MEI A Level Mathematics Year 1 (AS) - Full Worked Solutions

Practice questions: Pure Mathematics 1 (page 94)

1 (i) $\sqrt{2\frac{2}{3}} = \sqrt{\frac{8}{3}}$

The mixed number is not easy to work with – change it to a top-heavy fraction.

$= \sqrt{\frac{4 \times 2}{3}} = \sqrt{4} \times \sqrt{\frac{2}{3}} = 2\sqrt{\frac{2}{3}}$

Rewrite 8 as the product of factors where one of the factors is a square number.

(ii) $\frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

Use the denominator to decide what to multiply the top and bottom by – use the conjugate, the same bracket but with the sign changed to rationalise the denominator.

$= \frac{3+2\sqrt{3}+1}{3-1}$

Take care expanding the brackets – many errors occur at this stage of the working.

$= \frac{4+2\sqrt{3}}{2} = \frac{2(2+\sqrt{3})}{2} = (2+\sqrt{3})$

To simplify the fraction, factorise the numerator first and cancel the common factor with the denominator.

2 (i) $2^{3x} = 4^{x+4}$

$2^{3x} = (2^2)^{x+4}$

The base numbers must match. Rewrite 4 in terms of 2.

$2^{3x} = 2^{2(x+4)}$

Using the laws of indices.

$3x = 2(x+4)$

$3x = 2x + 8$

$x = 8$

Now that the base numbers match, the powers can be equated and the equation solved.

Check $\text{LHS} = 2^{3\times 8} = 16777216$

$\text{RHS} = 4^{8+4} = 16777216$

Wherever possible, check your answer.

(ii) $10^x > 2^x$ is true for positive values of x

If it is not clear what to do, try values for x. Or you could use a graphical calculator if you have one to draw the two graphs on the same axes and compare.

Choose zero or a negative value e.g. when $x = -1$, $10^{-1} = 0.1$ and $2^{-1} = 0.5$

3 (i) At the points of intersection, the y values are equal $x^2 - 4x + 1 = 7 - x^2$

Eliminate y from the two equations to form a quadratic equation.

$2x^2 - 4x - 6 = 0$

$x^2 - 2x - 3 = 0$

Collect terms and simplify the equation.

$(x-3)(x+1) = 0$

$x = 3$ or -1

You must show the factors, or the substitution into the quadratic formula to show that you have not used your calculator to solve the equation.

When $x = 3$, $y = 3^2 - 4 \times 3 + 1 = -2$

Notice the wording of the question required both coordinates of each point to be found.

When $x = -1$, $y = (-1)^2 - 4 \times (-1) + 1 = 6$

Points of intersection are $(3, -2)$ and $(-1, 6)$

Check: when $x = 3$, $y = 7 - 3^2 = -2$

and $x = -1$, $y = 7 - (-1)^2 = 6$

Check your answers whenever possible.

(ii) The line $y = -2x$ intersects the curve $y = x^2 - 4x + 1$ when

$-2x = x^2 - 4x + 1$

$x^2 - 2x + 1 = 0$

$(x-1)(x-1) = 0$

Repeated root means that the line is a tangent at the point where $x = 1$.
When $x = 1$, $y = 1^2 - 4 + 1 = -2$
so point of contact is $(1, -2)$.

Again the question requires both coordinates.

4 (i) $x^2 + 6x + 7 = (x + 3)^2 - 9 + 7$

$= (x + 3)^2 - 2$

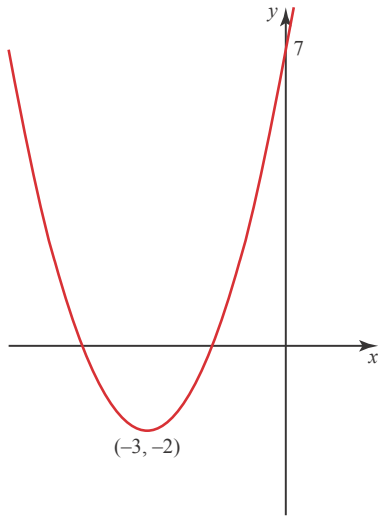
The number in the bracket is half the coefficient of x. Subtract the square of the number in the bracket.

There is no need to use the values a and b in your answer. An alternative method involves expanding $(x + a)^2 + b$ and equating coefficients.

(ii) Minimum point at $(-3, -2)$

The word 'state' in the question means no further working is required. The bracket is never negative, so the minimum value occurs when the bracket takes the value zero.

(iii) U-shaped graph with minimum point at $(-3, -2)$



Crosses y axis when $x = 0, y = 7$
Crosses x axis when $y = 0$, $(x + 3)^2 - 2 = 0$

This step could be left out and the curve drawn from the minimum point and one other point. In this question, these points are needed in the inequality, so could be included here.

$(x + 3) = \pm\sqrt{2}$

$x = -3 \pm \sqrt{2}$

You must show evidence of a non-calculator method. As the equation is already in completed square form, it is easiest to use it to solve the equation. You could use the quadratic formula but would need to show enough working to demonstrate that you have not used your calculator.

$x^2 + 6x + 7 > 0$

$x < -3 - \sqrt{2}$ or $x > -3 + \sqrt{2}$

The boundary values here are the roots of the equation $x^2 + 6x + 7 = 0$. To solve the inequality, choose the parts of the graph where the curve is above the x axis.

5 (i) The centre of the circle must be equidistant from A and B so it must lie on the perpendicular bisector of AB.

Show all the stages of working as the answer in given in the question.

To find the perpendicular bisector,
Midpoint of AB is at

$$\left(\frac{2 + (-1)}{2}, \frac{4 + 1}{2}\right) = \left(\frac{1}{2}, \frac{5}{2}\right)$$

You could just write down the midpoint by inspection.

Gradient of AB

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 1}{2 - (-1)} = \frac{3}{3} = 1$$

Gradient of perpendicular

$$m_2 = -\frac{1}{m_1} = -\frac{1}{1} = -1$$

The gradients of perpendicular lines are negative reciprocals of each other.

Equation of line is $y - y_1 = m_2(x - x_1)$

$$y - \frac{5}{2} = -1\left(x - \frac{1}{2}\right)$$

$$y = -x + \frac{1}{2} + \frac{5}{2}$$

$x + y = 3$

Show all the stages of working until you reach the given answer.

(ii) The centre of the circle is the point where $x + y = 3$ meets the x axis

Under examination conditions, use the given answer from (i) even if you did not get that answer.

When $y = 0, x = 3$ so C is $(3, 0)$
Radius is distance AC

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance BC could also be used. Mistakes can often be avoided by choosing the points which give fewer negative numbers in the calculation.

$r = \sqrt{(3-2)^2 + (0-4)^2} = \sqrt{1+4^2} = \sqrt{17}$

Equation of the circle

$(x-a)^2 + (y-b)^2 = r^2$

It is good practice to quote the formula and then substitute in the values.

