Version 2: 15/12/17

Chapter 1

Discussion points (page 5)

The value of r would be 5.57 cm (to 3 s.f.)

Exercise 1.1 (page 6)

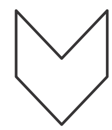
1 10, 10 and 12
2 8890 m^2 (3 s.f.)
3 6
4 6 kg
5 THE MEGGAN SPACE-FLEET IS APPROACHING PLEASE SEND HELP WE NEED MORE GALACTIC RUNABOUTS WITH HIGH SPEED LASER GUNS
6 17
7 18
8 (i) 110 minutes
(ii) The model does not allow for speeding up at the start of the journey and slowing down at the end.
(iii) 7 stops. Cannot be very confident as we don't know if there were delays on the journey. 7 stops suggests that it was 2 minutes late.
9 (i) (a) £30
(b) £75
(ii) More than 1250
(iii) For example, $C = 70 + 0.02n$

Discussion point (page 10)

$\Leftrightarrow$

Exercise 1.2 (page 11)

1 (i) $A \Rightarrow B$
(ii) $A \Leftarrow B$
(iii) $A \Leftrightarrow B$
(iv) $A \Leftrightarrow B$
(v) $A \Rightarrow B$
(vi) $A \Leftrightarrow B$
(vii) $A \Rightarrow B$
(viii) $A \Leftrightarrow B$
2 (i) If a triangle has two angles equal, then it has two sides equal True
(ii) If Alf is dead, then Fred murdered Alf False
(iii) Each of the angles of ABCD is 90° $\Rightarrow$ ABCD is a square False
(iv) A triangle with three equal angles has three equal sides True
(v) If Struan goes swimming, it is sunny False
3 (i) $P \Leftarrow Q$
(ii) $P \Leftarrow Q$
(iii) $P \Leftarrow Q$
(iv) $P \Rightarrow Q$
(v) $P \Leftarrow Q$
(vi) $P \Leftrightarrow Q$
(vii) $P \Rightarrow Q$
(viii) $P \Leftrightarrow Q$
4 (i) If x^2 is an integer, then x is an integer False
(ii) If $\angle PQR + \angle PSR = 180°$, the angles P, Q, R and S all lie on a circle True
(iii) If $x^2 = y^2$, then $x = y$ False
(iv) $\angle x = \angle y \Rightarrow$ *lines* l and m are parallel True
(v) $n > 2 \Rightarrow n$ is an odd prime number False
5 (i) No
(ii) Yes
6 (i) (a) True
(b) The six internal angles are all equal $\Rightarrow$ ABCDEF is a regular hexagon
(c) True
(ii) (a) True
(b) All the six sides are the same length $\Rightarrow$ ABCDEF is a regular hexagon
(c) False. The shape could look like this:



7 (i) True
(ii) Triangles ABC and XYZ are congruent $\Rightarrow$ Together AB = XY, BC = YZ and angle BAC = angle XYZ
(iii) False – for example it could be that AB = YZ, BC = XZ and angle BAC = angle XZY
8 0, 1 or 4

Discussion point (page 13)

Only the prime numbers were necessary

Exercise 1.3 (page 14)

1 (i) True
(ii) True
(iii) False, e.g. $x = -1$, or any negative number
(iv) True
(v) False, e.g. $n = 7$
(vi) True
2 (i) False, e.g. $n = 40$
(ii) False, e.g. $n = 41$
(iii) True
(iv) True
(v) True
(vi) True
3 (i) $w = 50\sqrt{2}$
4 Converse is: All numbers of the form $6n \pm 1$ are prime. The converse is false.

5 (i) True
(ii) (a) True
(b) False
(iii) True

6 Only the prime numbers were necessary

Chapter 2

Discussion points (page 23)

You cannot find a value for the answer when you substitute $x = 1, y = 1$. This is because in this case $x - \sqrt{y} = 0$ so you have multiplied the top and bottom by zero.

Exercise 2.1 (page 23)

1 (i) $2\sqrt{7}$
(ii) $5\sqrt{3}$
(iii) $8\sqrt{2}$

2 (i) $\sqrt{54}$
(ii) $\sqrt{125}$
(iii) $\sqrt{432}$

3 (i) $2\sqrt{2}$
(ii) $\frac{\sqrt{3}}{6}$
(iii) $\frac{3\sqrt{2}}{2}$

4 (i) $\frac{5}{7}$
(ii) $\frac{4\sqrt{2}}{9}$
(iii) $\frac{5}{7}$

5 (i) $8 + 3\sqrt{2}$
(ii) $4\sqrt{2}$
(iii) $8\sqrt{2} + 2\sqrt{5}$
(iv) $8\sqrt{5}$

6 (i) $2\sqrt{y}$
(ii) $2\sqrt{a} + 11\sqrt{b}$

7 (i) $11 + 6\sqrt{2}$
(ii) $11 - 6\sqrt{2}$
(iii) 7

8 (i) $3 - \sqrt{14}$
(ii) $3 + \sqrt{14}$
(iii) $p - 2q - \sqrt{pq}$
(iv) $p - 2q + \sqrt{pq}$

9 (i) $\frac{\sqrt{3} - 1}{2}$
(ii) $-\frac{(1 + \sqrt{5})}{4}$
(iii) $\frac{3(4 + \sqrt{2})}{14}$
(iv) $3(\sqrt{5} + 2)$

10 (i) $1 + \frac{1}{2}\sqrt{2}$
(ii) $\frac{3}{2}\sqrt{3} - \frac{5}{2}$
(iii) $1 - \frac{2}{5}\sqrt{5}$
(iv) $\frac{9}{7} + \frac{4}{7}\sqrt{2}$

11 (i) $4\sqrt{2}$ cm; 32 cm^2
(ii) $12\sqrt{2}$ cm

12 $4\sqrt{2}$ m

13 1 : 64 : 729

14 (i) $3 + \sqrt{6}$
(ii) $\frac{14 + 5\sqrt{10}}{18}$

16 (i) e.g. $a = 27, b = 12$
(ii) a and b are of the form nx^2 and ny^2

Exercise 2.2 (page 38)

1 (i) 3^4
(ii) 3^0
(iii) 3^{-4}
(iv) 3^{-2}

2 (i) $\frac{1}{32}$
(ii) 3
(iii) $\frac{1}{8}$
(iv) 1

3 (i) $x^{\frac{3}{2}}$
(ii) x^{-3}
(iii) x^3
(iv) $x^{\frac{1}{2}}$

4 (i) 4×10^9
(ii) 4000

5 (i) 3.78×10^{11} and 7.8×10^{10} metres
(ii) Greatest time 21 minutes, least time 4 minutes 20 seconds

6 (i) $4x^3$
(ii) $4x$
(iii) $64x^{-10}$

7 (i) $x^2 - x$
(ii) $a - a^{-2}$
(iii) $p - 1$

8 (i) $-9x^6$
(ii) $64x^9$

9 (i) $3\sqrt{2}$
(ii) $-3\sqrt[3]{4}$
(iii) $43\sqrt{7}$

10 $x = \frac{1}{2}$

11 (i) $(1 + x)^{\frac{1}{2}}(2 + x)$
(ii) $(x + y)^{\frac{1}{2}}(6 - 5x - 5y)$
(iii) $(x^2-2x+3)^{\frac{1}{2}}(-x^2+2x-2)$

12 $x = 12,\ y = 8$

13 $x = 0.5, y = -0.5; \frac{1}{5}, \sqrt{2}$

14 (i) 0.5, 0.25, 0.125, 0.0625, value approaches zero
(ii) 0, 0, 0, 0, value = 0
(iii) 1, 1, 1, 1, value = 1 (by definition)

Chapter 3

Activity 3.1 (page 35)

(i) $(x + 3)(x + 2)$
(ii) $(x + 2)(x + 3)$
(iii) $(x + 2)(x - 3)$
(iv) $(x - 3)(x - 2)$
(v) $(x - 3)(x + 2)$
(vi) $(x - 6)(x - 1)$
(vii) $(x - 6)(x + 1)$
(viii) not possible
(ix) $(x - 6)(x - 1)$
(x) not possible

(i) and (ii) give the same answer, (iii) and (v) give the same answer (vi) and (ix) give the same answer

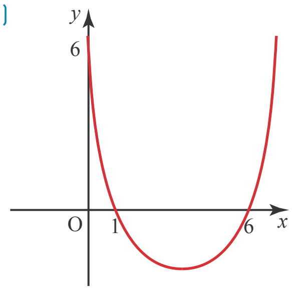
Exercise 3.1 (page 40)

1 (i) $(x + 4)(x + 2)$
(ii) $(x - 4)(x - 2)$
(iii) $(y + 4)(y - 2)$
(iv) $(y - 4)(y + 2)$
(v) $(r + 5)(r - 3)$
(vi) $(r - 5)(r + 3)$

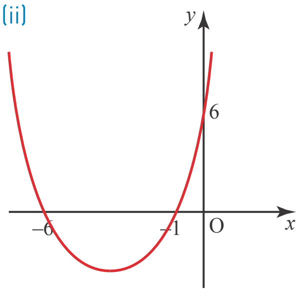
2 (i) $(s - 2)^2$
(ii) $(s + 2)^2$
(iii) $(p - 2)(p + 2)$
(iv) cannot be factorised
(v) $(a + 3)(a - 1)$
(vi) cannot be factorised

3 (i) $x = 1,\ 6$
(ii) $x = -1, -6$
(iii) $x = 2,\ 3$
(iv) $x = -2, -3$
(v) $x = 7, -1$
(vi) $x = -7,\ 1$

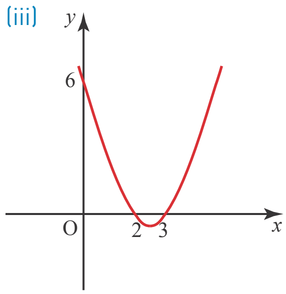
4 (i)



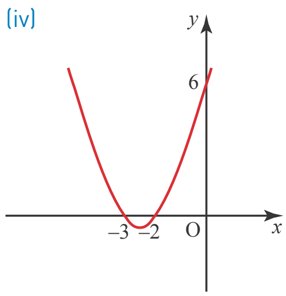
(ii)



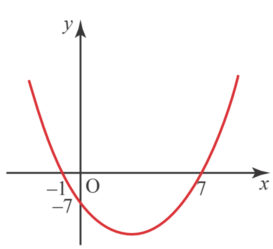
(iii)



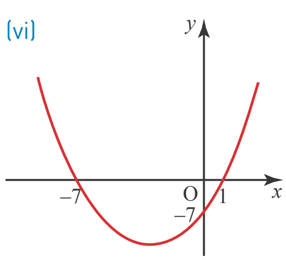
(iv)



(v)



(vi)

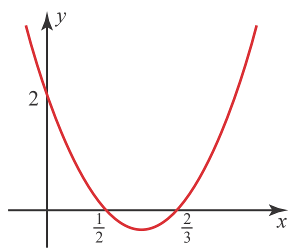


5 (i) $(4 - x)(1 + x)$
(ii) $(4 + x)(1 - x)$
(iii) $(4 - x)(3 + x)$
(iv) $(4 + x)(3 - x)$
(v) $(7 + x)(5 - x)$
(vi) $(7 - x)(5 + x)$

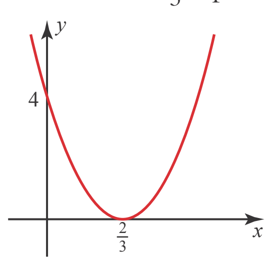
6 (i) $(x + 2)(2x + 1)$
(ii) $(x - 2)(2x - 1)$
(iii) $(5x + 1)(x + 2)$
(iv) $(5x - 1)(x - 2)$
(v) $2(x + 4)(x + 3)$
(vi) $2(x - 4)(x - 3)$

7 (i) $(1 + 3x)(1 - 2x)$
(ii) $(1 - 3x)(1 + 2x)$
(iii) $(1 + x)(5 - 2x)$
(iv) $(1 - x)(5 + 2x)$

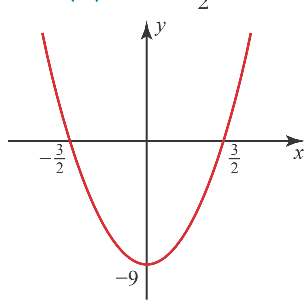
8 (i) $x = \frac{1}{2}$ or $x = \frac{2}{3}$



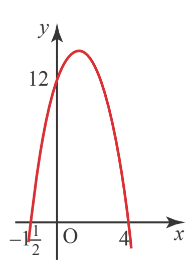
(ii) $x = \frac{2}{3}$ rep



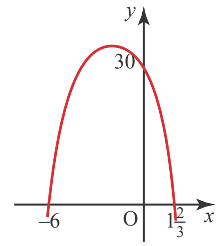
(iii) $x = \pm 1\frac{1}{2}$



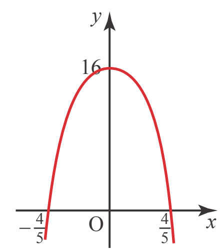
9 (i) $x = 4$ or $x = -1\frac{1}{2}$



(ii) $x = -6$ or $x = 1\frac{2}{3}$



(iii) $x = \pm\frac{4}{5}$



10 (i) $x = \pm 2, \pm 3$
(ii) $x = \pm\sqrt{2}$
(iii) $x = 1, 9$
(iv) $x = 1, -8$

11 (i) $A = w(30 + w)$
(ii) width = 80 m, perimeter = 380 m

12 (i) $A = 2\pi rh + 2\pi r^2$
(ii) $54\pi\,\text{cm}^3$
(iii) $250\pi\,\text{cm}^3$

13 19.1 cm

14 (i) $x = 2y$
(ii) 4, 3, 5; 8, 15, 17; 12, 35, 37
(iii) (a) $y = 10$
(b) The triangle is right angled but doesn't fit that pattern

15 (i) Volume $= 20x^2 - 600x + 4000\,\text{cm}^3$
(ii) 36 cm by 72 cm

Discussion point (page 44)

If the coefficient of x^2 is negative, then the quadratic function has the greatest value (i.e. the graph has a maximum point).

Activity 3.3 (page 45)

(i) Two numbers which multiply to make 2 must be either 1 and 2, or −1 and −2. Neither of these add to give −6.
(ii) 5.65 and 0.35
(iv) Substituting into the equation does not give exactly zero.

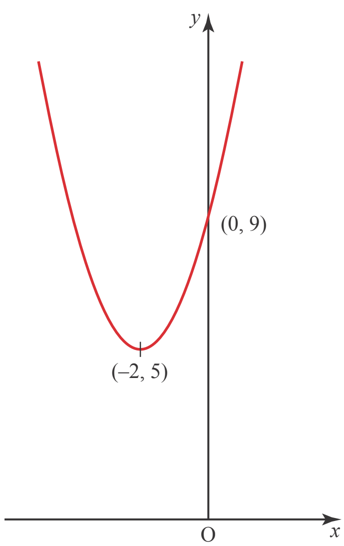
Exercise 3.2 (page 46)

1 (i) $x^2 + 4x + 1$
(ii) $x^2 + 8x + 12$
(iii) $x^2 - 2x + 3$
(iv) $x^2 - 20x + 112$
(v) $x^2 - x + 1$
(vi) $x^2 + 0.2x + 1$

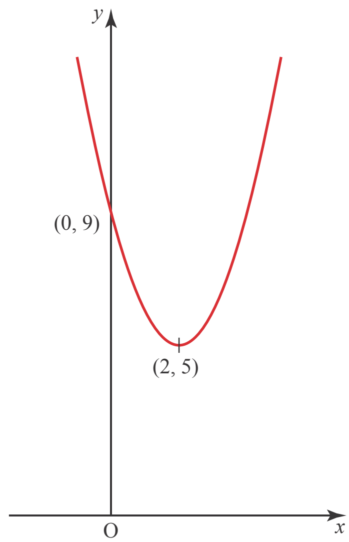
2 (i) $x = -1 \pm \sqrt{10}$
(ii) $x = 2 \pm \sqrt{5}$
(iii) $x = 3 \pm \sqrt{5}$
(iv) $x = \dfrac{-1 \pm \sqrt{3}}{2}$
(v) $x = \dfrac{3 \pm \sqrt{12}}{2}$
(vi) $x = \dfrac{1 \pm \sqrt{18}}{3}$

3 (i) $(x + 2)^2 + 1$
(ii) $(x - 3)^2 - 6$
(iii) $(x + 1)^2 - 6$
(iv) $(x - 4)^2 - 20$

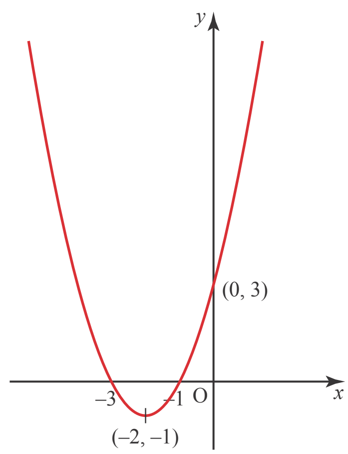
4 (i) (a) $y = (x + 2)^2 + 5$
(b) $x = -2, (-2, 5)$
(c)



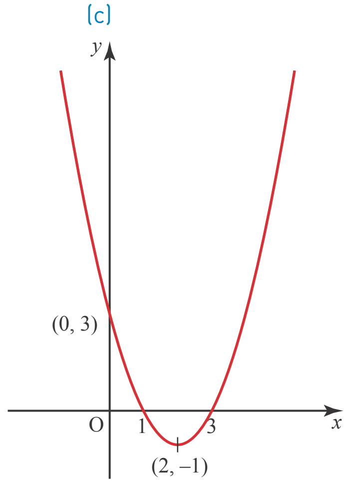
(ii) (a) $y = (x - 2)^2 + 5$
(b) $x = 2, (2, 5)$
(c)



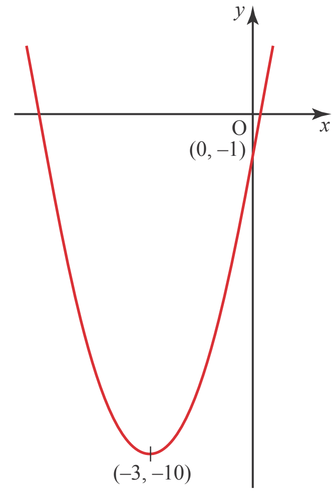
(iii) (a) $y = (x + 2)^2 - 1$
(b) $x = -2, (-2, -1)$
(c)



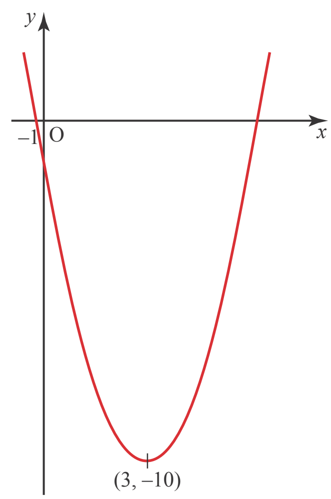
(iv) (a) $y = (x - 2)^2 - 1$
(b) $x = 2, (2, -1)$
(c)



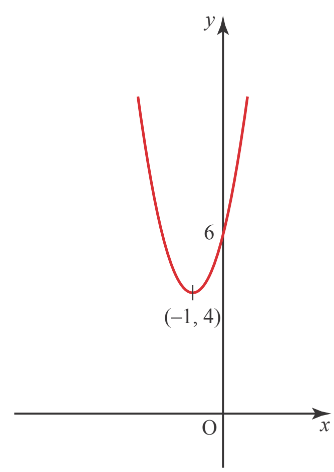
(v) (a) $y = (x + 3)^2 - 10$
(b) $x = -3, (-3, -10)$
(c)



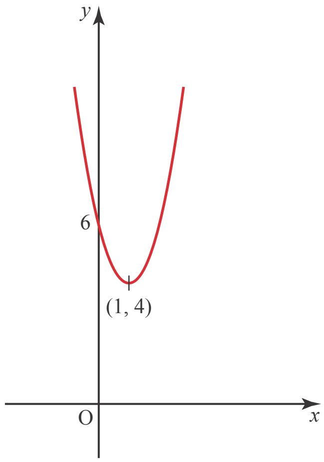
(vi) (a) $y = (x - 3)^2 - 10$
(b) $x = 3, (3, -10)$
(c)



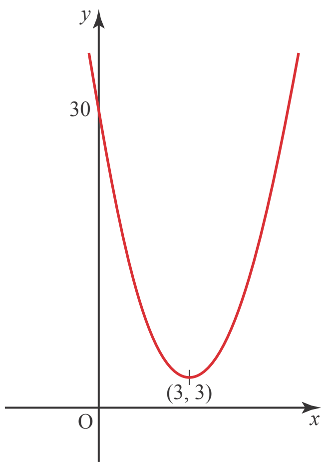
5 (i) (a) $y = 2(x + 1)^2 + 4$
(b) $(-1, 4), x = -1$



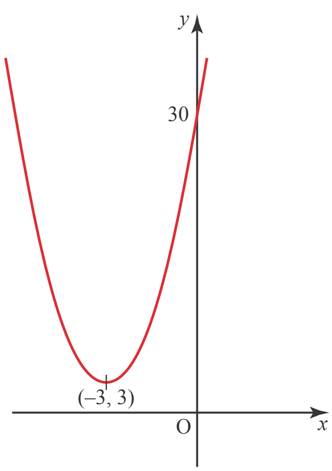
(ii) (a) $y = 2(x - 1)^2 + 4$
(b) $(1, 4)$, $x = 1$



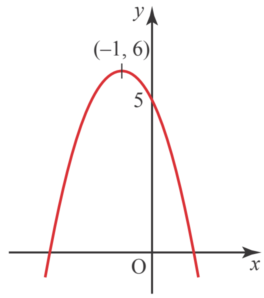
(iii) (a) $y = 3(x - 3)^2 + 3$
(b) $(3, 3)$, $x = 3$



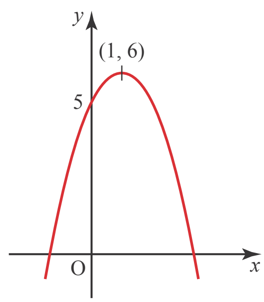
(iv) (a) $y = 3(x + 3)^2 + 3$
(b) $(-3, 3)$, $x = -3$



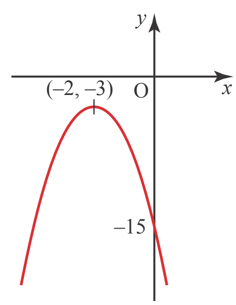
6 (i) (a) $y = -(x + 1)^2 + 6$
(b) $(-1, 6)$, $x = -1$



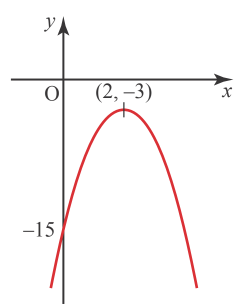
(ii) (a) $y = -(x - 1)^2 + 6$
(b) $(1, 6)$, $x = 1$



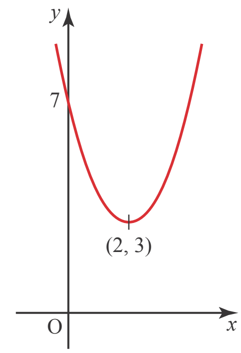
(iii) (a) $y = -3(x + 2)^2 - 3$
(b) $(-2, -3)$, $x = -2$



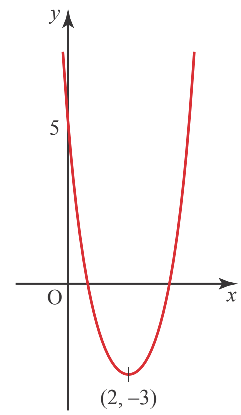
(iv) (a) $y = -3(x - 2)^2 - 3$
(b) $(2, -3)$, $x = 2$



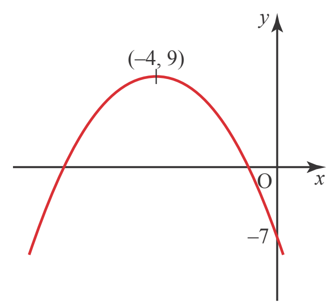
7 (i) (a) $y = (x - 2)^2 + 3$
(b) $(2, 3)$, $x = 2$



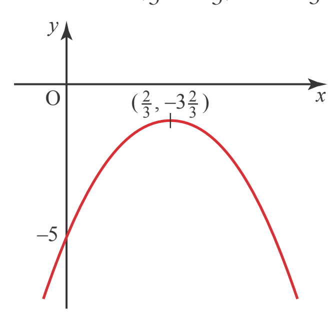
(ii) (a) $y = 2(x - 2)^2 - 3$
(b) $(2, -3)$, $x = 2$



(iii) (a) $y = -(x + 4)^2 + 9$
(b) $(-4, 9)$, $x = -4$

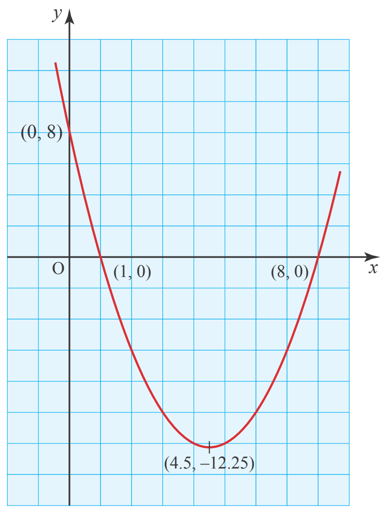


(iv) (a) $y = -3(x - \frac{2}{3})^2 - 3\frac{2}{3}$
(b) $(\frac{2}{3}, -3\frac{2}{3})$, $x = \frac{2}{3}$



8 (i) $b = -6,\ c = 10$
(ii) $b = 2,\ c = 0$
(iii) $b = -8,\ c = 16$
(iv) $b = 6,\ c = 11$

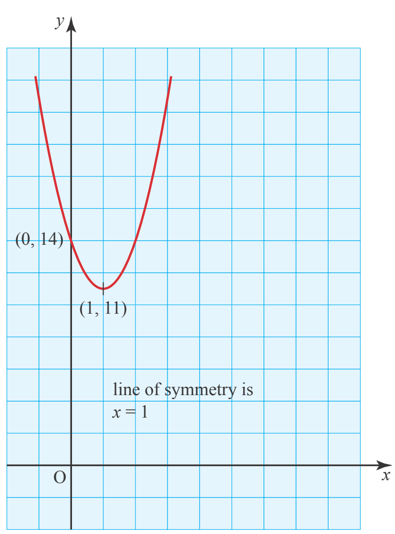
9 (i) $(x - 4.5)^2 - 12.25$
(ii) $(4.5, -12.25)$
(iii) $(1, 0), (8, 0), (0, 8)$
(iv)



10 (i) $(x - 2)^2 + 2; (2, 2)$
(ii) $c > 4$

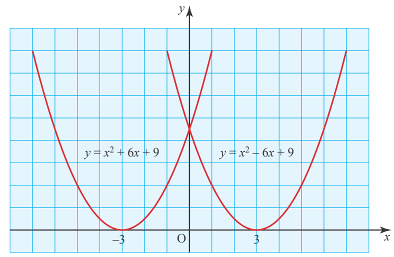
11 (i) (a) $(x + 11)^2 - 36$
(b) $x = -5$ or $x = -17$
(ii) (a) $(x - 12)^2 - 81$
(b) $x = 3$ or $x = 21$

12 (i) $3(x - 1)^2 + 11$
(ii) $x = 1; (1, 11)$



13 (i) $a = \pm 6$

(ii) Curves are reflections of each other in the y axis



Exercise 3.3 (page 50)

1 (i) $-0.683, -7.32$
(ii) $1.24, -3.24$
(iii) $7.52, -2.52$
(iv) $0.303, -3.30$
(v) $1.37, -0.366$
(vi) $1.57, -1.91$

2 (i) $-0.23, -1.43$
(ii) $1.43, 0.23$
(iii) $0.34, -5.84$
(iv) $-0.39, -5.11$
(v) $1.89, 0.11$
(vi) $2.10, -0.10$

3 (i) (a) 20
(b) 2
(c) No
(ii) (a) 0
(b) 1
(c) Yes
(iii) (a) -12
(b) 0
(c) No
(iv) (a) 49
(b) 2
(c) Yes
(v) (a) 0
(b) 1
(c) Yes
(vi) (a) -275
(b) 0
(c) No

4 (i) $2 \pm \sqrt{3}$
(ii) $\dfrac{-1 \pm \sqrt{13}}{6}$
(iii) $\dfrac{-1 \pm \sqrt{13}}{-6}$
(iv) $\dfrac{5 \pm \sqrt{21}}{2}$
(v) $4 \pm \sqrt{15}$
(vi) $\dfrac{-3 \pm \sqrt{89}}{8}$

5 (i) (a) $16 - 4c$
(b) 4
(c) $c \leqslant 4$
(ii) (a) $36 - 8c$
(b) 4.5
(c) $c \leqslant 4.5$
(iii) (a) $16 - 12c$
(b) $1\frac{1}{3}$
(c) $c \leqslant 1\frac{1}{3}$
(iv) (a) $4 + 20c$
(b) $-\frac{1}{5}$
(c) $c \geqslant -\frac{1}{5}$

6 (i) $A \Rightarrow B$
(ii) $A \Leftrightarrow B$
(iii) $A \Leftrightarrow B$
(iv) $A \Leftarrow B$
(v) $A \Leftrightarrow B$

7 (i) (a) $b^2 - 4ac = 0$
$\Rightarrow$ the graph of $y = ax^2 + bx + c$ touches the x axis
(b) True
(ii) (a) $ax^2 + bx + c$ cannot be factorised
$\Rightarrow b^2 - 4ac < 0$
(b) False
(iii) (a) $x^2 - 9 = 0$
$\Rightarrow x = 3$
(b) False

8 (i) $t = 1$ and 2
(ii) $t = 3.065$
(iii) 12.25 m

9 $k = 1$ or $k = 5$

11 (i) $k \leqslant -\frac{13}{9}$ or $k \geqslant 3$
(ii) $k = 3$, repeated root $= -4$; $k = -\frac{13}{9}$, repeated root $= \frac{8}{3}$

Chapter 4

Opening activity (page 53)

A packet of nuts costs £1.20 and a packet of crisps costs 80p.

In the second question, both statements give the same information, so there is not enough information to solve the problem.

Discussion points (page 54)

There are an infinite number of pairs of values for x and y that satisfy the equation. You need two equations.

Three equations

Activity 4.1 (page 55)

(1, 3)

Approximately (1.7, 1.2)

Discussion point (page 57)

There may be no points where the line meets the curve. If you try to solve the equations algebraically, you will obtain a quadratic equation with negative discriminant (so it has no real root).

There may be just one point where the line meets the curve, in which case the line will touch the curve without crossing it. If you try to solve the equations algebraically, you will obtain a quadratic equation with discriminant zero (so there is a repeated root).

Exercise 4.1 (page 57)

1 (i) $x = 7, y = 2$
(ii) $x = 8, y = 2$
(iii) $a = 5, b = -2$

2 (i) $x = 3, y = 7$
(ii) $x = 1, y = 7$
(iii) $x = 5.5, y = 1.5$
(iv) $a = 1, b = -2$

3 (i) $x = 0.5, y = 1$
(ii) $x = 3, y = -1$
(iii) $l = -1, m = -2$
(iv) $r = 5, s = -1$

4 (i) $5p + 8h = 20$
$10p + 6h = 20$
(ii) Paperback costs 80p, Hardback costs £2

5 (i) Adult costs £30, Child costs £15
(ii) £135

6 (i) $t_1 + t_2 = 4$
$110t_1 + 70t_2 = 380$
(ii) 275 km motorway, 105 km country roads

7 £6.80

8 (i) $x = 1, y = 1$
$x = 4, y = 16$
(ii) $x = 0.5, y = 2.5$
$x = -2, y = -5$

9 (i) $x = 3, y = 1$ or
$x = 1, y = 3$
(ii) $x = 4, y = 2$ or
$x = -4, y = -2$
(iii) $k = -1, m = -7$ or
$k = 4, m = -2$
(iv) $x = 1, y = -2$ or
$x = -2\frac{3}{7}, y = -\frac{2}{7}$

10 (ii) (0, 4) and (−1, 3)

11 (ii) (3, 4) and (5, 6)

12 (i) $(3x + 2y)(2x + y)\ \text{m}^2$
(iii) $x = \frac{1}{2}, y = \frac{1}{4}$

13 (i) $h + 4r = 100$,
$2\pi rh + 2\pi r^2 = 1400\pi$
(ii) 6000π or $\frac{98000}{27}\pi\ \text{cm}^2$

14 (i) $x = 1, y = 3$
(ii) Line is a tangent to the curve

15 (i) $x = \frac{5 \pm \sqrt{-19}}{2}$
(ii) No solution since there is a square root of a negative number.
(iii) The line and the curve don't meet.

