

Chapter 1

Discussion point (page 3)

Since odd numbers cannot be written as the sum of two odd numbers, the only way of writing an odd number as the sum of two primes is when one of the two primes is 2. There are many odd numbers that cannot be written as the sum of 2 and a prime number, e.g. 11.

Discussion point (page 3)

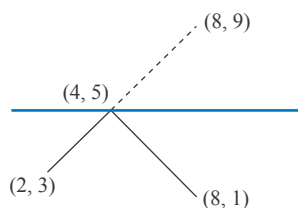
There are many prime numbers which are not 5.

Discussion point (page 3)

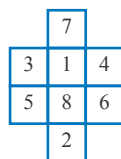
For example, 10 is even but 5 is not.

Exercise 1.1 (page 4)

- 1 (i) $\Rightarrow$
(ii) $\Leftrightarrow$
(iii) $\Rightarrow$
(iv) $\Leftarrow$
- 2 (i) No, because BC and AD may not be parallel
(ii) ABCD is a parallelogram $\Rightarrow$ AB is parallel to CD. The converse is true.
- 3 (ii) The total distance if R is (5,5) is $5 + \sqrt{13}$, but if R is (4,5) the total distance is $6\sqrt{2}$ which is smaller.
(iii) The best position for R is (4,5). The distance is equivalent to the case where Q is (8,9) (see diagram below), and (4,5) is on the straight line $y = x + 1$ between (2,3) and (8,1).



- 4 $a = 4, b = 12, c = 16, d = 5$
- 5 There are 4 different arrangements, one of which is shown below.



- 7 (i) $14 + 12^2 + 7^3$ and $15 + 19^2 + 5^3$

Discussion point (page 7)

No, reversing 90 does not give a two-digit number.

Exercise 1.2 (page 10)

- 1 True – direct proof
- 2 True – direct proof
- 3 True – proof by exhaustion
- 4 False – e.g. a hexagon (6 sides) has 9 diagonals
- 5 True – direct proof
- 6 True – proof by contradiction
- 7 (i) True – direct proof
(ii) True – direct proof
- 8 (i) True – direct proof
(ii) True – direct proof
- 9 False – e.g. $n = 10$
- 10 True – proof by contradiction
- 11 (i) True – direct proof
(ii) False – e.g. $1^2 + 2^2 + 3^2 + 4^2 = 30$
- 12 True – direct proof
- 16 Numbers of the form 2^n
- 17 (i) If N^2 is the sum of the squares of two integers, then N is the sum of the squares of two different integers.

- (ii) False – e.g. $N^2 = 225 = 12^2 + 9^2$, but $N = 15$ is not the sum of the squares of two integers.

Chapter 2

Opening activity (page 12)

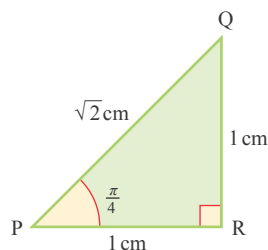
174.5 m

Exercise 2.1 (page 16)

- 1 (i) $\frac{\pi}{4}$
(ii) $\frac{\pi}{2}$
(iii) $\frac{2\pi}{3}$
(iv) $\frac{5\pi}{12}$
(v) $\frac{5\pi}{3}$
(vi) 0.4 radians
(vii) $\frac{5\pi}{2}$
(viii) 3.65 rad
(ix) $\frac{5\pi}{6}$
(x) $\frac{\pi}{25}$
- 2 (i) 18°
(ii) 135°
(iii) 114.6°
(iv) 80°
(v) 540°
(vi) 240°
(vii) 28.6°
(viii) 450°
(ix) 420°
(x) 77.1°
- 3 (i) (a) 1 cm
(b) $\sqrt{3}$ cm
(ii) (a) $\frac{\pi}{3}$
(b) $\frac{\pi}{6}$

- (iii) (a) $\frac{\sqrt{3}}{2}$
 (b) $\frac{1}{2}$
 (c) $\sqrt{3}$
 (d) $\frac{1}{2}$
 (e) $\frac{\sqrt{3}}{2}$
 (f) $\frac{\sqrt{3}}{3}$

4



$PQ = \sqrt{1^2 + 1^2} = \sqrt{2}$, the triangle is a right-angled so $\angle QPR + \angle PQR = \frac{\pi}{2}$ and the triangle is isosceles so $\angle QPR = \angle PQR = \frac{\pi}{4}$

- (i) $\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
 (ii) $\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
 (iii) $\tan \frac{\pi}{4} = \frac{1}{1} = 1$
- 5 $\sin \frac{9\pi}{4} = \cos \left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$
 $\tan \left(-\frac{5\pi}{3}\right) = \tan \frac{4\pi}{3} = \sqrt{3}$
 $\cos \frac{11\pi}{6} = \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$
 $\cos \pi = \tan \frac{3\pi}{4} = -1$
 $\sin \frac{5\pi}{6} = \cos \frac{5\pi}{3} = \frac{1}{2}$
- 6 $x = 0.6$ and $x = 2.5$ to 1 d.p.
- 7 $x = 0.8$ and $x = 3.9$ to 1 d.p.
- 8 (i) $k = \pi$
 (ii) $k = \frac{\pi}{2}$
 (iii) $k = \pi$
 (iv) $k = \frac{\pi}{2}$

- 9 (i) $x = \frac{2\pi}{7}$
 (ii) $y = \frac{8\pi}{5}$
 (iii) $k = \frac{1}{3}$

Activity 2.1 (page 18)

	Radians	Degrees
Angle	θ°	$\alpha^\circ \left(\alpha = \theta \times \frac{180}{\pi}\right)$
Arc length	$r\theta$	$r\alpha \times \frac{\pi}{180}$
Area of sector	$\frac{1}{2}r^2\theta$	$\frac{1}{2}r^2\alpha \times \frac{\pi}{180}$

Exercise 2.2 (page 20)

- 1 (i) (a) 3.14 cm
 (b) 9.14 cm
 (c) 4.71 cm²
 (ii) (a) 22.0 cm
 (b) 30.0 cm
 (c) 44.0 cm²

2

$r(\text{cm})$	$\theta(\text{rad})$	$s(\text{cm})$	$A(\text{cm}^2)$
4	$\frac{1}{2}$	2	4
$1\frac{1}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{3\pi}{8}$
5	0.8	4	10
1.875	0.8	1.5	1.41
3.46	$\frac{2\pi}{3}$	7.26	4π

- 3 (i) 140 yards
 (ii) 5585 square yards
- 4 (i) $15\pi \text{ cm}^2$
 (ii) 9 cm^2
 (iii) $(15\pi - 9) \text{ cm}^2$
- 7 1.05 cm^3
- 8 (i) 16.9 cm^2
 (ii) 19.7 cm^2
- 9 (i) 1.98 mm^2
 (ii) 43.0 mm

Discussion point (page 22)

Because the formula *area of a sector* $= \frac{1}{2} r^2 \theta$ assumes θ is in radians.

Discussion point (page 23)

By the shape and symmetry of the graphs, in each case the maximum percentage error will occur for $\theta = 0.1$ radians.

$y = \sin \theta$; $\theta = 0.1$ rad
 true value = 0.099 833
 approximate value = 0.1
 % error = 0.167%
 $y = \tan \theta$; $\theta = 0.1$ rad
 true value = 0.100 335
 approximate value = 0.1
 % error = 0.334%
 $y = \cos \theta$; $\theta = 0.1$ rad
 true value = 0.995 004
 approximate value = 0.995
 % error = 0.000 419%

Exercise 2.3 (page 24)

- 1 (i) θ^2
 (ii) $\frac{\theta^2}{2}$
 (iii) $(1 - 2\theta^2)$
 (iv) 2θ
- 2 (i) 2
 (ii) $1 - \frac{5\theta^2}{2}$
 (iii) $\frac{1}{2}$
 (iv) -3θ
- 3 (i) 5θ
 (ii) 5
- 4 (i) $\frac{\theta^2}{2}$
 (ii) $\frac{1}{8}$
- 5 (i) $8\theta^2$
 (ii) $4\theta^2$
 (iii) 2
- 6 $\theta \approx 0.02$ ($\theta = -10$ is not small)
- 7 0.102 radians; the other roots are not small angles
- 8 (ii) $\angle BAE = 90^\circ - \angle OAB$

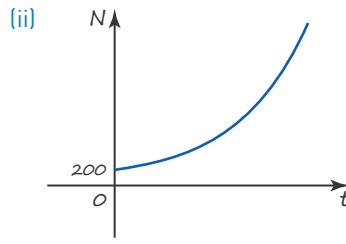
Review: Algebra (1)

Exercise R.1 (page 29)

- 1 (i) $6 - \sqrt{3}$
(ii) $-1 + 5\sqrt{2}$
(iii) $17 - 12\sqrt{2}$
(iv) 1
- 2 (i) $3\sqrt{3}$
(ii) $\sqrt{5} - 1$
(iii) $\frac{16 + 10\sqrt{2}}{7}$
(iv) $\frac{5(3\sqrt{5} + 4)}{29}$
- 3 (i) $\frac{1}{32}$
(ii) 3
(iii) 125
(iv) $\frac{3}{4}$
- 4 (i) x^7
(ii) x
(iii) x^{-6}
(iv) x^4
(v) x^4
- 5 (i) $6x^5y^5$
(ii) x^2y^4
(iii) $3x^4y^2$
(iv) $(1 + 3x)(1 + x)^2$

Exercise R.2 (page 33)

- 1 (i) $\log 9$
(ii) $\log 48$
(iii) $\log \frac{2}{3}$
- 2 (i) $2 \log x$
(ii) $6 \log x$
(iii) $\frac{3}{2} \log x$
- 3 (i) $x = -1.15$ (3 s.f.)
(ii) $x = 0.222$ (3 s.f.)
(iii) $x = 1.20$ (3 s.f.)
(iv) $x = 2$
- 4 (i) $x = 99999$
(ii) $x = \sqrt{5}$
(iii) $x = e^4 - 3$
(iv) $x = \sqrt{5}$
- 5 (i) $x = c^a + b$
(ii) $x = \frac{\ln s - q}{p}$
- 6 (i) 200



- (iii) 4017 bees
(v) The model is not suitable in the long term since not enough room, no allowance made for bees dying, etc.
- 7 (ii) $p = 1.4, q = 0.8$
8 (ii) $k = 5, n = 0.5$

Chapter 3

Discussion point (page 36)

- (i) (a) Avonford Savings
(b) 80 000, 160 000, 320 000, ...
(c) Exponential geometric sequence
(d) The sequence could go on but the family will not live forever.
- (ii) (a) Pizza outlet opening hours
(b) 10, 10, 10, 10, 12 ...
(c) There is a cycle that repeats every week.
(d) The sequence will continue as long as they keep the same opening hours.
- (iii) (a) The clock
(b) 0, -3.5, -5, -3.5, ...
(c) The pattern of 8 numbers keeps repeating.
(d) The sequence continues for as long as the clock is working.
- (iv) (a) The stairs
(b) 120, 140, 160, ...
(c) Increases by 20 each time
(d) The steps can't go on forever.

Exercise 3.1 (page 41)

- 1 (i) 19, 22, 25, 28, ...
Arithmetic, 1st term $a = 7$, common difference $d = 3$
(ii) 4, 3, 2, 1, ... Arithmetic, 1st term $a = 8$, common difference $d = -1$
(iii) 3.3, 3.1, 2.9, 2.7, ...
Arithmetic, 1st term $a = 4.1$, common difference $d = -0.2$
(iv) 48, 96, 192, 384, ...
Geometric, 1st term $a = 3$, common ratio $r = 2$
(v) 4, 2, 1, 0.5, ... Geometric, 1st term $a = 64$, common ratio $r = 0.5$
(vi) 16, -32, 64, -128, ...
Geometric, 1st term $a = 1$, common ratio $r = -2$
(vii) 2, 2, 2, 5, ... Periodic with period 4
(viii) 1, 3, 5, 3, ... Periodic with period 4, oscillating about 3
- 2 (i) 3, 5, 7, 9, ...
(ii) 6, 12, 24, 48, ...
(iii) 4, 8, 14, 24, ...
(iv) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$
(v) 4, 6, 4, 6, ...
- 3 (i) 12, 15, 18, 21, ...
(ii) -5, 5, -5, 5, ...
(iii) 72, 36, 18, 9, ...
(iv) 1, 4, 9, 16, ...
(v) 4, 6, 4, 6, ...
- 4 (i) 58
(ii) 90
(iii) 25
(iv) 8
(v) 40
- 5 (i) $\sum_{k=1}^{10} k$
(ii) $\sum_{k=1}^{10} (20 + k)$
(iii) $\sum_{k=1}^{10} (200 + 10k)$

- (iv) $\sum_{k=1}^{10} (200 + 11k)$
 (v) $\sum_{k=1}^{10} (200 - 10k)$
- 6** (i) 15
 (ii) 400
 (iii) -20
 (iv) 100
 (v) 220
- 7** (i) 0, 1, 1, 2, 3, ... After the first term this is the same as the sequence itself.
 (ii) 13, 21, 34, ...
 (iii) 1.0, 2.0, 1.5, 1.667, 1.6, 1.625, 1.615, 1.619, ...
 These values are oscillating and appear to be converging.
- 8** (i) 2, 6, 2, 6, 2, 6, ...
 (ii) Oscillating and periodic with period 2
 (iii) (a) 2, 8, -4, 20, -28, 68, ... Oscillating and diverging
 (b) $3\frac{1}{2}, 4\frac{1}{4}, 3\frac{7}{8}, 4\frac{1}{16}, 3\frac{31}{32}, 4\frac{1}{64}, \dots$
 Oscillating sequence converging towards 4
- 9** (i) $t_2 = 0, t_3 = -2, t_4 = 2, t_{100} = 2$
 (ii) $2^2 - 2 = 2; t_1 = -1$
 (iii) (a) Diverges
 (b) Converges to -1
 (c) The terms oscillate between $\frac{\sqrt{5}-1}{2}$ and $\frac{-\sqrt{5}-1}{2}$
- 10** (ii) First series: 118 terms, 294 terms, 10 793 terms, 23 445 terms
 Second series: 11 terms, 145 terms, 10 308 terms, 22 388 terms

Exercise 3.2 (page 46)

- 1** (i) Yes, 3, 32
 (ii) No

- (iii) Yes, -3, -22
 (iv) Yes, 4, 39
 (v) No
 (vi) Yes, 1.5, 15.5
- 2** (i) 34
 (ii) 29 terms
- 3** 38 terms
- 4** (i) 3
 (ii) 470
- 5** (i) 4
 (ii) -3, 1, 5, 9
 (iii) 375
- 6** (i) 3
 (ii) 15 150
- 7** (i) 120
 (ii) 22
- 8** (i) 5049
 (ii) 5100
 (iii) 51 terms in each sequence with the terms in the second one each 1 greater than those in the first.
- 9** 30
- 10** (i) $a = d = 2$
 (ii) 330
- 11** 267
- 12** (i) 16
 (ii) 2.5 cm
- 13** (i) $a = 1, d = 4$
 (ii) 270
- 14** (i) $S_n = \frac{d}{2}n^2 + \left(a - \frac{d}{2}\right)n$
 (ii) Subtracting the sum of the first $(n-1)$ terms from the sum of the first n terms will leave only the n th term.
 n th term $= p(2n-1) + q$
 (iii) First term $= (p+q)$; common difference $= 2p$

Discussion point (page 50)

If $r = +1$ then all of the terms are equal to the first term, a , and the sum of n terms is na so the sum is *divergent*.

If $r = -1$ then the terms alternate between $+a$ and $-a$ and the sum will alternate between $+a$ and 0.

Discussion point (page 52)

S is an infinite series with a common ratio of (-2) . Since this is outside the interval $(-1, 1)$ the sum to infinity does not have a unique value.

Discussion point (page 52)

Disease, availability of food, water and shelter.

Exercise 3.3 (page 53)

- 1** (i) Yes, 2, 160
 (ii) Yes, -1, -1
 (iii) No
 (iv) No
 (v) Yes, $\frac{1}{2}, \frac{3}{16}$
 (vi) Yes, -2, -64
- 2** (i) 48, $(-2), 3(-2)^{n-1}$
 (ii) $\frac{1}{8}, \frac{1}{2}, 2 \times \left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^{n-2}$
 (iii) ab^4, b, ab^{n-1}
- 3** (i) $5\frac{1}{3}$
 (ii) 8
 (iii) Diverges
 (iv) $4\frac{1}{2}$
 (v) Diverges
- 4** (i) 11
 (ii) 256
 (iii) 4094
- 5** (i) (a) 2, 12 (b) 20 475
 (ii) (a) 3, 11 (b) 177 146
 (iii) (a) $-\frac{1}{2}, 16$ (b) $5\frac{1365}{4096}$
 (iv) (a) $-\frac{1}{2}, 11$ (b) $26\frac{87}{128}$
- 6** 19 years
- 7** (i) 2
 (ii) 3
 (iii) 3069
 (iv) 19 terms
- 8** (i) $\frac{2}{3}$
 (ii) 7th
 (iii) 7 terms
 (iv) 27
- 9** (i) 83 terms
 (ii) 73 terms
- 10** (i) $\frac{x(1-x^n)}{(1-x)}$
 (ii) $\frac{(1-(-y)^n)}{(1+y)}$

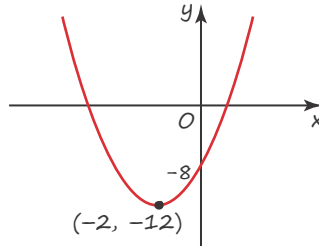
- (iii) $\frac{(1 - (-2a)^n)}{(1 + 2a)}$
- 12 (i) 20, 10, 5, 2.5, 1.25
(ii) 5 operations
(iii) 0, 10, 15, 17.5, 18.75
(iv) The sequence of amounts of water is geometric (first term 20, common ratio 0.5) but the sequence of antifreeze is not (a geometric sequence cannot have zero as its first term since that would imply that all terms are zero).
- 13 (i) 54
(ii) 120.41 cm
- 14 (i) $\frac{1}{\cos \theta}$
(ii) $\frac{31\sqrt{3}}{2}$
(iii) $2\sqrt{3}$
(iv) $\frac{1}{\cos \theta \sin^2 \theta}$
(v) No
- 15 (i) 27 cm, 36 cm, 48 cm, 64 cm, $85\frac{1}{3}$ cm
(ii) The length increases without bound (G.P. with $r = \frac{4}{3}$).
(iii) No – the figure is always inside the circle which circumscribes the original triangle.

Review: Algebra(2)

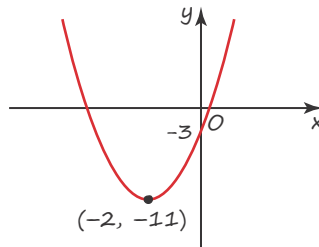
Review exercise R.1 (page 60)

- 1 (i) $x = 2, 6$
(ii) $a = -5, -6$
(iii) $x = \pm 4$
(iv) $p = -0.5, -2$
(v) $c = 1, -1\frac{3}{4}$
(vi) $x = 4, \frac{2}{3}$
- 2 (i) $x = 2.37, -3.37$
(ii) $x = 0.78, 3.22$
(iii) $x = -3.70, 2.70$

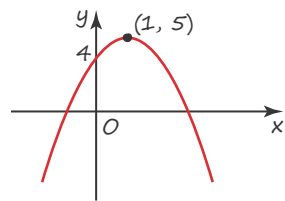
- 3 (i) $3(x - 2)^2 - 5$
(ii) $2(x + \frac{3}{2})^2 + \frac{1}{2}$
(iii) $21 - (x - 4)^2$
- 4 (i) $x = -2, (-2, -12)$



- (ii) $x = -2, (-2, -11)$



- (iii) $x = 1, (1, 5)$



- 5 (i) $x = 4, y = 3$
(ii) $x = 2, y = 3$
(iii) $x = -1, y = -2$
- 6 (i) $x = 4, y = 2$ or $x = -4, y = -2$
(ii) $x = 4, y = 1$ or $x = 1, y = 4$
(iii) $x = 1, y = 5$ or $x = 11, y = 25$

- 7 (i) $\{x: x < 4\} \cap \{x: x > -2\}$



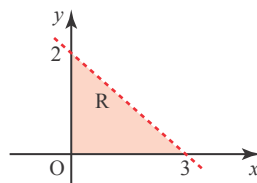
- (ii) $\{x: x \leq -1\} \cap \{x: x \geq -7\}$



- (iii) $\{x: x \geq 0\} \cup \{x: x \leq -4\}$

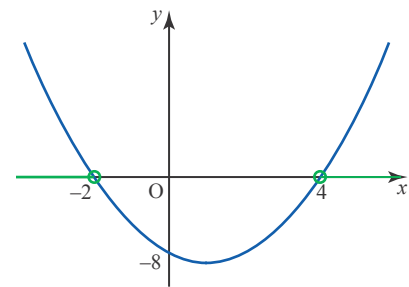


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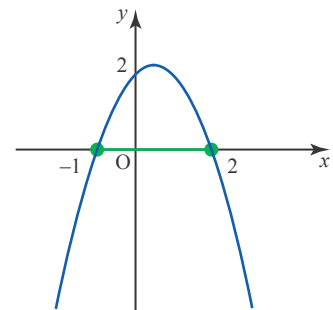


- 9 $(-\infty, -2] \cup [0.6, \infty)$.

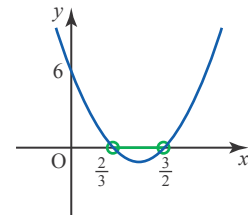
- 10 (i) $x < -2$ or $x > 4$



- (ii) $-1 \leq x \leq 2$

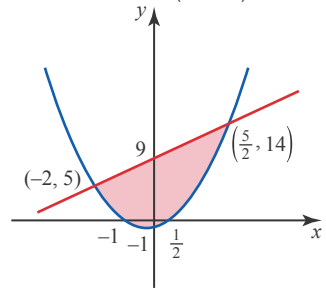


- (iii) $\frac{2}{3} < x < \frac{3}{2}$



- 11 $(x, y) = (-3, 5)$ or $(5, -7)$

- 12 (i) Points of intersection: $(-2, 5)$ and $(\frac{5}{2}, 14)$

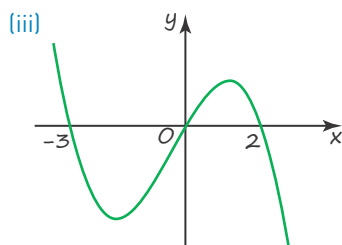
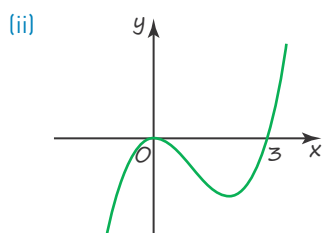
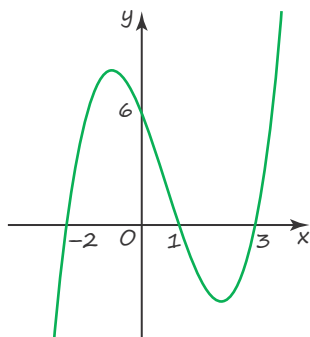


- 13 (i) $x = \pm 2$ or $x = \pm 1$
(ii) $x = \pm \frac{1}{2}$ or $x = \pm \sqrt{3}$
(iii) $x = \frac{9}{4}$ or $x = 25$
- 14 (i) $x = 0$ or $x = 1$
(ii) $x = -1.58$
(iii) $x = 0.208$

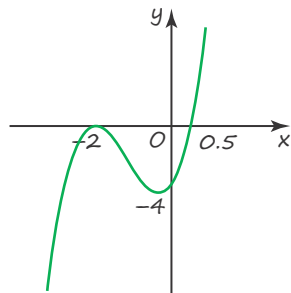
- 15 $(-1, 3)$ and $(\frac{20}{11}, -\frac{27}{22})$

Review exercise R.2 (page 62)

- 1 (i) $x^3 + 3x^2 - x + 1$
 (ii) $5x^4 + x^3 + 4x^2 - 2x - 4$
 (iii) $x^3 + x^2 + 3x - 6$
 (iv) $3x^3 + 7x^2 - 3x + 1$
- 2 (i) $2x^3 + 3x^2 - 5x + 12$
 (ii) $x^4 - 2x^2 + 1$
 (iii) $x^2 + 3x + 2$
 (iv) $x^2 - 3x - 4$
- 3 (i)



- 4 (i) $x = -2$ (repeated), $x = \frac{1}{2}$
 (ii)

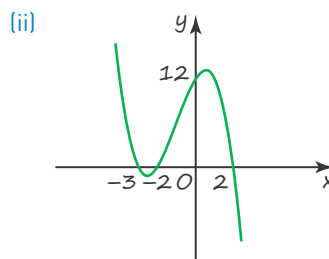
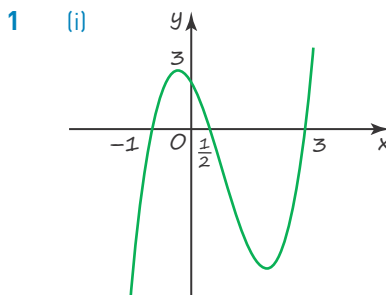


- 5 (i) 0, 17
 (ii) $f(x) \equiv (2x - 3)(x^2 + 4x + 5)$ and $x^2 + 4x + 5 = 0$ has no real roots, so $x = 1.5$ is the only real root.

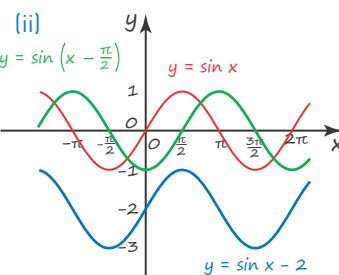
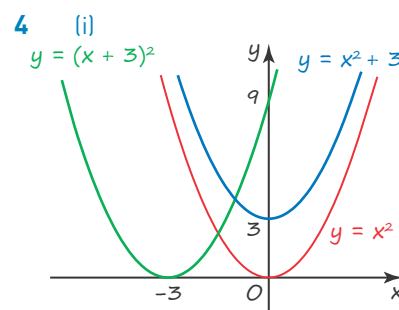
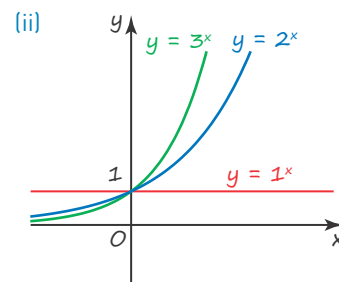
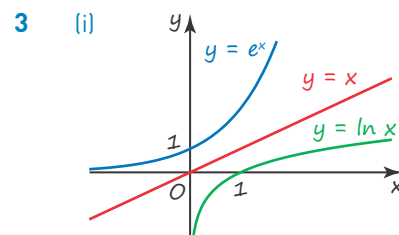
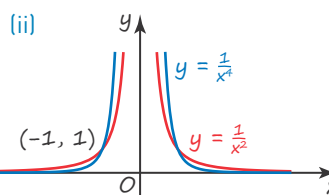
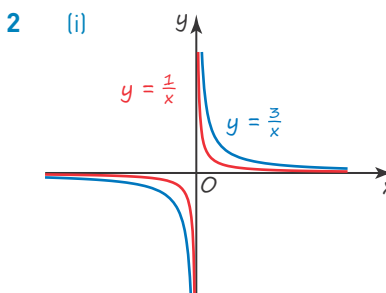
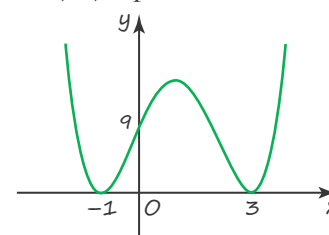
- 6 $y = 2x^3 + 4x^2 - 22x - 24$
 7 (i) $x = 1, -\frac{1}{2}, \frac{3}{2}$
 (ii) $x = -2, 3, -\frac{1}{3}$
 8 $x^3 - 2x^2 + 4x - 8$

Chapter 4

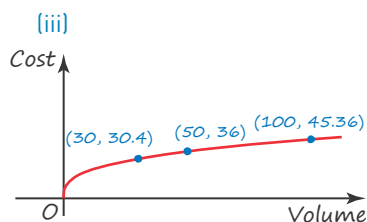
Review exercise (page 69)



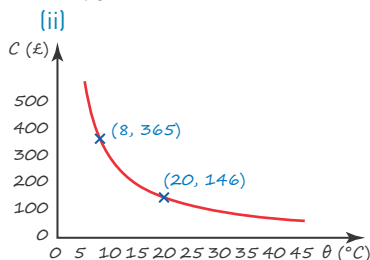
- (iii) (-1) repeated,
 (-3) repeated



- 5 (i) $x^2(x + 3)$;
 translation $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$
 (ii) $3(x - 2)^2(x + 1)$;
 stretch scale factor 3 in the y direction
 (iii) $(x - 2)^2(x + 1) - 2$;
 translation $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$
 (iv) $-(x - 2)^2(x + 1)$;
 reflection in the x -axis
- 6 (i) 3.11 cm
 (ii) £30.36
 (ii) £45.36

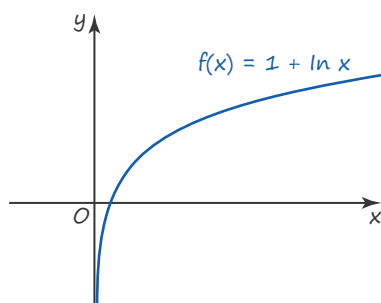


7 (i) £146



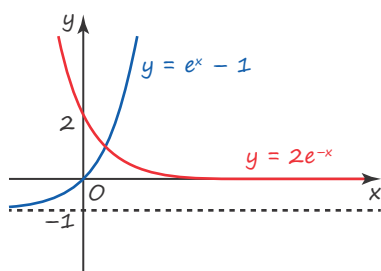
(iii) Not good for extreme temperatures, since it implies an infinite bill for a temperature of 0°C .

8 (i)



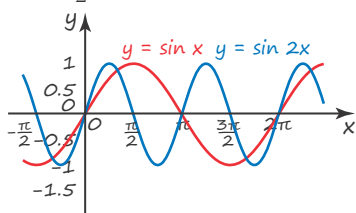
(ii) $(e^{-1}, 0)$

9 (i)

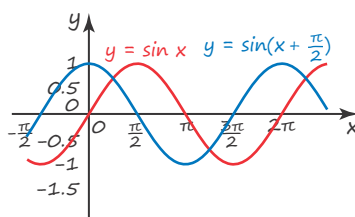


(ii) $(\ln 2, 1)$

10 (i) Can be obtained from by a stretch scale factor $\frac{1}{2}$ parallel to the x -axis.



(ii) Can be obtained from by a translation through $\begin{pmatrix} -\frac{\pi}{2} \\ 0 \end{pmatrix}$



Activity 4.1 (page 75)

- (i) Different
- (ii) Same
- (iii) Same
- (iv) Different
- (v) Same

Discussion point (page 77)

Translation 2 units to the right, reflection in the x -axis, translation 5 units vertically upwards

Exercise 4.1 (page 78)

- 1 (i) One-one, yes
- (ii) Many-one, yes
- (iii) Many-many, no
- (iv) One-many, no
- (v) Many-many, no
- (vi) One-one, yes
- 2 (i) (a) Examples: one $\rightarrow$ 3; word $\rightarrow$ 4
- (b) Many-one
- (ii) (a) Examples: 1 $\rightarrow$ 4; 2.1 $\rightarrow$ 8.4
- (b) One-one
- (iii) (a) Examples: 1 $\rightarrow$ 1; 6 $\rightarrow$ 4
- (b) Many-one
- (iv) (a) Examples: 1 $\rightarrow$ -3; -4 $\rightarrow$ -13
- (b) One-one
- (v) (a) Examples: 4 $\rightarrow$ 2; 9 $\rightarrow$ 3
- (b) One-one
- 3 (i) (a) One-one
- (b) Domain $\mathbb{R}^+$, Range $\mathbb{R}^+$

- (ii) (a) Many-many
- (b) Domain $\mathbb{R}^+$, Range $\mathbb{R}^+$

- (iii) (a) One-one
- (b) Domain $\mathbb{R}^+$, Range $\mathbb{R}^+$

- (iv) (b) Many-one
- (c) Domain $\mathbb{R}$, Range $\mathbb{R}^+$

- 4 (i) (a) -5
- (b) 9
- (c) -11
- (ii) (a) 3
- (b) 5
- (c) 10
- (iii) (a) 32
- (b) 82.4
- (c) 14
- (d) -40

- 5 (i) $f(x) \leq 2$
- (ii) $y \in \{2, 3, 6, 11, 18\}$
- (iii) $\mathbb{R}$
- (iv) $y \in \mathbb{R}^+$
- (v) $\frac{1}{\sqrt{2}} \leq y \leq 1$

- 6 (i) $\{0.5, 1, 2, 4\}$
- (ii) $0 < f(x) \leq 1$
- (iii) $f(x) \geq 3$

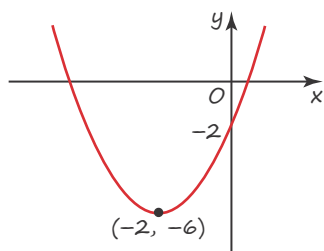
7 For f , every value of x , (including $x = 3$) gives a unique output, whereas $g(2)$ can equal either 4 or 6.

- 8 (i) Translation $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$; $x = 2$
- (ii) Stretch parallel to the y -axis of scale factor 3 and translation $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ in either

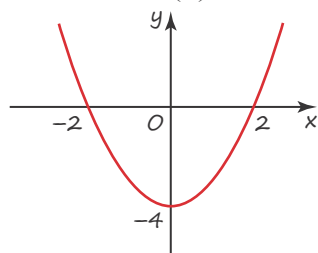
order; $x = 2$

(iii) Rewrite as $3[(x-2)^2 - \frac{14}{3}]$; translation $\begin{pmatrix} 2 \\ -\frac{14}{3} \end{pmatrix}$; then stretch parallel to the y -axis of scale factor 3; $x = 1$

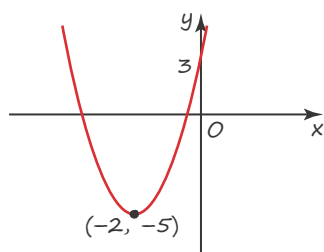
- 9 (i) Translation $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$



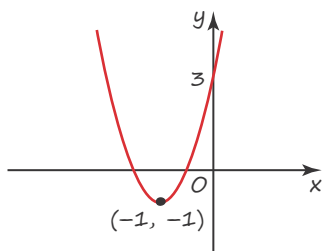
- (ii) Translation $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$



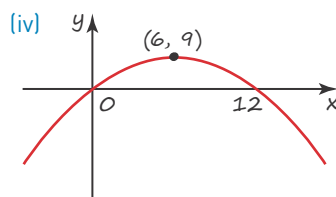
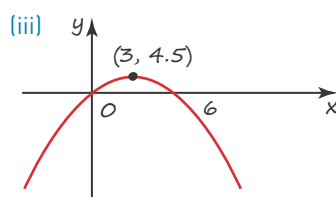
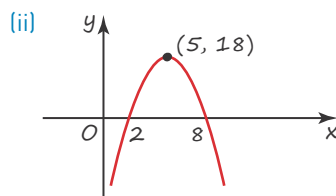
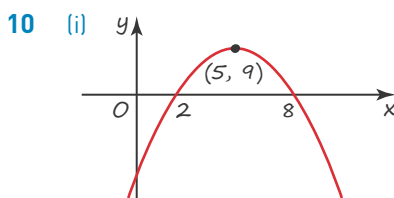
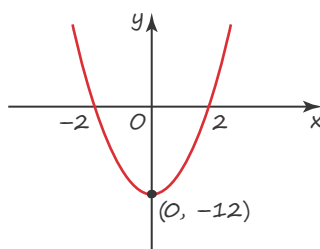
- (iii) Stretch parallel to the y -axis, scale factor 2, then translation $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$



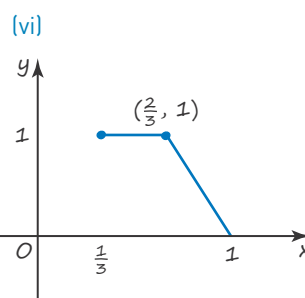
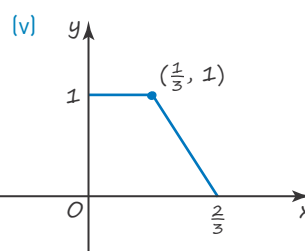
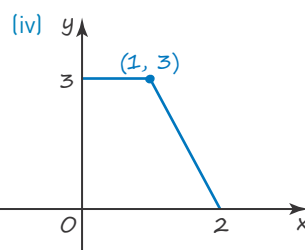
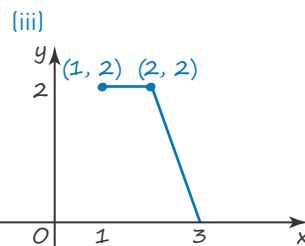
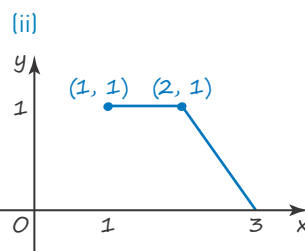
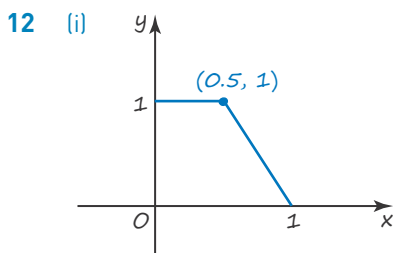
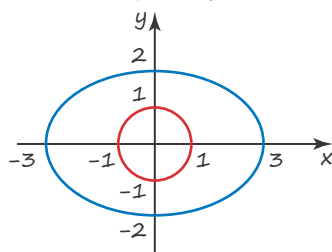
- (iv) Stretch parallel to the x -axis, scale factor $\frac{1}{2}$, then translation $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$



- (v) Translation $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ then stretch parallel to the y -axis scale factor 3

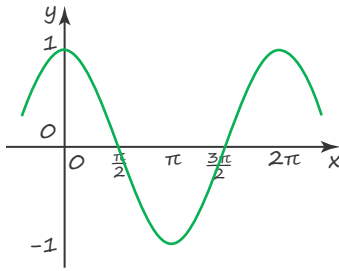


- 11 Ellipse is $\frac{x^2}{9} + \frac{y^2}{4} = 1$



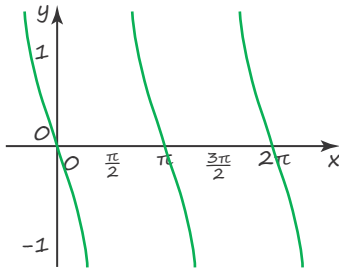
- 13 (i) Stretch scale factor 3 in the y direction.
(ii) Stretch scale factor $\frac{1}{3}$ in the x direction and translation $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ in either order.
(iii) Translation $\begin{pmatrix} -30^\circ \\ 0 \end{pmatrix}$ followed by a stretch scale factor $\frac{1}{3}$ in the x direction.

14 (i) (a)



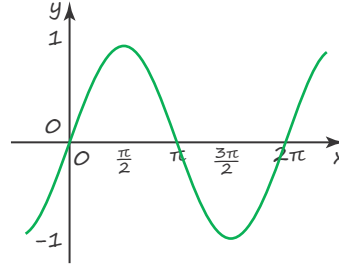
(b) $y = \cos x$

(ii) (a)



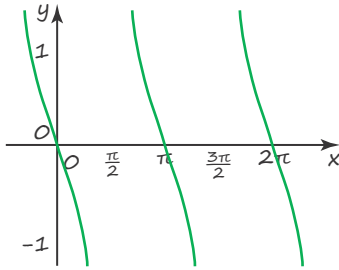
(b) $y = -\tan x$

(iii) (a)



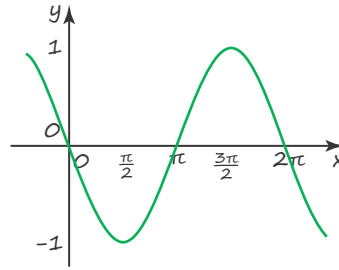
(b) $y = \sin x$

(iv) (a)



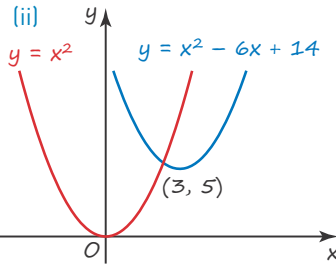
(b) $y = -\tan x$

(v) (a)



(b) $y = -\sin x$

15 (i) $a = 3, b = 5$



translation $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$

(iii) $y = 6x - x^2 - 14$

16 Reflection in the x -axis and translation $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ in either order; $x = 1$.