$(x-3)^2 + (y-0)^2 = (\sqrt{17})^2$

$(x-3)^2 + y^2 = 17$

There is no value in expanding the brackets here. Avoid unnecessary algebra.

6 (i) The triangle is equilateral, so OC must be a line of symmetry.
A is $(-2, 0)$ and B is $(2, 0)$
C is $(0, h)$ and distance BC = 4

You could also use Pythagoras in the triangle AOC.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$4 = \sqrt{(2-0)^2 + (0-h)^2}$

$4 = \sqrt{4 + h^2}$

$16 = 4 + h^2$

$h^2 = 12$

$h = \sqrt{12} = 2\sqrt{3}$

C is $(0, 2\sqrt{3})$

Exact surd answer is required here. There is a clue in the wording of the question that your answer should be a surd.

(ii) Gradient of the line BC is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2\sqrt{3} - 0}{0 - 2} = -\sqrt{3}$$

Equation of the line BC is

$y - y_1 = m(x - x_1)$

$y - 0 = -\sqrt{3}(x - 2)$

$y = \sqrt{3}(2 - x)$

You could use the coordinates of C but there would be more work to do with the surds.

Rewrite your answer into the form given in the question.

(iii) E is the point $(x, 0)$ and F is $(x, \sqrt{3}(2-x))$.

Find an algebraic expression for the area and then consider the maximum value it can take.

Area of the rectangle is $2x \times \sqrt{3}(2 - x)$
$= 2\sqrt{3}(2x - x^2)$.

Length of the base is double OB and the height is the y coordinate of F.

To find maximum value, find completed square form.

When you have studied all the topics, calculus can also be used to find the maximum value.

$= 2\sqrt{3}(2x - x^2) = -2\sqrt{3}(x^2 - 2x)$

$-2\sqrt{3}((x-1)^2 - 1)$

Minimum value occurs when $x = 1$ and maximum area $-2\sqrt{3}(-1) = 2\sqrt{3} = \sqrt{12}$

The value for x here is not enough. The question asks for the value for maximum area.

7 (i) (a) The points for thinking distance lie on a straight line through the origin and so thinking distance is directly proportional to speed.

(b) The thinking time will be the same at every speed. It is reasonable to think that the driver will react in the same time whatever speed they are travelling.

Read the question carefully – the question has moved from thinking distance to thinking time.

(c) The equation of the line is $d = mx + c$
The line goes through the origin, so $c = 0$.

If you were not sure that the line goes through the origin, you could use the equation of the line joining two other points on the line.

When $x = 20$, $d = 6$ so $m = \frac{6}{20} = 0.3$

The line is $d = 0.3x$

(ii) (a) When $x = 10$ the model gives
$y = 1.6771 \times 10 - 26.38 = -9.609$.

To be sure to achieve the method marks, show the substitution. You could use table mode here, if your calculator has it, and use the y values for larger values of x as a check.

(b) The distance will not be negative, so the model gives an unsuitable value for the stopping distance.

(iii) (a) When $x = 20$, the new model gives
$y = 0.0157 \times 20^2 + 0.2629 \times 20 + 0.6$
$= 12.138$

(b) The values in the Highway Code have been given as rounded values and the model gives values which are fairly close but not exactly the same.

Practice questions: Pure Mathematics 2 (page 184)

1 **(i)** The graph has positive y values for large negative x values, so the order of the polynomial must be even.

The behaviour of the graph for large positive and negative values of x gives information about the largest power in the equation.

(ii) The graph crosses the x axis at -1 and 0 and touches it at 2.
$(x + 1)$ and x must be factors and $(x - 2)$ must be a repeated factor.
So one possible equation is $y = x(x + 1)(x - 2)^2$.

We do not know the equation for certain, as any stretch of this graph would also cross and touch the axes in the same way.

2 (i) $y = x^3 - ax^2 + ax - 1$
When $x = 0$, $y = 0^3 - a \times 0^2 + a \times 0 - 1 = -1$ for all values of a

Use the information from the graph to determine which two points Holly is referring to.

When $x = 1$, $y = 1^3 - a \times 1^2 + a \times 1 - 1 = 0$ for all values of a
which proves Conjecture A.

(ii) When $a = 3$, $y = x^3 - 3x^2 + 3x - 1$.

Substitute $a = 3$.

$(x - 1)$ is a factor since $x = 1$ is a root of the equation $y = 0$

$y = x^3 - 3x^2 + 3x - 1 = (x - 1)(x^2 - 2x + 1)$

The quadratic factor can be found by inspection or by algebraic division.

$= (x - 1)(x - 1)^2 = (x - 1)^3$

The graph of $y = (x - 1)^3$ is a translation of $y = x^3$ to the right 1 unit, which proves Conjecture B.

(iii) When $a = -1$ $y = x^3 + x^2 - x - 1$

Substitute $a = -1$.

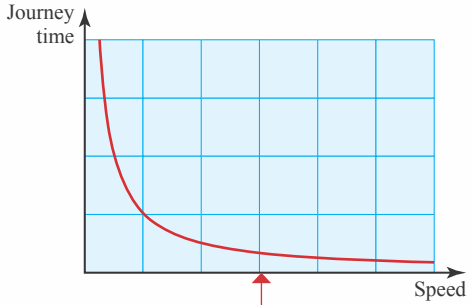
$y = x^3 + x^2 - x - 1 = (x - 1)(x^2 + 2x + 1)$
$= (x - 1)(x + 1)^2$
The graph has repeated zero at $x = -1$ which proves conjecture C.

The graph of this function can be seen in the diagram, and this can be used as a check.

3 (i) The model assumes the speed is constant and maximum speed can be maintained for the whole journey.

(ii) Journey time = distance ÷ speed

Graph needs to be the shape of $y = \frac{1}{x}$.



The question says no scales are needed but the axes should be labelled.

(iii) No of seconds in a day $= 24 \times 60 \times 60 = 86400$

The quantities must have compatible units.

EITHER Distance travelled in 175 days at 11000 m s^{-1} is $175 \times 86400 \times 11000$
$= 1.66 \times 10^{11}$ m
Journey is 1.65×10^{11} m so the model suggests there is enough food for the journey.

OR Time taken $= \frac{1.65 \times 10^{11}}{11000} = 1.5 \times 10^7$ seconds
$= \frac{1.5 \times 10^7}{86400} = 173.6$ days, so the model suggests there is enough food.

(iv) The calculated time is very close to the maximum time and does not take into account acceleration and deceleration at the ends of the journey, so I would not trust the model.

4 (i) Curve A is $y = 1 - \cos x$ because the graph has a period of 360°

The period of the graph is the interval for one complete wave.

(ii) $2\sin^2 x = 1 - \cos x$

$2(1 - \cos^2 x) = 1 - \cos x$

Use the identity $\sin^2 x + \cos^2 x = 1$ to obtain an equation with only $\cos x$.