Activity 4.2 (page 59)

$60m + 40d \leqslant 2500$

$m + d \geqslant 50$

She could buy 25 of each, or she could buy fewer muffins and more doughnuts, e.g. if she bought 23 muffins she could either buy 27 or 28 doughnuts.

Exercise 4.2 (page 62)

1 (i) $x \geqslant 4$
(ii) $x < 3$
(iii) $-2 \leqslant x \leqslant 5$
(iv) $-4 < x < -1$
(v) $5 \leqslant x < 7$
(vi) $-6 < x \leqslant 0$

2 (i)


(ii)


(iii)


(iv)

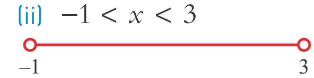

(v)

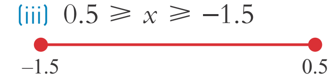

(vi)

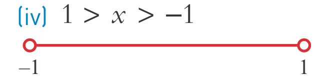

3 (i) $-2 \leqslant x \leqslant 3$
(ii) $4 < x < 7$
(iii) $x \leqslant -1$ or $x \geqslant 2$

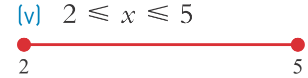
4 (i) $x < 7$
(ii) $x > 3$
(iii) $x \leqslant 4$
(iv) $x \geqslant -2$
(v) $p < 7$
(vi) $s \geqslant 6$

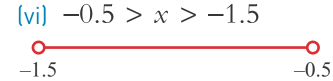
5 (i) $2 \leqslant x \leqslant 4$


(ii) $-1 < x < 3$


(iii) $0.5 \geqslant x \geqslant -1.5$


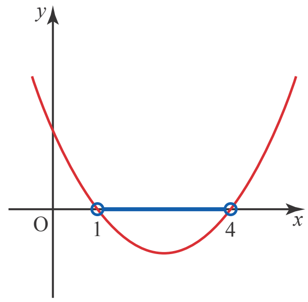
(iv) $1 > x > -1$


(v) $2 \leqslant x \leqslant 5$


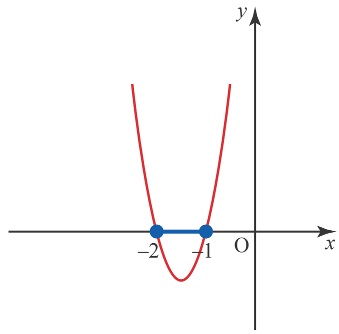
(vi) $-0.5 > x > -1.5$


6 (i) $c > -2$
(ii) $d \leqslant -\frac{4}{3}$
(iii) $e > 7$
(iv) $f > -1$
(v) $g \leqslant 1.4$
(vi) $h < 0$

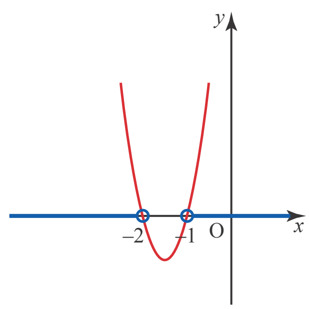
7 (i) $1 < p < 4$



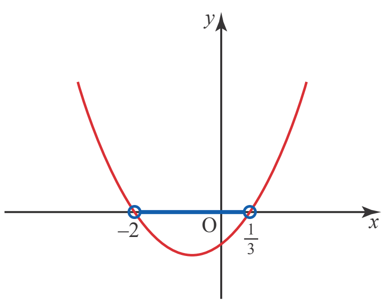
(ii) $-2 \leqslant x \leqslant -1$



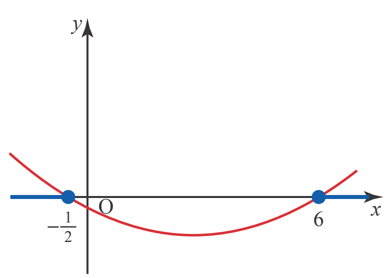
(iii) $x < -2$ or $x > -1$



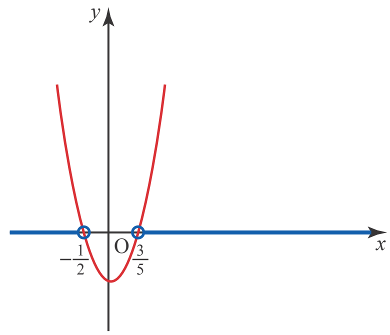
(iv) $-2 < x < \frac{1}{3}$



(v) $x \leqslant -\frac{1}{2}$ or $x \geqslant 6$



(vi) $x < -\frac{1}{2}$ or $x > \frac{3}{5}$



8 (i) $\{y : y < -1 \cup y > 3\}$

(ii) $\{z : z \geqslant -4 \cap z \leqslant 5\}$

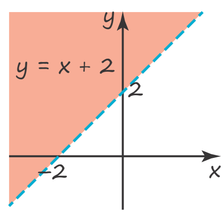
(iii) $\{x : x \leqslant 1 \cup x \geqslant 3\}$

(iv) $\{a : a < -3 \cup a > 2\}$

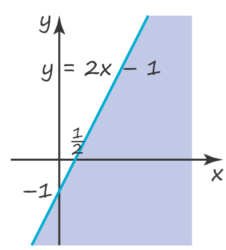
(v) $\{a : a \geqslant -4 \cap a \leqslant 2\}$

(vi) $\{s : s > -1 \cap s < \frac{1}{3}\}$

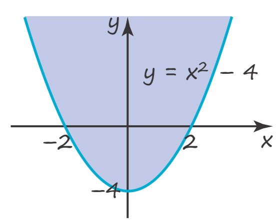
9 (i)



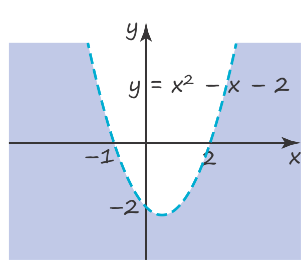
(ii)



(iii)



(iv)



10 $0 < k < 4$

11 (i) Single point solution $a = 2$

(ii) No real value of b

(iii) Doesn't factorise – quadratic formula needed

12 36 to 57 years

Chapter 5

Opening activity (page 65)

Assuming the ant takes the shortest route, it travels 11 m.

Assuming the mouse takes the shortest route, it travels $3 + 4\sqrt{2} = 8.66$ m.

The bee travels $\sqrt{53} = 7.28$ m.

Activity 5.1 (page 66)

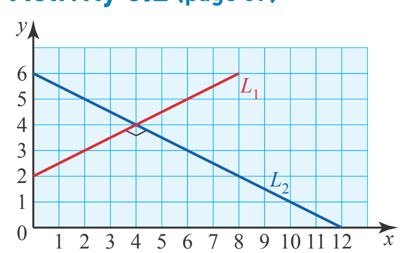
(i) M(5, 3)

(ii) $\sqrt{52}$

Discussion point (page 67)

No, the top and bottom lines of the fraction in the gradient formula would have the same magnitude but the opposite sign, so m would be unchanged.

Activity 5.2 (page 67)



$m_1 = \frac{1}{2}$; $m_2 = -2$

The gradients are negative reciprocals of each other; $m_1 m_2 = -1$.

This is true for any pair of perpendicular lines.

Exercise 5.1 (page 70)

1 (i) (a) (4, 6.5)

(b) 5

(c) $\frac{3}{4}$

(d) $-\frac{4}{3}$

(ii) (a) $(-4, -6.5)$

(b) 5

(c) $\frac{3}{4}$

(d) $-\frac{4}{3}$

(iii) (a) $(2, 1.5)$

(b) $\sqrt{233}$

(c) $\frac{13}{8}$

(d) $-\frac{8}{13}$

(iv) (a) $(2, -1.5)$

(b) $\sqrt{233}$

(c) $-\frac{13}{8}$

(d) $\frac{8}{13}$

2 $q = 5$

3 $y = 1$

4 (i) $\dfrac{2y}{x}$

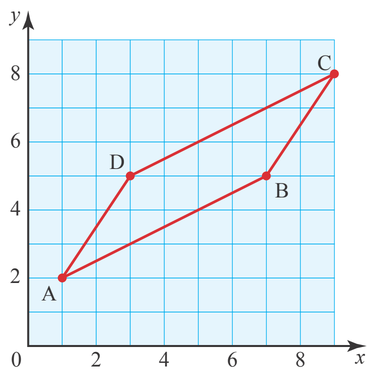
(ii) $(2x, 3y)$

(iii) $\sqrt{4x^2 + 16y^2}$

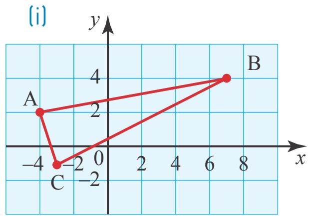
5 (i) AB: $\frac{1}{2}$, BC: $\frac{3}{2}$, CD: $\frac{1}{2}$, DA: $\frac{3}{2}$

(ii) Parallelogram

(iii)



6 (i)



(ii) $AB = BC = \sqrt{125}$

(iii) $(-3\frac{1}{2}, \frac{1}{2})$

(iv) 17.5 square units

7 (i) 6

(ii) $AB = \sqrt{20}$, $BC = \sqrt{5}$

(iii) 5 square units

8 (i) 1 or 5

(ii) 7

(iii) 9

(iv) 1

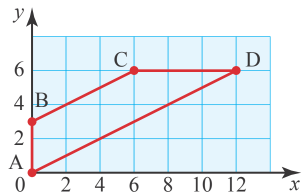
9 (i) 18

(ii) -2

(iii) 0 or 8

(iv) 8

10 (i)



(ii) Gradient BC = Gradient AD = $\frac{1}{2}$

(iii) $(6, 3)$

11 Diagonals have gradients $\frac{2}{3}$ and $-\frac{3}{2}$ so are perpendicular. Mid-points of both diagonals are $(4, 4)$ so they bisect each other.

52 square units.

12 (i) $\angle ABE = 90° - \theta$ so $\angle DBC = \theta$ which is the same as $\angle BAE$

Both triangles have a right angle.

Since AB = BC the triangles ABE and BCD are congruent (*AAS*: two angles and a side are equal).

(ii) Hence AE = BD and BE = DC

So $m_1 = \dfrac{BE}{AE}$ and

$m_2 = -\dfrac{BD}{DC} = -\dfrac{AE}{BE}$

So

$m_1 m_2 = \dfrac{BE}{AE} \times \left(-\dfrac{AE}{BE}\right)$

$= -1$ as required.

Activity 5.3 (page 73)

A $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$

Multiply both sides by $\dfrac{y_2 - y_1}{x - x_1}$ gives

$\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$

B $m = \dfrac{3 - 4}{5 - 2} = -\dfrac{1}{3}$

Now substitute into $y - y_1 = m(x - x_1)$

$$y - 4 = -\tfrac{1}{3}(x - 2)$$

$$3y - 12 = 2 - x$$

$$x + 3y - 14 = 0$$

Using $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$ gives:

$$\frac{y - 3}{4 - 3} = \frac{x - 5}{2 - 5}$$

$$\Rightarrow \frac{y - 3}{1} = \frac{x - 5}{-3}$$

$$\Rightarrow -3(y - 3) = x - 5$$

$$\Rightarrow -3y + 9 = x - 5$$

$$x + 3y - 14 = 0$$

Using $\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$ gives:

$$\frac{y - 3}{x - 5} = \frac{4 - 3}{2 - 5}$$

$$\frac{y - 3}{x - 5} = \frac{1}{-3}$$

$$-3(y - 3) = x - 5$$

$$-3y + 9 = x - 5$$

$$x + 3y - 14 = 0$$

Discussion point (page 73)

Equivalent forms include:

$3y = -x + 14$

$y = -\frac{1}{3}x + \frac{14}{3}$

It is tidier to write the equation of the line in a form that doesn't involve fractions.

Discussion points (page 75)

Interest earned on savings in a bank account (gradient gives the multiplier for the interest rate; simplifying assumption: money is not credited or debited from account, interest is simple; these assumptions are not realistic in real life, i.e. a savings account would normally have compound interest which does not fit a linear model)

Tax paid versus earnings (gradient gives tax rate; simplifying assumption: tax paid only at the lower rate)

Mass of candle versus length of time it is burning (gradient gives rate of change of mass;

simplifying assumption: candle is uniform thickness)

Cost of apples versus mass of apples (gradient gives cost per unit mass of apples; simplifying assumption: no discount offered for a bulk buy)

Distance travelled by car against time (gradient gives speed of car; simplifying assumption: car is travelling at constant speed; these assumptions are not realistic in real life, i.e. a car is unlikely to travel at constant speed)

Mobile phone bill against number of texts sent (gradient gives cost of one text; simplifying assumption: each text costs same amount, no calls or data used; these assumptions would only apply to a very simple pay-as-you-go tariff with no 'bonuses' or 'rewards' for topping up)

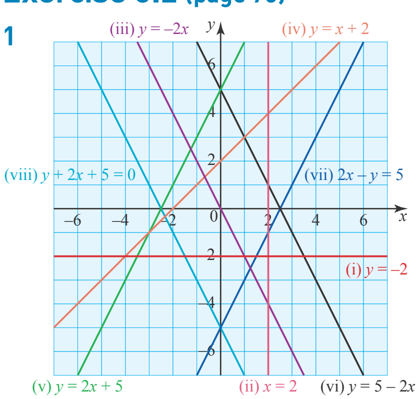
Profit of ice-cream seller against number of sales (gradient gives profit per ice-cream sold; simplifying assumption: profit on each sale is the same)

Mass of gold bars against volume of gold bars (gradient gives density of gold bars; simplifying assumption: all gold bars have same purity)

Length of spring versus mass of weights attached (gradient gives the extension of the spring per unit mass added)

Exercise 5.2 (page 75)

1



2 (i) $x = 7$
(ii) $y = 5$
(iii) $y = 2x$
(iv) $x + y = 2$
(v) $x + 4y + 12 = 0$

3 (i) $y = 3x$
(ii) $y = 3x - 1$
(iii) $y = 1 - 2x$
(iv) $y = 3x - 17$
(v) $x + 2y + 12 = 0$

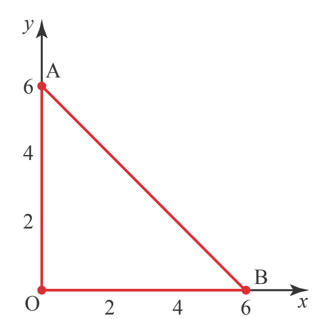
4 (i) $3y + x = 0$
(ii) $y = -\frac{1}{2}x + 5$
(iii) $y = \frac{1}{2}x - 5$
(iv) $y = -2x - 5$
(v) $y = \frac{3}{2}x + 3$

5 (i) $y = 3x - 8$
(ii) $y = 3x + 8$
(iii) $y = -3x - 8$
(iv) $y = 8 - 3x$
(v) $3y = x + 8$

6 The gradient of the lines $y = \frac{2}{3}x + 1$ and $3y - 2x + 1 = 0$ is $\frac{2}{3}$ so these lines are parallel.
The gradient of the lines $y = 1 - \frac{3x}{2}$ and $2y + 3x + 5 = 0$ is $-\frac{3}{2}$ so these lines are parallel.
$\frac{2}{3}\times\left(-\frac{3}{2}\right) = -1$ so the lines form a quadrilateral with 2 pairs of parallel lines and four right angles which is a rectangle.

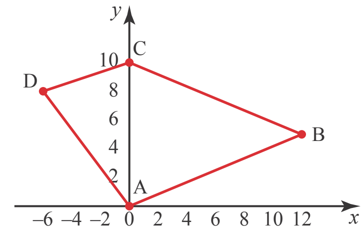
7 (i) $y = 7 - x$
(ii) $y = 7 - x$
(iii) $y = -x - 7$
(iv) $y = x + 7$
(v) $y = x - 7$

8 (i)



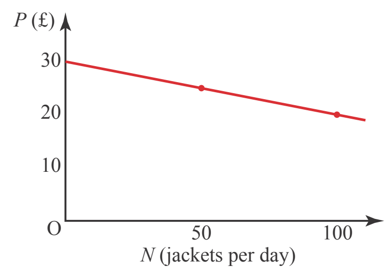
(ii) $y = x$; $x + 2y - 6 = 0$; $2x + y - 6 = 0$

9 (i)



(ii) AB: $\frac{5}{12}$; BC: $-\frac{5}{12}$; CD: $\frac{1}{3}$; AD: $-\frac{4}{3}$
(iii) AB $=13$; BC $=13$; CD $= \sqrt{40}$; AD $= 10$
(iv) AB: $5x - 12y = 0$; BC: $5x + 12y - 120 = 0$; CD: $x - 3y + 30 = 0$; AD: $4x + 3y = 0$
(v) 90 square units

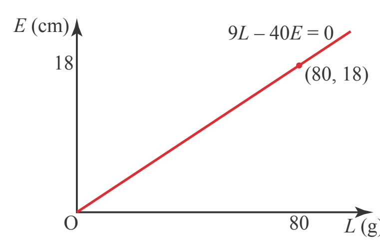
10 (i)



(ii) $10P + N = 300$
(iii) £21.20
(iv) 63

11 (i) $2x + y - 5 = 0$
(ii) 5 m
(iii) 5 m 59 cm

12 (i)



(ii) 10.8 cm
(iii) $44\frac{4}{9}$ g
(iv) $133\frac{1}{3}$ g

13 Take (x_1, y_1) to be $(0, b)$ and (x_2, y_2) to be $(a, 0)$.
The formula gives

$$\frac{y-b}{0-b} = \frac{x-0}{a-0}$$

$$\Rightarrow \frac{y-b}{-b} = \frac{x}{a}$$

$$\Rightarrow \frac{y-b}{b} = -\frac{x}{a}$$

$$\Rightarrow \frac{y}{b} - \frac{b}{b} = -\frac{x}{a}$$

$$\Rightarrow \frac{y}{b} - 1 = -\frac{x}{a}$$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} = 1$$

Discussion point (page 77)

The lines are parallel so there is no point of intersection.

Exercise 5.3 (page 78)

1 (i) $(\frac{1}{2}, 4)$
(ii) $(-2, 8)$
(iii) $(1, \frac{1}{2})$

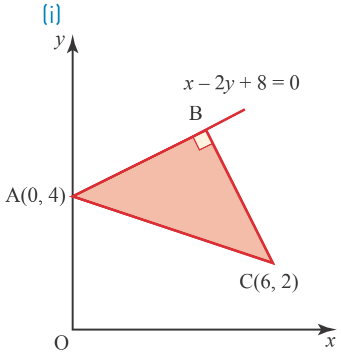
2 (i) (a) $(3, 2)$
(b) $(1, 3)$
(ii) $y = -\frac{1}{2}x + 1$ or
$y = -\frac{1}{2}x + 6$
(iii) 5 square units

3 (i) A $(1, 1)$; B $(5, 3)$;
C $(-1, 10)$
(ii) $BC = AC = \sqrt{85}$

4 (i) $y = \frac{1}{2}x + 1, y = -2x + 6$
(ii) Gradients $= \frac{1}{2}$ and -2
$\Rightarrow$ AC and BD are perpendicular.
Intersection $= (2, 2) =$ midpoint of both AC and BD.
(iii) $AC = BD = \sqrt{20}$
(iv) Square

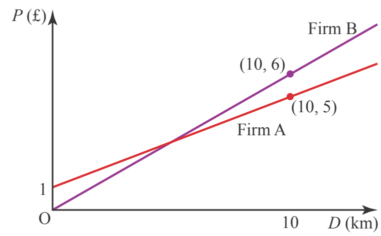
5 (i) $A(\frac{2}{5}, 0)$ B $(-2, 0)$
(ii) P(2, 8)
(iii) $\frac{48}{5}$ square units

6 (i)



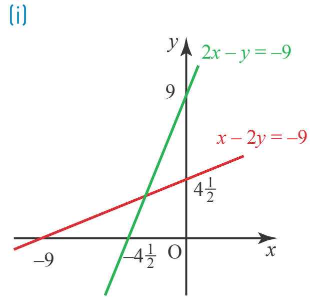
(ii) AC: $x + 3y - 12 = 0$,
BC: $2x + y - 14 = 0$
(iii) $AB = \sqrt{20}$, $BC = \sqrt{20}$,
area = 10 square units
(iv) $\sqrt{10}$

7 (i)



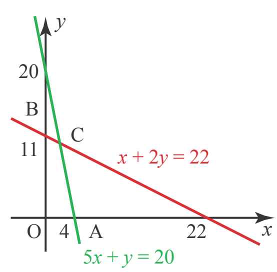
(ii) A: $2D - 5P + 5 = 0$,
B: $3D - 5P = 0$
(iii) 5 km
(iv) A

8 (i)



(ii) $(-3, 3)$
(iii) $2x - y = 3$; $x - 2y = 0$
(iv) $(-6, -3)$; $(5, 7)$

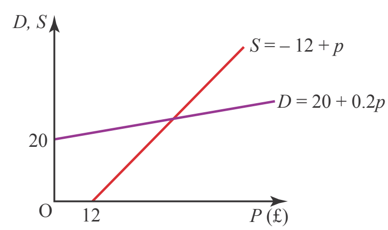
9 (i)



(ii) A: $(4, 0)$; B: $(0, 11)$;
C: $(2, 10)$
(iii) 11
(iv) $(-2, 21)$

10 (i) Supply:
$L - 500W + 500 = 0$;
Demand:
$L + 750W - 4750 = 0$
(ii) $L = 1600$; $W = 4.2$
(iii) Wage rate is the independent variable.

11 (i)



(ii) £40; 28 articles

12 (i) $(2, 4)$
(ii) $(0, 3)$

13 7.5 square units

Activity 5.4 (page 81)

(i) For the graph to look like a circle, the same scale needs to be used for both axes.

(ii) Sophie has used $y = \sqrt{(9 - x^2)}$ and so only has the part of the circle which lies above the x axis, i.e. where y is positive.

Activity 5.5 (page 81)

Multiplying out gives
$x^2 - 2ax + a^2 + y^2 - 2by + b^2 = r^2$.

Rearranging gives $x^2 + y^2 - 2ax - 2by + (a^2 + b^2 - r^2) = 0$

Discussion points (page 82)

Here are some hints: for the angle in a semicircle, join O to the vertex and use the fact that the triangles are isosceles, together with the angle sum of a triangle and the angle on a straight line.

For the perpendicular bisecting the chord, join O to both ends of the chord and prove that the triangles are congruent. For the tangent and radius, use a symmetry argument.

When triangle ABC has a right angle at B, then AC forms the diameter of a circle.

The centre of a circle lies on the perpendicular bisector of a chord.

The radius of a circle through a point is perpendicular to the tangent at that point.

Exercise 5.4 (page 85)

1 (i) $(x-2)^2+(y-3)^2=1$

(ii) $(x-2)^2+(y+3)^2=4$

(iii) $(x+2)^2+(y-3)^2=9$

(iv) $(x+2)^2+(y+3)^2=16$

2 (i) (a) (0, 0)

(b) radius: 1

(ii) (a) centre: (0, 2)

(b) radius: $\sqrt{2}$

(iii) (a) centre: (2, 0)

(b) radius: $\sqrt{3}$

(iv) (a) centre: (−2, −2)

(b) radius: 2

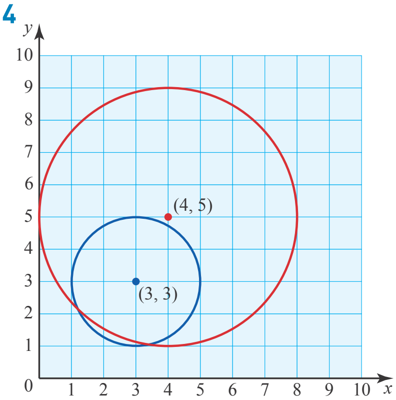
(v) (a) centre: (2, −2)

(b) radius: $\sqrt{5}$

3

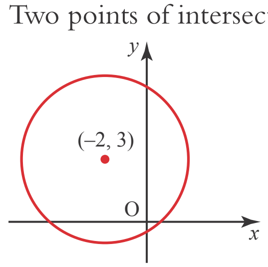
Point	Inside	Outside	On
(3, −2)	✓		
(−2, −5)		✓	
(6, −6)	✓		
(4, 3)			✓
(0, 2)	✓		
(−2, −3)			✓

4



Two points of intersection.

5



$x=-6, x=2, y=-1,$
$y=7$

6 (i) (±5, 0), (0, ±5)

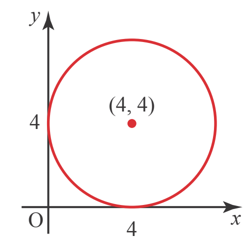
(ii) (0, −2), (0, −8), (4, 0)

(iii) (0, 0), (0, 16), (−12, 0)

7 $(x-1)^2+(y-7)^2=169$

8 $r=2$; (−1, 2); 2

9



$(x-4)^2+(y-4)^2=16$

11 (i) (a) (3, 1)

(b) $r=4$

(ii) (a) (−1, −3)

(b) $r=4$

(iii) (a) (1, −4)

(b) $r=3$

12 (i) $AB=\sqrt{20}$, $BC=\sqrt{45}$, $AC=\sqrt{65}$

(ii) $AB^2+BC^2=65=AC^2$

⇒ triangle ABC is right-angled, with the right angle at B (converse of Pythagoras' theorem)

⇒ B is angle in a semi-circle (converse of the angle in a semi-circle is a right angle)

i.e. AC is a diameter of the circle, as required.

(iii) 15

13 (i) (2, 11); $\sqrt{10}$

(ii) $(x-2)^2+(y-11)^2=10$

14 (i) Centre: (3, −1); B: (5, 0)

15 $\frac{169}{24}$ square units

16 $(x-5)^2+(y-4)^2=25$ or $(x-5)^2+(y+4)^2=25$

17 (i) $\sqrt{45}$

(ii) $(x-11)^2+(y-8)^2=50$

18 $(x-3)^2+(y-2)^2=25$

Discussion point (page 88)

Look for any like terms. You can see that x^2 appears in both equations, so you can eliminate x^2 by rewriting $y=x^2-4$ as $x^2=y+4$ and then substituting into the equation of the circle. This method is more efficient.

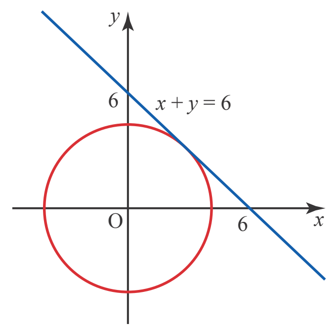
Exercise 5.5 (page 88)

1 (2, 7)

2 (1, 1); $(-\frac{1}{5}, -\frac{7}{5})$

3 (1, −2); the line forms a tangent to the circle

4 (ii)



(iii) (−3, −3); $x+y=-6$

5 (i) (1, 2); (−5, −10)

(ii) no real roots

6 (0, 5); (−3, 4). (0, 1); (1, 2); (2, 3)

7 (i) 14 cm

(ii) $x^2+(y-8)^2=100$

(iii) 264 cm² to 3 s.f., or 84π

8 (i) (−3, 4) and (4, −3); $\sqrt{98}$

(ii) No; the diameter of the circle is 10 and AB is $\sqrt{98}<10$, hence AB is a chord not a diameter.

9 (i) $k=2$

(ii) (2, 2)

10 $k=-6$ or $k=2$

11 5 square units

Practice questions 1 (page 94)

1 (i)

$$\sqrt{2\frac{2}{3}}=\sqrt{\frac{8}{3}}$$

$$=\sqrt{\frac{4\times 2}{3}} \quad [1]$$

$$=2\sqrt{\frac{2}{3}} \quad [1]$$

(ii) $\dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

$= \dfrac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$ [1]

$= \dfrac{4+2\sqrt{3}}{2} = 2+\sqrt{3}$ [1]

2 (i) $2^{3x} = 2^{2(x+4)}$ [1]

$3x = 2x + 8$ [1]

$x = 8$ [1]

(ii) Any value of x such that $x < 0$ [1]

3 (i) $x^2 - 4x + 1 = 7 - x^2$ [1]

$2x^2 - 4x - 6 = 0$ [1]

$x^2 - 2x - 3 = 0$ [1]

$(x-3)(x+1) = 0$ [1]

$(3, -2)$ and $(-1, 6)$ [1]

(ii) $x^2 - 4x + 1 = -2x$ [1]

$x^2 - 2x + 1 = 0$ [1]

$(x-1)^2 = 0$ [1]

Exactly one solution so the line is a tangent to the curve.

$(1, -2)$ [1]

4 (i) $(x^2 + 6x) + 7$ [1]

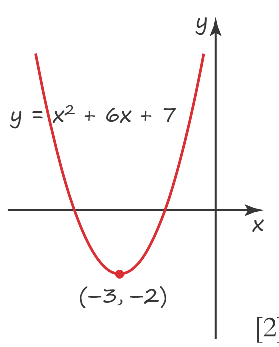
$((x+3)^2 - 9) + 7$ [1]

$(x+3)^2 - 2$ [1]

(ii) $(-3, -2)$ [1]

Minimum [1]

(iii)



[2]

$(x+3)^2 - 2 > 0$

$(x+3)^2 > 2$

$x + 3 > \sqrt{2}$ or

$x + 3 < -\sqrt{2}$

$x > -3 + \sqrt{2}$ or [1]

$x < -3 - \sqrt{2}$ [1]

5 (i) Centre lies on perpendicular bisector of AB [1]

Midpoint of AB is $(0.5, 2.5)$ [1]

Gradient of AB is $\frac{3}{3} = 1$ [1]

Gradient of perpendicular bisector is -1 [1]

Equation of perpendicular bisector is

$(y - 2.5) = -(x - 0.5)$ [2]

Centre on $x + y = 3$ [1]

(ii) Centre $(3, 0)$ [1]

$\text{Radius}^2 = 1^2 + 4^2 = 17$ [2]

Equation

$(x-3)^2 + y^2 = 17$ [2]

6 (i) $A(-2, 0)$ [1]

$B(2, 0)$ [1]

$OC^2 = BC^2 - OB^2$ [1]

$OC^2 = 16 - 4 = 12$ [1]

$C(0, \sqrt{12})$ or $(0, 2\sqrt{3})$ [1]

(ii) Gradient $-\dfrac{\sqrt{12}}{2} = -\sqrt{3}$ [1]

$y = \sqrt{12} - x\sqrt{3}$ [1]

$y = 2\sqrt{3} - x\sqrt{3}$

$= \sqrt{3}(2 - x)$ [1]

(iii) **Method 1**

GFC is an equilateral triangle (hence similar to ABC)

Suppose $GF = 4d$ [1]

The height of triangle GFC is $d\sqrt{12}$ [1]

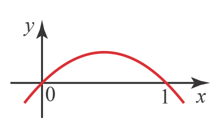
$FE = \sqrt{12} - d\sqrt{12}$

$= \sqrt{12}(1 - d)$ [1]

Area of rectangle

$= 4d\sqrt{12}(1 - d)$ [1]

$d(1 - d)$ is a quadratic [1]



Max at line of symmetry, i.e. when $d = 0.5$ [1]

Max area $= \sqrt{12}$ [1]

Method 2

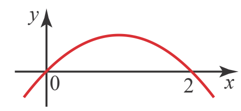
F has coordinates

$(x, \sqrt{3}(2 - x))$ [1]

Area of rectangle

$= 2x\sqrt{3}(2 - x)$ [1]

$x(2 - x)$ is a quadratic [1]



[2]

Max at line of symmetry, i.e. when $x = 1$ [1]

Max area $= \sqrt{12}$ [1]

7 (i) (a) The points for thinking distance lie in a straight line. [1]

(b) Distance travelled is proportional to speed at constant time so this means the thinking time is the same for all speeds. [1]

Reasonable as it is to do with reaction time. [1]

(c) $d = 0.3x$ [1]

(ii) (a) -9.609 m [1]

(b) It is not possible to have negative stopping distance. [1]

(iii) (a) 12.138 m [1]

(b) Possible reason such as

- The model does not include all factors.
- Distances in the Highway Code have been rounded. [1]

Chapter 6

Opening activity (page 97)

- Angle of elevation and height of lighthouse.
- Angle between train and mountain at each point, the angles of elevation of the point on the mountain as viewed from the train and the speed of the train, hence the distance between the points.