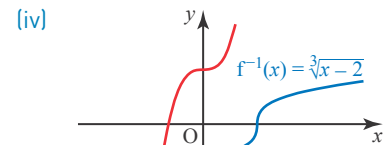
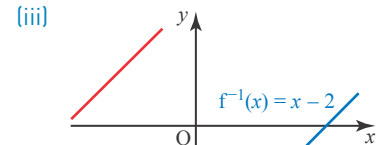
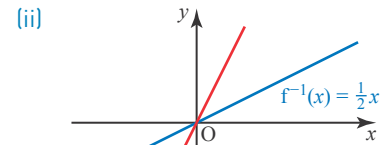
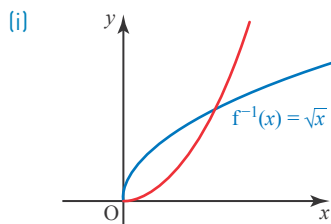
17 $f(x) = (x-1)^2(x-3) - 4$;
 $g(x) = 4 - (x-1)^2(x-3)$.

18 $g(x) = \ln(9-2x)$.

Discussion point (page 83)

- (i) (a) Function with an inverse function.
- (b) $f: C \rightarrow \frac{9}{5}C + 32$;
 $f^{-1}: F \rightarrow \frac{5}{9}(F - 32)$
- (ii) (a) Function but no inverse function since one grade corresponds to several marks.
- (iii) (a) Function with an inverse function.
- (b) 1 light year $\approx 6 \times 10^{12}$ miles or almost 10^{16} metres.
 $f: x \rightarrow 10^{16}x$ (approx.);
 $f^{-1}: x \rightarrow 10^{-16}x$ (approx.)
- (iv) (a) Function but no inverse function since fares are banded.

Activity 4.2 (page 85)



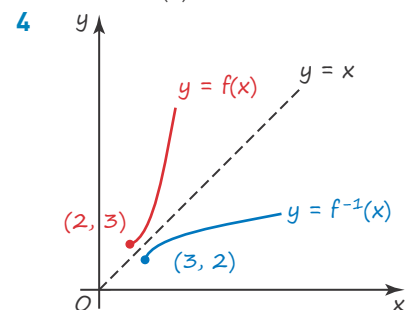
$y = f(x)$ and $y = f^{-1}(x)$ appear to be reflections of each other in the line $y = x$

Discussion point (page 88)

- (i) For all values of x , $\sqrt{x^2}$ will give a positive answer.
- (ii) The answer will be -19° since the arcsin function gives the principal value.

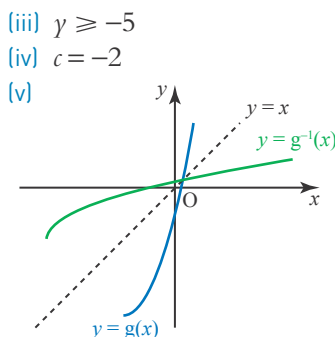
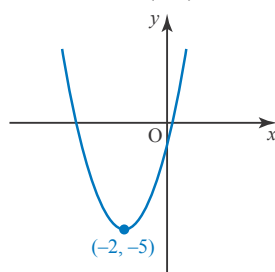
Exercise 4.2 (page 88)

- 1 (i) 4
(ii) 5
(iii) 9
(iv) 25
- 2 (i) $8x^3$
(ii) $2x^3$
(iii) $(x+2)^3$
(iv) $x^3 + 2$
(v) $4x$
(vi) $x + 4$
- 3 (i) $f^{-1}(x) = \frac{x-7}{2}$
(ii) $f^{-1}(x) = 4 - x$

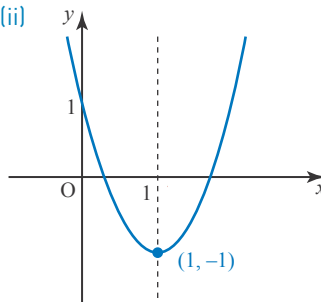


- 5 (i) fg
 (ii) g^2
 (iii) fg^2
 (iv) gf
- 6 (i) $\frac{3\pi}{2}$
 (ii) $\frac{\pi}{2}$
- 7 (i) $8(x+2)^3$
 (ii) $2(x^3+2)$
 (iii) $[(x+2)^3+2]^3$
- 8 (i) One-way stretch, parallel to the y -axis, with scale factor 3, followed by translation $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$
 (ii) One-way stretch, parallel to the x -axis, with scale factor $\frac{1}{2}$, and translation $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$ in either order
 (iii) translation $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$, followed by one-way stretch, parallel to the x -axis, with scale factor $\frac{1}{5}$.
- 9 $y = \left[\frac{1}{3}(x+4)\right]^3 + 7$
- 10 (i) $f^{-1}(x) = \frac{2x-4}{x}$
 (ii) $f^{-1}(x) = \sqrt{x+3}; x \geq -3$
- 11 (i) $f(x)$ not defined for $x=4$;
 $h(x)$ not defined for $x>2$
 (ii) $f^{-1}(x) = \frac{4x+3}{x}$;
 $h^{-1}(x) = 2-x^2; x \geq 0$
 (iii) g is a many-to-one function
 (iv) $\mathbb{R}^+$
 (v) No, the domain of fg excludes $x=2$ and $x=-2$, whereas the domain of gf excludes $x=4$
- 12 (i) x
 (ii) $\frac{1}{x}$
 (iii) $\frac{1}{x}$
 (iv) $\frac{1}{x}$

- 13 (i) $a=2, b=-5$
 (ii) Translation $\begin{pmatrix} -2 \\ -5 \end{pmatrix}$

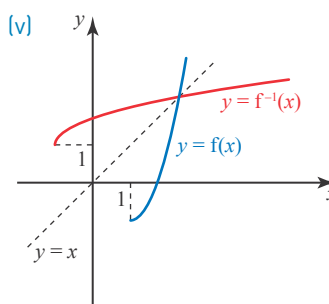


- 14 (i) $a=2, b=-1$
 (ii)



Line of symmetry is $x=1$, minimum point $(1, -1)$

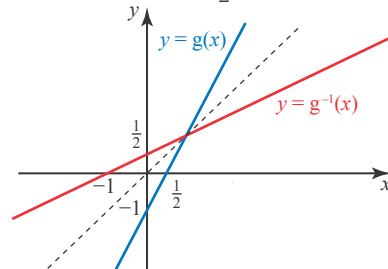
- (iii) $f(x) \geq -1$
 (iv) f is not a 1-1 function;
 $x \geq 1$



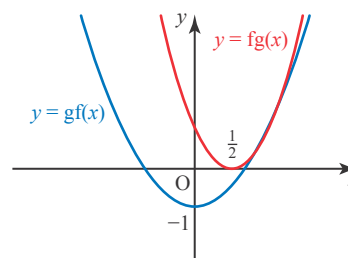
- (vi) $\left(\frac{5+\sqrt{17}}{4}, \frac{5+\sqrt{17}}{4}\right)$

- 15 (i) Range of $f(x)$ is $\mathbb{R}^+$.
 Range of $g(x)$ is $\mathbb{R}$.
 $f(x)$ is not 1-1

- (ii) $g^{-1}(x) = \frac{1}{2}(x+1)$



- (iii) $gf(x) = 2x^2 - 1$;
 $fg(x) = (2x-1)^2$;
 $x=1$ (repeated).



- (iv) $a > \frac{1}{4}$

Discussion point (page 90)

$$g(3) = 3, g(-3) = 3$$

$$|3+3| = 6, |3-3| = 0,$$

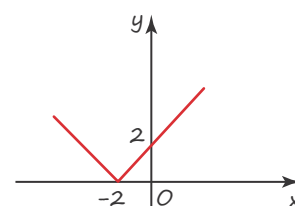
$$|3| + |3| = 6, |3| + |-3| = 6$$

Discussion point (page 91)

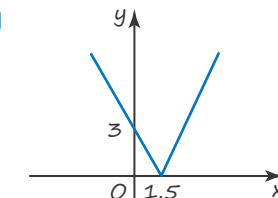
$y = |x|$ lies below the line $y=2$ for $-2 < x < 2$

Exercise 4.3 (page 94)

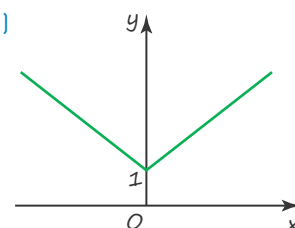
- 1 (i)



- (ii)



- (iii)

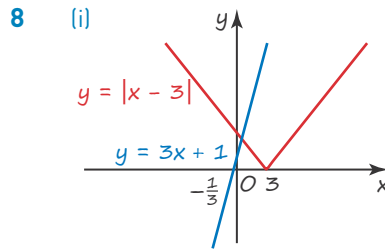
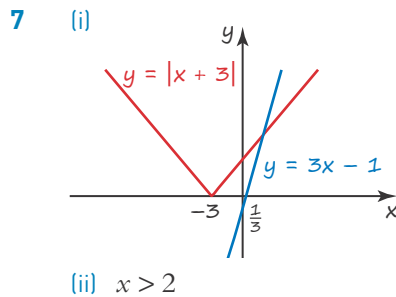
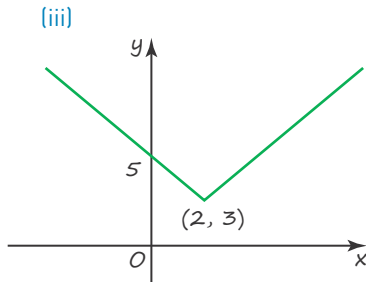
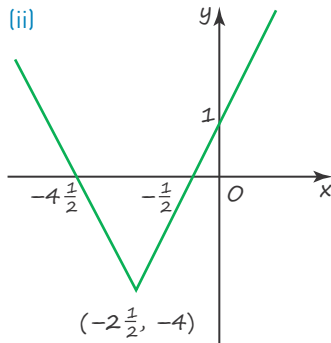
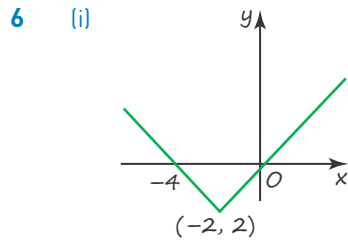


- 2 (i) $-8 < x < 2$
 (ii) $0 \leq x \leq 4$
 (iii) $x < -1$ or $x > 11$
 (iv) $x \leq -3$ or $x \geq 1$

- 3 (i) $|x - 1| < 2$
 (ii) $|x - 5| < 3$
 (iii) $|x - 1| < 3$

- 4 (i) $-2 < x < 5$
 (ii) $-\frac{2}{3} \leq x \leq 2$
 (iii) $x > 1$ or $x < -4$
 (iv) $x \leq -3\frac{1}{3}$ or $x \geq 2$

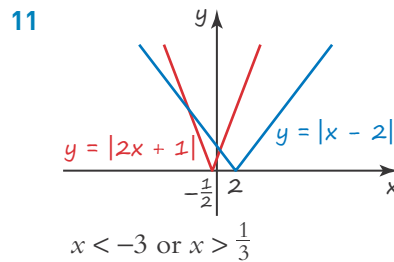
- 5 (i) $|x - 2.5| < 3.5$
 (ii) $|x - 10| < 0.1$
 (iii) $|x - 4| < 3.5$



- (ii) $x > \frac{1}{2}$

- 9 (i) $\Leftrightarrow$
 (ii) $\Leftrightarrow$
 (iii) $\Leftarrow$

- 10 $x < 1$



- 12 $-0.4 \leq x \leq 4$

- 13 $\pm 2, \pm\sqrt{14}$

- 14 $(\ln 7) - \pi, (\ln 3) - \pi$

Chapter 5

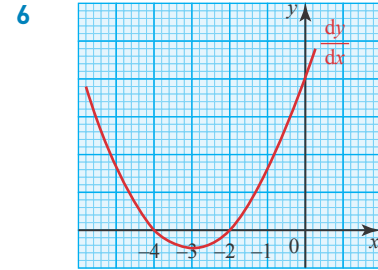
Opening activity (page 96)

- (i) The rate of change of the height with respect to the time and the rate of change of the volume with respect to height.
 (ii) The rate of change of the volume with respect to time.

Review exercise (page 101)

- 1 (i) 4
 (ii) 9
 (iii) 2
- 2 5890 thousand bacteria per second
- 3 Tangent is $4x + y - 9 = 0$
 Normal is $x - 4y - 15 = 0$
- 4 (i) $\frac{dy}{dx} = 12x^3 - x$,
 $\frac{d^2y}{dx^2} = 36x^2 - 1$
 (ii) $\frac{dy}{dx} = 8x^3 - \frac{1}{3}$,
 $\frac{d^2y}{dx^2} = 24x^2$

(iii) $\frac{dy}{dx} = \frac{1}{\sqrt{x}} - 2x - \frac{1}{x^2}$,
 $\frac{d^2y}{dx^2} = -\frac{1}{2x\sqrt{x}} - 2 + \frac{2}{x^3}$



- 7 (i) $x < -2$
 (ii) $x < -4, x > 4$
 (iii) $0 < x < \frac{2}{3}$

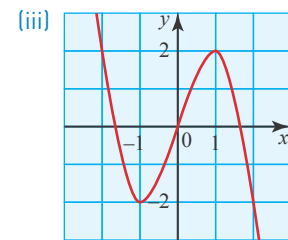
8 (i) $\frac{dy}{dx} = 5x\sqrt{x} - \frac{2}{\sqrt{x}}$,
 $\frac{d^2y}{dx^2} = \frac{15\sqrt{x}}{2} + \frac{1}{x\sqrt{x}}$

(iii) $\frac{dy}{dx} = 15x^2 - \frac{3}{2}$,

$\frac{d^2y}{dx^2} = 30x$

9 (i) $\frac{dy}{dx} = 3 - 3x^2$,
 $\frac{d^2y}{dx^2} = -6x$

- (ii) Minimum at $(-1, -2)$ and maximum at $(1, 2)$



10 $8x + 7y - 95 = 0$

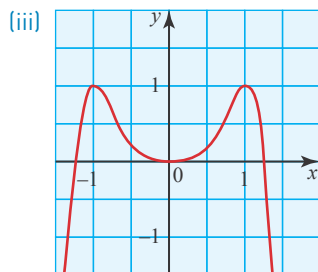
11 Minimum at $(9, -21)$

12 (i) $a = 1, b = 3$

- (ii) No real solutions to $x^2 + 2x + 5 = 0$

13 (i) $\frac{dy}{dx} = 12x^3 - 12x^5$,
 $\frac{d^2y}{dx^2} = 36x^2 - 60x^4$

- (ii) Maximum point at $(-1, 1)$ and $(1, 1)$, minimum point at $(0, 0)$.



14 $c = -\frac{19}{2}, c = \frac{34}{3}$

15 $V = 486, x = 3$

Discussion point (page 102)

Gradient is zero but point is not a turning point.

Activity 5.1 (page 105)

- (i) (a)
 (b)
 (c)
 (ii) (a) concave upwards
 (b) concave downwards

Discussion point (page 105)

The gradient is constant, i.e. a straight line.

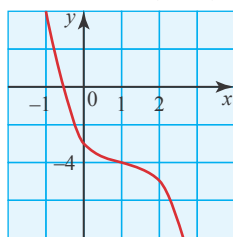
Discussion point (page 106)

No, the second differential is sometimes zero at maximum points and minimum points.

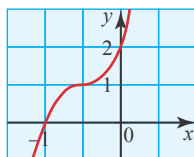
Exercise 5.1 (page 110)

- 1 (i) $x < 3$
 (ii) $x < -7$
 (iii) $x > \frac{1}{2}$
 2 (i) $x > -\frac{1}{3}$
 (ii) $x > 4$
 (iii) $x < \frac{2}{3}$

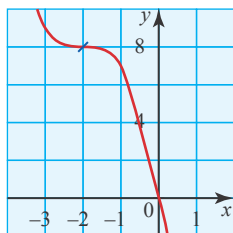
- 3 (i) Stationary point of inflection is $(1, -4)$



- (ii) Stationary point of inflection is $(-\frac{1}{2}, 1)$



- (iii) Stationary point of inflection is $(-2, 8)$



- 4 (i) $x < -1, x > 3$

- (ii) $-1 < x < 2$

- 5 (i) $\frac{dy}{dx} = 3x^2 - 2x,$

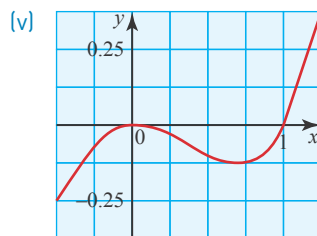
$$\frac{d^2y}{dx^2} = 6x - 2$$

- (ii) maximum at $(0, 0),$

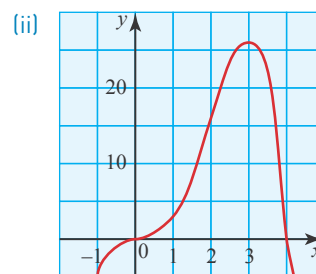
$$\text{minimum at } \left(\frac{2}{3}, -\frac{4}{27}\right)$$

- (iii) $\left(\frac{1}{3}, -\frac{2}{27}\right)$

- (iv) $-\frac{1}{3}$

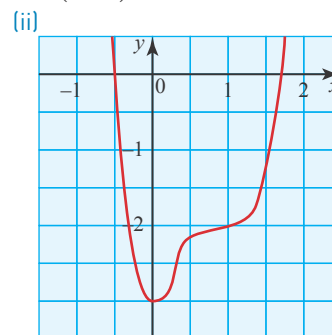


- 6 (i) Point of inflection at $(0, 0)$ and maximum at $(3, 27)$



- (iii) $(2, 16)$

- 7 (i) Minimum at $(0, -3)$ and point of inflection at $(1, -2)$



- (iii) $\left(\frac{1}{3}, -\frac{70}{27}\right)$

- 8 Minimum points at $(2, 16)$ and $(-1, -38)$. Maximum points at $(-2, -16)$ and $(1, 38)$. Points of inflection at $(0, 0), \left(\frac{\sqrt{10}}{2}, \frac{65\sqrt{10}}{8}\right)$ and $\left(-\frac{\sqrt{10}}{2}, -\frac{65\sqrt{10}}{8}\right)$

Discussion point (page 110)

- (i) Can be done easily by expanding the brackets.
 (ii) Can also be done by expanding the brackets, but this is time-consuming and tedious.
 (iii) and (iv) Cannot be done by expanding the brackets, and an alternative method is needed.

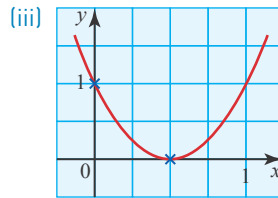
Activity 5.2 (page 111)

$y = f(x)$	u	$y = f(u)$
E.g. $y = \tan\left(\frac{1}{x^3}\right)$	$u = \frac{1}{x^3}$	$y = \tan u$
(i) $y = (x^3 + 3)^4$	$u = x^3 + 3$	$y = u^4$
(ii) $y = \sin(x^2 - 4)$	$u = x^2 - 4$	$y = \sin u$
(iii) $y = e^{2x+5}$	$u = 2x + 5$	$y = e^u$
(iv) $y = \ln(x^2 - 2x)$	$u = x^2 - 2x$	$y = \ln u$

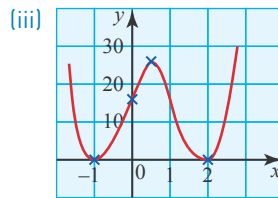
Exercise 5.2 (page 114)

- B; an alternative is to expand the brackets and differentiate directly
 - C; differentiate directly
 - A
- $\frac{dy}{dx} = -3(3x + 2)^{-2}$
 - $\frac{dy}{dx} = 28x(2x^2 + 6)^6$
 - $\frac{dy}{dx} = \frac{3}{\sqrt{6x - 2}}$
- $y = x^6 - 6x^4 + 12x^2 - 8$
 $\Rightarrow \frac{dy}{dx} = 6x^5 - 24x^3 + 24x$
 - $\frac{dy}{dx} = 6x(x^2 - 2)^2$
 - Expanding the answer to (ii) gives the answer to (i)
- 48
 - $\frac{3}{2}$
 - 3
- $\frac{dy}{dx} = 9(3x - 5)^2$
 - $y - 9x + 17 = 0$
- $\frac{dx}{dt} = \frac{2(\sqrt{t} - 1)^3}{\sqrt{t}}$
 - $\frac{dz}{dy} = \frac{1 - \frac{1}{y^2}}{2\sqrt{\frac{1}{y} + y}} = \frac{y^2 - 1}{2y^2\sqrt{\frac{1}{y} + y}}$
 - $\frac{dp}{dr} = \frac{-3\sqrt{r}}{(2r\sqrt{r} - 6)^2}$

- $\frac{dy}{dx} = 8(2x - 1)^3$
 - minimum at $(\frac{1}{2}, 0)$



- $\frac{dy}{dx} = 4(2x - 1) \times (x^2 - x - 2)^3$
 - Minimum at $(-1, 0)$, maximum at $(\frac{1}{2}, \frac{6561}{256})$, minimum at $(2, 0)$



- $\frac{dy}{dx} = 3(3x^2 - 2x) \times (x^3 - x^2 + 2)^2$
 - $a = \frac{2}{3}$
 - 12, $12x - y - 4 = 0$
- 72 minutes
 - 67 minutes
 - 1333 metres, 66 minutes

Activity 5.3 (page 116)

- $\frac{dy}{dx} = 3x^2$
- $x = y^{\frac{1}{3}}$

(iii) $\frac{dx}{dy} = \frac{1}{3}y^{-\frac{2}{3}}$

(iv) $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$

(v) Yes

Discussion point (page 116)

$\frac{dy}{dx} = 2, \frac{dx}{dy} = \frac{1}{2}$

Exercise 5.3 (page 118)

- $\frac{1}{6}$
 - $-\frac{1}{2}$
 - 12
- $-\frac{1}{3}$
 - $-\frac{3}{2}$
 - $\frac{3}{2}$
- $\frac{dA}{dx} = 10x$
 - $\frac{dA}{dt} = 5x$
 - 30
- $16\text{ cm}^2\text{ s}^{-1}$
- $0.01\pi\text{ m}^2\text{ per day}$
- -0.015 N s^{-1}
- $\frac{1}{5\pi}\text{ m s}^{-1}$
- $\frac{1}{48}\text{ m h}^{-1}$
- $1.2\text{ cm}^3\text{ s}^{-1}$
- $20.8\text{ mm}^2\text{ s}^{-1}$

Activity 5.4 (page 119)

- $y = x^9 \Rightarrow \frac{dy}{dx} = 9x^8$
- $u = x^3 \Rightarrow \frac{du}{dx} = 3x^2$
 $v = x^6 \Rightarrow \frac{dv}{dx} = 6x^5$
- No, $9x^8 \neq 18x^7$

Discussion point (page 120)

No

Activity 5.5 (page 120)

No, $\frac{dy}{dx} = 4x^3$, but

$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{7x^6}{3x^2} = \frac{7}{3}x^4$

Discussion point (page 121)

Yes

Exercise 5.4 (page 122)

1

Function	Product or quotient rule?	u	v
e.g. $y = \frac{(x+1)^6}{\sqrt[3]{x-1}}$	Quotient	$u = (x+1)^6$	$v = \sqrt[3]{x-1}$
(i) $y = \sqrt{(x^2-1)}(x^3+3)^2$	Product	$u = (x^2-1)^{\frac{1}{2}}$	$v = (x^3+3)^2$
(ii) $y = (2x^3-3)^5$	Neither	–	–
(iii) $y = \frac{\sqrt{x+1}}{x^2}$	Quotient	$u = (x+1)^{\frac{1}{2}}$	$v = x^2$
(iv) $y = x^2\sqrt{x-2}$	Product	$u = x^2$	$v = (x-2)^{\frac{1}{2}}$

2 (i) $\frac{dy}{dx} = 2x(2x-1)^3 \times (4x^2+2x-1)$

(ii) $\frac{dy}{dx} = \frac{3x-1}{2\sqrt{x-1}}$

(iii) $\frac{dy}{dx} = \frac{(7x-3)(x-3)^2}{2\sqrt{x}}$

3 (i) $\frac{dy}{dx} = \frac{3x^2-2x}{(3x-1)^2}$

(ii) $\frac{dy}{dx} = \frac{-3(1-2x)^2}{x^4}$

(iii) $\frac{dy}{dx} = \frac{\sqrt{x}+2}{2(\sqrt{x}+1)^2}$

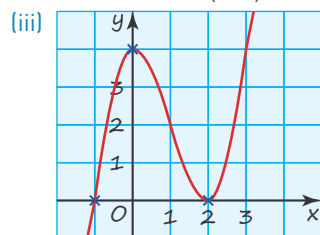
4 (i) $\frac{1}{4}$

(ii) -8

(iii) $-\frac{13}{108}$

5 (i) $\frac{dy}{dx} = 3x(x-2)$

(ii) Maximum at $(0, 4)$ and minimum at $(2, 0)$



6 (i) $\frac{dy}{dx} = \frac{-9}{(2x-3)^2}$

(ii) $y + 9x - 6 = 0$

(iii) $y = x$

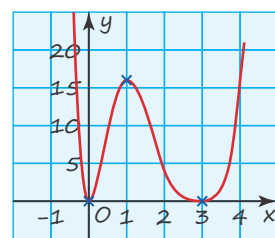
(iv) $(3, 3)$

(i) $\frac{dy}{dx} = \frac{2(x+2)(x+1)}{(2x+3)^2}$

(ii) $\frac{d^2y}{dx^2} = \frac{2}{(2x+3)^3}$

(iii) Turning points at $(-1, -2)$ and $(-2, -3)$

8 Minimum points at $(0, 0)$ and $(3, 0)$, maximum at $(1, 16)$



9 (i) $\frac{dy}{dx} = \frac{\sqrt{x}-2}{(\sqrt{x}-1)^2}$

(ii) $\frac{1}{4}$

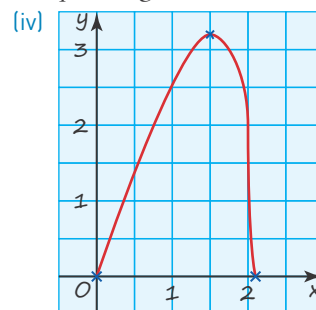
(iii) $(4, 8)$

(iv) Q is $(\frac{37}{4}, 8)$ and R is $(4, 29)$. Area of triangle is $\frac{441}{8}$

10 (i) $(\frac{3}{2}, \frac{9}{4}\sqrt{2})$

(ii) Gradient is 3 at the origin

(iii) $y = 0$, gradient is infinite



Practice questions 1

(page 125)

1 (i) $2r + r\theta = 3$ [1]

$\theta = \frac{3-2r}{r}$ [1]

Area = $\frac{1}{2}r^2\left(\frac{3-2r}{r}\right)$ [1]

$A = \frac{3}{2}r - r^2$

(ii) $\frac{dA}{dr} = \frac{3}{2} - 2r$ [1]

$\frac{dA}{dr} = 0$ when $r = \frac{3}{4}$ [1]

$\frac{d^2A}{dr^2} = -2$ so the area is a maximum [1]

$\theta = \frac{3-1.5}{0.75} = 2$ [1]

2 (i) $\cos\left(-\frac{\pi}{3}\right) = 0.5$ [1]

$\cos\left(-\frac{\pi}{3} - \frac{\pi}{6}\right) + 0.5$
 $= 0 + 0.5 = 0.5$

$\cos\left(-\frac{\pi}{2}\right) = 0$ [1]

$\cos\left(-\frac{\pi}{2} - \frac{\pi}{6}\right) + 0.5$
 $= -0.5 + 0.5 = 0$

(ii) $(\frac{5\pi}{3}, 0.5)$ OR

$(\frac{3\pi}{2}, 0)$ OR either of

these with a positive multiple of 2π added to the x coordinate. [1, 1]

(iii) Translation [1]

$\begin{pmatrix} \frac{\pi}{6} \\ 0.5 \end{pmatrix}$ [1]

3 (i) $\frac{dy}{dx} = 8x(x^2-1)^3$ [1]

$\frac{d^2y}{dx^2} = 8(x^2-1)^3 +$

$24x(x^2-1)^2 \cdot 2x$ [1, 1]

$$\frac{d^2y}{dx^2} = 8(x^2 - 1)^2(x^2 - 1 + 6x^2)$$

$$= 8(x^2 - 1)^2(7x^2 - 1) \quad [1]$$

$$(iii) \quad 7x^2 - 1 < 0 \quad [1]$$

$$-\frac{1}{\sqrt{7}} < x < \frac{1}{\sqrt{7}} \quad [1]$$

$$4 \quad (i) \quad f(x) = \frac{\frac{x+3}{x-1} + 3}{\frac{x+3}{x-1} - 1} \quad [1]$$

$$\frac{x+3+3(x-1)}{x+3-(x-1)} \quad [1]$$

$$\frac{4x}{4} = x \quad [1]$$

$$f^{-1}(x) = \frac{x+3}{x-1} \quad [1]$$

$$(iii) \quad y = x \quad [1]$$

$$(iii) \quad \frac{x+3}{x-1} = \frac{x-1+4}{x-1} \quad [1]$$

$$f(x) = 1 + \frac{4}{x-1} \quad [1]$$

$$(iv) \quad \text{Translation} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad [1]$$

Stretch parallel to y -axis,
scale factor 4 [1]

$$\text{Translation} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad [1]$$

$$5 \quad (i) \quad \text{Max} = a + b \quad [1]$$

$$\text{Min} = a - b \quad [1]$$

$$a + b = 16.75 \quad [1]$$

$$a - b = 7.583... \quad [1]$$

$$a = 12.166... \quad [1]$$

$$b = 4.5833... \quad [1]$$

$$(ii) \quad 21 \text{ June is day} \quad [1]$$

$$\text{number } 172 \quad [1]$$

$$\text{Model says} \quad [1]$$

$$12.1666 - 4.58333 \cos 172 \quad [1]$$

$$= 16.705 \text{ hours} \quad [1]$$

$$\text{Close to actual} \quad [1]$$

$$\text{value of } 16.75 \text{ hours} \quad [1]$$

$$(iii) \quad (a) \quad \text{Stretch to make} \quad [1]$$

$$\text{period } 365 \text{ days} \quad [1]$$

$$\left(\text{scale factor } \frac{365}{360} \right) \quad [1]$$

$$(b) \quad \text{Translation to make} \quad [1]$$

$$\text{maximum at } 21 \quad [1]$$

$$\text{June (oe)} \quad [1]$$

$$6 \quad \text{Let the terms be} \quad [1]$$

$$a - d, a, a + d \quad [1]$$

Suppose they form a
geometric sequence
and $d \neq 0$ [1]

$$\frac{a}{a-d} = \frac{a+d}{a} \quad [1]$$

$$a^2 = a^2 - d^2 \quad [1]$$

So $d^2 = 0$ and hence
 $d = 0$ but this is a
contradiction so the
terms cannot form a
geometric sequence. [1]

$$7 \quad \text{Nine terms are} \quad [1]$$

$$1, r, r^2, \dots, r^8 \quad [1]$$

$$1 + 2 + \dots + 8 = 36 \quad [1]$$

$$r^{36} = 262144 \quad [1]$$

$$r = \pm\sqrt[36]{262144} \quad [1]$$

$$\text{Sum} = \frac{(\sqrt[36]{262144})^9 - 1}{\sqrt[36]{262144} - 1}$$

$$= 15\sqrt[36]{262144} + 31$$

$$\approx 52.2132 \quad [1]$$

$$\text{OR } \frac{(-\sqrt[36]{262144})^9 - 1}{-\sqrt[36]{262144} - 1}$$

$$= -15\sqrt[36]{262144} + 31 \approx 9.7868$$

$$[1]$$

$$8 \quad (i) \quad \frac{dy}{dx} = 4x^3 - 3x \quad [1]$$

$$\frac{d^2y}{dx^2} = 12x^2 - 3 \quad [1]$$

$$\text{At A and B} \quad [1]$$

$$12x^2 - 3 = 0 \quad [1]$$

$$x = \pm 0.5 \quad [1]$$

$$(-0.5, 2.6875) \text{ and} \quad [1]$$

$$(0.5, 2.6875) \quad [1]$$

$$(ii) \quad \text{Gradient at A is } 1. \quad [1]$$

$$\text{Gradient at B is } -1. \quad [1]$$

$$1 \times -1 = -1 \text{ so AC is} \quad [1]$$

$$\text{perpendicular to BC.} \quad [1]$$

$$\text{By symmetry, C is the} \quad [1]$$

$$\text{point } (0, \frac{51}{16}). \quad [1]$$

$$\text{AC} = \text{BC} \quad [1]$$

$$\text{Angle CAD} = \text{Angle} \quad [1]$$

$$\text{CBD (tangent perpen-} \quad [1]$$

$$\text{dicular normal)} \quad [1]$$

All angles are 90° and
all sides are equal so it is
a square.

Review: The sine and cosine rules

Discussion point (page 129)

Yes, cosine rule leads to same
answer of 203.4° .

Review exercise R.1 (page 130)

- 1 (i) (a) 63.1°
(b) 42.8 cm^2
(ii) (a) 47.7°
(b) 13.0 cm^2
- 2 (i) 4.72 cm
(ii) 10.4 cm
(iii) 9.1 cm
(iv) 10.72 cm
- 3 114 m
- 4 120°
- 5 71.3° or 108.7°
- 6 $AD = 57.3 \text{ m}$, $CD = 70.2 \text{ m}$,
height = 46.9 m
- 7 3.28 km
- 8 14.8 cm
- 9 (i) 87°
(ii) 5.29 km
- 10 $6\sqrt{3} \text{ cm}^2$
- 11 $(14 + 2\sqrt{13}) \text{ cm}$
- 12 (i) (a) 0.5327 m
(b) 5.746 m
(c) 5.746 m^2
(ii) (b) 20.0 m
(c) 22.6 m^2

Chapter 6

Discussion point (page 136)

$$\text{Since } \tan \theta = \frac{y}{x} \text{ and } \sin \theta = \frac{y}{r}$$

$$\text{and } \cos \theta = \frac{x}{r} \text{ then } \tan \theta = \frac{\sin \theta}{\cos \theta},$$

$$\cos \theta \neq 0.$$

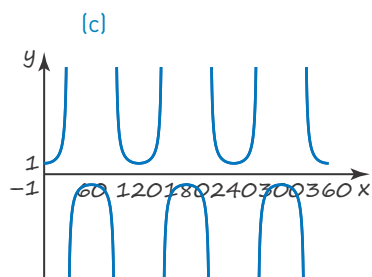
Using Pythagoras' theorem:

$$x^2 + y^2 = r^2$$

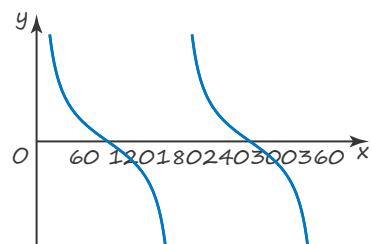
$$\text{So } \sin^2 \theta + \cos^2 \theta \equiv 1.$$

Exercise 6.1 (page 138)

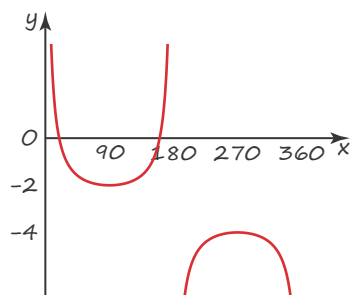
- 1 (i) (a) $\frac{\sqrt{2}}{2}$
(b) $-\frac{\sqrt{2}}{2}$
(c) -1
(d) $-\sqrt{2}$
(ii) (a) $-\frac{1}{2}$
(b) $-\frac{2\sqrt{3}}{3}$
(c) $-\frac{\sqrt{3}}{3}$
(d) 2
(iii) (a) $\sqrt{3}$
(b) $\frac{1}{2}$
(c) $\frac{\sqrt{3}}{2}$
(d) $-\frac{\sqrt{3}}{3}$
(iv) (a) $\frac{3}{4}$
(b) $\frac{1}{4}$
(c) 3
(d) $\frac{4}{3}$
- 2 (i) $B = 60^\circ, C = 30^\circ$
(ii) $\sqrt{3}$
- 3 (i) $L = 45^\circ, N = 45^\circ$
(ii) $\sqrt{2}, \sqrt{2}, 1$
- 4 (ii) 14.0°
- 5 (i) Domain: $x \in \mathbb{R}$,
 $x \neq \pm 90^\circ, \pm 270^\circ \dots$
range:
 $f(x) \geq 1, f(x) \leq -1$
(ii) Domain: $x \in \mathbb{R}$,
 $x \neq n180^\circ$ where n is an
integer; range: $f(x) \geq 1$,
 $f(x) \leq -1$
(iii) Domain: $x \in \mathbb{R}$,
 $x \neq n180^\circ$ where n
is an integer; range:
 $f(x) \in \mathbb{R}$
- 6 (i) $\frac{1}{2}$
(ii) $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
- 7 (i) $-\frac{\sqrt{7}}{4}$
(ii) $-\frac{4}{3}$
- 9 (i) (a) $f(x) \geq 1$ or
 $f(x) \leq -1$
(b) 120°



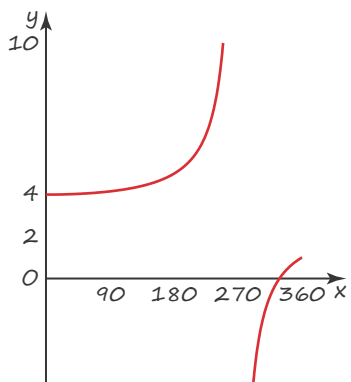
- (ii) (a) $f(x) \in \mathbb{R}$
(b) 180°
(c)



- (iii) (a) $f(x) \leq -4$,
 $f(x) \geq -2$
(b) 360°
(c)

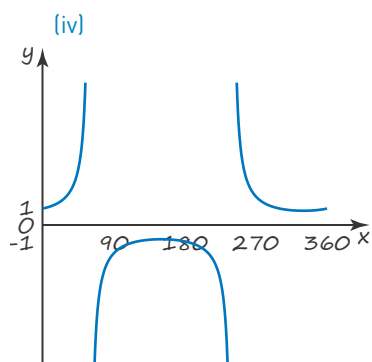
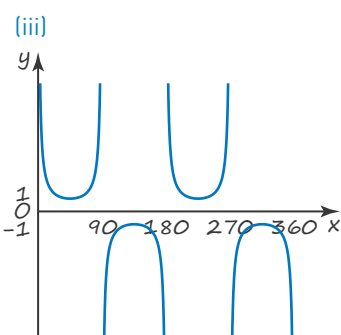
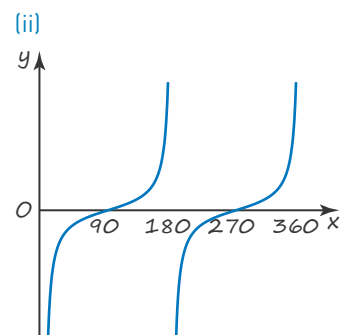
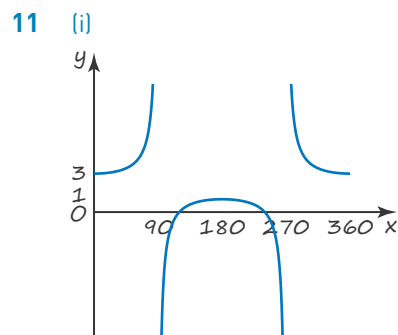


- (iv) (a) $f(x) \leq 2, f(x) \geq 4$
(b) 1080°
(c)



- 10 (i) One-way stretch, scale
factor -2 , parallel to y -axis
(ii) Reflection in x -axis and
then one-way stretch, scale
factor $\frac{1}{2}$, parallel to x -axis
(iii) Translation by $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ and
then reflection in y -axis

- (iv) One-way stretch, scale
factor 2 , parallel to
 x -axis and one-way
stretch, scale factor 2 ,
parallel to y -axis
(v) Translation by $\begin{pmatrix} 0 \\ -30 \end{pmatrix}$
and then reflection in
 x -axis
(vi) Reflection in x -axis
and then translation
by $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$



- 12 (i) $a = 1, b = -1$ and $c = 2$
 (ii) $203^\circ, 255^\circ (\pm 2^\circ)$
 (iii) 0 as $f(2x)$ is a one way stretch scale factor $\frac{1}{2}$, parallel to the x -axis.