$0 = 2\cos^2 x - \cos x - 1$

$0 = (2\cos x + 1)(\cos x - 1)$

Show all the steps in your non-calculator method to solve the quadratic equation.

$(2\cos x + 1) = 0$ or $(\cos x - 1) = 0$

$\cos x = -\frac{1}{2}$ or $\cos x = 1$

Use the exact values of the cosine of key angles – you should have learnt these.

$x = 120$ or $360 - 120 = 240$ or $x = 0, 360$

Each value of $\cos x$ gives two values for x in the range 0 to 360 inclusive.

5 (i) Using the sine rule with $c = 5, b = 8$ and $C = 20°$

Choose the sine rule as you have a pair of values (C and c).

$\frac{\sin\theta}{8} = \frac{\sin 20}{5}$

$\sin\theta = \frac{8\sin 20}{5}$

$\theta = \sin^{-1}\left(\frac{8\sin 20}{5}\right) = 33.18$

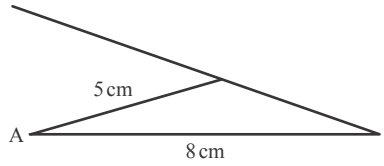
or $180 - 33.18 = 146.82$

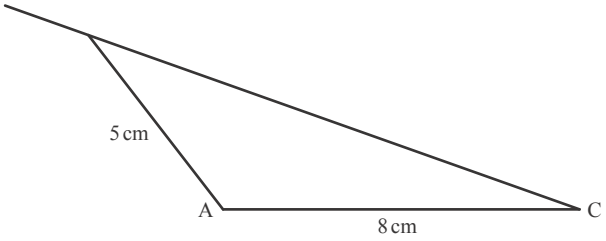
There are two possible values for θ here. The diagram gives a hint that the angle may be obtuse.

(ii) When $\theta = 33.18°$ $A = 126.82°$

Both the angles A and C are known. You can also construct a pair of triangles using a fixed angle of 20° and using compasses to show the two places on the line which are 5 cm from A.

When $\theta = 146.82°$ $A = 13.18°$





$\frac{1}{2}bc\sin A = \frac{1}{2}8 \times 5\sin 13.18 = 4.56$

$\frac{1}{2}bc\sin A = \frac{1}{2}8 \times 5\sin 126.82 = 16.010$

The formula for area uses two sides and the angle between those two sides, so the two different values of A are needed.

Area of larger triangle is 16.0 cm^2

The question asks for one, not the other.

6 (i) Graph A $y = (2x)^3 = 8x^3$ as this has the largest stretch in the y direction.

The graphs are easier to compare when written in the same way.

Graph B $y = 2x^3$

Graph C $y = x^3$

Graph D $y = \frac{x^3}{2}$

(ii) Graph D is $y = \sin x$ as it passes through (0, 0)

Look for the standard graphs by considering the points where they cross the y axis.

Graph B is $y = \cos x$ as it passes through (0, 1)

Graph C is $y = \sin(x + 30)$ it is a translation of –30 in the x direction of $y = \sin x$

Look for the translations by thinking about where the graphs cross the y axis or the position of the maximum points.

Graph A is $y = \cos(x + 30)$ it is a translation of –30 in the x direction of $y = \cos x$

(iii) Graph A is $y = x^4 + 1$ as it is positive for large negative values of x
Graph D is $y = -x^3$ as it goes through $(0, 0)$
Graph B is $y = x^3 + 1$ as it is positive for positive values of x and negative for large negative values of x
Graph C is $y = -x^3 + 1$ as it is positive for negative values of x and negative for large positive values of x

7 (i) Use cosine rule

The triangle is not right angled and the angle given is between the given sides, so the cosine rule must be used.

$x^2 = 1^2 + 2^2 - 2 \times 1 \times 2\cos 20$

$x = \sqrt{1.241229517} = 1.114\,\text{cm}$

Use the ANS key or the memory on your calculator to avoid problems with rounding too early.

(ii) In general

$x^2 = 1^2 + 2^2 - 2 \times 1 \times 2\cos\theta$

$= 5 - 4\cos\theta$

$R = x^2 = 5 - 4\cos\theta$

The question asks for R, the area of the square. Make sure you demonstrate that you know that R has the same value as x^2.

(iii) To find when $R < 4$, find the boundary values first.

Inequalities are more difficult to work with, especially, as $\cos\theta$ has a negative coefficient and is a decreasing function.

$5 - 4\cos\theta = 4$

$4\cos\theta = 1$

$\cos\theta = \frac{1}{4}$

$\theta = 75.5$

The area of the square increases as the angle increases so $\theta < 75.5$.

It's good to justify the direction of the inequality.

8 (i) The binomial coefficients are 1 3 3 1

$(2 + x)^3 = 1 \times 2^3 + 3 \times 2^2 x + 3 \times 2^1 x^2 + 1 \times x^3$

$(2 + x)^3 = 8 + 12x + 6x^2 + x^3$

$(1 - x)^3 = 1 \times 1^3 + 3 \times 1^2(-x) + 3 \times 1^1(-x)^2 + 1 \times (-x)^3$

$(1 - x)^3 = 1 - 3x + 3x^2 - x^3$

(ii) (a) $(2 + x)^3 + (1 - x)^3 = (8 + 12x + 6x^2 + x^3) + (1 - 3x + 3x^2 - x^3)$

$= 9 + 9x + 9x^2$, which is a quadratic function.
To find the line of symmetry, complete the square.

$9 + 9x + 9x^2 = 9(x^2 + x + 1)$

$= 9\left(\left(x + \frac{1}{2}\right)^2 - \frac{1}{4} + 1\right)$

$= 9\left(\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}\right)$

Minimum point when $x = -\frac{1}{2}$ so line of symmetry is $x = -\frac{1}{2}$

The original graph showing the two separate functions is also symmetrical with this line of symmetry, so you could also use a symmetry argument for the sum of the two functions.

(b) $(2 + x)^3 - (1 - x)^3$

$= (8 + 12x + 6x^2 + x^3) - (1 - 3x + 3x^2 - x^3)$

$= 7 + 15x + 3x^2 + 2x^3$, which is a cubic function.
When $x = -\frac{1}{2}$,

Substitute the given value into the cubic. Show all the stages of your working as the answer zero is given in the question.

$7 + 15x + 3x^2 + 2x^3 = 7 + 15\left(-\frac{1}{2}\right) + 3\left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right)^3$

$= 7 - \frac{15}{2} + \frac{3}{4} - \frac{2}{8} = 0$, so $x = -\frac{1}{2}$ is a root.

This value of x is the only point where the separate graphs cross and so is the only point where the difference between them is zero.

To find any other roots use $(2x + 1)$ as a factor.

You can use inspection or algebraic division to find the other factor.

$2x^3 + 3x^2 + 15x + 7$

$= (2x + 1)(x^2 + x + 7)$

To show that $(x^2 + x + 7)$ has no roots, find the discriminant.

