷ 14 is not a whole number
5 120°
6 $5x + 110 = 360$ leading to $x = 50$
The angles are 110°, 90°, 90°, 130°, 120°
7 135°
8 **a)** $2(a + b + c) = 720$ so $a + b + c = 360$
b) $c - 20 + c - 10 + c = 360$ leading to $c = 130$
c) 110°, 120°, 130°, 130°, 120°, 110°
d) No. A regular hexagon has all angles equal to 120°.
9 Check students' diagrams.
10 Check students' diagrams.

Exercise 11.4 (page 186)

1 Perimeter 24 cm, area 24 cm^2
2 Perimeter 23.9 cm, area 24.5 cm^2
3 Perimeter 30 cm, area 48 cm^2
4 Perimeter 40 cm, area 88 cm^2
5 Perimeter 42 cm, area 84.7 cm^2
6 Perimeter 28 cm, area 42 cm^2
7 Perimeter 56 cm, area 84 cm^2
8 Perimeter 11.8 cm, area 6.3 cm^2
9 Perimeter 26 cm, area 36 cm^2
10 Perimeter 34 cm, area 42 cm^2
11 Perimeter 34 cm, area 46 cm^2
12 Perimeter 32 cm, area 24 cm^2
13 Perimeter 148 mm, area 1208 mm^2
14 Perimeter 48 cm, area 136 cm^2
15 Trapezium
16 No − it could be a rhombus.
17 $x = 5$ cm
18 **a)** $x = 3$ **b)** 25 cm
19 **a)** Equilateral
b) $4x - 5 = 3x + 1$ leading to $x = 6$
c) 57 cm
20 **a)** $3x - 10 = x + 6$ leading to $x = 8$
b) $3y - 1 = 2y + 4$ leading to $y = 5$
c) 14, 14, 14, 14
d) Rhombus

Exercise 11.5 (page 191)

1 Surface area 600 cm^2, volume 1000 cm^3
2 **a)** 792 cm^2 **b)** 1440 cm^3
3 **a)** 30 cm^2 **b)** 180 cm^3 **c)** 240 cm^2
4 **a)** 440 cm^2 **b)** 35 200 cm^3
5 **a)** 1728 cm^3 **b)** 864 cm^2
6 **a)** 9000 cm^3 **b)** 2700 cm^2
7 **a)** 8.71 m^3 **b)** 20.68 m^2
8 **a)** 450 m^3 **b)** 450 000 litres
9 **a)** 22 cm along each edge
b) 2904 cm^2
10 **a)** 5 cm by 7 cm by 13 cm
b) 382 cm^2

Exercise 11.6 (page 194)

1 2 000 000 cm^3 **2** 0.5 m^2
3 3 km^3 **4** 6.6 cm^2
5 1 000 000 000 mm^3 **6** 0.035 m^3
7 24 000 cm^2
8 **a)** 20 000 cm^3 **b)** 0.02 m^3
9 **a)** 4.2 m^3 **b)** 4 200 000 cm^3
c) 4200 litres
10 **a)** 512 000 cm^3 **b)** 0.512 m^3
c) 0.8 m on each side

Review Exercise 11 (page 194)

1 $a = 52°$, $b = 38°$
2 **a)** $10y + 60 = 360$ leading to $y = 30$
b) 110°, 70°, 110°, 70°
c) Parallelogram
3 **a)** 54° **b)** 72°; isosceles triangle
4 30°, 150°
5 13 sides
6 90 sides
7 **a)** No **b)** Rectangle
8 **a)** $x = 4$ **b)** 34 cm **c)** 70 cm^2
9 **a)** 35 cm^2 **b)** 2 500 000 cm^3
10 **a)** 31° **b)** 135°
11 20 cm

12 1 cm by 28 cm, 2 cm by 14 cm, 4 cm by 7 cm
13 **a) i)** 109°
ii) Angles on a line add up to 180°
b) i) 24°
ii) Alternate to angle QSR
14 **a)** 700 cm^3 **b)** 13.51 kg
15 **a)** $e = 42°$ **b)** $f = 69°$
16 30°
17 **a)** $x = 60°$ **b)** $y = 120°$
18 $x^3 - 8x$
19 8 m^2
20 **a)** 20 000 cm^3 **b)** 4 minutes
21 **a)** $6x + 8$ **b)** $x = 7$
22 140°
23 **a)** $11 < 2x + 6 < 20$ so $5 < 2x < 14$
b) 3, 4, 5, 6
24 **a)** 16 cm **b)** 2600 cm^3
25 102 cm^2 **26** 142°
27 **a)** 60° **b)** 120° **c)** 12 cm^2
28 62.96 $m^2 \rightarrow$ 25.2 litres of paint $\rightarrow$ £75.30
(or 26 × £2.99 = £77.74)
29 **a)** $4x + 8$ **b)** 15.5 cm
30 24 cm

Internet Challenge 11 (page 203)

1 Francis Guthrie
2 Alfred Kempe
3 Peter Tait
4 1976
5 Kenneth Appel and Wolfgang Haken
6 It was computer-assisted.
7 Four colours (as before)
8 Seven colours
9 Gardner claimed to have found a map that required five colours.
10 A real map-maker might want to colour two separate regions (for example, Alaska and USA) in the same colour to indicate political association. This adds extra restrictions and thus might require extra colours.

Chapter 12: Circles, cylinders, cones and spheres

Starter 12 (page 204)

Calculator results for the approximations are:

$3 + \frac{1}{8} = 3.125$

$\frac{22}{7} = 3.142\,857\,143$

$\sqrt{10} = 3.162\,277\,66$

$3 + \frac{8}{60} + \frac{30}{60^2} = 3.141\,666\,667$

$\frac{333}{106} = 3.141\,509\,434$

$\left(\frac{2143}{22}\right)^{\frac{1}{4}} = 3.141\,592\,653$ is the closest

$\frac{88}{\sqrt{785}} = 3.140\,854\,685$

$\sqrt{2} + \sqrt{3} = 3.146\,264\,37$

$\frac{355}{113} = 3.141\,592\,92$

$\left(\frac{4}{3}\right)^4 = 3.160\,493\,827$

$\pi = 3.141\,592\,654$

Exercise 12.1 (page 207)

1 75.4 cm **2** 69.1 cm
3 1020 cm^2 **4** 104 cm^2
5 133.5 cm **6** 3447 cm^2
7 3.927 cm **8** 0.6504 cm^2
9 95.0 cm^2 **10** 2.27 mm^2
11 785 cm **12** 6.66 m
13 210 mm **14** 425 mm^2
15 **a)** 157 m **b)** 32 laps
16 111 cm^2
17 **a)** 12.6 cm^2 **b)** 64.9 cm^2
18 **a)** 356 cm **b)** 1730 cm^2 **c)** 1100 cm^2
19 **a)** 359.4 m **b)** 334.2 m **c)** 7.5% longer
20 **a)** 113 mm^2 **b)** 28 mm^2 **c)** 85 mm^2
d) 25%

Exercise 12.2 (page 211)

1 15.4 cm 10.1 cm^2
2 31.0 cm 58.7 cm^2
3 30.7 cm 58.9 cm^2
4 27.8 cm 46.3 cm^2
5 95.7 mm 169 mm^2
6 35.3 cm 66.0 cm^2
7 12.4 cm 9.27 cm^2
8 44.3 cm 103 cm^2
9 **a)** 140 cm^2, 122 cm^2, 52 cm^2
b) 30.5 cm
10 **a)** 72° **b)** 22.6 in^2

Exercise 12.3 (page 212)

1 2.47 cm **2** 4.07 cm
3 6.18 cm **4** 7.48 cm
5 0.231 cm **6** 32.9 cm
7 1.99 cm **8** 8.46 cm
9 **a)** 3848.4 cm^2 **b)** 35 cm
c) 94.4 cm
10 4.77 m

Exercise 12.4 (page 215)

1 8600 cm^3
2 62.8 cm^2
3 **a)** 3040 cm^3 **b)** 553 cm^2
4 509 cm^3
5 1810 cm^2
6 **a)** 2 120 000 cm^3 **b)** 2120 litres
7 3563 cm^3
8 14 cm
9 **a)** 73.6 cm^3 **b)** $73.6 < 75$ so not possible.
10 **a)** Nick is right.
b) Alan used 14 cm as radius, not diameter.

Exercise 12.5 (page 217)

1 Circumference $= 24\pi$ cm, area $= 144\pi$ cm^2
2 Circumference $= 22\pi$ cm, area $= 121\pi$ cm^2
3 **a)** 192π cm^2 **b)** 1152π cm^3
4 **a)** 12 cm **b)** 144π cm^2
5 **a)** 11 cm **b)** 22π cm
6 3 cm
7 30 cm
8 **a)** 64π cm^2 **b)** $32 + 8\pi$ cm
9 **a)** 18π cm^2 **b)** $144 + 72\pi$ cm^2
10 **a)** $\pi \times 6 \times 8 = 48\pi$ and $\pi \times 8 \times 6 = 48\pi$, so the same.
b) $\pi \times 3^2 \times 8 = 72\pi$ and $2\pi \times 4^2 \times 6 = 96\pi$, so B has the larger volume.

Exercise 12.6 (page 222)

1 3050 cm^3, 1020 cm^2
2 172 cm^3, 188 cm^2
3 286 cm^3, 267 cm^2
4 2304π cm^3, 576π cm^2
5 144π cm^3, 108π cm^2
6 12π cm^3, 24π cm^2
7 ($r = 15$) 4500π cm^3
8 ($r = 4$) 16π cm^2

Review Exercise 12 (page 223)

1 2460 cm^2 **2** 283 cm
3 11.94 cm **4** 3217 mm^2
5 **a)** 8 cm **b)** 16π cm
6 **a)** 942 cm^2 **b)** 314 cm^2
c) 1570 cm^2
7 **a)** 2.5 cm **b)** 19.6 cm^2
8 72.7 cm^2 **9** 81.7 m^2
10 **a)** 28.3 cm^2 **b)** 23.1 cm
11 88.4 cm^2 **12** 201 cm
13 218 cm^2 **14** 754 cm^3
15 7.7 cm **16** 58.8 cm
17 $18 + 9\pi$ cm
18 **a)** 12.6 cm **b)** 240 cm^2
19 137 000 cm^3
20 8.9 cm
21 $18\pi + 15\pi = 33\pi$
22 15π

Internet Challenge 12 (page 228)

1 The Earth's shadow on the Moon (during a lunar eclipse) is round.
2 The Flat Earth Society.
3 Diameter 12 756 km (7926 miles), circumference 40 074 km (24 900 miles)

4 Diameter 12 714 km (7900 miles), circumference 39 942 km (24 818 miles)
5 A Great Circle is a circle on the surface of a sphere, whose centre coincides with the centre of the sphere. The equator is a Great Circle.
6 Ferdinand Magellan, from August 1519 to September 1522, taking 3 years. (Magellan died during the voyage; the expedition was commanded by Juan Sebastian del Cano thereafter.)
7 Sir Ranulph Fiennes and Charlie Burton, from 1979 to 1982.
8 Round the world yacht races typically take over 50 000 km (over 32 000 miles). They do not complete a Great Circle, but they travel a greater equivalent distance, and cross every line of longitude.
9 Greek *geo* = Earth, *metron* = measure
10 Check students' answers.

Chapter 13: Geometric constructions

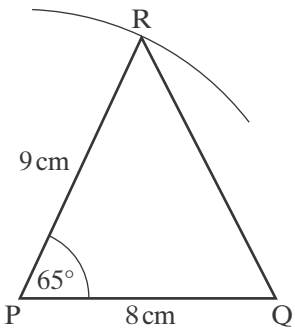
Starter 13 (page 229)

Check students' diagrams.

Exercise 13.1 (page 236)

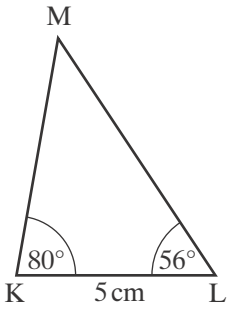
Diagrams are shown to scale but not full size.

1



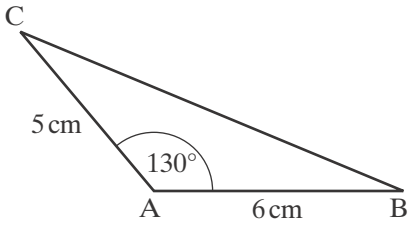
SAS

2



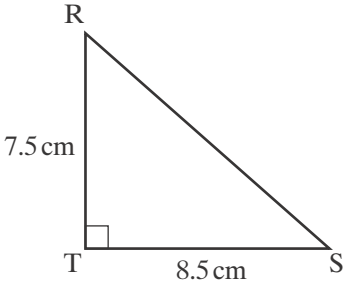
ASA

3



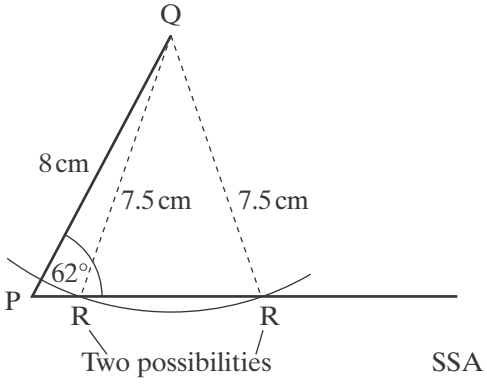
SAS

4



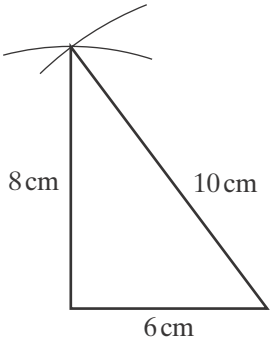
SSA

5



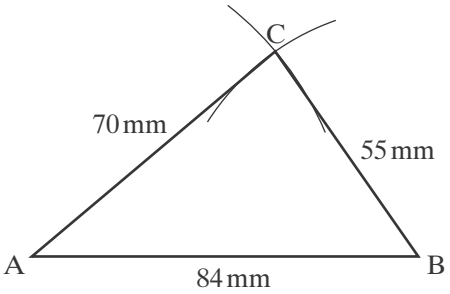
SSA

6

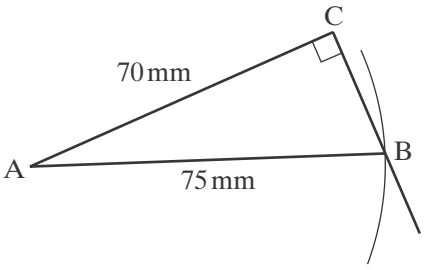


SSS

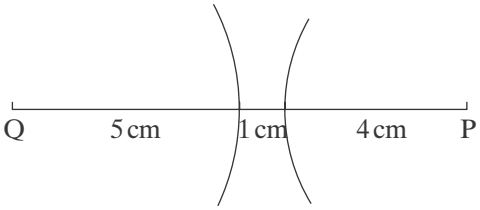
7



8

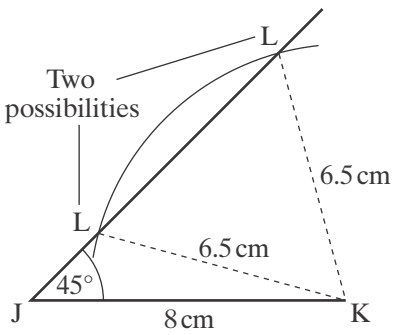


9



The arcs do not intersect because the sum of the shorter sides is less than the longest side.

10 a)

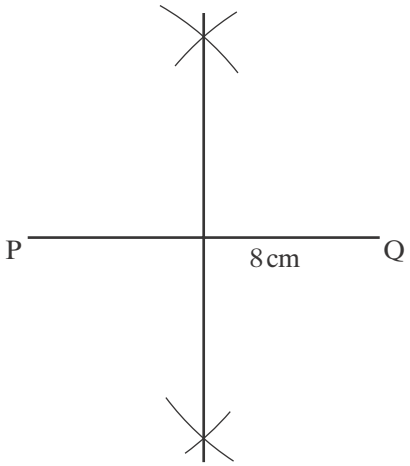


b) There are two possibilities.

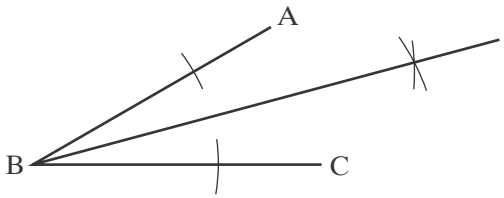
Exercise 13.2 (page 239)

Diagrams are shown to scale but not full size.

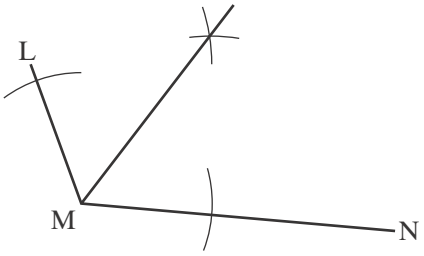
1



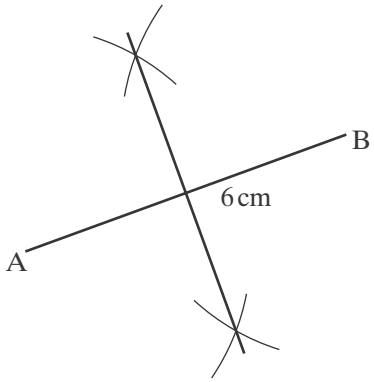
2



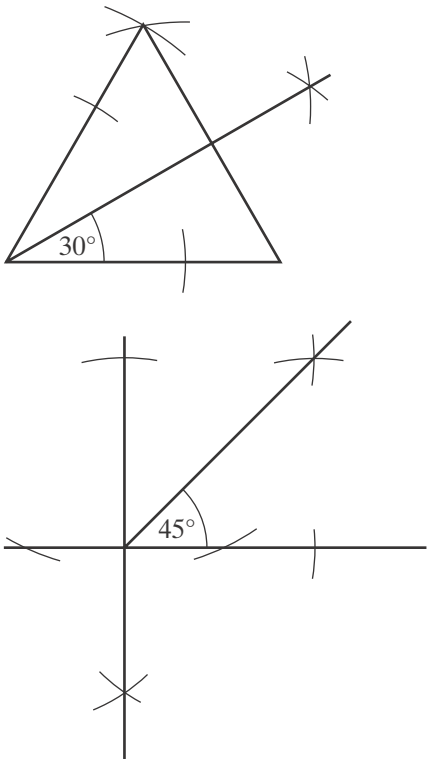
3



4



5



Exercise 13.3 (page 242)

1 **a)** Check students' diagrams.

b) Check students' diagrams.

c) 258°

2 **a)** 067°

b) 247°

3 **a)** 106°

b) 286°

4 **a)** 225°

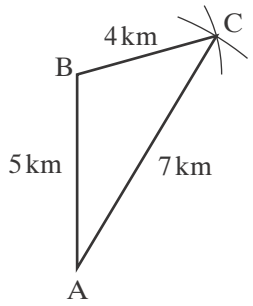
b) 045°

5 033°

Review Exercise 13 (page 243)

Diagrams are shown to scale but not full size.

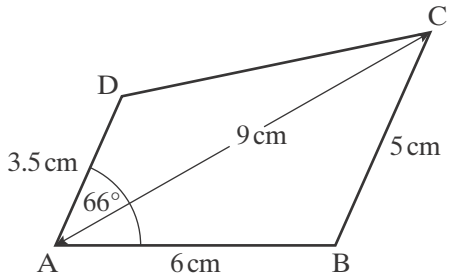
1 **a)**



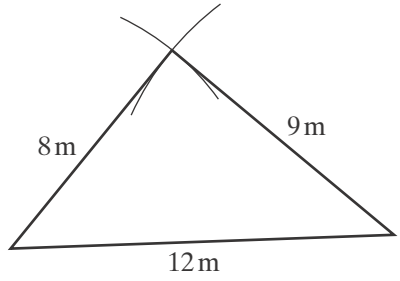
b) 34°

c) 214°

2

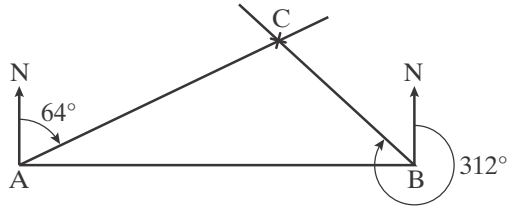


3

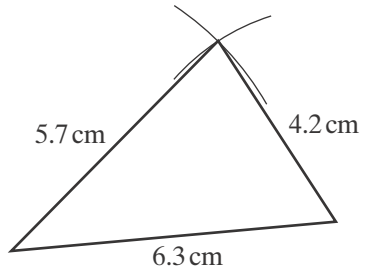


4 a) 3 km

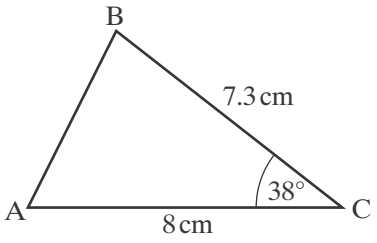
b)



5



6 a)



b) 64°

7 081°

Internet Challenge 13 (page 246)

1 There are five; we will never find more (Euclid proved there are only five).

2 The Greek mathematician Plato wrote extensively about them.

3 Check students' answers.

4 a) Check students' answers.

b) Such a football is not a Platonic solid, since it uses two different polygons for the faces.

5

Solid	Faces	Vertices	Edges
Tetrahedron	4	4	6
Cube	6	8	12
Octahedron	8	6	12
Dodecahedron	12	20	30
Icosahedron	20	12	30

Euler's formula: Faces + Vertices − Edges = 2

No; only true for convex polyhedra.