Exercise 6.2 (page 144)

- 1 (i) $30^\circ, 330^\circ$
 (ii) $60^\circ, 120^\circ$
 (iii) $60^\circ, 240^\circ$
 (iv) $150^\circ, 210^\circ$
 (v) $240^\circ, 300^\circ$
 (vi) $120^\circ, 300^\circ$
- 2 (i) 2
 (ii) 2
 (iii) 1
 (iv) 4
 (v) 4
 (vi) 4
- 3 (i) $-1 \leq x \leq 1$;
 $0 \leq f(x) \leq \pi$
 (ii) $-1 \leq x \leq 1$;
 $-\frac{\pi}{2} \leq f(x) \leq \frac{\pi}{2}$
 (iii) All real numbers;
 $-\frac{\pi}{2} < f(x) < \frac{\pi}{2}$
- 4 (i) 90°
 (ii) $60^\circ, 300^\circ$
 (iii) $14.0^\circ, 194.0^\circ$
 (iv) $109.5^\circ, 250.5^\circ$
 (v) $135^\circ, 315^\circ$
 (vi) $210^\circ, 330^\circ$
- 6 (i) $0^\circ, 180^\circ, 360^\circ$
 (ii) $45^\circ, 225^\circ$
 (iii) $60^\circ, 300^\circ$
 (iv) $54.7^\circ, 125.3^\circ, 234.7^\circ, 305.3^\circ$
 (v) $18.4^\circ, 71.6^\circ, 198.4^\circ, 251.6^\circ$
 (vi) $45^\circ, 135^\circ, 225^\circ, 315^\circ$

Exercise 6.3 (page 145)

- 1 (i) 2
 (ii) $\frac{2}{\sqrt{3}}$
 (iii) $\frac{1}{\sqrt{3}}$
 (iv) $\frac{2}{\sqrt{3}}$
 (v) 2
 (vi) $-\frac{1}{\sqrt{3}}$

- (vii) $-\sqrt{2}$
 (viii) $-\sqrt{2}$
 (ix) -1

- 2 (i) $\frac{\pi}{6}, \frac{11\pi}{6}$
 (ii) $\frac{\pi}{4}, \frac{5\pi}{4}$
 (iii) $\frac{\pi}{4}, \frac{3\pi}{4}$
 (iv) $\frac{7\pi}{6}, \frac{11\pi}{6}$
 (v) $\frac{3\pi}{4}, \frac{5\pi}{4}$
 (vi) $\frac{\pi}{3}, \frac{4\pi}{3}$
- 3 (i) 0.201 rads, 2.940 rads
 (ii) -0.738 rads, 0.738 rads
 (iii) -1.893 rads, 1.249 rads
 (iv) -2.889 rads, -0.253 rads
 (v) -1.982 rads, 1.982 rads
 (vi) -0.464 rads, 2.678 rads
- 4 (i) 0.32, 1.25, 3.46, 4.39
 (ii) 2.27, 6.02
 (iii) 0.19, 1.76, 3.33, 4.90
 (iv) $\frac{3\pi}{8}, \frac{5\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}$
 (v) $\frac{\pi}{3}$
 (vi) $\frac{\pi}{3}, \frac{2\pi}{3}$
- 5 (i) $\frac{\pi}{4}, \frac{3\pi}{4}$
 (ii) $\frac{\pi}{4}, \frac{3\pi}{4}$
 (iii) $\frac{\pi}{4}, \frac{3\pi}{4}$
 (iv) $\frac{\pi}{3}, \frac{5\pi}{6}$
 (v) $\frac{\pi}{3}, \frac{2\pi}{3}$
 (vi) $\frac{\pi}{3}, \pi$
 (vii) 0, 0.730, 2.412, π
 (viii) 1.23, $\frac{\pi}{3}$
 (ix) 0.245 and 2.55

- 6 (i) 06.30
 (ii) Between 01.59 and 11.01, and between 13.59 and 23.01
- 7 $0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi$
- 8 2.24, 4.05
- 9 (i) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Chapter 7

Activity 7.1 (page 149)

$$k = \frac{1}{2}, x = 0.18$$

Review exercise (page 150)

- 1 (i)
- | | | | | | | | | | | |
|---|---|----|----|----|---|---|--|--|--|--|
| | | | 1 | | | | | | | |
| | | | 1 | 2 | 1 | | | | | |
| | | 1 | 3 | 3 | 1 | | | | | |
| | 1 | 4 | 6 | 4 | 1 | | | | | |
| 1 | 5 | 10 | 10 | 5 | 1 | | | | | |
| 1 | 6 | 15 | 20 | 15 | 6 | 1 | | | | |
- (ii) Totals are 1, 2, 4, 8, 16, 32, 64. These can be written as $2^0, 2^1, 2^2, 2^3, 2^4, 2^5, 2^6$.
- (iii) The total would be $2^{12} = 4096$.
- 2 (i) $1 + 3x + 3x^2 + x^3$
 (ii) $1 + 6x + 12x^2 + 8x^3$
 (iii) $1 - 6x + 12x^2 - 8x^3$
- 3 (i) (a) $1 + 8x + 24x^2 + 32x^3 + 16x^4$
 (b) $1 - 4x + 6x^2 - 4x^3 + x^4$
 (ii) $1 + 4x - 2x^2$
- 4 (i) $12x^2$
 (ii) $24x^2$
 (iii) $720x^3$
- 5 $1 - \frac{7}{2}x + \frac{21}{4}x^2 - \frac{35}{8}x^3 + \frac{35}{16}x^4$

Activity 7.3 (page 151)

- (i) (a) $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \dots$
 (b) $1 - 2x + 3x^2 + \dots$
- (ii) (a) 1.04875
 (b) 0.83
- (iii) $\sqrt{1.1} = 1.048808\dots$ and $\frac{1}{1.1^2} = 0.8264\dots$, so the approximations are good.
- (iv) (a) The expansion gives -6.5 which is not a good approximation for $\sqrt{11}$.
 (b) The expansion gives 281 which is not a good approximation for $\frac{1}{11^2}$.

Discussion point (page 153)

It is because of the x^2 in the bracket.

Exercise 7.1 (page 155)

- 1 (i) $1 - 3x + 6x^2 - 10x^3$;
valid for $-1 < x < 1$
(ii) $1 - 6x + 24x^2 - 80x^3$;
valid for $-\frac{1}{2} < x < \frac{1}{2}$
(iii) $1 + 6x + 24x^2 + 80x^3$;
valid for $-\frac{1}{2} < x < \frac{1}{2}$
- 2 (i) $1 + \frac{1}{2}x - \frac{1}{8}x^2$; valid for
 $-1 < x < 1$
(ii) $1 - \frac{1}{2}x + \frac{3}{8}x^2$; valid for
 $-1 < x < 1$
(iii) $1 + \frac{1}{4}x - \frac{3}{32}x^2$; valid for
 $-1 < x < 1$
(iv) $1 - \frac{1}{4}x + \frac{5}{32}x^2$; valid for
 $-1 < x < 1$
- 3 (i) $1 - \frac{2}{3}x + \frac{1}{3}x^2 - \frac{4}{27}x^3$;
valid for $-3 < x < 3$
(ii) $1 - \frac{4}{3}x + \frac{4}{3}x^2 - \frac{32}{27}x^3$;
valid for $-1.5 < x < 1.5$
(iii) $1 + \frac{4}{3}x + \frac{4}{3}x^2 + \frac{32}{27}x^3$;
valid for $-1.5 < x < 1.5$
- 4 (i) (a) $1 - 2x + 3x^2$
(b) $|x| < 1$
(c) 0.43%
(ii) (a) $1 - 2x + 4x^2$
(b) $|x| < \frac{1}{2}$
(c) 0.8%
(iii) (a) $1 - \frac{x^2}{2} - \frac{x^4}{8}$
(b) $|x| < 1$
(c) 0.0000063%
- 5 (i) $1 - 3x + 3x^2 - x^3$
(ii) $1 - 4x + 10x^2$; valid
when $|x| < 1$
(iii) $a = -7, b = 25$
- 6 (ii) $1 - \frac{x}{8} + \frac{3x^2}{128}$; valid
when $|x| < 4$
(iii) $1 - \frac{9x}{8} + \frac{19x^2}{128}$
- 7 (i) $1 - 3x + 7x^2$
(ii) $|x| < \frac{1}{2}$
- 8 $\frac{3}{4} + \frac{11}{16}x - \frac{5}{64}x^2, |x| < 4$
- 9 (i) $a = 2, b = -1, c = \frac{3}{16}$
(ii) $|x| < 2$

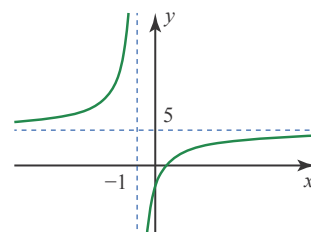
- 10 $2(1 - x); |x| < 1$
- 11 (i) $1 + 2x + 5x^2$
(ii) $|x| < \frac{1}{3}$
- 12 (i) $-4 - 10x - 16x^2$
(ii) $|x| < \frac{1}{2}$
- 13 (i) $3\sqrt{\frac{7}{9}}$ (there are other
options)
(ii) 2.64609 (5 d.p.)
(iii) % error 0.013% so very
good
- 14 (i) A possible rearrangement
is $\sqrt{x\left(1 - \frac{1}{x}\right)} = \sqrt{x}\left(1 - \frac{1}{x}\right)^{\frac{1}{2}}$
Since $x > 1$
 $\Rightarrow 0 < \frac{1}{x} < 1$; the
binomial expansion
could be used but the
resulting expansion
would not be a series
of positive powers of x .
(ii) 3.968

Exercise 7.2 (page 161)

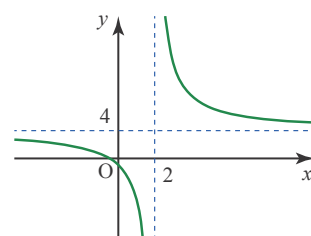
- 1 (i) $(x + 2)$
(ii) $\frac{1}{(x + 2)}$
(iii) $\frac{(x - 1)}{(x + 2)}$
(iv) $\frac{(x + 2)}{(x - 1)}$
- 2 (i) $\frac{x + 3}{x - 6}$
(ii) $\frac{2x - 5}{2x + 5}$
(iii) $\frac{a - b}{2a - b}$
- 3 (i) $\frac{9}{20x}$
(ii) $\frac{2a}{(a + 1)(a - 1)}$
(iii) $\frac{7x + 3}{12}$
- 4 (i) $-\frac{2}{(x + 2)(x - 2)}$
(ii) $-\frac{(3b + 1)}{(b + 1)^2}$
(iii) $\frac{3a - 4}{(a + 2)(2a - 3)}$
- 5 (i) $2(2x + 1)$
(ii) $x(2x + 1)$
(iii) $a(2x + 1)$

- 6 (i) $\frac{3x^2 - 4}{x(x - 2)(x + 2)}$
(ii) $\frac{9 + 6a - a^2}{a(3 - a)(3 + a)}$
(iii) $\frac{2b^2 - 2b + 1}{b(b - 1)^2}$

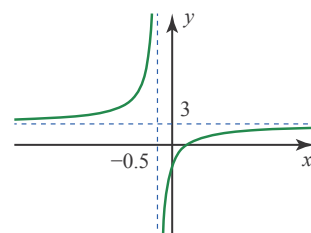
7 (i) $y = 5 - \frac{8}{x + 1}$



(ii) $y = 4 + \frac{10}{x - 2}$



(iii) $y = 3 - \frac{6}{2x + 1}$

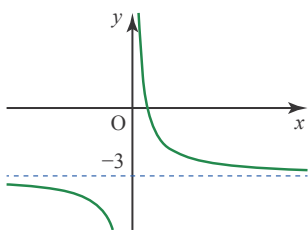


- 8 (i) $2x + 1 + \frac{4}{x + 1}$
(ii) $3x - 5 + \frac{4}{x + 2}$
(iii) $x - 5 + \frac{2}{x - 1}$
- 9 (i) Quotient $x^2 + 10x + 25$,
remainder 77
(ii) Quotient $x^3 - 2x^2 + 5x$
 $- 7$, remainder 19
- 10 (i) 0; $x = -2, 1.5, -0.5$
(ii) 0, 0; $x = -2, -0.5, 1.5, -5$
- 11 (i) $\frac{(x + 1)(x + 3)}{(x - 3)(x - 2)}$
(ii) $\frac{(x + 1)(x - 2)}{(2x - 3)(2x + 3)}$
(iii) $\frac{(x - 4)}{(x + 2)}$

12 (i) $\frac{1}{x(x-1)(x+1)}$
 (ii) $\frac{4}{(x-1)(x+3)}$

13 $3x^2 - 4x + 3 - \frac{10}{x+2}$

14 Write as $y = -3 + \frac{2}{x}$;
 Stretch scale factor
 2 in y direction and
 translation $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$.



Activity 7.4 (page 162)

- (i) $\frac{13x-3}{(3x+1)(2x-3)}$
 (ii) $1 - \frac{20}{3}x + \frac{158}{9}x^2$
 (iii) Probably expanding the two separate fractions.

Discussion point (page 163)

The identity is true for all values of x . Once a particular value of x is substituted you have an equation. Equating constant terms is equivalent to putting $x = 0$.

Exercise 7.3 (page 165)

- 1 (i) $\frac{1}{x-1} - \frac{1}{x}$
 (ii) $\frac{1}{x} - \frac{1}{x+1}$
 (iii) $\frac{1}{x-1} - \frac{1}{x+1}$
 2 (i) $\frac{1}{x-1} - \frac{1}{x+2}$
 (ii) $\frac{1}{x-2} - \frac{1}{x+3}$
 (iii) $\frac{1}{x-3} - \frac{1}{x+4}$
 3 (i) $\frac{1}{x-2} - \frac{2}{2x-1}$
 (ii) $\frac{1}{x-3} - \frac{3}{3x-1}$
 (iii) $\frac{1}{x-4} - \frac{4}{4x-1}$

- 4 (i) $\frac{1}{x-2} - \frac{1}{x}$
 (ii) $\frac{1}{x-3} - \frac{1}{x-1}$
 (iii) $\frac{1}{x-4} - \frac{1}{x-2}$
 5 (i) $\frac{1}{(x-1)^2} - \frac{1}{x-1} + \frac{1}{x+2}$
 (ii) $\frac{9}{1-3x} - \frac{3}{1-x} - \frac{2}{(1-x)^2}$
 6 (i) $\frac{12}{1+2x} - \frac{2}{1+x} - \frac{12}{1+3x}$
 (ii) $\frac{6}{2+x} - \frac{2}{1+x} - \frac{4}{3+x}$
 7 (i) $\frac{5}{x-1} - \frac{3}{x} - \frac{2}{x+1}$
 (ii) $\frac{8}{x} + \frac{3}{2-x} - \frac{5}{2+x}$
 8 (i) $\frac{9}{(1-3x)} - \frac{3}{(1-x)} - \frac{2}{(1-x)^2}$
 (ii) $4 + 20x + 72x^2$
 (iii) $|x| < \frac{1}{3}$
 9 (i) $\frac{7x-1}{x^2+1} - \frac{7}{x+1}$
 (ii) $-8 + 14x - 6x^2$ valid for $|x| < 1$
 10 (i) $\frac{2}{(x-1)} + \frac{1}{(x-1)^2} + \frac{3}{(x-1)^3}$
 (ii) $-4 - 9x - 17x^2$ valid for $|x| < 1$

Chapter 8

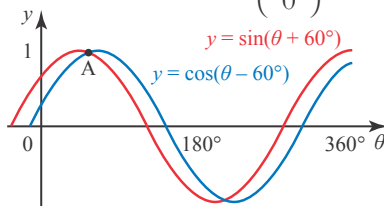
Opening activity (page 169)

Answers will vary.
 For example, $a \approx 1.6$ to 1.8 ,
 $b \approx 3 \Rightarrow y = 1.5 \sin 3x$

Activity 8.1 (page 170)

$y = \sin(\theta + 60^\circ)$ is obtained from $y = \sin \theta$ by a translation $\begin{pmatrix} -60^\circ \\ 0 \end{pmatrix}$

$y = \cos(\theta - 60^\circ)$ is obtained from $y = \sin \theta$ by a translation $\begin{pmatrix} 60^\circ \\ 0 \end{pmatrix}$



It appears that the θ coordinate of A is midway between the two maxima $(30^\circ, 1)$ and $(60^\circ, 1)$.

Checking: $\theta = 45^\circ \rightarrow$
 $\sin(\theta + 60^\circ) = 0.966$

$\cos(\theta - 60^\circ) = 0.966$.

If 60° is replaced by 35° , using the trace function on a graphic calculator would enable the solutions to be found.

Activity 8.2 (page 171)

1 (i) $\sin(\theta + \phi) =$
 $\sin \theta \cos \phi + \cos \theta \sin \phi$
 $\Rightarrow \sin[(90^\circ - \theta) + \phi] =$
 $\sin(90^\circ - \theta) \cos \phi +$
 $\cos(90^\circ - \theta) \sin \phi$
 $\Rightarrow \sin[90^\circ - (\theta - \phi)] =$
 $\cos \theta \cos \phi + \sin \theta \sin \phi$
 $\Rightarrow \cos(\theta - \phi) =$
 $\cos \theta \cos \phi + \sin \theta \sin \phi$
 (ii) $\Rightarrow \cos[\theta - (-\phi)] =$
 $\cos \theta \cos(-\phi) + \sin \theta$
 $\sin(-\phi)$
 $\Rightarrow \cos(\theta + \phi) =$
 $\cos \theta \cos \phi - \sin \theta \sin \phi$
 (iii) $\tan(\theta + \phi) =$
 $= \frac{\sin(\theta + \phi)}{\cos(\theta + \phi)}$
 $= \frac{\sin \theta \cos \phi + \cos \theta \sin \phi}{\cos \theta \cos \phi - \sin \theta \sin \phi}$
 $= \frac{\left[\frac{\sin \theta \cos \phi}{\cos \theta \cos \phi} + \frac{\cos \theta \sin \phi}{\cos \theta \cos \phi} \right]}{\left[\frac{\cos \theta \cos \phi}{\cos \theta \cos \phi} - \frac{\sin \theta \sin \phi}{\cos \theta \cos \phi} \right]}$
 $= \frac{\tan \theta + \tan \phi}{1 - \tan \theta \tan \phi}$

(iv) $\tan[\theta + (-\phi)] = \frac{\tan \theta + \tan(-\phi)}{1 - \tan \theta \tan(-\phi)}$

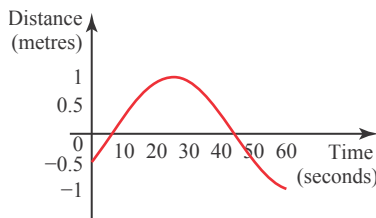
$\tan(\theta - \phi) = \frac{\tan \theta - \tan \phi}{1 + \tan \theta \tan \phi}$

- 2 Yes, the formulae are valid for all values of θ and ϕ in both degrees and radians.

Exercise 8.1 (page 174)

- 1 (i) $\frac{1}{\sqrt{2}}(\sin \theta + \cos \theta)$
 (ii) $\frac{1}{\sqrt{2}}(\cos \theta - \sin \theta)$
 (iii) $\frac{1}{\sqrt{2}}(\cos \theta + \sin \theta)$
 (iv) $\frac{1}{\sqrt{2}}(\cos \theta - \sin \theta)$
 (v) $\frac{\tan \theta + 1}{1 - \tan \theta}$
 (vi) $\frac{1 - \tan \theta}{1 + \tan \theta}$
- 2 $\sin 2\theta \cos \theta - \cos 2\theta \sin \theta$
 $= \sin \theta$
 $\cos \theta \cos 3\theta - \sin \theta \sin 3\theta$
 $= \cos 4\theta$
 $\cos \theta \cos \theta + \sin \theta \sin \theta = 1$
 $\cos \theta \cos \theta - \sin \theta \sin \theta = \cos 2\theta$
 $\cos 4\theta \cos 2\theta + \sin 4\theta \sin 2\theta$
 $= \cos 2\theta$
 $\cos 2\theta \sin \theta + \sin 2\theta \cos \theta$
 $= \sin 3\theta$
- 3 (i) 0
 (ii) $\frac{\sqrt{3}}{2}$
 (iii) $\frac{1}{2}$
 (iv) -1
- 4 (i) (a) $\frac{1 + \sqrt{3}}{2\sqrt{2}}$
 (b) $\frac{\sqrt{3} - 1}{2\sqrt{2}}$
 (c) $2 + \sqrt{3}$
 (ii) (a) $\frac{\sqrt{3} - 1}{2\sqrt{2}}$
 (b) $\frac{1 + \sqrt{3}}{2\sqrt{2}}$
 (c) $2 - \sqrt{3}$
- 5 (i) $\frac{\sqrt{3} - 1}{2\sqrt{2}}$
 (ii) $\frac{1 - \sqrt{3}}{2\sqrt{2}}$
 (iii) $-2 - \sqrt{3}$
- 6 (i) 15°
 (ii) 157.5°
 (iii) 0° or 180°
 (iv) 111.7°
 (v) 165°
- 7 (i) $\frac{\pi}{8}$
 (ii) $\frac{\pi}{2}$
 (iii) 2.79 radians

- 8 (ii) $\frac{\pi}{2}$
- 10 (i) $1 - \theta\sqrt{3} - \frac{\theta^2}{2}$
 (ii) $\theta \sin \alpha + \theta^2 \cos \alpha$
 (iii) $\frac{1 - \theta}{1 + \theta}$
- 11 (i) (a) $\frac{\theta^2\sqrt{3}}{2}$
 (b) $2\theta^2$
 (ii) $\frac{\sqrt{3}}{4}$
- 12 (i) -0.5 m
 (ii) $k = \frac{\pi}{6}$
 (iii) 2π seconds, 14π seconds
 (iv) 1 m; 8π seconds
 (v) No; $\sin\left(\frac{t}{12} - \frac{\pi}{6}\right) = -1$
 at $t = 20\pi$ and $20\pi > 60$
 (vi)



- 13 (i) $\sin x \cos h + \cos x \sin h$
 (ii) $\sin x + h \cos x - \frac{h^2}{2} \sin x$
 (iii) $\cos x - \frac{h}{2} \sin x$
 (iv) $\cos x$
 (v) Derivative of $\sin x$
- 14 (i) $\frac{1 + \theta}{1 - \theta}$
 (ii) $1 + 2\theta + \theta^2$

Discussion point (page 175)

For $\sin 2\theta$ and $\cos 2\theta$, substituting $\theta = 45^\circ$ is helpful.

You know that $\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$ and that $\sin 90^\circ = 1$ and $\cos 90^\circ = 0$.

For $\tan 2\theta$ you cannot use $\theta = 45^\circ$.

Take $\theta = 30^\circ$ instead;

$\tan 30^\circ = \frac{1}{\sqrt{3}}$ and $\tan 60^\circ = \sqrt{3}$.

No, checking like this is not the same as proof.

Exercise 8.2 (page 177)

- 1 (i) $\cos 80^\circ$
 (ii) $\frac{1}{2} \sin 80^\circ$
 (iii) $\tan 40^\circ$
- 2 $\cot \theta$
- 3 (i) (a) $\frac{4}{5}$
 (b) $\frac{3}{4}$
 (c) $\frac{24}{25}$
 (d) $\frac{7}{25}$
 (e) $\frac{24}{7}$
 (ii) (a) $-\frac{4}{5}$
 (b) $-\frac{3}{4}$
 (c) $-\frac{24}{25}$
 (d) $\frac{7}{25}$
 (e) $-\frac{24}{7}$
- 4 (i) $14.5^\circ, 90^\circ, 165.5^\circ, 270^\circ$
 (ii) $0^\circ, 35.3^\circ, 144.7^\circ, 180^\circ, 215.3^\circ, 324.7^\circ, 360^\circ$
 (iii) $90^\circ, 210^\circ, 330^\circ$
 (iv) $30^\circ, 150^\circ, 210^\circ, 330^\circ$
 (v) $0^\circ, 138.6^\circ, 221.4^\circ, 360^\circ$
- 5 (i) $-\pi, 0, \pi$
 (ii) $-\pi, 0, \pi$
 (iii) $-\frac{2\pi}{3}, 0, \frac{2\pi}{3}$
 (iv) $-\frac{3\pi}{4}, -\frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{2}$
 (v) $-\frac{11\pi}{12}, -\frac{3\pi}{4}, -\frac{7\pi}{12}, -\frac{\pi}{4}, \frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}, \frac{3\pi}{4}$
- 6 $3 \sin \theta - 4 \sin^3 \theta, \theta = 0,$
 $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, 2\pi$
- 7 $51^\circ, 309^\circ$
- 9 (ii) 63.4°

Activity 8.4 (page 178)

$y = a \sin \theta + b \cos \theta$ is a translation and vertical stretch of a sine or cosine graph.

(i) and (ii) Answers depend on students' choice of values of a and b .

Discussion point (page 179)

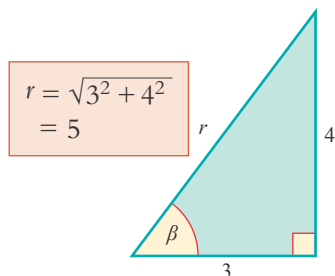
A translation by the vector

$$\begin{pmatrix} -36.9 \\ 0 \end{pmatrix}$$
 and a one-way

stretch, scale factor 5, parallel to the y -axis.

Activity 8.5 (page 179)

- (i) The expansion of $r \cos(\theta - \beta)$ is given by $r \cos(\theta - \beta) = r(\cos \theta \cos \beta + \sin \theta \sin \beta)$.



To compare this with $3 \cos \theta + 4 \sin \theta$
 $r = \sqrt{3^2 + 4^2} = 5$
 $\cos \beta = \frac{3}{5}$
 $\sin \beta = \frac{4}{5} \Rightarrow \beta = 53.1^\circ$.

So you can write

$3 \cos \theta + 4 \sin \theta$ in the form $r \cos(\theta - \beta) = 5 \cos(\theta - 53.1^\circ)$.

- (ii) $y = \cos x$ maps to $y = \sin x$ by a translation by the vector $\begin{pmatrix} 90 \\ 0 \end{pmatrix}$

So translating $y = \cos x$ by the vector $\begin{pmatrix} 53.1 \\ 0 \end{pmatrix}$ and translating $y = \sin x$ by the vector $\begin{pmatrix} -36.9 \\ 0 \end{pmatrix}$ result in the same curve.

Exercise 8.3 (page 182)

- 1 (a) $r = 3, \alpha = 41.8^\circ$
 (b) $r = \sqrt{5}, \alpha = 50.8^\circ$
- 2 (i) $\frac{3}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta$
 (ii) $\frac{3}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta$
 (iii) $\frac{3}{2} \sin \theta - \frac{\sqrt{3}}{2} \cos \theta$
 (iv) $\frac{3}{2} \sin \theta + \frac{\sqrt{3}}{2} \cos \theta$

- 3 (i) (a) max: 2; min: -2
 (b) $x = \frac{\pi}{3}$
 (ii) (a) max: $\sqrt{2}$; min: $-\sqrt{2}$
 (b) $x = \frac{5\pi}{3}$
 (iii) (a) max: $\frac{1}{\sqrt{2}}$; min: $-\frac{1}{\sqrt{2}}$

(b) $x = \frac{\pi}{6}$

- (iv) (a) max: $\frac{1}{2 - \sqrt{2}}$;
 min: $\frac{1}{2 + \sqrt{2}}$

(b) $x = \frac{7\pi}{6}$

- 4 (i) (a) $\sqrt{2} \cos(\theta - 45^\circ)$
 (b) $5 \cos(\theta - 53.1^\circ)$
 (ii) (a) $\sqrt{2} \sin(\theta - 45^\circ)$
 (b) $5 \sin(\theta - 53.1^\circ)$

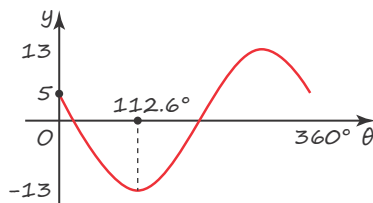
- 5 (i) (a) $\sqrt{2} \cos\left(\theta + \frac{\pi}{4}\right)$
 (b) $\sqrt{2} \sin\left(\theta - \frac{\pi}{4}\right)$

- (ii) (a) $2 \cos\left(\theta + \frac{\pi}{6}\right)$

(b) $2 \sin\left(\theta - \frac{\pi}{6}\right)$

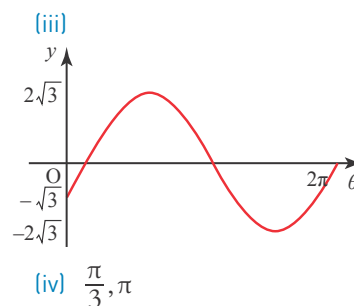
- 6 (i) $2 \cos(\theta - (-60^\circ))$
 (ii) $4 \cos(\theta - (-45^\circ))$
 (iii) $2 \cos(\theta - 30^\circ)$
 (iv) $13 \cos(\theta - 22.6^\circ)$
 (v) $2 \cos(\theta - 150^\circ)$
 (vi) $2 \cos(\theta - 135^\circ)$

- 7 (i) $13 \cos(\theta + 67.4^\circ)$
 (ii) Max 13, min -13
 (iii)

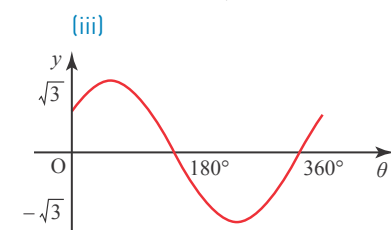


- (iv) $4.7^\circ, 220.5^\circ$

- 8 (i) $2\sqrt{3} \sin\left(\theta - \frac{\pi}{6}\right)$
 (ii) Max $2\sqrt{3}, \theta = \frac{2}{3}\pi$;
 min $-2\sqrt{3}, \theta = \frac{5}{3}\pi$

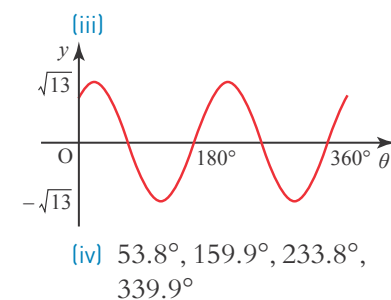


- 9 27.0°
- 10 (i) $(4 \sin \theta + 3 \sin \phi, 4 \cos \theta + 3 \cos \phi)$
 (iii) The cosine rule
- 11 (i) $\sqrt{3} \cos(\theta - 54.7^\circ)$
 (ii) Max $\sqrt{3}, \theta = 54.7^\circ$;
 min $-\sqrt{3}, \theta = 234.7^\circ$



- (iv) Max $\left(\frac{1}{3 - \sqrt{3}}\right), \theta = 234.7^\circ$; min $\left(\frac{1}{3 + \sqrt{3}}\right), \theta = 54.7^\circ$

- 12 (i) $\sqrt{13} \sin(2\theta + 56.3^\circ)$
 (ii) Max $\sqrt{13}, \theta = 16.8^\circ$;
 min $-\sqrt{13}, \theta = 106.8^\circ$



Summary exercise (page 183)

- 1 (i) $\sin 6\theta$
 (ii) $\cos 6\theta$
 (iii) 1
 (iv) $\cos \theta$
 (v) $\sin \theta$
 (vi) $\frac{3}{2} \sin 2\theta$
 (vii) $\cos \theta$
 (viii) -1

- 2 (i) $1 - \sin 2x$
 (ii) $\frac{1}{2}(5 \cos 2x - 1)$
- 4 (i) $4.4^\circ, 95.6^\circ$
 (ii) $199.5^\circ, 340.5^\circ$
 (iii) $-\frac{\pi}{6}, \frac{\pi}{2}$
 (iv) $-15.9^\circ, 164.1^\circ$
 (v) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$
 (vi) $20.8^\circ, 122.3^\circ$
 (vii) $76.0^\circ, 135^\circ$

Chapter 9

Discussion point (page 186)

$$f'(x)e^{f(x)}$$

Discussion point (page 187)

The graphs are parallel. They are translations in the y direction.

Discussion point (page 187)

$$\frac{f'(x)}{f(x)}$$

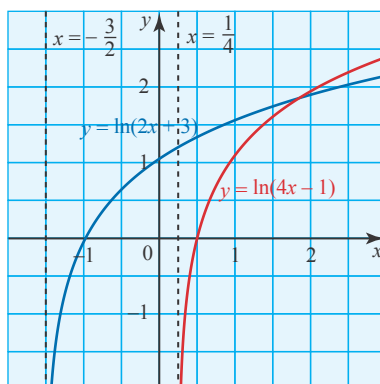
Exercise 9.1 (page 188)

- 1 (i) $\frac{dy}{dx} = 3e^{3x+1}$
 (ii) $\frac{dy}{dx} = (2x+3)e^{x^2+3x+1}$
- 2 (i) $\frac{dy}{dx} = 7^x \ln 7$
 (ii) $\frac{dy}{dx} = (5 \ln 7)7^{5x}$
- 3 (i) $\frac{dy}{dx} = \frac{1}{x}$
 (ii) $\frac{dy}{dx} = \frac{2x}{x^2+1}$
 (iii) $\frac{dy}{dx} = \frac{1}{2x}$
- 4 (i) $\frac{dy}{dx} = e^{4x}(4x+1)$
 (ii) $\frac{dy}{dx} = \frac{1-2 \ln x}{x^3}$
 (iii) $\frac{dy}{dx} = \frac{1-x}{e^x}$
- 5 (i) $\frac{4}{e}$
 (ii) 24
 (iii) $\frac{3}{8}$
- 6 (i) $\frac{dy}{dx} = e^x(x+1)$ and $\frac{d^2y}{dx^2} = e^x(x+2)$

(ii) $\left(-1, -\frac{1}{e}\right)$

(iii) $\left(-2, -\frac{2}{e^2}\right)$

- 7 (i) $(2, \ln 7)$
 (ii) $2x - 7y - 4 + 7 \ln 7 = 0$
 (iii)

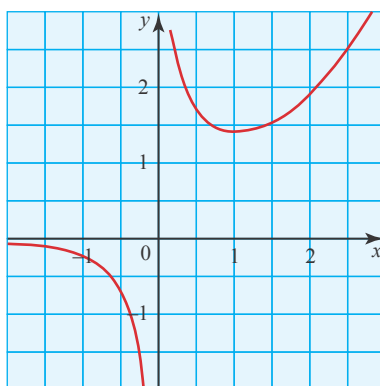


- 8 (i) Rotational symmetry of order 2 about origin, because $f(-x) = -f(x)$
 (ii) $f'(x) = \ln(x^2) + 2$ and $f''(x) = \frac{2}{x}$
 (iii) $\left(\frac{1}{e}, -\frac{2}{e}\right), \left(-\frac{1}{e}, \frac{2}{e}\right)$

- 9 (i) $a = 1$
 (ii) $x + 9y - 1 = 0$

- 10 (i) $\frac{dy}{dx} = 2x - 3 + \frac{1}{x}$, $\frac{d^2y}{dx^2} = 2 - \frac{1}{x^2}$
 (ii) Minimum at $(1, 0)$, maximum at $\left(\frac{1}{2}, \frac{3}{4} - \ln 2\right)$

- 11 Minimum at $(1, e)$ and asymptotes $x = 0, y = 0$



- 12 The proofs depend on the results that $\frac{dy}{dz} = \frac{-z}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$ and $\frac{d^2y}{dz^2} = \frac{(z^2-1)}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$

Discussion point (page 191)

No

Activity 9.2 (page 191)

$$\begin{aligned} y &= \frac{\sin x}{\cos x} \Rightarrow \frac{dy}{dx} \\ &= \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{\cos^2 x} \\ &= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} \\ &= \sec^2 x \end{aligned}$$

Activity 9.3 (page 192)

$$\begin{aligned} y &= \sin(f(x)) \Rightarrow \frac{dy}{dx} \\ &= f'(x) \cos(f(x)) \\ y &= \cos(f(x)) \Rightarrow \frac{dy}{dx} \\ &= -f'(x) \sin(f(x)) \end{aligned}$$

Exercise 9.2 (page 193)

- 1 (i) $\frac{dy}{dx} = \cos x - 2 \sin x$
 (ii) $\frac{dy}{dx} = 5 \cos x + 3 \sin x$
 (iii) $\frac{dy}{dx} = 3 \sec^2 x - 2$
- 2 (i) $\frac{dy}{dx} = 3 \sec^2 3x$
 (ii) $\frac{dy}{dx} = \frac{1}{2} \cos\left(\frac{x}{2}\right)$
 (iii) $\frac{dy}{dx} = 6 \sin(-6x)$
- 3 (i) $\frac{dy}{dx} = 2x \cos(x^2)$
 (ii) $\frac{dy}{dx} = e^{\tan x} \sec^2 x$
 (iii) $\frac{dy}{dx} = -\tan x$
- 4 (i) $\frac{dy}{dx} = \tan x + x \sec^2 x$
 (ii) $\frac{dy}{dx} = e^x(\cos x - \sin x)$
 (iii) $\frac{dy}{dx} = \cos^2 x - \sin^2 x$

$$\begin{aligned} 5 \quad (i) \quad \frac{dy}{dx} &= \frac{x \cos x - \sin x}{x^2} \\ (ii) \quad \frac{dy}{dx} &= \frac{e^x (\cos x + \sin x)}{\cos^2 x} \\ (iii) \quad \frac{dy}{dx} &= \frac{\sin x - x \cos x - 1}{\sin^2 x} \end{aligned}$$

$$\begin{aligned} 6 \quad (i) \quad \frac{dy}{dx} &= \frac{3 \cos 3x}{2\sqrt{\sin 3x}} \\ (ii) \quad \frac{dy}{dx} &= -2e^{\cos 2x} \sin 2x \\ (iii) \quad \frac{dy}{dx} &= \frac{\sin x \cos x - x \ln x}{x \sin^2 x} \end{aligned}$$

$$\begin{aligned} 8 \quad (i) \quad \frac{dy}{dx} &= \cos x - x \sin x \\ (ii) \quad &-1 \\ (iii) \quad &y + x = 0 \\ (iv) \quad &x - y - 2\pi = 0 \end{aligned}$$

$$\begin{aligned} 9 \quad (i) \quad \frac{dy}{dx} &= \cos x - 2 \sin 2x, \\ \frac{d^2y}{dx^2} &= -\sin x - 4 \cos 2x \end{aligned}$$

$$(ii) \quad \left(\frac{\pi}{2}, 0\right), (0.253, 1.125)$$

$$\begin{aligned} &(2.889, 1.125) \\ (iii) \quad \left(\frac{\pi}{2}, 0\right) &\text{ is a minimum,} \\ (0.253, 1.125) &\text{ is a} \\ &\text{maximum, } (2.889, 1.125) \\ &\text{is a maximum} \end{aligned}$$

$$(iv) \quad (0.883, 0.579),$$

$$(2.259, 0.579)$$

$$\begin{aligned} 11 \quad (i) \quad \frac{dy}{dx} &= \lim_{h \rightarrow 0} \left(\frac{\cos(x+h) - \cos x}{h} \right) \\ (ii) \quad \cos(x+h) &= \cos x \cos h - \sin x \sin h \end{aligned}$$

$$\begin{aligned} (iii) \quad \frac{dy}{dx} &= \lim_{h \rightarrow 0} \left(\frac{(\cos x) \left(1 - \frac{1}{2}h^2\right) - (\sin x)h - \cos x}{h} \right) \\ &= \lim_{h \rightarrow 0} \left(\frac{\cos x - \frac{1}{2}h^2 \cos x - h \sin x - \cos x}{h} \right) \end{aligned}$$

$$= \lim_{h \rightarrow 0} \left(-\frac{1}{2}h \cos x - \sin x \right)$$

$$(iv) \quad \lim_{h \rightarrow 0} \left(-\frac{1}{2}h \cos x - \sin x \right)$$

$$= -\sin x$$

$$\begin{aligned} 12 \quad (i) \quad &\left(\frac{\pi}{6}, \frac{\pi}{2} - \sqrt{3}\right) \\ (ii) \quad &\sqrt{3} + \frac{\pi}{6} \end{aligned}$$