When a question asks for roots of a quadratic, find the discriminant.

$b^2 - 4ac = 1^2 - 4 \times 1 \times 7 = -27 < 0$

Make a comment or inequality to make clear that the discriminant is negative.

so there are no other roots.

Make your conclusion clear from your working.

Practice questions: Pure Mathematics 3 (page 285)

1 ABCD is a trapezium if one pair of sides is parallel.

Vectors are parallel if one is a scalar multiple of the other.

$\mathbf{AB} = -2\mathbf{i} + 3\mathbf{j}$ and

$\mathbf{DC} = -\mathbf{i} + \frac{3}{2}\mathbf{j} = \frac{1}{2}(-2\mathbf{i} + 3\mathbf{j}) = \frac{1}{2}\mathbf{AB}$

So AB and DC are parallel sides and ABCD is a trapezium.

It is not necessary to show that the other sides are not parallel.

2 (i) Model $P = ae^{kt}$

When $t = 0$ $P = ae^{kt} = ae^0 = a = 52300$

The initial value always gives this number in the model.

$P = ae^{kt} = 52300e^{kt}$

When $t = 5$,

Substituting other values for t and P gives an equation from which k can be found.

$58500 = 52300e^{5k}$

$e^{5k} = \frac{58500}{52300}$

$5k = \ln\left(\frac{58500}{52300}\right)$

$k = 0.0224$

Rearrange to get the exponential term on its own then take natural logs to find k.

(ii) $P = 52300e^{0.0224t}$

When $t = 8$

Use the values of a and k found in (i) and substitute $t = 8$.

$P = 52300e^{0.0224 \times 8} = 62564.39655$

Population is 62 600 to the nearest 100.

The question specifies the rounding required. Make sure you check that your rounding is to the nearest 100.

3 (i) $\int_{-1}^{1}(3x - x^3)\,dx = \left[3 \times \frac{x^2}{2} - \frac{x^4}{4}\right]_{-1}^{1}$

Make sure that you show the working here even if your calculator will give a numerical answer for the definite integral. Use it only as a check.

$= \left[\left(3 \times \frac{1^2}{2} - \frac{1^4}{4}\right) - \left(3 \times \frac{(-1)^2}{2} - \frac{(-1)^4}{4}\right)\right]_{-1}^{1} = \frac{5}{4} - \frac{5}{4} = 0$

Take care with negative numbers. The value zero is obtained because of the symmetry of the graph and because the area lies below the axis for negative values of x.

(ii) $y = 3x - x^3$

$\frac{dy}{dx} = 3 - 3x^2$

The y coordinate needs to be verified.

When $x = 1, y = 3 \times 1 - 1^3 = 2$ and

$\frac{dy}{dx} = 3 - 3 \times 1^2 = 0$

The zero for $\frac{dy}{dx}$ needs to be shown.

The question uses the word 'verify' so it is enough to substitute the given values.
You could also do this question by solving the quadratic formed when $\frac{dy}{dx}$ is set to zero.

So (1, 2) is a turning point on the graph

When $x = -1$, $y = 3 \times (-1) - (-1)^3$

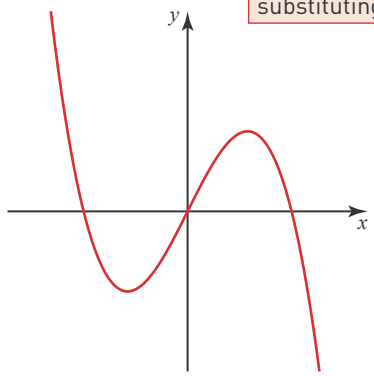
$= -3 + 1 = -2$ and

$\frac{dy}{dx} = 3 - 3 \times (-1)^2 = 0$

So (–1, –2) is another turning point on the graph.

Find the point where the graph crosses the y axis by substituting $x = 0$.

(iii) When $x = 0, y = 0$



When $y = 0$ $y = 3x - x^3 = 0$

It is good practice to show where the graph cuts the x axis but this sometimes gives an equation which is difficult to solve. You could sketch the graph without this section of working as the question does not ask for the points where the graph cuts the axes and the shape of the graph is clear from the turning points and the point on the graph (0, 0).

$x(3 - x^2) = 0$

$x = 0, \quad \pm\sqrt{3}$

Turning points at (–1, –2) and (1, 2).

(iv) The graph has rotational symmetry, so the area under the graph between 0 and 1 is the same size as the negative area under the graph between –1 and 0. The two areas add to give zero.

4 (i) Value in cell F3 is the gradient of the chord joining P (2, 12) to Q (2.1, 13.23)

The question requires you to identify what quantity is to be found in the cell F3.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{13.23 - 12}{2.1 - 2}$

$= \frac{1.23}{0.1} = 12.3$

You could just use the value of h as the denominator here instead of working it out from the gradient formula.

(ii) D5 has the value $2 + 0.001 = 2.001$

E5 has the value $3 \times 2.001^2 = 12.012003$

F5 has $\frac{12.012003 - 12}{2.001 - 2} = \frac{0.012003}{0.001}$

$= 12.003$

(iii) Limit as $h \to 0$ is 12

The smaller the value of h, the nearer to 12 the answers become.

(iv) $y = 3x^2$ so $\frac{dy}{dx} = 6x$

When $x = 2$ gradient $= \frac{dy}{dx} = 6 \times 2 = 12$

This is an opportunity to check that your answers are correct as this is the same as the answer in (iii).

5 (i) Area of the box $= 5x + 2 \times (5y) + 2 \times (xy)$

Area of the sleeve is $5(2x + 2y)$

Total area is $A = 15x + 20y + 2xy$

Read carefully which area is required here. Include the 5 faces of the box and the 4 faces of the sleeve.

(ii) Volume of the box $V = 5xy = 60$.

So $y = \frac{60}{5x} = \frac{12}{x}$

The given formula for A does not include y, so the formulae for V needs to be rearranged so that y can be eliminated.

Substitute for y: $A = 15x + 20 \times \frac{12}{x}$

$+ 2x \times \frac{12}{x}$

$A = 15x + \frac{240}{x} + 24$

(iii) To find minimum area $\frac{dA}{dx} = 0$.

A problem involving 'minimum' is usually a calculus problem.

$A = 15x + 24 + 240x^{-1}$

The x in the denominator needs to be rewritten as a negative power of x before differentiating.

$\frac{dA}{dx} = 15 + 240(-1)x^{-2} = 15 - \frac{240}{x^2} = 0$

The equation is more easily solved if the negative power is rewritten as a fraction.

$$\frac{240}{x^2} = 15$$

$$x^2 = \frac{240}{15} = 16$$

$$x = 4$$

$y = \frac{12}{x} = \frac{12}{4} = 3$ Dimensions of the box are 5 cm, 4 cm and 3 cm.