6, 7 Check students' answers

8 59

9 Double Planet = stellated octahedron,
Gravity = stellated dodecahedron

10 Check students' answers.

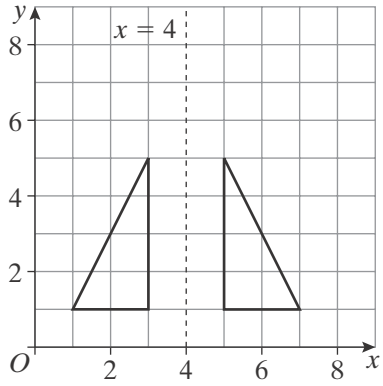
Chapter 14: Transformation and similarity

Starter 14 (page 247)

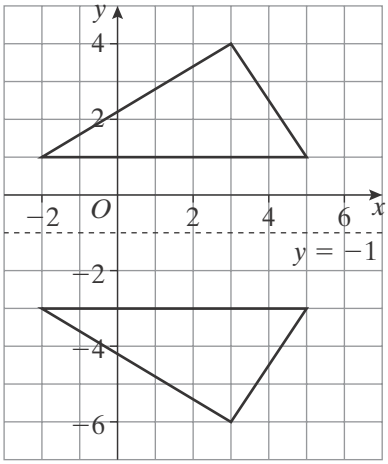
Seven of the nine monkeys are "the same", i.e. congruent; one other is "too long" and one is "too thin". Of the seven congruent monkeys, six are rotationally equivalent and one is flipped – or, if you like, there are six right-handed and one left-handed version of the same picture.

Exercise 14.1 (page 249)

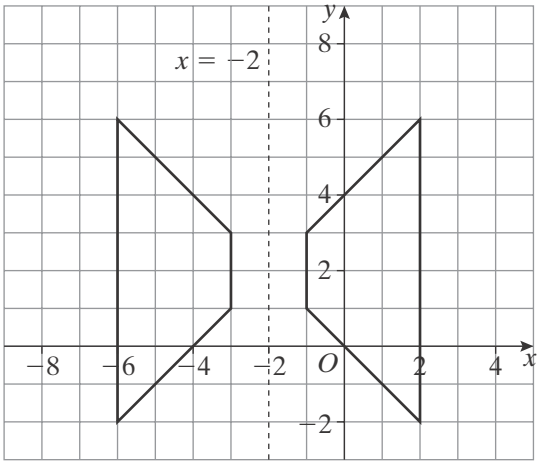
1



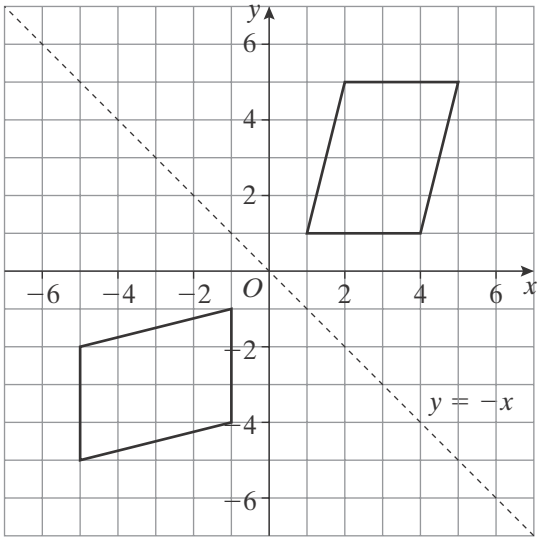
2



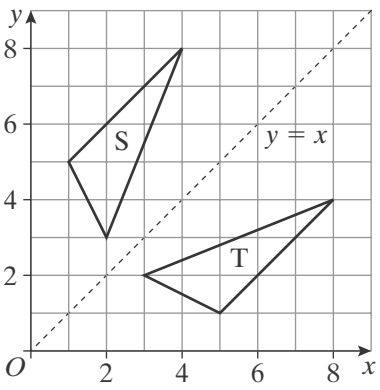
3



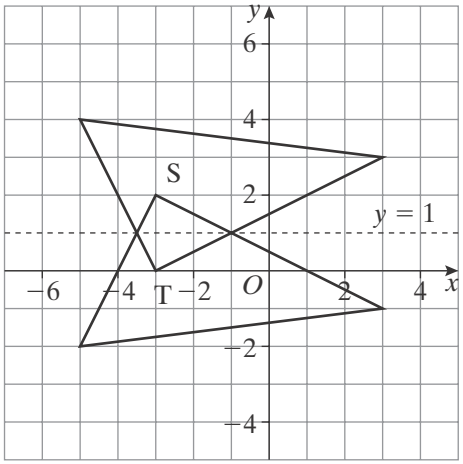
4



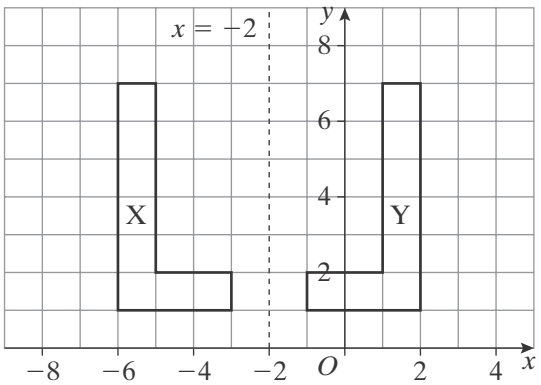
5



6



7

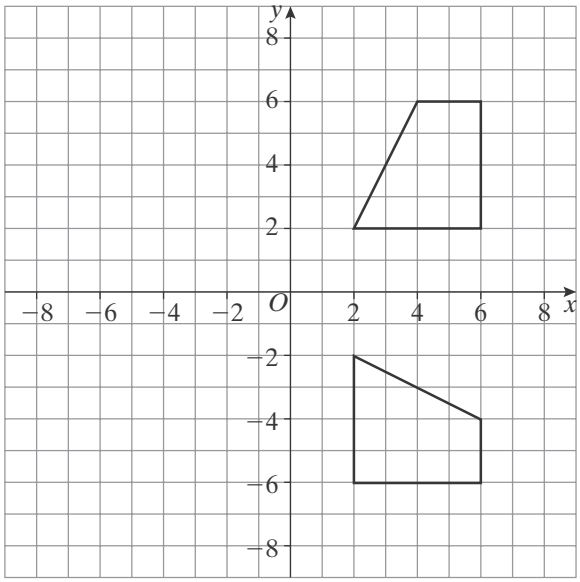


8 a) Two shapes are congruent if they are the same shape and size.
b) $y = -1$
c) B
d) $y = x$
e) $y = -x$

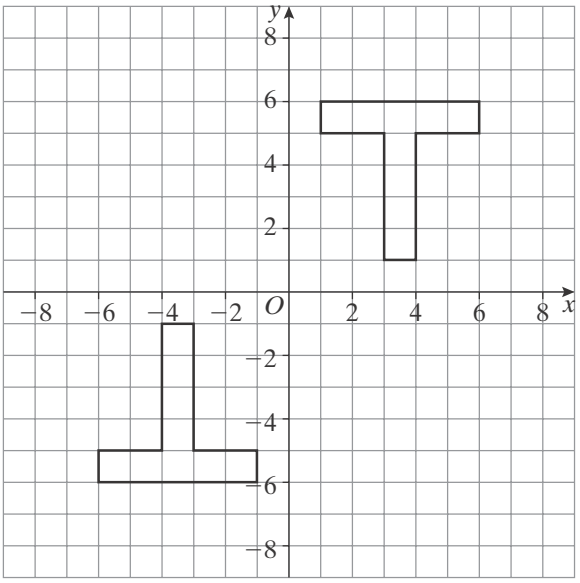
9 T and V are coincident (they are, in effect, the same triangle)

Exercise 14.2 (page 255)

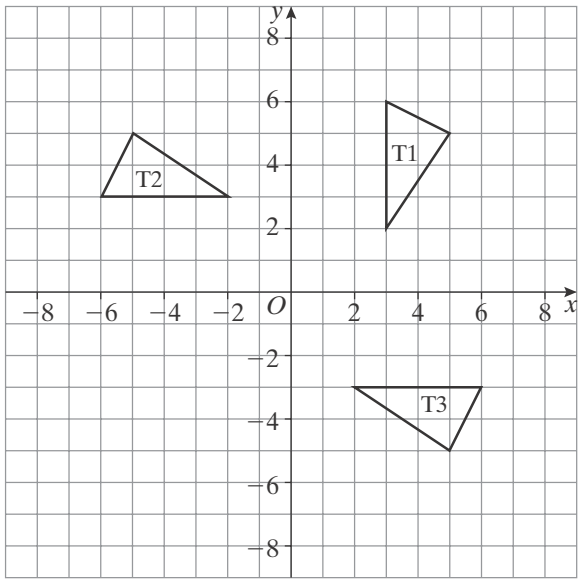
1



2

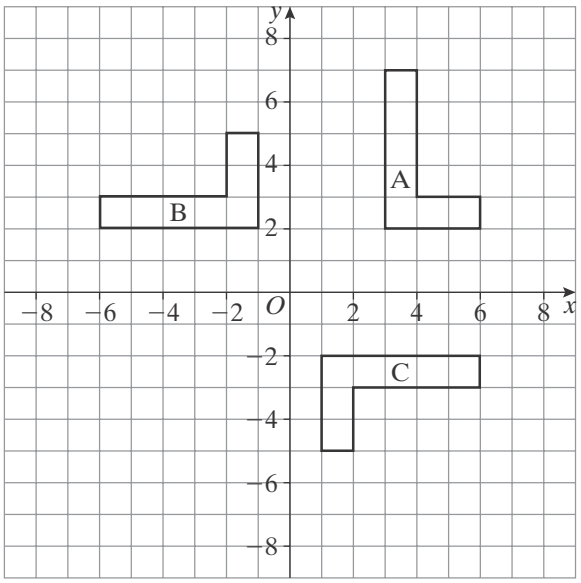


3 a) b)



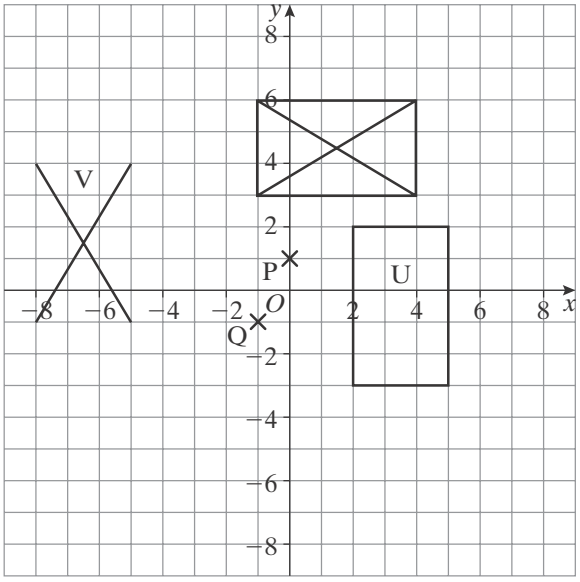
c) T1 → T3: a rotation of 90° clockwise about O.

4 a) b)

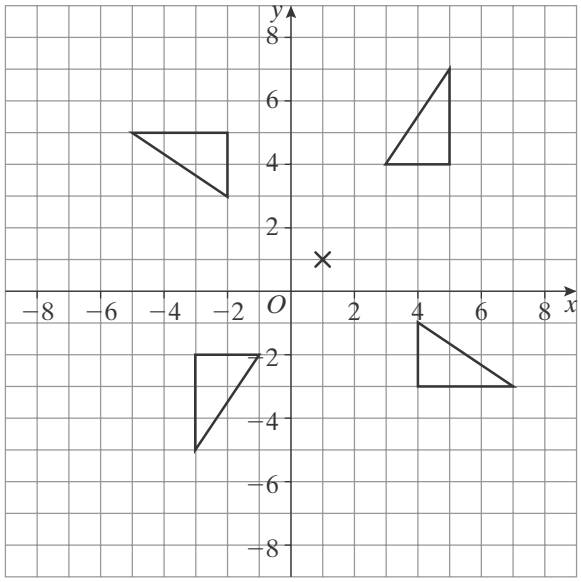


c) C → A: a rotation of 90° clockwise about (0, 1).

5 a) b)



6 a) b)

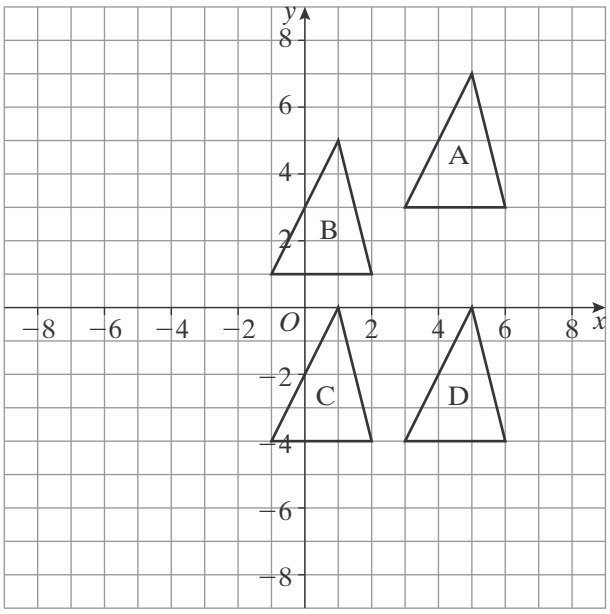


7 a) 90° anticlockwise. **b)** (2, 0)

8 Anita is wrong (for example, 90° anticlockwise and then 90° clockwise using two different centres is equivalent to a translation). Bella is (very) wrong. Therefore Cat is wrong too!

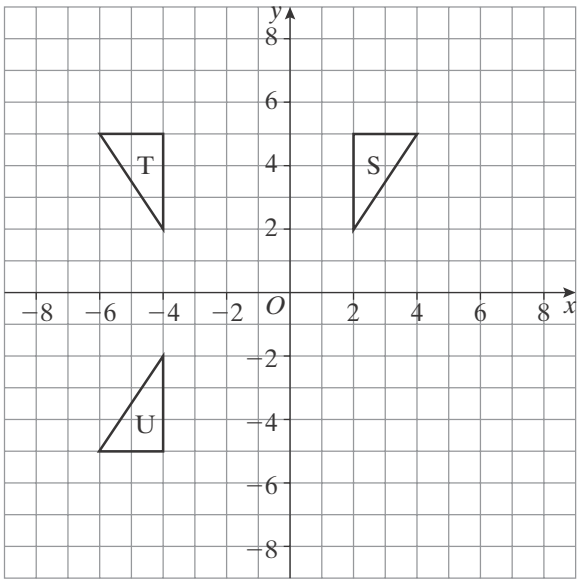
Exercise 14.3 (page 259)

1 a), b), c)



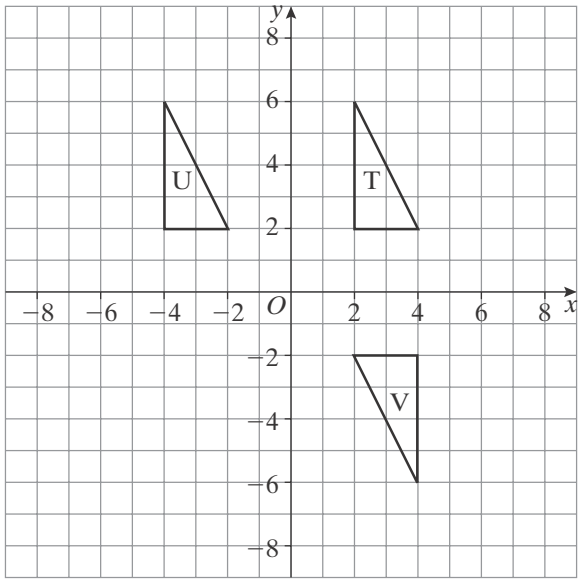
d) $\begin{pmatrix} 0 \\ 7 \end{pmatrix}$

2 a), b)



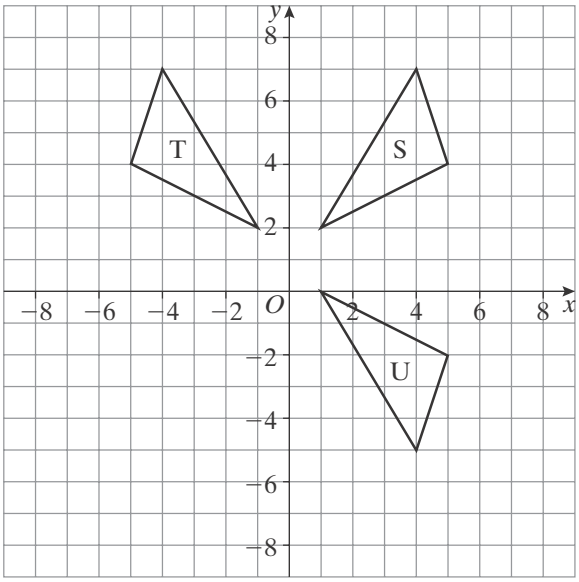
c) Rotation of 180° about (−1, 0)

3 a), b)



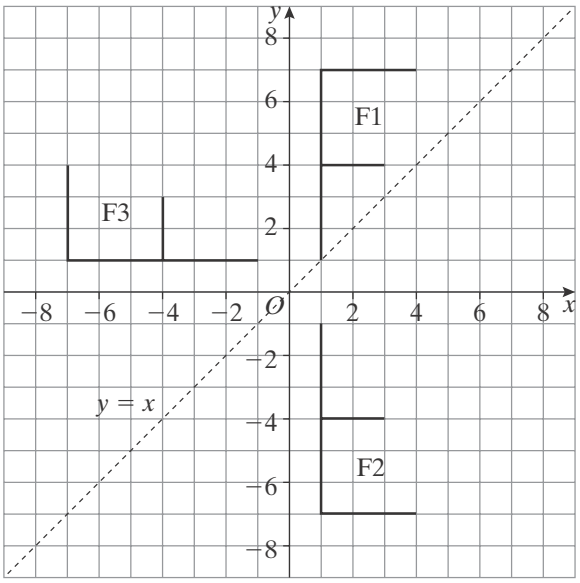
c) Rotation of 180° about (3, 0)

4 a), b)



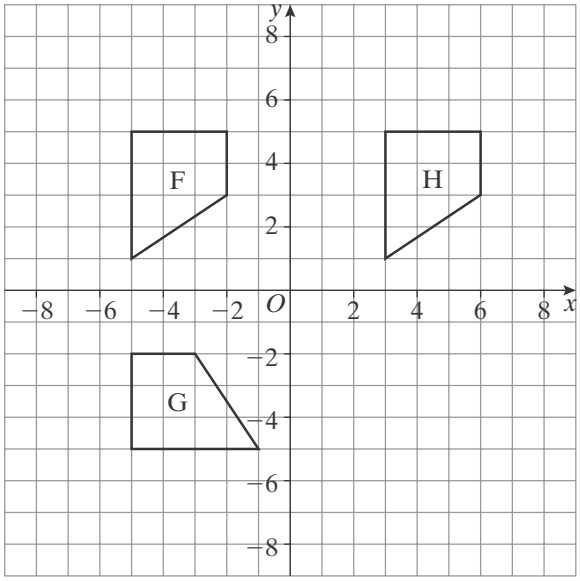
c) Rotation 180° about (0, 1)

5 a), b)



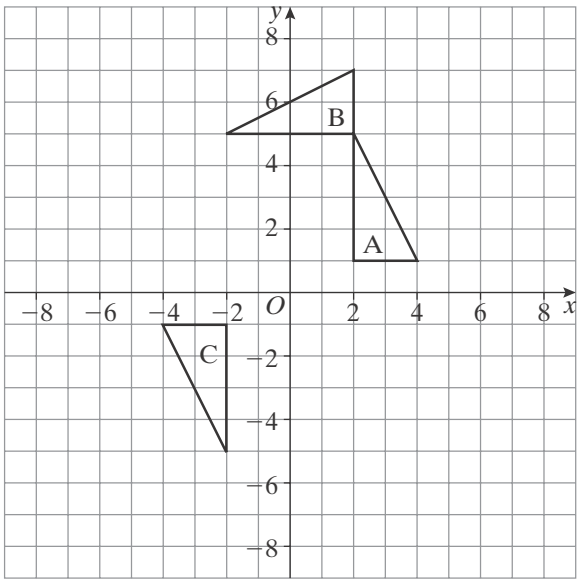
c) Rotation of 90° about (0, 0)

6 a), b)



c) Translation by $\begin{pmatrix} 8 \\ 0 \end{pmatrix}$

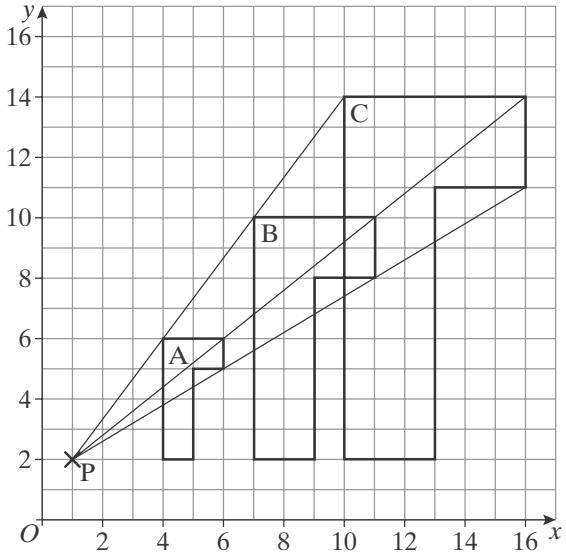
7 a), b)



c) Rotation of 90° about (3, 0)

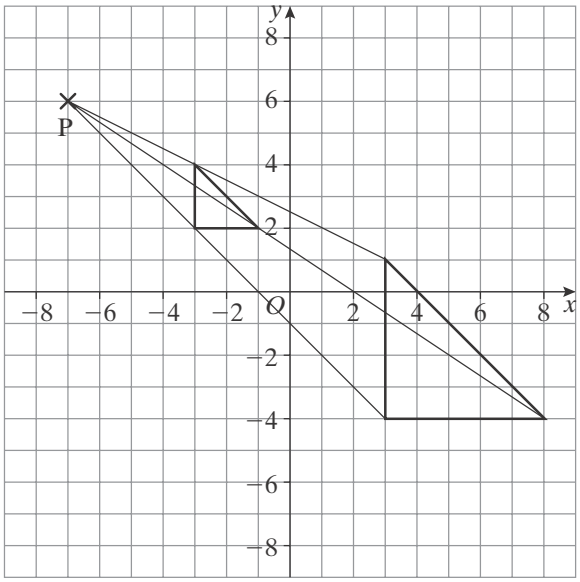
Exercise 14.4 (page 264)

1 a), b), c)

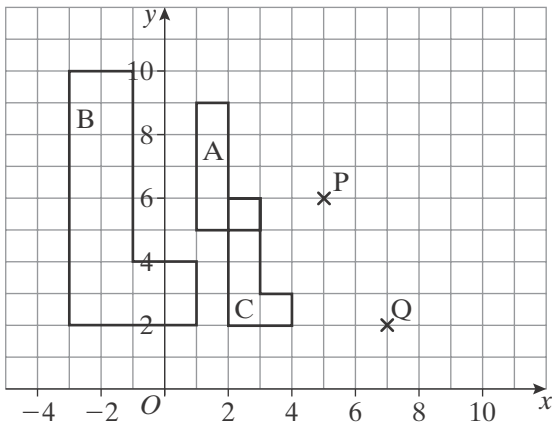


d) They are similar but not congruent.