Activity 9.4 (page 195)

$$(i) \quad 7x^6$$

$$(ii) \quad 10y^4 \frac{dy}{dx}$$

$$(iii) \quad -2z \frac{dz}{dx}$$

$$(iv) \quad 3 \frac{dz}{dy}$$

$$(v) \quad 8x^3 \frac{dx}{dz}$$

Exercise 9.3 (page 196)

$$1 \quad (i) \quad \frac{1}{2\sqrt{y}} \frac{dy}{dx}$$

$$(ii) \quad -\frac{8}{y^3} \frac{dy}{dx}$$

$$(iii) \quad 3z^2 \frac{dz}{dx} + 9y^8 \frac{dy}{dx}$$

$$2 \quad (i) \quad 2ye^{y^2} \frac{dy}{dx}$$

$$(ii) \quad 3 \frac{dy}{dx} \cos(3y - 2)$$

$$(iii) \quad \frac{2y}{y^2 + 2} \frac{dy}{dx}$$

$$3 \quad (i) \quad e^y \left(1 + x \frac{dy}{dx} \right)$$

$$(ii) \quad -\sin x \sin y \frac{dy}{dx} + \cos x \cos y$$

$$(iii) \quad \frac{3x^2 \sin y - x^3 \cos y \frac{dy}{dx}}{\sin^2 y}$$

$$4 \quad (i) \quad \frac{dy}{dx} = \frac{-(1+y)}{1+x}$$

$$(ii) \quad \frac{dy}{dx} = \frac{-y^2}{2(e^{2y} + xy)}$$

$$(iii) \quad \frac{dy}{dx} = \frac{-\sin x}{\sin y}$$

$$5 \quad \frac{1}{5}$$

$$6 \quad (i) \quad 0$$

$$(ii) \quad y = -1$$

$$7 \quad \text{Tangent is } x + y - 2 = 0$$

$$\text{and normal is } x - y + 2 = 0$$

$$\begin{aligned} 8 \quad (i) \quad \frac{dx}{dy} &= \cos y \\ \text{so } \frac{dy}{dx} &= \frac{1}{\cos y} \end{aligned}$$

$$(ii) \quad \cos y \frac{dy}{dx} = 1 \text{ so}$$

$$\frac{dy}{dx} = \frac{1}{\cos y}$$

$$\frac{1}{\cos y} = \frac{1}{\sqrt{1 - \sin^2 y}} = \frac{1}{\sqrt{1 - x^2}}$$

$$9 \quad \text{Write } x = \tan y:$$

$$(i) \quad \frac{dx}{dy} = \sec^2 y \text{ so}$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1 + \tan^2 y} = \frac{1}{1 + x^2}$$

$$(ii) \quad \sec^2 y \frac{dy}{dx} = 1 \text{ so}$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1 + \tan^2 y} = \frac{1}{1 + x^2}$$

$$10 \quad (1, -2) \text{ and } (-1, 2)$$

$$11 \quad (i) \quad \frac{dy}{dx} = \frac{y+4}{6-x}$$

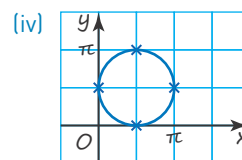
$$(ii) \quad x - 2y - 11 = 0$$

$$(iii) \quad \left(2, -\frac{9}{2}\right)$$

$$12 \quad (i) \quad \left(\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, \pi\right)$$

$$(ii) \quad \left(0, \frac{\pi}{2}\right), \left(\pi, \frac{\pi}{2}\right)$$

$$(iii) \quad x = \frac{\pi}{2}, y = \frac{\pi}{2}$$



Chapter 10

Review exercise (page 202)

$$1 \quad (i) \quad \frac{3}{2}x^8 + c$$

$$(ii) \quad \frac{4}{3}x^3 + x + c$$

$$(iii) \quad x^2 - \frac{x^5}{5} + \frac{x^9}{3} + c$$

$$2 \quad (i) \quad \frac{15}{2}$$

$$(ii) \quad \frac{75}{4}$$

$$(iii) \quad -26$$

3 (i) $\frac{4x\sqrt{x}}{3} + c$

(ii) $-\frac{3}{x} + c$

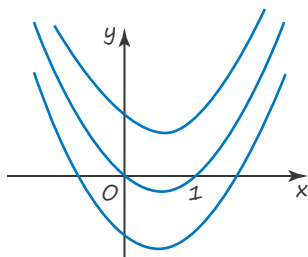
(iii) $\frac{6x^3\sqrt{x}}{7} + c$

4 (i) $-\frac{103}{12}$

(ii) $\frac{14}{9}$

5 (i) $y = x^2 - x + c$

(ii)



(iii) $y = x^2 - x + 1$

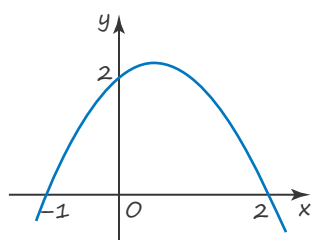
6 $y = 2 - \frac{2}{x^2} - 2\sqrt{x}$

7 (i) $\frac{y^3}{3} + 2y - \frac{1}{y} + c$

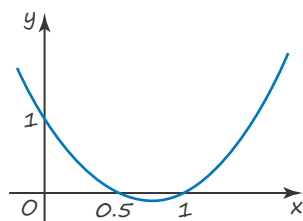
(ii) $\frac{t^4}{2} - 2\sqrt{t} + c$

8 $f(x) = x^3 - 2x^2 + x - 4$

9 $\frac{9}{2}$

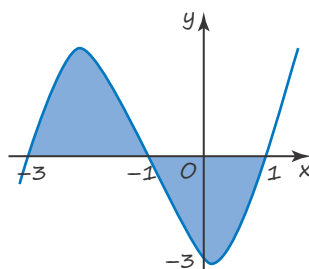


10 $\frac{1}{24}$



11 (i) $x^3 + 3x^2 - x - 3 = (x-1)(x+1)(x+3)$

(ii)

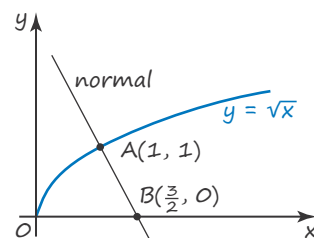


(iii) 8

12 (i) $y + 2x - 3 = 0$

(ii) $(\frac{3}{2}, 0)$

(iii)



(iv) $\frac{11}{12}$

Discussion point (page 205)

Each of these rectangles has a height which is the height of the top curve minus the height of the bottom curve. The enclosed region is the limit of the sum of the areas of these rectangles.

Discussion point (page 208)

The area under the red curve is partly above and partly below the x -axis. If you find the total area between the red curve and the x -axis, and then subtract this from the area under the blue curve, this will give the wrong answer. You would need to subtract the areas under the red curve between $x = 0$ and $x = 1$, and between $x = 3$ and $x = 4$, and add the area between $x = 1$ and $x = 3$.

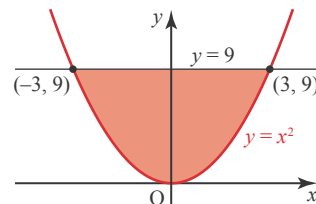
Exercise 10.1 (page 209)

1 $\frac{4}{3}$

2 (ii) 6 square units

3 (i) $(3, 9), (-3, 9)$

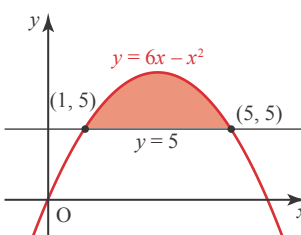
(ii)



(iii) 36 square units

(i) $(1, 5), (5, 5)$

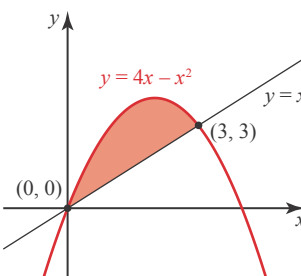
(ii)



(iii) $\frac{32}{3}$ square units

(i) $(0, 0), (3, 3)$

(ii)



(iii) 4.5 square units

6 (i) $\frac{4}{3}$ square units, $\frac{4}{3}$ square units, $\frac{4}{3}$ square units

(ii) $\frac{32}{3}$ square units

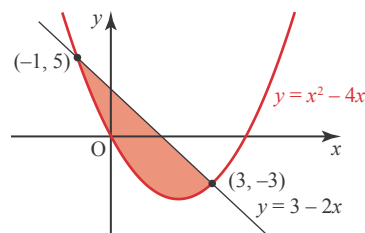
(iii) $\int_{-2}^2 (3 - (x^2 - 1)) dx = \int_{-2}^2 (4 - x^2) dx$

(iv) $\frac{32}{3}$ square units

7 $\frac{14}{3}$ square units

8 (i) $(-1, 5), (3, -3)$

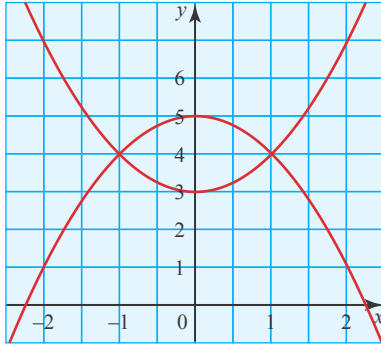
(ii)



(iii) $\frac{32}{3}$ square units

9 $\frac{64}{3}$

- 10 (i) $(1, 4), (-1, 4)$
(ii)

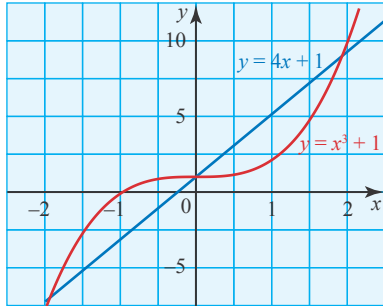


(iii) $\frac{8}{3}$ square units

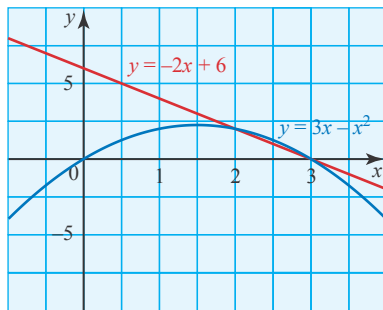
11 $\frac{4}{3}$ square units

12 72 square units

13 8 square units



14 (i)



(ii) $\sqrt{5}$

(iii) $\frac{1}{6}$

- 15 (i) 20 square units
(ii) It is impossible to tell as the strips are partly below and partly above the curve.
(iii) 20 square units
(iv) 19.5 square units
(v) $18\frac{2}{3}$ square units. The more rectangles used, the closer the estimate gets to the true answer.

- 16 (i) 21 square units
(ii) 21.25 square units
(iii) $21\frac{1}{3}$ square units
17 652:77

Discussion point (page 212)

$$\frac{1}{20} = \frac{1}{5 \times 4}$$

5 is the derivative of the bracket and 4 is the new power.

Activity 10.1 (page 212)

- (i) (a) $\frac{1}{14}(2x + 3)^7 + c$
(b) $-\frac{1}{12}(5 - 3x)^4 + c$
(c) $\frac{1}{3}(1 + 2x)^{\frac{3}{2}} + c$
(ii) $\frac{1}{a(n+1)}(ax + b)^{n+1} + c$

Activity 10.2 (page 214)

(i) $\frac{2(x^3 + 1)^{\frac{3}{2}}}{9} + c$

Exercise 10.2 (page 215)

- 1 (i) $\frac{1}{5}(x - 7)^5 + c$
(ii) $\frac{1}{10}(2x - 7)^5 + c$
(iii) $\frac{1}{25}(5x - 7)^5 + c$
2 (i) $\frac{1}{10}(x - 2)^{10} + c$
(ii) $\frac{1}{8}(2x + 3)^4 + c$
(iii) $-\frac{1}{36}(1 - 6x)^6 + c$
3 (i) 0.2
(ii) 20
(iii) 20
4 $\frac{812}{625}$
5 (i) $\frac{2}{3}(x + 3)^{\frac{3}{2}} + c$
(ii) $-\frac{1}{3(3x + 1)} + c$
(iii) $\sqrt{2x - 1} + c$
6 $y = \sqrt{3} + \sqrt{2x + 3}$
7 $\frac{4}{3}$ square units

8 $\frac{175}{12}$

- 9 (i) $\frac{1}{14}(x^2 - 1)^7 + c$
(ii) $\frac{2}{9}(x^3 + 1)^{\frac{3}{2}} + c$
(iii) $\frac{1}{4(1 - x^4)} + c$

10 $\frac{1}{6}$

11 (i) $(2x + 1)^4 = 16x^4 + 32x^3 + 24x^2 + 8x + 1$

$\int (2x + 1)^4 dx = \frac{16}{5}x^5 + 8x^4 + 8x^3 + 4x^2 + x + c$

- (ii) $\frac{1}{10}(2x + 1)^5 + k$
(iv) In (ii) the constant term is $\frac{1}{10} + k$ which is an arbitrary constant.
12 (i) A negative number does not have a square root.
(ii) $|\sqrt{x^2 - 1}| < |x|$
(iii) $2\sqrt{2} - \sqrt{3}$

Activity 10.3 (page 216)

- 1 (i) $\cos x$
(ii) $-\sin x$
(iii) $\sec^2 x$
2 (i) $\sin x + c$
(ii) $-\cos x + c$
(iii) $\tan x + c$
3 (i) $4 \cos 4x$
(ii) $-5 \sin 5x$
(iii) $5 \sec^2 5x$
4 (i) $\frac{1}{4} \sin 4x + c$
(ii) $-\frac{1}{5} \cos 5x + c$
(iii) $\frac{1}{5} \tan 5x + c$

Activity 10.5 (page 217)

- 1 (i) e^x
(ii) $3e^{3x}$
2 (i) $e^x + c$
(ii) $\frac{1}{3}e^{3x} + c$

Activity 10.6 (page 219)

- (i) $e^{f(x)} + c$
(ii) $\frac{(f(x))^{n+1}}{n+1} + c$

Exercise 10.3 (page 220)

- 1 (i) $\frac{1}{2}e^{2x} + c$
 (ii) $\frac{1}{3}e^{3x+5} + c$
 (iii) $-\frac{1}{2}e^{3-2x} + c$
- 2 (i) $-\frac{1}{6}\cos 6x + c$
 (ii) $\frac{1}{2}\sin(2x + 1) + c$
 (iii) $\frac{1}{3}\cos(1 - 3x) + c$
- 3 (i) $\sin x + \cos x + c$
 (ii) $e^x - e^{-x} + c$
 (iii) $-3\cos x - \frac{1}{2}e^{2x} + c$
- 4 (i) 1
 (ii) $\frac{\pi}{2} - 1$
 (iii) $2(e - 1)$
- 5 (i) $\frac{\pi}{4}$
 (ii) $\frac{3\pi}{4} + 2$
- 6 (i) $\frac{1}{2}e^{x^2+3} + c$
 (ii) $-\frac{1}{5}\cos^5 x + c$
- 7 (i) $\left(\frac{1}{2}, \frac{1}{2\sqrt{e}}\right)$,
 $\left(-\frac{1}{2}, -\frac{1}{2\sqrt{e}}\right)$
 (ii) 0.432
- 8 (i) $-e^{\cos x} + c$
 (ii) $\frac{1}{8}(e^x - 5)^8 + c$
- 9 (i) $(0, 0), (\sqrt{\ln 3}, 3\sqrt{\ln 3}),$
 $(-\sqrt{\ln 3}, -3\sqrt{\ln 3})$
 (ii) $3\ln 3 - 2$
- 10 (i) $2\sin^{-1}\left(\frac{x}{2}\right) + \frac{1}{2}x\sqrt{4 - x^2} + c$
 (ii) $\frac{1}{3}\tan x^{-1}\left(\frac{x}{3}\right) + c$
- 11 (ii) $-\cos x + \frac{1}{3}\cos^3 x + c$
- 12 (i) $\frac{3x}{8} + \frac{1}{4}\sin 2x$
 $+ \frac{1}{32}\sin 4x + c$
 (ii) $\frac{x}{8} - \frac{1}{32}\sin 4x + c$

Discussion point (page 221)

All three differentiate to $\frac{1}{x}$.

For $x > 0$, $\int \frac{1}{x} dx = \ln kx + c$
 where k is any positive constant.

This is possible because of the arbitrary constant: $\ln kx = \ln k + \ln x$, and $\ln k$ is a constant. $\ln x$ is only defined for positive values of x .

Discussion point (page 212)

k must be positive for $\ln k$ to be defined. Yes, for every value of c we can write $k = e^c$, and therefore $k > 0$ and $c = \ln k$.

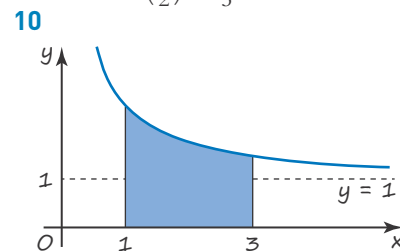
Activity 10.7 (page 225)

$$\begin{aligned}\frac{x+3}{(x+1)^2} &= \frac{x+1+2}{(x+1)^2} \\ &= \frac{x+1}{(x+1)^2} + \frac{2}{(x+1)^2} \\ &= \frac{1}{x+1} + \frac{2}{(x+1)^2}\end{aligned}$$

Exercise 10.4 (page 226)

- 1 (i) $2\ln x + c$
 (ii) $\frac{1}{2}\ln x + c$
 (iii) $\frac{1}{2}\ln(2x + 7) + c$
- 2 (i) $\ln|x - 4| + c$
 (ii) $-\ln|2 - x| + c$
 (iii) $\frac{1}{2}\ln|2x - 7| + c$
- 3 (i) $\ln 2$
 (ii) $\ln\left(\frac{2}{3}\right)$
 (iii) $\ln\left(\frac{9}{4}\right)$
- 4 $\ln|x^2 + 1| + c$
- 5 $\ln\left(\frac{2}{3}\right)$
- 6 (i) $\ln|x^3 - 4| + c$
 (ii) $-\frac{1}{2}\ln|2 - x^2| + c$

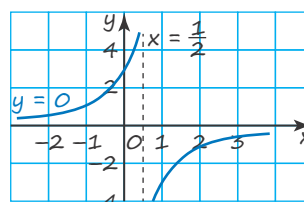
- 7 (i) $\frac{1}{3}\ln 6$
 (ii) $-\frac{1}{2}\ln 2$
- 8 (i) $\ln\left|\frac{3x-2}{1-x}\right| + c$
 (ii) $\ln\left|\frac{x}{1-x}\right| - \frac{1}{x} + c$
 (iii) $\ln\left|\frac{(x-1)^2}{\sqrt{2x+1}}\right| + c$
- 9 (i) $\frac{1}{2}\ln\left(\frac{3}{2}\right)$
 (ii) $\frac{2}{3} + \ln\left(\frac{21}{11}\right)$
 (iii) $\ln\left(\frac{5}{2}\right) - \frac{2}{5}$



Asymptotes are $x = 0$, $y = 1$

Area of region is $2 + \frac{3}{2}\ln 3$

- 11 (i) $y = 0$, $x = \frac{1}{2}$
 (ii)



- (iii) $\frac{3}{2}\ln\left(\frac{5}{3}\right)$
- 12 (i) $\frac{1}{1+x} + \frac{2}{1-2x}$
 (ii) 0.31845
- 13 (i) $\left(0, \frac{1}{2}\right)$
 (ii) As $x \rightarrow \infty$, $y \rightarrow 1$ (from below), as $x \rightarrow -\infty$, $y \rightarrow 0$ (from above)
 (iii) 1.31 square units
- 14 (i) $\frac{1}{2}\ln\left(\frac{x-1}{x+1}\right) + c$
 (ii) $\frac{1}{2}\ln|x^2 - 1| + c$
 (iii) $x + \frac{1}{2}\ln\left(\frac{x-1}{x+1}\right) + c$

- 15 (i) $\left(\frac{2\pi}{3}, \frac{\sqrt{3}}{3}\right)$
 (ii) Area of triangle = $\frac{\sqrt{3}}{6}\pi$
 (iii) Total area $\approx \frac{\sqrt{3}}{3}\pi$
 (iii) $2 \ln 3$. This is a little more than the answer from (iii)
 16 (i) $-\frac{1}{x} + c$
 (ii) There is a discontinuity at $x = 0$
 (iii) No, because the function is undefined at $x = 0$
 (iv) You cannot evaluate a definite integral if the function has a discontinuity within the interval of integration.

Activity 10.9 (page 228)

- (i) (a) $u = 1 + x$
 (b) $\ln|1+x| + \frac{1}{1+x} + c$
 (ii) (a) $u = 2 + x$
 (b) $\frac{2}{15}(2+x)^{\frac{3}{2}}(3x-4) + c$
 (iii) (a) $u = 1 + \sqrt{x}$
 (b) $2\sqrt{x} - 2\ln(1 + \sqrt{x}) + c$

Exercise 10.5 (page 229)

- 1 (i) $x - 2\ln|x+1| - \frac{1}{x+1} + c$
 (ii) $\frac{1}{15}(2x+1)^{\frac{3}{2}}(3x-1) + c$
 2 (i) $\frac{7}{8}$
 (ii) $100\frac{4}{5}$
 3 (i) (a) $u = 1 - x$
 (b) $\frac{-2(2+x)\sqrt{1-x}}{3} + c$
 (ii) (a) $u = x + 4$
 (b) $\frac{2(3x-8)(x+4)^{\frac{3}{2}}}{3} + c$
 (iii) (a) $u = 2x + 1$
 (b) $\frac{1}{4}\ln|2x+1| - \frac{x}{2} + c$

- 4 (i) $y = 2x - \ln|2x+1| + c$
 (ii) $y = 2x - 1 - \ln|2x+1|$
 5 (i) $\frac{-2}{(x-4)} - \frac{11}{2(x-4)^2} + c$
 $= \frac{5-4x}{2(x-4)^2} + c$
 (ii) $\frac{3}{2}x^{\frac{2}{3}} - 3\sqrt[3]{x} + 3\ln(1 + \sqrt[3]{x}) + c$
 6 (i) $a = 1$
 (ii) Minimum at $(-1, -\frac{1}{4})$
 (iii) $\ln 2 - \frac{1}{2}$
 7 $\frac{(1+\ln x)^2}{6}(2\ln x - 1) + c$
 8 (i) $k = -3$
 (ii) Minimum at $(-4.5, 3.93)$
 (iii) 1.72
 9 $y = \frac{1}{35}(2x+1)^{\frac{5}{2}}(5x-1) + 2$
 10 0.0271
 11 (ii) $\frac{\pi}{8}$
 12 (ii) $\frac{\pi}{6}$
 13 $8\sin^{-1}\frac{x}{4} + \frac{1}{2}x\sqrt{16-x^2} + c$

Activity 10.10 (page 230)

- (i) $\int x \sin x \, dx = -x \cos x + \sin x + c$
 (ii) $\int 2xe^{2x} \, dx = xe^{2x} - \frac{1}{2}e^{2x} + c$

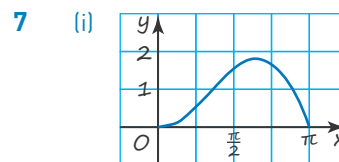
Activity 10.11 (page 231)

- (i) $\int x \sin x \, dx = \frac{x^2}{2} \sin x - \int \frac{x^2}{2} \cos x \, dx$
 (ii) Parts chosen the wrong way round

Exercise 10.6 (page 234)

- 1 $\frac{1}{2}x \sin 2x + \frac{1}{4} \cos 2x + c$
 2 $\frac{5}{e} - e$
 3 (i) $\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + c$
 (ii) $\frac{x^2}{2} \ln 3x - \frac{x^2}{4} + c$

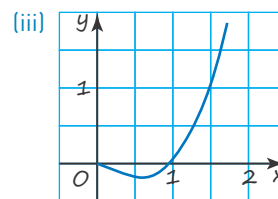
- 4 (i) $1 - \frac{2}{e}$ (ii) $2 + \pi$
 5 $\frac{1}{15}(5x+2)(x-2)^5 + c$
 6 (i) $(0, 2), (2, 0)$
 (ii) $e^{-2} + 1$



- (ii) π
 8 $\ln 3 - \frac{2}{3}$
 9 (i) 1
 (ii) $e - 1$
 (iii) e

This is the area of the rectangle formed by the red and blue regions.

- 10 (i) $-\frac{1}{2}x^2 \cos 2x + \frac{1}{2}x \sin 2x + \frac{1}{4} \cos 2x + c$
 (ii) $-e^{-x}(x^2 + 2x + 2) + c$
 11 (i) $(1, 0)$
 (ii) Minimum at $(\frac{1}{\sqrt{e}}, -\frac{1}{2e})$



- (iv) $\frac{8}{3} \ln 2 - \frac{7}{9}$
 12 (ii) $\frac{1}{2}(e^x \sin x - e^x \cos x) + c$

Summary exercise (page 235)

- 1 $-\frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x} + c$
 2 $\frac{1}{3}x^3 - \frac{5}{2}x^2 + 6x + c$
 3 $\frac{1}{22}(2x+5)^{11} + c$
 4 $\frac{1}{2}e^{x^2} + c$
 5 $\frac{1}{4}(1 + \sin x)^4 + c$
 6 $-\frac{1}{12(3x-4)^4} + c$
 7 $\frac{1}{3} \sin 3x + \frac{1}{9} \cos 3x + c$

- 8 $\frac{1}{4} \ln \left| \frac{x-2}{x+2} \right| + c$
 9 $x + 2 \ln |x-2| + c$
 10 $\frac{1}{3}(x+3)^3 + c$
 11 $\frac{1}{7}x^7 - 16x + c$
 12 $x \ln 3x - x + c$
 13 $\frac{1}{2} \tan^2 x + c$
 14 $\ln |x^3 + x| + c$
 15 $e^{\sin x} + c$
 16 $\frac{1}{2} \sin^2 x + c$
 17 $\tan x + c$
 18 $\ln \left| \frac{(x-3)^3}{(x-5)^2} \right| + c$
 19 $\frac{2}{27}(9x+4)^{\frac{3}{2}} + c$
 20 $x + \tan x + c$
 21 $\frac{1}{5}x^5 + \frac{1}{4}x^4 + \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + c$
 22 $2x + c$
 23 $\frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{1}{4}e^{2x} + c$
 24 $2 \sin x + c$
 25 $\frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{1}{2}x^2 + c$
 26 $\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + c$
 27 $\frac{1}{2} \ln |x^2 + 2 \ln x| + c$
 28 $\frac{1}{2}x^2 - x + \ln |x+1| + c$
 29 $x - \ln |\cos x| + c$
 30 $\frac{1}{8}e^{2x} - \frac{1}{8}e^{-2x} + \frac{1}{2}x + c$
 31 $\frac{-1}{18(3x-2)^2} + \frac{-5}{27(3x-2)^3} + c$

$$= \frac{-(9x+4)}{54(3x-2)^3} + c$$

 32 $2 \sin^{-1} \frac{x}{2} + \frac{1}{2}x\sqrt{4-x^2} + c$
 33 $2 \ln(1 + \sqrt{x}) + c$

Practice questions 2

(page 237)

- 1 $\int_2^4 \frac{1}{x} dx = [\ln x]_2^4$ [1]
 $= \ln 4 - \ln 2 = \ln 2$
 $\int_5^a \frac{1}{x} dx = \ln a - \ln 5$ [1]
 $\ln 2 = \ln \frac{a}{5}$ (oe) [1]
 $a = 10$ [1]

- 2 (i) Any correct justification,
 e.g. $y = 5 \sin x$ is the
 curve which passes
 through the origin
 since $5 \sin 0 = 0$. [1]
 (ii) Valid method to
 determine horizontal
 translation such as
 $4 \sin x - 3 \cos x = 0$ [1]
 $\tan x = \frac{3}{4}$ [1]
 Therefore the
 translation is described
 by the vector [1]
 $\begin{pmatrix} -\arctan \frac{3}{4} \\ 0 \end{pmatrix}$ [1]

- 3 (i) $(1-x)^{-2}$
 $= 1 + (-2)(-x) + \frac{(-2)(-3)}{2}(-x)^2 + \dots$ [1]
 $= 1 + 2x + 3x^2 + 4x^3$
 $+ 5x^4 + 6x^5 + \dots$ [2]
 ([1] if at least three
 correct terms)
 (ii) $\left(\frac{9}{10}\right)^{-2}$ [1]
 $= 1 + 0.2 + 0.03 + 0.004 +$
 $0.0005 + 0.00006 + \dots$ [1]

- 4 Area of triangle
 $AOB = \frac{1}{2} \cdot r \cdot r \sin 2\alpha$ [1]
 Area of triangle AOB
 $= 2 \times \text{area of triangle}$
 OMA (oe) [1]
 $= 2 \times \frac{1}{2} OM \cdot AM$
 $= r \cos \alpha \cdot r \sin \alpha$ [2]
 $\frac{1}{2} \cdot r \cdot r \sin 2\alpha =$
 $r \cos \alpha \cdot r \sin \alpha$
 $\sin 2\alpha = 2 \cos \alpha \cdot \sin \alpha$ [1]

- 5 (i) $\frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta} \equiv \frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta \sin^2 \theta}$ [1]
 $\equiv \frac{1}{\cos^2 \theta \sin^2 \theta}$
 $= \frac{4}{(2 \sin \theta \cos \theta)^2}$ [1]
 $= \frac{4}{\sin^2 2\theta}$ [1]

- (ii) This is equivalent to
 $\frac{4}{\sin^2 2\theta} = 4$ and so
 $\sin 2\theta = \pm 1$ [1]

Roots $2\theta =$
 $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \dots$ [1]
 at least two correct
 and so
 $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \dots$ [1]

- all correct
 6 (i) $x^2 \frac{dy}{dx} + 2xy + 2 \frac{dy}{dx} - 4 = 0$ [1]

for product rule; [1]
 for at least one correct
 appearance of $\frac{dy}{dx}$

$$\frac{dy}{dx}(x^2 + 2) = 4 - 2xy$$
 [1]

$$\frac{dy}{dx} = \frac{4 - 2xy}{2 + x^2}$$

- (ii) $\frac{dy}{dx} = 0 \Leftrightarrow$
 $4 - 2xy = 0$ [1]
 Substitute $y = \frac{2}{x}$
 or equivalent in
 $x^2 y + 2y - 4x = 0$ [1]

$$\frac{4}{x} - 2x = 0 \Rightarrow x = \pm \sqrt{2}$$

 $(x = \sqrt{2} \text{ OK})$ [1]
 Maximum point has
 coordinates $(\sqrt{2}, \sqrt{2})$ [1]

- 7 (i) $\frac{x-2}{(x+1)(x-3)} \equiv \frac{A}{x+1} + \frac{B}{x-3}$
 $x-2 \equiv A(x-3) + B(x+1)$ [1]
 $x=3: 1=4B \Rightarrow B=\frac{1}{4}$ [1]
 $x=-1: -3=-4A \Rightarrow A=\frac{3}{4}$ [1]

$$\frac{x-2}{(x+1)(x-3)} \equiv \frac{\frac{3}{4}}{x+1} + \frac{\frac{1}{4}}{x-3}$$

- (ii) $\int_0^2 \left(\frac{\frac{3}{4}}{x+1} + \frac{\frac{1}{4}}{x-3} \right) dx$ [1]
 $= \left[\frac{3}{4} \ln |x+1| + \frac{1}{4} \ln |x-3| \right]_0^2$
 [1] for $\ln$, [1] for $|x-3|$
 $= \left(\frac{3}{4} \ln 3 + \frac{1}{4} \ln 1 \right) - \left(\frac{3}{4} \ln 1 + \frac{1}{4} \ln 3 \right)$

$$= \left(\frac{3}{4} \ln 3\right) - \left(\frac{1}{4} \ln 3\right) \quad [1]$$

$$= \frac{1}{2} \ln 3 \quad [1]$$

- 8 (i) P clearly lies on the line $y = x$.
P lies on the curve since when $x = \frac{\pi}{2}, y = \frac{\pi}{2}, \sin \frac{\pi}{2} = \frac{\pi}{2}$ [1]

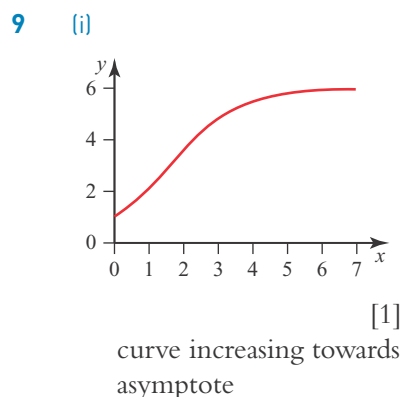
Evidence of the need to show that at P the gradient of the curve is 1. [1]

$$y = x \sin x \Rightarrow \frac{dy}{dx} = \sin x + x \cos x \quad [1]$$

$$\text{When } x = \frac{\pi}{2}, \frac{dy}{dx} = \sin \frac{\pi}{2} + \frac{\pi}{2} \cos \frac{\pi}{2} = 1 + 0 = 1 \quad [1]$$

$$\begin{aligned} \int x \sin x \, dx &= -x \cos x + \int \cos x \, dx \\ &= -x \cos x + \sin x + c \quad [1] \text{ by parts} \\ & \quad [1] \text{ accuracy} \end{aligned}$$

$$\begin{aligned} \frac{1}{2} \cdot \frac{\pi}{2} \cdot \frac{\pi}{2} - \int_0^{\frac{\pi}{2}} x \sin x \, dx & \text{ or } \int_0^{\frac{\pi}{2}} x - x \sin x \, dx \quad [1] \\ &= \frac{\pi^2}{8} - [-x \cos x + \sin x]_0^{\frac{\pi}{2}} \quad [1] \\ &= \frac{\pi^2}{8} - 1 \quad [1] \end{aligned}$$



$$\begin{aligned} \text{(ii)} \quad \frac{dP}{dt} &= [1] \text{ with attempt} \\ &= \frac{(5 + e^t)6e^t - 6e^t \cdot e^t}{(5 + e^t)^2} \quad [1] \end{aligned}$$

$$= \frac{30e^t}{(5 + e^t)^2} \quad [1]$$

$$\frac{d^2P}{dt^2} = 0 \quad [1] \text{ attempt}$$

$$\frac{(5 + e^t)^2 30e^t - 30e^t \cdot 2(5 + e^t)e^t}{(5 + e^t)^4} \quad [1]$$

$$= 30e^t \frac{(5 + e^t) - 2e^t}{(5 + e^t)^3} \quad [1]$$

$$5 = e^t \Rightarrow t = \ln 5 \quad [1]$$

$$\begin{aligned} \text{(iii)} \quad &\text{Substitute } P = 7 \text{ to} \\ &\text{obtain } 7 = \frac{6e^t}{5 + e^t} \quad [1] \\ &\text{Simplification to} \\ &e^t = -35 \quad [1] \end{aligned}$$

The equation has no solution so P cannot reach a value of 7. [1]

Review: Coordinate geometry

Discussion point (page 242)

$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2} \text{ CM} \times \text{MD} \\ &= \frac{1}{2} \times \sqrt{3^2 + 6^2} \times \sqrt{12^2 + 6^2} \\ &= \frac{1}{2} \times \sqrt{45} \times \sqrt{180} \\ &= \frac{1}{2} \times \sqrt{8100} = 45 \end{aligned}$$

Exercise R.1 (page 242)

- 1 (i) (a) -1
(b) (4, 3)
(c) $2\sqrt{2}$
(ii) (a) -1
(b) (3, 4)
(c) $2\sqrt{2}$
(iii) (a) $-\frac{3}{4}$
(b) (-1, -1)
(c) 10
(iv) (a) 1
(b) (-3, 4)
(c) $2\sqrt{2}$

- 2 (i) 3
(ii) -2
(iii) $-\frac{2}{3}$
(iv) $\frac{5}{2}$

- 3 (i) $y = 2x + 3$
(ii) $2y = x - 3$
(iii) $2y = -x - 3$
(iv) $2y = x + 3$
(v) $2y = 3 - x$

- 4 (i) (3, 5)
(ii) (-2, 3)
(iii) (-1, 4)
(iv) $(\frac{9}{8}, -\frac{1}{4})$

5 $k = 9$

- 6 (i) Gradient of AB is $\frac{\pi}{2}$ and gradient of BC is $-\frac{4}{3}$.
 $-\frac{4}{3} \times \frac{\pi}{2} = -1$, hence AB and BC are perpendicular and ABC is right-angled.

- (ii) 12.5 square units
(i) Gradient of AB is $-\frac{1}{2}$, gradient of CD is $-\frac{1}{2}$, gradient of BC is -3 and gradient of AD is $\frac{1}{3}$. Hence one pair of parallel sides $\Rightarrow$ ABCD is a trapezium.

AD = BC = $\sqrt{10} \Rightarrow$ ABCD is an isosceles trapezium.

(ii) $(\frac{5}{3}, \frac{7}{3})$

- 8 6.25
9 (i) $k = 2$
(ii) $3x - 2y - 27 = 0$
10 $2y + 3x = 0$
11 (4, 0), (0, 1.6)
12 6.5 square units
13 (2, -1), (-0.8, -6.6)
14 $\frac{13}{3}$ square units
15 (i) 86.72

(ii) Many factors may make the assumption of linear growth unrealistic (although it has been a good model for the rates of the last 40 years). 'Extrapolation is unreliable.'

Exercise R.2 (page 245)

- 1 (i) $(3, 1); 5$
(ii) $(3, -1); \sqrt{5}$
(iii) $(-3, -1); 5$
(iv) $(1, -3); \sqrt{5}$
- 2 $(x - 5)^2 + (y + 2)^2 = 16$
- 3 $x^2 + 6x + y^2 - 8y - 56 = 0$
- 4 $(x - 1)^2 + (y - 6)^2 = 10$
- 5 (i) $(x - 4)^2 + (y + 1)^2 = 10$
(ii) centre $(4, -1)$; radius $\sqrt{10}$
- 6 $(5, -5), (8, -2)$
- 7 $\frac{6}{5}\sqrt{5}$
- 8 $(\sqrt{2}, -2(\sqrt{2} - 1))$ and $(-\sqrt{2}, 2(1 + \sqrt{2}))$
- 9 (i) One point
(ii) Two points
(iii) No points
- 10 $k = 13$ or $k = -7$
- 11 (i) Use the circle theorem: 'angle in a semicircle is a right angle' to show PQR is a right-angled triangle with a right angle at R. Either show $PR^2 + QR^2 = PQ^2$ ($2 + 82 = 164$) or show $m_{PR} \times m_{RQ} = -1$ ($\frac{1}{9} \times -9 = -1$)
(ii) $(x - 3)^2 + (y + 5)^2 = 41$
(iii) $(0, -5 \pm 4\sqrt{2})$, $(-1, 0), (7, 0)$
- 12 (i) Centre lies on the perpendicular bisector of AB which is $2y = x + 4$
(ii) $(x - 2)^2 + (y - 3)^2 = 10$
- 13 $12\sqrt{5}$
- 14 $(2.4, 0.2); 5$

Chapter 11

Discussion point (page 251)

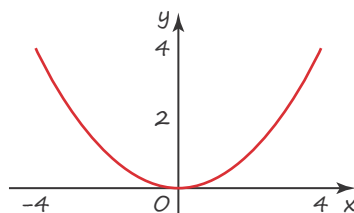
At points where the rate of change of gradient is greatest.