Exercise 6.1 (page 101)

1 (i) Converse of Pythagoras' theorem

(ii) $\frac{8}{17}, \frac{15}{17}, \frac{8}{15}$

2 (i) $\frac{3}{2}$

(ii) $\frac{1}{3}$

(iii) $\frac{1}{2}$

(iv) $\sqrt{3}$

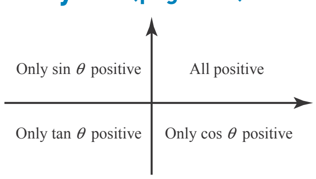
(v) 3

(vi) $\frac{3\sqrt{3}}{2}$

4 (i) 5 cm

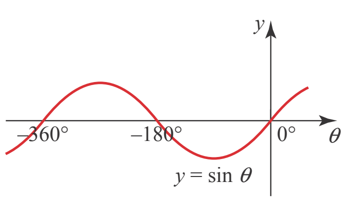
5 (i) $\frac{8}{9}\sqrt{3}$ cm

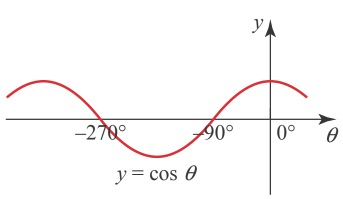
Activity 6.1 (page 103)



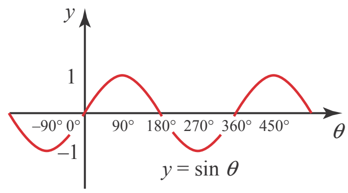
Discussion point (page 106)

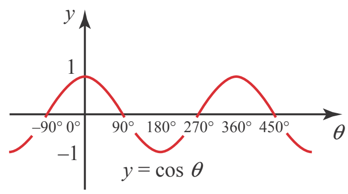
The oscillations continue to the left.

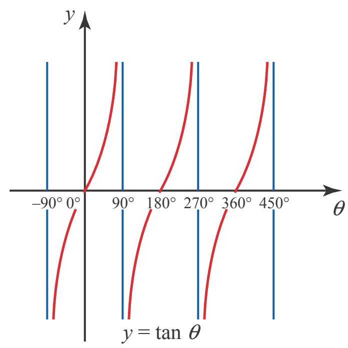




Activity 6.2 (page 107)







Discussion point (page 107)

$y = \sin\theta$: the graph has rotational symmetry order 2 about the origin.

$y = \cos\theta$: the graph is symmetrical about the y axis.

$y = \tan\theta$: the graph has rotational symmetry order 2 about the origin.

Exercise 6.2 (page 109)

1 (i) (a) $\frac{\sqrt{3}}{2}$

(b) $\frac{1}{2}$

(c) $\sqrt{3}$

(ii) (a) $-\frac{\sqrt{3}}{2}$

(b) $\frac{1}{2}$

(c) $-\sqrt{3}$

2 (i) (a) $\frac{\sqrt{3}}{2}$

(b) $-\frac{1}{2}$

(c) $-\sqrt{3}$

(ii) (a) $\frac{1}{2}$

(b) $-\frac{\sqrt{3}}{2}$

(c) $\frac{1}{\sqrt{3}}$

(iii) (a) $\frac{1}{\sqrt{2}}$

(b) $\frac{1}{\sqrt{2}}$

(c) 1

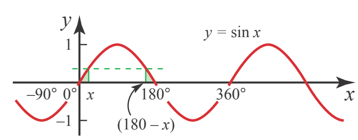
3 $\frac{25\sqrt{3}}{2}$

4 (i) α between 0° and 90°, 360° and 450°, 720° and 810°, etc. (and corresponding negative values)

(ii) No: since $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$, all must be positive or one positive and two negative

(iii) No: $\sin\alpha = \cos\alpha$ $\Rightarrow \alpha = 45°, 225°$, etc. but $\tan\alpha = \pm 1$ for these values of α, and $\sin\alpha = \cos\alpha = \frac{1}{\sqrt{2}}$

5 (i)



shaded areas are congruent

(ii) (a) False

(b) True

(c) False

(d) True

6 (i) $\frac{4}{5}$

(ii) $\frac{4}{3}$

7 (i) $-\frac{1}{2}$

(ii) $-\frac{1}{\sqrt{3}}$

8 (i) $-\frac{\sqrt{7}}{4}$

(ii) $-\frac{3}{\sqrt{7}}$

9 (i) $\sin\theta$

(ii) $\cos\theta$

(iii) 2

11 (i) $\sqrt{1-k^2}$

(ii) $\dfrac{\sqrt{1-k^2}}{k}$

12 n is an integer

(i) $(90°n - 45°)$

(ii) $(180°n - 90°)$ and $180°n$

(iii) $180°n - 90°$ and $180°n$

(iv) $180°n - 90°$

Discussion point (page 111)

Add or subtract multiples of 180°.

Discussion point (page 114)

The graph crosses any horizontal line (in this case $y = \sqrt{3}$) four times.

Exercise 6.3 (page 115)

1 (i) 120°, 300°
(ii) 30°, 210°
(iii) 60°, 300°
(iv) 120°, 240°
(v) 45°, 135°
(vi) 225°, 315°

2 (i) 36.9°, 143.1°
(ii) 216.9°, 323.1°
(iii) 53.1°, 306.9°
(iv) 126.9°, 233.1°
(v) 31.0°, 211.0°
(vi) 149.0°, 329.0°

3 (i) −45.6°
(ii) –158.2°
(iii) 53.1°

4 (i) 60°, 300°
(ii) 199.5°, 340.5°
(iii) 60°, 120°, 240°, 300°
(iv) 0°, 180°, 360°
(v) 18.4°, 71.6°, 198.4°, 251.6°
(vi) 180°

5 (i) 60°, 180°, 300°
(ii) 0°, 90°, 270°, 360°
(iii) 0°, 180°, 360°
(iv) 54.7°, 125.3°, 234.7°, 305.3°
(v) 60°, 300°
(vi) 120°, 240°

6 (i) 70°, 310°
(ii) 10°, 190°
(iii) 30°, 120°
(iv) 10°, 70°, 130°, 190°, 250°, 310°
(v) 120°, 240°
(vi) 60°, 120°, 240°, 300°
(vii) 20°, 100°, 140°, 220°, 260°, 340°
(viii) 120°, 150°, 300°, 330°
(ix) 0°, 60°, 120°, 180°, 240°, 300°, 360°

7 9°, 45°, 81°

8 A = (38.2°, 0.786), B = (141.8°, −0.786)

9 (i) 26.6°, 206.6°
(ii) 45°, 225°
(iii) 54.7°, 125.3°, 234.7°, 305.3°

10 (i) 45°, 75°, 225°, 255°
(ii) 13.3°, 103.3°, 193.3°, 283.3°
(iii) 15°, 75°, 135°, 195°, 255°, 315°
(iv) 75°, 165°, 255°, 345°
(v) 60°, 300°
(vi) 90°, 270°

11 90°, 270°

Discussion point (page 119)

$a^2 = b^2 + c^2 - 2bc\cos A$

$\Rightarrow 2bc\cos A = b^2 + c^2 - a^2$

$\Rightarrow \cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Exercise 6.4 (page 121)

1 (i) 8.0 m
(ii) 7.4 cm

2 (i) 10.14 cm
(ii) 5.57 cm

3 (i) 42.8°
(ii) 47.9°
The diagram shows that θ is acute, so 132.1° is not relevant.

4 (i) 57.1°
(ii) 97.4°

5 (i) 5 cm
(ii) 90.7°

6 8.8 km

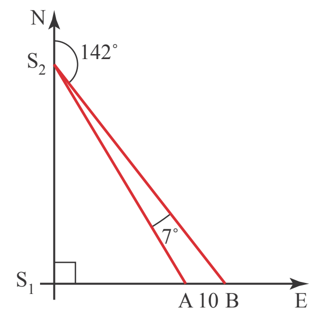
7 10.7 km

8 3.28 km

9 (i) 18.6°
(ii) 76.9 m
(iii) 35.6 m

10 (i) 1011 m
(ii) 1082 m
(iii) 065°

11 (i)



(ii) 64.7 km
(iii) 27.7 km h^{-1}

12 14.6 km/h

13 (i) 3.72 km
(ii) 3.32 km
(iii) 94.8 km h^{-1}

14 (i) QR = QT = a, RT = $a\sqrt{2-\sqrt{3}}$

Activity 6.3 (page 124)

1 (i) $\frac{1}{2}bc\sin A$
(ii) $\frac{1}{2}ac\sin B$
(iii) $\frac{1}{2}ab\sin C$

2 Each expression gives the area of the triangle which is the same however it is worked out.

3 $\frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C$;
$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$;
the sine rule

Exercise 6.5 (page 125)

1 (i) 7.21 cm²
(ii) 8.45 cm²
(iii) 6.77 cm²
(iv) 6.13 cm²

2 (i) 2.25 m²
(ii) 0.3375 m³

3 27.4°, 152.6°

4 77.94 cm²

5 (i) 4.35 m
(ii) 7.38 m
(iii) 47.38 m²
(iv) 7.29 m

6 11 011 m²

7 5412 m²

Chapter 7

Discussion points (page 131)

Order $m + n$

Line A: $x^3 + 3x - 2$ has been multiplied by x^2

Line B: $x^3 + 3x - 2$ has been multiplied by $-2x$

Line C: $x^3 + 3x - 2$ has been multiplied by -4

Line D: lines A, B and C have been added together

The multiplication has been laid out in columns so that each column contains a different power of x.

Discussion point (page 134)

If there is a factor $(x - a)^3$, the curve is horizontal at the x axis but crosses it. If there is a factor $(x - a)^4$, the curve touches the x axis, but is flatter than if there were a factor $(x - a)^2$.

Exercise 7.1 (page 136)

1 (i) 3
(ii) 4
(iii) 2

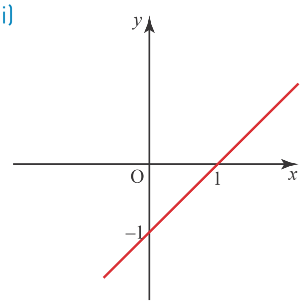
2 (i) $2x^3$
(ii) $2x^2 + 6x + 4$
(iii) $-2x^2 - 6x - 4$

3 (i) $2x^3 + 7x^2 + 9x + 10$
(ii) $9x^2 + 9x + 10$
(iii) $2x^3 + 7x^2 + 12x + 17$
(iv) $6x^2 + 7x + 9$

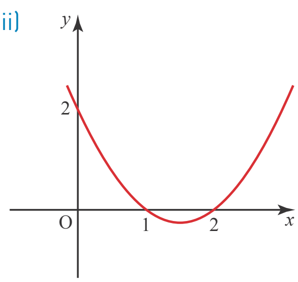
4 (i) $x^4 + 4x^3 + 6x^2 + 4x + 1$
(ii) $x^3 - 7x + 6$

5 (i) D
(ii) A
(iii) C
(iv) B

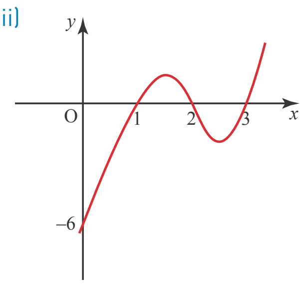
6 (i)



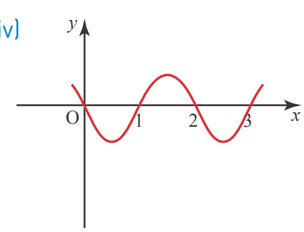
(ii)



(iii)



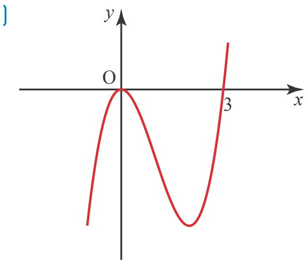
(iv)



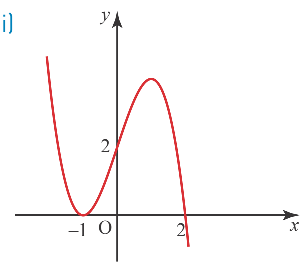
7 (i) $-2x^2 + 2x$
(ii) $10x^2$

8 (i) $x^3 - 7x - 6$
(ii) $-2x^3 + 5x^2 + 4x - 3$

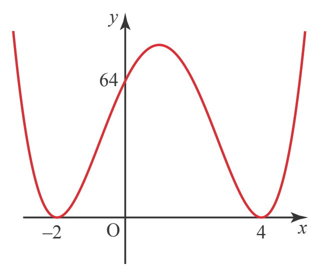
9 (i)



(ii)

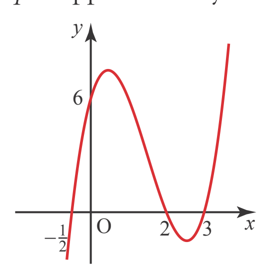


(iii)



10 (i) For example, $x = 7$ gives $y = 21$, which does not fit the graph.
(ii) k is approximately $\frac{1}{3}$.
(iii) The graph is fairly flat near the origin, and Fatima's equation gives a repeated root at $x = 0$.
(iv) p is approximately 0.05.

11 (i)



(ii) $f(x) = 2x^3 - 9x^2 + 7x + 6$

12 (i) Possibly $y = -(x - 2)^2(x - 4)$.
Could be any multiple of this since no y axis intercept is given.
(ii) Possibly $y = \frac{1}{2}(x + 1)^2(x - 2)^2$.

Other equations like $y = \frac{1}{2}(x+1)^4(x-2)^2$ would fit the intersections with the axes, but substituting $x = 1$ shows that this does not fit the rest of the graph.

Discussion points (page 138)

Order 3

Order $m - n$

Exercise 7.2 (page 139)

1 (i) $x + 3$
(ii) $x - 1$
(iii) $x^2 + 3x$

2 (i) $3x + 1$
(ii) $5x - 4$
(iii) $2x + 1$

3 (i) $2x - 1$
(ii) $4x - 3$
(iii) $2x - 3$

4 (i) $x^2 + x + 2$
(ii) set $x = 10$
(iii) set $x = 5$

5 (i) $x^2 + 3x + 2$
(ii) $2x^2 - 5x + 5$

6 (i) $x^2 + x + 2$
(ii) $2x^2 + 3$

7 (i) $x^2 + 3x$
(ii) $x^3 + x^2 + 2x + 2$
(iii) $2x^2 + 3$

8 (i) $x^3 - x^2 + 2x - 2$
(ii) $3x^3 + 2x^2 + 2x + 4$

Activity 7.2 (page 140)

1.689

Discussion point (page 142)

Any integer root must be a factor (positive or negative) of the constant term.

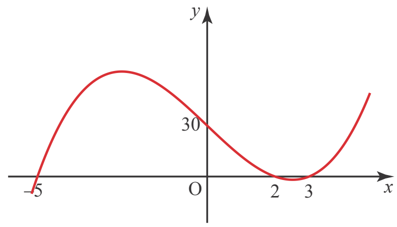
Exercise 7.3 (page 144)

3 $x = 0$

4 $k = -4$

5 Show substituting gives a non-zero answer in each case.

6 (i) $f(0) = 30; f(3) = 0; (x - 3)$
(ii) $p = 2; q = -15$
(iii) $x = 2, 3, -5$
(iv)



7 $a = 2, 3$

8 $a = 0,\ b = -7, (x + 3)$

9 $x = -\frac{1}{2}, 1, 3$

10 $4p + q = -26; 9p + q = 39;$
$p = 13, q = -78$

11 $(x + 2)(x + 3)(x - 4)$

12 (i) $f(x) = (x - 1)^2(x + 4)$
(ii) $A(-4, 0); B(1, 0); C(0, 4)$
(iii) $(2\frac{1}{4}, 9\frac{49}{64})$

13 (i) $\frac{18}{x^2}$
(iv) $x = 3$ or 3.62 (3 s.f.) or -6.62 (3 s.f.)
Dimensions $3 \times 3 \times 2$ or $3.62 \times 3.62 \times 1.37$ (3 s.f.)

Chapter 8

Discussion point (page 146)

For the solid curve you can consider it to be a two part function consisting of 2 quadratic functions.

$f(x) = 0.32x(1 - x)$ for $0 \leqslant x \leqslant 1$

and $f(x) = 0.32(x - 1)(x - 2)$ for $1 < x \leqslant 2$.

Likewise for the dashed curve:

$g(x) = -0.32x(1 - x)$ for $0 \leqslant x \leqslant 1$

and $g(x) = -0.32(x - 1)(x - 2)$ for $1 < x \leqslant 2$.

Activity 8.1 (page 149)

(i) A: $y = x^3$; B: $y = x^2$; C: $y = x^4$ and D: $y = x^5$

(ii) (a) When n is even the curve is U shaped. As n increases the curve is flatter at the origin and increases more steeply.

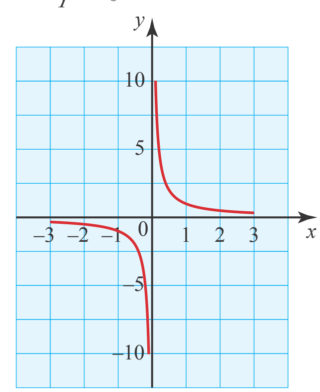
(b) When n is odd the curve is ~-shaped. As n increases the curve is flatter at the origin and increases more steeply.

(iii) They are reflections in the x axis of the curves of $y = x^n$.

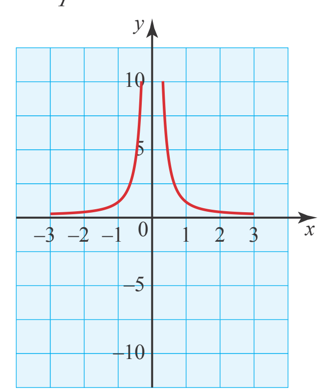
Exercise 8.1 (page 151)

1 1E, 2D, 3C, 4A, 5B

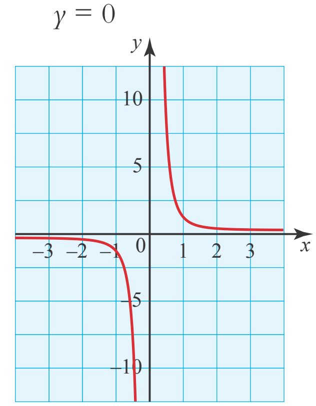
2 (i) asymptotes at $x = 0$ and $y = 0$



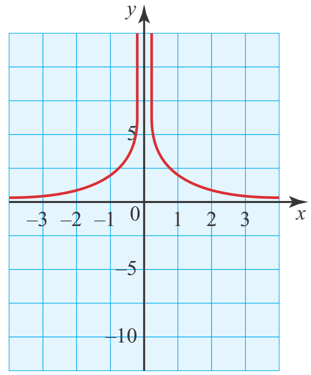
(ii) asymptotes at $x = 0$ and $y = 0$



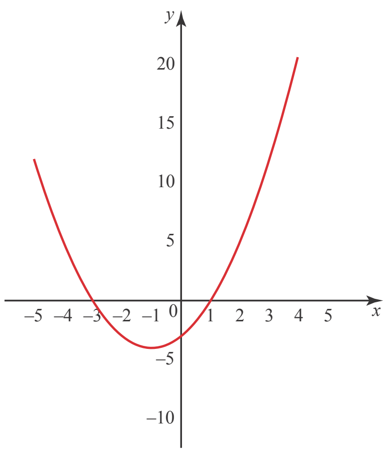
(iii) asymptotes at $x = 0$ and $y = 0$



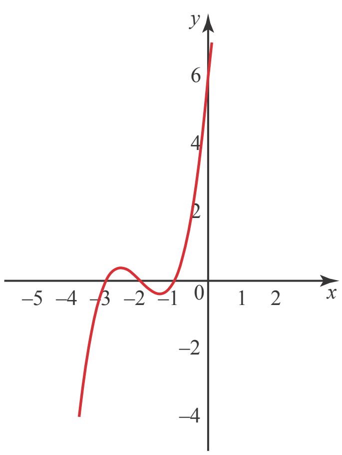
(iv) asymptotes at $x = 0$ and $y = 0$



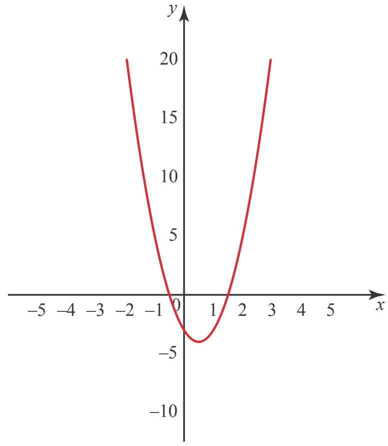
3 **(i)** intersects axes at $(1, 0)$, $(-3, 0)$ and $(0, -3)$



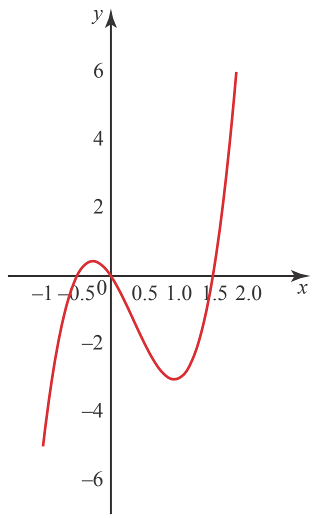
(ii) intersects axes at $(-1, 0)$, $(-2, 0)$, $(-3, 0)$ and $(0, 6)$



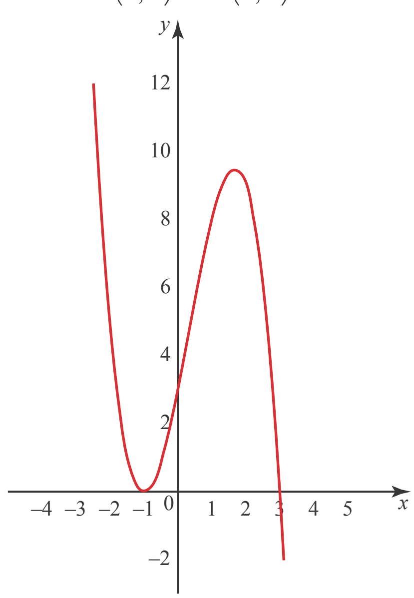
(iii) intersects axes at $(-\frac{1}{2}, 0)$, $(\frac{3}{2}, 0)$ and $(0, -3)$



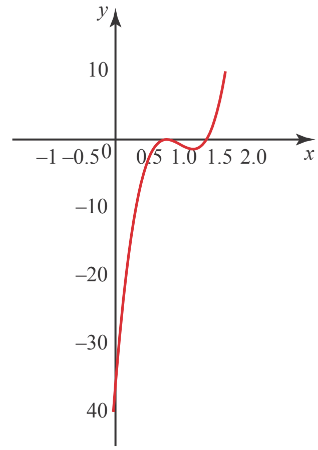
(iv) intersects axes at $(-\frac{1}{2}, 0)$, $(0, 0)$ and $(\frac{3}{2}, 0)$



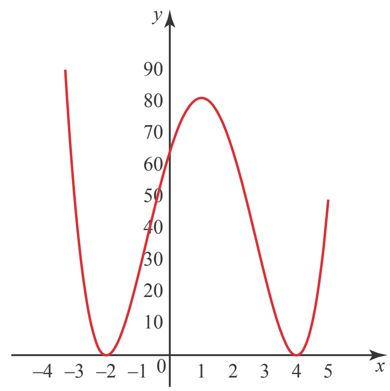
4 **(i)** intersects axes at $(-1, 0)$, $(3, 0)$ and $(0, 3)$



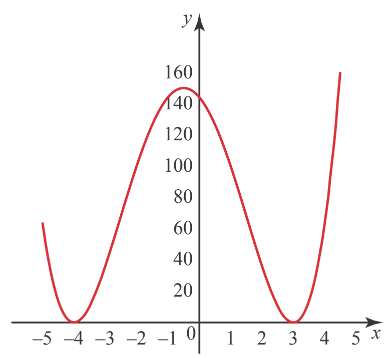
(ii) intersects axes at $(\frac{3}{4}, 0)$, $(\frac{4}{3}, 0)$ and $(0, -36)$



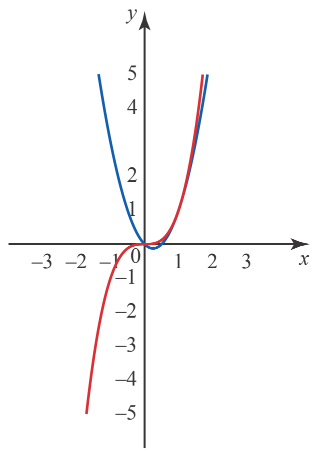
(iii) intersects axes at $(-2, 0)$, $(4, 0)$ and $(0, 64)$



(iv) intersects axes at $(-4, 0)$, $(3, 0)$ and $(0, 144)$



5 (i)



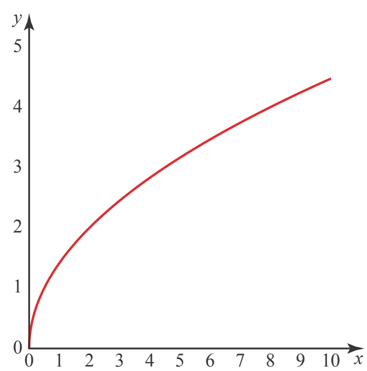
(ii) 2

(iii) (a) (0, 0)

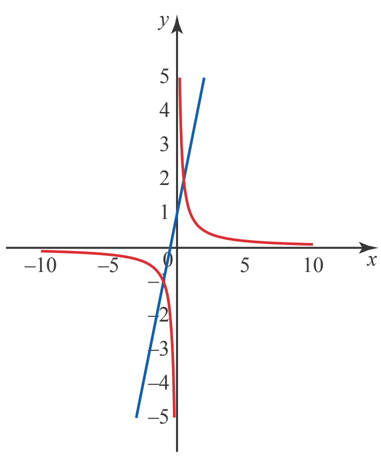
(b) (1, 1)

6 (i) $y = \sqrt{2x}$

(ii)



7 (i)



(ii) $(-1, -1), (\frac{1}{2}, 2)$

(iii) $y = \frac{1}{x}$ shows a proportional relationship as the product of x and y is a constant.

$y = 2x + 1$ does not show direct proportion as it doesn't pass through the origin.

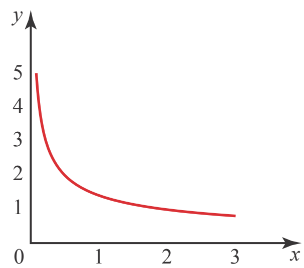
8 (i) $y = \frac{3}{x^2}$

(ii) $y = 0.03$

(iii) $x = \pm 2$

9 (i) $y = \sqrt{\frac{2}{x}}$

(ii)



10 $2, \left(\frac{\sqrt{2}}{2}, 2\sqrt{2}\right), \left(-\frac{\sqrt{2}}{2}, -2\sqrt{2}\right)$

Activity 8.2 (page 153)

(i) All the graphs are translations of each other in the y direction.

(ii) All the graphs are translations of each other in the x direction.

Activity 8.3 (page 155)

A $y = -f(x)$ is a reflection of $y = f(x)$ in the x axis

B $y = f(-x)$ is a reflection of $y = f(x)$ in the y axis

Activity 8.4 (page 157)

A The graphs are stretched in the y direction.

B The graphs are stretched in the x direction.

Discussion point (page 158)

A stretch in the negative direction, so this means that the graph will be reflected in the y axis as well as stretched.

Exercise 8.2 (page 159)

1

x	−3	−1	0	2
f(x)	3	5	2	−1
f(x) + 2	5	7	4	1
f(x) − 2	1	3	0	−3
2f(x)	6	10	4	−2
−2f(x)	−6	−10	−4	2

2 (i) translation by $\begin{pmatrix} -5 \\ 0 \end{pmatrix}$

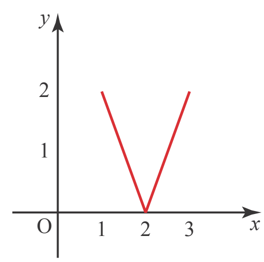
(ii) one way stretch, scale factor $\frac{1}{5}$, parallel to the x axis

(iii) one way stretch, scale factor 5, parallel to the y axis

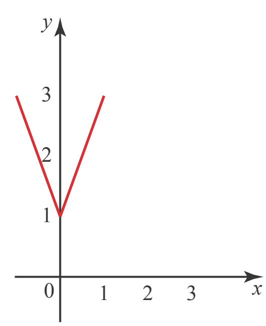
(iv) reflection in the y axis

(v) translation by $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$.

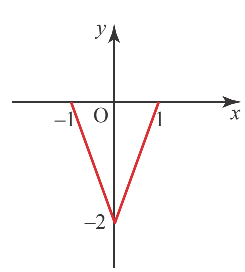
3 (i)



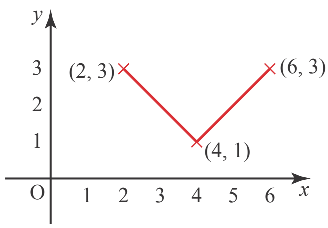
(ii)



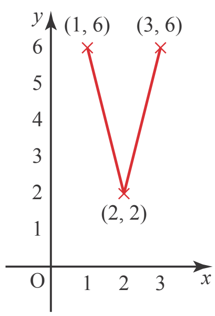
(iii)



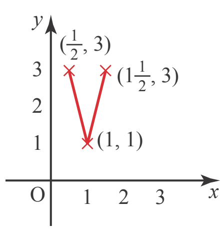
(iv)



(v)

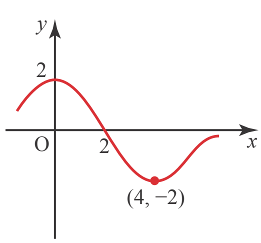


(vi)

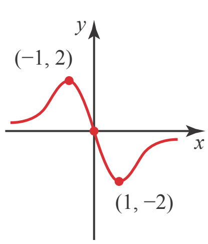


4 (i) 9
(ii) 5
(iii) 1
(iv) 13
(v) 15
(vi) 3.5

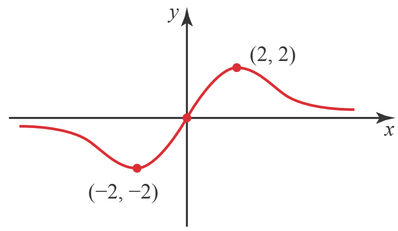
5 (i)



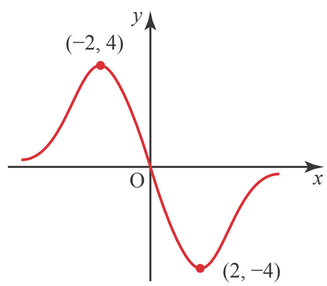
(ii)



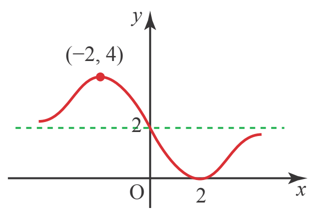
(iii)



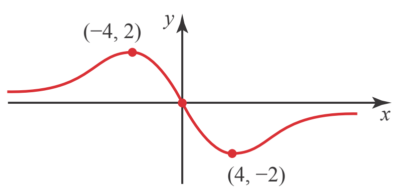
(iv)



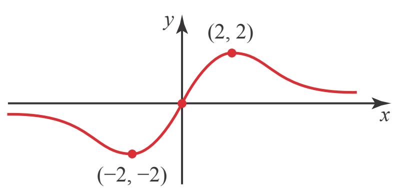
(v)



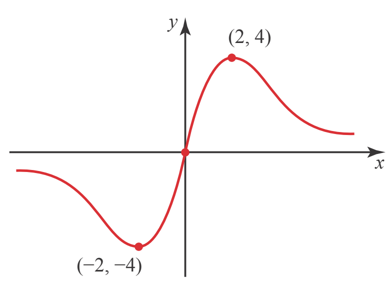
(vi)



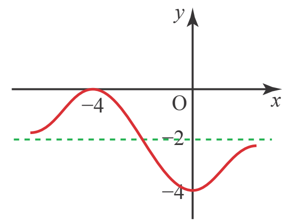
(vii)



(viii)

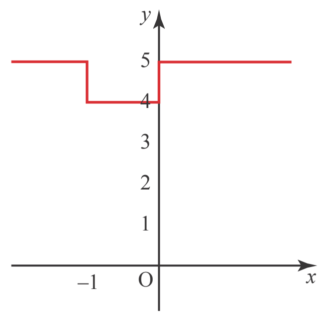


(ix)

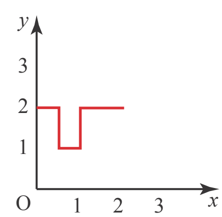


6 (i) (a) $y = \mathrm{g}(x) + 1$
(b) $y = \mathrm{g}(x - 1) - 1$
(c) $y = 2\mathrm{g}(x)$
(d) $y = \mathrm{g}(\frac{1}{3}x)$

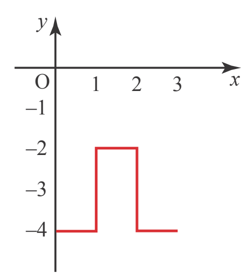
(ii) (a)



(b)



(c)



7 (i) (a) (1, 0), (−3, 0), (0, 3), (−1, 4)
(b) (−1, 0), (3, 0), (0, −3), (1, −4)
(c) $(-\frac{1}{3}, 0)$, (1, 0), (0, 3), $(\frac{1}{3}, 4)$
(d) (−1, 0), (3, 0), $(0, -\frac{3}{2})$, (1, −2)

(ii) $a = 1$

8 (i) $y = \mathrm{f}(x + 2)$
(ii) $y = -\mathrm{f}(x)$

(iii) $y = \mathrm{f}\left(\frac{x}{2}\right)$

(iv) $y = \mathrm{f}(x) - 3$

(v) $y = \mathrm{f}(-x)$ (or $y = 2 - \mathrm{f}(x)$)

(vi) $y = \frac{3}{2}\mathrm{f}(x)$

Discussion points (page 162)

Completing the square uses an approach starting from the algebra of the equation in the form $y = ax^2 + bx + c$. Using transformations starts best from the graph, identifying the equation as $y = (x - p)^2 + q$.