2



3 a), b)



c) A and B are similar but not congruent (same shape, different sizes).

d) A and C are congruent (same shape and size).

4 a) 3

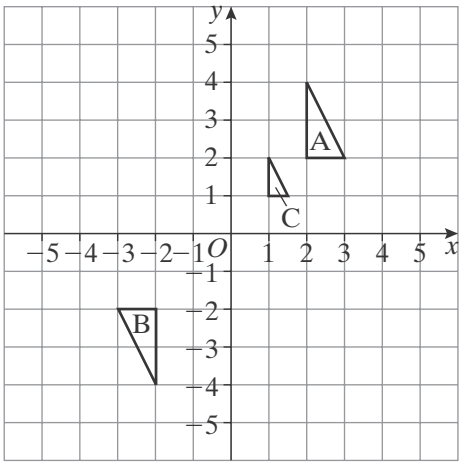
b) (2, 5)

Exercise 14.5 (page 267)

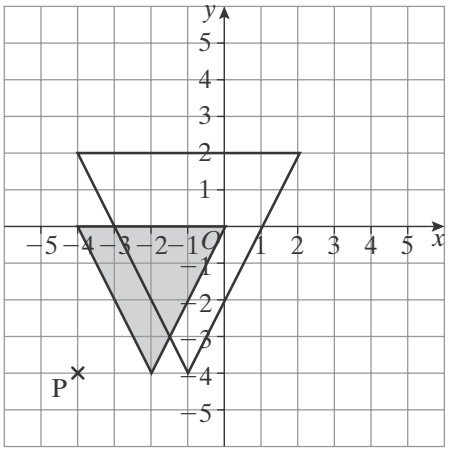
1 **a)** 17.6 cm **b)** 1 : 1.6 **c)** 1 : 2.56
2 54 cm^2 **3** 8.64 kg
4 **a)** RT = 11.25 cm, RQ = 20.25 cm **b)** 10.4 cm
5 **a)** 185 m **b)** 10 350 tonnes (4 s.f.)
6 x = 12 cm, y = 12 cm **7** 13.0 kg
8 **a)** Angle RPQ = angle RST (corresponding angles), and angle RQP = angle RTS (same reason), so each angle in one triangle is equal to the corresponding angle in the other one.
b) 12 cm **c)** 15 cm
9 **a)** 1.64 m **b)** 14.8 m^2
10 15.1 cm (3 s.f.)

Review Exercise 14 (page 270)

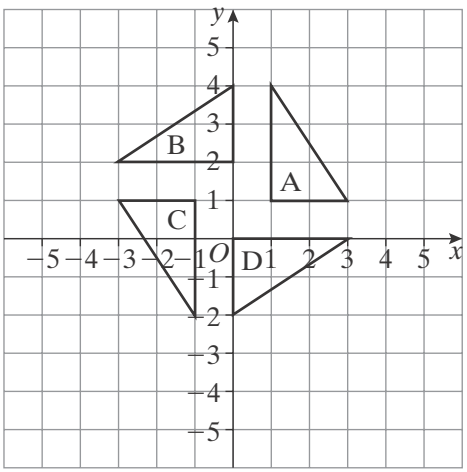
1 270 cm^3
2 **a) b)**



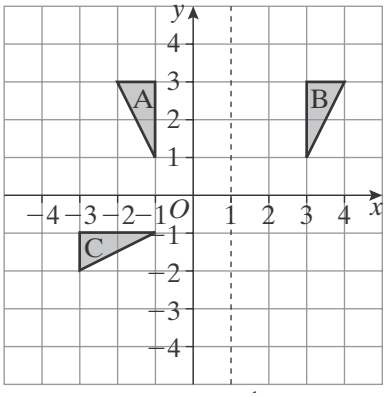
3 6
4 10 248 cm^2
5



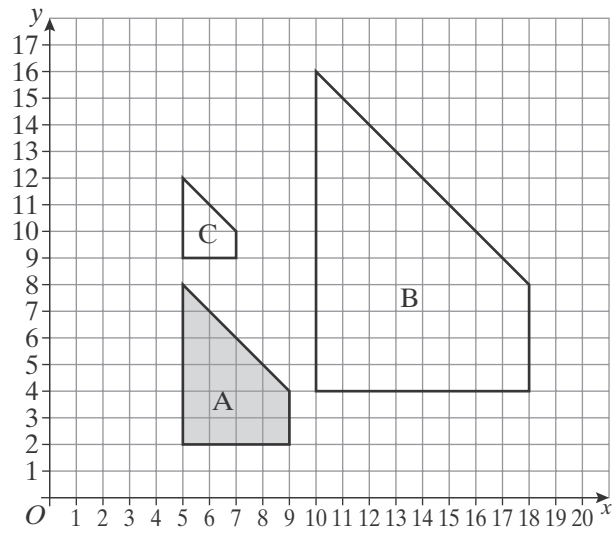
6 **a)**



b) Rotation of 180° about (0, 1).
7 **a)** **i)**



ii) x = 1
b) Rotation of 90° anticlockwise about (0, 0)
8 **a)** Reflection in the y axis
b) Rotation of 90° clockwise about (0, 0)
9 8.25 cm
10 **a)** 2
b), c)



11 23.4 buckets of sand to 3 s.f.
12 **a)** 13.5 cm **b)** 2.5 cm
13 317 mm
14 **a)** 12.5 cm **b)** 7.2 cm

Internet Challenge 14 (page 275)

1 Icosahedron **2** Cylinder
3 Congruent **4** Similar
5 Alternate **6** Parallel
7 Torus **8** Octagon
9 Hemisphere **10** Apex
11 Tetrahedron **12** Rhombus
13 Minute of arc **14** Radian
15 Truncated cone = frustum

Chapter 15: Pythagoras' theorem

Starter 15 (page 276)

Task 1: 16, 49, 6.25, 1.44, 0.64, 169, 36, 256
Task 2: 3.61, 3.16, 4, 4.74, 2.5, 7, 11.0, 11
Task 3: 3: 2.83, 3, 3.16, 3.32, 3.46, 3.61, 3.74, 3.87, 4, 4.12

Some whole numbers like 9 and 16 are the squares of whole numbers, and thus have exact whole number square roots.

Exercise 15.1 (page 278)

1 No **2** Yes
3 No **4** No
5 Yes, C **6** No
7 No **8** Yes, Q
9 No **10** No

Exercise 15.2 (page 279)

1 5.39 cm **2** 2.6 km
3 5.66 cm **4** 6.71 cm
5 4.22 mm **6** 7.91 cm
7 2.77 m **8** 4.72 cm
9 1 km **10** 7.81 cm
11 2 km **12** 8.94 mm

Exercise 15.3 (page 281)

1 5.29 cm **2** 4 km
3 5.03 m **4** 8.06 cm
5 24 mm **6** 4.45 cm
7 6.63 cm **8** 11.5 cm
9 2.24 cm **10** 7 cm
11 10.6 km **12** 14.0 cm
13 14.6 cm **14** 12.0 cm
15 11.8 cm

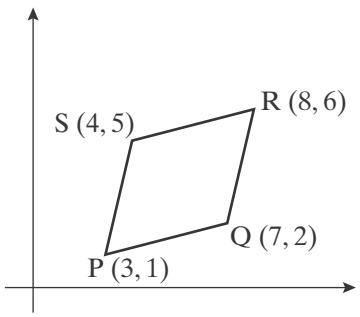
Exercise 15.4 (page 283)

1 a) 11.4 cm **b)** 12.1 cm
2 a) 9.43 cm **b)** 9.90 cm
3 b) 16.1 cm
4 Diagonal is 26.4 cm so the rod does fit.
5 a) 20.6 cm **b)** 21.0 cm
6 4.5 cm
7 13 cm

Exercise 15.5 (page 286)

1 a) 5
b) 17
c) 80 (approx 8.94)
2 a) AB = $\sqrt{37}$ BC = 5, CA = $\sqrt{34}$
a) Dee is wrong.

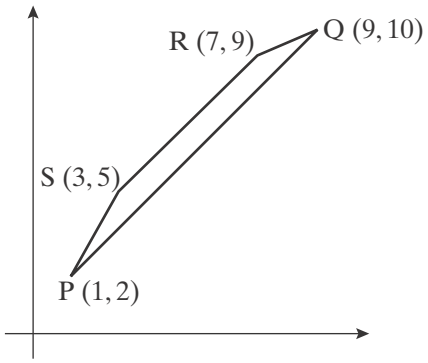
3 a)



b) PQ = QR = RS = SP = $\sqrt{17}$
c) PQRS is a rhombus

Review Exercise 15 (page 286)

1 3.91 cm **2** 9.93 cm
3 10.1 km **4** 56.0 cm
5 19.4 mm **6** 11.2 cm
7 a) 10 cm **b)** 6.6 cm
8 a) 5.15 cm, 7.91 cm, 6.96 cm
b) 8.29 cm
9 3.62 cm
10 a) 86.64 cm^2 **b)** 45.6 cm
11 13.7 cm
12 a) 21.25 cm^2 **b)** 9.86 cm
13 10
14 a)



b) PQ = $\sqrt{128} = 8\sqrt{2}$ RS = $\sqrt{32} = 4\sqrt{2}$
c) 2 : 1
d) QR = $\sqrt{5}$, PS = $\sqrt{13}$
e) The trapezium is not isosceles, since QR and PS are not the same length.
15 9.11 cm

Internet Challenge 15 (page 290)

1 $c = 17$
2 They are multiples of 3, 4, 5 and 5, 12 and 13.
3 3, 4, 5; 5, 12, 13; 6, 8, 10; 7, 24, 25; 8, 15, 17; 9, 12, 15; 12, 16, 20; 15, 20, 25
4 $n^2 - m^2$, $2mn$, $n^2 + m^2$ generates all the irreducible triples, and most of the others.
5 Yes
6 Yes, for example $3^2 + 4^2 + 12^2 = 13^2$
7 Fermat's Last Theorem was proved in 1994/1995 by Wiles (and Taylor).

Chapter 16: Introducing trigonometry

Starter 16 (page 291)

None of the triangles are similar.

The hypotenuse of the fifth triangle is exactly 3 units; this is because their lengths form a progression of $\sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, \sqrt{9}$.

The angles at the centre of the spiral become progressively smaller.

Exercise 16.1 (page 294)

1 4.23 cm **2** 3.66 cm **3** 7.78 cm
4 7.80 cm **5** 6.10 cm **6** 7.77 cm
7 5.28 cm **8** 28.9 mm **9** 58.9 mm
10 9.54 cm **11** 7.05 cm **12** 4.27 cm
13 35.2 mm **14** 86.4 mm **15** 42.4 mm

Exercise 16.2 (page 297)

1 9.27 cm **2** 2.5 cm **3** 3.05 cm
4 15.9 cm **5** 10.1 cm **6** 7.89 cm
7 2.64 cm **8** 14.8 mm **9** 53.6 mm
10 13.4 cm **11** 8.02 cm **12** 4.91 cm
13 108 mm **14** 60.4 mm **15** 57.2 cm

Exercise 16.3 (page 300)

1 7.50 cm **2** 8.58 cm **3** 6.64 cm
4 6.29 cm **5** 25.3 cm **6** 11.8 cm
7 9.45 cm **8** 159 mm **9** 55.4 mm
10 2.27 cm **11** 19.6 cm **12** 73.6 cm

Exercise 16.4 (page 302)

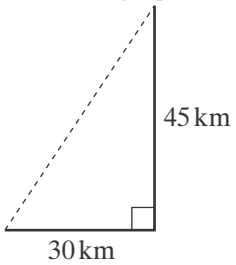
1 12.5 cm **2** 5.91 cm **3** 3.20 cm
4 2.90 cm **5** 7.93 cm **6** 8.30 cm
7 6.60 cm **8** 3.32 cm **9** 6.59 cm
10 8.29 cm

Exercise 16.5 (page 304)

1 36.9° **2** 41.8° **3** 17.5°
4 72.5° **5** 30° **6** 49.4°
7 47.1° **8** 47.8° **9** 41.3°
10 40.9° **11** 54.5° **12** 47.2°

Exercise 16.6 (page 307)

1 a)

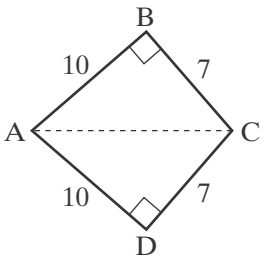


b) 54.1 km
c) 214°

2 a) 12.4 cm
b) 80.9 cm^2
c) 38.9°

3 a) Angle PMQ = angle RMQ, so must be $180 \div 2 = 90°$
b) 65.1°
c) 1.72 m (172 cm)

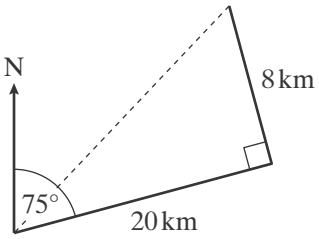
4 a)



b) AC = 12.2 cm (122 mm)
c) 70°

5 a) 9.44 cm
b) 59.1°

6 a)



b) 21.5 km
c) 075 − 022 = 053°

Exercise 16.7 (page 310)

1 54.5 m
2 a) 11.0°
b) 11.0°
3 51.6 m
4 a) 2500 m
b) 36.9°
5 a) 30 m
b) 45.6°

Review Exercise 16 (page 311)

1 $a = 6.74$ cm, $b = 9.18$ cm, $c = 6.10$ cm, $d = 11.0$ cm, $e = 5.54$ cm, $f = 5.91$ cm
2 $a = 44.3°$, $b = 54.3°$, $c = 33.5°$, $d = 53.1°$, $e = 49.9°$, $f = 22.6°$
3 35.1 m
4 27.7°
5 4.28 m
6 a) 5.29 cm **b)** 41.4° **c)** 12.4 cm
7 a) 20.6° **b)** 110.6°
8 50.2°
9 8.79 m
10 116 cm
11 a) 62.0° **b)** 19.1 m
12 a) 11.7 m **b)** 36.9°

Internet Challenge 16 (page 317)

1 Euclid **2** Pythagoras
3 Benoit Mandelbrot **4** Apollonius of Perga
5 Johann Kepler **6** Sir Isaac Newton
7 Leonhard Euler **8** Bernhard Riemann
9 Plato **10** Felix Klein

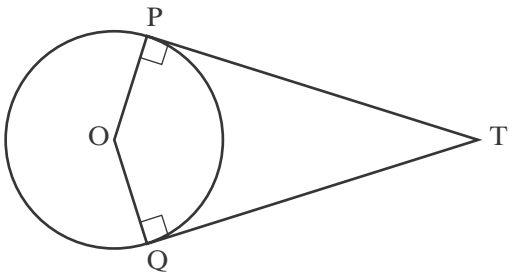
Chapter 17: Circle theorems

Starter 17 (page 318)

1 Radius **2** Circumference **3** Centre
4 Arc **5** Diameter **6** Chord
7 Tangent **8** Sector **9** Segment
10 a) Arc-en-ciel → curve in the sky → a good name
b) A 'segment' of an orange is really a sector → not a good name

Exercise 17.1 (page 322)

1 a) $x = 90°$ (tangent and radius at right angles)
b) $y = 61°$
2 RT = 25 − 7 = 18 cm
3 a) 136° (radius/tangent at 90° and quadrilateral angles add up to 360°)
b) Cyclic quadrilateral (opposite angles add up to 180°)
4 a) Isosceles **b)** 65° **c)** 50°
5 8 cm
6 18.8 cm (3 s.f.)
7 a)



b) Square
8 a) Radius bisects a chord at right angles
b) 17 cm **c)** 6 cm
d) 19.3 cm

Exercise 17.2 (page 330)

1 $a = 59°$, $b = 28°$ **2** $c = 48°$, $d = 44°$
3 $e = 42°$, $f = 45°$, $g = 51°$ **4** $h = 39°$
5 $i = 88°$ **6** $j = 58°$
7 $k = 63°$, $l = 75°$ **8** $m = 76°$
9 $n = 51°$ **10** $p = 36°$
11 $q = 72°$ **12** $r = 44°$, $s = 46°$
13 $t = 21°$ **14** $u = 15°$
15 $v = 30°$

Exercise 17.3 (page 334)

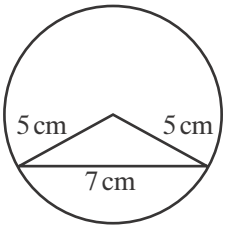
1 $a = 92°$, $b = 67°$ **2** $c = 105°$, $d = 95°$
3 $e = 109°$, $f = 78°$ **4** $g = 126°$, $h = 27°$
5 $i = 37°$ **6** $j = 29°$, $k = 61°$
7 $l = 114°$ **8** $m = 54°$
9 $n = 93°$ **10** $p = 162°$
11 $q = 41°$ **12** $r = 87°$, $s = 83°$
13 $t = 116°$ **14** $u = 150°$
15 $v = 132°$, $w = 42°$

Exercise 17.4 (page 337)

1 30 cm
2 6 cm
3 a) 18 cm **b)** 8 cm
4 a) 12 cm **b)** 7 cm
5 a) 12 cm **b)** 2 cm

Review Exercise 17 (page 338)

1 a)



b) 3.57 cm
2 a) 36°
b) Yes (opposite angles add up to 180° − cyclic quadrilateral)
3 a) 74°
b) No (74° and 32° do not add up to 180°)
4 a) 27° (tangent and radius meet at right angles)
b) 63° (alternate segment theorem)
5 a) 90° (angle in a semicircle)
b) 56° (same segment as angle PSQ)
c) 112° (angle at centre = twice angle at circumference)
6 a) 60° **b)** 35°
c) Yes (angle DAB = 65° + 25° = 90°)
7 a) 55° (angle ABC = 90°, angles in a triangle add up to 180°)
b) 35° **c)** 110°
8 a) 48° **b)** 18°
c) Angle CDA is not 90°
9 a) 40° **b)** 12°
c) Angle BAD = 78° so cannot be the angle in a semicircle
10 a) 27° (90° − 36° = 54°, 54° ÷ 2 = 27°)
153° (opposite angles in a cyclic quadrilateral)
11 a) i) 56° **ii)** 112°
b) i) 23° **ii)** 65°
12 8 cm
13 $x = 100$
14 17.4 cm

Internet Challenge 17 (page 344)

The line segment AP1 is perpendicular to BC, BP2 is perpendicular to CA and CP3 is perpendicular to AB.

The three line segments AP1, BP2, CP3 intersect at M.

P4 is the midpoint of AM, P5 is the midpoint of BM and P6 is the midpoint of CM.

P7 is the midpoint of BC, P8 is the midpoint of AC, P9 is the midpoint of AB.

Check students' constructions.

This construction was probably first made by Feuerbach, Poncelet or Brianchon around 1820.

Chapter 18: Sets

Starter 18 (page 345)

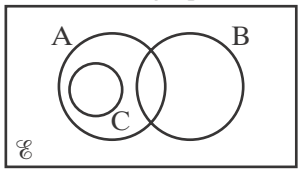
1 Andy is right.
2 Britney could be wrong: some children could write with both hands, so there might be fewer than 30 children in class 2.
3 Carlo could be wrong: some children may have other favourite sports, so there might be more than 30 children in class 3.
4 Donna could be wrong: some children might play neither the violin nor the piano, so there might be more than 30 children in class 2.