Exercise 11.1 (page 253)

- 1 (i) (a) $(4, 1)$
(b) $(7, 6)$
(c) $(-2, 3)$
(ii) $t = -3$
- 2 (i) $(1, 3)$
(ii) $(27, 27)$
(iii) $(8, 12), (-8, 12)$
- 3 (i) (a)

t	x	y
-2	-4	4
-1.5	-3	2.25
-1	-2	1
-0.5	-1	0.25
0	0	0
0.5	1	0.25
1	2	1
1.5	3	2.25
2	4	4

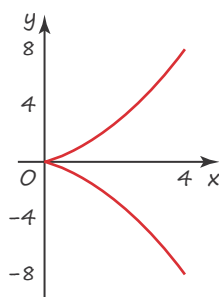
(b)



(ii) (a)

t	x	y
-2	4	-8
-1.5	2.25	-3.375
-1	1	-1
-0.5	0.25	-0.125
0	0	0
0.5	0.25	0.125
1	1	1
1.5	2.25	3.375
2	4	8

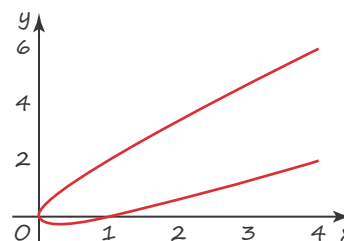
(b)



(iii) (a)

t	x	y
-2	4	6
-1.5	2.25	3.75
-1	1	2
-0.5	0.25	0.75
0	0	0
0.5	0.25	-0.25
1	1	0
1.5	2.25	0.75
2	4	2

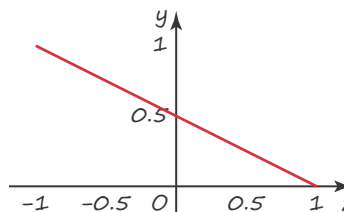
(b)



4 (i) (a)

θ	x	y
0°	1	0
30°	0.5	0.25
60°	-0.5	0.75
90°	-1	1
120°	-0.5	0.75
150°	0.5	0.25
180°	1	0
210°	0.5	0.25
240°	-0.5	0.75
270°	-1	1
300°	-0.5	0.75
330°	0.5	0.25
360°	1	0

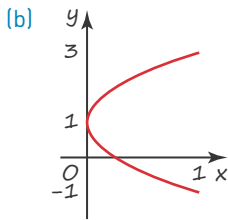
(b)



(c) Defined for $-1 \leq x \leq 1$ and $0 \leq y \leq 1$

(ii) (a)

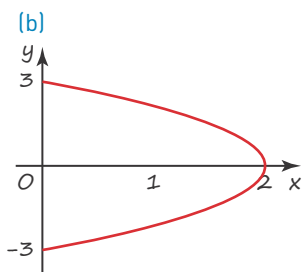
θ	x	y
0°	0	1
30°	0.25	2
60°	0.75	2.73
90°	1	3
120°	0.75	2.73
150°	0.25	2
180°	0	1
210°	0.25	0
240°	0.75	-0.73
270°	1	-1
300°	0.75	-0.73
330°	0.25	0
360°	0	1



(c) Defined for $0 \leq x \leq 1$ and $-1 \leq y \leq 3$

(iii) (a)

θ	x	y
0°	0	3
30°	0.5	2.6
60°	1.5	1.5
90°	2	0
120°	1.5	-1.5
150°	0.5	-2.6
180°	0	-3
210°	0.5	-2.6
240°	1.5	-1.5
270°	2	0
300°	1.5	1.5
330°	0.5	2.6
360°	0	3



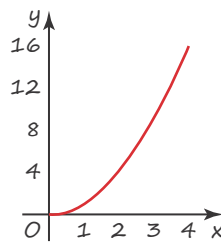
(c) Defined for $0 \leq x \leq 2$ and $-3 \leq y \leq 3$

5

(i)

t	x	y
-2	4	16
-1.5	2.25	5.0625
-1	1	1
-0.5	0.25	0.0625
0	0	0
0.5	0.25	0.0625
1	1	1
1.5	2.25	5.0625
2	4	16

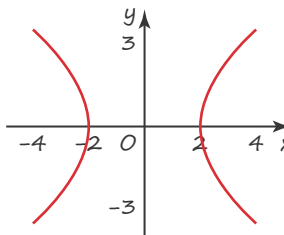
(ii)



(iii) Because it should also state 'for $x \geq 0$ '

6

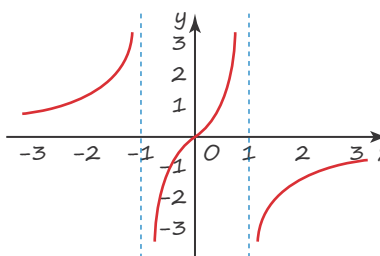
(i)



(ii) Undefined for $-2 < x < 2$

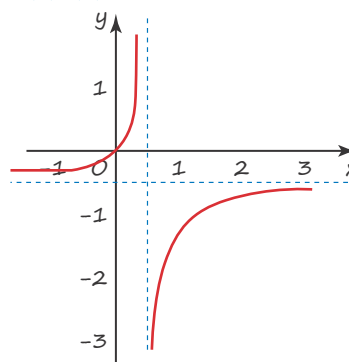
7

(i) (a)



(b) $x = -1, x = 1, y = 0$

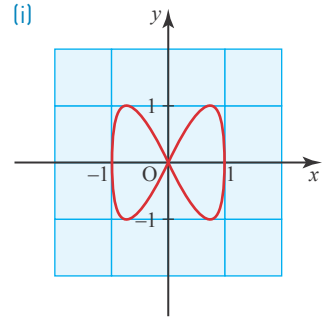
(ii) (a)



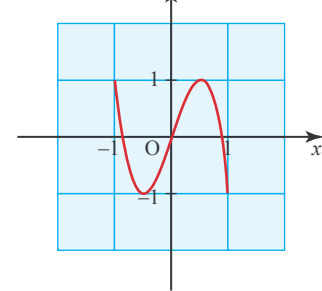
8

(i)

(b) $x = \frac{1}{2}, y = -\frac{1}{2}$



(ii)



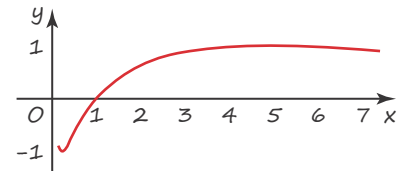
9

(i)

t	x	y
-2	0.14	-0.91
-1.5	0.22	-1.00
-1	0.37	-0.84
-0.5	0.61	-0.48
0	1	0
0.5	1.65	0.48
1	2.72	0.84
1.5	4.48	1.00
2	7.39	0.91

(ii) $x > 0$

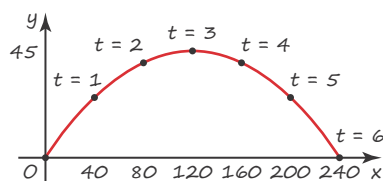
(iii)



(iv) The graph oscillates infinitely many times from -1 to $+1$ for $t < -2$, i.e. where $0 < x < 0.14$.

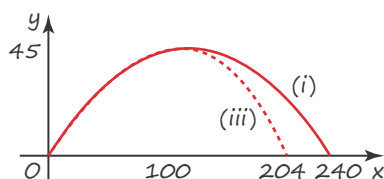
For $t > 2$ the graph oscillates infinitely many times from -1 to $+1$, but successive distances between a maximum and a minimum become increasingly large.

10 (i)



(ii) 240m

(iii)

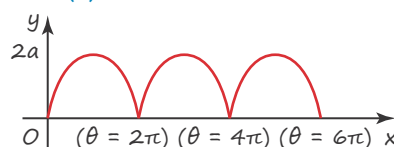


(iv) 36m

11 (i)

θ	x	y
0	0	0
$\frac{\pi}{3}$	$0.2a$	$0.5a$
$\frac{2\pi}{3}$	$1.2a$	$1.5a$
π	$3.1a$	$2a$
$\frac{4\pi}{3}$	$5.1a$	$1.5a$
$\frac{5\pi}{3}$	$6.1a$	$0.5a$
2π	$6.3a$	0
$\frac{7\pi}{3}$	$6.5a$	$0.5a$
$\frac{8\pi}{3}$	$7.5a$	$1.5a$
3π	$9.4a$	$2a$
$\frac{10\pi}{3}$	$11.3a$	$1.5a$
$\frac{11\pi}{3}$	$12.4a$	$0.5a$
4π	$12.6a$	0
$\frac{13\pi}{3}$	$12.7a$	$0.5a$
$\frac{14\pi}{3}$	$13.8a$	$1.5a$
5π	$15.7a$	$2a$
$\frac{16\pi}{3}$	$17.6a$	$1.5a$
$\frac{17\pi}{3}$	$18.7a$	$0.5a$
6π	$18.8a$	0

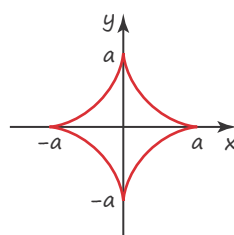
(ii)



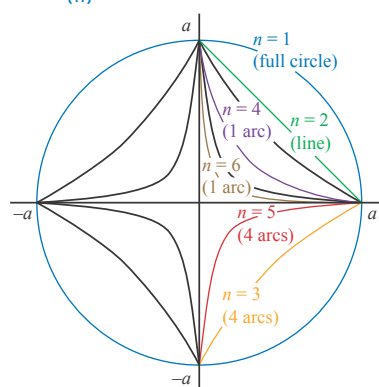
(iii) Periodic

12 (i)

θ	x	y
0	a	0
$\frac{\pi}{3}$	$0.13a$	$0.65a$
$\frac{\pi}{2}$	0	a
$\frac{2\pi}{3}$	$-0.13a$	$0.65a$
π	$-a$	0
$\frac{4\pi}{3}$	$-0.13a$	$-0.65a$
$\frac{3\pi}{2}$	0	$-a$
$\frac{5\pi}{3}$	$0.13a$	$-0.65a$
2π	a	0



(ii)



When $n = 0$ the curve becomes the single point (a, a) .

(iii) (a) The larger the value of n the closer the curve is to the axes. If the power is even, the curve is only in the first quadrant.

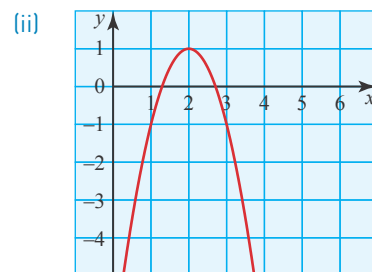
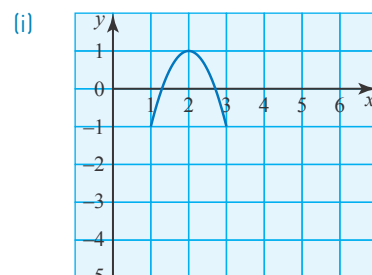
(b) If the power is odd, the curve is in all four quadrants.

Discussion point (page 257)

A one-way stretch scale factor 3 parallel to the y -axis followed by a one-way stretch scale

factor 4 parallel to the x -axis (transformations can be carried out in either order).

Discussion point (page 257)



They are different because in (i) $1 \leq x \leq 3$ as $\sin \theta$ lies between -1 and 1 and $-1 \leq y \leq 1$ as $\cos 2\theta$ lies between -1 and 1 and in (ii) there are no restrictions on the domain ($x \in \mathbb{R}$) and the range is $y \leq 1$

Activity 11.1 (page 258)

(i)

$$\begin{array}{ll} x = 2 + 5 \sin t & x = 2 + 5 \cos 2t \\ y = 3 + 5 \cos t & y = 3 + 5 \sin 2t \\ x = 2 - 5 \cos t & x = 2 + 5 \cos t \\ y = 3 - 5 \sin t & y = 3 + 5 \sin t \end{array}$$

$$\begin{array}{ll} x = 3 + 5 \sin t & x = 3 + 5 \cos \frac{1}{2}t \\ y = 2 + 5 \cos t & y = 2 + 5 \sin \frac{1}{2}t \end{array}$$

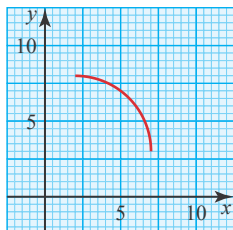
$$\begin{array}{ll} x = 2 + 5 \cos t & x = 2 + 5 \sin t \\ y = 3 + 5 \cos t & y = 3 + 5 \sin t \\ x = 2 + 5 \sin \frac{1}{2}t & \\ y = 3 + 5 \sin \frac{1}{2}t & \end{array}$$

$$\begin{array}{ll} x = 5 \cos \frac{1}{2}t & x = 5 \cos 2t \\ y = 5 \sin \frac{1}{2}t & y = 5 \sin 2t \\ x = 5 \cos t & \\ y = 5 \sin t & \end{array}$$

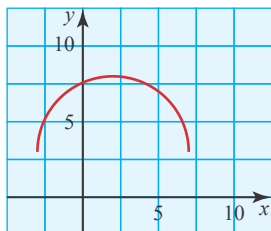
- (iii) Changing the interval for the parameter produces different sections of the circle.

For example, for $x = 2 + 5 \sin t$, $y = 3 + 5 \cos t$ the following curves are produced:

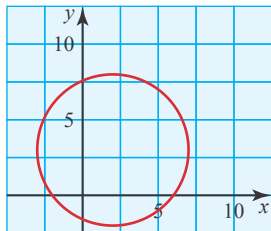
(a) $0 \leq t \leq \frac{\pi}{2}$



(b) $0 \leq t \leq \pi$



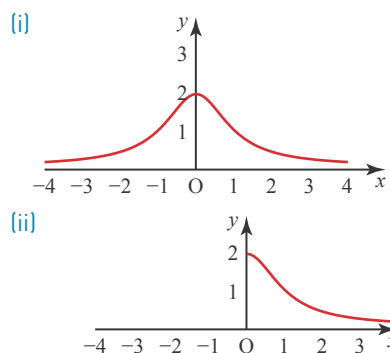
(c) $0 \leq t \leq 2\pi$



Discussion point (page 258)

Changing t to $2t$ traces out the circle more quickly, so a full circle is drawn in the interval $0 \leq t \leq \pi$ and it is described twice in the interval $0 \leq t \leq 2\pi$.

Discussion point (page 259)

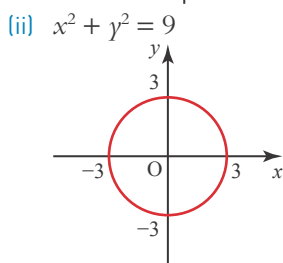
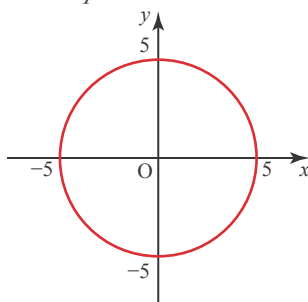


Since $e^t > 0$, the second graph only shows half of the full curve.

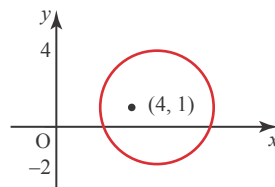
Exercise 11.2 (page 259)

1 $xy = 9$ and $\begin{matrix} x = 3t \\ y = \frac{3}{t} \end{matrix}$
 $y^2 = 12x$ and $\begin{matrix} x = 3t^2 \\ y = 6t \end{matrix}$
 $y = (1-x)^2$ and $\begin{matrix} x = 1-t \\ y = t^2 \end{matrix}$
 $xy = 3$ and $\begin{matrix} x = \frac{1}{t} \\ y = 3t \end{matrix}$
 $4y = 4 - x^2$ and $\begin{matrix} x = 2t \\ y = 1 - t^2 \end{matrix}$

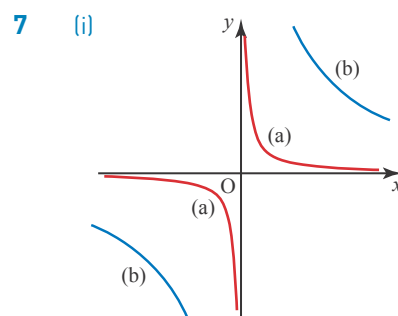
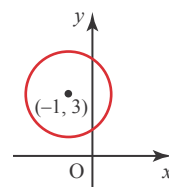
- 2 (i) $y = \frac{x^2}{4}$
 (ii) $y^2 = x^3$
 (iii) $y = x \pm \sqrt{x}$
- 3 (i) $x = 2t$, $y = 5 - 4t$ for $0 \leq t \leq 2$
 (ii) $x = u - 1$, $y = 7 - 2u$ for $1 \leq u \leq 5$
- 4 (i) $y = -1 + 4 \sin t$
 (ii) $y = -1 + 4 \cos t$
- 5 (i) A segment of $y = \frac{1-x}{2}$ where $-1 \leq x \leq 1$ and $0 \leq y \leq 1$
 (ii) Part of $(y-1)^2 = 4x$ where $0 \leq x \leq 1$ and $-1 \leq y \leq 3$
 (iii) Part of $y^2 = \frac{9}{2}(2-x)$ where $0 \leq x \leq 2$ and $-3 \leq y \leq 3$
- 6 (i) $x^2 + y^2 = 25$



(iii) $(x-4)^2 + (y-1)^2 = 9$



(iv) $(x+1)^2 + (y-3)^2 = 4$

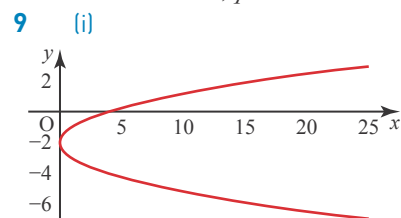


- (ii) (a) $xy = 1$
 (b) $xy = 16$
 (iii) The curve in (b) is an enlargement of the curve in (a), centre the origin, scale factor 4.

- 8 (i) $7 = 2 \times 3 + 1$
 (ii) (a) $\begin{pmatrix} 3 \\ 7 \end{pmatrix}$
 (b) $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ or any vector of the form $\begin{pmatrix} a \\ 2a \end{pmatrix}$, $a \neq 0$

(iii) $\begin{pmatrix} 3 \\ 7 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3+t \\ 7+2t \end{pmatrix}$

(iv) $x = 3 + t$, $y = 7 + 2t$



(ii) $y = -2$

(iii) $x = (y+2)^2$

10 (i) $x^2 - y^2 = 4$

(ii) $y = \frac{2x}{1 - x^2}$

11 (i) C(4, 6); P(9, 6)

(ii) $x = 4 + 5 \cos t$,

$y = 6 + 5 \sin t$

(iii) (a) $X\left(\frac{13}{2}, 6 + \frac{5}{2}\sqrt{3}\right)$,

$XY = 5\sqrt{2}$

(b) No

Exercise 11.3 (page 263)

1 (i) (a) $\frac{dx}{dt} = 6t$

(b) $\frac{dy}{dt} = 6t^2$

(c) $\frac{dy}{dx} = t$

(ii) (a) $\frac{dx}{dt} = 4$

(b) $\frac{dy}{dt} = 4t^3$

(c) $\frac{dy}{dx} = t^3$

(iii) (a) $\frac{dx}{dt} = 1 - \frac{1}{t^2}$

(b) $\frac{dy}{dt} = 1 + \frac{1}{t^2}$

(c) $\frac{dy}{dx} = \frac{t^2 + 1}{t^2 - 1}$

2 (i) (a) $\frac{dx}{dt} = 2(t + 1)$

(b) $\frac{dy}{dt} = 2(t - 1)$

(c) $\frac{dy}{dx} = \frac{t - 1}{t + 1}$

(ii) (a) (16, 4)

(b) $\frac{dy}{dx} = \frac{1}{2}$

(c) $2y = x - 8$

3 (i) $\frac{1 + \cos \theta}{1 + \sin \theta}$

(ii) $-\frac{2}{3} \cot \theta$

(iii) $-\tan \theta$

4 (i) (a) $\frac{(1+t)^2}{(1-t)^2}$

(b) 9

(ii) (a) $y = \frac{x}{1 - 2x}$

(b) $\frac{1}{(1 - 2x)^2}$

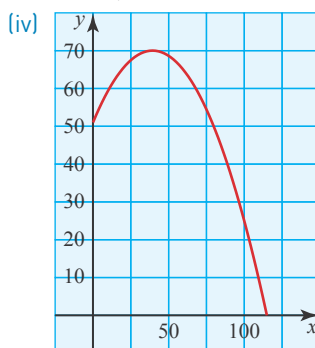
(iii) At $t = 2$, $x = \frac{2}{3}$ and

$\frac{dy}{dx} = \frac{1}{(1 - 2x)^2} = 9$

5 (i) (10, 58.75)

(ii) 70 m

(iii) 5.74 s; 114.8 m



6 (i) $\frac{dy}{dx} = \frac{-t(t^3 - 2)}{1 - 2t^3}$

(ii) $\left(2^{\frac{1}{3}}, 2^{\frac{2}{3}}\right)$

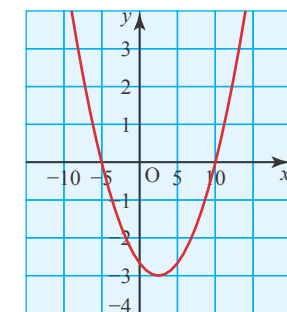
(iii) $\left(\frac{3}{2}, \frac{3}{2}\right)$; $\frac{3\sqrt{2}}{2}$; P is the farthest point on the curve from the origin

7 (i) $\frac{1}{2e^t}$

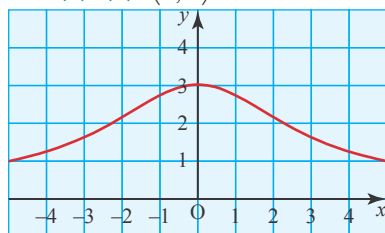
(ii) (a) $y = \frac{1}{2}x$

(b) $2ey = x + e^2 - 1$

8 (i) (a) (1, -3) minimum



(ii) (a) (0, 3) maximum



9 (i) 6

(ii) $y = 6x - \sqrt{3}$

(iii) $3x + 18y - 19\sqrt{3} = 0$

10 (i) $\frac{1}{2}r$ (ii) $15 \text{ cm}^3 \text{ s}^{-1}$

11 (i) $\left(\frac{1}{4}, 0\right)$ (ii) 2

(iii) $y = 2x - \frac{1}{2}$

(iv) $\left(0, -\frac{1}{2}\right)$

12 (i) $x - ty + at^2 = 0$

(ii) $tx + y = at^3 + 2at$

(iii) $(at^2 + 2a, 0)$, $(0, at^3 + 2at)$

13 (ii) (0, -1); minimum

14 (i) $-\frac{b}{at^2}$

(ii) $at^2y + bx = 2abt$

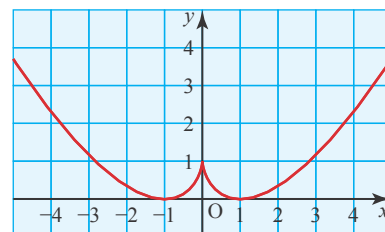
(iii) $X(2at, 0)$, $Y\left(0, \frac{2b}{t}\right)$

(iv) Area = $2ab$

15 (ii) (1, 0) and (-1, 0)

(iii) The tangent is vertical at the point (0, 1).

(iv)



16 (i) $-\frac{3 \cos t}{4 \sin t}$

(ii) $3x \cos t + 4y \sin t = 12$

(iii) $t = 0.6435 + n\pi$

Exercise 11.4 (page 267)

1 (i) $\frac{4t^3}{3} + c$

(ii) $\frac{t^2}{2} - 2 \ln t - \frac{1}{2t^2} + c$

(iii) $6t + 3 \sin 2t + c$

2 $\frac{13750}{3}$

3 πab

4 $\frac{3\pi}{2}$

5 $\frac{4}{3}$

6 (i) $\frac{1}{2}$

(ii) This is the area of the triangle with vertices (0, 0), (1, 0) and (0, 1).

Chapter 12

Opening activity (page 269)

Yes, by sailing at an angle to the wind. When a boat sails in the same direction as the wind it can never go faster than wind speed.

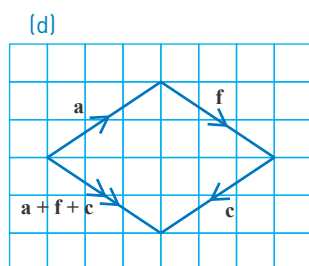
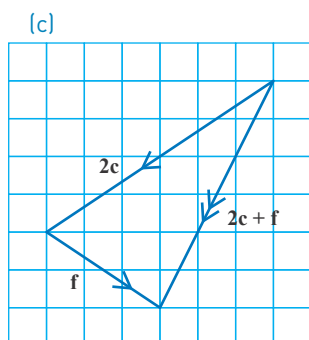
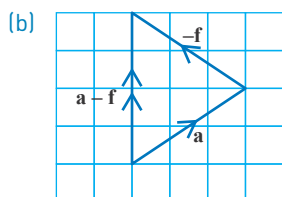
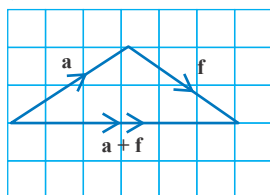
The keel stops the boat moving sideways, so only the forwards component of the wind comes into play. However, when a boat sails at an angle to the wind it can increase the relative wind velocity across the sails and so sail faster than the wind.

Discussion point (page 273)

For example, velocity, force, acceleration, weight, displacement.

Exercise 12.1 (page 275)

- 1 $\mathbf{a} = -2\mathbf{i} - 3\mathbf{j}$
 $\mathbf{b} = 2\mathbf{i}$
 $\mathbf{c} = \mathbf{i} + 3\mathbf{j}$
 $\mathbf{d} = -2\mathbf{i} + 2\mathbf{j}$
 $\mathbf{e} = -2\mathbf{j}$
 $\mathbf{f} = 3\mathbf{i} + 2\mathbf{j}$
- 2 (i) $\mathbf{b} = \mathbf{a}$, $\mathbf{c} = -\mathbf{a}$, $\mathbf{d} = 2\mathbf{a}$,
 $\mathbf{e} = -3\mathbf{a}$; $\mathbf{f}$ cannot be given in terms of $\mathbf{a}$
- (ii) (a)



- 3 (i) $(13, 67.4^\circ)$
(ii) $(13, 22.6^\circ)$
(iii) $(13, -67.4^\circ)$
(iv) $(13, 112.6^\circ)$
(v) $(13, 202.6^\circ)$
- 4 (i) 5
(ii) $\sqrt{21}$
(iii) $\sqrt{5}$
(iv) $\sqrt{14}$
- 5 (i) $\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$ (ii) $\begin{pmatrix} -2 \\ -2 \\ 4 \end{pmatrix}$
(iii) $\begin{pmatrix} -2 \\ -3 \\ 3 \end{pmatrix}$ (iv) $\begin{pmatrix} 16 \\ 1 \\ 1 \end{pmatrix}$
(v) $\begin{pmatrix} -7 \\ -5 \\ 8 \end{pmatrix}$ (vi) $\begin{pmatrix} 2 \\ 0 \\ -6 \end{pmatrix}$
- 6 (i) $\sqrt{\left(\frac{1}{3}\right)^2 + \left(-\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2}$
 $= \sqrt{\frac{1}{9} + \frac{4}{9} + \frac{4}{9}} = 1$
(ii) (a) $\frac{2}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}$
(b) $\frac{\sqrt{3}}{3}\mathbf{i} + \frac{\sqrt{3}}{3}\mathbf{j} + \frac{\sqrt{3}}{3}\mathbf{k}$
- 7 $\mathbf{a}$, $\mathbf{d}$, $\mathbf{e}$, $\mathbf{f}$, and $\mathbf{g}$ are parallel
 $\mathbf{c}$ and $\mathbf{h}$ are parallel
- 8 $x = 2$, $y = 3$
- 9 $a = -4.5$, $b = 10.5$
- 10 $k = 4$, 10.8 N
- 11 (i) $\overrightarrow{AC} = -3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$,
 $\overrightarrow{AB} = 5\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}$,
 $\overrightarrow{BC} = -8\mathbf{i} + 5\mathbf{j} - \mathbf{k}$
(ii) $\sqrt{98} \neq \sqrt{6} + \sqrt{46} - \sqrt{8}$
- 12 (i) $(6\mathbf{i} + \mathbf{j})\text{ N}$
(ii) $(4\mathbf{i} + 5\mathbf{j})\text{ N}$
(iii) $2\sqrt{5}\text{ N}$; 116.6° to the $\mathbf{i}$ direction
(iv) $\sqrt{10}\text{ N}$; 71.6° to the $\mathbf{i}$ direction
- 13 $a = -6$; $b = -5$
- 14 (i) $a = -3$; $b = -2$
(ii) $a = -6$; $b = 6$
(iii) $a = 0$; $b = 1$
- 15 (i) 12.8 km
(ii) 20 km h^{-1} , 5 km h^{-1}
(iii) After 40 minutes the boats meet.

Discussion point

(page 277)

Three different answers:

$$\overrightarrow{OD} = \begin{pmatrix} 10 \\ -1 \\ -3 \end{pmatrix}, \overrightarrow{OD} = \begin{pmatrix} -14 \\ 7 \\ -1 \end{pmatrix}$$

$$\text{or } \overrightarrow{OD} = \begin{pmatrix} 16 \\ 11 \\ 21 \end{pmatrix}$$

Exercise 12.2 (page 278)

- 1 (i)
 $\overrightarrow{AB} = -\overrightarrow{CD}$,
 $\overrightarrow{BC} = -\overrightarrow{DA} \Rightarrow \overrightarrow{AB} + \overrightarrow{CD}$
 $\quad \quad \quad + \overrightarrow{BC} + \overrightarrow{DA}$
 $= \overrightarrow{AB} - \overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{BC} = 0$
- (ii) Yes, closed loop.
- 2 (i) (a) $2\mathbf{b}$
(b) $\mathbf{c} - \mathbf{a}$
(c) $-\mathbf{a} - \mathbf{b}$
(d) $\mathbf{c}$ or $(\mathbf{b} - \mathbf{a})$
(ii) $\overrightarrow{AB} = \mathbf{c} = \mathbf{b} - \mathbf{a}$
 $\Rightarrow \mathbf{a} + \mathbf{c} - \mathbf{b} = 0$
- 3 (i) (a) $\mathbf{a}$
(b) $\mathbf{c} + \mathbf{a}$
(c) $\mathbf{c} - \mathbf{a}$
(d) $\mathbf{a} - \mathbf{c}$
(e) $-\mathbf{a} - \mathbf{c}$
(f) $\frac{1}{2}\mathbf{c}$
(g) $\mathbf{a} + \frac{1}{2}\mathbf{c}$
(h) $\frac{1}{2}\mathbf{c} - \mathbf{a}$
- (ii) $\overrightarrow{OP} = \frac{1}{2}(\mathbf{c} + \mathbf{a})$,
 $\overrightarrow{OQ} = \frac{1}{2}(\mathbf{a} + \mathbf{c})$;
diagonals of a parallelogram bisect each other.
- 4 (i) (a) $\mathbf{q}$
(b) $-\mathbf{p}$
(c) $\mathbf{q} + \mathbf{p}$
(d) $\mathbf{r} + \mathbf{p}$
(e) $\mathbf{p} + \mathbf{q} + \mathbf{r}$
(ii) $\mathbf{p} + \frac{3}{5}\mathbf{r}$
(iii) Assume the diagonals bisect at N, then
 $\overrightarrow{ON} = \frac{1}{2}\overrightarrow{OC}$
 $= \frac{1}{2}(\mathbf{p} + \mathbf{q} + \mathbf{r})$

Also,

$$\begin{aligned}\overrightarrow{ON} &= \mathbf{p} + \mathbf{q} + \frac{1}{2}\overrightarrow{BG} \\ &= \mathbf{p} + \mathbf{q} + \frac{1}{2}(\mathbf{r} - \mathbf{p} - \mathbf{q}) \\ &= \frac{1}{2}\mathbf{p} + \frac{1}{2}\mathbf{q} + \frac{1}{2}\mathbf{r} = \frac{1}{2}\overrightarrow{OC}\end{aligned}$$

as required.

5 Trapezium

6 11.74

7 (i) $\frac{3}{2}(\mathbf{b} - \mathbf{a})$

(ii) $\overrightarrow{AB} = (\mathbf{b} - \mathbf{a})$,

$\overrightarrow{PQ} = \frac{3}{2}(\mathbf{b} - \mathbf{a})$ and

$\overrightarrow{XY} = \frac{2}{3}(\mathbf{b} - \mathbf{a})$ since the vectors are all scalar multiples of $\mathbf{b} - \mathbf{a}$ they are all parallel.

(iii) 4:9

8 $\mathbf{v} = 7\mathbf{i} + 14\mathbf{j} - 3\mathbf{k}$,
 $\mathbf{a} = 20\mathbf{i} + 24\mathbf{j} - 4\mathbf{k}$

9 $\overrightarrow{AB} = \overrightarrow{DC} = \begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix}$,

so AB and CD are parallel and equal in length.

$\overrightarrow{AD} = \overrightarrow{BC} = \begin{pmatrix} 3 \\ -3 \\ -6 \end{pmatrix}$,

so AD and BC are also parallel and equal in length. Hence ABCD is a parallelogram as it has two pairs of equal, parallel sides.

10 $\overrightarrow{OD} = \frac{1}{2}(\mathbf{p} + \mathbf{q})$,

$\overrightarrow{OE} = \frac{1}{2}\mathbf{p} + \mathbf{q}$,

$\overrightarrow{DE} = \overrightarrow{OE} - \overrightarrow{OD} =$

$$\frac{1}{2}\mathbf{p} + \mathbf{q} - \frac{1}{2}(\mathbf{p} + \mathbf{q}) = \frac{1}{2}\mathbf{q}$$

11 (i) $\overrightarrow{OD} = \begin{pmatrix} 1 \\ 4 \\ 10 \end{pmatrix}$

(ii) $|\overrightarrow{AB}|^2 + |\overrightarrow{AD}|^2 = |\overrightarrow{BD}|^2$
 $\Rightarrow 24 + 35 = 59$
 $\Rightarrow$ triangle ABD is right-angled, hence ABCD is a rectangle.

$$(iii) \overrightarrow{OX} = \begin{pmatrix} 0.5 \\ 5.5 \\ 6.5 \end{pmatrix}$$

12 $\overrightarrow{OC} = \frac{1}{3}(\mathbf{a} + 2\mathbf{b})$ and

$\overrightarrow{OD} = \frac{1}{2}(\mathbf{a} + 2\mathbf{b})$, hence the vectors are scalar multiples of each other and so are parallel. Both vectors pass through O, so OCD is a straight line; 2:3

13 (i) $\overrightarrow{OM} = \begin{pmatrix} -0.25 \\ 3 \\ -1.25 \end{pmatrix}$

$$(ii) \overrightarrow{ON} = \begin{pmatrix} 0.8 \\ 4.4 \\ 1.2 \end{pmatrix}$$

14 $a = 4$ or -2

15 $\overrightarrow{OC} = \overrightarrow{OA} + \frac{s}{s+t}\overrightarrow{AB}$

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$

$$\overrightarrow{OC} = \mathbf{a} + \frac{s}{s+t}(\mathbf{b} - \mathbf{a})$$

$$= \mathbf{a} + \frac{s}{s+t}\mathbf{b} - \frac{s}{s+t}\mathbf{a}$$

$$= \frac{s+t}{s+t}\mathbf{a} - \frac{s}{s+t}\mathbf{a} + \frac{s}{s+t}\mathbf{b}$$

$$= \frac{t}{s+t}\mathbf{a} + \frac{s}{s+t}\mathbf{b}$$

16 No, a quadrilateral is a plane figure and these points do not lie on the same plane.

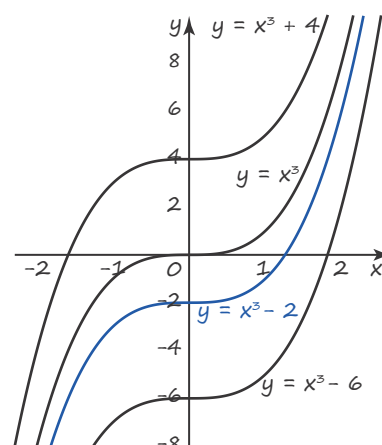
Chapter 13

Discussion point (page 282)

- (i) First
- (ii) Third
- (iii) First
- (iv) Second

Exercise 13.1 (page 285)

1 (i) $y = x^3 + c$



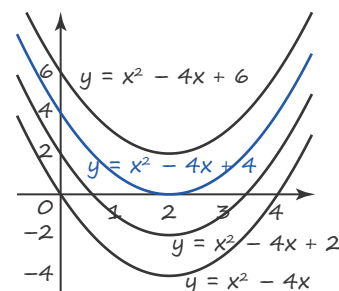
(ii) $y = x^3 - 2$

2 $\frac{dv}{dt}$ is the rate of change of the velocity with respect to time (the acceleration). The differential equation tells you that the acceleration is increasing and so is the velocity.

3 $\frac{dN}{dt} = kN$

4 The odd one out is $\dot{y}$. It is the rate of change of y with respect to time; all the others are the rate of change of y with respect to x .

5 (i) $y = x^2 - 4x + c$

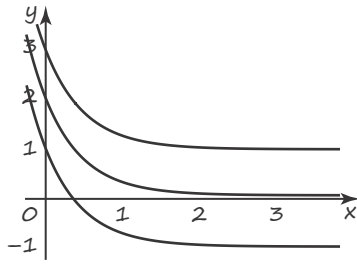


6 $\frac{dP}{dt} = k\sqrt{P}$

7 $\frac{dv}{dt} = \frac{4}{\sqrt{v}}$

8 $\frac{dT}{dx} = -\frac{x}{4}$

9 (i) $y = 2e^{-2x} + c$



(ii) $y = 2e^{-2x} - \frac{1}{2}$

10 $\frac{dT}{dt} = -\frac{(T-15)}{160}$

11 (i) $\frac{dr}{dt} = -k\sqrt{t}$ with $k = 1.5$

(ii) $r = 8 - t^{\frac{3}{2}}$

(iii) 4 weeks

12 (i) $\frac{dv}{dt} = \frac{2}{t+2}$

13 (i) $\frac{dM}{dt} = 0.05t(8-t)$

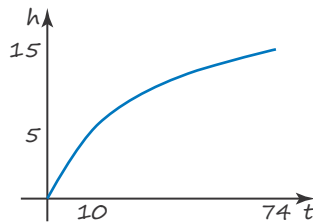
(ii) 4.27 kg

16 (i) It is growing at a constant rate of 0.5 metres per year. After 10 years it is 5 metres tall.

(ii) $\frac{dh}{dt} = \frac{k}{t}$ with $k = 5$

(iv) 74 years to the nearest year

(v)



Exercise 13.2 (page 290)

1 (i) $y = \sqrt[3]{\frac{3x^2}{2} + c}$

(ii) $y = \frac{2}{c-x^2}$

2 (i) $y = \ln\left(\frac{x^2}{2} + 1\right)$

(ii) $y = \sec x$

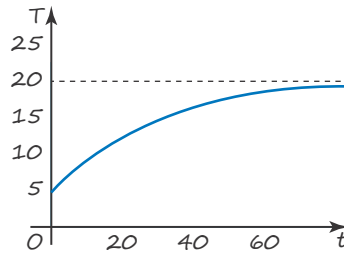
3 11.02 a.m.

4 $\frac{ds}{dt} = \frac{2}{s}$, $s = \sqrt{4t+c}$

5 (i) $\theta = 20 - Ae^{-2t}$

(ii) $\theta = 20 - 15e^{-2t}$

(iii)



(iv) 60 seconds

(v) Temperature becomes the same as the room, 20°C

6 $y = \pm\sqrt{\frac{5x^2+3}{2}}$

7 (i) $\frac{4}{2x-3} - \frac{1}{x-1}$

(ii) $\frac{dy}{dx} = \frac{A(2x-3)^2}{x-1}$

8 (i) $h = 5 + \frac{(20-3t)^2}{4}$

(ii) The water stops flowing

(iii) 163 seconds

9 (i) $\frac{dr}{dt} = \frac{k}{r^2}$

(ii) $k = 5000$, 141 m (3 s.f.)

(iii) $\frac{dr}{dt} = \frac{c}{r^2(2+t)}$,

$c = 10000$

(iv) 104 m (3 s.f.)

10 (i) $\frac{dv}{dt} = \frac{3}{v}$

(ii) $v = \sqrt{6t+4}$

(iii) 10 seconds

(iv) For large t , the velocity is roughly proportional to $\sqrt{t}$, so it increases indefinitely but at a decreasing rate.

11 (i) $v = 1 + \frac{2}{e^{kt} + 1}$

(ii) (a) v decreases and tends to 1

(b) v increases and tends to 3

12 (i) $P = 600e^{kt}$

(iii) $P = 600e^{(0.005t-0.4\sin(0.02t))}$,

very good fit with the data

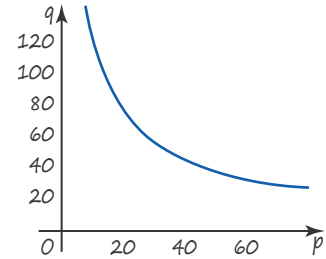
(iv) 549

13 (i) $q = Ap^{-\eta}$

(ii) (a) $\eta = 0.776$ (to 3 s.f.)