Yes. An identity is true for all values of the variable so any particular values can be substituted to give an equation.

Exercise 8.3 (page 163)

1 (i) (a) $y = x^3 + 4$

(b) $y = (x + 3)^3$

(c) $y = (x - 3)^3 - 4$

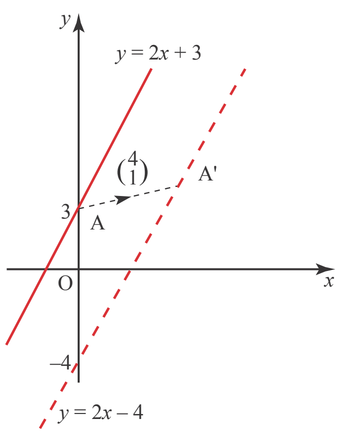
(ii) (a) $y = 2x^3$

(b) $y = \frac{1}{8}x^3$

(c) $y = 3x^3$

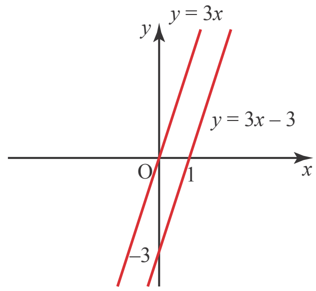
(d) $y = 8x^3$

2 (i)



(ii) $(y - 1) = 2(x - 4) + 3$; $y = 2x - 4$

3 (i)



(ii) -3

(iii) 1

4 (i) $y = 2x - 10$

(ii) $y = 2x + 10$

(iii) $y = 2x$

5 (i) one way stretch scale factor -2 parallel to the y axis

(ii) translation by the vector $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$

(iii) translation by the vector $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

(iv) translation by the vector $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$

6 (i) one way stretch, stretch factor 4 parallel to the y axis or one way stretch, stretch factor $\frac{1}{2}$ parallel to the x axis.

(ii) one way stretch, stretch factor $\frac{1}{3}$ parallel to the y axis or one way stretch, stretch factor $\sqrt{3}$ parallel to the x axis.

7 (i) $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$; $x = 0$

(ii) $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$; $x = -4$

(iii) $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$; $x = 0$

(iv) $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$; $x = 3$

(v) $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$; $x = 4$

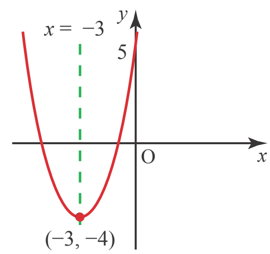
(vi) $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$; $x = -3$

(vii) $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$; $x = 2$

(viii) $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$; $x = 2$

8 (i) $a = 3$ and $b = -4$

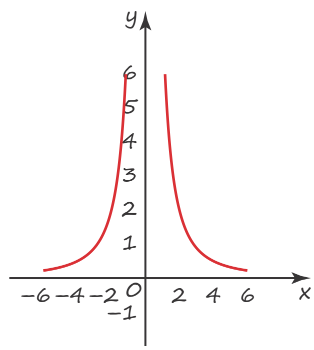
(ii)



9 $y = 3(x - 4)(x - 1)(x + 2)$

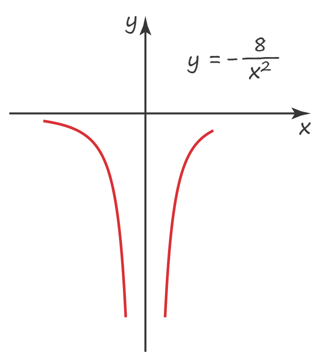
10 (i) $y = \frac{8}{x^2}$

(ii)



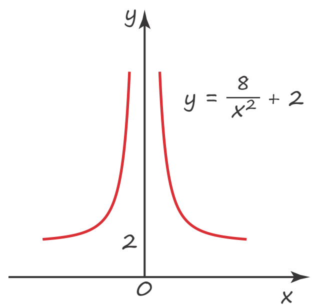
Asymptotes at $x = 0$ and $y = 0$

(iii) (a)



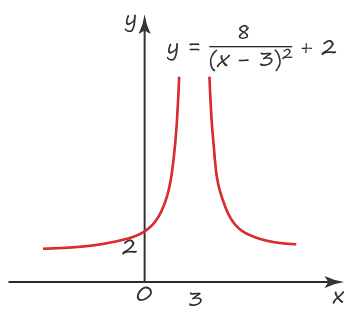
Asymptotes at $x = 0$ and $y = 0$

(b)



Asymptotes at $x = 0$ and $y = 2$

(c)



Crosses y axis at $(0, 2\frac{8}{9})$
Asymptotes at $x = 3$ and $y = 2$

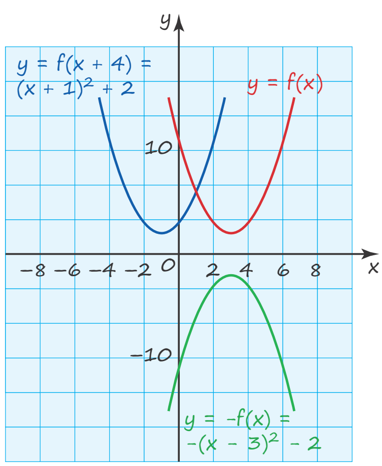
(iv) (a) $x = 5$

(b) $(5, 4)$

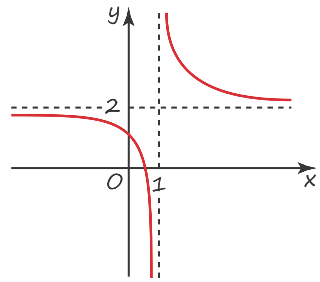
11 $y = x^2 + 20x + 91$

12 (i) $p = 3$ and $q = 2$

(ii)

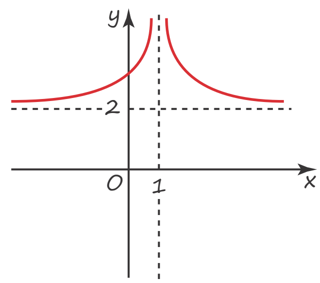


13 (i)



$y = \frac{1}{x-1} + 2$. Asymptotes $x = 1$, $y = 2$

(ii)



$y = \frac{1}{(x-1)^2} + 2$.

Asymptotes $x = 1$, $y = 2$

Exercise 8.4 (page 167)

1 (i) $-1 \leqslant f(x) \leqslant 1$

(ii) $-3 \leqslant f(x) \leqslant 3$

(iii) $-2 \leqslant f(x) \leqslant 0$

(iv) $2 \leqslant f(x) \leqslant 4$

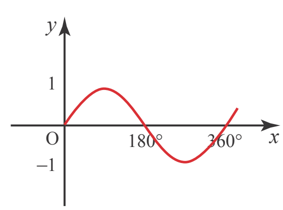
2 (i) 360°

(ii) 180°

(iii) 720°

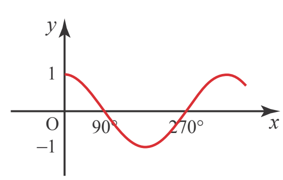
(iv) 120°

3 (i) (a)



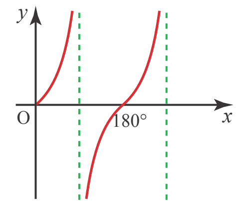
(b) $y = \sin x$

(ii) (a)



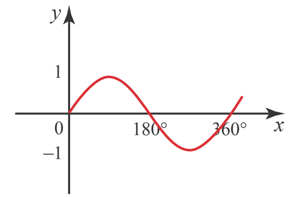
(b) $y = \cos x$

(iii) (a)



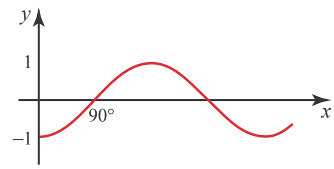
(b) $y = \tan x$

(iv) (a)



(b) $y = \sin x$

(v) (a)



(b) $y = -\cos x$

4 (i) Translation $\begin{pmatrix} 90° \\ 0 \end{pmatrix}$

(ii) One way stretch parallel to x axis of s.f. $\frac{1}{2}$

(iii) One way stretch parallel to y axis of s.f. $\frac{1}{2}$

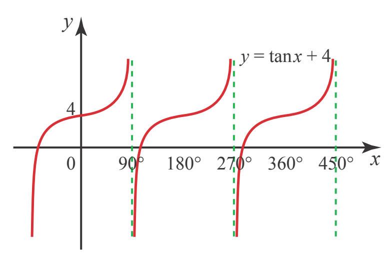
(iv) One way stretch parallel to x axis of s.f. 2

(v) Translation $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

(vi) Reflection in the x axis

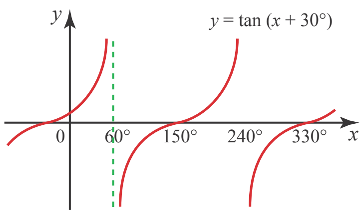
5 (i) (a) $y = \tan x + 4$

(b)



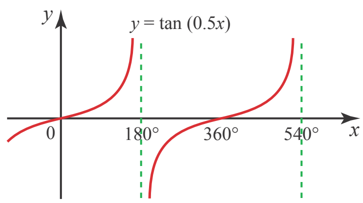
(ii) (a) $y = \tan(x + 30°)$

(b)



(iii) (a) $y = \tan 0.5x$

(b)



6 (i) $(120°, 0), (0, \frac{\sqrt{3}}{2})$

(ii) $(60°, 0), (0, -\sqrt{3})$

(iii) $(90°, 0), (0, \frac{1}{2})$

(iv) $(360°, 0), (0, 0)$

(v) $(180°, 0), (0, 2)$

7 (i) $y = \cos(x - 45°)$; $y = \sin(x + 45°)$

(ii) P $(0, \frac{\sqrt{2}}{2})$, Q$(45, 1)$

8 (i) $y = 4\sin x$

(ii) $-2\sqrt{3}$

9 (i) False

(ii) True

(iii) True

(iv) False

(v) True

Chapter 9

Opening activity (page 170)

6 routes from A to B

4 routes from A to C

Discussion points (page 176)

Yes

No

A Pascal puzzle (page 176)

1.61051. This is $1 + 5 \times (0.1) + 10 \times (0.1)^2 + 10 \times (0.1)^3 + 5 \times (0.1)^4 + 1 \times (0.1)^5$ and 1, 5, 10, 10, 5, 1 are the binomial coefficients for $n = 5$.

Exercise 9.1 (page 177)

1 (i) $1 + 3x + 3x^2 + x^3$

(ii) $x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$

(iii) $x^4 + 8x^3 + 24x^2 + 32x + 16$

2 (i) $27 + 27x + 9x^2 + x^3$

(ii) $16x^4 - 96x^3 + 216x^2 - 216x + 81$

(iii) $x^5 + 10x^4y + 40x^3y^2 + 80x^2y^3 + 80xy^4 + 32y^5$

3 (i) 15

(ii) 1

(iii) 220

(iv) 1

(v) 36

4 (i) 35

(ii) −120

(iii) 180

(iv) 135

(v) 70

5 $12x + 18x^2 + 36x^3 + 15x^4$

6 $1 - 9x + 27x^2 - 27x^3$; $2 - 17x + 45x^2 - 27x^3 - 27x^4$

7 (i) $x^3 - 6x^2 + 12x - 8$

(ii) $x = 0, 2$

8 (i) $1 - 6x + 12x^2 - 8x^3$

(ii) $1 - 3x - 6x^2 + 28x^3 - 24x^4$

9 (i) $1 + 3x + 3x^2 + x^3$

(ii) $1 + 3y - 5y^3 + 3y^5 - y^6$

10 (i)

1	2					
1	3	2				
1	4	5	2			
1	5	9	7	2		
1	6	14	16	9	2	
1	7	20	30	25	11	2

(ii) Row sums are 3, 6, 12, 24, 48, 96, i.e. start with 3 and double, whereas the 'normal' Pascal's triangle starts with 2 and doubles.

(iii)

1	3					
1	4	3				
1	5	7	3			
1	6	12	10	3		
1	7	18	22	13	3	
1	8	25	40	35	16	3

Row sums are 4, 8, 16, …, i.e. start with 4 and double

(iv) Starting with 1, n has row sums $(1 + n), 2(1 + n), 4(1 + n), 8(1 + n)$, etc.

Discussion point (page 178)

Gary could have put the bricks in order by chance. A probability of $\frac{1}{120}$ is small but not very small. What would be really convincing is if he could repeat the task whenever he was given the bricks.

Discussion point (page 179)

No, it does not matter.

Discussion point (page 180)

Multiply top and bottom by 53!

$$\frac{59 \times 58 \times 57 \times 56 \times 55 \times 54}{6!} \times \frac{53!}{53!}$$

$$= \frac{59!}{6!53!}$$

This works out to be 45 057 474.

Discussion point (page 180)

By following the same argument as for the National Lottery example but using n instead of 59 and r instead of 6.

Exercise 9.2 (page 181)

1 24

2 $\frac{1}{593775}$

3 40 320

4 715

5 (i) 120

(ii) $\frac{1}{120}$

6 280

7 (i) 14!

(ii) $\frac{1}{14!}$

8 (i) 715

(ii) 5

(iii) $\frac{1}{143}$

(iv) The applicants are all equally suitable and so the jobs are given at random.

9 (i) 126

(ii) (a) $\frac{1}{126}$

(b) $\frac{45}{126}$

10 (i) $\frac{1}{120}$

(ii) $\frac{1}{7893600}$

11 (i) $\frac{1}{10!}$

(ii) Lose 7.2p

(iii) £362 000 (to the nearest £1000)

12 (i) 495

(ii) 45

(iii) $\frac{1}{11}$

Practice questions 2

(page 184)

1 (i) Even, because $y > 0$ for negative and positive large x [1]

(ii) x as factor [1]

$(x + 1)$ as factor [1]

$(x - 2)$ as factor [1]

$y = x(x + 1)(x - 2)^2$ [1]

2 (i) $y(1) = 1 - a + a - 1 = 0$

…so goes through (1, 0) [1]

$y(0) = 0 - 0 + 0 - 1 = -1$

… so goes through (0, − 1) [1]

(ii) Goes through (1, 0) so try translation by 1 to the right $y = (x - 1)^3$ [1]

$= x^3 - 3x^2 + 3x - 1$

which is the equation with $a = 3$ [1]

(iii) $y = x^2 + x^2 - x - 1$

Root at (1, 0) so factor $(x - 1)$ [1]

$x^3 + x^2 - x - 1 = (x - 1)$[1]

$(x^2 + 2x + 1)$

$(x - 1)(x + 1)^2$ so repeated root [at $x = -1$] [1]

3 (i) E.g. space craft travels at maximum speed for the whole journey [1]

Acceleration and deceleration can be ignored. Distance travelled is 2.25×10^9 metres whatever the speed. [1]

(ii) Graph showing inverse proportionality [1]

(iii) Journey time = $\frac{1.65 \times 10^{11}}{11000}$ =

1.5×10^7 seconds [1]

$= \frac{1.5 \times 10}{24 \times 60 \times 60}$

= 173.6 days [1]

which is less than 175 so supplies appear sufficient

(iv) 173.6 is close to 175, so probably need a better model to be sure that supplies are sufficient. Would not trust model. [1]

4 (i) $y = 1 - \cos x$: period 360° [1]

(ii) $2\sin^2 x = 1 - \cos x$

$2 - 2\cos^2 x = 1 - \cos x$ [1]

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$ [1]

$\cos x = -\frac{1}{2}$ or 1 [1]

$x = 0°, 120°, 240°, 360°$ [1, 1]

5 (i) $\frac{\sin\theta}{8} = \frac{\sin 20°}{5}$ [1]

$\theta = \arcsin 0.547$

= 33.2° [1]

or 146.8° [1]

(ii) Sketch(es) showing both triangles [1]

A = 180° − (33.2° + 20°)

= 126.8°

Or

A = 180° − (146.8° + 20°)

= 13.2° [1]

Area =

$\frac{1}{2} \times 5 \times 8 \times \sin A$ [1]

Correct choice of triangle [1]

Area = 16.0 units ^{2} [1]

6 (i) A $y = (2x)^3$; B $y = 2x^3$;

C $y = x^3$; D $y = \frac{x^3}{2}$

[1, 1, 1]

(ii) A $y = \cos(x + 30)$;

B $y = \cos x$;

C $y = \sin (x + 30)$;

D $y = \sin x$ [1, 1, 1]

(iii) A $y = x^4 + 1$;

B $y = x^3 + 1$;

C $y = -x^3 + 1$;

D $y = -x^3$ [1, 1, 1]

7 (i) $x^2 = (1^2 + 2^2 - 2 \times 1 \times 2 \times \cos 20°)$ [1]

$x = 1.114$ [1]

(ii) $x^2 = (1^2 + 2^2 - 2 \times 1 \times 2 \times \cos\theta)$ [1]

$R = x^2 = 5 - 4\cos\theta$ [1]

(iii) $5 - 4\cos\theta = 4$ (or inequality) [1]

$\cos\theta = \frac{1}{4}$ (or inequality) [1]

So $R < 4$ for

$[0 <]\ \theta < 75.5°$ [1]

8 (i) $(2 + x)^3 = 2^3 + 3.2^2 x + 3.2.x^2 + x^3$ [1]

$= 8 + 12x + 6x^2 + x^3$ [1]

$(1 - x)^3 = 1 - 3x + 3x^2 - x^3$ [1, 1]

(ii) (a) $y = 9 + 9x + 9x^2$

$= 9(x^2 + x + 1)$

$= 9((x + \frac{1}{2})^2 + \frac{3}{4})$ [1,1]

Which is quadratic with line of symmetry

$x = -\frac{1}{2}$ [1]

(v) (b) $y = 2x^3 + 3x^2 + 15x + 7$

$y(-\frac{1}{2}) = -\frac{1}{4} + \frac{3}{4} - \frac{15}{2} + 7 = 0$

so root at $x = -\frac{1}{2}$ [1]

$y = (2x + 1)(x^2 + x + 7)$ [1, 1]

Discriminant of quadratic factor =

$1 - 28 < 0$ [1]

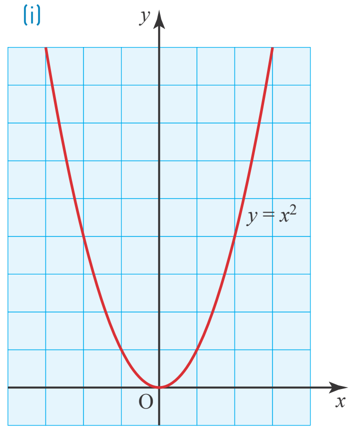
So quadratic factor has no roots

So cubic has only one root, [at $x = -\frac{1}{2}$] [1]

Chapter 10

Activity 10.1 (page 189)

(i)



For the curve $y = x^2$, the gradient is positive for $x > 0$, and negative for $x < 0$. The gradient is zero at $x = 0$.

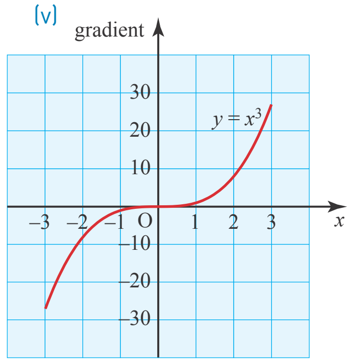
(ii) For positive values of x, as x gets bigger the gradient also increases.

(iii) For negative values of x, as x gets more negative the gradient also becomes more negative.

(iv) Because the curve is symmetrical about the y axis, the gradient at $x = -2$ has the same magnitude as the gradient at $x = 2$ but the opposite sign. In general for the curve $y = x^2$

$(\text{gradient at } x = a)$
$= -(\text{gradient at } x = -a)$

(v)



For the curve $y = x^3$, the gradient is positive for both $x > 0$ and $x < 0$. The gradient is zero at $x = 0$.

For positive values of x, as x gets bigger the gradient also increases. For negative values of x, as x gets more negative the gradient increases. The gradient at $x = -2$ is the same as the gradient at $x = 2$, and in general for $y = x^3$

$(\text{gradient at } x = a)$
$= (\text{gradient at } x = -a)$

Activity 10.2 (page 190)

(x_1, y_1)	(x_2, y_2)	Gradient $= \dfrac{y_2 - y_1}{x_2 - x_1}$
(1,1)	(3,9)	4
(1,1)	(2,4)	3
(1,1)	(1.5,2.25)	2.5
(1,1)	(1.25,1.5625)	2.25
(1,1)	(1.1,1.21)	2.1
(1,1)	(1.01,1.0201)	2.01
(1,1)	(1.001, 1.002001)	2.001

1 The gradients are getting closer and closer to 2.

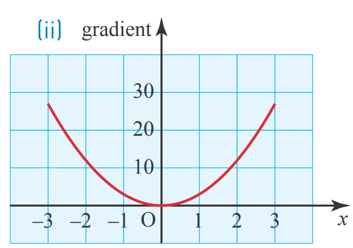
2 The gradient of $y = x^2$ at the point (1,1) appears to be 2.

Exercise 10.1 (page 191)

1 (i)

(x_1, y_1)	Gradient of $y = x^3$ at x_1
(–3,–27)	27
(–2,–8)	12
(–1,–1)	3
(0,0)	0
(1,1)	3
(2,8)	12
(3,27)	27

(ii)

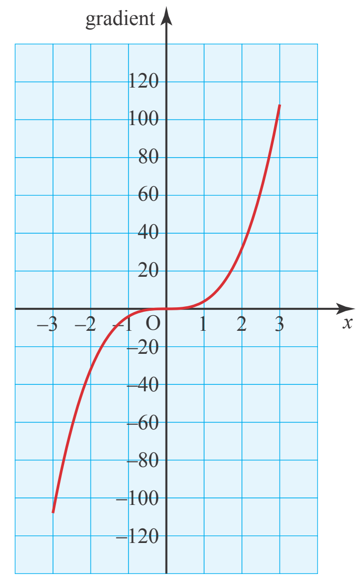


(iii) $f'(x) = 3x^2$.

2 (i)

(x_1, y_1)	Gradient of $y = x^4$ at x_1
(–3,81)	–108
(–2,16)	–32
(–1,1)	-4
(0,0)	0
(1,1)	4
(2,16)	32
(3,81)	108

(ii)



(iii) $f'(x) = 4x^3$

3 (i) $f'(x) = 5x^4$

(ii) 80

(iii) Yes

4 (i) (a) $f'(x) = 6x^5$

(b) $f'(x) = 7x^6$

(ii) $f'(x) = nx^{n-1}$

Discussion point (page 193)

The transformation stretches the graph by a scale factor of a parallel to the y axis. This multiplies the gradient by a.

Discussion points (page 193)

(i) This follows from the fact that the graph of $y = c$ is a horizontal line with gradient zero.

(ii) This follows from the fact that the graph of $y = kx$ is a straight line with gradient k.

Exercise 10.2 (page 195)

1 (i) $\dfrac{dy}{dx} = 7x^6$

(ii) $\dfrac{dy}{dx} = 11x^{10}$

(iii) $\dfrac{dy}{dx} = 14x^6 - 33x^{10}$

2 (i) $\dfrac{dV}{dx} = 3x^2$

(ii) $\dfrac{dx}{dt} = 4t - 5$

(iii) $\dfrac{dz}{dl} = 15l^4 - 2l + 5$

3 (i) -1

(ii) 9

(iii) 0

4 (i) -8

(ii) 4

(iii) $\dfrac{35}{3}$

5 $(-1, 8)$ and $(4, -57)$

6 (i) $5x^3 - x^2 + 15x - 3$

(ii) $\dfrac{dy}{dx} = 15x^2 - 2x + 15$

(iii) He has differentiated each bracket separately and then multiplied the results.

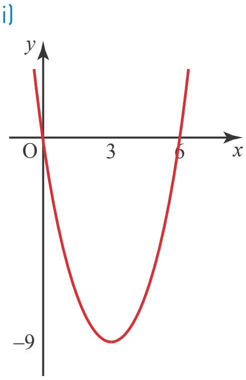
7 (i) $2x + 3$

(ii) $\dfrac{dy}{dx} = 2$

8 (i) $\dfrac{dy}{dx} = 4x^3 + 6x^2 - 4$

(ii) $\dfrac{dy}{dx} = 6x - \dfrac{3}{2}$

9 (i)

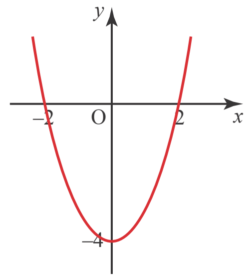


(ii) $\dfrac{dy}{dx} = 2x - 6$

(iii) Gradient is 0

(iv) Turning point of curve

10 (i)



(ii) $(2, 0)$ and $(-2, 0)$

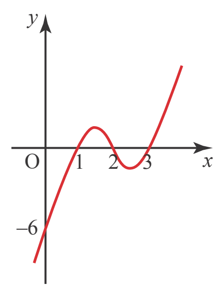
(iii) $\dfrac{dy}{dx} = 2x$

(iv) At $(2, 0)$ gradient is 4, at $(-2, 0)$ gradient is -4

(v) $(0, -8)$

11 (i) $x^3 - 6x^2 + 11x - 6 = (x - 1)(x - 2)(x - 3)$.
Curve crosses x axis at $(1, 0)$, $(2, 0)$, $(3, 0)$.

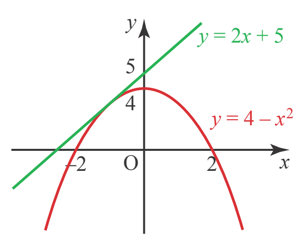
(ii)



(iii) $\dfrac{dy}{dx} = 3x^2 - 12x + 11$

(iv) Gradients are 2, -1 and 2. Tangents are parallel at $(1, 0)$ and $(3, 0)$

12 (i)

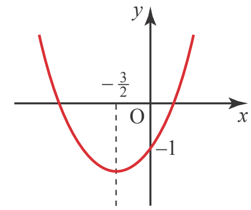


(iii) $\dfrac{dy}{dx} = -2x$, gradient is 2

(iv) Yes

(v) Yes

13 (i)



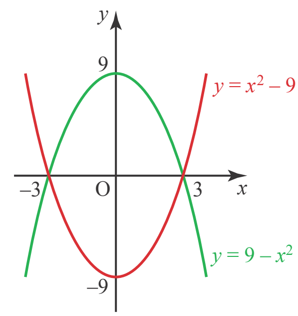
(ii) $\dfrac{dy}{dx} = 2x + 3$

(iii) $(1, 3)$

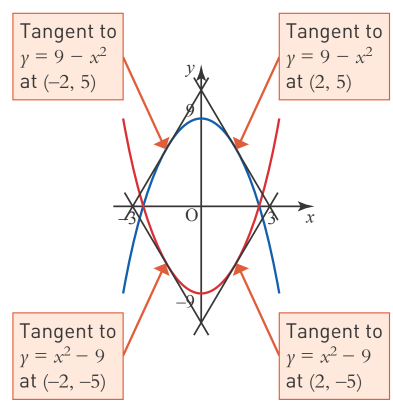
(iv) No

14 $a = 2, b = -3$

15 (i)



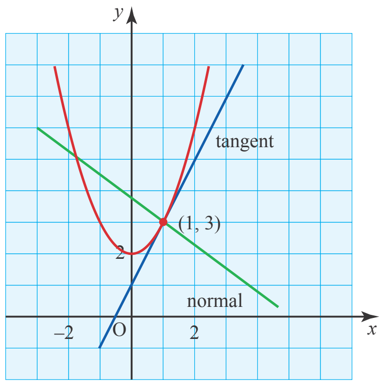
(ii)



Rhombus – opposite sides parallel, all sides of equal length

Exercise 10.3 (page 198)

1 (i) and (v)

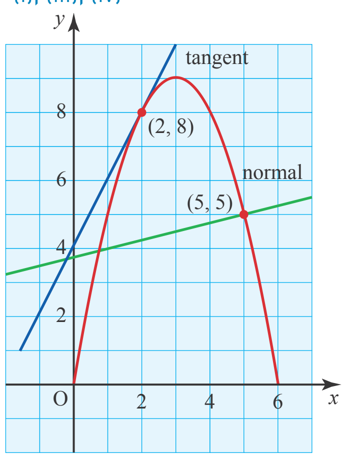


(ii) $\frac{dy}{dx} = 2x$

(iii) 2

(iv) $-\frac{1}{2}$

2 (i), (iii), (iv)

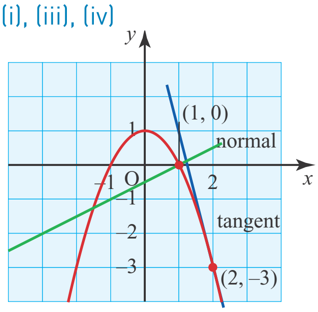


(ii) $\frac{dy}{dx} = 6 - 2x$

(iii) 2

(iv) $\frac{1}{4}$

3 (i), (iii), (iv)



Draw the curve $y = 1 - x^2$, using the same scale on both axes.