Exercise 18.1 (page 348)

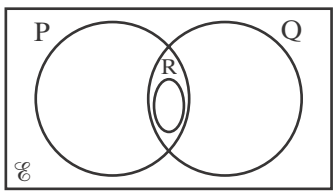
1 a) {21, 23, 25, 27, 29, 31}
b) 6
2 Multiples of 5 from 5 to 25 inclusive
3 a) A = {2, 3, 4, 5, 6, 7}
b) B = {6, 7, 8, 9, 10, 11}
c) C = {21, 22, 23, 24, 25}
4 a) 45 ∉ {multiples of 7}
b) 24 ∈ {factors of 144}
c) 2 ∈ {all prime numbers}
5 Hardback fiction books belonging to John
6 a) The set of all whole numbers (integers)
b) Empty set
7 a) (i) 16 **(ii)** 10
b) Multiples of 15 between 1 and 50, i.e. {15, 30, 45}
8 a) Quadrilaterals have 4 sides, triangles have 3
b) Equilateral triangles
c) Squares and rhombuses
9 a) (i) 4 **(ii)** 4 **(iii)** 0
b) Empty set
10 a) {11, 12}
b) {x: $5 \leqslant x \leqslant 20$, where x is an integer}

Exercise 18.2 (page 352)

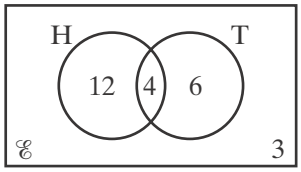
1



2

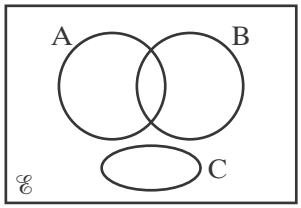


3 a)

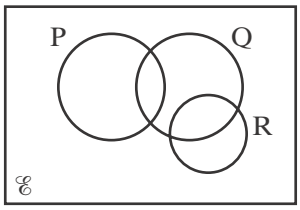


b) 3

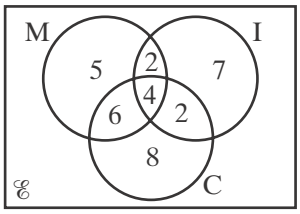
4



5

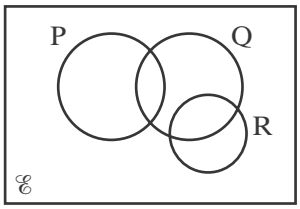


6 a)



b) 15 **c)** 6

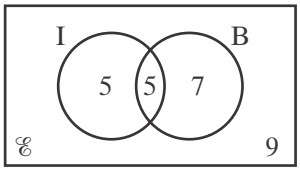
7 a)



b) Contains just the number 2
c) The empty set

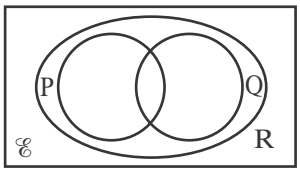
8 a) 10 + 12 + 9 − 26 = 5 liked both

b)

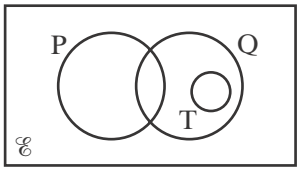


c) 12

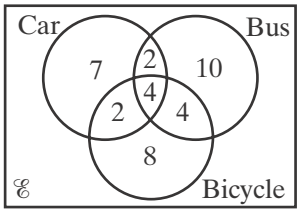
9 a)



b)



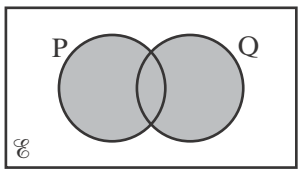
10 a)



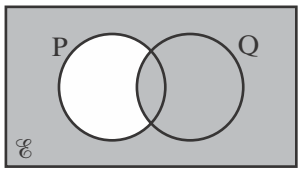
b) 7 + 2 + 2 + 4 + 10 + 4 + 8 = 37, and 50 − 37 = 13 but this is the number of people who do not use car, bus or bicycle. They could use other forms of transport (train, motorcycle etc) so Fred's statement is not correct.

Exercise 18.3 (page 356)

1 a)

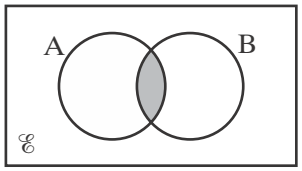


b)

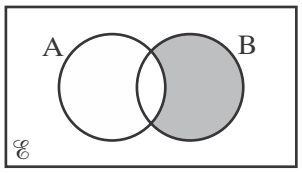


2 a) {2, 4, 6, 8, 10, 12, 14}
b) {odd numbers from 1 to 15 inclusive}
c) {2, 4, 8, 10, 14}

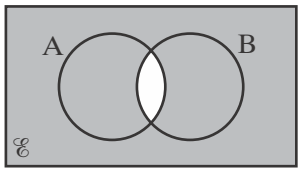
3 a)



b)

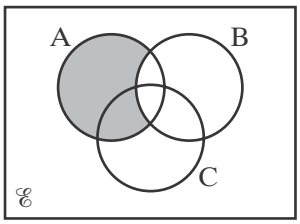


c)

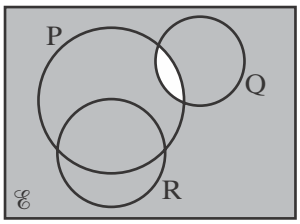


4 a) {consonants}
b) 3

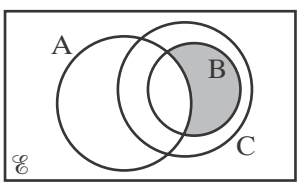
5



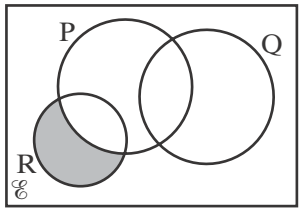
6



7



8



Review Exercise 18 (page 357)

1 a) {2, 4, 7, 8, 9, 10}
b) {5, 6}

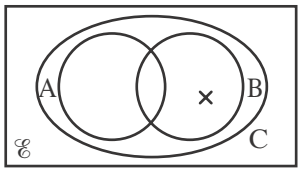
2 a) $P \cap Q = \varnothing$ **b)** $Q \subset P$

3 a) 27 **b)** 5
c) 6 **d)** 62

4 a) {3, 6, 9, 12, 15, 18, 21, 24, 27, 30}
b) 7 is neither a member of A nor B, so it is not true
c) {12, 24}

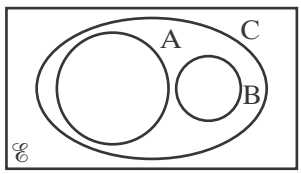
5 $\mathscr{E}$ = {whole numbers}
A = {multiples of 2}
B = {multiples of 3}

a), b)



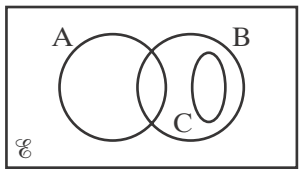
6 a) {1, 2, 3, 4, 5, 6, 8} **b)** {2, 4, 6} **c)** 8

7



8 a) {counties in England without a coastline}
b) There is at least one English county beginning with the letter C that has a coastline (e.g. Cornwall)

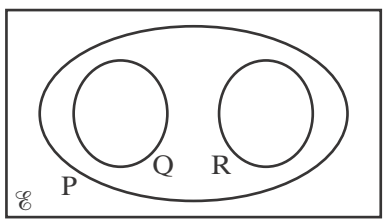
9



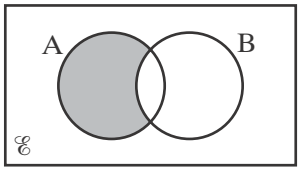
10 a) There are 20 multiples of 5 between 1 and 100 inclusive. There are 14 multiples of 7 between 1 and 100 inclusive.
b) Some multiples of 5 are also multiples of 7
c) {35, 70}

11 a) i) {1, 3} **ii)** {1, 2, 3, 4, 5}
b) The number 3 is an element of set A

12

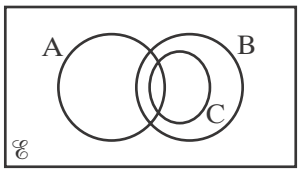


13 a) i)



ii) {5, 10, 15} (or any other three numbers that are multiples of 5 but not 3)

b) i)



ii) {15, 45, 75} (or any other three multiples of 15 that are not multiples of 10)

Internet Challenge 18 (page 361)

1 Bertrand Arthur William Russell
2 18 May 1872
3 Trinity
4 Alfred North Whitehead
5 Sir Isaac Newton
6 1 + 1 = 2
7 Nobel Prize for Literature
8 CND (Campaign for Nuclear Disarmament)
9 Four
10 2 February 1970

Chapter 19: Working with data

Starter 19 (page 362)

Medical research: The y axis should be drawn all the way down to zero, then the recent growth may be seen to be much more gradual.

Milk bottles: The bottle for Sykes Farm is 60% taller and 60% wider, making it look much more than just 60% more volume.

Staff cars: The yellow sector looks much bigger because it is at the front of the pseudo-3-D diagram.

Exercise 19.1 (page 368)

1 a) 40 **b)** 2.55

2 a) 35.2 **b)** 35 to 39

c) Largest possible range is $44 - 25 = 19$

3 24.7 cm

4 40.2 years

5 a) 17.25 minutes

b) Exact raw values are not recorded.

Exercise 19.2 (page 370)

1 9 kg **2** 10 kg **3** 30 runs **4** 115.5

5 Billy is right; his method works because there are equal numbers of rods in each batch.

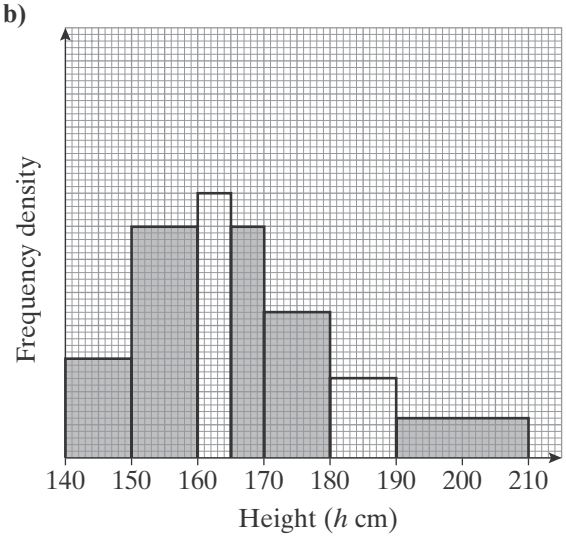
6 43 years **7** 90 marks **8** 6

Exercise 19.3 (page 375)

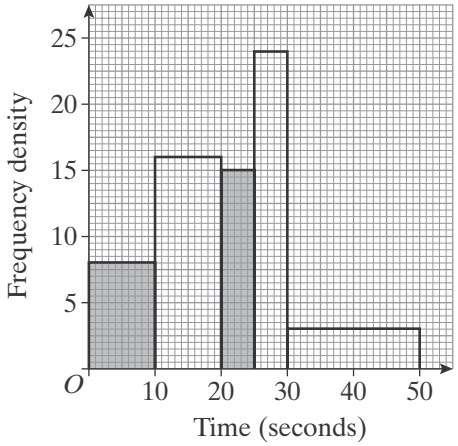
1 a)

Height (h cm)	Frequency
$140 \leqslant h < 150$	15
$150 \leqslant h < 160$	35
$160 \leqslant h < 165$	20
$165 \leqslant h < 170$	18
$170 \leqslant h < 180$	22
$180 \leqslant h < 190$	12
$190 \leqslant h < 210$	12

b)



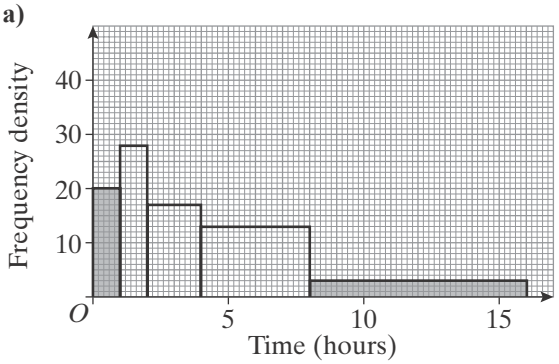
2 a)



b)

Time (t seconds)	Frequency
$0 < t \leqslant 10$	10
$10 < t \leqslant 20$	18
$20 < t \leqslant 25$	14
$25 < t \leqslant 30$	10
$30 < t \leqslant 50$	8

3 a)



b)

Time (t seconds)	Frequency
$0 < t \leqslant 1$	20
$1 < t \leqslant 2$	28
$2 < t \leqslant 4$	34
$4 < t \leqslant 8$	52
$8 < t \leqslant 16$	24

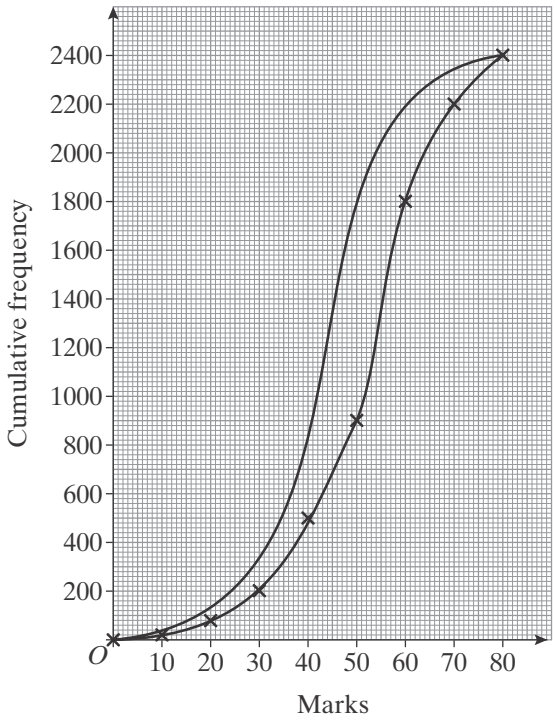
4 a) 97 **b)** 34 **c)** Between 40 grams and 60 grams

Exercise 19.4 (page 379)

1 a) 70 **b)** 18 seconds **c)** 21% approx

2 a) 1200

b)

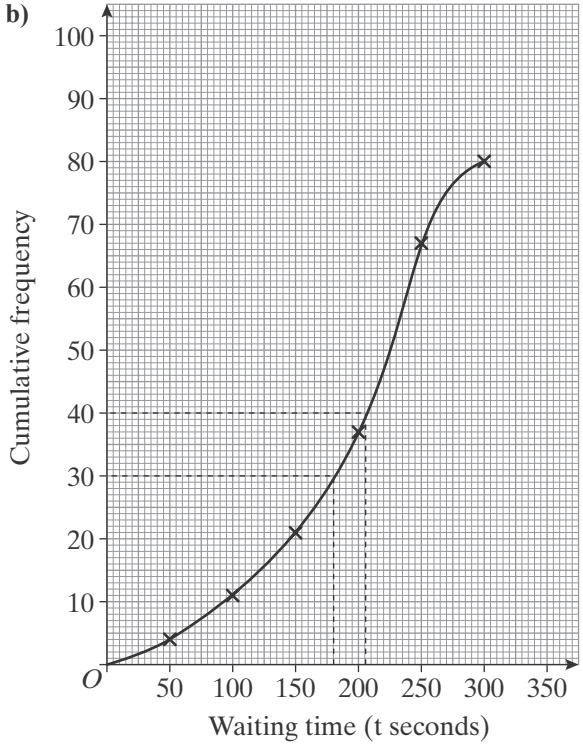


c) 53 approx

3 a)

Waiting time (t seconds)	Cumulative frequency
$0 \leqslant t < 50$	4
$0 \leqslant t < 100$	11
$0 \leqslant t < 150$	21
$0 \leqslant t < 200$	37
$0 \leqslant t < 250$	67
$0 \leqslant t < 300$	80

b)



i) 205 seconds **ii)** 80 − 30 = 50 people

Exercise 19.5 (page 383)

1 a) 124 minutes

b) UQ = 133 minutes, LQ = 115 minutes, IQR = 18 minutes

2 a) 0.98, 1.16, 1.17, 1.22, 1.22, 1.23, 1.28, 1.29, 1.32, 1.33, 1.45; median = 1.23 volts

b) LQ = 1.17 volts, UQ = 1.32 volts

3 a) 63

b) IQR = 81 − 45 = 36

c) The medians for boys and girls are similar, so on average they scored about the same; the IQR for the girls is much smaller than for the boys, so their scores were closer together (more consistent)

4 a) 2°C

b) LQ = 1, UQ = 5

5 a) 12 runs

b) Interquartile range = 44 runs

Review Exercise 19 (page 383)

1 a) 11 to 15 **b)** 1230 ÷ 75 = 16.4

2 £53 000 ÷ 50 = £1060

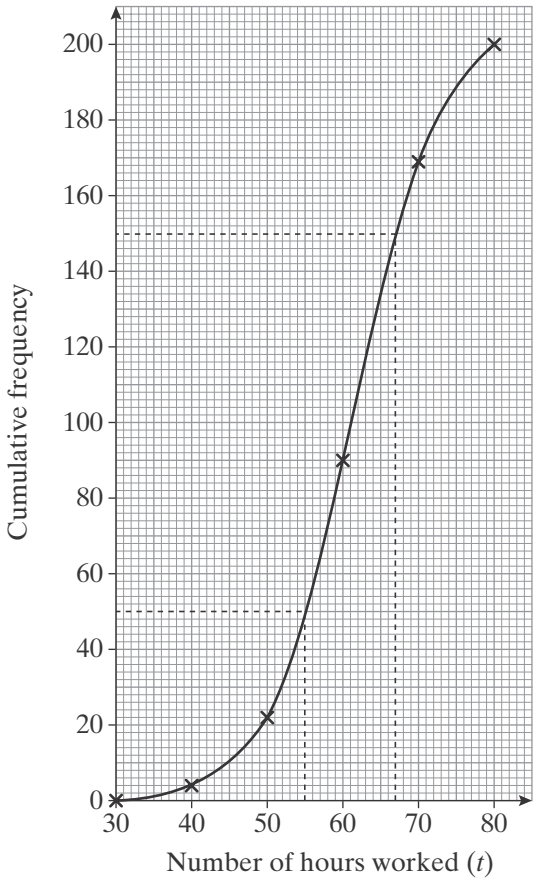
3 988 ÷ 10 = 98.8 grams

4 a) 12 150 ÷ 200 = 60.75 hours

b)

Number of hours worked (t)	Cumulative Frequency
$0 < t \leqslant 30$	0
$0 < t \leqslant 40$	4
$0 < t \leqslant 50$	22
$0 < t \leqslant 60$	90
$0 < t \leqslant 70$	169
$0 < t \leqslant 80$	200

c)



d) 67 − 55 = 12 hours

5 a) IQR for Class A is 11; IQR for Class B is 8
b) The marks for Class B are closer together (more consistent)

6 a) Class A has an IQR 4 (and a median of 5)
b) Class B's scores were higher, on average, but less spread out (more consistent)

7 79.3

8 £380

9 a) 1.8
b) p and q might be equal.
Both lists might contain the same number of items.

10 a) 29 minutes
b) A: 12, B: 20, C: 18

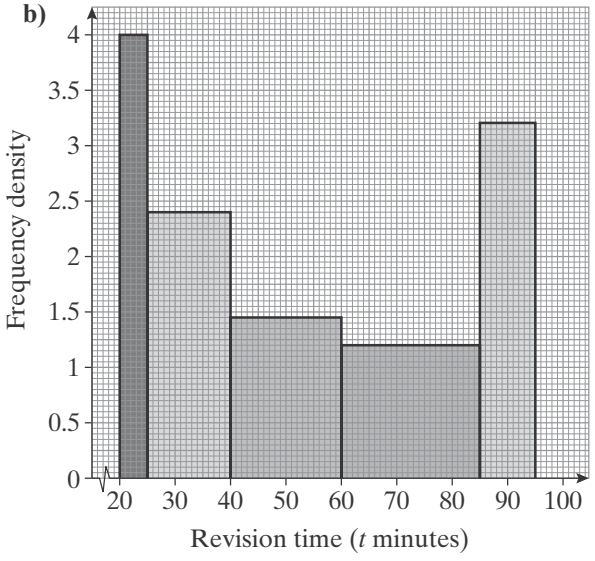
11

Time (t minutes)	Frequency
$0 < t \leqslant 10$	20
$10 < t \leqslant 15$	18
$15 < t \leqslant 30$	45
$30 < t \leqslant 50$	52
Total	135

12

Revision time (t minutes)	Frequency
$20 \leqslant t < 25$	20
$25 \leqslant t < 40$	36
$40 \leqslant t < 60$	29
$60 \leqslant t < 85$	30
$85 \leqslant t < 95$	32

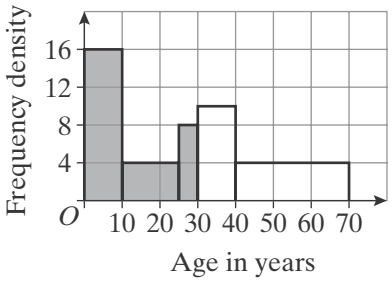
b)



13 a)

Age (x) in years	Frequency
$0 < x \leqslant 10$	160
$10 < x \leqslant 25$	60
$25 < x \leqslant 30$	40
$30 < x \leqslant 40$	100
$40 < x \leqslant 70$	120

b)



14 a) 100 **b)** 55
c) Between 30 minutes and 50 minutes

Internet Challenge 19 (page 389)

1 Benjamin Disraeli
2 Mark Twain
3 Oscar Wilde
4 W.I.E. Gates
5 Rex Stout
6 J.R. 'Bob' Dodds
7 H.G. Wells
8 Abraham Maslow
9 Joseph Stalin
10 Albert Einstein

Chapter 20: Probability

Starter 20 (page 390)

One dice: All six outcomes should occur with similar frequencies.
Two dice: 7 should occur most often, with 6 and 8 also having high frequencies.

Exercise 20.1 (page 393)

1 a) $\frac{15}{40} = \frac{3}{8}$ **b)** $\frac{16}{40} = \frac{2}{5}$

2 a) $\frac{1}{7}$ **b)** $\frac{6}{7}$ **c)** 0

3 0.78

4 a) $\frac{12}{50} = \frac{6}{25}$ **b)** 96

5 a) $\frac{10}{30} = \frac{1}{3}$ **b)** $\frac{20}{30} = \frac{2}{3}$ **c)** $\frac{5}{30} = \frac{1}{6}$

6 a)

	Science	History	Total
Hardback	10	18	28
Paperback	20	32	52
Total	30	50	80

b) $\frac{10}{30} = \frac{1}{3}$ **c)** $\frac{32}{52} = \frac{8}{13}$

7 a) $\frac{46}{50} = \frac{23}{15}$ **b)** 28

8 a) $\frac{3}{50}$ **b)** $\frac{21}{29}$

c) Disagree – data from one day, so not representative (may be biased).

9 a)

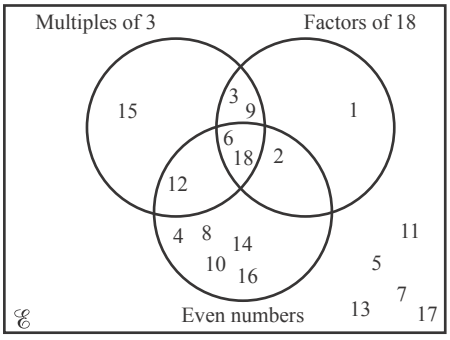
	Tea	Coffee	Other	Total
Morning	78	32	0	110
Afternoon	22	48	20	90
Total	100	80	20	200

b) $\frac{32}{110} = \frac{16}{55}$ **c)** $\frac{22}{90} = \frac{11}{45}$ **d)** 520

10 a) $\frac{18}{30} = \frac{3}{5}$ **b)** 288

c) Class 3G might not be representative of the school as a whole.