(b) $q = 817p^{-0.776}$

(c)



14 (i) $\frac{dP}{dt} = \lambda P$

(ii) $P = P_0e^{2t}$

(iii) 499 minutes

(iv) $P = P_0e^{\sin 2t}$

(v) 551 minutes

(vi) In the first model, the population increases exponentially with time. In the second model, the population oscillates with time, going above then below the initial population of P_0

15 (ii) $\frac{1}{c}$

Chapter 14

Activity 14.1 (page 295)

(i) Yes: $x = 1$, $x = 3$

(ii) Yes: $x = -5 \pm \sqrt{17}$

(iii) No: $x \approx -1.6$, $x \approx 0.6$, $x \approx 1.3$

(iv) Yes: $x = 0$, $x = -1$, $x = 1$

(v) No: $x \approx 0.35$, $x \approx 2.15$

Discussion points (page 295)

The first root is between 0.3 and 0.4, and the second is between 2.1 and 2.2.

The first root looks as if it is closer to 0.4, so you can be fairly confident to state that the root is 0.4 to 1 d.p., but you would have to zoom in further to show that the root is to the right of 0.35, to be sure. It is not clear whether the

second root is nearer to 2.1 or 2.2 – you would have to zoom in to see on which side of 2.15 the root lies.

Discussion point (page 297)

Decimal search is reasonably efficient but a computer programme might need to carry out several calculations before finding a change of sign. If you are doing it yourself on a calculator, you can speed it up: if you know that a root lies in the interval $[1, 2]$ you could try 1.5 first, to find out which half of the interval it is in, rather than working through 1.1, 1.2, etc.

Discussion point (page 298)

Interval bisection is more efficient when programming a computer because it follows the rule of halving the interval each time, so only one point needs to be tested at each stage. However, the numbers to be tested quickly become long, so this method is more difficult to carry out by hand.

Activity 14.2 (page 301)

The iterations either diverge, or they converge to the root already found in the interval $[0, 1]$.

Activity 14.3 (page 302)

- (i) $[0.45, 0.55]$
- (ii) $[-0.3695, -0.3685]$
- (iii) $[21.63415, 21.63425]$

Discussion point (page 304)

Often it is not so much a failure of the method as a failure to use an appropriate starting point or an appropriate re-arrangement of the equation.

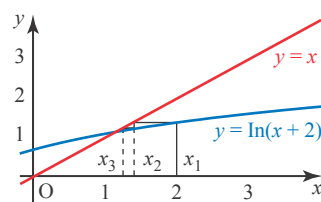
Discussion point (page 304)

The iteration diverges because of the gradient of the curve; it is steeper than the line $y = x$.

Exercise 14.1 (page 305)

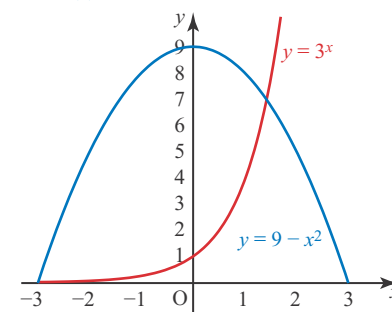
- 1 $f(x) = e^x - 5x$
 $f(2) = -2.61... < 0$
 $f(3) = 5.08... > 0$
 Sign change and continuity, therefore root in the interval.
- 2 Equation 1 – Diagram (b). The iterations are converging to the root α .
 Equation 2 – Diagram (c). The iterations are converging to the root α .
 Equation 3 – Diagram (a). The iterations are diverging.
 Equation 4 – Diagram (d). The iterations are diverging.
- 4 (i) $f(x) = x^3 - x - 2$
 $f(1) = -2 < 0$
 $f(2) = 4 > 0$
 Sign change and continuity, therefore root in the interval
 (iii) $x_1 = 1,$
 $x_2 = 1.442249...,$
 $x_3 = 1.509897...,$
 $x_4 = 1.519724...,$
 $x_5 = 1.521141...$
 (iv) $f(1.5205) = -0.005 < 0$
 $f(1.5215) = 0.0007... > 0$
 Sign change and continuity, therefore root in the interval $[1.5205, 1.5215]$.
 Therefore root is 1.521 accurate to 3 d.p.
- 5 0.739.
 $0.7385 - \cos(0.7385)$
 $= -0.00098.$
 $0.7395 - \cos(0.7395) = 0.00069$
- 6 (i) $f(x) = e^x - x - 2$
 $f(1) = -0.28... < 0$
 $f(2) = 3.38... > 0$
 Sign change and continuity, therefore root in the interval
 (iii) $x_1 = 2,$
 $x_2 = 1.386294...,$
 $x_3 = 1.219736...$

(iv) and (v)



- 7 (i) $f(x) = 6 - x - \ln x$
 $f(4) = 0.61... > 0$
 $f(5) = -0.60... < 0$
 Sign change and continuity, therefore root in the interval
 (ii) $f(4.5) = -0.004... < 0$
 Sign change between $x = 4$ and $x = 4.5$ therefore root lies in this interval. So root is closer to $x = 4$.

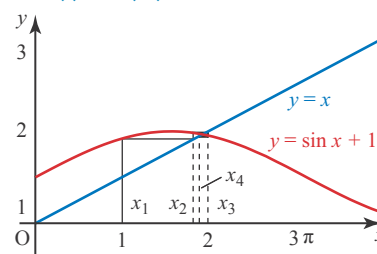
8 (i) Two roots



(iv) $x = 1.6549$ (4 d.p.)

9 Converges to α

10 (i) and (iii)



- (ii) $x_1 = 1,$
 $x_2 = 1.84147098...,$
 $x_3 = 1.96359072...,$
 $x_4 = 1.92384305...$
- (iv) You can only give the root as $x = 2$ to the nearest whole number.
- (v) $x = 1.93$ (2 d.p.)

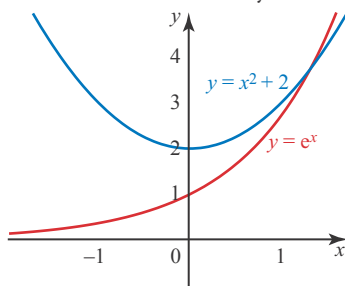
- 11 (ii) $f(0.2) = 0.208$; $f(0.3) = -0.173$. Change of sign, the function is continuous, so there is a solution between $x = 0.2$ and $x = 0.3$.
 $f(1.8) = -0.368$, $f(1.9) = 0.259$. Change of sign, the function is continuous, so there is a solution between $x = 1.8$ and $x = 1.9$.

- (iii) (a) $f'(x) = \frac{3}{4}x^2$
 $f'(0.2) = 0.03$ which is less than 1 so this should converge;
 $f'(1.6) = 1.96$ which is greater than 1 so this should not converge.

(b) $f'(x) = \frac{4}{3}(4x - 1)^{-\frac{2}{3}}$

$f'(0.3) \approx 3.9$ which is greater than 1 so this should not converge;
 $f'(1.9) \approx 0.38$ which is less than 1 so this should converge.

- 12 (ii) Curves cross only once



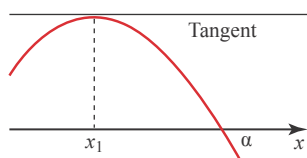
(iii) 1.32

- 13 (ii) 3.1623

- 14 0.747

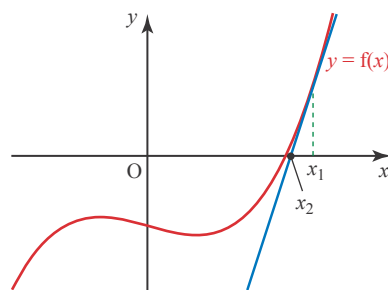
Discussion point (page 309)

$f'(x_1) = 0$ so the method cannot proceed. The tangent never crosses the x -axis. The formula cannot work because it would involve division by zero



Exercise 14.2 (page 309)

- 1 (i) $f'(x) = 3x^2 - 1$
(ii) $x_{n+1} = x_n - \frac{x_n^3 - x_n - 3}{3x_n^2 - 1}$
(iii) 1.727...



- 2 (i) $f(x) = x^3 - 6x^2 + 12x - 11$
 $f(3) = -2 < 0$
 $f(4) = 5 > 0$
Sign change and continuity, therefore the root is in the interval.

- (ii) $x_2 = 3.444$,
 $x_3 = 3.442$ (3 d.p.)
(iii) $f(3.435) < 0$ and $f(3.445) > 0$, so the root lies in the interval $[3.435, 3.445]$ and so the root is 3.44 to 3 s.f.

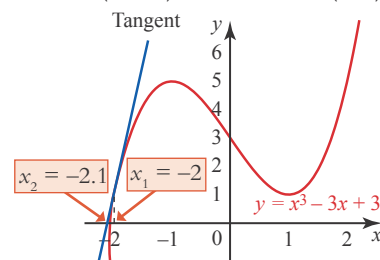
- 3 (i) $f(x) = e^{2x} - 15x - 2$
 $f(1.5) = -4.41... < 0$
 $f(1.7) = 2.46... > 0$
Sign change and continuity, therefore the root is in the interval.

- (ii) $x_2 = 1.64$ (3 s.f.)
(iii) $f(x) = e^{2x} - 15x - 2$
 $f(1.635) = -0.21... < 0$
 $f(1.645) = 0.16... > 0$
Sign change and continuity, therefore the root is in the interval $[1.635, 1.645]$ and is therefore 1.64 to 3 s.f.

- 4 (i) $f'(x) = 6x^2 + 5$
 $f'(x)$ is positive for all values of x , so the graph of $y = f(x)$ has no turning points and therefore crosses the x -axis once only.

- (ii) $f(-1) = -5 < 0$
 $f(0) = 2 > 0$
Sign change and continuity, therefore the root is in the interval
(iii) $x_2 = -0.3846153...$,
 $x_3 = -0.3783533...$
(iv) $f(-0.3785) < 0$,
 $f(-0.3775) > 0$, so root lies in the interval $[-0.3785, -0.3775]$ and is therefore -0.378 correct to 3 d.p.

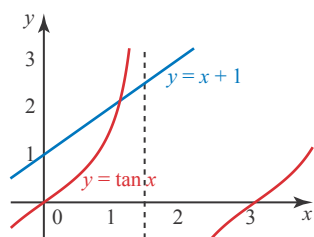
- 5 (i) and (iii) Maximum at $(-1, 5)$, minimum at $(1, 1)$



- (ii) $-2.111111...$
(iv) $f(-2.115) < 0$,
 $f(-2.105) < 0$, so the root does not lie in the interval $[-2.115, -2.105]$ and therefore the root is not -2.11 correct to 2 d.p.
 $f(-2.15) < 0$,
 $f(-2.05) > 0$, so the root does lie in the interval $[-2.15, -2.05]$ and therefore the root is -2.1 correct to 1 d.p.
(v) $f'(1) = 0$, and $f'(x)$ is also zero at $x = -1$.

- 6 (i) $f(x) = x^4 - 7x^3 + 1$
 $f(0) = 1 > 0$
 $f(1) = -5 < 0$
Sign change and continuity, therefore the root is in the interval
(ii) 0.54 (2 d.p.)
(iii) Tangent to the curve at $x = 0$ is horizontal, so never meets the x -axis. Second approximation is undefined

7



1.13 (3 s.f.)

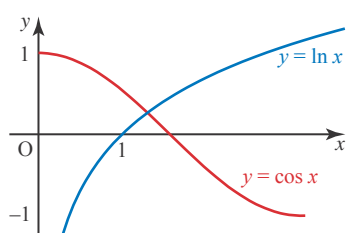
$$f(x) = \tan x - x - 1$$

$$f(1.125) = -0.032... < 0$$

$$f(1.135) = 0.012... > 0$$

Sign change and continuity, therefore the root is in the interval $[1.125, 1.135]$ and is therefore 1.13 correct to 3 s.f.

8



1.303

$$f(x) = \ln x - \cos x$$

$$f(1.3025) = -0.0008... < 0$$

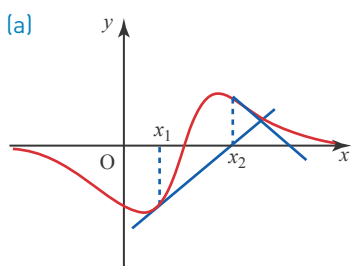
$$f(1.3035) = 0.0009... > 0$$

Sign change and continuity, therefore the root is in the interval $[1.3025, 1.3035]$ and is therefore 1.303 correct to 3 d.p.

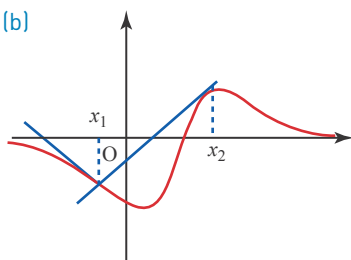
9

For some initial values to the right of the left-hand turning point, successive iterations may diverge to $+\infty$ (see (a)), or diverge to $-\infty$ (see (b)). If the initial value is to the left of the left-hand turning point, the process will diverge to $-\infty$ (see (c)). If any value is at one of the turning point, the process breaks down (see (d))

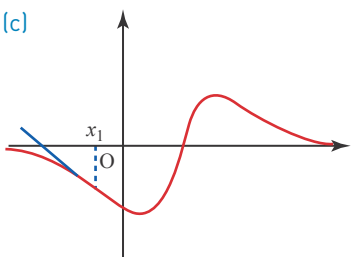
(a)



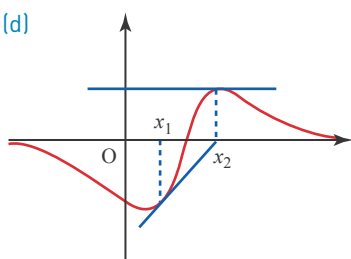
(b)



(c)



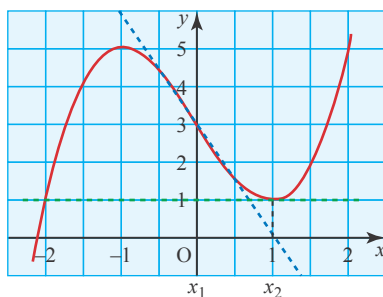
(d)



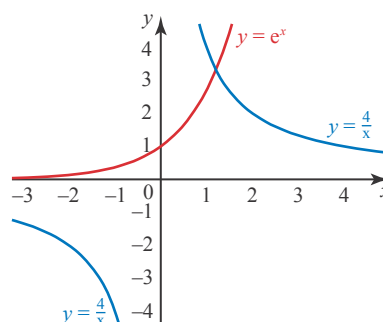
10 (i) $x_2 = 1, x_3$ is not defined.

(ii) At $x_2 = 1$ the gradient is zero, so the tangent at that point is horizontal and x_3 cannot be found.

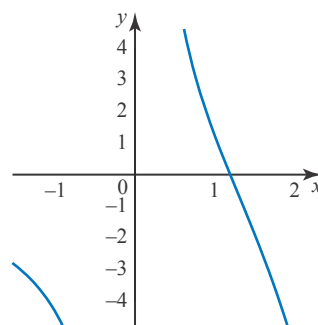
(iii) -2.1038



11 (i) One point of intersection



(ii)



(iii) 1.202 (3 d.p.)

(iv) Still converges rapidly to root

12 (i) 0.25, 0.200075; 0.19658; 0.196564; 0.196564.

Therefore 0.197 is a solution correct to 3 decimal places.

(ii) (a) $-1 < f'(\alpha) < 1$

(b) $f'(x) = -3e^{3x}$;
 $f'(0.197) \approx -5.4$
which is less than -1
so this should not converge.

13 Equation is
 $3x^{12} - 750x + 747 = 0$
 $x = 1.49513$ (5 d.p.)

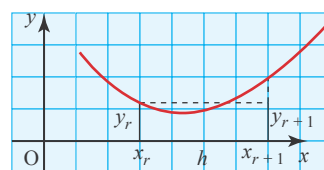
Discussion point (page 312)

As you increase the number of strips, the accuracy of the approximation increases.

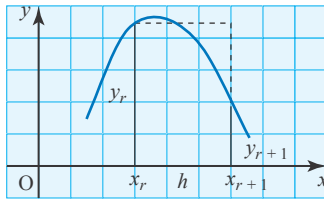
Discussion point (page 315)

If there is a turning point then the rectangle using the smaller y value may not lie entirely below the curve, and the top of the rectangle using the bigger y value may not be entirely above the curve, making your values unreliable as bounds.

Rectangle using smaller y value:



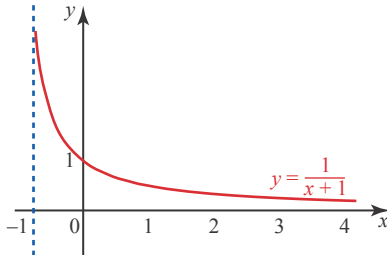
Rectangle using bigger y value:



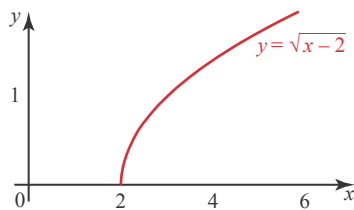
As you increase the number of strips, the accuracy of the approximation increases.

Exercise 14.3 (page 317)

- 1 54.75
- 2 (i) 0.8194
(ii) 0.8317
- 3 0.512, overestimate because curve is concave upwards.



- 4 1.22, underestimate because curve is concave downwards.



- 5 (i) 458 m
(ii) Trapezium rule approximates curve with straight lines, values are not given to a very high degree of accuracy.

x	y
0	1
0.2	0.9615
0.4	0.8621
0.6	0.7353
0.8	0.6098
1	0.5

- (ii) Upper bound is 0.8337
Lower bound is 0.7337
- (iii) 0.8 to 1 d.p.

- (iv) Increase the number of rectangular strips

- 7 (i) 0.229 383 0
(ii) $2.3 \ln 2.3 - 2 \ln 2 - 0.3, 0.0059\%$
- 8 Trapezium rule with 8 strips gives 0.3749, sum of 8 rectangles below the curve gives 0.3667, so the area is 0.37 to 2 d.p.
- 9 (i) 0.683

Practice questions 3

(page 322)

$$1 \quad \frac{dm}{dt} = \frac{k}{\sqrt{m}} \quad [1]$$

When $t = 0$, $m = 9$, and

$$\frac{dm}{dt} = 10, \text{ so } 10 = \frac{k}{3} \\ \Rightarrow k = 30, \text{ so } \frac{dm}{dt} = \frac{30}{\sqrt{m}}. \quad [2]$$

$$2 \quad \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \\ (2a\mathbf{i} + 3b\mathbf{j}) + (-b\mathbf{i} + a\mathbf{j}) + \\ (10\mathbf{i} - 2\mathbf{j}) = \\ (2a - b + 10)\mathbf{i} + (3b + a - 2)\mathbf{j} \\ = 0 \quad [1] \\ \Rightarrow 2a - b = -10, a + 3b = 2 \\ \Rightarrow b = 2, a = -4 \quad [3]$$

$$3 \quad (i) \quad \overline{AB} = \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}; \\ \overline{AC} = \begin{pmatrix} 2 \\ -2 \\ -6 \end{pmatrix}; \\ \overline{BC} = \begin{pmatrix} 4 \\ -3 \\ -5 \end{pmatrix} \quad [2]$$

(ii)

$$AB = \sqrt{(-2)^2 + 1^2 + (-1)^2} = \sqrt{6}; \\ AC = \sqrt{(2)^2 + (-2)^2 + (-6)^2} = \sqrt{44}; \\ BC = \sqrt{4^2 + (-3)^2 + (-5)^2} = \sqrt{50}, \\ \text{so } AB^2 + AC^2 = 6 + 44 = 50 \\ = BC^2, \text{ and by Pythagoras' theorem triangle ABC is right angled.} \quad [3]$$

$$4 \quad (i) \quad \overline{OD} = \mathbf{v} + \frac{1}{3}\mathbf{u}; \\ \overline{OE} = \frac{3}{4} \left(\mathbf{v} + \frac{1}{3}\mathbf{u} \right) \\ = \frac{3}{4}\mathbf{v} + \frac{1}{4}\mathbf{u}; \\ \overline{CE} = -\mathbf{v} + \frac{3}{4}\mathbf{v} + \frac{1}{4}\mathbf{u} \\ = \frac{1}{4}\mathbf{u} - \frac{1}{4}\mathbf{v} \quad [3]$$

$$(ii) \quad \overline{CA} = \mathbf{u} - \mathbf{v}, \text{ so} \\ \overline{CE} = \frac{1}{4}\overline{CA} \text{ and E must therefore lie on AC (as CE is in the same direction as CA).} \quad [2]$$

$$5 \quad (i) \quad f'(x) = 8 \cos 2x - 3x^2 \quad [1]$$

$$x_{n+1} = x_n - \frac{4 \sin 2x_n - x_n^3}{8 \cos 2x_n - 3x_n^2} \quad [1] \\ x_0 = 1, 1.41667, \\ 1.29723, 1.28864, \\ 1.28859, \text{ so} \\ x = 1.29 \text{ (3 s.f.)} \quad [2]$$

$$(ii) \quad f(1.285) = 0.042..., \\ f(1.295) = -0.0755... \\ \text{and change of sign means root is correct to 3 s.f.} \quad [1]$$

$$6 \quad (i) \quad \text{Interval is } [-1.5, -1.4] \text{ as} \\ f(-1.5) = -1.59375 < 0 \\ \text{and } f(-1.4) = 0.42176 > 0. \quad [2]$$

$$(ii) \quad \text{Using list function or otherwise:} \\ f(-1.43) = -0.11971, \\ f(-1.42) = 0.066466, \\ \text{so root lies in} \\ [-1.43, -1.42]. \quad [2]$$

$$(iii) \quad f(-1.425) = -0.0259, \\ \text{or interval to 3 d.p. is} \\ [-1.424, -1.423] \\ \text{so root is closer to} \\ -1.42, \text{ to 2 d.p.} \quad [2]$$

$$(i) \quad x = \frac{1}{2}(x^3 - 5), \text{ so } a = 0.5 \\ \text{and } b = -2.5 \quad [1]$$

$$(ii) \quad x_1 = -2, x_2 = -6.5, \\ x_3 = -139.8125. \quad [1]$$

Divergent [1]

$$(iii) \quad x = \sqrt[3]{(2x + 5)}, \\ \text{so } c = 2 \text{ and } d = 5. \quad [1]$$

$$(iv) \quad x_1 = 1.912931183, \\ x_2 = 2.066580768, \\ x_3 = 2.090292423. \quad [2]$$

$$(v) \quad f(2.085) = -0.106..., \\ f(2.095) = 0.005...,$$

change of sign so root lies in interval

[2.085, 2.095] and

root is 2.09 to 2 d.p. [2]

- 8 (i) A is when $t = -2$,
 $x = -3$, $y = 0$; B is
 when $t = 2$, $x = 5$,
 $y = 8$. [4]

(ii) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2+2t}{2}$. [2]

$\frac{dy}{dx} = 0$ when

$2 + 2t = 0 \Rightarrow t = -1$ [1]

when $t = -1$, $x = -1$,

$y = -1$, so turning

point is $(-1, -1)$. [2]

(iii) $t = \frac{x-1}{2}$

$\Rightarrow y = 2 \times \frac{x-1}{2} + \left(\frac{x-1}{2}\right)^2$

$= x - 1 + \frac{1}{4}x^2 - \frac{1}{2}x + \frac{1}{4}$

$= \frac{1}{4}x^2 + \frac{1}{2}x - \frac{3}{4}$ [3]

- 9 (i)

$\int \frac{3dv}{v(v-3)} = -\int ktdt$ [1]

$\frac{3}{v(v-3)} = \frac{A}{v} + \frac{B}{v-3}$

$\Rightarrow 3 = A(v-3) + Bv$

$v = 0 \Rightarrow 3 = -3A$,

so $A = -1$

$v = 3 \Rightarrow 3 = 3B$,

so $B = 1$

so $\frac{3}{v(v-3)} = \frac{1}{v-3} - \frac{1}{v}$ [3]

$\Rightarrow \int \frac{3dv}{v(v-3)} = \int \left[\frac{1}{v-3} - \frac{1}{v} \right] dv$

$= \ln(v-3) - \ln v$ [1]

so $\ln(v-3) - \ln v = -kt + c$

$\Rightarrow \frac{v-3}{v} = e^{-kt+c} = Ae^{-kt}$ [2]

when $t = 0$, $v = 10 \Rightarrow A = 0.7$ [1]

$\Rightarrow 1 - \frac{3}{v} = 0.7e^{-kt}$

$\Rightarrow \frac{3}{v} = 1 - 0.7e^{-kt}$

$\Rightarrow v = \frac{3}{1 - 0.7e^{-kt}}$ as required. [2]

- (ii) As $t \rightarrow \infty$, $e^{-kt} \rightarrow 0$, so

$v \rightarrow 3$, and terminal

velocity is 3 ms^{-1} [2]

Review: Working with data

Discussion point (page 328)

The same They are illustrating the same data

Different Labels

Frequency chart The vertical scale is labelled 'Frequency'

Histogram The vertical scale is labelled 'Frequency density'

Scales on the vertical axes

Frequency chart The vertical scale is the frequency

Histogram The vertical scale is such that the areas of the bars represent frequency.

Class intervals

Frequency chart The class intervals are all the same, 10 seconds

Histogram The class intervals are not all the same; some are 10 seconds but one is 50 seconds.

Discussion point (page 331)

$S_{xx} = 8$, $s^2 = \frac{8}{9} = 0.888\dots$,
 $s = \sqrt{0.888\dots} = 0.943$ (3 d.p.)

(vii) 19th or 20th

3 (i) 1.623 m and 0.152 m

(ii) 31.7°C and 0.61°C

4 (i) Positively skewed

(ii) Median = 0, $Q_1 = 0$,
 $Q_3 = 1$

(iii) Since the lower quartile and median are both zero you can't draw the box properly and there is no whisker on the left.

(iv) The large frequency for 0 gold medals means that this group dominates displays such as vertical line charts and stem and leaf diagrams, making it difficult to see the details.

5 A FALSE. 309 medals were awarded and

between them the top 4 countries won 118 of them, so less than half.

B TRUE. There were 309 medals and 207 countries, giving a mean of $\frac{309}{207} = 1.492\dots = 1.5$ to 1 d.p.

Exercise R.1 (page 332)

1 R.8

2 (i) The mean per capita GDP is much higher in Western Europe than it is in Eastern Europe.

(ii) (a) 41 000 US\$

(b) 8000 US\$

(iii) (a) Both 30 000 to 40 000 US\$ and 40 000 to 50 000 US\$

(b) 20 000 to 30 000 US\$

(c) 20 000 to 30 000 US\$

(iv) Modal group

(v) Western Europe 39 000 US\$ Eastern Europe 16 000 US\$

(vi) (a) 15 941 US\$

(b) They do not take the sizes of the populations of the various countries into account.

Weighted means would be better.

- C** TRUE. $148 + 21 = 169$ countries got 0 or 1 gold medals. This is 81.6% of the 207 countries.
- D** UNCERTAIN. The information is not provided to draw this conclusion. It was in fact false with several large countries, e.g. India, Pakistan and Nigeria, getting no gold medals but that information was not given.
- E** TRUE. $1.49 + 2 \times 4.79 = 11.07$ so countries with 12 or more medals are identified as outliers.
- F** FALSE. It is true that the mode is 0 but the midrange is $\frac{0 + 46}{2} = 23$ and not 0. So the combined statement is false.
- 6** (i) 77
(ii) 73. You do not know the exact numbers in the ranges 250 to 260 or 270 to 272.
(iii) (a) Large frequency densities (FDs) in the middle, decaying to small FDs at the extremities; some symmetry
(b) Not very symmetric; rather higher FD in the lowest two ranges.
- 7** (i) 4.78
(ii)

$$\begin{aligned}\sum (x - \bar{x})^2 &= \sum (x^2 - 2x\bar{x} + \bar{x}^2) \\ &= \sum x^2 - 2\sum (x\bar{x}) + \sum \bar{x}^2 \\ &= \sum x^2 - 2\bar{x}\sum x + n\bar{x}^2\end{aligned}$$

$$\begin{aligned}&= \sum x^2 - 2\bar{x} \times (n\bar{x}) + n\bar{x}^2 \\ &= \sum x^2 - 2n\bar{x}^2 + n\bar{x}^2 \\ &= \sum x^2 - n\bar{x}^2\end{aligned}$$

- (iii) If you are working by hand and the mean, $\bar{x}$, has several decimal places, working out the individual deviations can be tedious. So in such cases it is easier to use the $\sum x^2 - n\bar{x}^2$ form.
- 8** Cleaning involves looking through a data set to check for the following:
- Outliers: Example – Bob Roberts's age is given as 138
- Errors: Example – Debbie Lane's distance from home is 75 km
- Missing entries: Example – the time of Simon Sefton's accident is missing
- Format: Example – use of 12 hour and 24 hour clock for times of accidents
- 9** Averages could be calculated for age, distance from home; nights in hospital and possibly time of accident. It would be best to look at the distributions before deciding which average. The other fields are categorical (including the number of the police officer reporting the accident), and so no average is appropriate. However for some of these the modal class may be helpful, for example, the day of the week.
- 10** Robin would want his case studies to be as representative as possible so he should look for different groups, or strata within the data.

Obvious fields are: age, cause and injuries. Next he should decide which of these he thinks is the most important, e.g. age, and classify the data for it. Then he should write down some possible stratified samples. He should then classify the data in his other chosen fields and see in which of the possible samples those strata are as well represented as possible, given the small size of the sample he is selecting. Note that a random sample may not turn out to be all that representative when, like this, the size is small. So, in this case, a stratified sample is preferable.

- 11** With the exception of people's names, all the fields could be relevant to this comparison, some more obviously so than others. So the first thing is to decide which fields you are going to use. Then, starting with the most important field, divide the data for each field into two subsets: younger cyclists and older cyclists. Compare their total frequencies and their distributions. This comparison may show there are no differences, which is interesting because it suggests that both groups may be equally at risk. On the other hand, the comparison may show up differences and that too is interesting. It suggests respects in which one group is more at risk than the other. This in turn could lead to intervention policies to reduce the risk.

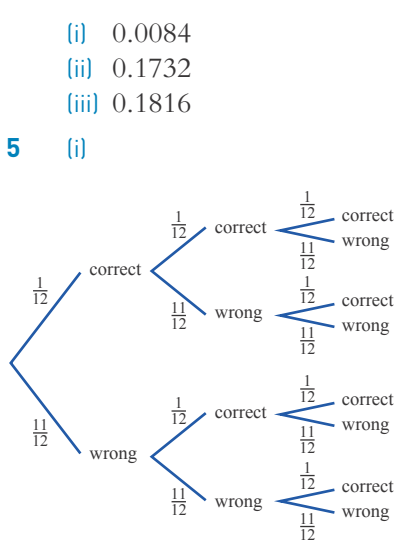
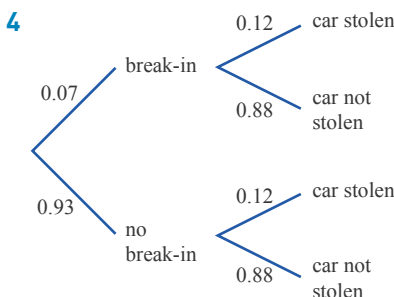
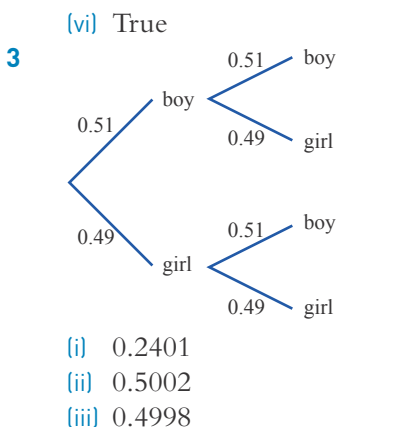
Chapter 15

Opening activity (page 338)

- (i) A civil case between two parties is decided on the 'balance of probabilities'.
- (ii) In a criminal case there is an assumption that the accused is innocent and must be proved guilty 'beyond reasonable doubt', i.e. with a very high probability.

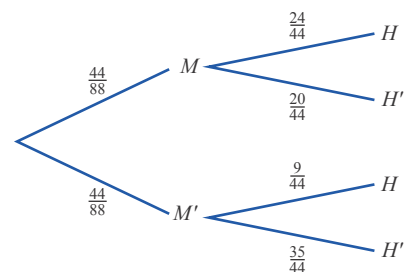
Review exercise (page 342)

- 1 (i) $\frac{2}{6}$
 (ii) $\frac{3}{6}$
 (iii) $\frac{1}{6}$
 (iv) 1
 (v) 0
- 2 (i) True
 (ii) False. There are two ways of selecting those two cards: first 7♦, then 8♥ or first 8♥ then 7♦. Each has a probability of $\frac{1}{52} \times \frac{1}{51} = \frac{1}{2652}$ so the overall probability is $2 \times \frac{1}{2652} = \frac{2}{2652}$.
- (iii) True
 (iv) False. There are 4 fours and 13 hearts but the 4♥ is repeated. So the total number is $4 + 13 - 1 = 16$ and the probability is $\frac{16}{52}$.
- (v) False. The probability that the first card is a spade is $\frac{13}{52} = \frac{1}{4}$ but for the second card there would be only 12 spades left out of 51 cards and $\frac{12}{51} \neq \frac{1}{4}$.



- 6 0.93
- 7 (i) 0.2436
 (ii) 0.7564
 (iii) 0.2308
 (iv) 0.4308

- 8 (i) 0.0741
 (ii) 0.5787
 (iii) 0.5556
- 9 For a sequence of events you multiply the probabilities. However, $\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6}$ gives the probability of six 6s in six throws. To find the probability of at least one 6 you need $1 - P(\text{No 6s})$ and that is $1 - \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = 0.665$.
- 10 0.5833
- 11 0.31
- 12 (i) 0.1667
 (ii) 0.1389
 (iii) 0.1157
 (iv) 0.5787
- 13 (i)



- (ii) (a) $\frac{24}{88}$
 (b) $\frac{55}{88}$
- (iii)
- | | H | H' | Total |
|-------|----|----|-------|
| M | 24 | 20 | 44 |
| M' | 9 | 35 | 44 |
| Total | 33 | 55 | 88 |
- (iv) The probability of being kept in hospital after an accident involving a motor vehicle, $\frac{24}{44}$, is greater than that after an accident not involving a motor vehicle, $\frac{9}{44}$.

Discussion point (page 348)

It seems unrealistic to think that any witness would select someone in an identity parade entirely at random, let alone all four doing so. It is also unlikely that any court would convict someone on the evidence of only one out of four witnesses.

However, this example raises an important statistical idea about how we make judgements and decisions.

Judgements are usually made under conditions of uncertainty and involve us in having to weigh up the plausibility of one explanation against that of another. Statistical judgements are usually made on such a basis. We choose one explanation if we judge the alternative explanation to be sufficiently unlikely, that is, if the probability of its being true is sufficiently small. Exactly how small this probability has to be will depend on the individual circumstances and is called the significance level.

Somebody on trial is assumed innocent until shown to be guilty beyond reasonable doubt; reasonable doubt must mean a very small probability that the person is innocent.

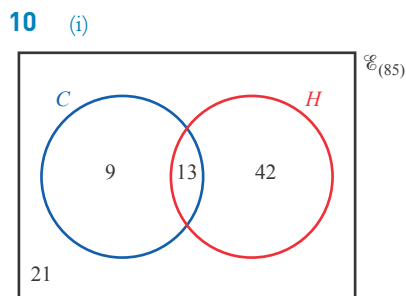
Exercise 15.1 (page 349)

- 1 0.9
- 2 0.42
- 3 0.28
- 4 0.57
- 5 (i) $x = 3, y = 14, z = 11$
 (ii) $\frac{17}{30}, \frac{25}{30}, \frac{28}{30}, \frac{14}{30}$
 (iii) $\frac{28}{30} + \frac{14}{30} = \frac{42}{30}$
 $= \frac{17}{30} + \frac{25}{30}$
 (iv) Since $P(B \cap F) \neq 0$ both B and F can occur. There are nights when both badgers and foxes visit the garden.

- 6 0.6
- 7 (i) The outcome of each attempt is independent of the outcome of all other attempts, *or* the probability of a success is the same on each attempt
 (ii) Jack may have only a limited number of passwords to choose from, in which case (a) he might eliminate some possibilities first, increasing the probability of getting subsequent ones right, *or* (b) he might try the most likely ones first, reducing the probability of getting subsequent ones right. The model in these cases might be improved by building in points (a) and/or (b) as assumptions.

[Other answers are possible]

- 8 Those who are better in general may be more likely to get both questions right. The teacher should attempt to find out how many did get both right, or at least see if the questions were sufficiently similar to suggest that someone who got the easier question right would be very likely to get the harder question right too.



- (ii) (a) $\frac{63}{85} = 0.741$
 (b) $\frac{13}{85} = 0.153$
 (iii) $\frac{64}{85} = \frac{22}{85} + \frac{55}{85} - \frac{13}{85}$
 (iv) $\frac{21}{85} = 1 - \frac{64}{85}$
 (v) 30% of those not wearing helmets suffered from concussion compared with 24% of those wearing helmets. You would expect lower concussion rates among those wearing helmets and this is what the data indicate. However the actual percentages, 30% and 24%, are too close for it to be safe to draw any conclusions on statistical grounds, particularly given the small number of cyclists involved.

Discussion point (page 352)

$$P(T | S) = \frac{109}{169} = 0.645$$

$$P(T | S') = \frac{43}{87} = 0.494$$

So $P(T | S) \neq P(T | S')$

Exercise 15.2 (page 355)

- 1 (i) 0.6
 (ii) 0.556
 (iii) 0.625
 (iv) 0.047
 (v) 0.321
 (vi) 0.075
 (vii) 0.028
 (viii) 0.0022
 (ix) 0.00095
 (x) 0.48
- 2 (i) (a) 0.031
 (b) 0.078
 (c) 0.043
 (d) 0.061
 (e) 0.00086
 (f) 0.11
 (g) 0.014

- (ii) Those sentences for motoring offences would probably have shorter sentences than others so are likely to represent less than 1.5% of the prison population at any time.

3 (i) $\frac{35}{100}$

(ii) $\frac{42}{100}$

(iii) $\frac{15}{65}$

4 (i) $\frac{1}{6}$

(ii) $\frac{5}{12}$

(iii) $\frac{2}{5}$

5 (i) 30

(ii) $\frac{7}{40}$

(iii) $\frac{7}{10}$

(iv) $\frac{7}{15}$

- 6 (i) A and B are independent

- (ii) Older students are more likely to be at least six feet tall than younger ones.

- (iii) $P(B|A) > P(B|A')$, so $P(A \cap B) = P(B|A) \times P(A)$ is likely to be greater than $P(B) \times P(A)$.

- 7 (i)

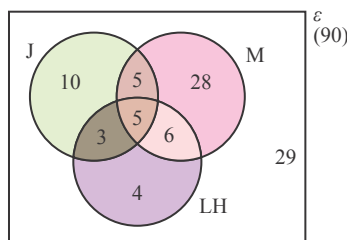
	Hunter dies	Hunter lives	Total
Quark dies	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{1}{2}$
Quark lives	$\frac{2}{12}$	$\frac{1}{3}$	$\frac{1}{2}$
Total	$\frac{1}{4}$	$\frac{3}{4}$	1

(ii) $\frac{1}{12}$

(iii) $\frac{5}{12}$

(iv) $\frac{5}{6}$

- 8 (i)



Key:
J = juniors
M = males
LH = left-handed players

(ii) (a) $\frac{1}{4}$

(b) $\frac{1}{6}$

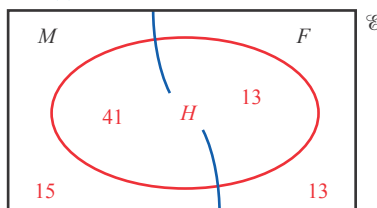
(c) $\frac{28}{45}$

(d) $\frac{4}{5}$

(e) $\frac{19}{24}$

(f) $\frac{10}{39}$

- 9 (i)



(ii) (a) $\frac{26}{82}$

(b) $\frac{54}{82}$

(c) $\frac{13}{54}$

(d) $\frac{13}{26} = \frac{1}{2}$

- (iii) Robin might say, 'Our figures suggest that males are more likely to be wearing a helmet than females. Using them we estimate that the probability that someone is wearing a helmet given that the person is male, $\frac{41}{56} = 0.73$, and the probability that someone is wearing a helmet given that the person is female, $\frac{13}{26} = 0.5$ '

Chapter 16

Opening activity (page 359)

For accurate estimates you must leave your thought experiment behind and go out and collect some real data.