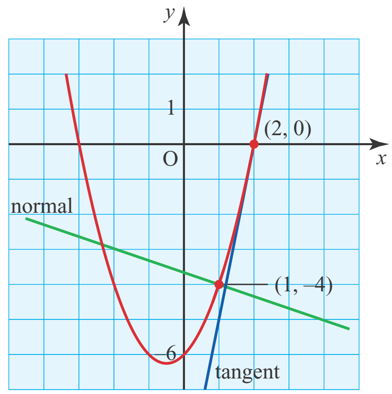
(ii) $\frac{dy}{dx} = -2x$

(iii) $y + 4x - 5 = 0$

(iv) $2y - x + 1 = 0$

(v) $\left(\frac{11}{9}, \frac{1}{9}\right)$

4 (i), (iii), (iv)



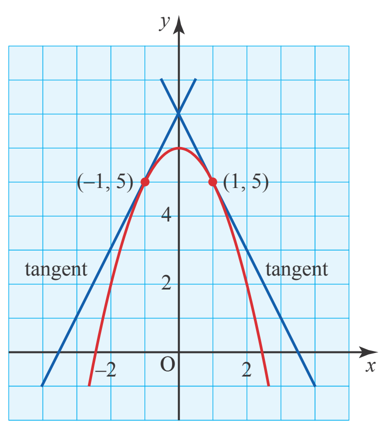
(ii) $\frac{dy}{dx} = 2x + 1$

(iii) $y - 5x + 10 = 0$

(iv) $3y + x + 11 = 0$

(v) $\frac{845}{32}$

5 (i) and (iii)

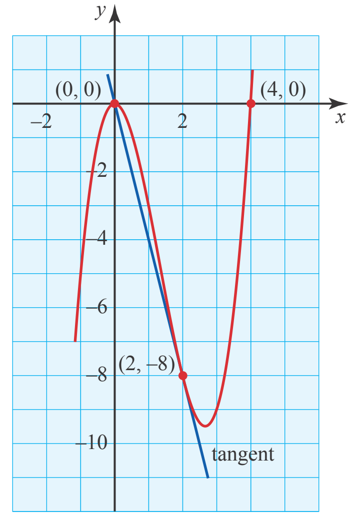


(ii) 2, –2

(iii) $y - 2x - 7 = 0$,
$y + 2x - 7 = 0$

(iv) (0, 7)

6 (i) and (iv)



(ii) $\frac{dy}{dx} = 3x^2 - 8x$

(iii) –4

(iv) $y = -4x$

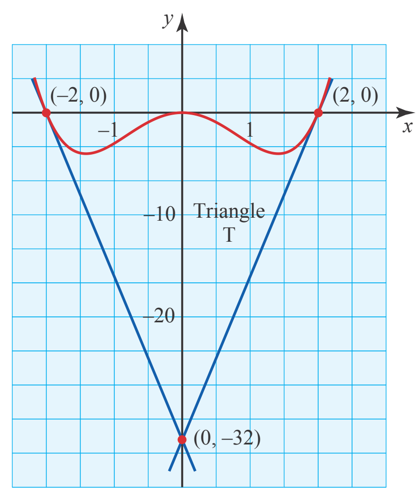
(v) (0, 0)

7 (i) $y - 6x - 28 = 0$

(ii) (3, 45)

(iii) $6y + x - 273 = 0$

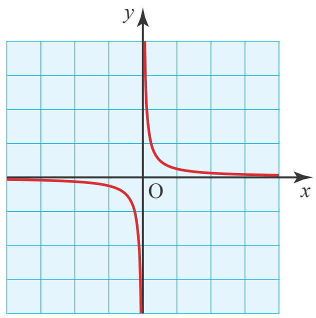
8 64



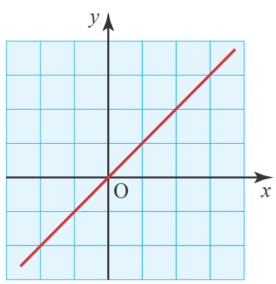
Activity 10.3 (page 200)

(i) and (ii)

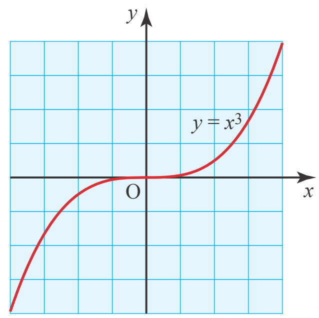
(a) Increasing for no values of x



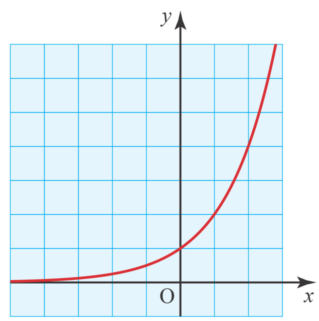
(b) Increasing for all values of x



(c) Increasing for some values of x (all except $x = 0$)



(d) Increasing for all values of x



Activity 10.4 (page 201)

	What are the coordinates of the turning point?	Is the gradient just to the left of the turning point positive or negative?	What is the gradient at the turning point?	Is the gradient just to the right of the turning point positive or negative?
Maximum	$(-1, 2)$	Positive	0	Negative
Minimum	$(1, -2)$	Negative	0	Positive

Exercise 10.4 (page 203)

1 (i) $-1 < x < 3$

(ii) $x < -1,\ x > 3$

2 (i) $x > 0$

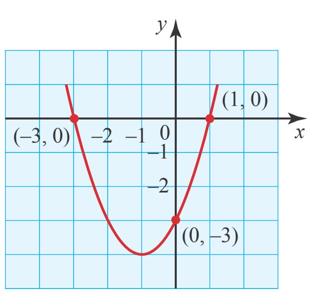
(ii) $x > 3$

(iii) $x > -5$

3 (i) $x > -1$

(ii) $x = -3,\ x = 1$

(iii)



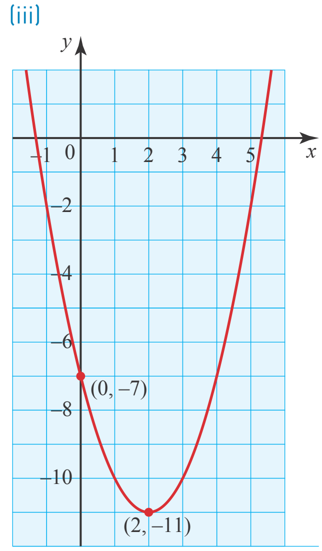
(iv) By symmetry, turning point is at $x = -1$. Curve increasing after turning point

4 Maximum at $(-2, 18)$ and minimum at $(2, -14)$

5 (i) $x < 2$

(ii) $y = (x - 2)^2 - 11$, $(2, -11)$

(iii)



(iv) Function decreasing before turning point at $x = 2$

6 (i) $\dfrac{dy}{dx} = 3x^2 + 6x - 24$

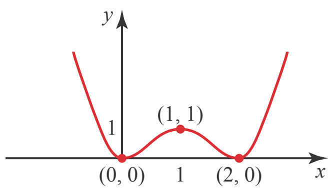
(ii) $-4 < x < 2$

7 (i) $y = x^4 - 4x^3 + 4x^2$,

$\dfrac{dy}{dx} = 4x^3 - 12x^2 + 8x$

(ii) $(0, 0)$ minimum,
$(1, 1)$ maximum,
$(2, 0)$ minimum

(iii)



8 $x < 3, x > 7$

10 (i) $\dfrac{dy}{dx} = 3x^2 - 12x + 15$

(ii) $\dfrac{dy}{dx} = 3(x-2)^2 + 3 > 0$

11 $\dfrac{dy}{dx} = 5x^4 - 30x^2 + 50$

$= (5\ (x^2 - 3)^2 + 1)$

since $(x^2 - 3) \geqslant 0$ for all real n

$\dfrac{dy}{dx} \geqslant 5 \Rightarrow f(x)$ is an increasing function

Discussion point (page 205)

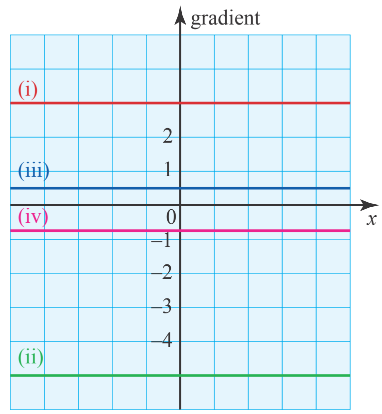
(i) $m = \frac{5}{2}$. Gradient graph is horizontal line $y = \frac{5}{2}$

(ii) Horizontal line $y = m$

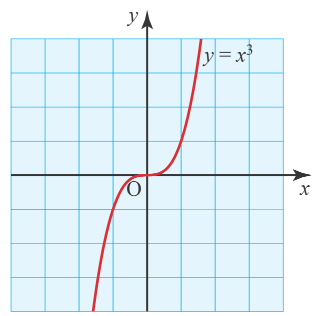
(iii) The line $y = 3$ is horizontal and therefore has a constant gradient of zero. The gradient curve would lie along the x axis. The line $x = 3$ is vertical and has an infinite gradient. It is not possible to draw a gradient curve for this straight line.

Exercise 10.5 (page 206)

1



2



(iv) The gradient is 0 when $x = 0$; everywhere else it is positive.

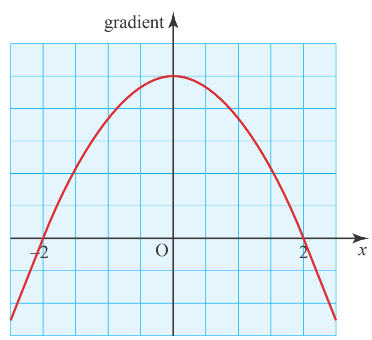
3 C

4 (i) $(-2, -8)$, $(2, 8)$

(ii) $-2 < x < 2$

(iii) $x < -2$, $x > 2$

(iv)



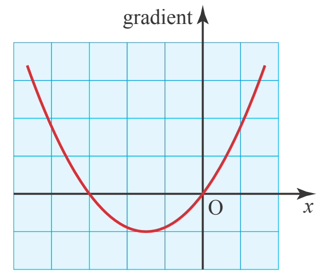
5 (i) (a) $\Rightarrow$

(b) $\Leftarrow$

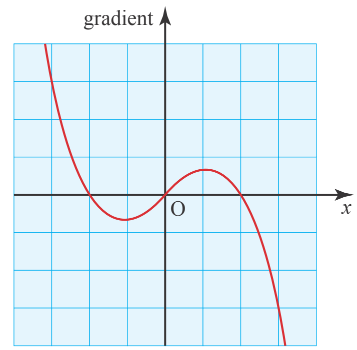
(c) $\Rightarrow$

(d) none

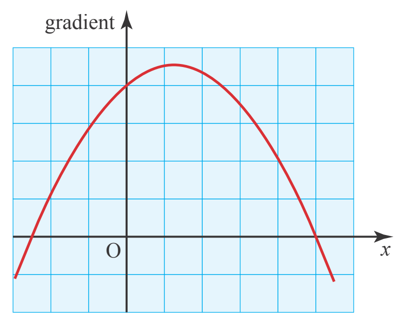
(ii)



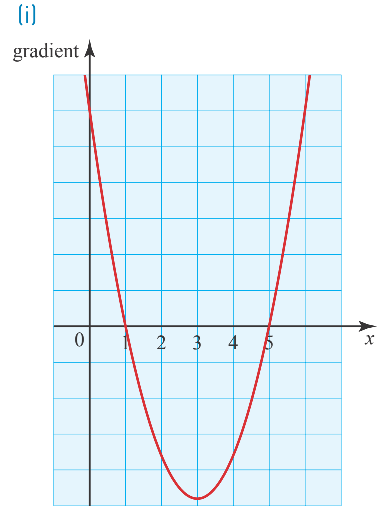
6 (i)



(ii)

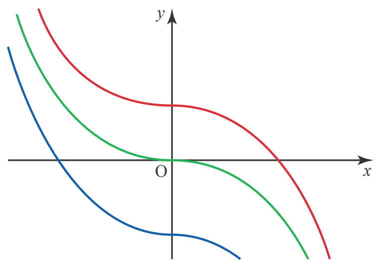


7 (i)



(ii) $\dfrac{dy}{dx} = 3x^2 - 18x + 15$

8



The graphs are the same curve translated up or down – each graph is a translation by $\begin{pmatrix} 0 \\ a \end{pmatrix}$ for some value a of every other graph.

Activity 10.5 (page 208)

(i) (a) Using graphing software, the gradient of the tangent at $x = 4$ is 0.25.

(b) Using the formula $\frac{dy}{dx} = \frac{1}{2\sqrt{x}} = \frac{1}{4}$. The results are the same.

(c) At $x = 1$, using graphing software, the gradient of the tangent is 0.5. Using the formula $\frac{dy}{dx} = \frac{1}{2}$

(ii) The result is true for other values of n.

Exercise 10.6 (page 209)

1 (i) $\frac{dy}{dx} = -20x^{-6}$

(ii) $\frac{dy}{dx} = 3x^{-\frac{1}{2}}$

(iii) $\frac{dy}{dx} = -\frac{2}{3}x^{-\frac{5}{3}}$

2 (i) $\frac{dz}{dt} = -21t^{-4}$

(ii) $\frac{dx}{dt} = -2t^{-\frac{3}{2}}$

(iii) $\frac{dp}{dr} = -12r^{-3} + 3$

4 (i) $-\frac{1}{8}$

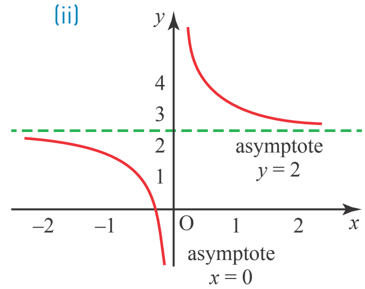
(ii) 2

5 (i) $\frac{dy}{dx} = \frac{7x^2\sqrt{x}}{2} + \frac{1}{x^2}$

(ii) $\frac{dy}{dx} = -\frac{1}{2x\sqrt{x}} + \frac{5}{x^6}$

6 (i) $\left(-\frac{1}{2}, 0\right)$

(ii)



(iii) $\frac{dy}{dx} = -\frac{1}{x^2}$

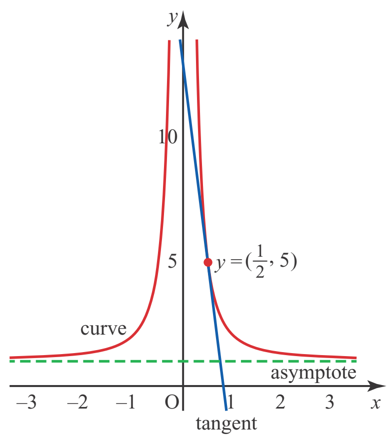
(iv) -4

7 (i) $\frac{dy}{dx} = -\frac{8}{x^3} + 1$

(iii) 2

(iv) (2, 3)

8 (i) and (iv)



(iii) $\frac{dy}{dx} = \frac{-2}{x^3}$, -16

(iv) $y = -16x + 13$

9 (i) $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

(ii) $\left(\frac{1}{16}, -\frac{3}{4}\right)$

(iii) No

10 (i) $y = -2$

(ii) $\frac{dy}{dx} = 3 + \frac{2}{x^3}$

(iii) 1

(iv) $y - x + 1 = 0$

11 (i) $\frac{dy}{dx} = 2x - \frac{1}{x^2}$

(ii) 1

(iv) $\left(-1 + \sqrt{2}, 4 - \sqrt{2}\right)$, $\left(-1 - \sqrt{2}, 4 + \sqrt{2}\right)$

12 $\frac{3}{4}$

Discussion point (page 211)

(i) No

(ii) By differentiating for a third time; third derivative; rate of change of $\frac{d^2y}{dx^2}$ with respect to x.

Discussion point (page 214)

(i) $\frac{dy}{dx} = 0$

Exercise 10.7 (page 214)

1 (i) $\frac{dy}{dx} = 4x^3$, $\frac{d^2y}{dx^2} = 12x^2$

(ii) $\frac{dy}{dx} = 15x^4 - 6x^2 + 2x$,

$\frac{d^2y}{dx^2} = 60x^3 - 12x + 2$

2 (i) $\frac{dy}{dx} = 2x + \frac{1}{x^2}$,

$\frac{d^2y}{dx^2} = 2 - \frac{2}{x^3}$

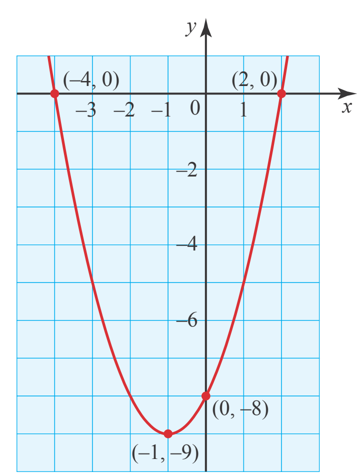
(ii) $\frac{dy}{dx} = 3\sqrt{x}$, $\frac{d^2y}{dx^2} = \frac{3}{2\sqrt{x}}$

3 (i) $\frac{dy}{dx} = 2x + 2$, $(-1, -9)$

(ii) $y = (x + 1)^2 - 9$

(iii) $\frac{d^2y}{dx^2} = 2$, minimum, yes

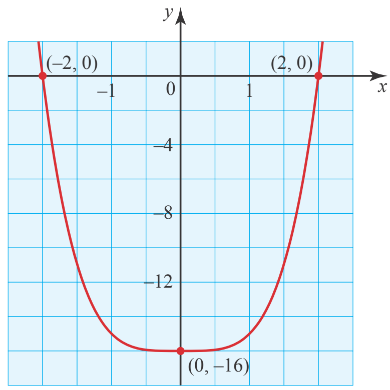
(iv)



4 (i) $\dfrac{dy}{dx} = 4x^3$

(ii) Minimum at $(0, -16)$

(iii)



5 (i) $y = x^3 - 5x^2 + 7x - 3,$

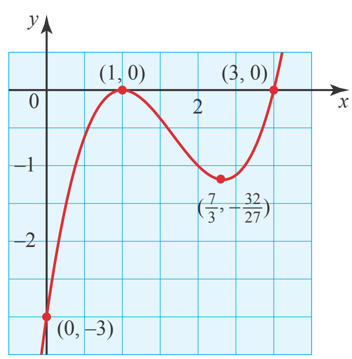
$\dfrac{dy}{dx} = 3x^2 - 10x + 7,$

$\dfrac{d^2y}{dx^2} = 6x - 10$

(ii) $(1, 0)$ maximum,

$\left(\frac{7}{3}, -\frac{32}{27}\right)$ minimum

(iii)



6 (i) $\dfrac{dy}{dx} = 1 - \dfrac{2}{\sqrt{x}},$

$\dfrac{d^2y}{dx^2} = \dfrac{1}{x\sqrt{x}}$

(ii) $(4, -4)$ minimum

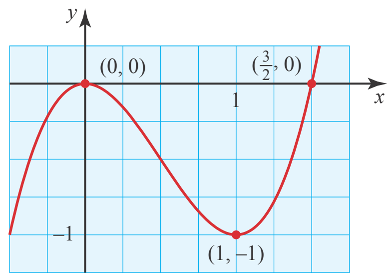
7 $x = 2$

8 (i) $p = 2, q = -3$

(ii) minimum

(iii) $(0, 0)$ maximum

(iv)



Exercise 10.8 (page 217)

1 (i) $y = 20 - x$

(ii) $P = x(20 - x)$

(iii) $\dfrac{dP}{dx} = 20 - 2x,$

$\dfrac{d^2P}{dx^2} = -2$

(iv) $P = 100$

2 (i) $y = \dfrac{400}{x}$

(ii) $S = x + \dfrac{400}{x}$

(iii) $\dfrac{dS}{dx} = 1 - \dfrac{400}{x^2},$

$\dfrac{d^2S}{dx^2} = \dfrac{800}{x^3}$

(iv) $S = 40$

3 (i) $y = 8 - x$

(ii) $S = 2x^2 - 16x + 64$

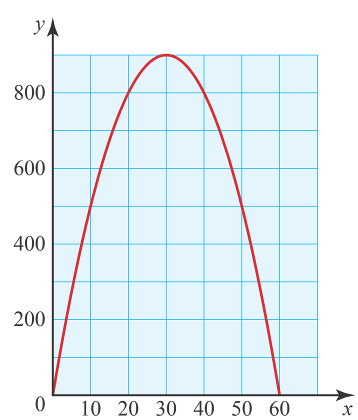
(iii) $\dfrac{dS}{dx} = 4x - 16,$

$\dfrac{d^2S}{dx^2} = 4$

4 (i) $y = 60 - x$

(ii) $A = x(60 - x)$

(iii)



(iv) $\dfrac{dA}{dx} = 60 - 2x,$

$\dfrac{d^2A}{dx^2} = -2$

(v) Maximum value

$x = 30, y = 30, A = 900$

5 (i) $A = xy$

(ii) $T = 2x + y$

(iv) $\dfrac{dT}{dx} = 2 - \dfrac{18}{x^2},$

$\dfrac{d^2T}{dx^2} = \dfrac{36}{x^3}$

(v) $x = 3, y = 6, T = 12$

6 (i) $4 - 2x,\ (4 - 2x)^2$

(iii) $x = \frac{8}{7}$

(iv) $A = \frac{48}{7}$

8 $144\,\text{cm}^3$

9 (i) $y = \dfrac{16}{x}$

(ii) $S = x^2 + \dfrac{256}{x^2}$

(iii) $S = 32$

(iv) $4\sqrt{2}$

10 (i) $r = \dfrac{15 - 2x}{\pi}$

(iii) $\dfrac{120}{4 + \pi}, \dfrac{30\pi}{4 + \pi}$

11 $\dfrac{40}{3}\sqrt{\dfrac{40}{3\pi}}$

12 £2052

Exercise 10.9 (page 221)

1 $h + 6$

2 (i) 1

(ii) $(1 + h)^3$

(iii) $h^2 + 3h + 3$

3 (i) -2

(ii) $2(-1 + h)^2 - 4$

(iv) -4

4 (i) 18

(ii) $(3 + h)^2 + 3(3 + h)$

(iii) $h + 9$

(iv) 9

5 (i) $y_1 = x_1^3$

$y_2 = (x_1 + h)^3 =$

$x_1^3 + 3x_1^2h + 3x_1h^2 + h^3$

(iii) $3x_1^2$

6 (i) $y_1 = 2x_1^3 + 1$

$y_2 = 2(x_1 + h)^3 + 1 = 2x_1^3 + 6x_1^2h + 6x_1h^2 + 2h^3 + 1$

(iii) $6x_1^2$

7 (i) $y_1 = x_1^2 + 5x_1$

(iii) $2x_1 + h + 5$

(iv) $2x_1 + 5$

8 (i) $x_1^2 - x_1 - 6$, $(x_1 + h)^2 - (x_1 + h) - 6 = x_1^2 + 2x_1h + h^2 - x_1 - h - 6$

(ii) $2x_1 + h - 1$

(iii) $2x_1 - 1$

Chapter 11

Opening activity (page 227)

They all have gradient function $\frac{dy}{dx} = 3x^2$

$y = x^3 + k$

The gradient is the same for all four curves for every value of x.

e.g. $y = x^3 + 100$, $y = x^3 - \pi$, $y = x^3 + \sqrt{2}$

$y = x^3 + c$, where c is any constant

Activity 11.1 (page 228)

(i) e.g. $y = x^2$, $y = x^2 + 1$, $y = x^2 - 1$

(ii) the curves are parallel in the sense that they have the same gradient for every value of x.

(iii) $y = x^2 + c$, where c is any constant

(iv) (a) $y = x^3 + c$,

(b) $y = \frac{x^3}{3} + c$,

(c) $y = x^4 + c$,

(d) $y = \frac{x^4}{4} + c$

(v) $y = \frac{x^{n+1}}{n+1} + c$. It does not work for $n = -1$, because you cannot divide by zero.

Discussion point (page 228)

Write 1 as x^0, and then apply the rule with $n = 0$.

Discussion point (page 229)

c represents an arbitrary constant and does not have an assigned value. So it is not correct to say that the constant in part (iv) is three times the constant in the other three parts.

Activity 11.2 (page 230)

(i) $y = x^3 + c$

(ii) (a) $y = x^3 + 3$

(b) $y = x^3 + 12$

(c) $y = x^3 - 7$

(iii) (a) $y = x^3 + 2$

(b) $y = x^3 + 2$

(c) $y = x^3 + 2$

Exercise 11.1 (page 231)

1 (i) $\frac{x^7}{7} + c$

(ii) $\frac{x^8}{4} + c$

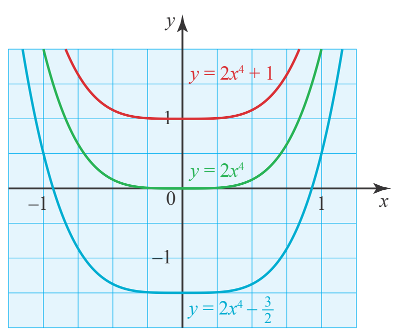
(iii) $\frac{x^7}{7} + \frac{x^8}{4} + c$

(iv) $\frac{5x^7}{7} + c$

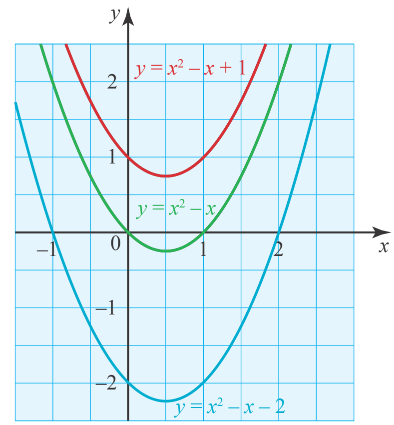
$\int (x^6 + 2x^7)\,dx = \int x^6\,dx + \int 2x^7\,dx$

$\int 5x^6\,dx = 5\int x^6\,dx$

2 (i) $y = 2x^4 + c$



(ii) $y = x^2 - x + c$

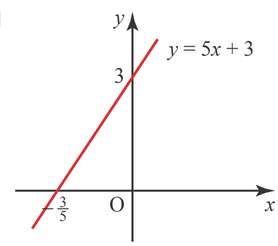


3 $f(x) = \frac{3x^2}{2} - \frac{x^3}{6} + c$

4 (i) $y = 5x + c$

(ii) $y = 5x + 3$

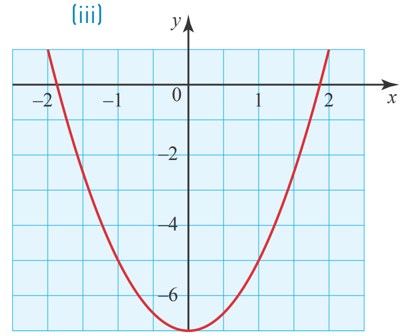
(iii)



5 (i) $y = 2x^2 - 7$

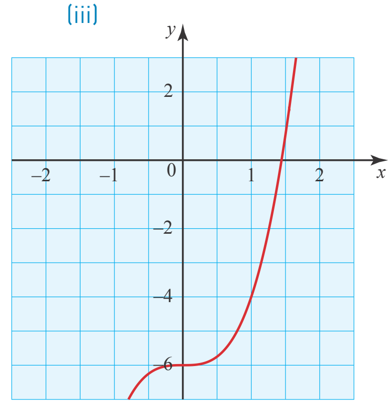
(ii) $y = -5$

(iii)



6 (i) $y = 2x^3 - 6$

(iii)



7 (i) $y = 4x^3 + 2x^2 + c$

(ii) $y = 4x^3 + 2x^2 + 3$

8 (i) $\dfrac{x^3}{3} + x^2 - 3x + c$

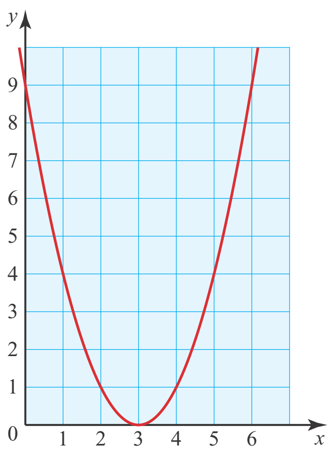
(ii) $\dfrac{x^2}{2} + 7x - \dfrac{2x^5}{5} + c$

9 (i) $2z^3 - \dfrac{11z^2}{2} + 3z + c$

(ii) $\dfrac{2t^5}{5} - \dfrac{4t^3}{3} + \dfrac{t^2}{2} + c$

10 (i) $y = x^2 - 6x + 9$

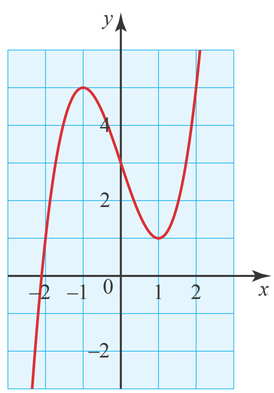
(ii) Through the point



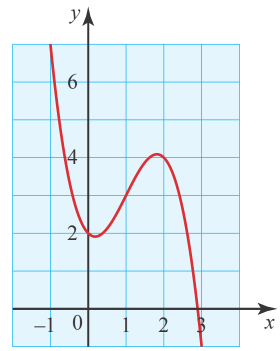
11 (i) Minimum at $x = 1$, maximum at $x = -1$

(ii) $y = x^3 - 3x + 3$

(iii)



12 $y = -x^3 + 3x^2 - x + 2$



13 $y = x^4 + 4x^3 - 2x^2 - 12x + 1$

Exercise 11.2 (page 234)

1 (i) 16

(ii) 52

(iii) 7

2 (i) $\frac{7}{3}$

(ii) $\frac{19}{3}$

(iii) $\frac{26}{3}$

The answer to (iii) is the sum of the answers to (i) and (ii)

3 (i) $\frac{32}{5}$

(ii) 32

The answer to (ii) is 5 times the answer to (i)

4 (i) (a) $\frac{19}{3}$

(b) $\frac{19}{3}$

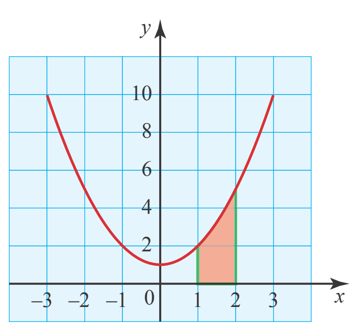
(ii) Answers the same because curve is symmetrical in the y axis

5 (i) 15

(ii) 40

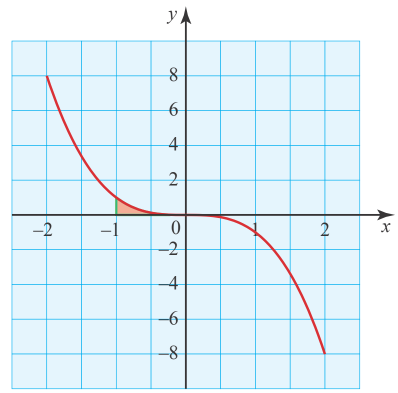
(iii) −8

6 (i) and (iii)



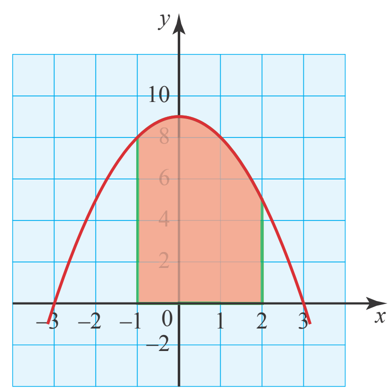
(ii) $3\frac{1}{3}$

7 (i) and (ii)



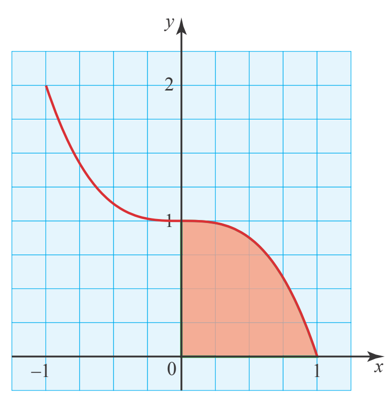
(iii) $\frac{1}{4}$

8 (i)



(ii) 24

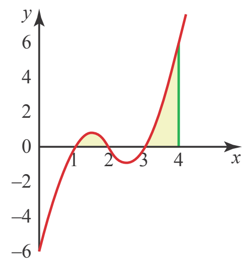
9 (i)



(ii) $\frac{3}{4}$

10 (i) and (ii)

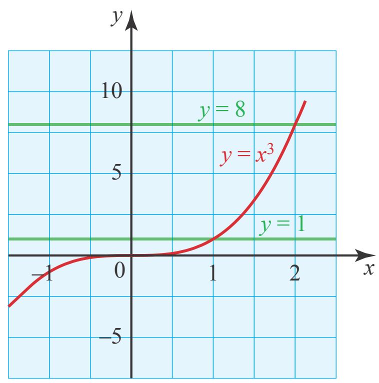
$x^3 - 6x^2 + 11x - 6$
$= (x - 1)(x - 2)(x - 3)$



(iii) $\frac{1}{4}, \frac{9}{4}$

(iv) $\frac{9}{64}$. Maximum is between 1 and 1.5

11



$\frac{45}{4}$

12 (i) 45, 135

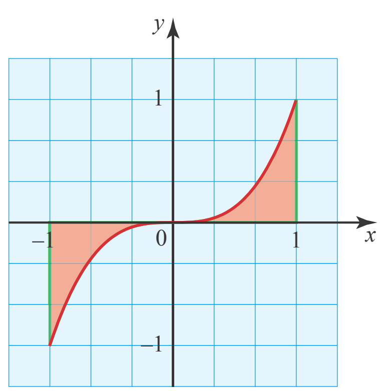
(ii) No, in general

$$\int_a^b kf(x)\ dx \neq \int_{ka}^{kb} f(x)\ dx$$

Activity 11.3 (page 235)

(i) 0

(ii)



(iii) Integral between −1 and 0 is negative, integral between 0 and 1 is positive, and they cancel each other out.

(iv) $2 \times \int_0^1 x^3\ dx = 2 \times \frac{1}{4} = \frac{1}{2}$

Discussion point (page 236)

An area cannot be negative. The integral coming out as a negative number tells you that the area is below the x axis; the area is the positive value.

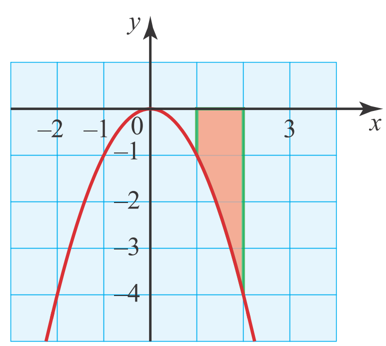
Discussion point (page 237)

$\frac{15}{2}$

Negative answer for region below x axis is cancelling out some of the positive answer for region above x axis.