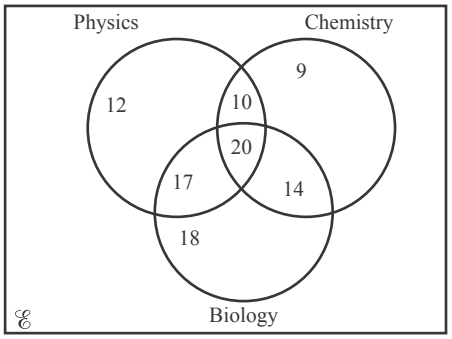
11 a)



b) i) $\frac{6}{18}$ or $\frac{1}{3}$ **ii)** $\frac{3}{18}$

iii) $\frac{4}{18}$ or $\frac{2}{9}$ **iv)** $\frac{2}{18}$ or $\frac{1}{9}$

12 a)



b) i) 0.69 **ii)** 0.39 **iii)** 0.23

Exercise 20.2 (page 396)

1 a) 0.2 **b)** 0.9 **c)** 0.2 **d)** 0.4

2 a)

Activity	Cinema	Pizza	Stay in
Frequency	0.25	0.45	0.3

b) Pizza

c) 0.75

3 a)

Type of bird	Blackbird	Sparrow	Starling	Robin
Frequency	0.35	0.25	0.3	0.1

b) Blackbird

c) 0.65

d) 0.55

4 a) $\frac{4}{7}$ **b)** $\frac{6}{7}$

5 a) 0.3 **b)** 0.4

6 a) $\frac{1}{3}$ **b)** $\frac{2}{3}$

7 a) 0.7 **b)** 0.3 **c)** 0.15

8 a) Fred has added the two probabilities together.

b) Julie thinks the two events might not be mutually exclusive; a car can have unsafe lights and unsafe tyres.

9 a) 0.31 **b)** 144

10 a) 0.2 **b)** 20

c) $\frac{7}{25}, \frac{11}{25}, \frac{2}{25}, \frac{5}{25}$ **d)** 25

Exercise 20.3 (page 399)

1 a) 0.09 **b)** 0.11

2 a)

+	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

b) $\frac{2}{16} = \frac{1}{8}$ **c)** $\frac{1}{2}$

3 a) $\frac{2}{5}$ **b)** $\frac{4}{25}$

4 a)

+	1	2	3	4
5	6	7	8	9
6	7	8	9	10

b) $\frac{2}{8} = \frac{1}{4}$ **c)** 5

5 a) $\frac{1}{6}$ **b)** $\frac{5}{36}$ **c)** $\frac{1}{36}$

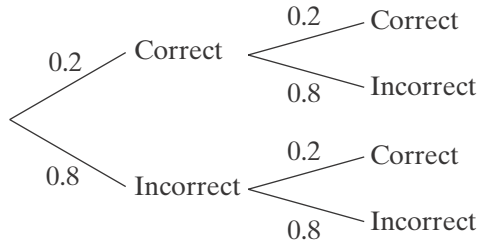
6 a) 0.7 **b)** 0.09 **c)** 0.063

7 a) $\frac{11}{144}$ **b)** $\frac{121}{144}$

8 a) 0.24 **b)** 0.01 **c)** 15 days

Exercise 20.4 (page 401)

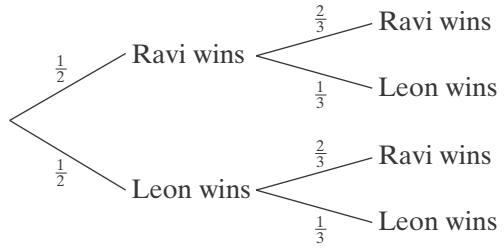
1 a)



b) 0.04

c) 0.32

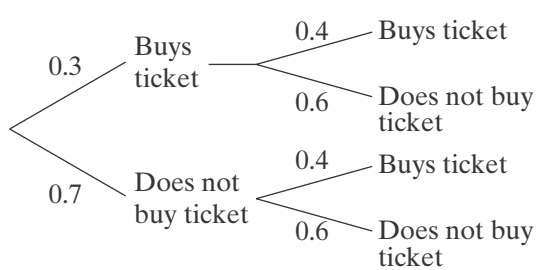
2 a)



b) $\frac{4}{9}$ **c)** $\frac{2}{9}$ **d)** $\frac{4}{9}$

3 a)

Joan **Simi**

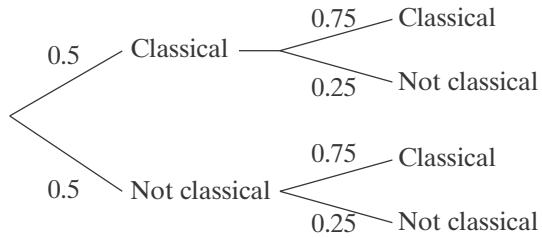


b) 0.46 **c)** 0.42

4 a) 0.5 **b)** 0.75

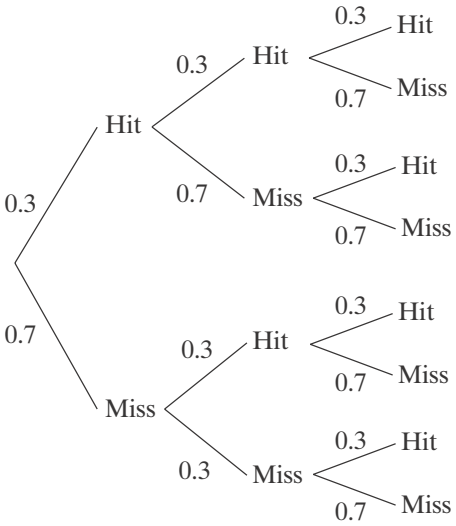
c)

Jesse **Camilla**



d) 0.5

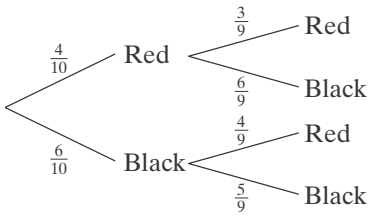
5 a)



b) 0.027 **c)** 0.441

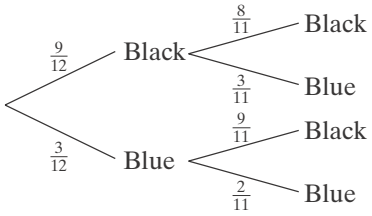
Exercise 20.5 (page 404)

1 a)



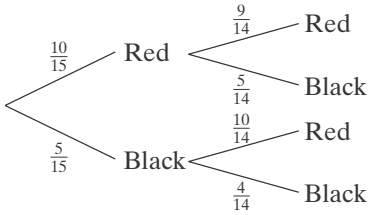
b) $\frac{30}{90} = \frac{1}{3}$ **c)** $\frac{42}{90} = \frac{7}{15}$

2 a)



b) $\frac{72}{132} = \frac{6}{11}$ **c)** $\frac{78}{132} = \frac{13}{22}$

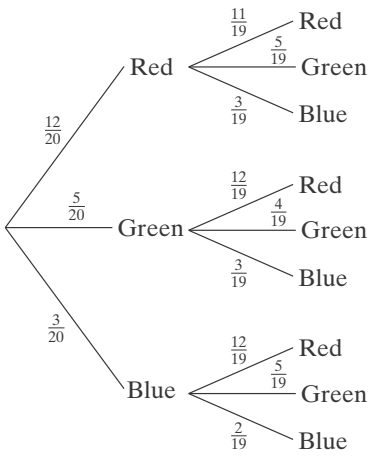
3 a)



b) $\frac{100}{210} = \frac{10}{21}$

4 $\frac{10}{18} \times \frac{9}{17} \times \frac{8}{16} = \frac{5}{34}$ or 0.147 (3 d.p.)

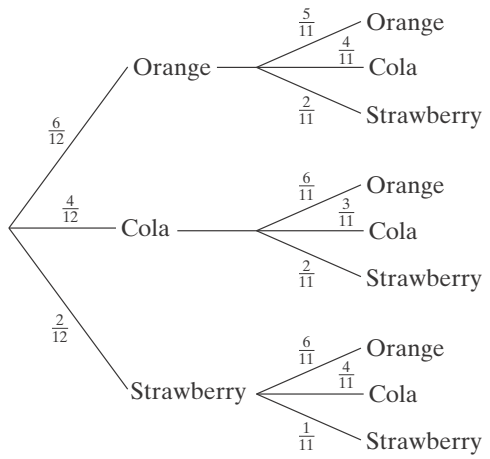
5 a)



b) $\frac{132+20+6}{380} = \frac{158}{380} = \frac{79}{190}$

6 a)

b)



$\frac{12}{132} = \frac{1}{11}$

c) $\frac{24 + 12 + 24 + 8 + 12 + 8}{132} = \frac{88}{132} = \frac{2}{3}$

7 a) $\frac{6}{16} \times \frac{5}{15} = \frac{1}{8}$ or 0.125

b) $\frac{10}{16} \times \frac{9}{15} \times \frac{8}{14} = \frac{3}{14}$ or 0.214 (3 d.p.)

Review Exercise 20 (page 405)

1 a) 0.5 **b)** 0.35

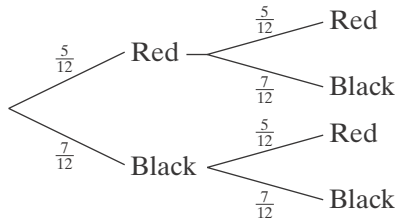
2 $\frac{3}{8}$

3 a) (i) 0.2 **(ii)** 0

b) 40 times

4 a) 0.48 **b)** 0.2

5 a)



b) (i) $\frac{25}{144}$ **(ii)** $\frac{35}{72}$

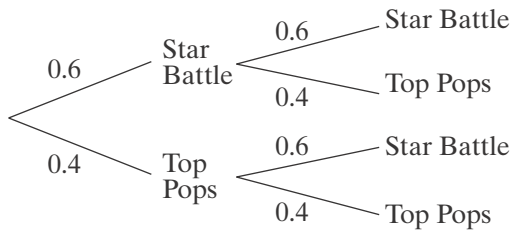
6 a) (1, Heads), (2, Heads), (3, Heads), (4, Heads), (5, Heads)
(1, Tails), (2, Tails), (3, Tails), (4, Tails), (5, Tails)

b) (i) 0.14 **(ii)** 0

(iii) 1 **(iv)** 0.25

c) 0.125

7 a) **Monday** **Tuesday**



b) 0.36 **c)** 0.48

8 a) 0.16 **b)** 4000

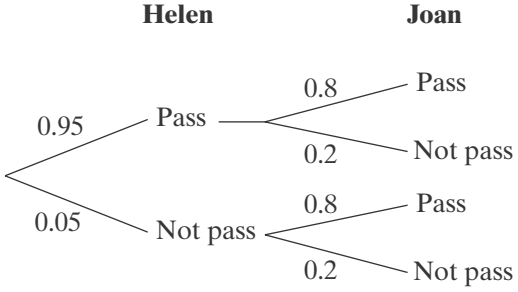
c) 212

9 a) $\frac{4}{24} = \frac{1}{6}$ **b)** $\frac{6}{24} = \frac{1}{4}$

10 a) 0.91 **b)** 3

c) (i) 0.0081 **(ii)** 0.1638

11 a)



b) 0.76 **c)** 0.23

12 a)

Spinner B

Spinner A ×	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12

b) $\frac{2}{12} = \frac{1}{6}$ **c)** $\frac{4}{12} = \frac{1}{3}$

13 $\frac{41}{95}$ (approx 0.432)

14 a) $\frac{42}{90} = \frac{7}{15}$

b) $\frac{1}{3}$

15 $0.05 \times 0.06 = 0.003$, Which is not equal to 0.011 Thus Fred is not correct; the events are not independent.

16 $\frac{33}{50} = 0.66$

Internet Challenge 20 (page 410)

The probability of initially selecting the right door is $\frac{1}{3}$, and remains at this value if the contestant does not switch. Thus it is better to switch, since the probability of winning increases to $1 - \frac{1}{3} = \frac{2}{3}$, i.e. is doubled.

Chapter 21: Direct and inverse proportion

Starter 21 (page 411)

1 49 mm – but only assuming the plant continues to grow at the same rate
2 $8 \times 12 \div 6 = 16$ nights
3 $9 \times 15 \times 20 \div (18 \times 6) = 25$ minutes
4 1
5 10 minutes

Exercise 21.1 (page 414)

1 $y = 3x$; $y = 39$
2 $y = 2.5x$; $x = 16$
3 **a)** $y = 2x$; 6 **b)** $y = 4x$; 4, 4.5
c) $y = \frac{x}{6}$; 2, 42, 17 **d)** $y = \frac{5x}{8}$; 1.25, 56
4 **a)** $y = \frac{2x}{3}$
b) i) 40 **ii)** 37.5
5 **a)** $y = \frac{x^2}{5}$
b) i) 180 **ii)** ± 25
6 $y = 4x^2$; $y = 36$
7 $y = 0.4x^3$; $y = 204.8$
8 **a)** $y = 4\sqrt{x}$
b) i) $y = 24$
ii) $x = 9$
9 **a)** $F = \frac{\sqrt{s}}{2}$
b) i) $F = 2$
ii) $s = 4$
10 **a)** $T = \frac{c^2}{6.4}$ **b)** 22.5 minutes **c)** 24 cities
11 **a)** $t = \frac{n^2}{50\,000\,000}$ **b)** 20 000 seconds
c) 173 000
12 **a)** £4.80 **b)** 30 cm

Exercise 21.2 (page 417)

1 25
2 3
3 **a)** 5, 2 **b)** 8, 36 **c)** 0.2, 0.25 **d)** 0.8, 0.4
4 **a)** $r = \frac{294}{t}$
b) (i) 21 **(ii)** 6
5 **a)** $p = \frac{180}{s}$
b) (i) 20 **(ii)** 3
6 $y = 20.25$
7 2
8 117; 7.75 m
9 $v = \frac{8.424}{m}$; 58.5 km/h
10 **a)** 2401 **b)** 9603

Exercise 21.3 (page 420)

1 Neither **2** Inverse proportion
3 Neither **4** Neither
5 Neither **6** Direct proportion

Exercise 21.4 (page 422)

1 350 km
2 15 km
3 6 km/h
4 0.01 m/s
5 1 h 20 minutes
6 208 m
7 **i)** 10.4 m/s **ii)** 37.6 km/h
8 **a)** 2.38 cm^3
b) 11.9 cm^3
c) 20.0 g
9 **a)** 2.7 g/cm^3 **b)** 2700 kg/m^3
10 8 Pascals
11 170 g
12 **a)** 650 people/km^2
b) 36 000 km^2
c) 17 000

Review Exercise 21 (page 423)

1 39
2 2
3 9
4 320
5 **a)** $T = \frac{2x}{3}$ **b)** 24 **c)** 72
6 **a)** $T = 0.2\sqrt{l}$ **b)** 1.18 seconds
7 **a)** $d = \frac{L^3}{168\,750}$ **b)** 136
8 **a)** $y = \frac{48}{x^2}$ **b)** 1.92
9 **a)** $y = 9x^2$ **b)** $c = 18$ and $n = -\frac{1}{2}$
10 **a)** $y = \frac{72}{x}$
b) (i) 12 **(ii)** 15
11 **a)** $d = 5t^2$ **b)** 245 **c)** 3
12 **a)** $F = \frac{36}{x^2}$ **b)** 9 **c)** 0.75
13 **a)** 41.4 m **b)** $d = 0.1V^2 + 0.5V$
14 **a)** $S = \frac{8000}{f^2}$ **b)** 500
15 **a)** 737 000 cm^3 **b)** 275 grams

Internet Challenge 21 (page 427)

1–4

Planet/dwarf planet	Mean distance, *d*, from Sun	Orbital period, *T*
Mercury	0.387	88 days
Venus	0.723	225 days
Earth	1	1 year
Mars	1.524	1.88 years
Jupiter	5.203	11.86 years
Saturn	9.529	29.41 years
Uranus	19.19	84.0 years
Neptune	30.06	164.8 years
Pluto	39.53	248.5 years

5 Planets move in orbits that are ellipses. Planets move such that the line between the Sun and the planet sweeps out the same area in the same time, no matter where in the orbit.
6 11 500 years
7 Scientists are unsure of how to classify them.

Chapter 22: Quadratic equations , curves and inequalities

Starter 22 (page 428)

1 $x = 3$ **2** $x = -1, x = 1$
3 $x = 2, x = 5$ **4** $x = 4$
5 $x = -1, x = 1$ **6** $x = 3$
7 $x = -2, x = 1$ **8** $x = -6, x = 6$
9 $x = -5, x = 5$ **10** $x = 1, x = 2, x = 3$
Yes, they do.

Exercise 22.1 (page 430)

1 $-1, -2$ **2** $-1, -5$
3 $1, -8$ **4** $1, -2$
5 $2, -4$ **6** $2, -6$
7 $3, 4$ **8** $3, 5$
9 $4, -2$ **10** 2 (twice)
11 $-\frac{1}{2}, -1$ **12** $\frac{1}{2}, -3$
13 $-2, -\frac{1}{3}$ **14** $1, -1\frac{1}{2}$
15 $-2, -\frac{2}{3}$ **16** $3, 1\frac{1}{2}$
17 $-1, -1\frac{2}{3}$ **18** $2, 2\frac{1}{2}$
19 $-5, -\frac{1}{5}$ **20** $-\frac{1}{2}$ (twice)
21 $\frac{1}{3}, -\frac{1}{2}$ **22** $0, \frac{1}{5}$
23 $\frac{1}{2}, -\frac{1}{2}$ **24** $0, 1$
25 $\frac{1}{3}, \frac{1}{4}$ **26** $0, \frac{1}{10}$
27 $\frac{1}{2}, \frac{3}{4}$ **28** $1, \frac{3}{8}$
29 $-1\frac{1}{2}$ (twice) **30** $-1\frac{1}{2}, 1\frac{1}{2}$
31 $x^2 - 6x - 7 = 0$; $7, -1$
32 $x^2 - 13x + 40 = 0$; $5, 8$
33 $x^2 + 13x + 30 = 0$; $-3, -10$
34 $x^2 + 7x - 44 = 0$; $4, -11$
35 $2x^2 - 11x - 6 = 0$; $6, -\frac{1}{2}$
36 $3x^2 + 23x - 8 = 0$; $-8, \frac{1}{3}$
37 $3x^2 + x - 2 = 0$; $-1, \frac{2}{3}$
38 $4x^2 - 8x + 3 = 0$; $1\frac{1}{2}, \frac{1}{2}$
39 $6x^2 + 5x - 6 = 0$; $\frac{2}{3}, -1\frac{1}{2}$
40 $4x^2 - 25 = 0$; $-2\frac{1}{2}, 2\frac{1}{2}$

Exercise 22.2 (page 433)

1 **a)** $(x + 7)^2$
b) i) $(x + 7)^2 + 11$ **ii)** $(x + 7)^2 + 6$
iii) $(x + 7)^2 - 9$ **iv)** $(x + 7)^2 - 59$

2 **a)** $(x - 3)^2$
b) i) $(x - 3)^2 + 1$ **ii)** $(x - 3)^2 + 6$
iii) $(x - 3)^2 - 6$ **iv)** $(x - 3)^2 - 1$

3 **a)** $(x+3)^2 - 24$ **b)** $(x+3)^2 - 29$
c) $(x+1)^2 + 5$ **d)** $(x+5)^2 - 34$
e) $(x-2)^2 - 3$ **f)** $(x-1)^2 - 6$
g) $(x+\frac{1}{2})^2 + \frac{3}{4}$ **h)** $(x-\frac{1}{2})^2 + \frac{3}{4}$
i) $(x+\frac{7}{2})^2 - \frac{89}{4}$ **j)** $(x+\frac{3}{2})^2 - \frac{25}{4}$
k) $(x+\frac{5}{2})^2 + \frac{7}{4}$ **l)** $(x-\frac{3}{2})^2 + \frac{15}{4}$

4 **a)** $2(x+1)^2 + 3$ **b)** $2(x+2)^2 - 11$
c) $2(x+\frac{3}{2})^2 + \frac{9}{2}$ **d)** $3(x+2)^2 - 17$
e) $3(x+\frac{3}{2})^2 - \frac{23}{4}$ **f)** $4(x+\frac{5}{4})^2 - \frac{5}{4}$

5 **a) i)** $(x+2)^2 - 11$ **ii)** $x = 1.32$ or $x = -5.32$
b) i) $(x+4)^2 - 28$ **ii)** $x = 1.29$ or $x = -9.29$
c) i) $(x+5)^2 - 17$ **ii)** $x = -9.12$ or $x = -0.877$
d) i) $2(x+1)^2 - 5$ **ii)** $x = 0.581$ or $x = -2.58$
e) i) $2(x+\frac{3}{2})^2 - \frac{19}{2}$ **ii)** $x = 0.679$ or $x = -3.68$
f) i) $3(x+1)^2 - 1$ **ii)** $x = -1.58$ or $x = -0.423$

Exercise 22.3 (page 435)

1 $-0.438, -4.562$ **6** $0.721, -1.387$
2 $-0.757, -9.243$ **7** $0.193, -5.193$
3 $5.898, 1.102$ **8** $2.351, -0.851$
4 $0.422, -5.922$ **9** $0.558, -0.358$
5 $6.854, 0.146$
10 $0.212, -4.712$
11 $x^2 + 5x - 7 = 0$; $1.14, -6.14$
12 $2x^2 - 3x - 1 = 0$; $1.78, -0.281$
13 $3x^2 - 4x - 5 = 0$; $2.12, -0.786$
14 $x^2 + 10x - 2 = 0$; $0.196, -10.2$
15 $2x^2 + 11x - 1 = 0$; $0.0895, -5.59$
16 $3x^2 - 12x - 1 = 0$; $4.08, -0.0817$
17 $5x^2 - 2x - 4 = 0$; $1.12, -0.717$
18 $7x^2 - 21x - 1 = 0$; $3.05, -0.0469$
19 $6x^2 + 17x + 4 = 0$; $-0.259, -2.57$
20 $9x^2 - x - 2 = 0$; $0.530, -0.419$