Check that you have given the answer to the question and not just the value of x as your answer.

6 (i) $\overrightarrow{CB} = \overrightarrow{OA} = \mathbf{u}$ and $\overrightarrow{AB} = \overrightarrow{OC} = \mathbf{v}$

Vectors which are parallel and equal in length are equal vectors.

$$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD} = \mathbf{v} + \frac{1}{2}\overrightarrow{CB} = \mathbf{v} + \frac{1}{2}\mathbf{u}$$

Travel from O to D via C, adding the vectors.

D is the midpoint of CB so it is halfway from C to B.

$$\overrightarrow{OE} = \frac{2}{3}\overrightarrow{OD} = \frac{2}{3}\left(\mathbf{v} + \frac{1}{2}\mathbf{u}\right)$$

$$= \frac{2}{3}\mathbf{v} + \frac{1}{3}\mathbf{u}$$

E divides the line OD in the ration of 2 : 1 and so is two thirds of the way from O to D.

(ii) To find the position vector of F

$$\overrightarrow{OF} = \overrightarrow{OA} + \overrightarrow{AF} = \overrightarrow{OA} + \frac{2}{3}\overrightarrow{AC}$$

$$= \mathbf{u} + \frac{2}{3}(-\mathbf{u} + \mathbf{v}) = \frac{1}{3}\mathbf{u} + \frac{2}{3}\mathbf{v} = \overrightarrow{OE}$$

which means that E and F are the same point.

7 (i) P is $(\ln 800, \ln 100.91) = (6.685, 4.614)$ to 3 d.p.

Make sure you round to 3 d.p., as required in the question.

The natural logs of the values in the table are needed.

Q is $(\ln 10000, \ln 1577.53) = (9.210, 7.364)$ to 3 d.p.

(ii) B is the gradient

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7.364 - 4.614}{9.210 - 6.685} = 1.089$$

(1.09 to 2 d.p.)

Make sure you round to 2 d.p., as required in the question.

P is on the line $\ln t = a + b \ln d$

The intercept, a, cannot be read from the graph as the scale on the x axis does not go to zero. The intercept must be found from the coordinates of one of the points on the line.

$$4.614 = a + 1.089 \times 6.685$$

$$a = -2.666 \text{ (}-2.67 \text{ to 2dp)}$$

(iii) $\ln t = a + b \ln d$

Use the equation of the line as a starting point.

$$\ln t = -2.67 + 1.09 \ln d$$

$$\ln t - 1.09 \ln d = -2.67$$

$$\ln t - \ln d^{1.09} = -2.67$$

$$\ln\left(\frac{t}{d^{1.09}}\right) = -2.67$$

Collect log terms together. Use the laws of logs to write as the difference of two logs – the multiplying number becomes the power. Use the laws of logs to replace the difference of logs with the log of a fraction.

$$\frac{t}{d^{1.09}} = e^{-2.67}$$

The exponential function is the inverse of the natural log function.

$$t = 0.069d^{1.09}$$

Evaluate $e^{-2.67}$.

(iv) 100 m time is 9.58 s

The model when $d = 100$ gives

$$t = 0.069d^{1.09} = 0.069 \times 100^{1.09} = 10.44$$

The model gives a value 10.44 which is quite different from the actual value 9.58.

Quote the values as you compare them.

The model may not be suitable for values of d which are not between 800 and 10 000 which are the smallest and largest values in the data set.

The data is for middle and long distance races but 100 m is a sprint.

8 (i) To find APR when $p = 2$

Amount owing after a year is

Be clear about what quantity the formula gives.

$$100\left(1 + \frac{p}{100}\right)^{12} = 100\left(1 + \frac{2}{100}\right)^{12}$$

$$= 100(1.02)^{12} = 126.824$$

The amount owing is £126.82 which is an increase of 26.82% on £100.

Interpret the amount owing as a percentage increase on £100.

(ii) For APR = 50%, the amount owing at the end of a year is £150.

The formula gives the amount owing, so £150 is needed.

$$150 = 100\left(1 + \frac{p}{100}\right)^{12}$$

Use the information about amount owing after a year to build an equation for p.

$$1.50 = \left(1 + \frac{p}{100}\right)^{12}$$

METHOD 1

$$1.50^{\frac{1}{12}} = \left(1 + \frac{p}{100}\right)$$

To undo the power 12 we need the 12th root, which can be expressed as a fractional index.

$$\left(1 + \frac{p}{100}\right) = 1.034366$$

$$\frac{p}{100} = 0.034366$$

$$p = 3.4366$$

$p = 3.4$ to 1 d.p.

METHOD 2

$$1.50 = \left(1 + \frac{p}{100}\right)^{12}$$

You can also deal with a power by taking logs of both sides. You can use log base 10 or natural logs in exactly the same way.

$$\ln 1.50 = \ln\left(1 + \frac{p}{100}\right)^{12}$$

$$= 12\ln\left(1 + \frac{p}{100}\right)$$

$$\frac{\ln 1.50}{12} = \ln\left(1 + \frac{p}{100}\right)$$

$$\left(1 + \frac{p}{100}\right) = e^{\frac{\ln 1.50}{12}}$$

$$= 1.0344366$$

Which gives $p = 3.4$ as above.

Practice questions: Statistics (page 395)

1 (i) P(all parcels delivered next day)
$= 0.75^3 = 0.421875$

This assumes that the delivery times are independent of each other.

(ii) P(book arrives on third day)
$= 1 - 0.75 - 0.2 = 0.05$
P(delivered the same day)
$= 0.75^3 + 0.2^3 + 0.05^3 = 0.43$

Think how this could happen – they all come in one day, or all in two days or all in three days.

2 (i) The data does not use the total number of goals, so a pie chart is not the correct graph to use.

A pie chart is always used to show proportions of the total and not the figures themselves.

Pie charts do not make it easy to compare numbers, especially when the numbers are close together.

(ii) A bar chart would make it easier to compare the numbers. A graph does not add anything in this situation, and a list would be just as good.

A bar chart is the simplest graph that displays the data most clearly.

3 (i) A simple random sample is one in which every sample is equally likely to be chosen. It requires a complete list of customers for the sampling frame and each has an equal probability of being included in the sample. In this case, there is no such list, so a simple random sample is not possible.

Learn the definitions and then apply them to the situation described in the question.

(ii) For opportunity sampling, an interviewer would go to one of the supermarkets and use as the sample any 200 customers who arrive in the supermarket while the interviewer is there. Quota sampling means that each interviewer would need to carry out 200 interviews, and have a target number of interviews with

Use the information in the question in your answer – it is not enough just to quote definitions you have learned.

people from different groups, e.g. males and females, young and old. Quota sampling is better as it is more likely to be representative of the whole population – opportunity sampling could produce a biased sample depending on the time or the day of the week when the interviews were conducted.

Give a reason for your answer that is relevant to the context of the question.