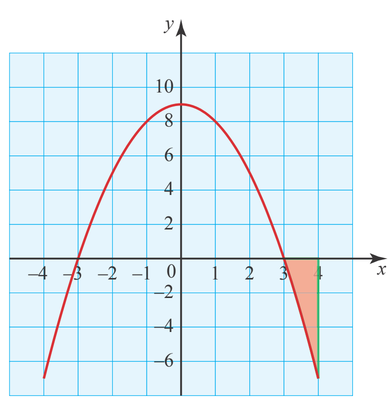
Exercise 11.3 (page 238)

1 (i) and (ii)



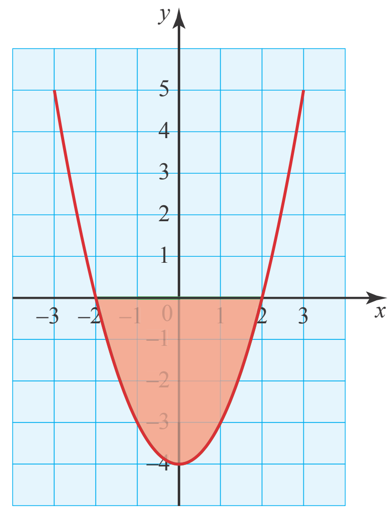
(iii) $\frac{7}{3}$

2 (i) and (ii)



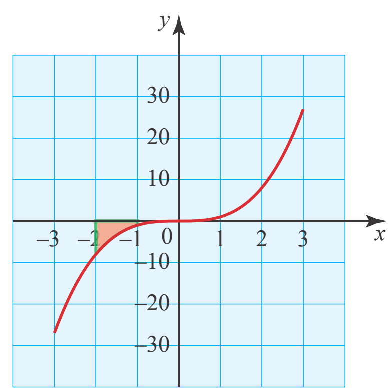
(iii) $\frac{10}{3}$

3 (i) and (ii)



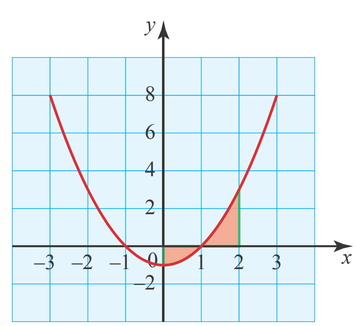
(iii) $\frac{32}{3}$

4 (i) and (ii)



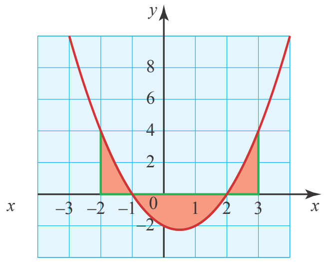
(iii) $\frac{15}{4}$

5



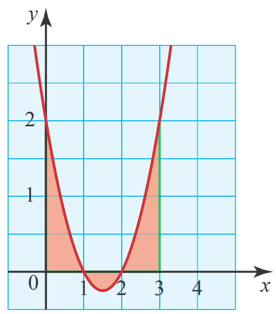
Area = 2 square units

6



Area $= \frac{49}{6}$ square units

7



Area $= \frac{11}{6}$ square units

8 (i) $-\frac{20}{3}$

(ii) $\frac{28}{3}$

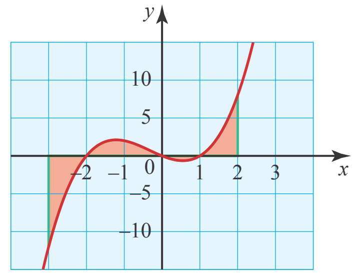
(iii) Below the x axis, y is negative, resulting in a negative integral. Answer to (i) is Area B − Area A − Area C, not Area A + Area B + Area C.

9 (i) $\frac{dy}{dx} = 20x^3 - 5x^4$, $(0,0), (4, 256)$

(ii) $\frac{3125}{6}$

(iii) 0, region between $x = 5$ and $x = 6$ is below the x axis, and the same area as the shaded region

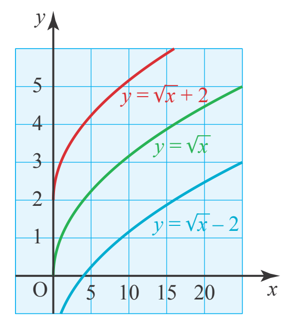
10 $x^3 + x^2 - 2x$
$= x(x+2)(x-1)$



Area $= \frac{133}{12}$ square units

Exercise 11.4 (page 241)

1 $y = \sqrt{x} + c$



2 (i) $f(x) = -\frac{1}{2}x^{-2} + c$

(ii) $f(x) = -\frac{1}{2}x^{-4} + c$

(iii) $f(x) = -\frac{1}{3x^3} + c$

(iv) $f(x) = -\frac{2}{3x^6} + c$

3 (i) $\frac{3}{4}x^{\frac{4}{3}} + c$

(ii) $\frac{8}{3}x^{\frac{3}{4}} + c$

(iii) $\frac{4}{5}x^{\frac{5}{4}} + c$

(iv) $-\frac{4}{\sqrt{x}} + c$

4 (i) 4

(ii) $\frac{64}{5}$

(iii) $-\frac{3}{2}$

5 (i) $\frac{1}{3}$

(ii) $\frac{2}{3}$

(iii) $\frac{1}{3}$

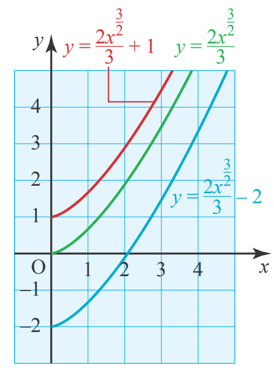
6 (i) $\frac{5}{3}x^3 + \frac{1}{2}x^{\frac{2}{3}} + c$

(ii) $y = 6x - \frac{2}{3}x^{\frac{3}{2}} - \frac{1}{4x^2} + c$

(iii) $\frac{5x^{\frac{8}{3}}}{8} + c$

(iv) $y = -\frac{3}{2x^2} + \frac{1}{3x^{\frac{3}{2}}} + c$

7 (i) $y = \frac{2x^{\frac{3}{2}}}{3} + c$



(ii) x cannot be negative because you cannot take the square root of a negative number

(iii) $y = \frac{2x^{\frac{3}{2}}}{3} + 2$

8 $x = \frac{4}{3}$

9 $v = 4t^{\frac{1}{2}} + 1$

10 (i) $\frac{707}{192}$

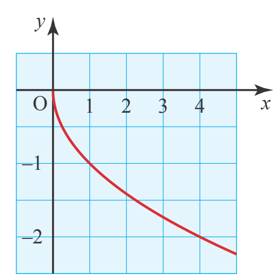
(ii) $\frac{335}{12}$

(iii) $\frac{6237}{10}$

11 (i) $P(-4,3), Q(-2,0), R(2,0), S(4,3)$

(ii) 8

12 (i)



(ii) $y - 2x + 3 = 0$

(iii) $\left(\frac{3}{2}, 0\right)$

(iv) $\frac{11}{12}$

13 $\frac{5}{4}$

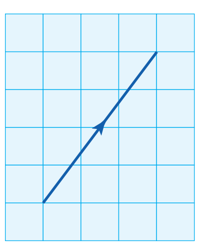
Chapter 12

Exercise 12.1 (page 248)

1 (i) (a) $3\mathbf{i} + 2\mathbf{j}$
(b) $\sqrt{13}$
(ii) (a) $5\mathbf{i} - 4\mathbf{j}$
(b) $\sqrt{41}$
(iii) (a) $3\mathbf{i}$
(b) 3
(iv) (a) $-3\mathbf{i} - \mathbf{j}$
(b) $\sqrt{10}$
(v) (a) $2\mathbf{j}$
(b) 2

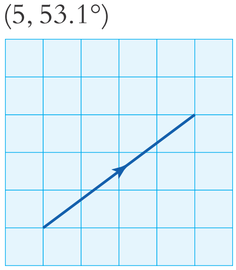
2 Three – first he needs to say where to start from (for example, one corner of the field) and then he needs to give a distance and a direction.

3 (i)



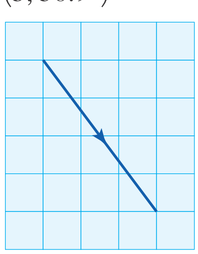
(5, 53.1°)

(ii)



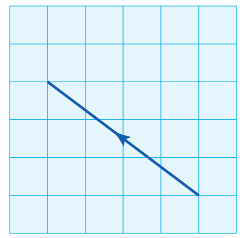
(5, 36.9°)

(iii)



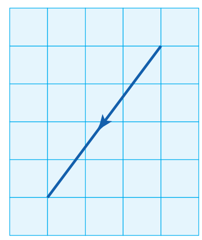
(5, 306.9°)

(iv)



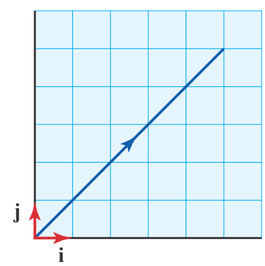
(5, 143.1°)

(v)



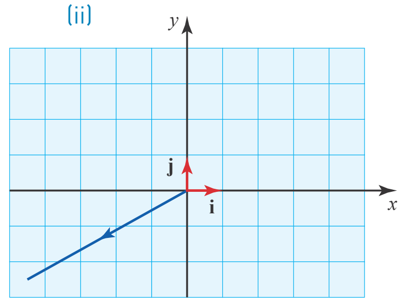
(5, 233.1°)

4 (i)



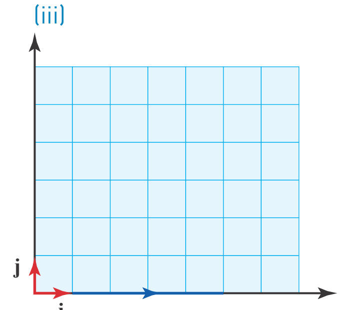
$7.07\mathbf{i} + 7.07\mathbf{j}$

(ii)



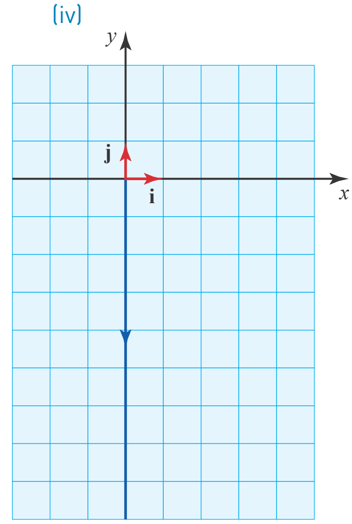
$-4.33\mathbf{i} - 2.5\mathbf{j}$

(iii)



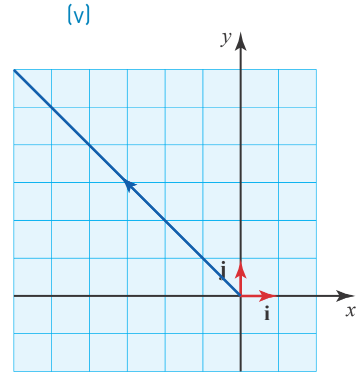
$5\mathbf{i}$

(iv)



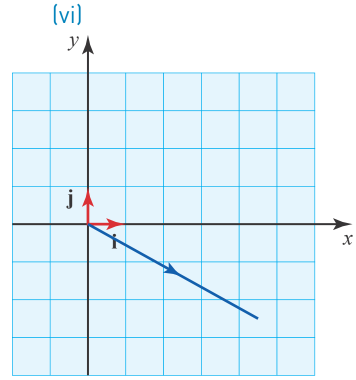
$-10\mathbf{j}$

(v)



$-7.07\mathbf{i} + 7.07\mathbf{j}$

(vi)



$4.33\mathbf{i} - 2.5\mathbf{j}$

5 (i) $2\mathbf{i} - 2\mathbf{j}$
(ii) $-2\mathbf{i} + 2\mathbf{j}$
(iii) $6\mathbf{i} + 4\mathbf{j}$
(iv) $-2\mathbf{i} - 2\mathbf{j}$
(v) $-2\mathbf{i} + 2\mathbf{j}$
(vi) $2\mathbf{i} - 2\mathbf{j}$

6 (i) (a) Median at A: $\begin{pmatrix}1.5\\3\end{pmatrix}$;

median at B: $\begin{pmatrix}-3\\-1.5\end{pmatrix}$;

median at C: $-\begin{pmatrix}1.5\\-1.5\end{pmatrix}$

(b) Magnitude of median at A is $\frac{3}{2}\sqrt{5}$; magnitude of median at B is $\frac{3}{2}\sqrt{5}$; magnitude of median at C is $\frac{3}{2}\sqrt{2}$

(ii) (a) $|\overrightarrow{AX}| = \sqrt{5}$

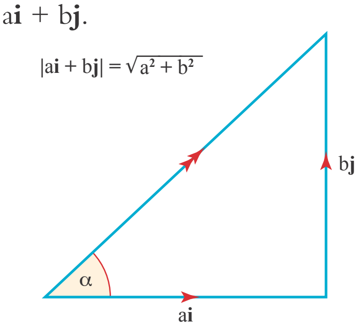
7 (i) Possible – R is on the perpendicular bisector of PQ

(ii) Possible – PQR is a straight line and R is on the same side of both P and Q

(iii) Not possible – if (i) is true then R is between P and Q, so (ii) cannot be true.

Activity 12.1 (page 254)

The diagram shows the vector a**i** + b**j**.



Magnitude of

$$\frac{a}{\sqrt{a^2+b^2}}\mathbf{i} + \frac{b}{\sqrt{a^2+b^2}}\mathbf{j} \text{ is}$$

$$\sqrt{\left(\frac{a}{\sqrt{a^2+b^2}}\right)^2 + \left(\frac{b}{\sqrt{a^2+b^2}}\right)^2}$$

$$= \frac{a^2}{a^2+b^2} + \frac{b^2}{a^2+b^2}$$

$$= \frac{a^2+b^2}{a^2+b^2}$$

$$= 1$$

Direction of $\frac{a}{\sqrt{a^2+b^2}}\mathbf{i} + \frac{b}{\sqrt{a^2+b^2}}\mathbf{j}$ is given by

$$\tan\alpha = \frac{\dfrac{b}{\sqrt{a^2+b^2}}}{\dfrac{a}{\sqrt{a^2+b^2}}} = \frac{b}{a}$$

Direction of a**i** + b**j** is given by $\tan\alpha = \frac{b}{a}$

Hence the two vectors have the same direction.

Exercise 12.2 (page 255)

1 (i) $\begin{pmatrix}6\\8\end{pmatrix}$ (ii) $\begin{pmatrix}-1\\-1\end{pmatrix}$

(iii) $\begin{pmatrix}0\\0\end{pmatrix}$ (iv) $\begin{pmatrix}-7\\-2\end{pmatrix}$

(v) $2\mathbf{i} + \mathbf{j}$

(vi) $2\mathbf{i} + 7\mathbf{j}$

3 (i) $\begin{pmatrix}2\\3\end{pmatrix}$ (ii) $\begin{pmatrix}1\\0\end{pmatrix}$ (iii) $\begin{pmatrix}0\\1\end{pmatrix}$

(iv) $\begin{pmatrix}0\\0\end{pmatrix}$ (v) $\begin{pmatrix}1\\4\end{pmatrix}$ (vi) $\begin{pmatrix}3\\2\end{pmatrix}$

4 (i) $a = 4, b = 6$

(ii) $p = -1, q = 2$

5 (i) $x = 4$

(ii) $x = -16$

(iii) $x = -3$

6 (i) $\frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j}$

(ii) $\begin{pmatrix}-\frac{3}{5}\\ \frac{4}{5}\end{pmatrix}$

(iii) $\frac{4}{5}\mathbf{i} - \frac{3}{5}\mathbf{j}$

(iv) $\frac{2}{\sqrt{13}}\mathbf{i} + \frac{3}{\sqrt{13}}\mathbf{j}$

(v) $\begin{pmatrix}\frac{3}{\sqrt{13}}\\ \frac{2}{\sqrt{13}}\end{pmatrix}$

(vi) $\begin{pmatrix}-\frac{3}{\sqrt{13}}\\ -\frac{2}{\sqrt{13}}\end{pmatrix}$

(vii) $\mathbf{i}$

(viii) $-\mathbf{j}$

(ix) $\begin{pmatrix}\frac{6}{\sqrt{72}}\\ -\frac{6}{\sqrt{72}}\end{pmatrix}$

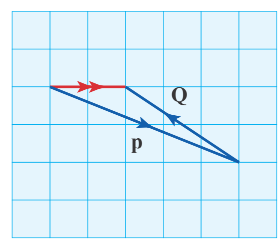
7 (i) $\begin{pmatrix}\mathbf{F}_2\\ \mathbf{F}_1\end{pmatrix}$

(ii) $\begin{pmatrix}\mathbf{F}_6 - \mathbf{F}_5\\ \mathbf{F}_4\end{pmatrix}$

(iii) $\begin{pmatrix}\mathbf{F}_8 - \mathbf{F}_{10}\\ \mathbf{F}_7 - \mathbf{F}_9\end{pmatrix}$

8 (i) $2\mathbf{i}$

(ii)



9 $\begin{pmatrix}16\\9\end{pmatrix}$

10 (i) $\begin{pmatrix}\cos\alpha\\ \sin\alpha\end{pmatrix}$

(ii) $\begin{pmatrix}\cos\beta\\ \sin\beta\end{pmatrix}$

11 (i) O, A and B are not in a straight line.

(ii) O, A and B all lie on a straight line.

Exercise 12.3 (page 258)

1 $-4\mathbf{i} + 6\mathbf{j}, 2\mathbf{i} - 3\mathbf{j}$,
$\frac{2}{\sqrt{13}}\mathbf{i} - \frac{3}{\sqrt{13}}\mathbf{j}$;
$6\mathbf{i} + 9\mathbf{j}, 2\mathbf{i} + 3\mathbf{j}$;
$3\mathbf{i} + 2\mathbf{j}, -\frac{3}{\sqrt{13}}\mathbf{i} - \frac{2}{\sqrt{13}}\mathbf{j}$,
$-12\mathbf{i} - 8\mathbf{j}$

2 (i) (a) $\begin{pmatrix}3\\7\end{pmatrix}$

(b) $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$

(c) $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$

(d) $\sqrt{10}$

(ii) (a) $\begin{pmatrix} -3 \\ -7 \end{pmatrix}$

(b) $\begin{pmatrix} -2 \\ -4 \end{pmatrix}$

(c) $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

(d) $\sqrt{10}$

(iii) (a) $\begin{pmatrix} -3 \\ -7 \end{pmatrix}$

(b) $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$

(c) $\begin{pmatrix} 5 \\ 11 \end{pmatrix}$

(d) $\sqrt{146}$

(iv) (a) $\begin{pmatrix} 3 \\ 7 \end{pmatrix}$

(b) $\begin{pmatrix} -2 \\ -4 \end{pmatrix}$

(c) $\begin{pmatrix} -5 \\ -11 \end{pmatrix}$

(d) $\sqrt{146}$

(v) (a) $\begin{pmatrix} 3 \\ -7 \end{pmatrix}$

(b) $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$

(c) $\begin{pmatrix} -5 \\ 11 \end{pmatrix}$

(d) $\sqrt{146}$

3 (i) A: $2\mathbf{i} + 3\mathbf{j}$, C: $-2\mathbf{i} + \mathbf{j}$

(ii) $\overrightarrow{AB} = -2\mathbf{i} + \mathbf{j}$, $\overrightarrow{CB} = 2\mathbf{i} + 3\mathbf{j}$

(iii) (a) $\overrightarrow{AB} = \overrightarrow{OC}$

(b) $\overrightarrow{CB} = \overrightarrow{OA}$

(iv) A parallelogram

4 (i) $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 4 \\ 9 \end{pmatrix}$, $\overrightarrow{OC} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$

(ii) $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} -7 \\ -14 \end{pmatrix}$

(iii) $\overrightarrow{BC} = -\frac{7}{2} \times \overrightarrow{AB}$ therefore the vectors have the same direction. Both vectors pass through B, hence points are collinear

5 (i) $\overrightarrow{PR} = \mathbf{a} + \mathbf{b}$, $\overrightarrow{QS} = -\mathbf{a} + \mathbf{b}$

(ii) $\overrightarrow{PM} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$, $\overrightarrow{QM} = \frac{1}{2}(-\mathbf{a} + \mathbf{b})$

(iii) For a parallelogram PQRS, $\overrightarrow{PM} = \frac{1}{2}\overrightarrow{PR}$, and $\overrightarrow{QM} = \frac{1}{2}\overrightarrow{QS}$

6 (i) (a) $\mathbf{i}$

(b) $2\mathbf{i}$

(c) $\mathbf{i} - \mathbf{j}$

(d) $-\mathbf{i} - 2\mathbf{j}$

(ii) $|\overrightarrow{AB}| = |\overrightarrow{BC}| = \sqrt{2}$, $|\overrightarrow{AD}| = |\overrightarrow{CD}| = \sqrt{5}$

7 (i) $6\mathbf{q} - 9\mathbf{p}$

(ii) $2\mathbf{q} - 3\mathbf{p}$

(iii) $6\mathbf{p} - 4\mathbf{q}$

(iv) $6\mathbf{p} + 2\mathbf{q}$

8 (i) $\overrightarrow{BC} = -\mathbf{p} + \mathbf{q}$, $\overrightarrow{NM} = -\frac{1}{2}\mathbf{p} + \frac{1}{2}\mathbf{q}$

(ii) $\overrightarrow{NM} = \frac{1}{2}\overrightarrow{BC}$, and similarly $\overrightarrow{NL} = \frac{1}{2}\overrightarrow{AC}$ and $\overrightarrow{ML} = \frac{1}{2}\overrightarrow{AB}$

So the sides ABC and LMN are similar triangles, with the lengths of the sides of LMN half the corresponding lengths of the sides of ABC.

9 (i) ABCD is a quadrilateral and $\overrightarrow{AB}$ and $\overrightarrow{DC}$ are parallel

(ii) $\overrightarrow{AC} = 2\mathbf{a} + \mathbf{b}$, $\overrightarrow{BC} = -\mathbf{a} + \mathbf{b}$

(iii) $\overrightarrow{AP} = \mu(2\mathbf{a} + \mathbf{b})$, $\overrightarrow{BP} = \mu(2\mathbf{a} + \mathbf{b}) - 3\mathbf{a}$

(iv) $\mu = \lambda = \frac{3}{5}$

Chapter 13

Opening activity (page 261)

32

1024

About 20 days

Activity 13.1 (page 262)

(i) (a) A = (2, 4), B = (4, 16)

(b) $0 < x < 2$ and $x > 4$

(iii) (a) Yes (b) Yes

Discussion point (page 263)

If the graphs are translated vertically by k units, the horizontal asymptote will be $y = k$ and the graphs will go through the point $(0, k)$,

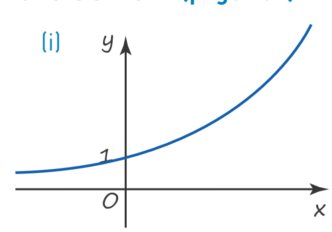
If the graphs are translated horizontally by k units, the horizontal asymptote will be $y = 0$ and the graphs will both go through $(k, 1)$.

If the graphs are stretched horizontally, the horizontal asymptote will be $y = 0$ and the graphs will both go through (0, 1).

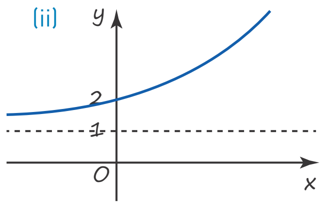
If the graphs are stretched vertically by a scale factor k, the horizontal asymptote will be $y = 0$ and the graphs will both go through $(0, k)$.

Exercise 13.1 (page 264)

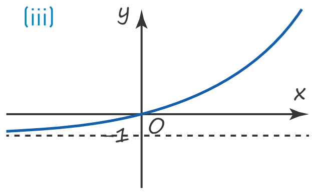
1 (i)



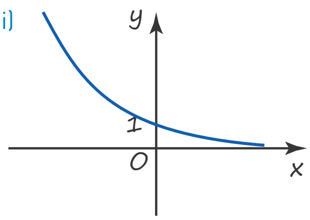
(ii)



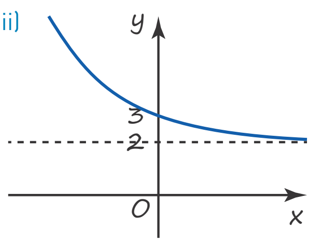
(iii)



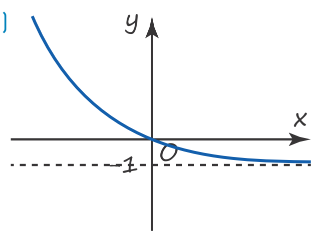
2 (i)



(ii)

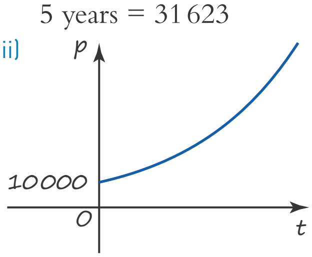


(iii)



3 (i) Initial population = 10 000; population after 5 years = 31 623

(ii)



4 (i) 1 m

(ii) 4.61 m

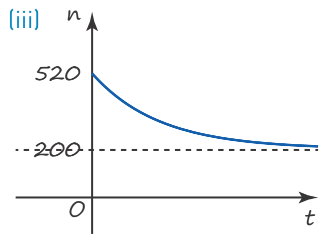
(iii) Just over 6 years

(iv) 20 m

5 (i) 520

(ii) 210

(iii)



(iv) 200 birds

6 (ii) $x = 1.8$

7 (i) 1976 Hz

(ii) A, 3 octaves below the middle

(iii) 48 (from E two octaves below the middle, up to C 5 octaves above the middle

Exercise 13.2 (page 269)

1 (i) $\log_3 9 = x; x = 2$

(ii) $\log_4 64 = x; x = 3$

(iii) $\log_2 \frac{1}{4} = x; x = -2$

(iv) $\log_5 \frac{1}{5} = x; x = -1$

(v) $\log_7 1 = x; x = 0$

(vi) $\log_{16} 2 = x; x = \frac{1}{4}$

2 (i) $3^y = 81; y = 4$

(ii) $5^y = 125; y = 3$

(iii) $4^y = 2; y = \frac{1}{2}$

(iv) $6^y = 1; y = 0$

(v) $5^y = \frac{1}{125}; y = -3$

3 (i) $\log 10$

(ii) $\log 2$

(iii) $\log 36$

(iv) $\log \frac{1}{7}$

(v) $\log 3$

4 (i) $2 \log x$

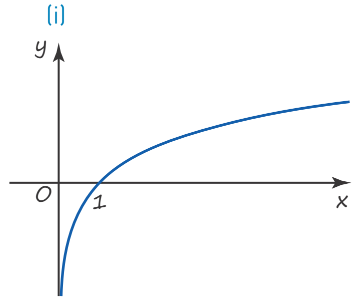
(ii) $3 \log x$

(iii) $\frac{1}{2} \log x$

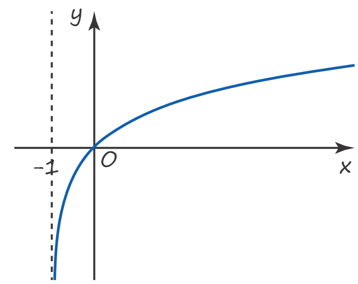
(iv) $6 \log x$

(v) $\frac{5}{2} \log x$

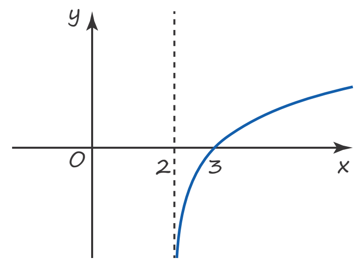
5 (i)



(ii)



(iii)



6 (i) $\log 4$

(ii) $\log 4$

(iii) $\log \frac{1}{3}$

(iv) $\log \frac{1}{2}$

(v) $\log 12$

7 (i) $\log \frac{x^2}{7}$

(ii) $x = 21$

8 (i) $x = 19.93$

(ii) $x = -9.97$

(iii) $x = 9.01$

(iv) $x = 48.32$

(v) $x = 1375$

9 (i) 6.6

(ii) 1.58×10^{12}

(iii) 794 times more energy

10 (i) 1.259 (3 d.p.)

(iii) 3 decibels

(iv) It should be

$$\frac{10^{3.7} - 10^{3.5}}{10^{3.5}} \times 100 = 58.5\%$$

11 (i) (a) 7

(b) 5.30

(c) 7.80

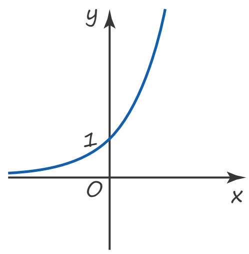
(ii) 3.98×10^{-5} and 3.16×10^{-6}

Discussion point (page 271)

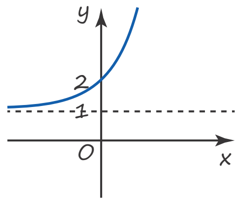
£21.05

Exercise 13.3 (page 273)

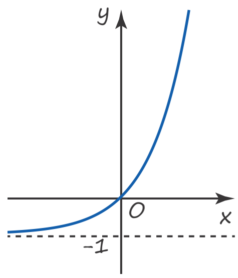
1 (i)



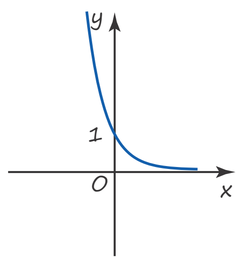
(ii)



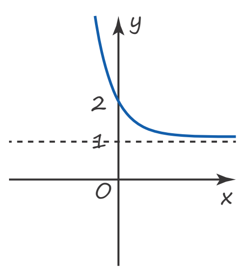
(iii)



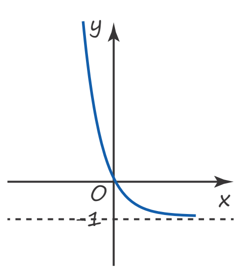
2 (i)



(ii)



(iii)



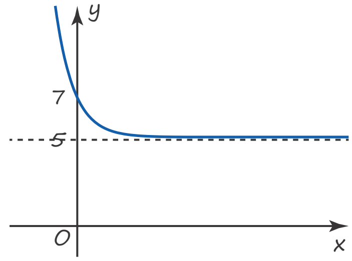
3 (i) $\dfrac{dy}{dx} = 2e^{2x}$

(ii) $\dfrac{dy}{dx} = -3e^{-3x}$

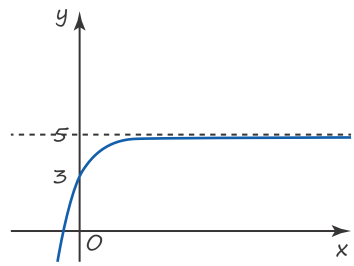
(iii) $\dfrac{dy}{dx} = 0.5e^{0.5x}$

(iv) $\dfrac{dy}{dx} = -6e^{-6x}$

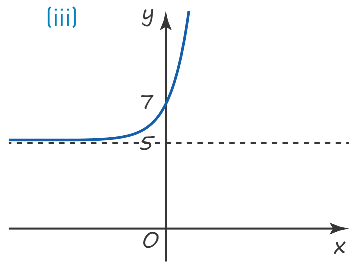
4 (i)



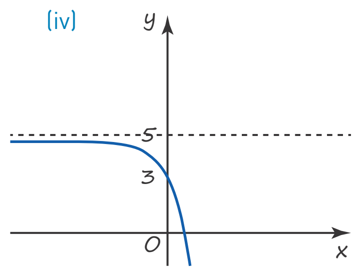
(ii)



(iii)



(iv)



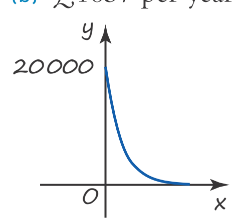
5 (i) £20 000

(ii) £14 816

(iii) (a) £2000 per year

(b) £1637 per year

(iv)



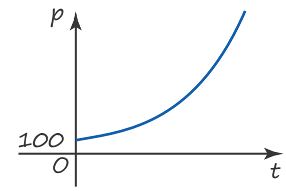
(v) The value of the car would probably fall more quickly in practice. The model implies that the value of the car would eventually reach zero, which is not the case as it would have some value as scrap metal.

6 (i) £2210

(ii) £7788

7 (i) 100

(ii)



(iii) 8.24 people per year

(iv) 1218

(v) 165

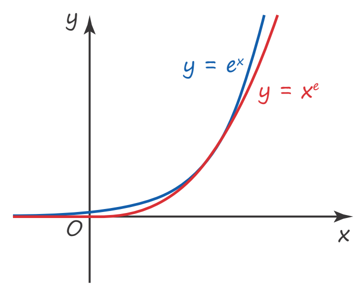
8 0.231

9 (i) £12649

(ii) 9.12 years

10 (i) Gradient of both graphs is e^e

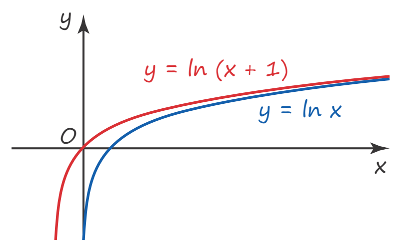
(ii)



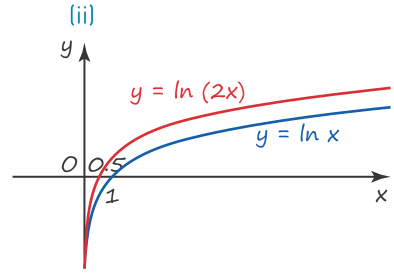
(iii) e^π is greater

Exercise 13.4 (page 276)

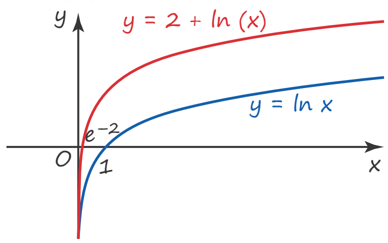
1 (i)



(ii)



(iii)



2 (i) $x = x_0 e^{kt}$

(ii) $t = \frac{1}{k}\ln\left(\frac{s_0}{s}\right)$

(iii) $p = 25e^{-0.02t}$

(iv) $x = \ln\left(\frac{y-5}{y-y_0}\right)$

3 (i) $x = \ln 6$

(ii) $x = \frac{1}{2}\ln 6$

(iii) $x = 1 + \ln 6$

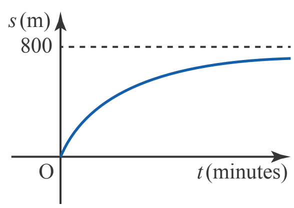
(iv) $x = e^5$

(v) $x = 2e^5$

(vi) $x = \frac{1}{2}e^5$

4 (0.34, 0.5)

5 (i)



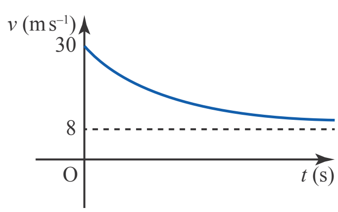
(ii) 621.5 m

(iii) 8.07 a.m.