Exercise 22.4 (page 436)

1 **a)** $x(x + 7) = 144$
c) 9 and 16 or -16 and -9
2 **a)** $x(2x - 5) = 3000$
c) $x = 40$. The field is 40 m by 75 m
3 **a)** $x(x + 3) = 180$ **c)** 12 and 15
4 **a)** $x^2 + x(2x + 1) = 114$ **c)** 6
5 **a)** $(3x + 1)(2x + 5) - 2x^2 = 55$
c) $x = 2$. The rectangle is 7 cm by 9 cm
6 **a)** $x(2x + 3)$ and $(x + 3)(x + 4)$
b) $x(2x + 3) = (x + 3)(x + 4)$ which becomes $x^2 - 4x - 12 = 0$
c) $x = 6$. The rectangles are 6 cm by 15 cm and 9 cm by 10 cm
7 $n = 20$
8 $n = 32$ or $n = 49$

Exercise 22.5 (page 439)

1 $y = x^2 - 5$

x	−3	−2	−1	0	1	2	3
y	4	−1	−4	−5	−4	−1	4

2 $y = 2x^2 + 3$

x	−2	−1	0	1	2	3
y	11	5	3	5	11	21

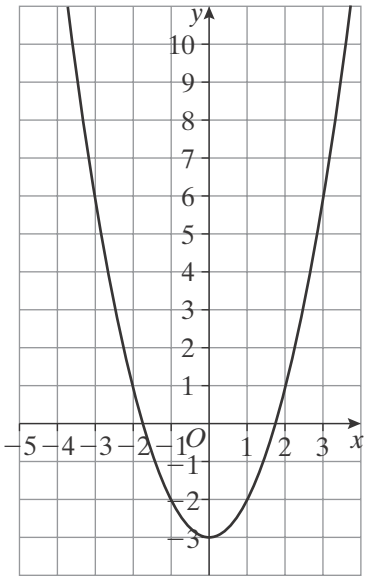
3 $y = 3x^2 + x$

x	−2	−1	0	1	2	3
y	10	2	0	4	14	30

4 $y = x^2 - 4x$

x	−1	0	1	2	3	4	5
y	5	0	−3	−4	−3	0	5

5 a), b) $y = x^2 - 3$

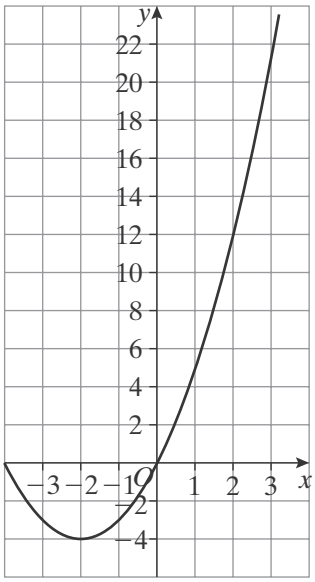


c) −0.75 **d)** (0, −3)

6 a)

x	−3	−2	−1	0	1	2	3
y	−3	−4	−3	0	5	12	21

b) $y = x^2 + 4x$

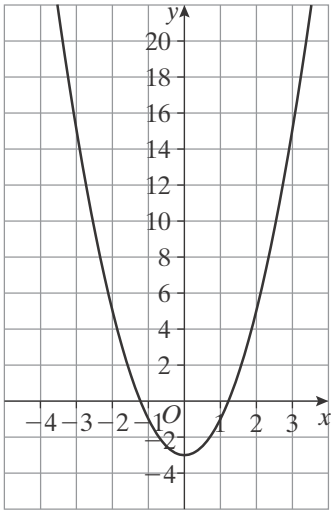


c) (−2, −4)

7 a)

x	−3	−2	−1	0	1	2	3
y	15	5	−1	−3	−1	5	15

b) $y = 2x^2 - 3$

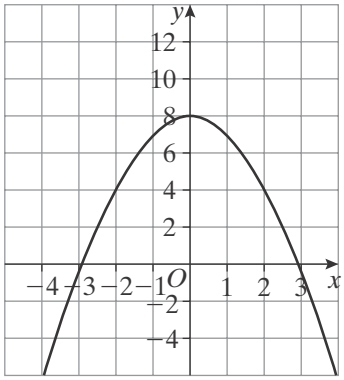


c) 1.2, −1.2

8 a)

x	−3	−2	−1	0	1	2	3
y	−1	4	7	8	7	4	−1

b) $y = 8 - x^2$

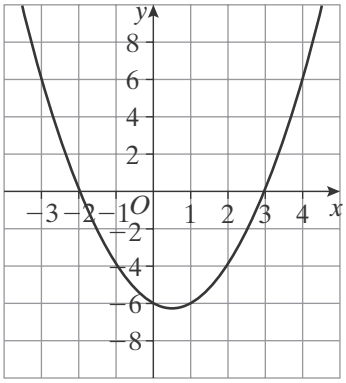


c) (0, 8) **d)** −2.8, 2.8

9 a)

x	−3	−2	−1	0	1	2	3	4
y	6	0	−4	−6	−6	−4	0	6

b) $y = x^2 - x - 6$



c) −2, 3
d) (0.5, −6.25)

Exercise 22.6 (page 441)

1 $2 < x < 3$
2 $x \leqslant 2$ or $x \geqslant 3$
3 $x \leqslant -4$ or $x \geqslant -3$
4 $-5 \leqslant x \leqslant 3$
5 $-\frac{1}{2} < x < 2$
6 $x \leqslant -1$ or $x \geqslant \frac{2}{3}$
7 $-3 < x < 2$
8 $x < -3$ or $x > \frac{1}{2}$

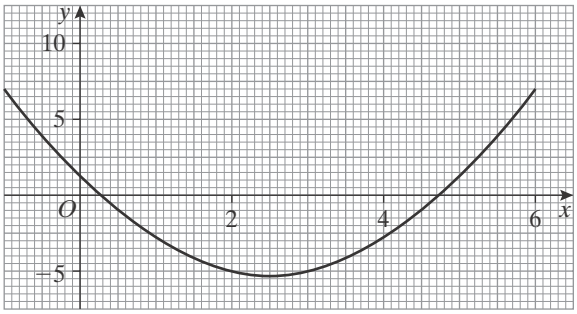
Review Exercise 22 (page 442)

1 a) $(x - 2)(x - 4)$
b) 2, 4

2 a)

x	–1	0	1	2	3	4	5	6
y	**7**	**1**	–3	**–5**	**–5**	**–3**	1	**7**

b)

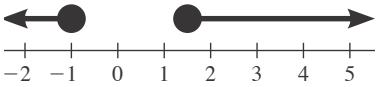


c) 0.2 and 4.8 **d)** (2.5, –5.25)

3 6.5, −3.5
4 4.236, −0.236
5 b) 2.19, −3.19
6 a) $(x + 4)(x - 3) = 78$
b) (ii) 9, −10
(iii) 13 cm, 6 cm
7 b) 9.93, −3.93. Radius of circle is 9.93 cm.
8 c) $2\frac{2}{3}$, 2
9 b) 12.361
10 a) $(x + 6)^2 - 28$
b) $x = -0.708$ or $x = -11.3$
11 a) $2(x + \frac{3}{2})^2 - \frac{23}{2}$
b) $x = -3.90$ or $x = 0.898$
12 a) $4 < x < 5$



b) $x \leqslant -1$ or $x \geqslant \frac{3}{2}$



13 a) i) $(x - 14)(2x - 7)$ **ii)** 14, 3.5
b) i) $\frac{7}{n+7}$
ii) $\frac{7}{n+7} = \frac{2}{5}$ gives $n = 10.5$
But n must be an integer.
d) $n = 14$ so 7 white balls out of 21, giving $\frac{1}{9}$

Internet Challenge 22 (page 445)

1 Parabola
2 The other three are the circle, ellipse and hyperbola.
3 Yes, a parabola.
4 If the orbit is closed it must be an ellipse (or a circle).
Some comets probably have open orbits; these could be parabolas or hyperbolas.
5 Good method but requires some skill!

Chapter 23: Advanced algebra

Starter 23 (page 446)

By counting, the numbers of squares/rectangles are 9, 36, 30. Thus $k = 4$.

To prove the formula, select one corner of a square/rectangle at random. On an m by n grid, there are $m + 1$ possible choices for the x coordinate and $n + 1$ for the y coordinate, giving $(m + 1)(n + 1)$ possibilities altogether.

Now choose a second corner, not in the same row or column as before; this can be done in mn ways. Thus there would seem to be $m(m + 1)n(n + 1)$ choices altogether.

However, each different square/rectangle gets counted four times in this way. Thus the number is $(m + 1)(n + 1) \div 4$ and the result is proved.

Exercise 23.1 (page 450)

1 $3\sqrt{2}$ **2** $4\sqrt{2}$ **3** $5\sqrt{2}$
4 $3\sqrt{5}$ **5** $5\sqrt{6}$ **6** $2\sqrt{6}$
7 $3\sqrt{11}$ **8** $6\sqrt{3}$ **9** $6\sqrt{3}$
10 $7\sqrt{2}$ **11** $3\sqrt{3}$ **12** $7\sqrt{2}$
13 $4\sqrt{11}$ **14** $4\sqrt{2}$ **15** $12+4\sqrt{3}$
16 $8+7\sqrt{2}$ **17** 22 **18** $22+11\sqrt{5}$
19 $\frac{2+3\sqrt{5}}{5}$ **20** $2-\sqrt{7}$ **21** **a)** -1
b) $-3(1-\sqrt{2})$
23 $-3(2+\sqrt{5})$ **24** **a)** $8+3\sqrt{5}$ **b)** $10-4\sqrt{3}$
25 **a)** $4+2\sqrt{7}$ **b)** $18+2\sqrt{7}$ **c)** $6+6\sqrt{7}$
26 $(12\sqrt{7}-16)$cm **27** $-2\pm\sqrt{11}$
28 $\frac{-1\pm\sqrt{5}}{2}$ **29** $\frac{-3\pm\sqrt{13}}{2}$ **30** $\frac{-4\pm\sqrt{10}}{2}$
31 $\frac{5\pm\sqrt{17}}{2}$ **32** $x=-3-\sqrt{11}$ or $x=\sqrt{11}-3$

Exercise 23.2 (page 453)

1 $\frac{8x + 3}{15}$ **2** $\frac{7x + 2}{24}$
3 $\frac{5x + 2}{10}$ **4** $\frac{5x}{6}$
5 $\frac{13x + 4}{20}$ **6** $\frac{11x + 2}{12}$
7 $\frac{2x + 5}{x(x + 1)}$ **8** $\frac{5x + 7}{(x + 1)(x + 2)}$
9 $\frac{5x + 5}{(x + 3)(2x + 1)}$ **10** $\frac{3x + 10}{(x + 3)(x + 4)}$
11 $\frac{x + 19}{(x - 2)(x + 5)}$ **12** $\frac{4x + 9}{(x + 1)(x + 2)}$
13 3 **14** 2
15 3 **16** -2
17 3 **18** 5
19 $4, \frac{2}{3}$ **20** $2, -\frac{5}{14}$

Exercise 23.3 (page 455)

1 $\frac{x + 3}{2x + 1}$ **2** $\frac{3x + 5}{2x + 1}$
3 $\frac{4x + 6}{3}$ **4** $\frac{x^2 + 5}{2x}$
5 $\frac{x + 2}{x}$ **6** $5(x + 3)^4$
7 $\frac{x + 10}{5}$ **8** $\frac{3x + 2}{x}$
9 $\frac{x - 5}{x}$ **10** $\frac{4}{(2x + 1)^2}$
11 $\frac{x + 8}{2}$ **12** $\frac{x}{2}$
13 $\frac{x}{x + 2}$ **14** $\frac{x + 2}{x + 7}$
15 $\frac{x + 2}{x + 4}$ **16** $\frac{x + 5}{x + 3}$
17 $\frac{x + 3}{x + 4}$ **18** $\frac{x + 4}{x + 2}$
19 $\frac{x + 5}{x - 3}$ **20** $\frac{1}{x - 4}$

Exercise 23.4 (page 457)

1 $x = 2$ and $y = 2$ or $x = -1$ and $y = -1$
2 $x = 3$ and $y = 10$ or $x = -2$ and $y = 5$
3 $x = 3$ and $y = 19$ or $x = -1$ and $y = 3$
4 $x = 2$ and $y = 20$ or $x = \frac{1}{5}$ and $y = \frac{1}{5}$
5 $x = 4$ and $y = 17$ or $x = 0$ and $y = 1$
6 $x = 2$ and $y = 0$ or $x = -1$ and $y = -3$
7 $x = 3$ and $y = 1$ or $x = -1$ and $y = -3$
8 $x = 2$ and $y = 2$ or $x = -\frac{2}{5}$ and $y = -2\frac{4}{5}$
9 $x = 3$ and $y = -1$ or $x = 1$ and $y = -3$
10 $x = 1$ and $y = -6$ or $x = 6$ and $y = -1$
11 $x = 5$ and $y = 13$ or $x = -3$ and $y = -3$
12 $x = 6$ and $y = 1$ or $x = -5$ and $y = -10$
13 $x = 1$ and $y = 2$ or $x = 2$ and $y = 4$
14 $x = 2$ and $y = -3$ or $x = 3$ and $y = -1$
15 $x = 5$ and $y = 3$ or $x = 0.6$ and $y = -5.8$
16 $x = 1$ and $y = 2$ or $x = -2$ and $y = -1$

Exercise 23.5 (page 458)

1 $x = \frac{5}{3 - m}$ **2** $x = \frac{d - b}{a - c}$
3 $x = \frac{2k}{2 - k}$ **4** $y = \frac{1 - 2d}{d - 1}$
5 $t = \frac{bc - a}{1 - c}$ **6** $x = \frac{n + 2}{3 - k}$
7 $x = \frac{ab}{1 - 5b}$ **8** $x = \frac{3}{a - 2}$
9 $x = \frac{ka}{1 - k}$ **10** $u = \frac{vf}{v - f}$

Exercise 23.6 (page 459)

1 Let the numbers be n and $n + 1$.
Their sum is $2n + 1$ which is odd.
2 Let the numbers be $2n$ and $2m$.
Their product is $4mn = 2 \times 2mn$, hence even.
3 Let the numbers be $2n + 1$ and $2m + 1$.
Their product is $(2n + 1)(2m + 1) = 4mn + 2n + 2m + 1 = 2 \times (2mn + n + m) + 1$, hence odd.

4 Let the numbers be n , $n + 1$ and $n + 2$.
Their sum is $n + n + 1 + n + 2 = 3n + 3$
$= 3 \times (n + 1)$,
hence a multiple of 3.

5 Let the numbers be $2n + 1$ and $2m + 1$.
Then
$(2n + 1)^2 - (2m + 1)^2 = [4n^2 + 4n + 1]$
$- [4m^2 + 4m + 1]$
$= 4n^2 + 4n + 1 - 4m^2 - 4m - 1$
$= 4n^2 + 4n - 4m^2 - 4m$
$= 4(n^2 + n - m^2 - m)$,
hence a multiple of 4.

6 a) $4 \times \frac{1}{2}ab = 2ab$
b) i) $c^2 + 2ab$ **ii)** $a^2 + b^2 + 2ab$
d) Pythagoras' theorem

7 b) Setting $x = 3$ gives $301 \times 299 = 89\,999$, so not prime.

8 Let the consecutive odd numbers be $2n - 1$ and $2n + 1$.
Then
$(2n + 1)^2 - (2n - 1)^2 = [4n^2 + 4n + 1] - [4n^2 - 4n + 1]$
$= 4n^2 + 4n + 1 - 4n^2 + 4n - 1$
$= 8n$, hence a multiple of 8.

Review Exercise 23 (page 460)

2 $a = 3$

3 a) 10 **b)** 3 **c)** 2

4 $\sqrt{22}$

5 a) i) 3.5 **ii)** 1
b) 3

6 a) 4 **b)** 2 **c)** $83\frac{1}{3}\%$

7 a) $\dfrac{3x}{(x - 2)(x + 4)}$ **b)** 8, -1

8 $10\frac{1}{2}$

9 a) $5n$

b) Two consecutive multiples of 5 are $5n$ and $5(n + 1)$

i) $5n + 5(n + 1) = 5n + 5n + 5 = 10n + 5$; $10n$ is even for all integer values of n, hence $10n + 5$ is odd as 5 is odd and an even number + odd number = odd number.

ii) $5n \times 5(n + 1) = 25n(n + 1)$; when n is odd then $(n + 1)$ is even and when you multiply by an even number the result is even; likewise, when n is even the product is even.

10 $(n + 1)^2 - (n - 1)^2 = (n^2 + 2n + 1) - (n^2 + 2n + 1) = 4n$
$4n$ is a multiple of 4 for all values of n

11 a) $(2a - 1)^2 - (2b - 1)^2 = (4a^2 - 4a + 1) - (4b^2 - 4b + 1)$
$= 4a^2 - 4b^2 - 4a + 4b$
$= 4(a - b)(a + b) - 4(a - b)$
$= 4(a - b)(a + b - 1)$

b) The difference between the squares of any two odd numbers is
$(2a - 1)^2 - (2b - 1)^2 = 4(a - b)(a + b - 1)$
which is a multiple of 4.

To prove $(2a - 1)^2 - (2b - 1)^2$ is a multiple of 8 then need to show that $(a - b)(a + b - 1)$ is a multiple of 2 (since $8 = 4 \times 2$).

When a and b are either both even or both odd then $(a - b)$ is even and so $(a - b)$ is a multiple of 2.

When only one of a or b is even then $(a + b - 1)$ is even and so $(a + b - 1)$ is a multiple of 2.

Hence $4(a - b)(a + b - 1)$ is a multiple of 8 as required.

12 2 is prime and $2^2 + 3 = 7$.

13 a) $(x + 1)(2x + 5)$ **b)** $\dfrac{11x + 15}{(x + 1)(2x + 5)}$

14 a) $23 - 6x$ **b)** $32x^5y^{15}$ **c)** $\dfrac{2(n - 1)}{n - 2}$

15 a) 7 **b)** $\dfrac{2x}{2x + 3}$

16 $y = \dfrac{2k}{4 + 3k}$

17 $x = \dfrac{ay}{y + 1}$

18 $x = 2$ and $y = 5$ or $x = -1.4$ and $y = -5.2$

19 a) If $y = 6$ then $x^2 = -11$ so Bill must be wrong.
b) $x = 3$ and $y = 4$ or $x = -1.4$ and $y = -4.8$

Internet Challenge 23 (page 463)

1 Pythagoras' theorem
2 Circumference of a circle
3 Area of a trapezium
4 Voltage = Current × Resistance
5 Volume of a cone
6 Quadratic equation formula
7 Energy = mass × (speed of light)^2
8 Surface area of a sphere
9 Distance s in terms of initial speed u, acceleration a and time t
10 Periodic time for a pendulum of length l
11 Euler's formula for faces, edges and vertices of a polyhedron
12 Conversion from degrees Fahrenheit to degrees Celsius
13 Kinetic energy
14 Potential energy
15 Optics formula, u = object distance, v = image distance, f = focal length
16 Electrical resistance (resistors in parallel)
17 Simple interest
18 Area of a triangle
19 Gravitational force of attraction
20 Work done by a force F moving over a distance d

Chapter 24: Functions and function notation

Starter 24 (page 464)

a) $2 + 3 = 5$ and $5 \times 4 = 20$
b) 5, since $5 + 3 = 8$ and $8 \times 4 = 32$
c) Alison started with 8 and did $8 \times 4 = 32$ then $32 + 3 = 35$
She should have done $8 + 3 = 11$ then $11 \times 4 = 44$

Exercise 24.1 (page 466)

1 a) $f : x \to 5x - 2$
b) (i) 13 **(ii)** 48
2 a) $g(x) = 2(x + 3)$
b) (i) 30 **(ii)** 4
3 a) 9 **b)** 13 **c)** 27
4 a) 7 **b)** 28 **c)** 7
5 a) 8 **b)** -1 **c)** 9.5

Exercise 24.2 (page 467)

1 {3, 5, 7, 9}
2 {3, 4, 6, 12}
3 {0, 1, 2, 3, 4, 5}
4 {–5, 5, 15, 25}
5 {6, 10, 14, 18, 22}
6 {1, 2, 5, 10, 17}
7 –1
8 All values of x smaller than 3
9 All real numbers greater than or equal to 6
10 $\{x: 1 \leqslant x \leqslant 41\}$

Exercise 24.3 (page 469)

1 a) 23 **b)** 8
2 7
3 $f^{-1}(x) = \dfrac{x - 11}{2}$
4 $h^{-1} : x \to 2(x - 3)$
5 a) (i) 11 **(ii)** 26
b) (i) 2 **(ii)** 7
6 a) $f^{-1}(x) = \dfrac{x - 7}{5}$ **b)** $g^{-1}(x) = \dfrac{1}{x - 3}$
c) $h^{-1}(x) = 2 - \dfrac{3}{x}$
7 a) 15 **b)** –3
8 a) 5 **b)** 7
c) $f^{-1}(x) = \sqrt{x - 1}$
9 a) 15 **b)** 1.5
c) $g^{-1}(x) = \dfrac{x - 3}{4}$
10 $x = 1.25$

Exercise 24.4 (page 471)

1 a) (i) 62 **(ii)** 70
b) $fg(x) = 15x + 2$
2 a) (i) 17 **(ii)** 52
b) $gf : x \to 2x^2 + 2$
3 a) $pq(x) = 4x + 5$
b) 0.5
4 $x = 1.25$
5 a) 6 **b)** –6 **c)** 49
6 $x = 3$ or -3
7 $x = 3.5$
8 a) $pq(x) = x$
b) p and q are inverses of each other

Review Exercise 24 (page 471)

1 a) (i) 8 **(ii)** 2 **(iii)** 0.1
b) (i) –1.5 **(ii)** –15 **(iii)** –2
2 a) (i) $\frac{1}{7}$ **(ii)** 4
b) $fg(x) = \dfrac{1}{x^2 - 2}$
c) 2
3 The function f is defined as $f(x) = 3x + 1$.
a) (i) 13 **(ii)** 2
b) $9x + 4$, i.e. $a = 9$ and $b = 4$
4 a) f {all real numbers}
g {all real numbers $\geqslant 0$}
h {all real numbers between 0 and 1 inclusive}
b) 5 and –3
c) 30
5 a) {all real numbers from 3 to 15 inclusive}
b) $m = 1$ and $n = 4$
6 a) (i) 4 **(ii)** 2
b) $pq(x) = x$
c) p and q are inverses of each other
7 a) (i) 5 **(ii)** 1
b) $f^{-1} : x \to \dfrac{1}{x}$
c) (i) $fg : x \to \dfrac{1}{4x - 1}$ **(ii)** 0.25
8 a) 0.5
b) $\{x: 0 \leqslant x \leqslant 1\}$
c) $x = 6$ or -1
9 a) (i) 5 **(ii)** 0
b) $f^{-1} : x \to \dfrac{x + 1}{2}$
c) (i) $gf : x \to \dfrac{3}{2x - 1}$ **(ii)** 0.5
10 $x = 10$ or 3

Internet Challenge 24 (page 474)

Α α	alpha	CONSTELLATION
Β β	beta	COMPUTER SOFTWARE
Γ γ	gamma	FACTORIAL
Δ δ	delta	QUADRATIC EQUATION
Ε ε	epsilon	BRAVE NEW WORLD
Ζ ζ	zeta	NUMBER THEORY
Η η	eta	DEDEKIND and DIRICHLET
Θ θ	theta	UNKNOWN ANGLE
Ι ι	iota	SEVEN VOWELS
Κ κ	kappa	JOHN BARROW
Λ λ	lambda	WAVELENGTH
Μ μ	mu	ONE MILLIONTH
Ν ν	nu	FREQUENCY
Ξ ξ	xi	SIXTY
Ο ο	omicron	SEVENTY
Π π	pi	CIRCUMFERENCE to DIAMETER
Ρ ρ	rho	STATISTICS
Σ σ	sigma	SUMMATION
Τ τ	tau	TORQUE
Υ υ	upsilon	PLANETARY SYSTEM
Φ ϕ	phi	NULL SET
Χ χ	chi	GOODNESS OF FIT
Ψ ψ	psi	PSYCHOLOGY
Ω ω	omega	SWAN or HORSESHOE

Chapter 25: Further trigonometry

Starter 25 (page 475)

By calculation, the height is 22.0 metres, to 3 significant figures. Scale drawings will scatter around this value.