4 (i) Eruptions last between 1.5 minutes and 5.5 minutes with intervals between eruptions between 40 and 95 minutes. The data seems to fit into two groups, the first with short eruptions and short intervals to follow, and those with long eruptions and long intervals to follow. In general, the longer the eruption, the longer the interval before the next eruption.

Use the context of the question in your answer. The question is worth three marks here – find three separate points to discuss in your answer.

If you are not sure whether your sentence makes the point clear, draw a sketch diagram to illustrate your point.

(ii) Ann looks at the data as one set. Ben notices that the data splits into two groups and that there is little correlation between the length of eruption and the interval before the next within each group separately.

(iii) The histogram would be bimodal with a peak for the short eruptions and another for long ones. The time to the next eruption would not show the bimodal feature as there is no gap between one group and the other.

A diagram could be helpful here too.

(iv) The mean of 3.5 minutes is not a typical value for the eruption – it falls between the two peaks of the histogram and an eruption of 3.5 minutes is unlikely to occur. However, it is a useful as it lies between the values for the long and short values, and it gives information about the proportion of the time that the eruptions are happening.

The question uses the word 'discuss' so it is important to give two opposing statements.

5 (i) In these rows, the number of birds observed is very small, so the sample may not be representative of the population.

The question is worth two marks, which indicates that you should include a reason for why small samples should be discarded.

(ii) The mass of the birds increases in the first part of the day (from 6 am to 9 am). The mass then stays almost constant until 3 pm and then decreases to the original value by 7 pm. This may be due to feeding, and then burning off energy or excreting waste towards the end of the day.

Include information such as the times in your answer. The question is worth 4 marks, so you need to make 4 different points.

It is worth thinking about the context to suggest a reason for the changes you have described. You could suggest other reasons that are plausible in the context.

(iii) The point at 1 pm is very slightly lower than the others, and is most likely due to chance as it depends on which birds are included in the sample. It is only slightly less – the scales on the axes make it look much bigger than it is.

(iv) The data does not fit a straight line. However, the three sections in turn might fit three separate straight lines, with the middle one horizontal, and the other two showing the rate of increase and decrease in mass in the other two sections. It would also be possible to fit a U-shaped graph to the whole data set.

The word 'discuss' indicates that several contrasting points need to be made.

6 (i) (a) To find the mean $\bar{x} = \frac{\Sigma x}{n} = \frac{35059}{3496}$

$= 10.02831808$ so the mean is 10.0 km

Quote the formula you are using and then substitute the values from the question.

To avoid problems with rounding, write down an unrounded value and then consider separately how to round it. In general, the mean needs to be one more decimal place than the original data.

Remember to include the units with your rounded answer.

To find standard deviation

$sd = \sqrt{\frac{S_{xx}}{n-1}}$ where $S_{xx} = \sum x^2 - n\bar{x}^2$

$S_{xx} = 8704723 - 3496 \times 10.02831808^2$

$= 8353140.197$

Use the unrounded value for mean here. The ANS key on your calculator is useful.

$sd = \sqrt{\frac{8353140.197}{3495}} = 48.88789019$ so the standard deviation is 48.9 km.

It is unusual for the standard deviation to be so much bigger than the mean – use this as the basis of a checking strategy; if the value of your answer surprises you, check your work.

(b) Removing the magpies with $x = 0$, changes n to $3496 - 1694 = 1802$.

The value of n changes but the totals are unchanged.

$\bar{x} = \frac{\Sigma x}{n} = \frac{35059}{1802} = 19.45560488$, so the mean is 19.5 km.

$S_{xx} = 8704723 - 1802 \times 19.4556^2$

$= 8022628.948$

$sd = \sqrt{\frac{8022628.948}{1801}} = 66.7423504$, so the standard deviation is 66.7 km

(ii) Outliers are more than $2 \times sd$ from the mean.

Choose this definition of outlier, as these are the values calculated so far.

Less than $10.0 - 2 \times 48.9 = -87.8$ so no values are outliers here.
More than $10.0 + 2 \times 48.9 = 107.8$ so there are 39 outliers here.
The data has a strong positive skew.
The values 471, 1197 and 2210 are hugely greater than this threshold and are very extreme outliers.

(iii) The proportion of magpies with $x = 0$ is $\frac{1694}{3496} = 0.4846$ so almost half of the magpies are recaptured less than 0.5 km from where they were first ringed. For the others, the mean distance is quite small (10.0 km). These support the model that magpies generally do not migrate far.
However, there are a few individuals $\frac{39}{3496} = 0.0112$ (about 1%) whose behaviour does not fit the model as they are found more than 110 km away.

Use all the information given in the question to support your answer.

Interpret the figure $x = 0$ in the context of the question – you may need to reread how x is defined to be clear about this.

The word 'discuss' in the question suggests that opposing views should be included in your answer.

7 (You could begin this question by drawing the 6×6 table with all 36 equally likely possible outcomes and count to calculate the probabilities.)

Scores	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	2	3	4	5	6
3	3	3	3	4	5	6
4	4	4	4	4	5	6
5	5	5	5	5	5	6
6	6	6	6	6	6	6

(i) Scores on two dice need to be (2, 1), (2, 2) or (1, 2) out of 36 possible outcomes.

In this context, 2 on the second die and 1 on the first are not the same outcomes.

$P(X = 2) = \frac{3}{36} = \frac{1}{12}$

(ii) $P(X = 2) = \frac{1}{12} = 3k$ giving $k = \frac{1}{36}$

The question uses the word 'state', so you can just write down the answer without working.

$P(X = 6) =$

$1 - \left(\frac{1}{36} + \frac{3}{36} + \frac{5}{36} + \frac{7}{36} + \frac{9}{36}\right) = 1 - \frac{25}{36}$

$= \frac{11}{36}$

r	1	2	3	4	5	6
$P(X=r)$	$k = \frac{1}{36}$	$3k = \frac{3}{36}$	$5k = \frac{5}{36}$	$7k = \frac{7}{36}$	$9k = \frac{9}{36}$	$11k = \frac{11}{36}$

You can give your answer in terms of k or as a fraction.

(iii) $P(X = Y) = P(\text{both 1, or both 2 … or both 6}) =$

List the ways in which the event can happen and add the probabilities.

$k^2 + (3k)^2 + (5k)^2 + (7k)^2 + (9k)^2 + (11k)^2 =$

$k^2 + 9k^2 + 25k^2 + 49k^2 + 81k^2 + 121k^2$

$= 286k^2$

When the answer is given in the question, it is important to show all the stages of working.

EITHER

P(one player wins) $= 1 - \text{P}(X = Y) =$

$1 - 286k^2 = 1 - 286 \times \left(\frac{1}{36}\right)^2 = 1 - \frac{143}{648}$

$= \frac{505}{648} = 0.77932$

A and B are equally likely to win so

P(B wins) $= 0.77932 \div 2 = 0.38966$

(0.390 to 3 d.p.)