(iv) No

(v) Never, according to this model.

6 (i)



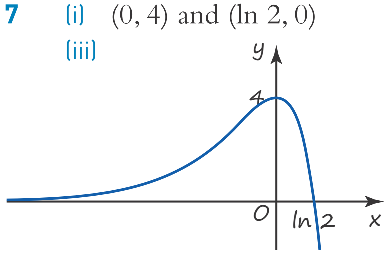
(ii) $30\,\text{m}\,\text{s}^{-1}$, $8\,\text{m}\,\text{s}^{-1}$

(iii) $8.33\,\text{m}\,\text{s}^{-1}$

(iv) 8.7 seconds

7 (i) (0, 4) and (ln 2, 0)

(iii)



Exercise 13.5 (page 280)

1 (ii) compared to $y = mx + c$

(iii) $m = \log b$, $c = \log k$

(iv) $k = e^c$

2 (i) $\log P = \log k + n \log V$

(ii) compared to $y = mx + c$

(iii) $m = n$

(iv) $k = e^c$

3 (ii) $b = 1.4$, $k = 0.9$

(iii) (a) 2.4 days

(b) $2.9\,\text{cm}^2$

4 (ii) $n = 1.5$, $k = 1200$

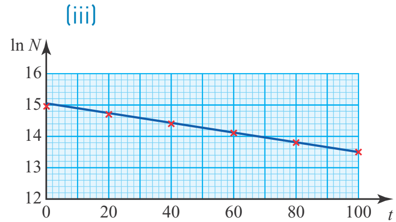
(iii) 2600 m

(iv) The train won't continue to accelerate for that length of time.

5 (ii)

t	0	20	40	60	80	100
$\ln N$	14.98	14.71	14.40	14.11	13.82	13.51

(iii)



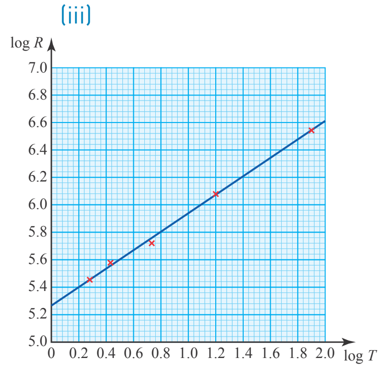
(iv) $a = 0.98$, $k = 3\,400\,000$

(v) In the year 2005

6 (ii)

Moon	Tethys	Dione	Rhea	Titan	Iapetus
$\log R$	5.46	5.58	5.72	6.09	6.55
$\log T$	0.278	0.431	0.732	1.201	1.899

(iii)



(iv) $n = 0.68$, $k = 180\,000$

(v) 0.7 days

7 (ii) $y = 400 \times 0.63^t$

(iii) $y = 0.4$. The infection is under control.

8 (i) $P = 4000 \times t^{0.2}$

(ii) 4000

(iii) The model predicts growth in local user numbers without limit which is impossible. However, the growth becomes so slow that this may well not be a problem.

Practice questions 3

(page 285)

1 $\overrightarrow{AB} = -2\mathbf{i} + 3\mathbf{j}$,

$\overrightarrow{DC} = -1\mathbf{i} + \frac{3}{2}\mathbf{j}$ [1]

so $\overrightarrow{DC} = \frac{1}{2}\overrightarrow{AB}$, [1]

and DC is therefore parallel to AB, and ABCD is a trapezium. [1]

2 (i) $52\,300 = ae^0 = a$ [1]

$58\,500 = 52\,300\,e^{5k}$ [1]

$5k = \ln\left(\frac{58\,500}{52\,300}\right)$ [1]

$k = 0.0224$ [1]

(ii) When $t = 8$,

$P = 52\,300\,e^{0.0224 \times 8}$

$= 62\,600$ to the nearest 100 [1]

3 (i)

$\int_{-1}^{1}(3x - x^3)\,dx = \left[\frac{3}{2}x^2 - \frac{1}{4}x^4\right]_{-1}^{1}$ [1]

$= \left(\frac{3}{2}1^2 - \frac{1}{4}1^4\right) - \left(\frac{3}{2}(-1)^2 - \frac{1}{4}(-1)^4\right)$ [1]

$= 0$ [1]

(ii) $\frac{dy}{dx} = 3 - 3x^2$ [1]

when $x = 1$, $\frac{dy}{dx} = 3 - 3 = 0$;

when $x = -1$,

$\frac{dy}{dx} = 3 - 3(-1)^2 = 0$ [2]

(or solve $3 - 3x^2 = 0$ to get $x = -1$ or 1)

When $x = 1$,

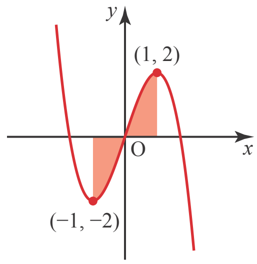
$y = 3 \times 1 - 1^3 = 2$;

when $x = -1$,

$y = 3 \times (-1) - (-1)^3 = -2$ [2]

So $(-1, -2)$ and $(1, 2)$ are stationary points.

(iii)



[2]

(iv) There are equal areas above and below the x axis, which cancel each other out when working out the integral. [1]

4 (i) $\frac{(13.23 - 12)}{0.1}$ [2]

(ii) 2.001, 12.012, 12.003 [3]

(iii) 12 [1]

(iv) $y = 3x^2$, so $\frac{dy}{dx} = 6x$ [1]

When $x = 2$,

$\frac{dy}{dx} = 6 \times 2 = 12$ [1]

5 (i) Box area

$= xy + xy + 5y + 5y + 5x$

$= 2xy + 5x + 10y$ [1]

sleeve area

$= 5y + 5x + 5y + 5x$

$= 10x + 10y$ [1]

so $A = 2xy + 15x + 20y$ [1]

(ii) $V = 5xy = 60$ [1]

so $y = \frac{12}{x}$, and

$A = 24 + 15x + \frac{240}{x}$

as required [1]

(iii) $\frac{dA}{dx} = 15 - \frac{240}{x^2} = 0$ [1]

so $\frac{240}{x^2} = 15, x^2 = \frac{240}{15} = 16$,

$x = 4$; [2]

$y = \frac{12}{4} = 3$, so dimensions of box are 3 cm by 4 cm by 5 cm. [1]

$\frac{d^2A}{dx^2} = \frac{480}{x^3} > 0$ for $x = 4$,

So the volume is a minimum

6 (i) $\overrightarrow{OD} = \mathbf{v} + \frac{1}{2}\mathbf{u}$, so

$\overrightarrow{OE} = \frac{2}{3}(\frac{1}{2}\mathbf{u} + \mathbf{v})$

$= \frac{1}{3}\mathbf{u} + \frac{2}{3}\mathbf{v}$ [2]

(ii) $\overrightarrow{OF} = \overrightarrow{OA} + \overrightarrow{AF}$

$= \overrightarrow{OA} + \frac{2}{3}\overrightarrow{AC}$ [1]

$= \mathbf{u} + \frac{2}{3}(\mathbf{v} - \mathbf{u})$

$= \frac{1}{3}\mathbf{u} + \frac{2}{3}\mathbf{v}$ [2]

So $\overrightarrow{OE} = \overrightarrow{OF}$, and E and F are the same point. [1]

7 (i) P is (6.685, 4.614), Q is (9.210, 7.364) [2]

(ii) $b = \frac{(7.364 - 4.614)}{(9.210 - 6.685)}$

$= 1.089$

$= 1.09$ (2 d.p.) [2]

$4.614 = a + 1.089 \times 6.685$ so $a = -2.67$ (2 d.p.) [2]

(iii) $t = e^{-2.67} \times d^{1.09}$

$= 0.069 \times d^{1.09}$ [2]

(iv) When $d = 100$, $t = 10.44$, [1]

so the record is below what is predicted by the model. [1]

The model is for middle and long distances but seems not to be accurate for sprints. [1]

8 (i) $100\left(1 + \frac{2}{100}\right)^{12}$ [1]

$= 126.82$, so the APR is 26.82% [1]

(ii) $100\left(1 + \frac{p}{100}\right)^{12} = 150$ [1]

$\Rightarrow \left(1 + \frac{p}{100}\right)^{12} = 1.5$ so

$\ln\left[\left(1 + \frac{p}{100}\right)^{12}\right] = \ln 1.5$ [1]

$\Rightarrow 12 \ln\left[\left(1 + \frac{p}{100}\right)\right] = \ln 1.5$ [1]

$\Rightarrow \ln\left[\left(1 + \frac{p}{100}\right)\right] = \frac{\ln 1.5}{12}$

$= 0.033788\ldots$ [1]

$\Rightarrow = \left(1 + \frac{p}{100}\right) = e^{0.033788\ldots}$

$= 1.034\ldots$

$\Rightarrow p = 3.4\%$ [1]

The same calculation could be done using log rather than ln, in which case

$\left(1 + \frac{p}{100}\right) = 10^{0.01467\ldots} = 1.034$, etc

Chapter 14

Discussion point (page 292)

The fairest answer is that there is not enough information. Ignoring the journalistic prose, '. . . our town council rate somewhere between savages and barbarians. . . ', the facts given are broadly correct. However, to say whether or not the council is negligent one would need to compare accident statistics with other *similar* communities. Also, one would need to ask who is responsible for a cyclist's risk of having or not having an accident. Perhaps parents should ensure there is adequate training given to their children and so on.

Exercise 14.1 (page 293)

2 (i) 30

(ii) 34 under the age of 18

(iii) approx. 3.4 km (depending on how the data is cleaned)

(iv) approx. 33% (depending on how the data is cleaned)

(v) approx. 47%

(vi) approx. 37%

(vii) (a) 74%

(b) approx. 27%

(viii) From this data 39014 and 54211 never reported whether the cyclist was wearing a helmet. On one occasion 78264 did not record this information.

3 While this is the correct average of the police officers' numbers it is quite meaningless. The numbers are a way of identifying the officers. There is no more sense in taking their average than there would be in trying to find an average name for them. So, she is not right.

4 and 5 There are various correct answers to these questions. Students could compare their diagrams with other students and discuss which graphs show the information in the best way.

6 He must establish well defined classes so that each accident can be assigned to one of them. These may include 'None of these' and 'Multiple'.

7 Calculations and displays showing the distribution of those having accidents among the age groups.

8 Robin should address the question of how likely it is that a cyclist, particularly a child, is involved in a preventable accident. His article should be based on data and not mere rhetoric. A key question is how many of the accidents were caused by cars, lorries and other heavy traffic.

9 Robin is focusing on two aspects of the investigation: he is looking at cycling accidents in the area over a period of time and he is considering the distribution of ages of accident victims.

He should also try to estimate the overall number of cyclists, not just those who have accidents, and their age distribution.

Another thing he might consider is to investigate accidents in a similar community in order to be able to make comparisons.

He should also look at the traffic management in the area and the availability (or otherwise) of cycle paths.

He could also look at whether adequate safe cycling training is available in Avonford or nearby.

Discussion point (page 294)

There is no information on how the sample was selected but with a size of only 30 it cannot be representative of all the different groups: male and female; different ages; different income levels; etc.

A sample of 30 is nowhere near large enough. Samples for polls about voting intentions are typically over 1000.

It may well matter that the right question was not asked; it certainly would not be safe to imagine otherwise.

Discussion point (page 295)

The population is made up of the MP's constituents. The sample is part of that population of constituents. Without information relating to how the constituents' views were elicited, the views obtained seem to be biased towards those constituents who bother to write to their MP.

The population is made up of Manchester households. We are not told how the sample is chosen. Even if a random sample of households were chosen the views obtained are still likely to be biased as the interview timing excludes the possibility of obtaining views of most of the residents in employment.

Discussion point (page 297)

1 Each student is equally likely to be chosen but samples including two or more students from the same class are not permissible so not all samples are equally likely.

2 Yes

Exercise 14.2 (page 299)

1 Systematic sampling

2 (i) Simple random sampling

(ii) $\frac{1}{25}$

3 (i) Cluster sampling

(ii) No. The streets are chosen at random and then 15 houses are chosen at random. However not every sample of 15 (throughout the town) can be chosen.

4 (i) Quota sampling

(ii) No

(iii) The sample size is 160 which should be large enough to give some useful information, providing they are reasonably representative of people who might possibly shop at Tracey's boutique.

5 In most of the situations there are several possible sampling procedures and so no single right answer.

(i) Cluster sampling. Choose representative streets or areas and sample from these streets or areas.

(ii) Stratified sample. Identify routes of interest and randomly sample trains from each route.

(iii) Stratified sample. Choose representative areas in the town and randomly sample from each area as appropriate.

(iv) If the candidate has a list of the electorate (which they should do), simple random sampling, or systematic sampling. Otherwise the candidate might take a number of locations and use cluster sampling.

(v) Depends on method of data collection. If survey is, say, via a postal enquiry, then a random sample may be selected from a register of addresses.

(vi) Cluster sampling. Routes and times are chosen and a traffic sampling station is established to randomly stop vehicles to test tyres.

(vii) Cluster sampling. Areas are chosen and households are then randomly chosen.

(viii) Cluster sampling. A period (or periods) is chosen to sample and speeds are surveyed.

(ix) Cluster sampling. Meeting places for 18-year-olds are identified: night clubs, pubs, etc. and samples of 18-year-olds are surveyed, probably via a method to maintain privacy. This might be a questionnaire to ascertain required information.

(x) Simple random sampling. The school student register is used as a sampling frame to establish a simple random sample within the school.

6 (i) $\frac{1}{10}$

(ii) Years 1 and 2: 14 students each; Years 3 and 4: 10 students each; Year 5: 12 students.

7 (i) 28 light vans, 2 company cars and 1 large-load vehicle.

(ii) Randomly choose the appropriate number of vehicles from each type. This is proportional stratified sampling.

8 (i) $\frac{1}{8}$

(ii) 0–5: 5; 6–12: 10; 13–21: 13 or 14; 22–35: 25 or 26; 36–50: 22 or 23; 51+: 3 or 4

9 (i) Systematic sampling. Easy to set up but may be difficult to track down the student once they have been identified.

(ii) Self-selected sample. The responses may be from those with strong views that are not representative of the majority of the students.

(iii) Opportunity sampling. The sample will be biased. Easy to survey. Those using the canteen will be surveyed.

(iv) Cluster sampling. Assumes first and second year students are representative of the whole college. (If there are only first and second year students this will be true. The sampling procedure is then stratified.) Similar to (i), that is, once students have been chosen from the lists they have to be located to seek their views.

10 Justification: $\frac{1153}{1235} = 93\%$ – and this is from people who chose to buy the product so it must be the brand they prefer!

Issues: only a quarter of the 5000 responded, which means that three quarters either didn't feel it was better, or didn't feel it was significantly better and worth replying. Only 80% who replied actually think the product is better than the current moisturiser. The group was self-selecting which means that they already had a preference (in this case to the company's products) and would be expected to be favourably inclined towards the new product.

11 (i) $\dfrac{5 \times 24 \times 7}{5 \times 100\,000} = 0.0084$

(ii) It is a systematic sample. It is easy to apply. It allows the output of each machine to be checked on a regular schedule.

12 All production lines are identified. If it is judged they are equivalent then one (or more) can be chosen to produce a sample. This is cluster sampling. From this (or these) production line(s) a day (or days) is chosen to be the time when a sample is taken. A reasonable number of strip lights is chosen and then tested to destruction, that is, tested until they are exhausted. An estimate is found from the mean life of the sample chosen.

13 The map of the forest is covered with a grid. Each grid square is numbered. A sample is chosen by randomly selecting the squares. The tree (or trees) in each of the chosen squares is sampled.

14 Depending on the number of staff, one could carry out a census of all staff or, if more appropriate, a stratified sample based on part-time staff, full-time staff, academic staff, support staff, etc.

15 Identify different courses in your school/college. Access students from each of these courses, choosing them at random in order to elicit their views. This is a stratified sample.

Chapter 15

Exercise 15.1 (page 307)

1 (i) Heart and coronary disease

(ii) They refer to the percentages of deaths in the various categories which can be attributed to smoking.

(iii) (a) Heart and coronary disease

(b) Heart and coronary disease and lung cancer.

2 (i) Product A (it has approx. doubled in size and increased by over £750,000)

(ii) Sales decreased in 2013 but have continued to increase since.

3 (i) Tertiary

(ii) Secondary

(iii) Primary education has probably remained universal. The increase in expenditure is due to inflation and an increased population. Secondary education is now universal or nearly so. The expenditure on it has increased more than that on primary education and is now slightly more than on primary education, possibly because it is more expensive to run. Tertiary education is now much more widespread but the lower expenditure on it suggests it is not universal.

4 (Typical answers)

(i) Comment about overall number of NEETs – general falling trend, only increases in Oct–Dec 2014. Comment about gender differences, e.g. female NEETs always higher than male NEETs, biggest difference in Oct–Dec 2014.

(ii) Figure 15.8 shows both time periods and gender differences so patterns over time can be identified. Figure 15.9 shows the total NEETs and allows comparison of gender differences.

(iii) Pie charts can be used to show how a total quantity is divided into categories. This is not suitable for the male NEET data as individuals may appear in more than one of the time periods and so would be double-counted in the sectors of the pie chart. The total quantity is not meaningful.

5 (i) Compound bar chart

(ii) PE & Sport has the largest change – a decrease of 40%. Classics show the smallest change – an increase of 16.7%.

(iii) The numbers are much larger for the new subjects – a new scale would be needed, which may mean that small changes are difficult to detect.

6 A TRUE – if you add together the sections covering those with two adults, it is more than half.

B TRUE – although there are slightly more females, the split is roughly equal.

C FALSE – although most people own at least part of their home, about half of those who do have a mortgage so the lending body owns part of their home until this is paid off.

D UNCERTAIN – there are about 4m households in the category 'Single 65 and over' and half of the population is male. However there is not enough evidence to say that these two are independent and therefore 2m men over 65 live on their own. A longer life expectancy for women makes it likely half of the 4m households are occupied by men.

E FALSE – although the housing status is roughly equally shared, there is not enough evidence to say that this applies equally across all household types. For instance young adults are more likely to live in rented accommodation, while older couples are more likely to have finished paying off their mortgage.

7 A good chart for proportions is a pie chart when viewed from above; for comparing numbers a bar chart is better. 3-D charts often mislead people as they give prominence to those groups which are 'closer' to the viewer as you see more of the side of these sectors. Colour can also be used to mislead people as darker colours tend to dominate.

Discussion point (page 310)

The leaves of a stem-and-leaf diagram provide a first sort of the data. If you then go on to order the figures in the leaves all that remains is to assign a rank to each of them.

Discussion point (page 311)

Range = 182 − 74 = 108 minutes

IQR = 119 − 77 = 42 minutes

The IQR is the better measure. The range was unduly influenced by Sally who took much longer than all the other athletes.

Exercise 15.2 (page 312)

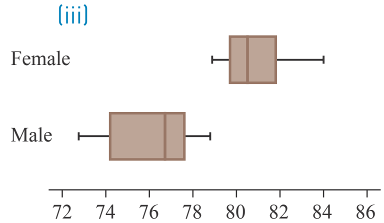
1 (i)

	Life expectancy for people born in different London boroughs		Ranking	
	Female	Male	Female	Male
Barking and Dagenham	79.7	74.7	9	7
Barnet	81.8	77.6	3	2.5
Camden	80.5	74.3	6.5	8
Croydon	80.5	76.7	6.5	5.5
Greenwich	80.1	74.2	8	9
Islington	79.1	73.5	10	10
Kensington & Chelsea	84.0	78.8	1	1
Kingston upon Thames	81.3	77.6	5	2.5
Redbridge	81.5	76.7	4	5.5
Tower Hamlets	78.9	72.7	11	11
Westminster	82.7	77.0	2	4

(ii)

	Female	Male
Minimum	78.9	**72.7**
Q_1	79.7	**74.2**
Q_2	80.5	**76.7**
Q_3	**81.8**	**77.6**
Maximum	84.0	**78.8**

(iii)



(iv) The distribution of female life expectancies is higher than the males. All of the female values, minimum, Q_1, Q_2, Q_3 and maximum are higher than the male equivalent. Even the **maximum** value for the male life expectancy (Kensington and Chelsea, 78.8) is below the **minimum** value for females (Tower Hamlets, 78.9). On average (median) a female in London is expected to outlive a male by approximately 4 years.

(v) (a) As different boroughs have different costs linked to living within them (such as housing, travel) and different environmental conditions (such as being central with few green spaces or towards the outer edges with more access to green spaces) there are different standards of living which could attribute to life expectancy.

(b) Generally women are found to live longer than men across the UK and across the world.

2 (i) 0.74 g

(ii) 0.845 − 0.7 = 0.145 g

(iii) The majority of the cereals have a salt content which is 0.8 g or less. The mid-point of the amber range for salt (0.3 g − 1.5 g) is 0.90 g. Half the cereals are below 0.74 g or $\frac{3}{4}$ of the cereals are below 0.89 g related to the amber range of (0.3 g − 1.5 g) for salt. Very few of the cereals are coloured green and so deemed healthy.

3 (i)

	1st	2nd	3rd	4th
A	Scotland	Ireland	England	Wales
B	England	Ireland	Wales	Scotland
C	Scotland	Ireland	England	Wales
D	Scotland	Ireland	England	Wales

(ii) The median ranks are based upon the times.

(iii) A and B both take the full team into account. A does not penalise a disaster as much as B so is perhaps fairer.

4 (i)

Europe		A & C	
Q_1	56.5	Q_1	27.5
Q_2	70	Q_2	49
Q_3	87	Q_3	77
IQR	30.5	IQR	49.5

(ii) Europe has a higher median, indicating that on average a greater proportion of the population in Europe has access to the internet. The IQR for America and the Caribbean is greater which suggests that there is a greater spread in the data.

The distribution for Europe is bimodal, suggesting that there may be two sorts of countries involved. These might, for example, be those in the west and the east of the continent; however, without the original data it is impossible to say.

The distribution for America and the Caribbean has three modes. Since some of these countries are small island states and others, like the USA and Canada, have large populations, the variability shown by the stem-and-leaf plot is unsurprising. However, as with the European countries, the original data are not available so no explanation of the distribution can be more than speculation.

5 A TRUE. The lowest wage in 2011(£445) is above the highest wage in 1997 (£400).

B TRUE. These can be read from the box plots.

C FALSE. The range in 1997 was approx. £115 while in 2011 it is approx. £205. Similarly the interquartile range has increased from about £15 to about £30.

D FALSE. In both years the median is closer to the lower quartile than the upper quartile, suggesting the higher earning regions have a greater spread. Similarly for the ranges.

E UNCERTAIN. It is true that the vertical scale has been truncated; it starts at £280 instead of zero. It is also true that this gives a false impression, making the increase from 1997 to 2011 look proportionately greater. However there is no information to indicate that this was done deliberately.

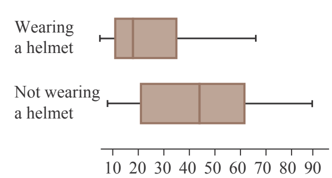
6 For a list of n items of data, an *Excel* spreadsheet uses the 'method of hinges'.

It places the median, Q_2, at position $\frac{n+1}{2}$, the lower quartile, Q_1, at position $\frac{1}{2}\left(1+\frac{n+1}{2}\right)=\frac{1}{2}+\frac{n+1}{4}$ and the upper quartile, Q_3, at position $\frac{1}{2}\left(\frac{n+1}{2}+n\right)=\frac{3(n+1)}{4}-\frac{1}{2}$.

Whilst the quartiles Q_1 and Q_3 differ from those obtained with a graphical calculator, either method is acceptable.

With a large set of data, results are likely to be less affected than with a small data set, but different methods should be taken into account when analysing data and checking on the results of others.

7



Possible responses include:

Younger people tend to

wear helmets; the box plot is skewed towards the younger ages.

The oldest people don't wear helmets.

We don't know the relative sizes of the groups: 55 were wearing helmets and 30 were not wearing helmets, but box plots don't show the relative sizes of the groups.

Activity 15.1 (page 315)

	Numbers of goals scored by teams	Goals	Frequency
0	𝍸𝍸𝍸𝍸𝍸𝍸𝍸𝍷	0	36
1	𝍸𝍸𝍸𝍸𝍸𝍸𝍸𝍸𝍷𝍷𝍷𝍷	1	44
2	𝍸𝍸𝍸𝍸𝍸𝍷𝍷	2	27
3	𝍸𝍸𝍷𝍷	3	12
4	𝍸𝍷	4	6
5	𝍷𝍷	5	2
6		6	0
7	𝍷	7	1

Discussion point (page 315)

A stem-and-leaf diagram is useful when there are lots of different items of data which can be grouped according to the initial digit and recorded so that there is no loss of raw data. If only a few different values are present a tally chart will not lose any of the original data.

Discussion point (page 316)

(One among many possible answers)

You know the mean GDP per person for each country in the world and want to find the mean GDP per person for the whole world. In this case you would find a weighted mean, multiplying the figure for each country by its population, adding all these figures together to get the total world income and then dividing by the world's population.

Discussion points (page 317)

It makes no difference mathematically but seems more natural to rank the best performance 1.

Discussion point (page 319)

You would treat it as an ordinary frequency table, with the range of the scores in the left hand column replaced by their mid-points.

Mid-point, x	Frequency, f	xf
4.5	1	4.5
14.5	2	29.0
24.5	4	98.0
34.5	7	241.5
44.5	14	623.0
54.5	15	817.5
64.5	17	1096.5
74.5	13	968.5
84.5	5	422.5
94.5	2	189.0
Σ	**80**	**4490**

Estimated mean =

$$\frac{\sum xf}{\sum f} = \frac{4490}{80} = 56$$

(to the nearest whole number).

Discussion points (page 329)

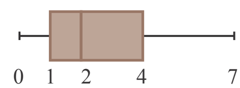
You would take all the items in any class to be at the mid-point and then work out the weighted mean.

There may be too much data for it to be practical to enter each item on a stem-and-leaf diagram and the variable may take too many values for a vertical line chart to be suitable.
For example, the marks obtained by candidates on a public examination with an entry of several thousand people.

The data may cover too large a number of possible values of the variable.
For example, the cost in £ of a sample of second-hand cars.
The data may be spread out over too wide a range.
For example, people's winnings in £ in the National Lottery one Saturday.

Exercise 15.3 (page 319)

1 (i)



(ii) The vertical line graph as it retains more data for this small sample.

2 (i)

Wickets	0	1	2	3	4	5	6
Matches	2	5	0	5	7	5	2

(ii) Mode: 4; mean: 3.3; midrange: 3

(iii) She will use the mode as it is the largest.

3 (i) These 12 females produce 28 fledglings; they do so 3 times so produce 84 fledglings. 42 of these are females, 25% survive which is 10 to the nearest bird.

So at the next generation these 12 females will be replaced by just 10.

So the birds will reduce in numbers.

(ii) The birds eat insects and the farmer is using insecticides so this is a possible cause.

There are many other possible causes, e.g. predation. The study is quite small and the data cover only one year.

[OCR]

4 (i) The mode is the most useful average to use in this situation as this is the shoe size which is most often bought. The median might also be the mode, but this just tells you which size is the middle size, not whether it is popular, while the mean may not even be a recognised shoe size.

(ii) He could ask every student in the sixth form but this may take a long time and not everyone might accurately know their height.

He could measure a sample of 60 students as they enter sixth form one morning and record their heights in a table. He should ensure that the groups are not overlapping and he may want to consider using narrower groups for the central values which are likely to be more common.

5 (i) (a) US\$8160 (b) US\$6964

(ii) The weighted mean

6 The mean wages are £10.91 per hour, which would justify the manager's claim. However, only about one third of the workers are paid over this, so a fairer advertisement would be to use the median or mode, both of which are £7.50 per hour.

7 (i) 14

(ii) 196

(iii) 0.7

(iv) The mode and median are both zero so are not representative. The midrange is 2 which would suggest more men were being killed than was the case. So the mean is the most representative measure.

8 A UNCERTAIN It is true that there was no day on which all 8 hens laid so it is possible that one hen did not lay any eggs at all, but it is also possible that they all laid some eggs but happened not to do so on the same day.

B FALSE The totals of the four columns are 90, 92, 92 and 91 making the 365 days in the year. The February was in 2015 which was not a leap year.

C FALSE The midrange is the mean of the greatest and least values actually attained so it is

$$\frac{7+0}{2} = 3.5.$$

D TRUE The total number of eggs was 1085 and $1085 \div 365 = 2.972\ldots$, which is 2.97 when rounded to 2 d. p.

E FALSE A pie chart could be used to illustrate and compare the total numbers of eggs in the four seasons but it would not be showing 'All the data in the table'; some of the detail would have been lost.

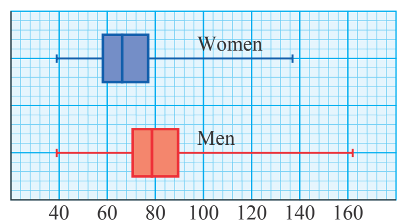
9 mean = 30 (rounded);
median = 22;
midrange = 46.5;
mode = 9
The best to use may depend on what you are trying to state. In Chapter 14, Robin was asked to report on the growing local concern about accidents involving children. In this case, the mode supports this argument. However, it can be argued that the mean is better as it uses all of the data or that the median is better as the extreme ages reported do not affect this measure.

Discussion point (page 325)

No. You worked out the frequency densities to draw the histogram. The modal class is that with the highest value.

Exercise 15.4 (page 329)

1 (i)



(ii) The median mass of the men is greater than the upper quartile of the women, so 50% of the men are heavier than 75% of the women. The central 50% of the women's masses is less symmetrical than the men's with the median towards the lower end. Both sets of data have long upper tails, suggesting both men and women's data have a few individuals who are heavier.

(iii) It is easier to see what is going on with the boxplots. They show you the actual distributions of the data whereas the cumulative frequency curves do not.

2 (i) 133 g
(ii) Median 135, lower quartile 118, upper quartile 156, IQR 38
(iii) The point for 160 should be plotted at 247
(iv) David's error affects the upper quartile and the IQR. They should be 144 and 26.
(v) This graph on its own is not very useful; a box plot (which can be generated from the graph) would be a better way of showing where the median lies and how spread out the data are.

3 (i) It is not clear in which interval a weight of, say, 2.5 kg should be recorded.
The table does not allow for very small or very large babies. The intervals are wide and so there is a risk of losing important information.
(ii) The majority of cases are within the usual weight range. There are some babies which are large and may cause issues during the birth process. There are many babies born with a low (below 2.5 kg) birth weight.
(iii) The doctors may need to know whether this trend is similar to others in similar areas. There may be other medical factors such as multiple births (twins and triplets), whether there are links to other diseases, whether these are first-born babies.

4 A FALSE It is true that the data have been grouped, and you could describe that process as rounding. At first sight the vertical line for, say, 100 could mean 'At least 95 and less than 105' which would correspond to rounding to the nearest 10. However, if you look at the very first vertical line this is marked 0 and that would correspond to an interval of −5 to 5. Clearly you cannot have a negative number of mobile phone accounts, so this interpretation must be wrong. The alternative that, for example, 100 means 'At least 100 and less than 110' must be the case.
B FALSE The modal class is 110 to 120. The mode is a single value. If the statement had been 'The mode of the numbers displayed' is 110, it would have been true.
C TRUE The total number of dots on the lines to the left of 100 is 89 and this is less than half of the total of 189.
D UNCERTAIN It may well be that the statement is true but the diagram does not provide you with the information to know it.
E FALSE Each dot represents one country and different countries have different populations.
F TRUE The raw data are means per 100 people, so they are numbers to 2 decimal places. Such data can be regarded as continuous so could indeed be represented by a frequency chart or a histogram.

5 On weekdays a histogram will show two maxima, one for each 'rush hour' when most people are travelling. This is more marked if school age children who have accidents during the holidays are omitted from the sample.
If the other data are grouped into weekends and children during school holidays the times are more evenly distributed.