Exercise 25.1 (page 478)

1 $a = 6.14$ cm, $b = 5.70$ cm
2 $c = 11.7$ cm, $d = 11.9$ cm
3 $e = 5.71$ cm **4** $f = 7.15$ cm **5** $g = 5.15$ cm
6 $h = 1.70$ cm **7** $i = 6.77$ cm **8** $j = 4.71$ cm
9 $p = 33.2°$ **10** $q = 48.7°$ **11** $r = 41.9°$
12 $s = 56.2°$

Exercise 25.2A (page 482)

1 6.27 cm **2** 5.85 cm **3** 6.16 cm
4 21.9 cm **5** 9.94 cm **6** 4.63 cm
7 60.6° **8** 56.6°
9 $r = 129.0°$, $s = 29.4°$ **10** 24.8°

Exercise 25.2B (page 483)

1 9.11 cm **2** 10.1 cm **3** 16.3 cm
4 13.9 cm **5** 20.2 cm **6** 5.48 cm
7 112.3° **8** 70.7° or 109.3°
9 58.5° **10** 50.8°

Exercise 25.3 (page 486)

1 86.8 cm^2 **2** 31.6 cm^2
3 a) 48.6° **b)** 19.0 cm
4 a) 117.3° **b)** 21.3 cm^2
5 a) 97.2° **b)** 35.2 cm^2
6 a) 31.6° **b)** 16.7 cm
7 a) 452 cm^2 **b)** 13.0 cm^2 **c)** 374 cm^2
8 45.3 cm^2

Exercise 25.4 (page 489)

1 a) 43.9 cm **b)** 34.7°
2 a) 28.3 cm **b)** 28.7 cm **c)** 60.5°
3 a) 6.3° **b)** 99.0° **c)** 5.8°
4 a) 24 cm **b)** 44.9°
5 a) 35.4 cm **b)** 36.2 cm **c)** 12.7°

Review Exercise 25 (page 490)

1 152 m^2
2 a) 28.9 cm^2 **b)** 9.40 cm
3 a) 11.7 cm **b)** 42.5°
4 18.3 cm **5** 177 m
6 a) 56.4 cm^2 **b)** 7.84 cm
7 b) 41.6 m **c)** 22.04 m
8 a) 9.11 cm **b)** 19.2°
9 22.2°

Internet Challenge 25 (page 493)

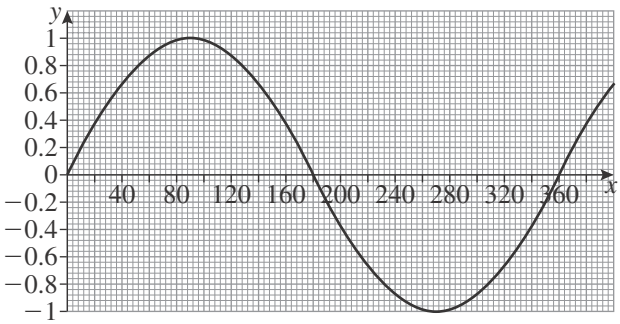
1 Area $= \sqrt{s(s-a)(s-b)(s-c)}$ where the triangle has sides a, b, c and semiperimeter s.
2 26.98 cm^2
3 85.45 cm^2, 87.00 cm^2, 44.90 cm^2 so triangle B is largest.
4 The formula was proved by Heron of Alexandria in the 1st century AD, but is thought to be rather older than this.
5 Various proofs exist, including one based on the cosine rule.

Chapter 26: Graphs of curves

Starter 26 (page 494)

x	0	30	45	60	90	120	135	150	180
$\sin x$	0	0.5	0.71	0.87	1	0.87	0.71	0.5	0

x	210	225	240	270	300	315	330	360	390
$\sin x$	−0.5	−0.71	−0.87	−1	−0.87	−0.71	−0.5	0	0.5



Exercise 26.1 (page 497)

1 $y = x^3 + x$

x	−2	−1	0	1	2	3
y	−10	−2	0	2	10	30

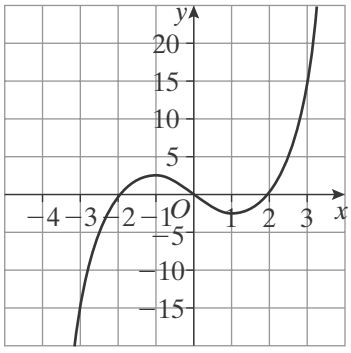
2 $y = x + \frac{1}{x}$

x	−2	−1	−0.5	0	0.5	1	2
y	−25	−2	−2.5	not defined	2.5	2	2.5

3 a)

x	−3	−2	−1	0	1	2	3
y	−15	0	3	0	−3	0	15

b) $y = x^2 - 4x$



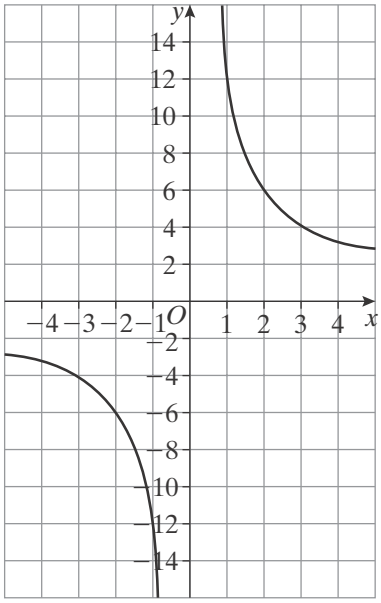
c) −2, 0, 2

d) 2.46

4 a)

x	−3	−2	−1	0	1	2	3	4
y	−4	−6	−12	not defined	12	6	4	3

b), c), d) $y = \frac{12}{x}$

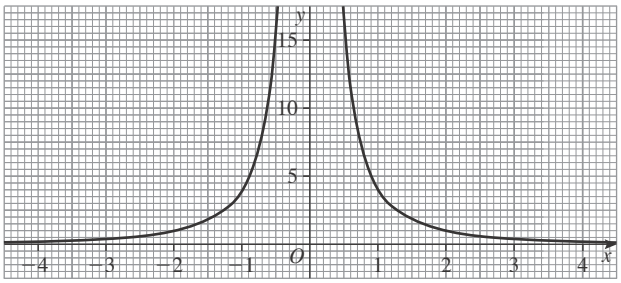


e) 1.33

5 a)

x	−4	−2	−1	−0.5	0.5	1	2	4
y	0.25	1	4	16	16	4	1	0.25

b)

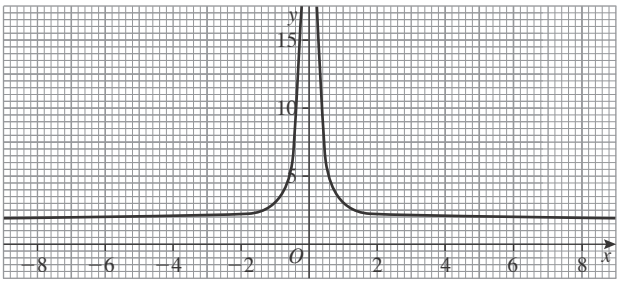


c) $x = 0.9$ or $x = -0.9$

6 a)

x	−4	−2	−1	−0.5	−0.25	0.25	0.5	1	2	4
y	2.06	2.25	3	6	18	18	6	3	2.25	2.06

b)



c) $x = 2.2$

7 a) 1.7, 5.3

b) 0.6, 6.4

8 1.25

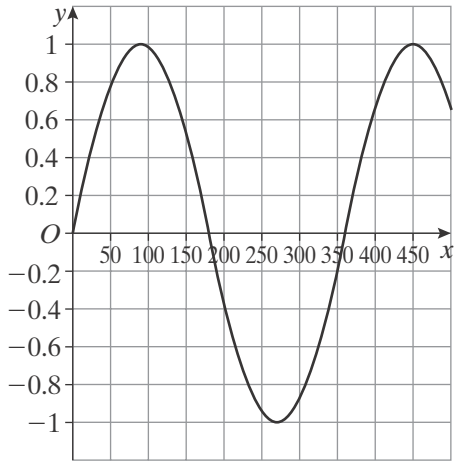
9

Equation	Graph
$y = 3 - x^2$	A
$y = x^3$	C
$y = \frac{3}{x^2}$	E
$y = \frac{3}{x}$	B
$y = x^2 - 3$	D
$y = -x^3$	F

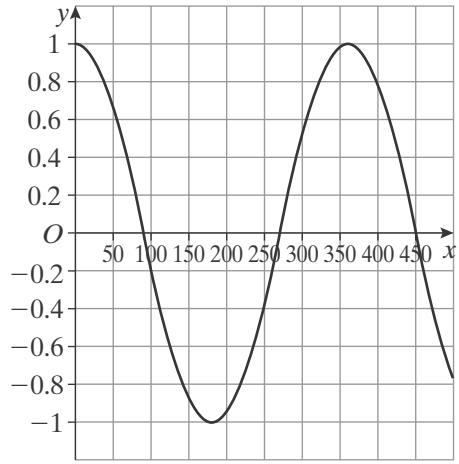
10 **a)** $y = 3$
b) $y = 2x - 1$
c) $y = 2 - 2x$

Exercise 26.2 (page 503)

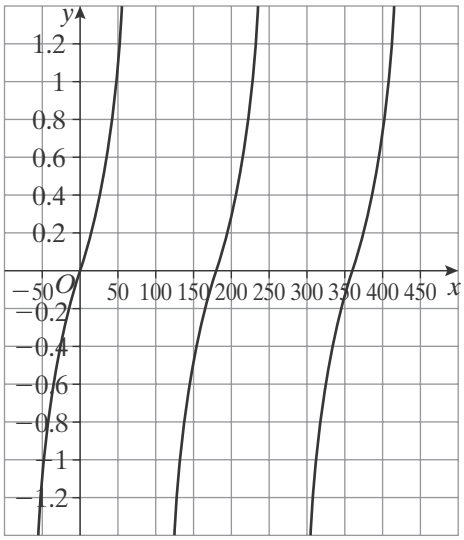
1 **a)** $y = \sin x$



b) $y = \cos x$



c) $y = \tan x$



2 **a)** True
b) False
c) False
d) False
e) True

3 **a)** $y = \cos x$
b) $y = \tan x$
c) $y = \sin x$
d) $y = \cos x$
e) $y = \sin x$

Exercise 26.3 (page 507)

1 **a)** translation by the vector $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$

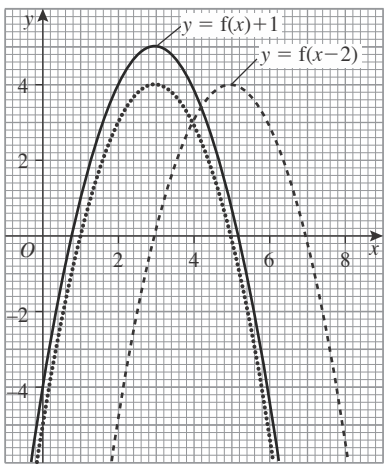
b) translation by the vector $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$

c) translation by the vector $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$

d) translation by the vector $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$

e) translation by the vector $\begin{pmatrix} 3 \\ -3 \end{pmatrix}$

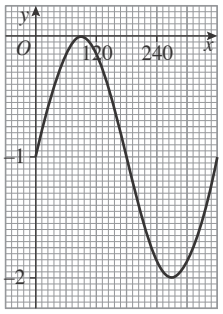
2



3 **a)** (3. 0)
b) (5, −1)
c) (5, 0)

4 **a)** $a = 2$

b)



5 $A(0, \frac{1}{2})$, $B(30, 0)$, $C(120, -1)$, $D(210, 0)$, $E(300, 1)$

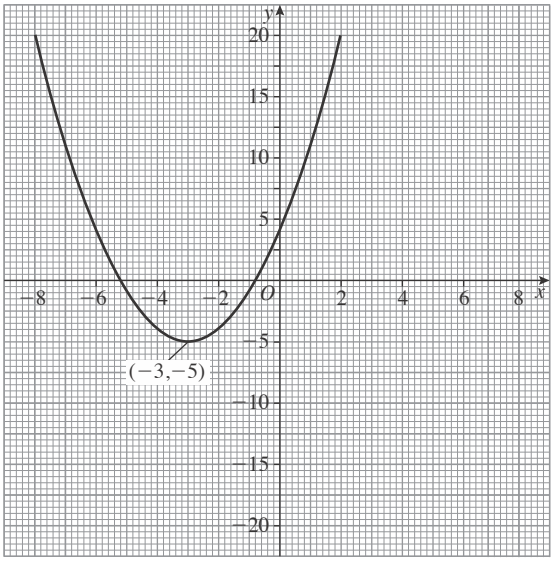
6 **a)** $y = 2x + 2$
b) $y = 2x - 7$
c) $y = 2x + 3$
d) $y = 2x + 2$

7 **a)** $y = x^2 - 1$
b) $y = (x - 1)^2$
c) $y = (x - 1)^2 + 1$

8 **a)** $y = (x - 2)^2 + 3$
b) A translation by the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$
c) (2, 3)
d) The turning point is a minimum, and (2, 3) is above the x axis. Hence the graph never intersects the x axis.

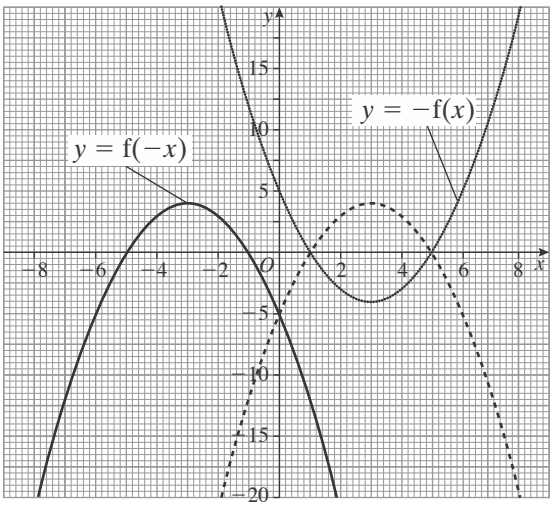
9 **a)** $y = (x + 3)^2 - 5$
b) A translation by the vector $\begin{pmatrix} -3 \\ -5 \end{pmatrix}$

c)



Exercise 26.4 (page 516)

1



2 **a)** (−4, −7) **b)** (−2, −21)
c) (2, 7) **d)** (−2, −7)

3 **a)** $-f(x) = 2 - 3x$ **b)** $f(-x) = -3x - 2$
c) $3f(x) = 9x - 6$ **d)** $f(2x) = 6x - 2$

4 **a)** $f(-x) = x^2 - x$ **b)** $-f(x) = -x^2 - x$
c) $2f(x) = 2x^2 + 2x$ **d)** $f(2x) = 4x^2 + 2x$

5 **a)** $y = 1 - 4x$
b) $y = -4x - 1$

6 **a)** $a = ½$ **b)** $b = 2$

Review Exercise 26 (page 517)

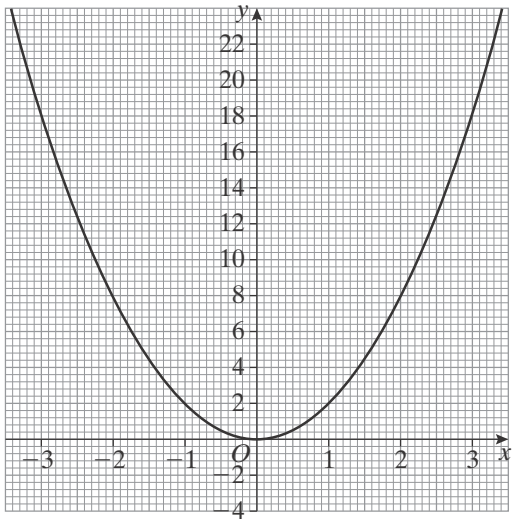
1

Equation	Graph
$y = \sin x$	B
$y = \cos x$	E
$y = \frac{1}{x^2}$	D
$y = \frac{2}{x}$	C
$y = \tan x$	F
$y = x^2 - 3x + 1$	A

2 **a)**

x	−3	−2	−1	0	1	2	3
y	18	8	2	0	2	8	18

b) $y = 2x^2$

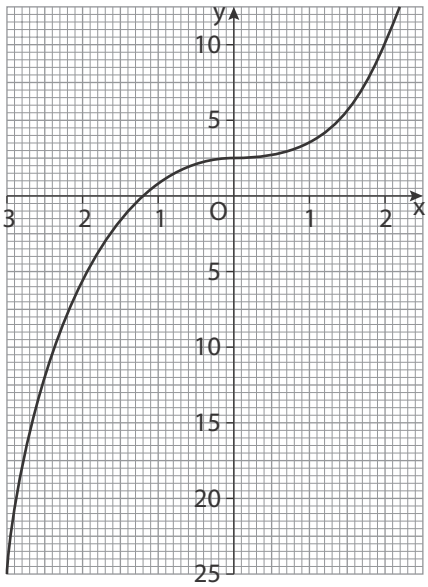


c) i) 12.5
ii) 2.45 and −2.45

3 a)

x	−3	−2	−1	0	1	2
$y = x^3 + 2$	−25	−6	1	2	3	10

b) $y = x^2 + 2$
c) i) −1.3
ii) 1.8

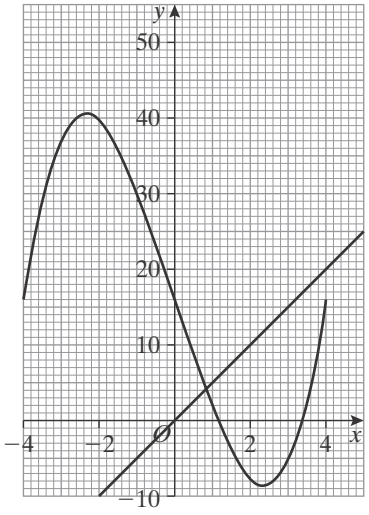


4 a) 0.2, 2.1 **b)** 2.6

5 a)

x	–4	–3	–2	–1	0	1	2	3	4
y	16	**37**	40	**31**	16	**1**	–8	**–5**	**16**

b)



c) 0.8 **d)** $x^3 - 21x + 16 = 0$

6 a) 0.4 and 2.6 **b)** 0.1 and 4.9

7 a) $y = 2\sin x^\circ$ **b)** $y = \frac{1}{2}\sin x^\circ$

8 a) $y = \cos\frac{1}{2}x^\circ$ **b)** $y = \cos 2x^\circ$

9 a) A translation by the vector $\begin{pmatrix}0\\1\end{pmatrix}$.

b) A translation by the vector $\begin{pmatrix}1\\0\end{pmatrix}$.

c) A translation by the vector $\begin{pmatrix}-1\\0\end{pmatrix}$.

d) A translation by the vector $\begin{pmatrix}1\\1\end{pmatrix}$.

e) A reflection in the y axis
f) A reflection in the x axis

10 a) (2, 3)
b) (2, 8)
c) (2, −4)
d) (4, 5)
e) (−2, 4)
f) (1, 12)

11 a) $y = 2x - 2$
b) $y = 2x - 11$
c) $y = -2x - 5$
d) $y = 6x - 15$

Internet Challenge 26 (page 524)

conchoid	equiangular spiral	double folium
trifolium	Archimedean spiral	cardioid
limacon of Pascal	rose curve	lemniscate

Chapter 27: Vectors

Starter 27 (page 525)