The question specifies the rounding required. Read the question again to check that you have rounded as required.

OR

You could work out this answer by listing the possible combination of scores for which B wins and adding the probabilities of each.

It is often better to use the previous part of the question as a hint to the best method.

P($B = 6, A = 1,2,3,4,5$) +

as the sample($B = 5, A = 1,2,3,4$)

$+ \cdots +$ as the sample$(B = 1) \times 0$

8 X is the number of cars with at least one faulty tyre. $X \sim \text{Bin}(25, 0.15)$

Begin this question by defining the random variable and stating its distribution.

(i) $\text{E}(X) = np = 25 \times 0.15 = 3.75$

Use the standard formula for expected value.

(ii) $P(X = x) = {}^{25}C_x p^x (1-p)^{25-x}$

$\text{P}(X < 3.75) = \text{P}(X \leq 3)$

$= \text{P}(X = 0,1,2,3)$

$= 0.0171 + 0.0759 +$

$0.1607 + 0.2174 = 0.4708$

Rewrite the question here with a numerical value for expected value and list the values of X for which the event occurs.

You could use cumulative binomial tables or the cumulative distribution function on your graphical calculator if you have one.

(iii) X is the number of cars with at least one faulty tyre. $X \sim \text{Bin}(50, p)$

$H_0 : p = 0.15$

$H_1 : p < 0.15$

The null hypothesis is always that the value of p is unchanged. The test is to find out whether the campaign has reduced the proportion of cars with faulty tyres, so a 1-tailed test is required.

Under H_0

$\text{P}(X \leq k) = \text{P}(X \leq 5) = 0.21935$

Calculate the cumulative probability that the observed value and more extreme values would occur. Use cumulative binomial tables if available or the cumulative distribution on your graphical calculator if you have one.

$0.21935 > 5\%$

Compare your answer with the significance level. It is important to write this comparison explicitly.

The observed result is not in the critical region, so there is insufficient evidence that the proportion of cars with faulty tyres has reduced.

Make sure you state all the stages of the argument and that you interpret the result in the context of the question.

(iv) The number of cars with faulty tyres is Bin$(100, p)$ where $p = 0.15$ under H_0

Critical region is the largest k for which

$\text{P}(X \leq k) < 0.01$

You can use cumulative tables if available. The inverse binomial function on your graphical calculator if you have one will find the value of k for which the probability is closest to 0.01.

$\text{P}(X \leq 6) = 0.004702 < 1\%$

State clearly the probabilities either side of 0.01.

$\text{P}(X \leq 7) = 0.0121 > 1\%$

So the possible values of k are 0, 1, 2, 3, 4, 5 or 6.

The question asks for a list of possible values – make sure you make your answer a clear list.

Practice questions: Mechanics (page 478)

1 Uniform acceleration – suvat equations

$s = 2800\text{m}$
$u = 0$
$v = 70\text{ms}^{-1}$
$a =$
$t = ?$

$s = \frac{1}{2}(u + v)t$

$2800 = \frac{1}{2}(0 + 70)t$

$t = \frac{2800}{35} = 80\text{s}$

Substitute the given values and rearrange.

Write all the values given in the questions and indicate which one is to be found. Choose the suvat equation without a.

2 To find an expression for velocity

Differentiate displacement to obtain an expression for velocity.

$x = 5 + 2.1t^2 - 0.07t^3$

$v = \frac{dx}{dt} = 4.2t - 0.21t^2$

When $t = 7$ — Substitute the given value for t.

$v = 4.2 \times 7 - 0.21 \times 7^2 = 19.11ms^{-1}$

$(19.1\,\text{ms}^{-1}$ to 3 s.f.)

It's good practice to round your answers to 3 significant figures.

3 Total force = 0

Total force = 0 when the forces are in equilibrium. Take care with signs.

$2\mathbf{F}_1 - 3\mathbf{F}_2 + \mathbf{F}_3 = 0$

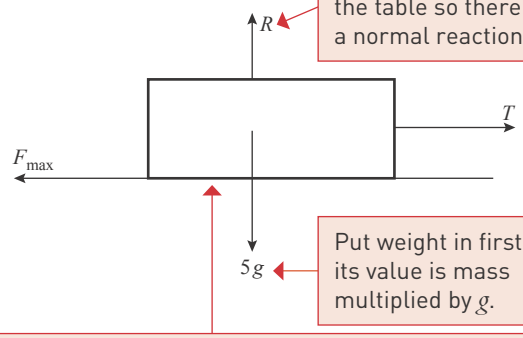
$2(7\mathbf{i} - 2\mathbf{j}) - 3(9\mathbf{i} - 3\mathbf{j}) + \mathbf{F}_3 = 0$

$14\mathbf{i} - 4\mathbf{j} - 27\mathbf{i} + 9\mathbf{j} + \mathbf{F}_3 = 0$

$-13\mathbf{i} + 5\mathbf{j} + \mathbf{F}_3 = 0$

$\mathbf{F}_3 = 13\mathbf{i} - 5\mathbf{j}$

4 (i) For 5 kg box on table

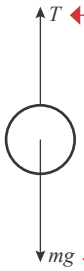


Box in contact with the table so there is a normal reaction R.

Put weight in first – its value is mass multiplied by g.

The box experiences a tension in the string which may or may not be equal to the weight of the hanging mass. The friction acts to oppose the likely motion. It is the maximum value as the question says that the box is on the point of sliding.

For hanging mass



The tension T must have the same value as the tension in the previous diagram as it is a single string over a smooth pulley.

The weight acts downwards and is equal to mass multiplied by g.

(ii) Hanging block in equilibrium

Look at the vertical forces.

$T = mg = 1.25g$

Box in equilibrium — Look at the horizontal forces.

$T = F$

So $F = 1.25g$ (= 12.3 N to 3 s.f.)

You can leave your answer in terms of g or you can use $g = 9.8$ to get an answer to 3 significant figures.

(iii) Inextensible string

If the string does not stretch, the displacement of the block matches the displacement of the box, so velocity and acceleration also both match. It does not matter here that the string is light.

(iv) N2L for block

Notice that, in this new situation, the value of T has changed even though F has not.

Use Newton's second law to link the forces and the acceleration.

$T - F = 5a$

$T - 1.25g = 5a$

N2L for hanging mass

$2g - T = 2a$

Adding

Solving simultaneous equations by adding to eliminate T.

$2g - 1.25g = 7a$

$a = 1.05$

$a = 1.05\,\text{m s}^{-2}$

5 (i) Sunil's distance

Displacement = area under graph. Area of the triangle = half base × height.

$$d = \frac{1}{2} \times 7 \times 9 = 31.5\,\text{m}$$

(ii) Distance to go

$100 - 31.5 = 68.5\,\text{m}$

Time at maximum velocity

At constant speed, time = distance ÷ speed.

$$\frac{68.5}{9} = 7.61\,\text{s}$$

Total time = 7 + 7.61 = 14.61 s (14.6 to 3 s.f.)