Review exercise (page 360)

1 (i) $X \sim B(100, 0.25)$

(ii) 25 (given by np)

(iii) 0.049

(iv) 0.149

2 (i) $B\left(7, \frac{1}{6}\right)$ where p is the probability of scoring a 6

(ii) ${}^7C_7 \left(\frac{1}{6}\right)^7 \left(\frac{5}{6}\right)^0 =$

0.00000357

(iii) 280 000

3 (i) (a) 0.282

(b) 0.526

(ii) 75

(iii) 0.049

- (iv) They may well be related and so their characteristics (i.e. the colour of their spots) may not be independent.

4 (i) (a) 20

(b) 0.3

(c) 0.7

(ii) It is $1 - 0.952 = 0.048$, so 4.8%

(iii) $a = 3$

5 $P(X \leq 11) = 0.697$

6 (i) 0.392

(ii) 0.556

(iii) 3

- (iv) It must be assumed that the probability of the train being late is the same on each day, and that whether or not the train is late in one day does not influence whether or not it is late on another day. The first assumption may not be unreasonable for a particular time of the year

(but it might be argued that lateness is more likely on certain days of the week, for various reasons). The second is more dubious; if a train is late on one day it may be due to a cause which is also present the next day, such as bad weather, engineering works, track problems, etc.

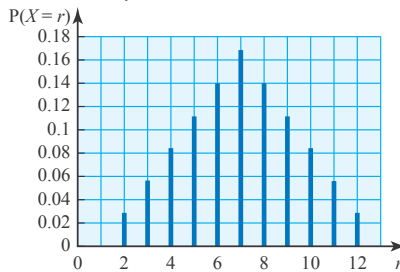
- 7 (i) (a) 0.01659
(b) 0.00369
(c) 0.00039
(d) 0.02067
- (ii) If 70% of this age group are wearing helmets, then the probability of 20 or more out of a sample of 22 wearing helmets is given by the answer to part (i)(d); it is 0.02067. This is indeed small so it is very likely that the true figure is greater than 70%. However, it is not certain, as the organiser claims.
- (iii) 70.4% for the 15 to 29 group and 44% for those over 29. These lower figures suggest that the campaign is influencing the behaviour of young people, particularly children, but is not very effective with older people. However, they are based on the small numbers of cyclists involved in accidents so can only be estimates of the overall figures in Avonford, where the campaign has been held.

Exercise 16.1 (page 366)

1 (i)

r	$P(X = r)$
2	$\frac{1}{36}$
3	$\frac{2}{36}$
4	$\frac{3}{36}$
5	$\frac{4}{36}$
6	$\frac{5}{36}$
7	$\frac{6}{36}$
8	$\frac{5}{36}$
9	$\frac{4}{36}$
10	$\frac{3}{36}$
11	$\frac{2}{36}$
12	$\frac{1}{36}$

- (ii) The distribution is symmetrical.



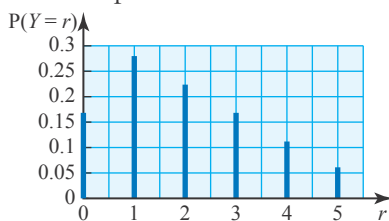
- (iii) (a) $\frac{5}{18}$
(b) $\frac{1}{2}$
(c) $\frac{2}{3}$

- 2 (i) $\frac{1}{20}$
(ii) Uniform distribution
(iii) 0.5

3 (i)

r	0	1	2	3	4	5
$P(Y = r)$	$\frac{3}{18}$	$\frac{5}{18}$	$\frac{4}{18}$	$\frac{3}{18}$	$\frac{2}{18}$	$\frac{1}{18}$

- (ii) The distribution has positive skew.



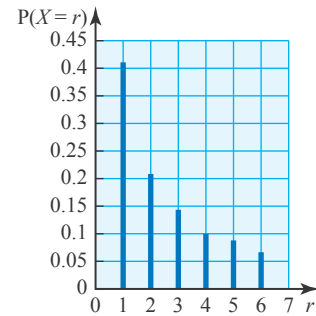
- (iii) (a) $\frac{2}{3}$
(b) $\frac{1}{2}$

4 (i) $k = 0.4$

r	2	4	6	8
$P(X = r)$	0.1	0.2	0.3	0.4

- (ii) (a) 0.3
(b) 0.35

- 5 (i) $k = \frac{20}{49}$

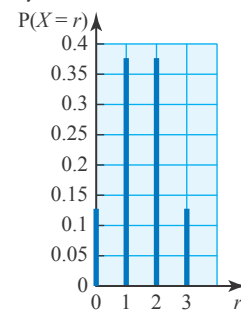


- (ii) 0.248 (to 3 s.f.)

6 (i)

r	0	1	2	3
$P(X = r)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

- (ii) The distribution is symmetrical.



- (iii) $\frac{1}{2}$

- (iv) No. Could get 2H, 2T

7 (i)

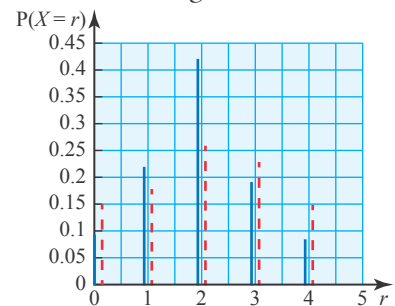
r	1	2	3	4	6	8	9	12	16
$P(X = r)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

- (ii) $\frac{1}{4}$

- 8 (i) $a = 0.42$

- (ii) $k = \frac{1}{35}$

- (iii) Since the probability distributions look quite different, the model is not a good one.



- 9 (i) $P(X = 1) = \frac{1}{216}$

- (ii) $P(X \leq 2) = \frac{8}{216}$

(iii) $P(X \leq 3) = \frac{27}{216},$

$P(X = 3) = \frac{19}{216};$

$P(X \leq 4) = \frac{64}{216},$

$P(X = 4) = \frac{37}{216};$

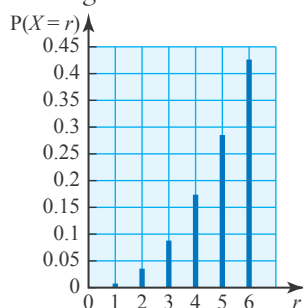
$P(X \leq 5) = \frac{125}{216},$

$P(X = 5) = \frac{61}{216};$

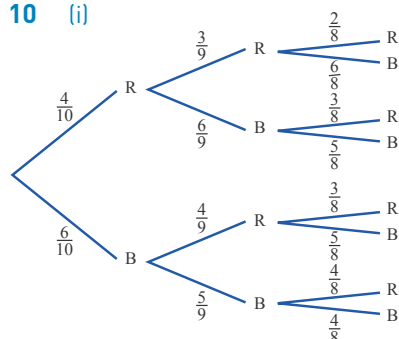
$P(X \leq 6) = 1,$

$P(X = 6) = \frac{91}{216}$

(iv) The distribution has negative skew.



10 (i)



(ii)

r	0	1	2	3
P(X = r)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

11 (i)

Nights	0	1	2	3	4	5	>5	No data
Frequency	58	23	5	2	0	2	2	1

(ii)

$$\begin{aligned} \text{Sum} &= (1-p)(1+p+p^2+\dots) \\ &= (1-p) \times \frac{1}{(1-p)} = 1 \end{aligned}$$

(iii)

Nights	0	1	2	3	4	5	>5
Frequency	59.8	20.9	7.3	2.6	0.9	0.3	0.2

- (iv) They are a good match, but the real data have more high values, including the two outliers, 8 and 40.
- (v) p is the probability that someone who is at hospital in the daytime will be detained overnight, and $(1-p)$ is the probability that the person will be discharged. So if there are N people at hospital on one day, $N(1-p)$ of them will be discharged and Np of them will be kept in overnight. So at the start of the next day Np of the people will be in hospital. Of these $Np(1-p)$ will be discharged and Np^2 will be kept for a further night, and so on. The initial value of N is the number of people in the sample, in this case 92.

Discussion point (page 374)

In both cases the vertical scale is frequency density and so the area under the graph represents frequency.

Exercise 16.2 (page 381)

1 (i) 0.631

(ii) 0.252

(iii) 0.117

2 (i) 0.8413

(ii) 0.0228

(iii) 0.1359

3 74, 86

4 N (5.349, 4.825)

5 (i) 0.0668

(ii) 0.6915

(iii) 0.2417

6 (i) 0.0668

(ii) 0.1587

(iii) 0.7745

7 (i) 78.9%

(ii) 0.5

8 (i) Unlikely to be symmetric; likely to be some very long times not matched by corresponding times less than the mean.

(ii) Complete cover (100%) or cloudless sky (0%) are quite likely and the distribution is not likely to have its mode in the middle. (This is an unusual scenario in which the probability of the individual values 0% and 100% are not zero.)

(iii) You would expect only about 5% of results to be outside the range $(\mu \pm 2\sigma)$.

9 (i) 0.246

(ii) 0.0796

(iii) 0.0179

(iv) The normal distribution is used for continuous data and so covers non-integer values that have to be rounded to find the nearest integer values of the binomial distribution.

10 (i) 0.0038

(ii) 0.5

(iii) 0.495

11 (i) 31, 1.98

(ii) 2.0, 13.5, 34.4, 34.4, 13.5, 2.0

(iii) More data would need to be taken to say reliably that the weights are normally distributed.

12 (i) 57 (0.0575)

(ii) 10 (0.0499)

13 (i) (a) 0.315

(b) 0.307

(ii) Assuming the answer to part (i)(b) is correct, there is a 7.6% error. Worse.

(iii) 0.5245

- 14 (i) The areas of the bars give the numbers of people they represent.
 $14 + 25 + 16 + 10 + 6 + 19 + 1 = 91$
- (ii) It is not symmetrical (positively skewed). It is bimodal.
- (iii)

Age, a years	Frequency density
$0 \leq a < 10$	0.836
$10 \leq a < 20$	1.363
$20 \leq a < 30$	1.743
$30 \leq a < 40$	1.743
$40 \leq a < 50$	1.363
$50 \leq a < 70$	0.618
$70 \leq a < 90$	0.098

- (iv) It would represent
 $8.36 + 13.63 + 17.43 + 17.43 + 13.63 + 12.37 + 1.96 = 84.81$.
 The left hand tail of the normal distribution (i.e. the part for negative ages) has been excluded.
- (v) It would give a partial explanation. However, a normal population with mode in the 60s would inevitably be quite small. Taking it away from the main population would still leave a positive skew.

Chapter 17

Opening activity (page 386)

Medicines are only licensed after undergoing extensive trials supported by rigorous statistical tests.

Review exercise (page 390)

- 1 (i) 2; 1.195
- (ii) $P(2 \text{ defectives in } 10) = 0.302$; in 50 samples of 10, the expected

number of samples with two defectives is 15.1 which agrees well with the observed 15

- (iii) H_0 :
 $P(\text{mug defective}) = 0.2$;
 H_1 :
 $P(\text{mug defective}) < 0.2$;
 $n = 20$

$P(0 \text{ or } 1 \text{ defective mug}) = 0.0692$; Accept H_0 since $0.0692 > 5\%$

It is not reasonable to assume that the proportion of defective mugs has been reduced.

- (iv) Opposite conclusion since $0.0692 > 10\%$

- 2 (i) Let $p = P(\text{business operates no smoking policy})$ $H_0: p = 0.7$,
 $H_1: p < 0.7$
- (ii) $k \leq 10$
- (iii) $k \leq 1$
- (iv) For 19 businesses,
 $P(H_0 \text{ rejected}) = 0.1855$

For 4 businesses,
 $P(H_0 \text{ rejected}) = 0.1265$

The 19 business test is preferable because it gives a greater probability of rejecting H_0 when it should be rejected.

- 3 (i) 0.430
- (ii) 0.9619
- (iii) 0.0046
- (iv) $H_0: p = 0.9, H_1: p < 0.9$
- (v) $n = 17$;
 $P(X \leq 13) = 0.0826 > 5\%$;
 not sufficient evidence to reject H_0 .

- (vi) Critical region is $X \leq 12$, since
 $P(X \leq 12) = 0.0221$

- 4 (i) (a) 0.0278
 (b) 0.0384
- (ii) Let $p = P(\text{blackbird is male})$ $H_0: p = 0.5$,
 $H_1: p > 0.5$

- (iii) Result is significant at the 5% significance level.

Critical region is $X \geq 12$.

- (iv) You would be more reluctant to accept H_1 . Although H_0 is still $p = 0.5$, the sampling method is likely to give a non-random sample.

- 5 (i) (a) 0.0991
 (b) 0.1391
- (ii) Let $p = P(\text{seed germinates})$
 $H_0: p = 0.8$,
 $H_1: p > 0.8$, since a higher germination rate is suspected.
- (iii) Critical region is $X \geq 17$, since
 $P(X \geq 17) = 0.0991 < 10\%$ but
 $P(X \geq 16) = 0.2713 > 10\%$.

- (iv) (a) When $p = 0.8$ he reaches the wrong conclusion if he rejects H_0 , i.e. if $X \geq 17$, with probability 0.0991.
- (b) When $p = 0.82$ he reaches the wrong conclusion if he fails to reject H_0 , i.e. if $X \leq 16$, with probability
 $1 - 0.1391 = 0.8609$.

- 6 (i) 0.0417
- (ii) 0.0592
- (iii) 0.0833
- (iv) 0.1184
- (v) Let $p = P(\text{man selected})$
 $H_0: p = 0.5, H_1: p \neq 0.5$
 $P(X \leq 4 \text{ or } X \geq 11) = 0.1184 > 5\%$

There is not sufficient evidence to reject H_0 , so it is reasonable to suppose that the process is satisfactory.

- (vi) $4 \leq w \leq 11$

- 7 (i) Let p be the proportion of those in this age group having accidents who are boys.
 $H_0: p = 0.5$ Boys and girls of this age group are equally likely to have cycling accidents
 $H_1: p > 0.5$ Boys are more likely than girls to have accidents.
 1-tail test, using binomial $B(20, 0.5)$
 Significance level 10%
 The p -value for the observed number of boys is $P(X \geq 13)$, where $X \sim B(20, 0.5)$ and this is 0.1316.
 Since $0.1316 > 0.10$, the result is not significant. H_0 is accepted.
 The data do not provide sufficient evidence to support Sally's conjecture.
- (ii) Overall 66% of the accidents are males. Among the under-13s the figure is 65%. This suggests that this age group are no different in this respect from people of all ages.
- (iii) The data do not provide evidence as to the cause of the disparity. Further data, directly related to the suggestions, would need to be collected to establish to what extent they are valid.

Activity 17.1 (page 397)

	1-tail	2-tail
10%	1.282	1.645
5%	1.645	1.960
$2\frac{1}{2}\%$	1.960	2.240
1%	2.320	2.576

Exercise 17.1 (page 399)

- 1 (i) $Z = 1.53$, not significant
 (ii) $Z = -2.37$, significant
 (iii) $Z = 1.57$, not significant
 (iv) $Z = 2.25$, significant
 (v) $Z = -2.17$, significant
- 2 (i) 0.3085
 (ii) 0.0062
 (iii) $H_0: \mu = 4.00$ g,
 $H_1: \mu > 4.00$ g
 $z = 3$, significant
- 3 (i) $H_0: \mu = 72.7$ g,
 $H_1: \mu \neq 72.7$ g;
 Two-tail test.
 (ii) $z = 1.84$, not significant
 (iii) No, significant
- 4 (i) $H_0: \mu = 23.9^\circ$,
 $H_1: \mu > 23.9^\circ$
 (ii) $z = 1.29$, significant
 (iii) 4.54; this is much greater than 2.3 so the ecologist should be asking whether the temperature has become more variable.
- 5 (i) You must assume it has a normal distribution.
 (ii) $H_0: \mu = 470$ days,
 $H_1: \mu > 470$ days
 (iii) $z = 3.02$, significant
 (iv) More time to produce offspring
- 6 (i) You must assume that the speeds are normally distributed.
 (ii) $H_0: \mu = 80$ mph,
 $H_1: \mu \neq 80$ mph
 $z = 2.28$, significant
 (iii) Yes: $z = 1.33$, not significant
- 7 (i) You must assume that the visibilities are normally distributed.
 (ii) $H_0: \mu = 14$ sea miles,
 $H_1: \mu < 14$ sea miles
 (iii) $z = -2.284$, significant
 (iv) Choosing 36 consecutive days to collect data is not a good idea because weather patterns will ensure that the data are not independent.
 A better sampling procedure would be to choose every tenth day. In this way the effects of weather patterns over the year would be eliminated.
- 8 (i) 998.6, 7.055
 (ii) $H_0: \mu = 1000$,
 $H_1: \mu < 1000$
 (iii) $z = -1.59$, not significant
- 9 $H_0: \mu = 0$, $H_1: \mu \neq 0$;
 $z = 0.98$, not significant
- 10 (i) 16.2, 5.231
 (ii) $H_0: \mu = 15$, $H_1: \mu > 15$
 (iii) $z = 1.986$, not significant
- 11 (i) 1.977, 0.132
 (ii) $H_0: \mu = 2$, $H_1: \mu < 2$
 (iii) $z = -1.68$, not significant
- 12 (i) 104.7, 3.019
 (ii) $H_0: \mu = 105$,
 $H_1: \mu \neq 105$
 (iii) $z = -0.89$, not significant
- 13 (i) Probability = $\frac{1}{7}$,
 Expectation = 13.143, the distribution is uniform.
 (ii) At the 10% significance level the evidence suggests the distribution is not uniform, but at the 5%, 2.5% and 1% levels the evidence is not strong enough for the hypothesis that the distribution is uniform to be rejected. So it is marginal whether or not the distribution is uniform.
 (iii) Patterns of cycling are different. On weekdays some people cycle to work, and students cycle to school or college. At the weekend people cycle more for leisure.

- (iv) The number in the Avonford data set is 92.

$$\text{Standard deviation} = \sqrt{92 \times \frac{5}{7} \times \frac{2}{7}} = 4.333,$$

$$\text{mean} = 92 \times \frac{5}{7} = 65.714$$

Using continuity correction

$$z = \frac{71.5 - 65.714}{4.333} = 1.3352$$

$$\Phi(1.335) = 0.9090$$

$$\text{Probability} = 1 - 0.9090 = 0.0910$$

(v) $H_0: p = \frac{5}{7}$

$$H_1: p > \frac{5}{7}$$

1-tail test at 5% significance level

Since $0.0910 > 0.05$, the null hypothesis is accepted.

The evidence does not suggest that accidents are more common on weekdays than at weekends.

Discussion point (page 402)

- A** Negative correlation, providing both variables are random
- B** Positive correlation, providing both variables are random
- C** Negative association
- D** Two distinct groups showing neither correlation nor association

Activity 17.3 (page 404)

The calculation in the example followed these steps.

Work out $\sum (x_i - 17)(y_i - 10)$. Then divide it by the number of points. Then divide by the rmsd for x and the rmsd for y .

In this 17 is the mean value of x so can be replaced by $\bar{x}$. Similarly 10 can be replaced by $\bar{y}$. The number of points is n .

The root mean squared deviation of x is given by

$$\text{rmsd} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$

and similarly for y .

So the calculation becomes

$$\frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n} \div \left[\sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} \times \sqrt{\frac{\sum (y_i - \bar{y})^2}{n}} \right]$$

The n cancels out and so this becomes

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \times \sum (y_i - \bar{y})^2}}$$

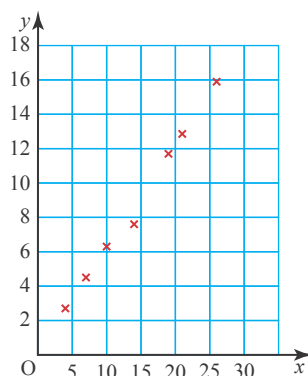
Activity 17.4 (page 409)

This can be done by calculator or by using the method in Example 17.5.

Exercise 17.2 (page 411)

- 1 (i) (a) Negative correlation
(b) Countries with high life expectancy tend to have low birth rates.
- (ii) (a) Positive association
(b) Countries with high GDP per capita tend to have high life expectancy.
- (iii) (a) Negative association
(b) Countries with high GDP per capita tend to have low birth rates.
- 2 (i) (a) Positive association
(b) Both variables are random but the relationship is not linear.
- (ii) (a) Positive association
(b) This is a time series and so correlation

is not appropriate because the time (in this case X) is not random.

- (iii) (a) Neither
(b) The two variables are independent so there is neither correlation nor association.
- (iv) (a) Positive correlation
(b) Both variables are random and the relationship is linear.
- (v) (a) Neither
(b) X is controlled and so not random and the value of Y is determined by it so is not random either.
- (vi) (a) Negative correlation
(b) Both variables are random but the relationship is linear.
- 3 (i) Points lie fairly close to a straight line with positive gradient
(ii) Points lie very close to a straight line with negative gradient
(iii) Points do not lie close to a straight line
- 4 (i) 
(ii) (a) $(14, 7.6)$
(b) ≈ 1
(c) ≈ 0.99
(it is in fact 0.996)
- 5 (i) Positive correlation
(ii) $\bar{x} = 8, \bar{y} = 6$
- 6 Sachin is not correct. The data clearly belong to two distinct groups and so it is

not appropriate to calculate a correlation coefficient. In fact there is no significant correlation in either of the two groups; if you work out the values of r , they are 0.316 for $x < 15$ and -0.1 for $x > 15$.

- 7** (i) $H_0: \rho = 0, H_1: \rho > 0$
 (ii) Accept H_1 . The evidence suggest that there is a positive correlation between performance in the high jump and the long jump.
- 8** (i) $H_0: \rho = 0, H_1: \rho < 0$
 (ii) Accept H_1
 (iii) Correlation does not imply causation. There may be other explanations for the high divorce rates in some countries and the low rates in others. Perhaps Charlotte should collect some data herself to highlight the dangers of drinking alcohol, e.g. wine consumption/liver disease.
- 9** (ii) $H_0: \rho = 0, H_1: \rho > 0$
 (iii) Accept H_1
 (iv) Giving more training to employees does tend to keep staff with the company.
- 10** The chart suggests that a single line is not best for the whole data. It seems that the relationship is approximately linear up to a certain value ν_0 of ν but for $\nu > \nu_0$ it becomes non-linear (or possibly linear with a different gradient). The values of r suggest that ν_0 is around 25 or 30.
- 11** Quantity is > 1511.5 or < -1511.5 .
- 12** (i) Outliers have been considered. Missing data items have been excluded.
 (ii) Both are random.

- (iii) There are places where several points lie on straight horizontal lines corresponding to whole numbers of kilometres. Ages are rounded down to the nearest integer.
- (iv) The independent variable is Age and the dependent variable is Distance from home. It is reasonable to think that people cycle further from home because they are older. It is not reasonable to think that people are older because they cycle further from home.
- (v) H_0 : There is no association between the distance from home at which people have accidents and their ages. H_1 : There is a positive association between the distance from home at which people have accidents and their ages. The p -value provides extremely strong evidence in favour of the alternative hypothesis. There is virtually no chance of seeing data like these unless there is an association in the underlying population.
- (vi) The scatter diagram suggests that the data are not drawn from a bivariate normal distribution; the departure from normality looks clear and considerable. Consequently, no useful interpretation can be made of the value of the product-moment

correlation coefficient. Knowing the value of the pmcc can add nothing new to Robin's understanding of the situation.

Practice questions: Statistics (page 417)

- 1** (i) Only 5 countries show a value over 50 minutes, and 24 countries show a value below. Of the two countries with the biggest populations, India is considerably above 50 minutes and China is just below. It isn't possible to be certain, but the figure may be plausible, particularly if it is rounded. [2]
 (ii) 50 minutes per day is about 304 hours per year. This is roughly 39 days of 7.8 hours, so 39 working days. [2]
- 2** It should say 'in a normal distribution (about) 95% of the distribution is within 2 standard deviations of the mean' (or about $\frac{2}{3}$ within 1 standard deviation). [1]
 Replace 2σ with 4σ (or change the percentage) in the diagram. [1]
- 3** (i) There may be fewer slugs to be seen in the winter months. There may be fewer observers looking for slugs in the winter months. [2]
 (ii) 93 (in month 8) is an anomalous dip. It could be just randomness in the sampling. Or, noting that the other observations in month 8 are low, it could be that

there are fewer observers in August: perhaps they are on holiday. [2]

- (iii) The rise and fall in the figures is consistent with an annual life cycle. However the low figures are also consistent with other hypotheses – for example that slugs hibernate in the winter – so it isn't possible to say. [2]

- 4 (i) Use $X \sim B(10, 0.65)$ to find $P(X > 5) = 0.751$. [2]

- (ii) $H_0: p = 0.65$ where p is the probability of choosing the quickest check-out when there are three available.

$$H_1: p < 0.65 \quad [2]$$

- (iii) $P(X < 8) = 0.0060$,
 $P(X < 9) = 0.020$, so
 $k = 0, 1, 2, \dots, 7$. [3]

- (iv) The first 20 visits might be sufficiently random if, for example, they were at different times of day. However, patterns of behaviour, by staff and customers, might be different immediately after opening a third check-out. Also, it is possible that the person asking the question is still learning in the first 20 trials. [2]

- 5 (i) (a) $0.4 \times 0.7 \times 0.2$
 $+ 0.4 \times 0.3 \times 0.8$
 $+ 0.6 \times 0.7 \times 0.8$
 $= 0.488$ [2]

$$(b) 1 - 0.6 \times 0.3 \times 0.2 = 0.964 \quad [2]$$

- (iii) $P(\text{successful on all 3})$
 $= 0.4 \times 0.7 \times 0.8 = 0.224$
 $P(\text{successful on all 3} \mid$
 $\text{successful on at least 1})$

$$= \frac{P(\text{successful on all 3} \cap \text{successful on at least 1})}{P(\text{successful on at least 1})}$$

$$= \frac{0.224}{0.964} = 0.232 \quad [3]$$

- 6 (i) $4 = \mu + 1.0364\sigma$ and
 $2 = \mu - 0.8416\sigma$ [2]

$$\text{Solve: } \mu = 2.90$$

$$\text{and } \sigma = 1.06 \quad [2]$$

- (ii) $H_0: \mu = 3.2$, where μ is the population mean yield for variety B
 $H_1: \mu < 3.2$ [2]

- (iii) Sample mean is 3.06 kg. Critical ratio is
 $\frac{3.06 - 3.2}{0.9 \div \sqrt{100}} = -1.556$ [2]

$$\text{Compare with } z = -1.645 \quad [1]$$

Accept H_0 ; insufficient reason to suppose his yield will be below 3.2. [1]

- (iv) Assumption: the SD is still 0.9. This is the best information available, but the SD for the farmer's crop could be different. [2]

- (v) Assumption: the farmer's crop is a random sample (of typical conditions on his land). If, for example, all the trial plants were grown in the same location this might be untypical of the land as whole. In that case the mean yield is not a reliable estimate. [2]

- 7 (i) $k\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) = 1$,
so $k = \frac{60}{125} = \frac{12}{25}$. [2]

- (ii) Vertical line chart. [1]

$$(iii) P(X_1 = X_2) = k^2 \left(\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16}\right) = \frac{41}{125} \quad [2]$$

$$P(X_1 < X_2) = \frac{1}{2} \left(1 - \frac{41}{125}\right) = \frac{42}{125} \quad [2]$$

- 8 (i) Mean wing lengths differ by about 2 SDs; mean weights differ by about $\frac{1}{4}$ of an SD. So there is much more overlap of weights than there is of wing lengths. So the comment is correct. [3]

- (ii) Females' weights will vary according to whether or not they have recently laid eggs. Males' weights will not have this element of variation. [1]

- (iii) The vertical 'striping' in the data corresponds to the wing lengths being recorded to the nearest millimetre. [1]

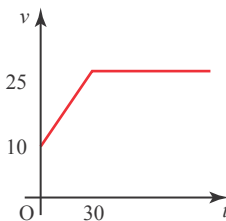
- (iv) The data cloud looks broadly elliptical. So it is appropriate to carry out the standard test on the pmcc. [2]

- (v) There is almost no chance (probability less than 1 in 10^6) that so strong a correlation would be obtained in a sample of this size if there were no underlying correlation in the population. [2]

- (vi) It seems very likely that there would be strong evidence for a correlation in the male blackbirds too. However, it is not certain. [1]
It would *not* have been sensible to work with males and females as a single sample. Combining two separate samples can often lead to spurious or misleading results. [1]

Chapter 18

Review exercise (page 429)

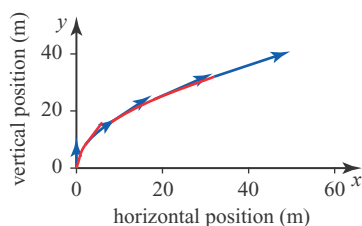
- 1 (i) 
 - (ii) 30 s
 - (iii) 2275 m
 - (iv) $v = 10 + 0.5t; 0 \leq t \leq 30$
 $v = 25; t \geq 30$
- 2 (i) 3 ms^{-1}
(ii) 2.25 ms^{-1}
- 3 (i) 200 m
(ii) 30 s
(iii) 150 m
- 4 (i) 1.875 ms^{-2}
(ii) 14.6 ms^{-1}
- 5 (i) 3 ms^{-1}
(ii) 50 m
(iii) 9.6 ms^{-2}
(iv) 35 m
(v) 5.25 m
- 6 78.4 m
- 7 (i) 4.11 s
(ii) -20.2 ms^{-1}
- 8 16 s
- 9 (i) $3.5 \text{ ms}^{-1}, -3 \text{ ms}^{-2}$
(ii) $21\frac{1}{3} \text{ m}$
- 10 (i) $v = 3t^2 - 8t + 6$;
 $s = t^3 - 4t^2 + 6t + 4$
(ii) $v = -10t + 12$;
 $s = -5t^2 + 12t - 2$

Discussion point (page 433)

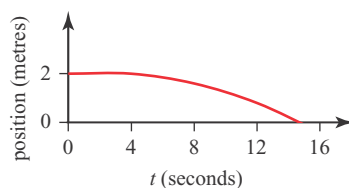
$y = 0$ when $t = 2.4$ and then the projectile hits the ground.

Exercise 18.1 (page 438)

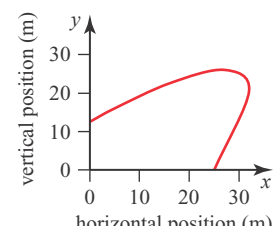
- 1 (i) $\mathbf{v} = 4t\mathbf{i} + 8\mathbf{j}$
(ii)



- 2 $\mathbf{v} = \begin{pmatrix} -4 \\ -5 \end{pmatrix}; \mathbf{a} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$
- 3 $4.47 \text{ ms}^{-2}; -153^\circ$
- 4 (i) $\frac{1}{20}t^2\mathbf{i} + \frac{1}{30}t^3\mathbf{j}$
(ii) $5\mathbf{i} + 33\frac{1}{3}\mathbf{j}$
- 5 $\mathbf{v} = 2t^2\mathbf{i} + (6t - t^2)\mathbf{j}$;
 $\mathbf{r} = \frac{2}{3}t^3\mathbf{i} + (3t^2 - \frac{1}{3}t^3)\mathbf{j}$
- 6 15.3°
- 7 8.11 ms^{-1}
- 8 (i) initial velocity =
 $3.54\mathbf{i} - 3.54\mathbf{j}$
(ii) $\mathbf{v} = 8.54\mathbf{i} + 11.46\mathbf{j}$;
 $\mathbf{r} = 52.0\mathbf{i} + 14.6\mathbf{j}$
- 9 (i) $\mathbf{v} = \begin{pmatrix} 15 \\ 16 - 10t \end{pmatrix}$;
 $\mathbf{a} = \begin{pmatrix} 0 \\ -10 \end{pmatrix}$
(ii) 1.6 s
(iii) 22.8 ms^{-1}
(iv) $y = 2 + \frac{16}{15}x - \frac{1}{45}x^2$
- 10 (i)



- (0, 2); (4, 2); (8, 1.6);
(12, 0.8); (16, -0.4)
- (ii) $\mathbf{v} = 20\mathbf{i} + (1 - 10t)\mathbf{j}$;
 $20\mathbf{i} - \mathbf{j}$
- (iii) $-10\mathbf{j}$
- (iv) $y = 2 + \frac{1}{20}x - \frac{1}{80}x^2$

- 11 (i) 
 - (ii) 5 s
 - (iii) $33.5 \text{ ms}^{-1}, 63.4^\circ$
(or 116.6°)
 - (iv)
 $\mathbf{a} = (12 - 6t)\mathbf{i} + (9 - 6t)\mathbf{j}$
is never 0
- 12 (i) 16.9 ms^{-1}
(ii)
 $\mathbf{r} = \begin{pmatrix} 2 - 3t + t^2 \\ -5 + 2t - 0.5t^2 \\ 8 + 2t^2 \end{pmatrix}, \begin{pmatrix} 6 \\ -5 \\ 40 \end{pmatrix}$
- 13 (i) A: $v \sin 35^\circ\mathbf{i} + v \cos 35^\circ\mathbf{j}$;
B: $-8.66\mathbf{i} + 5\mathbf{j}$
(ii) A: $vt \sin 35^\circ\mathbf{i} + vt \cos 35^\circ\mathbf{j}$;
B: $(5 - 8.66t)\mathbf{i} + 5\mathbf{j}$
(iii) 6.1 km h^{-1}
(iv) $0.4111\dots$ hours =
24.7 min
- 14 $20\mathbf{i} + 10\mathbf{j}; T = 5 \text{ s}; 125 \text{ m}$
- 15 (i) 16 cm
(ii) 20 cm
(iii) $0 \text{ cm s}^{-1}, 0 \text{ cm s}^{-1}$;
 $20 \text{ cm s}^{-1}, -16 \text{ cm s}^{-1}$
(iv) $t = 0, \mathbf{r}_A = \mathbf{r}_B = 16\mathbf{j}$;
 $t = 2, \mathbf{r}_A = \mathbf{r}_B = 20\mathbf{i}$
(v) All components are zero,
so model B is better.
- 16 (i) $2\mathbf{i} + \mathbf{j} + \mathbf{k}$
(ii) $(2t - 5)\mathbf{i} + (t + 1)\mathbf{j} + t\mathbf{k}$
(iii) 1.5 s; 3.5 m
- 17 $x = 6 - 4y + y^2$
- 18 $\mathbf{r} = (2 - \sin \omega t)\mathbf{i} + 2 \cos \omega t \mathbf{j}$;
 $+(\sin \omega t - 3)\mathbf{k}$;
 $\mathbf{v} = -\omega \cos \omega t \mathbf{i} - 2\omega \sin \omega t \mathbf{j}$;
 $+\omega \cos \omega t \mathbf{k}$

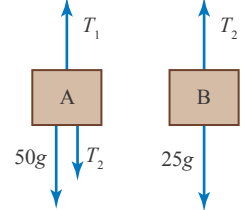
Chapter 19

Opening activity (page 443)

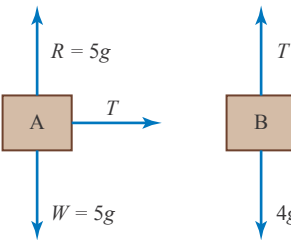
There are a number of forces acting on the car which cancel each other out, resulting in no motion. In order for that to be possible the cable must make small angles with the horizontal so that the vertical components of the tension cancel out the weight of the cable car. In that case the tensions in the cable will be greater than the weight of the car.

Review exercise (page 451)

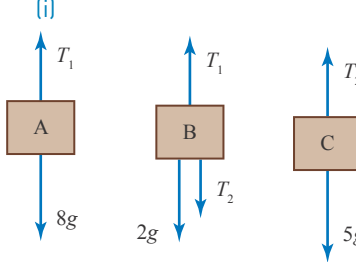
- 1 (i) 0.3 ms^{-2}
(ii) 500 ms^{-2}
- 2 65 N
- 3 5 s
- 4 (i) 3 ms^{-2} ; $2\frac{2}{3} \text{ s}$
(ii) 1440 N
- 5 (i) 29.5 N
(ii) 19.5 N
(iii) 24.5 N

- 6 (i)
 

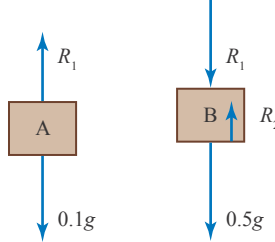
(ii) A: $50g + T_2 - T_1 = 100$
B: $25g - T_2 = 50$
(iii) 585 N , 195 N

- 7 (i)
 

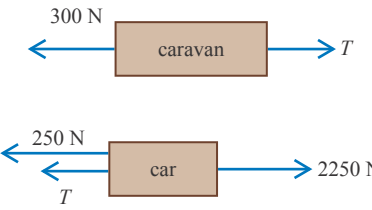
(ii) A: $T = 5a$;
B: $4g - T = 4a$
(iii) A 4.36 ms^{-2} horizontally to right, B 4.36 ms^{-2} vertically downwards

- 8 (i)
 

(ii) A: $8g - T_1 = 8a$;
B: $T_1 - T_2 - 2g = 2a$
C: $T_2 - 5g = 5a$
(iii) A 0.653 ms^{-2} vertically downwards. Both B and C 0.653 ms^{-2} vertically upwards

- 9 (i)
 

(ii) A: $R_1 - 0.98 = 0.08$;
B: $R_2 - R_1 - 4.9 = 0.4$
(iii) $R_1 = 1.06 \text{ N}$,
 $R_2 = 6.36 \text{ N}$

- 10 (i)
 

(ii) 1 ms^{-2}
(iii) 1100 N
- 11 (i) $2t^2\mathbf{i} + 4t\mathbf{j}$
(ii) $\frac{2}{3}t^3\mathbf{i} + 2t^2\mathbf{j}$
(iii) $\frac{1}{6}t^4\mathbf{i} + \frac{2}{3}t^3\mathbf{j}$
(iv) speed 9.61 ms^{-1} ; 56.3° to $\mathbf{i}$ direction.
- 12 (i) $18000 - 900 = 90000a$
 $\Rightarrow a = 0.19$
(ii) 4000 N
(iii) 8000 N
(iv) -0.21 ms^{-2}
(v) -4000 N (thrust)

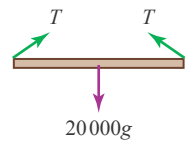
Discussion point (page 456)

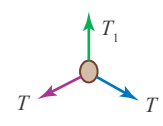
Draw a vertical line to represent the weight, $10g = 98 \text{ N}$. Then add the line of the force T_2 at 45° to the horizontal (note the length of this vector is unknown), and then the line of the force T_1 at 30° to the horizontal (60° to the vertical). C is the point at which these lines meet.

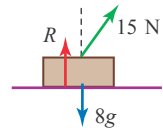
Discussion point (page 457)

The angles in the triangle are $180^\circ - \alpha$, $180^\circ - \beta$ and $180^\circ - \gamma$. The sine rule holds and $\sin(180^\circ - \alpha) = \sin \alpha$, $\sin(180^\circ - \beta) = \sin \beta$ and $\sin(180^\circ - \gamma) = \sin \gamma$.

Exercise 19.1 (page 459)

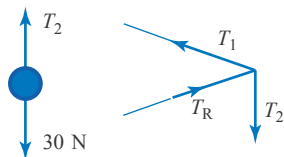
- 1 (i) 30 N , 36.9° ; 65 N , 67.4°
(ii) $\begin{pmatrix} 49 \\ 78 \end{pmatrix}$; 92.1 N , 57.9°
- 2 (i)
 

(ii) $T \cos 40^\circ$, $T \sin 40^\circ$
(iii) $T = \frac{20000 \times 9.8}{2 \times \sin 40^\circ} = 152460.9 \text{ N}$
(iv)
 

(v) 196000 N
(vi) Resolve vertically for whole system. It is the same as the weight of the girder.
- 3 (i) 78.4 N
(ii)
 

(iii) 66.1 N , 8.60 N

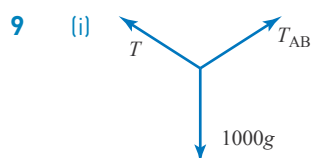
- 4 9.04 N
5 5.39 N; 21.8°
6 (i)



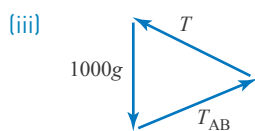
- (ii) Rod: 56.4 N, compression, Cable 1: 59.1 N, tension

- 7 4.40 N, 3.26 N

- 8 (i) 15.04 kg
(ii) Both read 10 kg
(iii) Both read 7.64 kg
(iv) Method A or C



- (ii) A force to the right is required to balance the horizontal component of T .