There are many ways of completing a knight's tour; here is one way:

Partly complete

2	7	4	11				
13	10	3	8				
6	1	12	15				
17		9	4				
	5	16					

Complete

2	7	4	11	28	39	46	43
13	10	3	8	45	42	29	40
6	1	12	15	38	27	44	47
17	22	9	4	53	48	41	30
64	5	16	21	26	37	54	49
23	18	59	62	57	52	31	34
60	63	20	25	36	33	50	55
19	24	61	58	51	56	35	32

Exercise 27.1 (page 527)

1 a) $\begin{pmatrix}2\\3\end{pmatrix}$ **b)** $\begin{pmatrix}3\\-2\end{pmatrix}$

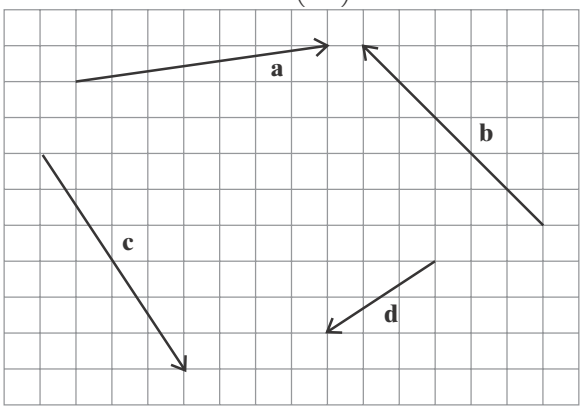
c) $\begin{pmatrix}-2\\-2\end{pmatrix}$ **d)** $\begin{pmatrix}0\\3\end{pmatrix}$

e) $\begin{pmatrix}1\\-4\end{pmatrix}$ **f)** $\begin{pmatrix}-4\\1\end{pmatrix}$

g) $\begin{pmatrix}1\\-5\end{pmatrix}$ **h)** $\begin{pmatrix}3\\2\end{pmatrix}$

i) $\begin{pmatrix}7\\-2\end{pmatrix}$ **j)** $\begin{pmatrix}-2\\4\end{pmatrix}$

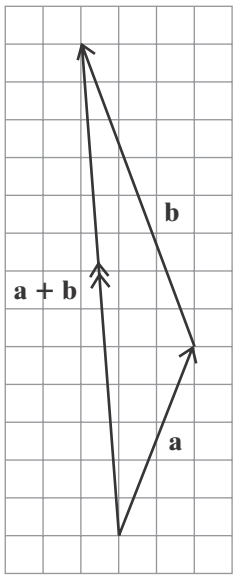
2



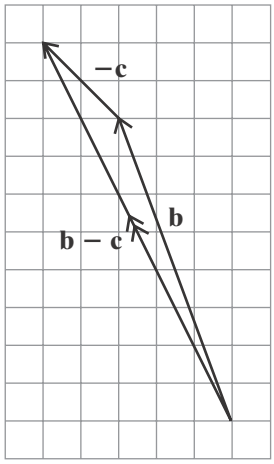
3 a) 5 **b)** 13 **c)** 15 **d)** 25 **e)** 17

Exercise 27.2 (page 529)

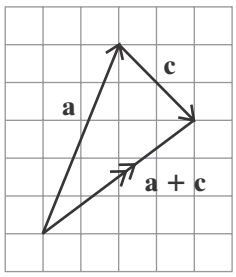
1 $\begin{pmatrix}-1\\13\end{pmatrix}$



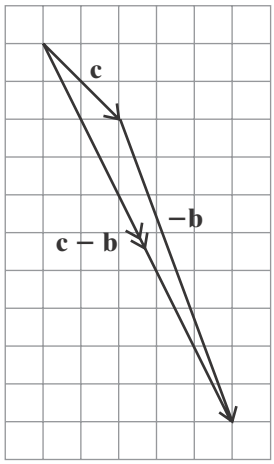
2 $\begin{pmatrix}-5\\10\end{pmatrix}$



3 $\begin{pmatrix}4\\3\end{pmatrix}$

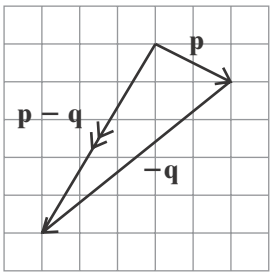


4 $\begin{pmatrix}5\\-10\end{pmatrix}$

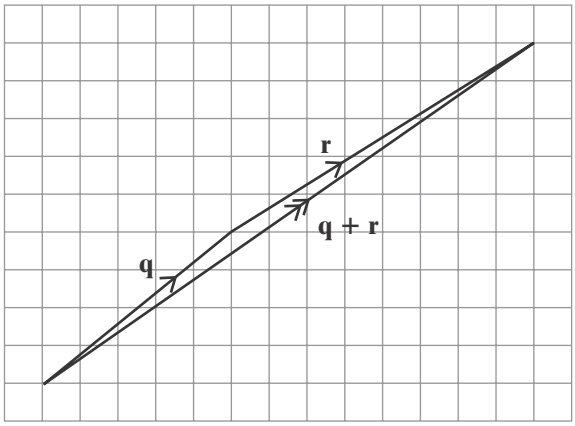


5 $\begin{pmatrix} -3 \\ 15 \end{pmatrix}$ **6** $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$

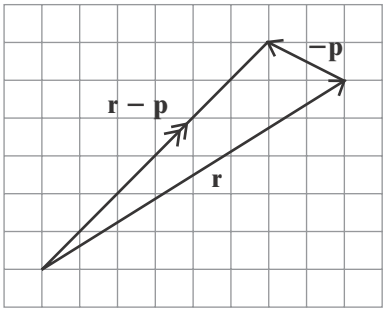
7 $\begin{pmatrix} -3 \\ -5 \end{pmatrix}$



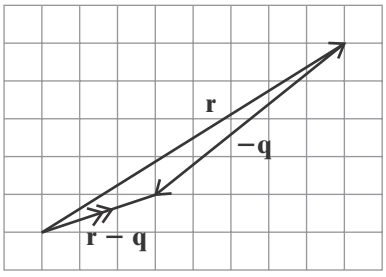
8 $\begin{pmatrix} 13 \\ 9 \end{pmatrix}$



9 $\begin{pmatrix} 6 \\ 6 \end{pmatrix}$



10 $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$



11 $\begin{pmatrix} 15 \\ 8 \end{pmatrix}$ **12** $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$

13 $x = 3$
14 $x = 9, y = 7$
15 $x = -3, y = 15$

Exercise 27.3 (page 530)

1 $\begin{pmatrix} 12 \\ -6 \end{pmatrix}$ **2** $\begin{pmatrix} 1 \\ 11 \end{pmatrix}$ **3** $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

4 $\begin{pmatrix} -8 \\ -22 \end{pmatrix}$ **5** $\begin{pmatrix} 21 \\ 17 \end{pmatrix}$ **6** $\begin{pmatrix} 12 \\ -8 \end{pmatrix}$

7 $\begin{pmatrix} -10 \\ 25 \end{pmatrix}$ **8** $\begin{pmatrix} -9 \\ 3 \end{pmatrix}$ **9** $\begin{pmatrix} 12 \\ -17 \end{pmatrix}$

10 $\begin{pmatrix} -5 \\ 27 \end{pmatrix}$ **11** $\begin{pmatrix} 12 \\ -17 \end{pmatrix}$ **12** $\begin{pmatrix} -13 \\ 17 \end{pmatrix}$

13 $x = -3$
14 $x = 1, y = 22$
15 $x = 3, y = -1$

Exercise 27.4 (page 533)

1 a) $\mathbf{q}$ **b)** $\mathbf{p} + \mathbf{q}$ **c)** $2\mathbf{q}$ **d)** $\mathbf{p} + 2\mathbf{q}$

2 a) $-\mathbf{a}$ **b)** $\mathbf{a} + \mathbf{b}$ **c)** $2\mathbf{b}$ **d)** $-\mathbf{a} + 2\mathbf{b}$

3 a) i) $-\mathbf{p} + \mathbf{q}$ **ii)** $3\mathbf{p}$
iii) $3\mathbf{q}$ **iv)** $-3\mathbf{p} + 3\mathbf{q}$

b) $\overrightarrow{PQ} = -\mathbf{p} + \mathbf{q}$, $\overrightarrow{BC} = -3\mathbf{p} + 3\mathbf{q} = 3 \times \overrightarrow{PQ}$ so BC and PQ are parallel.

4 a) i) $\begin{pmatrix} 4 \\ 7 \end{pmatrix}$ **ii)** $\begin{pmatrix} 4 \\ 7 \end{pmatrix}$

b) AB and DC are the same length, and parallel.

c) i) $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$ **ii)** $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$

d) Parallelogram

5 a) i) $-\mathbf{a} + \mathbf{b}$ **ii)** $-\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$
iii) $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

b) SR is parallel to PQ, and equal in length, since PQRS is a parallelogram.

c) $\mathbf{a} + \mathbf{b}$

d) Since $\overrightarrow{PR} = 2 \times \overrightarrow{PE}$ it follows that E is the midpoint of PR. But E is also the midpoint of QS (given). Thus the diagonals of a parallelogram bisect each other.

6 a) i) $\begin{pmatrix} 12 \\ 6 \end{pmatrix}$ **ii)** $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$

b) Since $\overrightarrow{AB} = 3 \times \overrightarrow{DC}$ it follows that AB and DC are parallel.

c) Trapezium

7 a) $\overrightarrow{QR} = \overrightarrow{QP} + \overrightarrow{PS} + \overrightarrow{SR} = -2\mathbf{a} + 2\mathbf{b} + 2\mathbf{c}$
b) $\mathbf{b} + \mathbf{c}$
c) $\mathbf{b} + \mathbf{c}$
d) They are parallel, and the same length.
e) Parallelogram

Review Exercise 27 (page 535)

1 $x = 6, y = 11$

2 a) $\overrightarrow{PQ} = \begin{pmatrix} -6 \\ 8 \end{pmatrix}$ and $\overrightarrow{QP} = \begin{pmatrix} 6 \\ -8 \end{pmatrix}$

b) 10

3 a) (i) $\begin{pmatrix} -4 \\ -3 \end{pmatrix}$ **(ii)** 5

b) $\begin{pmatrix} -2 \\ 6 \end{pmatrix}$

c) $(-6, 2)$

4 a) $\frac{1}{2}\mathbf{p} + \frac{1}{2}\mathbf{q}$

b) $\overrightarrow{RS} = \frac{1}{2}\mathbf{q} = \frac{1}{2}\overrightarrow{OQ}$ so RS and OQ are parallel.

5 a) $-\mathbf{a} + \mathbf{b}$ **b)** $\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$

6 a) $2\mathbf{a} + 4\mathbf{c}$

b) $\overrightarrow{OM} = 3\mathbf{a} + 6\mathbf{c} = 1\frac{1}{2} \times \overrightarrow{OP}$ so OP is parallel to OM. Since both OP and OM pass through O and are parallel, OPM is a straight line.

7 a) $\mathbf{a} + \frac{1}{2}\mathbf{b}$ **b)** $-\frac{2}{3}\mathbf{a} + \frac{1}{2}\mathbf{b}$

8 a) i) $-\mathbf{a} + \mathbf{b}$ **ii)** $2\mathbf{b} + 2\mathbf{c}$

iii) $-2\mathbf{a} + 2\mathbf{b} + 2\mathbf{c}$ **iv)** $-\mathbf{a} + \mathbf{b}$

b) KN and LM are parallel and equal in length.

9 a) i) $-6\mathbf{a} + 6\mathbf{b}$ **ii)** $6\mathbf{a}$

b) $-3\mathbf{a} + 12\mathbf{b}$

c) $\overrightarrow{EY} = -4\mathbf{a} + 16\mathbf{b}$ so $\overrightarrow{EY} = \frac{4}{3}\overrightarrow{EX}$

10 a) i) $\mathbf{a} + \mathbf{b}$ **ii)** $2\mathbf{a} - \mathbf{b}$

b) $2\mathbf{a} + \frac{1}{2}\mathbf{b}$

c) $\mathbf{a} + \mathbf{b}$

11 a) 2.5

b) 10

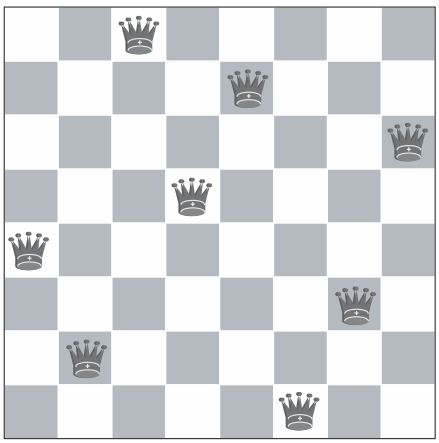
12 a) Vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are parallel

b) 18 cm

Internet Challenge 27 (page 539)

1 12 distinct solutions (ignoring rotations/reflections)

2 1, illustrated below.

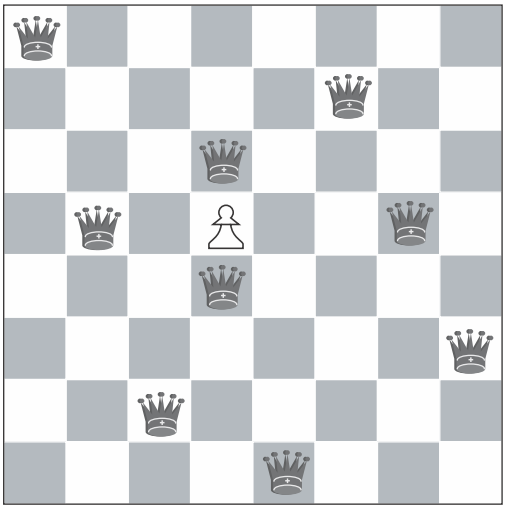


3 A Latin square is an n by n square grid in which each row (or column) contains the same n distinct symbols; any particular symbol occurs exactly once in each row and column (like a Sudoku). The eight Queens puzzle resembles the Latin square in some ways, although it is not a true Latin square.

4 32 knights (an easy solution is to put them all on squares of the same colour).

5 14 bishops

6



Chapter 28: Calculus

Starter 28 (page 540)

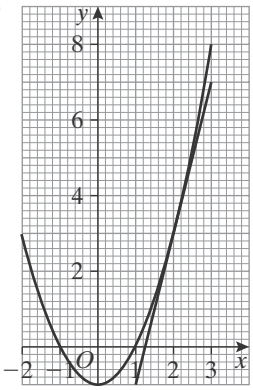
D (−0.4), A ($\frac{1}{2}$), C (1), B (2)

Exercise 28.1 (page 543)

1 a) 0 **b)** 2

2 a) 1 **b)** −3

3 a)



b) 4

4 a) 8 seconds **b)** 48 metres

c) 8.6 metres per second

Exercise 28.2 (page 545)

1 $3x^2 + 12x$ **2** $4x^3 - 6x^2 + 2x$

3 $8x^3 + 9x^2 + 10$ **4** $5x^4 - 10x$

5 $12x^3 - 12x^2$ **6** $2x - 6$

7 $10x^9 + 9x^2$ **8** $18x^2 + 12$

9 $27x^2 + 4$ **10** $3x^2 - 4x^3 + 10x^4$

Exercise 28.3 (page 546)

1 $8x + 5 - \frac{1}{x^2}$ **2** $8 - \frac{8}{x^3}$

3 $2x + \frac{3}{x^2}$ **4** $2 - \frac{4}{x^2} - \frac{10}{x^3}$

5 a) $2x^2 + 7x + 3$ **b)** $2x + 7$

6 a) $2x^2 - 3x - 5$ **b)** $4x - 3$

7 $\frac{2x + 5}{3}$ **8** $\frac{6x^2 + 6x - 5}{6}$

9 2 **10** −1

Exercise 28.4 (page 550)

1 (3, 2) **2** (2, –1)

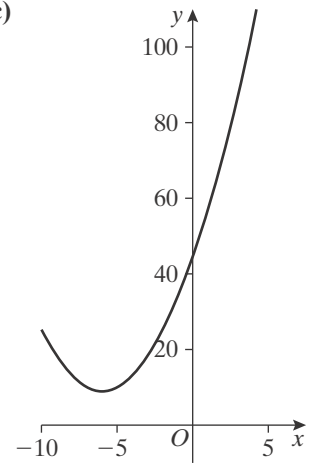
3 (5, 2) **4** (1, 4)

5 a) $6x + 6$ **b)** (–1, 2) **c)** Minimum

6 a) (4, 4) **b)** Maximum

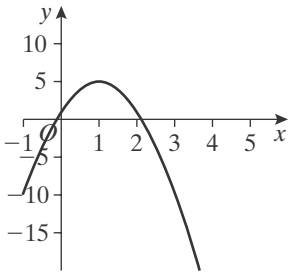
7 a) $2x + 12$ **b)** (–6, 9)

c)



8 a) $-8x + 8$ **b)** (1, 5)

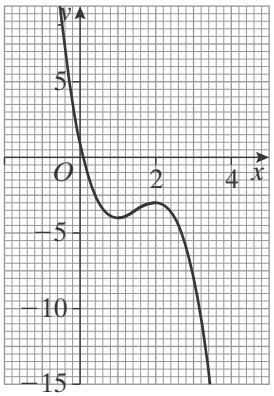
c)



9 a) $\frac{dy}{dx}$ $-12 + 18x - x^2$

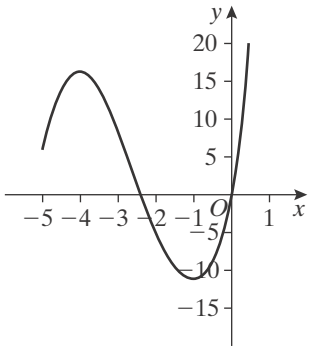
b) (2, –3) and (1, –4)

c)



10 a) $6x^2 + 30x + 24$ **b)** (–4, 16) and (–1, –11)

c)



11 ($\frac{1}{2}$, 12)

12 a) ($\frac{1}{2}$, 4), ($-\frac{1}{2}$, −4)

b) minimum: ($\frac{1}{2}$, 4); maximum: ($-\frac{1}{2}$, −4)

13 a) $y = x^3 - 6x^2 + 12x - 8$

b) $\frac{dy}{dx} = 3x^2 - 12x - 12$

c) (2, −4)

Exercise 28.5 (page 553)

1 a) $-2v + 100$ **b)** 50

2 a) 2000 **b)** $80 - 2t$ **c)** 3600

3 a) (i) 150 **(ii)** 75

b) $2n - 40$

c) Minimum value of P is 50, when the value of $n = 20$

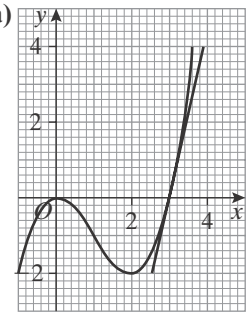
4 a) (i) 91 **(ii)** 96
b) $16 - 2t$ **c)** 100
5 a) $4 - 2z$
b) Maximum value of Q is 100, when $z = 2$
6 b) $V = 8x^2 - 2x^3$ and so $\frac{dV}{dx} = 16x - 6x^2$
c) 18.96 cm^3
7 a) $60 - 2x$
c) $A = 60x - 2x^2$ and so $\frac{dA}{dx} = 60 - 4x$
d) 15 **e)** 15 metres by 30 metres
8 a) $1200 - \frac{19\,200}{t^2}$
b) Minimum value of M is 9600, when the value of $t = 4$

Exercise 28.6 (page 555)

1 a) (i) $v = 3t^2 + 12t + 5$
(ii) $a = 6t + 12$
b) 41 m s^{-1}
c) 30 m s^{-2}
2 a) $v = 3t^2 + 16$ **b)** 43 m s^{-1} **c)** 24 m s^{-2}
3 a) 111 m s^{-1} **b)** 40 m s^{-2}
4 a) 2 m s^{-1} **b)** 4 seconds
5 a) 2 m s^{-1} **b)** 9.8 m s^{-2}

Review Exercise 28 (page 555)

1 a)



b) 4.5
2 $3x^2 + 10x$ **3** $2x + 8$
4 $\frac{x^2}{3} + \frac{3x}{2} - \frac{2}{5}$ **5** $8x + 10 - \frac{1}{x^2}$
6 $-\frac{2}{x^2} - \frac{2}{x^3}$ **7** $\frac{3x^2 + 10x + 4}{10}$
8 48
9 $2x^2 + 11x + 15$, $4x + 11$
10 3
11 $6x^2 - 11x + 3$, $12x - 11$
12 a) $2x - 10$ **b)** 5 **c)** –24
13 a) $2x - 10$ **b)** 5 **c)** (5, –21)
14 a) $4 - 2x$ **b)** (2, 9)
c) Maximum, since x^2 coefficient is negative
15 a) $2x^2 + x - 15$, $4x + 1$ **b)** –0.25
c) Parabola A, since x^2 coefficient is positive
16 a) $3z^2 - 12z$ **b)** 0 and 4 **c)** 0 and –32
17 2
18 a) 17 m s^{-1} **b)** 30 m s^{-2}
19 a) 36 m s^{-1} **b)** 0.5 seconds
20 a) 4 m s^{-1} **b)** –4 m s^{-2}
c) 2 m s^{-1}, when the time is 3 seconds
21 a) (i) $2x - 4$ **(ii)** (2, –3)
b) Minimum, since x^2 coefficient is positive
c) $x = 2$
22 a) $v = 3t^2 + 8t - 5$ **b)** 20 m s^{-2}

Internet Challenge 28 (page 559)

						1 **D**	I	F	F	E	R	E	N	T	I	A	T	I	O	N
						2 **L**	E	I	B	N	I	Z								
					3 **I**	N	T	E	G	R	A	T	I	O	N					
						4 **M**	A	X	I	M	U	M								
5 **P**	O	S	I	T	I	V	E													
	6 **C**	O	N	S	T	A	N	T												
						7 **T**	A	N	G	E	N	T								
					8 **M**	I	N	I	M	U	M									
						9 **V**	E	L	O	C	I	T	Y							
					10 **N**	E	W	T	O	N										