It is really easy to forget this last stage – make it part of your checking strategy to reread the question and make sure that you have done all that is required.

(iii) Integrate

An expression for displacement can be found by integrating the expression for velocity. Using the times at the start and end of the interval as limits means that there is no need for the constant ($+c$).

$$\int_0^6 0.9t^2 - 0.1t^3\,dt = \left[0.9\frac{t^3}{3} - 0.1\frac{t^4}{4}\right]_0^6$$

$= 32.4\,\text{m}$

(iv) METHOD 1 find time for Mo to run 100 m

Remaining distance to cover

$100 - 32.4 = 67.6$ m

$$\text{Time} = \frac{67.6}{10.8} = 6.26\ \text{s}$$

Total time = 6 + 6.26 = 12.26 s (12.3 s to 3 s.f.)

which is less than 14.61 s for Sunil, so Mo will win.

Make sure that you compare your two answers so that it is clear how you reached your conclusion.

METHOD 2 Find the distance Mo covers by the time Sunil finishes (14.6 s)

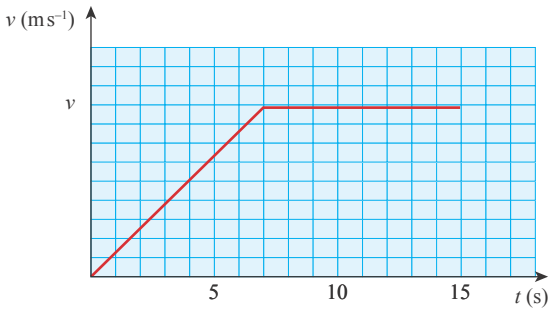
Distance =

$32.4 + 10.8 \times (14.6 - 6) = 125$

125 m is beyond the finish, so Mo will win.

It is important to make your argument clear – how do your answers support your conclusion?

(v) For Sunil to win, he must decrease his time to 12.26 s for 100 m.



Area under the trapezium

You may find it easier to find the area of the triangle for the first 7 seconds and the area of the rectangle from 7 to 12.6 seconds.

$$100 = \frac{1}{2}(12.26 + (12.26 - 7))V$$

$$V = \frac{200}{17.52} = 11.4$$

Velocity must be $11.4\,\text{m s}^{-1}$

6 Assume that air resistance is negligible, assume stone is a particle, assume gravity is constant, assume so horizontal motion

(i) Returns when $s = 0$

There is no need to look at the motion to the top and the downward motion separately.

$s = 0$

$u = 25$

$v =$

$a = -g$

$t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$s = 25t - 4.9t^2 = 0$$

$$t(25 - 4.9t) = 0$$

$t = 0, 5.1\,\text{s}$

$t = 0$ is the start, so it takes 5.1 s to return.

Write all the values given in the questions and indicate which one is to be found. Choose the suvat equation without v.

You could form the equation and rearrange, then use your calculator to solve the equation, if it has equation mode.

It is important to give both roots of the quadratic and select the one required, especially if you use your calculator to solve the quadratic.

(ii) At maximum height $v = 0$

$s = h$
$u = u$ $\quad v^2 = u^2 + 2as$
$v = 0$ $\quad 0 = u^2 - 2gh$
$a = -g$ $\quad h = \frac{u^2}{2g}$
$t =$

Write all the values given in the questions even if algebraic. Choose the suvat equation without t.

(iii) Height above point of projection
$= h$ – distance fallen in t seconds.
For journey falling from the top point:

At the top, velocity = 0. This is u in the equation.

$s =$ distance fallen $= ut + \frac{1}{2}at^2$
$u = 0$ $\quad = 0 - \frac{1}{2}gt^2$
$v =$
$a = -g$
$t = t$

Displacement is negative as it is below the starting point.

height above ground =

$$h - \frac{1}{2}gt^2 = \frac{u^2}{2g} - \frac{1}{2}gt^2$$

(iv) Height of the second stone t seconds after launch $s = ut - \frac{1}{2}gt^2$

Stones cross when heights are equal.

Equating the heights gives an equation for t. This tells us when and not where the stones cross.

$$\frac{u^2}{2g} - \frac{1}{2}gt^2 = ut - \frac{1}{2}gt^2$$

$$t = \frac{u}{2g}$$

Height $s = ut - \frac{1}{2}gt^2 = u\left(\frac{u}{2g}\right) - \frac{1}{2}g\left(\frac{u}{2g}\right)^2$

Use the value for t to find the height.

$$= \frac{u^2}{2g} - \frac{u^2}{8g} = \frac{3u^2}{8g} = \frac{3}{4}\left(\frac{u^2}{2g}\right) = \frac{3}{4}h$$

7 (i) In cell C3, we need acceleration during the 200 m journey.

You need to look at the column and row headings to know which quantity is required here.

$s = 200\text{m}$ $\quad v^2 = u^2 + 2as$
$u = 22\text{ms}^{-1}$ $\quad 18^2 = 22^2 + 2 \times 200a$
$v = 18\text{ms}^{-1}$ $\quad a = \frac{18^2 - 22^2}{400} = -0.4$
$a = ?$
$t =$

Write all the values given in the questions and indicate which one is to be found. Choose the suvat equation without t.

In cell D3, we need magnitude of the resistance force.

Newton's second law needed here.

$= ma = 800 \times 0.4 = 320$ N

(ii) Consider the next 200 m; the initial speed is the final speed for the previous 200 m.

You can work this as a new journey as written here. Alternatively, you can consider the 400 m journey from the beginning ($u = 22$ and $s = 400$).

$s = 200$
$u = 18\text{ms}^{-1}$
$v =$ $\quad v^2 = u^2 + 2as$
$a = -0.4\text{ms}^{-2}$ $\quad v^2 = 18^2 - 2 \times 0.4 \times 200$
$t =$ $\quad v = \sqrt{164} = 12.8\text{ms}^{-1}$

(iii) Consider the journey from the beginning to coming to rest:

Make it clear which journey you are considering in your answer.

$s =$
$u = 22\text{ms}^{-1}$ $\quad v = u + at$
$v = 0$ $\quad 0 = 22 - 0.4t$
$a = -0.4$ $\quad t = \frac{22}{0.4} = 55\text{s}$
$t =$

(iv) Predicted time (55 s) is much shorter than the actual time (74.2 s) so the model A is not a suitable model.

It is important to include values in your argument here and not just to give the phrase 'not suitable'.

(v) Calculation – for example:

There are many ways of showing that the resistance must decrease. You could look at other parts of the journey.

to come to rest in 74.2 s, constant acceleration model gives

$v = u + at$

$0 = 22 + 74.2a$

$a = -\frac{22}{74.2} = -0.296$

Resistance = $800 \times 0.296 = 237$N which is less than the force in the first 200 m, so the force must decrease as the car slows down.