- (iv) (a) 9800 N, 13859 N, 9800 N
(b) 9800 N, 9800 N, 9800 N

- 10 (i) $\begin{pmatrix} 60 \cos 75^\circ \\ -60 \sin 75^\circ \end{pmatrix}$
 $\begin{pmatrix} 15.5 \\ -58.0 \end{pmatrix}$

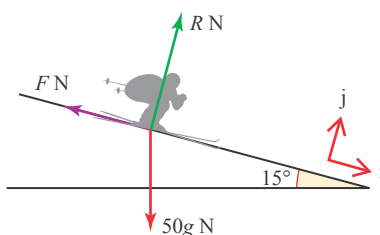
- (ii) (a) $\begin{pmatrix} 56.1 \\ 61.2 \end{pmatrix}$

- (b) 83.0 N, 47.5°

- (iii) 30.8 N, -121°

- 11 (i) Cable 1 (5638 N, 2052 N); Cable 2: ($T_2 \cos 30^\circ$, $T_2 \sin 30^\circ$)
(ii) 4104 N
(iii) 9193 N

- 12 (i)

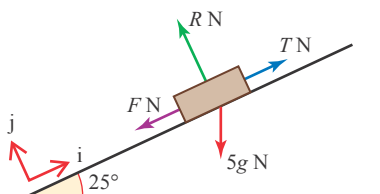


- (ii) $-F\mathbf{i}$, $R\mathbf{j}$, $127\mathbf{i} - 473\mathbf{j}$

- (iii) 473 N, 127 N

- (iv) 254 N

- 13 (i)



- (ii) $T\mathbf{i}$, $-F\mathbf{i}$, $R\mathbf{j}$, $-20.7\mathbf{i} - 44.4\mathbf{j}$

- (iii) $T = 29.4$ N, 8.69 N

- (iv) 1.23 kg

- 14 (i) 42.4 N, 24.5 N

- (ii) 45.2 N, 39.9 N

- 15 (i) 11.0 N, 63.4°

- (ii) A circle centre A radius 1 m. No, 2 parallel forces and a third non-parallel force cannot form a triangle.

- 16 (i) The wall can only push outwards.

- (ii) $T \sin 35^\circ = R \sin \alpha$,
 $T \cos 35^\circ + R \cos \alpha = 80g$

- (iii) $T = 562.92 \dots$,
 $R = 457$ N

- (iv) 563 N down along the rope

- (v) 563 N

- (vi) 1074 N

Discussion point (page 464)

Parallel to the slope, up the slope.

Discussion point (page 464)

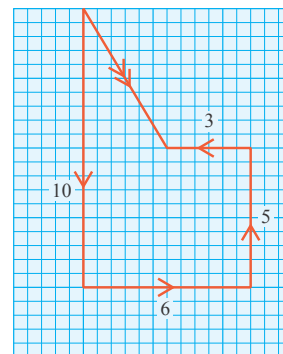
Start with AB and BC. Then draw a line in the right direction for CD and another perpendicular line through A. These lines meet at D.

Discussion point (page 464)

- (i) The sledge accelerates up the hill.
(ii) The sledge is stationary or moving with constant speed (forces in equilibrium).
(iii) The acceleration is downhill.

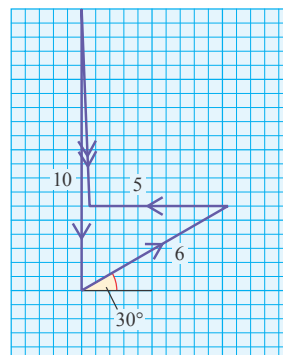
Exercise 19.2 (page 467)

- 1 (i)



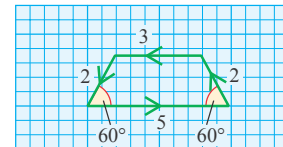
- (iii) $3\mathbf{i} - 5\mathbf{j}$; 5.83 N, -59°

- 2 (i)



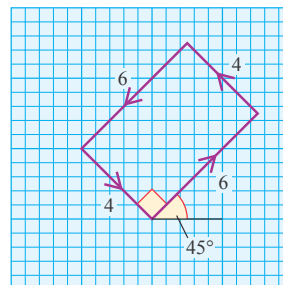
- (iii) $0.196\mathbf{i} - 7\mathbf{j}$;
7.00 N, -88.4°

- 3 (i)

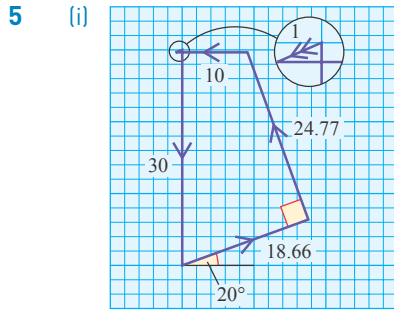


- (iii) Equilibrium

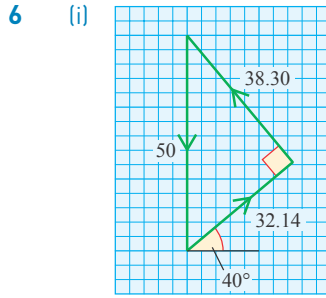
- 4 (i)



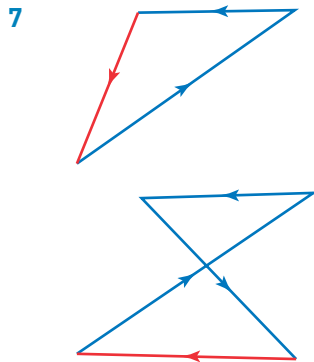
- (iii) Equilibrium



(iii) $-\mathbf{i}$; 1 N down incline



(iii) Equilibrium



8 (i) $\mathbf{D} = \begin{pmatrix} 8 \\ -11 \end{pmatrix} \text{N}$
 (iii) 13.6 N, on a bearing of 144°

9 5.73 N; -49.7°

10 (i) $6.04\mathbf{i} + 17.40\mathbf{j}$
 (ii) 18.4 N, 70.9°

11 6.79 N and 3.92 N

12 $3.77\mathbf{i} + 2.60\mathbf{j}$; 4.58 N, 34.6°

13 $P\sqrt{3}$

Discussion point (page 469)

Down the slope.

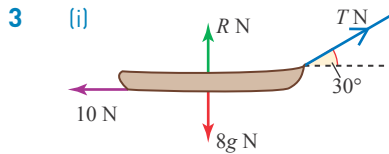
Discussion point (page 471)

Sam and the sledge are a particle. There is no friction and the slope is straight. Friction would reduce both accelerations so Sam would not travel as far on either leg of his journey.

Exercise 19.3 (page 473)

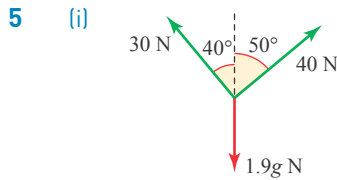
1 (i) $1.5\mathbf{i} - \mathbf{j}$
 (ii) 1.80 m s^{-2}

2 (i) $4\mathbf{i} + 11\mathbf{j}$
 (ii) $8\mathbf{i} + 8\mathbf{j}$, $2\mathbf{i} + 2\mathbf{j}$

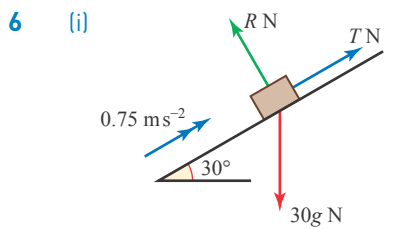


(ii) 11.55 N
 (iii) 1 m s^{-2}
 (iv) 0.4 s

4 (i) The 12 kg block: magnitude 0.368 m s^{-2} down the slope, the 7 kg block 0.368 m s^{-2} vertically upwards
 (ii) 71.2 N



(ii) 11.4 N, 30.1°
 (iii) 16.9 m s^{-2} at 69°
 (iv) The fish swings sideways as it moves up towards Jones.

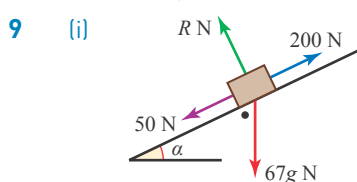


(ii) $T\mathbf{i}$, $R\mathbf{j}$, $-30g \sin 30^\circ \mathbf{i} - 30g \cos 30^\circ \mathbf{j}$
 (iii) 169.5 N

(iv) The crate slows down to a stop and then starts sliding down the slope.

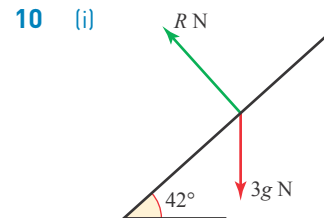
7 3.36 m s^{-1} , 0.1 N in compression

8 0.71 m s^{-2} , 600 N



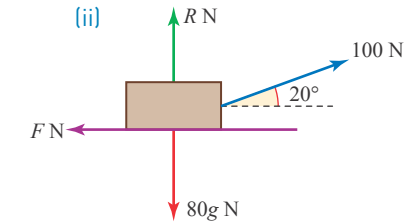
(ii) 13.2°

(iii) 0.87 m s^{-2} , 35.9 m
 (iv) 9.37 m s^{-1}

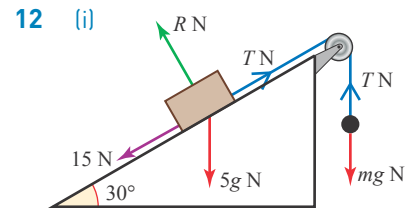


(ii) 6.56 m s^{-2}
 (iii) 1.75 s
 (iv) 13.0 N

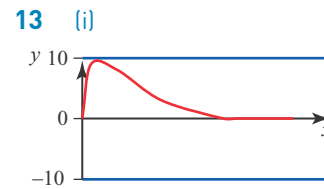
11 (i) Horizontal component of tension in the rope needs a balancing force.



(iii) 94.0 N, 750 N
 (iv) 128 N
 (v) 0.144 m s^{-2}



(ii) 4.03 kg, 39.5 N
 (iii) 1.75 m s^{-2} , 48.3 N



No it doesn't, maximum $y = 9.48$ when $t = 2.67$

(ii) $\mathbf{v} = 8t\mathbf{i} + \left(8 - 4t + \frac{3}{8}t^2\right)\mathbf{j}$;

$\mathbf{a} = 8\mathbf{i} + \left(-4 + \frac{3}{4}t\right)\mathbf{j}$;

$0 \leq t \leq 8$

$\mathbf{v} = 64\mathbf{i}$; $\mathbf{a} = 0\mathbf{i} + 0\mathbf{j}$;

$8 < t \leq 20$

(iii) $1200\mathbf{i} + (-600 + 112.5t)\mathbf{j}$

(iv) Because of air resistance

(v) The motorcycle leaves the ground

- 14 5.10 m
 15 $a = -0.121 \text{ m s}^{-2}$
 $\Rightarrow s = 413 \text{ m}$
 16 319 m

Chapter 20

Discussion point (page 480)

The tool shown on the left of Figure 20.8 works with one hand but has less leverage than the tool on the right.

Discussion point (page 483)

The man is standing with two feet on the plank, on either side of the 0.5 m mark; it is modelled as if he were standing on one foot at 0.5 m.

The points of contact between the plank and the ground are not, in fact, single points but they are modelled as being so.

Discussion point (page 485)

- (i) $P + Q$ line of action parallel to P and Q and in same direction; distance from O is $a + \frac{bQ}{(P+Q)}$ (between P and Q).
- (iii) $P - Q$ line of action parallel to P and Q and in direction of larger; distance from O is $a - \frac{bQ}{(P-Q)}$ (to the left of P for $P > Q$).

Discussion point (page 485)

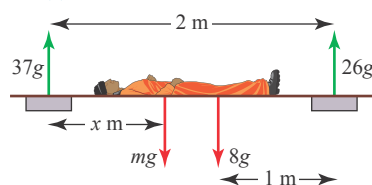
You produce equal and opposite couples using friction between the other hand and the jar so that they turn in opposite directions. Pressing increases the normal contact forces and hence the maximum friction possible.

Discussion point (page 486)

The working is simpler using A because two of the forces pass through it and so have no moment about it.

Exercise 20.1 (page 487)

- 1 (i) 15 Nm
 (ii) -22 Nm
 (iii) 18 Nm
 (iv) -28 Nm
 2 (i) 2.1 Nm
 (ii) 6.16 Nm
 (iii) -0.1 Nm
 (iv) 0.73 Nm
 3 28.6 N; 20.4 N
 4 (i) 1225 N, 1225 N
 (ii) 1449 N, 1785 N.
 5 (i) $P = \frac{75}{8}g \approx 91.9 \text{ N}$
 (ii) $P = \frac{175}{8}g \approx 214 \text{ N}$
 6 96.5 N, 138.5 N.
 7 (i)



- 55 kg
 (ii) 0.8 m
 8 (i) $P = 27.5g$, $Q = 147.5g$
 (ii) $P = 2.5g$, $Q = 172.5g$
 (iii) If child is less than 0.95 m from the adult, $P < 0$ so the bench tips unless A is anchored to the ground.
 (iv) The bench tips if A is not anchored.
 9 (i) 15 g N, 30 g N
 (ii) 90 g, 5 g
 (iii) 0
 (iv) $\frac{2}{3} \text{ m}$
 10 (i) $0.5g(30 - x) \text{ kN}$;
 $0.5g(20 + x) \text{ kN}$
 (ii) Its centre of mass
 (iii) Constant 15 g kN each
 11 Tension = $5g \approx 49 \text{ N}$
 12 (i) There will be no horizontal force equal and opposite to the reaction from the wall.
 (ii) $T = \frac{15}{16}g \approx 9.19 \text{ N}$
 13 6 g; $9\frac{1}{3} \text{ cm}$
 14 $20\frac{5}{7} \text{ kg}$
 15 (i) 35 g N, 75 g N

- (ii) No
 (iii) 36 kg
 16 2 N upwards, 9 m to the left of A .
 17 (i) 2262 N, 7538 N
 (ii) 6
 (iii) 784 N
 18 (i) 3600 N
 (ii) 0.0017 m s^{-2}
 (iii) 195 N

Chapter 21

Discussion point (page 492)

- (i) Not powered, and mass does not change.
 (ii) Modelled as a particle, and no air resistance.
 (iii) No air resistance, and perhaps modelled as a particle.
 (iv) Gravity is constant and no air resistance (and perhaps others).

Activity 21.1 (page 494)

- (i) (a) $v_y = 0$
 (b) $y = 0$
 (ii) These remain the same throughout.

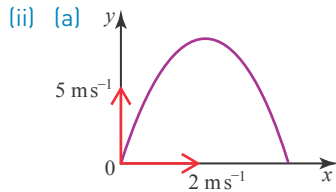
Discussion point (page 495)

- 1 Yes for a parabolic path.
 $u_y - gt = 0$ when $t = \frac{u_y}{g}$
 and $u_y t - \frac{1}{2}gt^2 = 0$ when $t = 2\frac{u_y}{g}$.
 2 The balls and the bullet can be modelled as projectiles when there is no spin or wind and air resistance is negligible. Also a rocket with no power. The air affects the motion of the others.

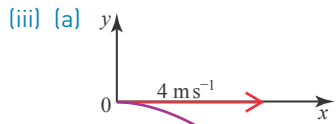
Exercise 21.1 (page 495)

- 1 (i) (a)
 (b) $u_x = 8.2$, $u_y = 5.7$

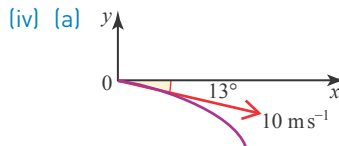
(c) $v_x = 8.2$,
 $v_y = 5.7 - 9.8t$
 (d) $x = 8.2t$,
 $y = 5.7t - 4.9t^2$



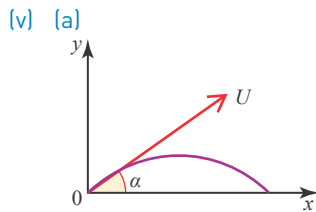
(b) $u_x = 2$, $u_y = 5$
 (c) $v_x = 2$,
 $v_y = 5 - 9.8t$
 (d) $x = 2t$,
 $y = 5t - 4.9t^2$



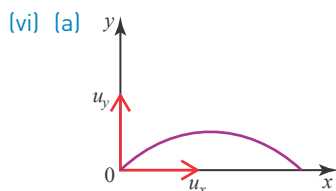
(b) $u_x = 4$, $u_y = 0$
 (c) $v_x = 4$, $v_y = -9.8t$
 (d) $x = 4t$, $y = -4.9t^2$



(b) $u_x = 9.74$,
 $u_y = -2.25$
 (c) $v_x = 9.74$,
 $v_y = -2.25 - 9.8t$
 (d) $x = 9.74t$,
 $y = -2.25t - 4.9t^2$



(b) $u_x = U \cos \alpha$,
 $u_y = U \sin \alpha$
 (c) $v_x = U \cos \alpha$,
 $v_y = U \sin \alpha - gt$
 (d) $x = Ut \cos \alpha$,
 $y = Ut \sin \alpha - \frac{1}{2}gt^2$



(b) u_x , u_y
 (c) $v_x = u_x$,
 $v_y = v_x - gt$
 (d) $x = u_x t$,
 $y = v_x t - \frac{1}{2}gt^2$

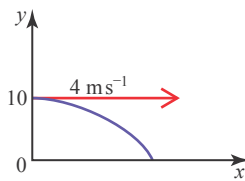
2 (i) (a) 1.5 s
 (b) 11.025 m
 (ii) (a) 0.51 s
 (b) 1.28 m

3 (i) (a) 4 s
 (b) 80 m
 (ii) (a) 0.88 s
 (b) 2.21 m

4 (i) $\frac{u_y^2}{2g}$
 (ii) $\frac{2u_y}{g}$
 (iii) $\frac{2u_x u_y}{g}$

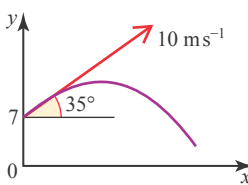
Exercise 21.2 (page 499)

1 (i) (a)



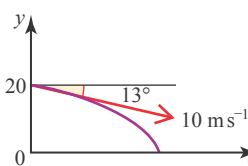
(b) $\mathbf{v} = \begin{pmatrix} 4 \\ -9.8t \end{pmatrix}$
 (c) $\mathbf{r} = \begin{pmatrix} 4t \\ 10 - 4.9t^2 \end{pmatrix}$

(ii) (a)



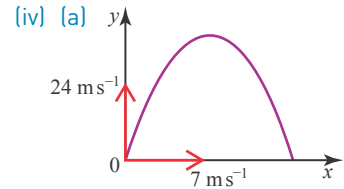
(b) $\mathbf{v} = \begin{pmatrix} 8.2 \\ 5.7 - 9.8t \end{pmatrix}$
 (c) $\mathbf{r} = \begin{pmatrix} 8.2t \\ 7 + 5.7t - 4.9t^2 \end{pmatrix}$

(iii) (a)



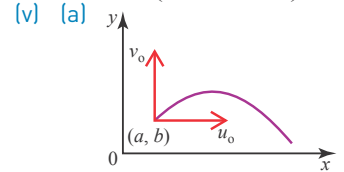
(b) $\mathbf{v} = \begin{pmatrix} 9.74 \\ -2.25 - 9.8t \end{pmatrix}$

(c) $\mathbf{r} = \begin{pmatrix} 9.74t \\ 20 - 2.25t - 4.9t^2 \end{pmatrix}$



(b) $\mathbf{v} = \begin{pmatrix} 7 \\ 24 - 9.8t \end{pmatrix}$

(c) $\mathbf{r} = \begin{pmatrix} 7t \\ 24t - 4.9t^2 \end{pmatrix}$

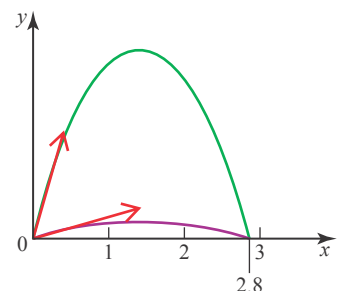


(b) $\mathbf{v} = \begin{pmatrix} u_0 \\ v_0 - gt \end{pmatrix}$

(c) $\mathbf{r} = \begin{pmatrix} a + u_0t \\ b + v_0t - \frac{1}{2}gt^2 \end{pmatrix}$

2 (i) (a) 1.5 s
 (b) 26.025 m
 (ii) (a) 0.31 s
 (b) 10.46 m

3 (i) 2.86 m
 (ii) 2.86 m
 (iii)



4 $T = \frac{u \sin \theta + \sqrt{u^2 \sin^2 \theta + 2gh}}{g}$;

$R = \frac{u^2 \sin \theta \cos \theta}{g} + \frac{u \cos \theta}{g} \sqrt{u^2 \sin^2 \theta + 2gh}$

Discussion point (page 504)

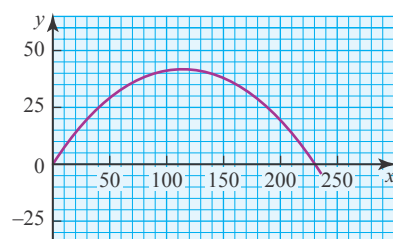
They land together because u , s and a in the vertical direction are the same for both.

Exercise 21.3 (page 504)

- 1 (i) 17.3 m s^{-1} , 10 m s^{-1}
(ii) 0 , -9.8 m s^{-2}
(iii) 35.3 m
(iv) 1.02 s
(v) 5.10 m
- 2 (i) 25.8 m
(ii) 4.59 s
(iii) 178.9 m
- 3 (i) 41.0 m s^{-1} , 28.7 m s^{-1}
(ii)

t	0	1	2	3	4	5	6
x	0	41	82	123	164	205	246
y	0	24	38	42	36	21	-4.3

(iii)



- (iv) 42.0 m , 239.7 m
(v) The ball is a particle, no spin, no air resistance, so acceleration = g .
- 4 (i) 17.21 , 8.03 m s^{-1}
(ii) 1.64 s
(iii) 28.2 m
(iv) 0.82 s
(v) 3.29 m
(vi) 2.72 m , probably no
- 5 (i) 10.32 , 14.74 m s^{-1}
(ii) 2.91 s
(iii) Into goal
(iv) No
- 6 (i) 45.2 s
(ii) 13.55 km
(iii) 535 m s^{-1}
(iv) 55.9°
- 7 (i) 3.2 m , vertical component of velocity is always less than 8 m s^{-1}
(ii) 5.54 m
(iii) 52.2°

- 8 (i) 1.74 s
(ii) 3.50 m , hits Juliet's window
(iii) 9.12 m s^{-1}
- 9 (i) 2.02 s
(ii) No, height is 0.2 m
(iii) 21.57 m s^{-1}
- 10 (i) 0.47 s
(ii) 0.64 s
(iii) 25.44 m s^{-1}
(iv) 28.8 m s^{-1}
- 11 (i) Yes, the range is 70.4 m
(ii) 32.7 m s^{-1}
- 12 19.8 m s^{-1} ; 17.7 m s^{-1}
- 13 7.84 m s^{-1} at 71.4° above the horizontal. (Note that the easiest way to do this question is to reverse the path of the ball, so it starts at the net with the given velocity reversed.)
- 14 (i) (a) 34.6 m
(b) 39.4 m
(c) 40 m
(d) 39.4 m
(e) 34.6 m
(ii) $800 \sin \alpha \cos \alpha = 800 \cos(90^\circ - \alpha) \sin(90^\circ - \alpha)$
(iii) 57.9°
(iv) -30 cm ; $+31 \text{ cm}$. The lower angle is slightly more accurate.
- 15 (i) 26.06 m s^{-1}
(ii) 27.35 m s^{-1}
(iii) $26.15 < u < 26.88$
- 16 25.48 m

Discussion point (page 507)

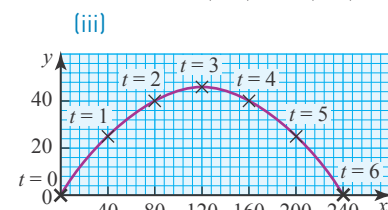
$20\mathbf{i} + 30\mathbf{j}$; $(0, 6)$; 10 m s^{-2}

Discussion point (page 508)

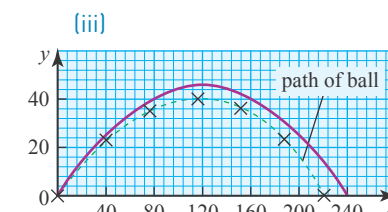
- (i) It is a parabola
- (ii) 51 m (to 3 s.f.)
- (iii) 124 m (to 3 s.f.)
- (iv) 6 m
- (v) Yes (apart from the missing bit at the start); $x = 60$ is a line of symmetry.

Exercise 21.4 (page 508)

- 1 (i) $y = \frac{5}{16}x^2$
(ii) $y = 6 + 0.4x - 0.2x^2$
(iii) $y = -14 + 17x - 5x^2$
(iv) $y = 5.8 + 2.4x - 0.2x^2$
(v) $y = 2x - \frac{gx^2}{2u^2}$
- 2 (i) $x = 40t$
(ii) $t = \frac{x}{40}$;
 $y = 30\left(\frac{x}{40}\right) - 5\left(\frac{x}{40}\right)^2$



- 3 (i) $y = 0.75x - \frac{1}{320}x^2$
(ii) Air resistance would reduce x .



Yes; horizontal acceleration = -0.5 m s^{-2}

- 4 (ii) 11.54 m s^{-1} ; 30.96°

Discussion point (page 511)

The projectile is a particle. There is no air resistance or wind. The particle is projected from the origin.

Exercise 21.5 (page 512)

- 1 (i) $(21.21t, 21.21t - 4.9t^2)$
(ii) $t = \frac{x}{21.21} \Rightarrow y = x - \frac{9.8}{900}x^2$
(iii) $8.9\mathbf{i}$
(iv) 29.4 or 62.4
- 2 (i) $y = 1.5 + x \tan 14^\circ - \frac{4.9}{100}(1 + \tan^2 14^\circ)x^2$
(ii) 7.0 m

3 (i) $y = 2.5 + x \tan 5^\circ$
 $- \frac{4.9}{625}(1 + \tan^2 5^\circ)x^2$

(ii) 24.2 m

(iii) Yes, $y = 2.43$ m

4 (i) $y = 1 + 0.70 \dots x$
 $- \frac{7.30 \dots}{u^2} x^2$

(ii) $u > 7.73 \text{ ms}^{-1}$

(iii) $u < 8.41 \text{ ms}^{-1}$

5 (i) 1 m, (25, 0)

(ii) 10 m

(iii) 0.6 s

(iv) $u_x = 25 \text{ ms}^{-1}$, 1 s

(v) 25.7 ms^{-1}

6 79 cm

7 $\tan^{-1} 4$ or $1 = 76.0^\circ$ (3 s.f.)
or 45°

8 (i) 59.4° or 29.1°

(ii) Lower is faster (higher might be required to go over the head of anyone in the way!)

9 $\tan^{-1} 1.25 = 51.3^\circ$ (3 s.f.)

10 (i) The equation for θ is $0.4 \tan^2 \theta - \tan \theta + 0.9 = 0$ and this equation has no real solutions as $b^2 - 4ac = -0.44$.

(ii) 0.225, $\tan^{-1} 1.25 = 51.3^\circ$ (3 s.f.)

11 8.12 ms^{-1}

12 56.3° , 18.43 ms^{-1}

13 (ii) 1.8, 0.6

(iii) 22.5 m when $\tan \alpha = 1.2$

(iv) 15.4 m

14 (ii) 39.2° , 72.6°

Chapter 22

Opening activity (page 518)

Assumptions: motorcycle is a particle, uniform frictional force with road, horizontal, linear motion with constant deceleration.

Discussion point (page 520)

1 Downward slope would extend skid so u is an overestimate; opposite for upward slope.

2 u would be an underestimate if the motor cycle was not actually at rest when it hit the car.

3 If the deceleration is not uniform then u could either be an over or under-estimate.

4 If the motorcycle were not a particle it would be affected by air resistance. Air resistance would reduce skid so u is an underestimate. Smaller μ would extend skid so u is an overestimate.

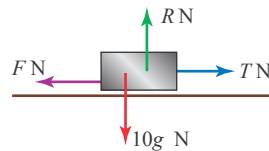
The model gives a good indication that the motorcyclist was speeding, but it would be sensible to take further measurements to try to improve the model, e.g. slope of road, a more accurate estimation of μ .

Discussion point (page 521)

Friction is forwards when pedalling, backwards when freewheeling.

Exercise 22.1 (page 525)

1



(i) 0.102

(ii) 0.051

2 (a) 0.61 ms^{-2} , $T = 45.9 \text{ N}$;
 $F = 36.8 \text{ N}$; 151.5 N

(b) 2.61 ms^{-2} , $T = 7.19 \text{ N}$;
 $F = 1.96 \text{ N}$; 19.7 N

(c) 0.31 ms^{-2} ; $T_1 = 76.0 \text{ N}$,
 $T_2 = 40.4 \text{ N}$;
 $F = 29.4 \text{ N}$; 198.2 N

(d) $a = 0$; $T_1 = 9.8 \text{ N}$,
 $T_2 = 29.4 \text{ N}$;
 $F = 19.6 \text{ N}$; 80.8 N

3 4802 N

4 (i) 42.7 N

(ii) 0.32

(iii) 44.8 N

5 0.82

6 (i) (-1.02 ms^{-2})

(ii) 0.102 N

(iii) 0.104

(iv) 49 m (independent of mass)

7 (i) smoother contact

(ii) 0.204

(iii) 140 N

8 (i) 7.35 ms^{-2}

(ii) 17.7 ms^{-1}

(iii) 59.9 m

9 $\frac{10g}{3} \approx 32.7 \text{ N}$;
 $\frac{3}{4}$

10 (i) 32.0°

(ii) 4.10 m

11 (i) 58.8 N

(ii) 62.5 N

12 (i) 0.577

(ii) 35.0°

(iii) 2.14

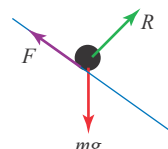
(iv) 50.2°

13 (i) 4.5 ms^{-2}

(ii) $F = 2.30 \text{ N}$, $R = 18.4 \text{ N}$,
 $\mu = 0.125$

(iii) 2.20 ms^{-2} , 4.20 ms^{-1}

14 (i)



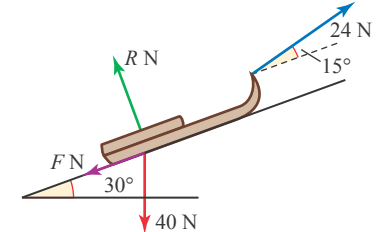
(ii) 4.42 ms^{-2}

(iii) 5.15 ms^{-1}

(iv) 5.42 m

15 Greater than, equal to, less than 16.7° , respectively.

16 (i)



(ii) $F = 3.18 \text{ N}$,
 $R = 28.4 \text{ N} \Rightarrow \mu = 0.11$

(iii) 3.46 N, 4.05 ms^{-2}

17 (i) 1.4 ms^{-2}

(ii) 1.20 s

(iii) 0.92 ms^{-1}

18 $144.1 < T < 1810.9$

- 19 (i) (a) 37.94 N
(b) 37.16 N
(c) 37.52 N

(ii) $\frac{40}{\cos \alpha + 0.4 \sin \alpha}$
(iii) 21.8°

- 20 Least force to move object up plane = $W(\sin \alpha + \mu \cos \alpha)$
Least force to prevent object sliding down plane = $W(\sin \alpha - \mu \cos \alpha)$

Practice questions: Mechanics (page 529)

1 Method 1

Weight = $0.5g$ [1]
Resolve horizontally and vertically
 $-T_A \cos 60 + T_B \cos 30 = 0$
 $T_A \sin 60 + T_B \sin 30 = 0.5g$
[1, 1]

Solve simultaneous equations
 $T_A = 4.24 \text{ N}$, $T_B = 2.45 \text{ N}$
[1, 1]

Method 2 – note the strings are perpendicular

Weight = $0.5g$ [1]
Resolve along the strings [1]
 $T_A = 0.5g \cos 30 = 4.24 \text{ N}$
 $T_B = 0.5g \cos 60 = 2.45 \text{ N}$
[1, 1, 1]

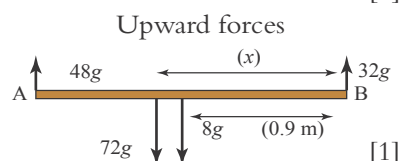
Method 3 triangle of forces

Weight $0.5g$ [1]
Diagram arrows forming a loop and right angle marked [1]

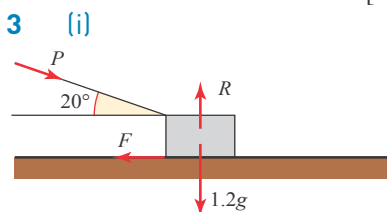
T_A
 $0.5g$
 T_B [1]

$T_A = 0.5g \cos 30 = 4.24 \text{ N}$ [1]
 $T_B = 0.5g \sin 30 = 2.45 \text{ N}$ [1]

- 2 (i) Contact force $72g$ [1]
(ii) Downward forces [1]



- (iii) Take moments about B [1]
 $(72g)x + (8g) \times 0.9 = (48g) \times 1.8$
 $x = 1.10$ [1]
Centre of mass is 1.1 m above his feet. [1]



Weight and P shown [1]
normal reaction [1]
Arrows showing friction opposing P [1]

- (iii) Resolve vertically [1]
 $R = P \sin 20^\circ + 1.2g$
 $F_{\max} = \mu R = 0.4(P \sin 20^\circ + 1.2g)$ [1]
On the point of sliding when [1]

$P \cos 20^\circ = F_{\max}$
 $P \cos 20^\circ = 0.4(P \sin 20^\circ + 1.2g)$ [1]
 $P(\cos 20^\circ - 0.4 \sin 20^\circ) = 0.48g$ [1]
 $P = \frac{0.48g}{\cos 20^\circ - 0.4 \sin 20^\circ} = 5.86[1]$

- 4 (i) Stationary when both components of $\mathbf{v} = 0$
 $-0.4t + 8 = 0$
 $t = 20$ [1]
When $t = 20$
 $-0.03(20)^2 + 0.8(20) - 0.4 = 3.6 \neq 0$
So boat is never stationary. [1]

(ii) $\mathbf{r} = \int \mathbf{v} dt$
 $= \begin{pmatrix} -0.01t^3 + 0.4t^2 - 0.4t \\ -0.2t^2 + 8t + 0 \end{pmatrix}$ [1, 1]

(iii) When $t = 5$,
 $\mathbf{r} = \begin{pmatrix} -0.01 \times 5^3 + 0.4 \times 5^2 - 0.4 \times 5 \\ -0.2 \times 5^2 + 8 \times 5 \end{pmatrix}$
 $= \begin{pmatrix} 6.75 \\ 35 \end{pmatrix}$ [1]

$\arctan\left(\frac{35}{6.75}\right) = 79.1$
Bearing = 011° [1, 1]

- (iv) Travelling NE when components of velocity equal (and positive) [1]
 $-0.03t^2 + 0.8t - 0.4 = -0.4t + 8$
 $-0.03t^2 + 1.2t - 8.4 = 0$
 $t = 30.95, 9.0$ [1]
 $30.95 > 20$ so time 9.0 s [1]

- 5 (i) In equilibrium for the bucket
 $T = mg$ [1]
In equilibrium for the block resolve up the slope
 $T = 1.5g \sin 40^\circ$ [1]
 $m = \frac{1.5g \sin 40^\circ}{g}$
 $= 0.964 \text{ kg}$ [1]

- (ii) Resolve downwards for the bucket N2L
 $1.7g - T_1 = 1.7a$ [1]
Resolve up the slope for the block
 $T_1 - 1.5g \sin 40 = 1.5a$
[1]

Add equations
 $1.7g - 1.5g \sin 40 = 3.2a$
 $a = 2.25$

Acceleration is 2.25 ms^{-2} [1]

- (iii) To find velocity when the bucket reaches the ground [1]
 $s = 0.8$

$u = 0$ $v^2 = u^2 + 2as$
 $v = ?$ $v^2 = u^2 + 2as$
 $= 4.5 \times 0.8$
 $= 3.6$
 $a = 2.25$ $v = 1.90$ [1]

$t =$
When tension removed
 $-1.5g \sin 40 = 1.5a$
 $a = -g \sin 40$ [1]

$s = ?$
 $u = 1.90$ $v^2 = u^2 + 2as$: $v^2 = 3.6$
 $v = 0$ $0 = 3.6 - 2 \times (g \sin 40)s$
 $a = -g \sin 40$ $s = \frac{3.6}{2g \sin 40} = 0.286$
 $t =$ [1]

Block travels another 28.6 cm. [1]

6 (i) Vertical motion

$$s = 0 \quad s = ut + \frac{1}{2}at^2$$

$$u = u \sin \alpha \quad 0 = u \sin \alpha - \frac{1}{2}gt^2$$

$$v =$$

$$a = -g \quad 0 = t(u \sin \alpha - \frac{1}{2}gt) \quad [1]$$

$$t = ? \quad t = 0, \quad \frac{u \sin \alpha}{\frac{1}{2}g} = \frac{2u \sin \alpha}{g}$$

Horizontal motion

$$\text{when } t = \frac{2u \sin \alpha}{g} \quad [1]$$

$$R = (u \cos \alpha)t = u \cos \alpha \left(\frac{2u \sin \alpha}{g} \right) \\ = \frac{2u^2 \sin \alpha \cos \alpha}{g} \quad [1]$$

(ii) Negligible air resistance

$$\sin 2\theta = 2 \sin \theta \cos \theta \quad [1]$$

$$R = \frac{2u^2 \sin \alpha \cos \alpha}{g} \\ = \frac{u^2 (2 \sin \alpha \cos \alpha)}{g} = \frac{u^2 \sin 2\alpha}{g} \quad [1]$$

$\sin 2\alpha$ takes maximum value of 1 when

$$2\alpha = 90^\circ \quad [1]$$

$$R_{\max} = \frac{u^2}{g} \text{ when}$$

$$\alpha = 45^\circ \quad [1]$$

(iv) Using $\alpha = 30$ to find time and range

For B2 time

$$t = \frac{2 \times 10 \sin 30}{g} = 1.02 \quad [1]$$

For C2 range

$$x = \frac{2 \times 10^2 \sin 30 \cos 30}{g} \\ = 8.84 \quad [1]$$

(v) Maximum range

$$\frac{u^2}{g} = 7.1 \quad [1]$$

$$u = \sqrt{7.1 \times 9.8} = 8.34 \quad [1]$$

(vi) The measurements give the maximum range when the angle is 40° . Changing the value for u will still have the maximum range at 45° . [1]

(vii)

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \begin{pmatrix} 10 \cos \alpha - 3t \\ 10 \sin \alpha - 9.8t \end{pmatrix} \quad [1]$$

When $t = 0$

$$\mathbf{v} = \begin{pmatrix} 10 \cos \alpha - 0 \\ 10 \sin \alpha - 0 \end{pmatrix} \quad [1]$$

Speed =

$$\sqrt{(10 \cos \alpha)^2 + (10 \sin \alpha)^2} \\ = \sqrt{100(\cos^2 \alpha + \sin^2 \alpha)} \\ = \sqrt{100 \times 1} = 10 \quad [1]$$

(viii) To find T in new model

y component of $\mathbf{r} = 0$

$$10t \sin \alpha - 4.9t^2 = 0$$

$$T = \frac{2u \sin \alpha}{g} \text{ which}$$

is exactly the same equation as in the standard model in

part (iii) [1]

(ix) For E5, we need

horizontal component

when $t = 1.44$

$$x = 10 \times 1.44 \cos 45^\circ - 1.5 \times 1.44^2 \\ = 7.07 \quad [2]$$

(x) The vector model is an improvement. It fits all the measurements better than the standard model and it gives the maximum range of 7.53 for 35° as compared with the measured value of 7.4 m occurring at 40° . [1]