Discussion point (page 332)

Place is ranked, Team is categorical and the other 7 variables are all discrete numerical.

Discussion point (page 333)

Manchester City and Liverpool are quite a long way from the rest of the teams so could be considered outliers. To be safe you should check that they are correctly shown on the scatter diagram.

Exercise 15.5 (page 335)

1 A positive correlation
B strong negative correlation
C no correlation
D positive correlation

2 C −0.461
A 0.560
D 0.802
Missing is B: estimate between −0.9 and −0.95

3 (i) There is a weak positive correlation between wingspan and weight.
(ii) One blackbird has wingspan of 122 mm and has weight of 128 g; this blackbird is much heavier than other birds with similar wingspans.

Another blackbird has wingspan 134 mm but a weight of only 81 g. This blackbird seems underweight compared with the rest of the sample.

(iii) This bird is clearly underweight. Maybe it is sick, or hungry, or both.

(iv) (a) 98 g. Yes it is consistent with the data on the scatter diagram.

(b) The points on the scatter diagram for a wingspan of 125 mm correspond to weights between 88 and 108 g so these are ±10 g on a value of 98, so about ±10%. It might be safer to give wider margins, say ±15%.

4 (i) The data are bivariate; they allow correlation/association between the variables to be identified.

(ii) Positive correlation: as the maximum monthly temperature increases the total monthly rainfall increases.

(iii) (14.4, 215). The total rainfall was unusually high for the maximum monthly temperature.

(iv) C = 15.6 (Note: the actual value that year was 15.0°C.)

(v) 269 mm

1975 is quite a long time before the data used to calculate the regression lines were collected. So there is an assumption that the climate has not changed in the intervening years.

5 (i) (a) The distinct sections are female elephant seals and male elephant seals.

(b) The outlier at the top right of the graph is a very large elephant bull seal, probably a dominant bull seal. The outlier near the middle of the graph (length 4.2 m) is possibly a young male.

(ii) A regression line would not be suitable due to there being two distinct groups within the sample. These two groups do not follow the same general pattern. Also, 15 is a small sample from which to produce a regression line.

(iii) $w = 775 \Rightarrow l = 2.48$. On the scatter diagram this point is clearly among the female elephant seals.

6 A TRUE The probabilities have been calculated correctly, dividing the number of homicides by the population in the same year.

B FALSE The number of homicides in given years is random variable but the years themselves are not.

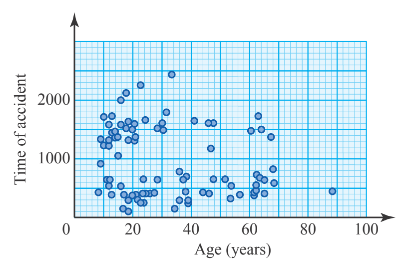
C TRUE The source of the data is not given but the National Census is carried out every 10 years, when the dates end in 1. So it was carried out in the years quoted, 1901, 1911, 1991 and 2001.

D FALSE The estimate is based on extrapolation well beyond the period covered by the data.

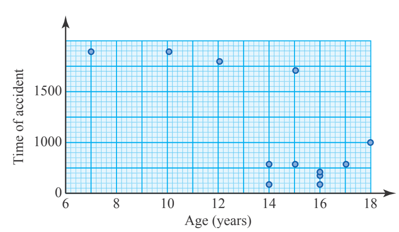
E FALSE The data form two islands on the scatter diagram and in that situation it is quite wrong to infer correlation from a line joining them (see page 334). Within each of the subsets the points do not fit a straight line particularly well, and there is a large gap in the data between 1911 and 1989.

A different issue is that the variables are not both random.

7 Using all of the data, you get a scatter graph like this which shows some areas or islands of data.



The effect becomes more noticeable if you look at the data from children who had accidents on weekdays not in school holidays.



Discussion points (page 340)

$$\sum(x-\bar{x})^2 = \sum(x^2 - 2x\bar{x} + \bar{x}^2)$$
$$= \sum x^2 - 2\bar{x}\sum x + \sum \bar{x}^2$$
$$= \sum x^2 - 2\bar{x}\left(n \times \sum \frac{x}{n}\right) + n\bar{x}^2$$

$\sum \frac{x}{n} = \bar{x}$ $\qquad$ $\sum \bar{x}^2 = n\bar{x}^2$

$$= \sum x^2 - 2n\bar{x}^2 + n\bar{x}^2$$
$$= \sum x^2 - n\bar{x}^2$$

If you are working by hand and the mean, $\bar{x}$, has several decimal places, working out the individual deviations can be tedious. So in such cases it is easier to use the $\sum x^2 - n\bar{x}^2$ form. However, it is good practice to get into the habit of using the statistical functions on your calculator or spreadsheet.

Activity 15.2 (page 341)

$$S_{xx} = 6,\ s^2 = \frac{6}{9} = 0.666...,$$
$$s = \sqrt{0.666...} = 0.816...$$

Discussion point (page 342)

No, standard deviation cannot be negative. If you work out a standard deviation and get a negative value, you have definitely made a mistake.

Exercise 15.6 (page 343)

1 (i) 0.14
(ii) 1.88

Number of (mm) away from 1 m	Mid point x	Fre-quency f	$f \times$ midpt	fx^2
$-5 \leqslant (l) < -3$	-4	6	-24	96
$-3 \leqslant (l) < -1$	-2	12	-24	48
$-1 \leqslant (l) < 1$	0	58	0	0
$1 \leqslant (l) < 3$	2	17	34	68
$3 \leqslant (l) < 5$	4	7	28	112
		100	14	324
		(i)	$\bar{x}$	0.14
		(ii)	σ	1.8

(iii) The estimates in (i) and (ii) assumed that all of the values in a group were the same value, the midpoint. The sample mean and standard deviation from the raw data was calculated using the actual measurements.

2 Both students have used the same data so the variation in plant height within the sample is the same. The different figures for the standard deviation are as a result of the different units of measurement chosen by the two students. In Alisa's case the standard deviation is 7.46 mm whilst Bjorn's standard deviation is 0.746 cm, these are equivalent distances.

3 (ii) Mean 58, standard deviation 14.9
(iii) (a) $58 + 2 \times 14.9$
$= 87.8 < 96$
(b) Outlier

4 (i) The box shows that the central part of the distribution is very nearly symmetrical about the median. The whiskers show that the distribution is skewed to the right.
(ii)

50–60	16×10	= 160
60–70	51×10	= 510
70–80	80×10	= 800
Total		1470

(iii) $81.7 \pm 2 \times 15.7$ so 50.3 to 113.1
(iv) $3152 - (16 \times 9.7 + 51 \times 10 + 80 \times 10 + 81 \times 10 + 48 \times 10 + 21 \times 10 + 3.1 \times 8) = 162$
(v) There are more very heavy people. The boxplot shows the distribution is skewed and this can also be seen on the histogram with more very high values than very low ones.

5 (i) Total weight is 1963.951 g
(ii) $\sum x^2 = 33819.03$
(iii) $n = 202$,
$\sum x = 3196.285$,
$\sum x^2 = 53748.02$
(iv) Mean = 15.823 g, standard deviation = 3.978

6 Estimated numbers of patients are as follows:
within 1 sd of the mean 2170, so 68.8% – very close to 68%
within 2 sds of the mean 2990, so 94.9% – extremely close to 95%
within 3 sd of the mean 3126. so 99.2% – very close to 99.75%.

7 (These values are rounded and answers may vary according to how the data is cleaned.)
The mean is 30 with a standard deviation of 20 (rounded); the median is 22 with a semi-IQR of 16. The mean takes extreme values into account, so the very young and very old can make a big difference to the mean – this is demonstrated by the large standard deviation. The median with the smaller semi-IQR may give a better representative value for the data set in this case. In this case the standard deviation and semi-IQR are both relatively large suggesting a varied set of data.

Chapter 16

Discussion point (page 360)

You would work out the probability that it is not flooded in 5 years and then subtract this from 1.

So for 5 years

P(the street is flooded in at least one year)

$= 1 - \text{P(it is not flooded in 5 years)}$

$= 1 - \left(\frac{29}{30}\right)^5 \approx \frac{1}{6.4}$

So the risk is once in every 6 or 7 years.

Exercise 16.1 (page 360)

1 $\frac{66}{534}$, assuming each faulty torch has only one fault

2 A TRUE – $\frac{90}{120} = \frac{3}{4}$

B UNCERTAIN – not enough information but it seems unlikely

C TRUE – the expected number is $\frac{80}{30} = 2\frac{2}{3}$ fledglings

D UNCERTAIN – not enough information. You don't know how many of the existing adult females and the female fledglings will have died by next year.

E TRUE – $\frac{1}{2}$ of 80 is 40 and $\frac{40}{120} = \frac{1}{3}$

3 (i) $\frac{1}{13}$

(ii) $\frac{36}{52} = \frac{9}{13}$

(iii) $\frac{9}{13}$

(iv) $\frac{16}{52} = \frac{4}{13}$

4 (i) $\frac{12}{108}$

(ii) $\frac{58}{108}$

(iii) $\frac{50}{108}$

(iv) $\frac{44}{108}$

(v) $\frac{66}{108}$

(vi) $\frac{4}{108}$

5 (i) 0.4

(ii) 0.5

(iii) 2.3

6 (i) 0.2

(ii) 0.6

(iii) 0.26. The readings are independent

7 (ii) Approximately $\frac{19}{60} = 0.32$

(iii) Approximately $\frac{19}{60} \times \frac{19}{60} = 0.1$

(iv) Consecutive journey times are independent of each other. However, the same roadworks could cause delays to both journeys, so the times would not be independent.

8 (i) $\frac{5}{2000}$

(ii) $\frac{1995}{2000}$

(iii) Lose £100, if all tickets are sold

(iv) 25p

(v) 2500

9 (i) 0.35

(ii) They might draw

(iii) 0.45

(iv) 0.45

10 (i) $k = 0.4$

r	2	4	6	8
$P(X = r)$	0.1	0.2	0.3	0.4

(ii) (a) 0.3

(b) 0.35

11 (i)

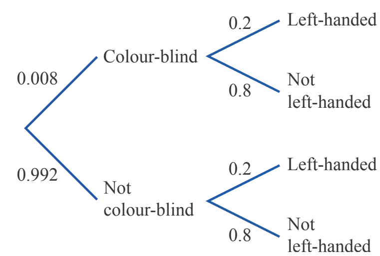
		First die					
	+	1	2	3	4	5	6
Second die	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

(ii) $\frac{3}{36} = \frac{1}{12}$

(iii) 7

(iv) The different outcomes are not all equally probable.

12



(i) 0.0016

(ii) 0.0064

(iii) 0.2064

(iv) 0.7936

13 1 in 2.6

14 (i) 0.0199

(ii) 0.65

(iii) 0.99

15 (i) (a) $\frac{15}{365}$

(b) $\frac{20}{365}$

(c) $\frac{330}{365}$

(ii) $\frac{55}{730\,000} = 0.000\,075\ldots$

(iii) Heat presents the greater risk.

16 (i) $k = 0.08$

r	0	1	2	3	4
$P(X = r)$	0.2	0.24	0.32	0.24	0

(ii) Let Y represent the number of chicks

r	0	1	2	3
$P(Y = r)$	0.351 04	0.449 28	0.184 32	0.015 36

Chapter 17

Exercise 17.1 (page 372)

1 $\frac{15}{64}$
2 0.271
3 0.214
4 0.294
5 (i) 0.146
(ii) Poor visibility might depend on the time of day, or might vary with the time of year. If so, this simple binomial model would not be applicable.
6 (i) $\frac{1}{8}$
(ii) $\frac{3}{8}$
(iii) $\frac{3}{8}$
(iv) $\frac{1}{8}$
7 (i) 0.246
(ii) Exactly 7 heads
8 (i) (a) 0.058
(b) 0.198
(c) 0.296
(d) 0.448
(ii) 2
9 (i) (a) 0.264
(b) 0.368
(c) 0.239
(d) 0.129
(ii) Assumed the probability of being born in January = $\frac{31}{365}$. This ignores the possibility of leap years and seasonal variations in the pattern of births throughout the year.
10 The three possible outcomes are not equally likely: 'one head and one tail' can arise in two ways (HT or TH) and is therefore twice as probable as 'two heads' or 'two tails'.

Exercise 17.2 (page 376)

1 (i) (a) 0.000129
(b) 0.0322
(c) 0.402
(ii) 0 and 1 are equally likely
2 (i) 2
(ii) 0.388
(iii) 0.323
3 (i) (a) 0.240
(b) 0.412
(c) 0.265
(d) 0.512
(e) 0.384
(f) 0.096
(g) 0.317
(ii) Assumption: the men and women in the office are randomly chosen from the population (as far as their weights are concerned).
4 (i) (a) $\frac{1}{81}$
(b) $\frac{8}{81}$
(c) $\frac{24}{81}$
(d) $\frac{32}{81}$
(e) $\frac{16}{81}$
(ii) 2 min 40 s
5 (i) He must be an even number of steps from the shop. (The numbers of steps he goes east or west are either both even or both odd, since their sum is 12, and in both cases the difference between them, which gives his distance from the shop, is even.)
(ii) 0.00293
(iii) At the shop
(iv) 0.242
(v) 0
(vi) 2.71
6 (i) 0.0735
(ii) 2.7
(iii) 0.267
(iv) The outcome of each trial (i.e. observing a moth that has been caught) is independent of any other trial. The probability that a moth is dark is the same for each trial.
7 (i) (a) 0.349
(b) 0.387
(c) 0.194
(ii) 0.070
(iii) 0.678

Chapter 18

Discussion point (page 380)

Assuming both types of parents have the same fertility, boys born would outnumber girls in the ratio 3 : 1. In a generation's time there would be a marked shortage of women of child-bearing age.

Discussion point (page 384)

It does seem a bit harsh when the number 1 came up more often than the others. You cannot actually prove a result with statistics but even trying to show it beyond reasonable doubt, in this case using a 5% significance level, can be difficult. This is particularly so if the sample size is small; it would have been better is the die had been thrown a lot more than 20 times.

Exercise 18.1 (page 386)

1 (i) Null hypothesis: $p = 0.25$; alternative hypothesis: $p > 0.25$
(ii) 0.0139
(iii) 5%
(iv) Yes
2 0.1275 Accept H_0
3 0.0547 > 5% Accept H_0
4 H_0: probability that toast lands butter-side down = 0.5
H_1: probability that toast lands butter-side down > 0.5
0.240 Accept H_0
5 0.048 Reject H_0. There is evidence that the complaints are justified at the 5% significance level, though Mr McTaggart might object that the candidates were not randomly chosen.

6 0.104 Accept H_0.
Insufficient evidence at the 5% significance level that the machine needs servicing.

7 (i) 0.590
(ii) 0.044
(iii) 0.0000712
(iv) 0.0292
(v) H_0: P(long question right) = 0.5; H_1: P(long question right) > 0.5
(vi) No

Discussion point (page 388)

$X \leqslant 4$

Exercise 18.2 (page 392)

1 (i) Henry is only interested in whether tails is more likely so he should carry out a 1-tail test. Mandy is interested in knowing about any bias so should carry out a two-tail test.
(ii) Henry: for 29 tails $p = 0.8987$ and for 30 tails $p = 0.9405$ so 30 is the critical value
Mandy: for 18 tails $p = 0.0325$ and for 19 tails $p = 0.0595$
For 30 tails $p = 0.9405$ and for 31 tails $p = 0.9675$ so her critical values are 18 and 31
(ii) Henry will accept any number of tails up to and including 29; Mandy will accept any number of tails between and including 19 and 30.
(iii) Henry will say the coin is biased if there are 30 or more tails.
(iv) Mandy will say the coin is biased if there are 18 or fewer tails or if there are 31 or more tails.
(v) Henry has only looked at whether tails is more likely; he has no way of knowing if heads is more likely. Mandy will be able to say whether either heads or tails is more likely and therefore whether the coin is biased.

2 $P(X \leqslant 7) = 0.1316 > 5\%$
Accept H_0

3 $P(X \geqslant 13) = 0.0106 < 2\frac{1}{2}\%$
Reject H_0

4 $P(X \geqslant 9) = 0.0730 > 2\frac{1}{2}\%$
Accept H_0

5 $P(X \geqslant 10) = 0.0139 < 5\%$
Reject H_0, but data not independent

6 (i) $P(X \geqslant 6) = 0.1018 > 5\%$
Accept H_0
(ii) Critical region $X > 6$; acceptance region $X \leqslant 6$

7 (i) $0.0395 < 5\%$
Reject H_0
(ii) $0.0395 > 2\frac{1}{2}\%$
Accept H_0

8 $\leqslant 1$ or > 9 males

9 $\leqslant 1$ or > 8 correct

10 Critical region is $\leqslant 3$ or $\geqslant 13$ letter Zs

11 (i) 20
(ii) 0.0623
(iii) Complaint justified

Practice questions Statistics (page 395)

1 (i) $0.75^3 = \frac{27}{64}$ (or 0.421 875) [2]
(ii) $0.75^3 + 0.2^3 + 0.05^3 = 0.43$ [2]

2 (i) Doughnut charts (ring charts, circle charts) are used to represent proportions of a total. In this case there is no such total as not all goals scored are shown. [1]
This type of chart makes comparisons difficult, particularly when the quantities are similar to one another. [1]
(ii) A bar chart would provide easier visual comparison. [1]
In this case there is little to be gained from having any sort of chart. The list of goal scorers and their numbers of goals is sufficient. [1]

NB: other sensible answers are possible.

3 (i) Simple random sampling is sampling in which all possible samples of the required size have an equal chance of selection. [1]
In this case simple random sampling would not be possible because there is no sampling frame, that is, there is no list of all customers from which a sample could be constructed. [1]
(ii) In opportunity sampling the interviewer(s) would interview any convenient customers until a total of 200 had been reached. [1]
In quota sampling the interviewer(s) would be given target numbers of interviewees in different groups, e.g. 100 male, 100 female; equal numbers under and over 40 years of age. [1]
Quota sampling is preferable. [1]
Because it attempts to make the sample representative of the target population. [1]

4 (i) Eruptions typically last between 1.5 and 5.5 minutes, with intervals between eruptions typically being between 40 and 95 minutes. [1]

There appear to be two different types of eruption, short and long. Similarly there are short and long intervals between eruptions. [1]

Short eruptions are typically followed by short intervals, long eruptions by long intervals. [1]

(ii) The data points separate out into two fairly distinct clusters. Within each cluster there is only a slight tendency for longer eruptions to be followed by longer times to the next eruption. [1]

(iii) The histogram of *Length of eruption* would be bimodal. [1]

The histogram of *Time to next eruption* would show little or no bimodality. [1]

(iv) The mean, 3.5 minutes, represents a length of eruption that is very unlikely to occur. So in that sense it could be misleading. [1]

However, it could be useful in other ways. For example, it is a figure that would be needed to estimate the proportion of the total time for which the geyser was erupting. [1]

NB: other sensible answers are possible.

5 (i) The sample sizes on these rows are very small in comparison with the rest. [1]

Very small sample sizes give unreliable information. [1]

(ii) There are three distinct phases to the graph. [1]

From 0600 to 0900, the blackbirds are gaining in mass, presumably through feeding. [1]

From 0900 to 1400 or 1500 the blackbirds are still gaining mass, but at a much lower rate. [1]

From 1500 to 1900 the blackbirds are losing mass; presumably they are not feeding (much), and they are expending energy and excreting. [1]

(iii) It is unlikely that this data point represents a real drop in blackbirds' masses; almost certainly this is just a random fluctuation in the data. (The actual drop is of very small magnitude; it looks bigger because the vertical axis does not start at zero.) [1]

(iv) There would be no merit in fitting a straight line model to the data set. [1]

It might be possible to find a simple curve (e.g. a quadratic) that gives a good model for the data. [1]

It would probably be best, however, to recognise that there are three distinct phases in the data and to model these phases separately.[1]

6 (i) (a) mean 10.0, standard deviation 48.9. [2]

(b) same values of Σx and Σx^2 but $n = 1802$. [1]

mean 19.5, standard deviation 66.7. [2]

(ii) The distribution has (large) positive skew. [1]

All 39 values of x above 110 are outliers (using mean plus twice SD). [1]

The two largest values are extreme even within the outliers. [1]

(iii) The model is supported by the fact that about half the magpies are recaptured very close to where they were ringed ($x = 0$, within 0.5 km). [1]

However there are clearly substantial numbers of exceptions too. [1]

NB: other sensible answers are possible.

7 (i) (1, 2), (2, 1), (2, 2). [1]

36 equally likely outcomes so the probability is $\frac{3}{36} = \frac{1}{12}$. [1]

(ii) $k = \frac{1}{36}$ [1]

$P(X = 6) = 11k = \frac{11}{36}$ [1]

(iii) $P(X = Y) = k^2 + (3k)^2 + (5k)^2 + (7k)^2 + (9k)^2 + (11k)^2 = 286k^2$ [1]

$P(X \neq Y) = 1 - 286k^2$ $(= 0.77932)$ [1]

Players A and B are equally likely to win. [1]

So P(B beats A) = 0.390 to 3 d.p. [1]

8 (i) Expected number is $0.15 \times 25 = 3.75$ cars. [1]

(ii) Use $X \sim B(25, 0.15)$ to find $P(X \leqslant 3)$ [1]

$P(X \leqslant 3) = 0.471$ (to 3 d.p.) [1]

(iii) $H_0: p = 0.15$, $H_1: p < 0.15$, where p is the proportion of cars with unsafe tyres. [2]

Use $X \sim B(50, 0.15)$ to find $P(X \leqslant 5) = 0.219$ [1]

Observe that $0.219 > 0.05$ so the observed result is not in the 5% critical region, hence insufficient evidence that p has reduced. [1]

(iv) Use $X \sim B(100, 0.15)$ to find $P(X \leqslant 7) = 0.012 > 0.01$ and $P(X \leqslant 6) = 0.005 < 0.01$. [2]

Hence the possible values of k are 0, 1…,6. [1]

Chapter 19

Discussion point (page 400)

−4, 0, −5

(i) +4

(ii) −5

Discussion point (page 401)

The marble is below the origin.

Exercise 19.1 (page 401)

1 (i) +1 m

(ii) +2.25 m

2 (i) 3.5 m, 6 m, 6.9 m, 6 m, 3.5 m, 0 m

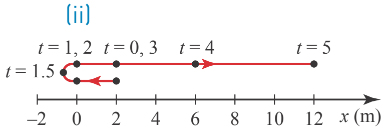
(ii) 0 m, 2.5 m, 3.4 m, 2.5 m, 0 m, −3.5 m

(iii) (a) 3.4 m

(b) 10.3 m

3 (i) 2 m, 0 m, −0.25 m, 0 m, 2 m, 6 m, 12 m

(ii)

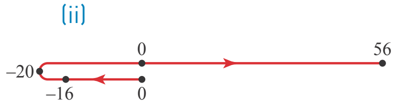


(iii) 0 m, −2 m, −2.25 m, −2 m, 0 m, 4 m, 10 m

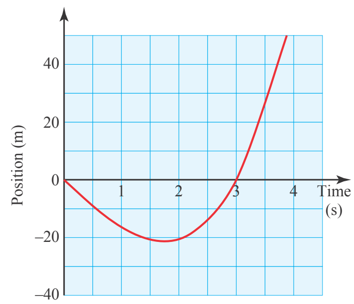
(iv) 14.5 m

4 (i) 0 m, −16 m, −20 m, 0 m, 56 m

(ii)

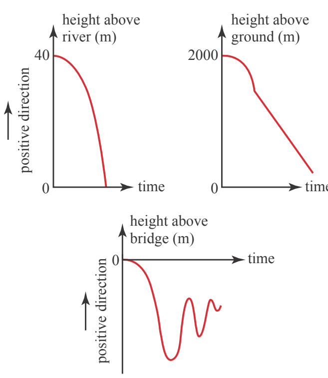


(iii)



(iv) $t = 0$ moving backwards, $t = 3$ moving forwards.

5



6 (i) The ride starts at $t = 0$. At A it changes direction and returns to pass the starting point at B, continuing past to C, where it changes direction again, returning to its initial position at D.

(ii) An oscillating ride such as a swing boat.

Discussion point (page 403 upper)

10, 0, −10. The gradient represents the velocity.

Discussion point (page 403 lower)

The graph would curve where the gradient changes. Little effect over this period.

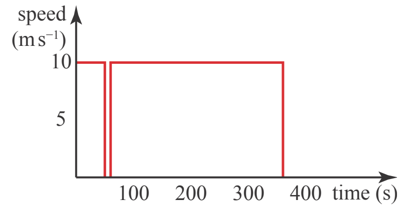
Discussion point (page 404)

$+5\,\text{m s}^{-1}, 0\,\text{m s}^{-1}, -5\,\text{m s}^{-1}, -6\,\text{m s}^{-1}$.

The velocity decreases at a steady rate.

Exercise 19.2 (page 405)

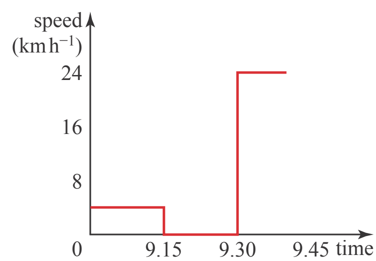
1



2 (i) The person is waiting at the bus stop.

(ii) It is faster.

(iii)



(iv) constant speed, infinite acceleration

3 (i) (a) 2 m, 8 m

(b) 6 m

(c) 6 m

(d) $2\,\text{m s}^{-1}, 2\,\text{m s}^{-1}$

(e) $2\,\text{m s}^{-1}$

(f) $2\,\text{m s}^{-1}$

(ii) (a) 60 km, 0 km

(b) −60 km

(c) 60 km

(d) $-90\,\text{km h}^{-1}, 90\,\text{km h}^{-1}$

(e) $-90\,\text{km h}^{-1}$

(f) $90\,\text{km h}^{-1}$

(iii) (a) 0 m, −10 m

(b) −10 m

(c) 50 m

(d) OA: $10\,\text{m s}^{-1}, 10\,\text{m s}^{-1}$; AB: $0\,\text{m s}^{-1}, 0\,\text{m s}^{-1}$; BC: $-15\,\text{m s}^{-1}$, $15\,\text{m s}^{-1}$

(e) $-1.67\,\text{m s}^{-1}$

(f) $8.33\,\text{m s}^{-1}$

(iv) (a) 0 km, 25 km

(b) 25 km

(c) 65 km

(d) AB: $-10\,\text{km h}^{-1}$, $10\,\text{km h}^{-1}$; BC: $11.25\,\text{km h}^{-1}$, $11.25\,\text{km h}^{-1}$

(e) 4.167 km h^{-1}

(f) 10.83 km h^{-1}

4 10.44 m s^{-1}, 37.58 km h^{-1}

5 20.59 km h^{-1}

6 1238.71 km h^{-1}

7 40 km h^{-1}

8 32 km h^{-1}; 35.7 km h^{-1}

9 (i) (a) 56.25 km h^{-1}

(b) 97.02 km h^{-1}

(c) 46.15 km h^{-1}

(ii) The ratio of distances must be in the ratio 10 : 3

10 (i) 4.48 km h^{-1}, 36.73 km h^{-1}, 18.32 km h^{-1}; 26.15 km h^{-1}

(ii) B finishes first

Discussion point (page 407)

(i) D

(ii) B, C, E

(iii) A

Exercise 19.3 (page 408)

1 (i) (a) 0.8 m s^{-2}

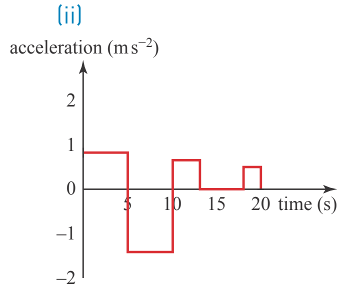
(b) -1.4 m s^{-2}

(c) 0.67 m s^{-2}

(d) 0 m s^{-2}

(e) 0.5 m s^{-2}

(ii)



2 (i) 20 m s^{-1}

(ii) 7.5 s

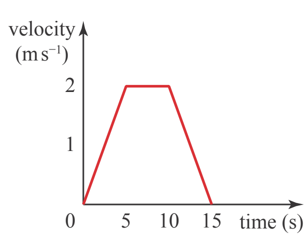
3 (i) 6 m s^{-2}

(ii) 6 m s^{-1}

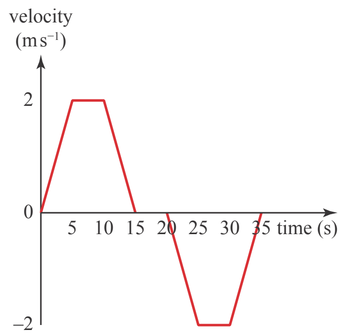
(iii) $a = 3t$

(iv) $t = 5$

4 (i)

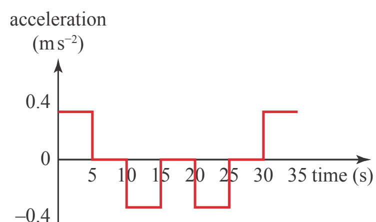


(ii)



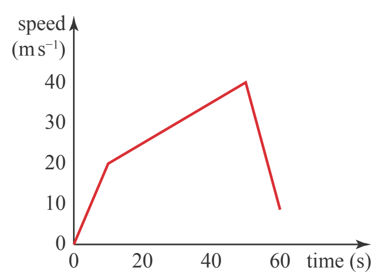
(iii) +0.4 m s^{-2}, 0 m s^{-2}, -0.4 m s^{-2}, 0 m s^{-2}, -0.4 m s^{-2}, 0 m s^{-2}, +0.4 m s^{-2}

(iv)



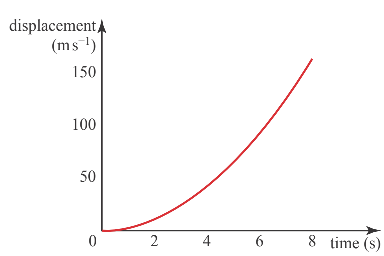
5 (i) 20 m s^{-1}, 40 m s^{-1}, 10 m s^{-1}

(ii)

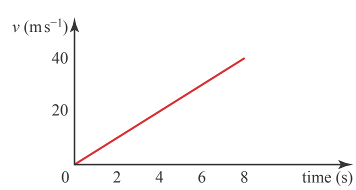


(iii) 1450 m

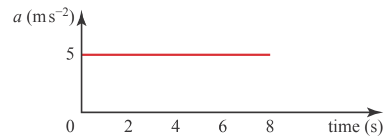
6 (i)



(ii) (a)



(b)



Discussion point (page 409)

(i) 5

(ii) 20

(iii) 45

They are the same.

Discussion point (page 410)

It represents the displacement.

Discussion point (page 411)

Approximately 460 m.

Discussion point (page 412)

No, as long as the lengths of the parallel sides are unchanged, the trapezium has the same area.

Exercise 19.4 (page 412)

1 (i) (A) 0.4 m s^{-2}, 0 m s^{-2}, 3 m s^{-2}

(B) -1.375 m s^{-2}, -0.5 m s^{-2}, 0 m s^{-2}, 2 m s^{-2}

(ii) (A) 62.5 m

(B) 108 m

(iii) (A) 4.17 m s^{-1}

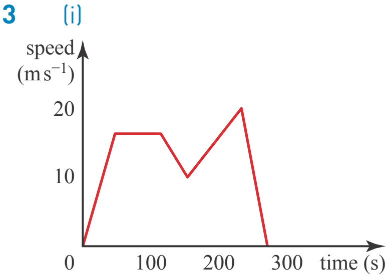
(B) 3.6 m s^{-1}

2 (i) Enters motorway at 10 m s^{-1}, accelerates to 30 m s^{-1} and maintains this speed for about 150 s. Slows down to a stop after a total of 400 s.

(ii) approx. 0.4 m s^{-2}, -0.4 m s^{-2}

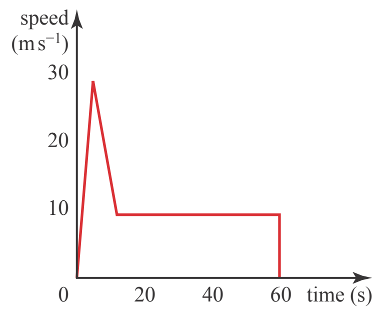
(iii) approx. 9.6 km, 24 m s^{-1}

3 (i)



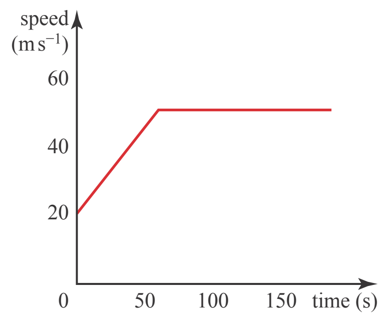
(ii) 3562.5 m

4 (i)



(ii) 558 m

5 (i)

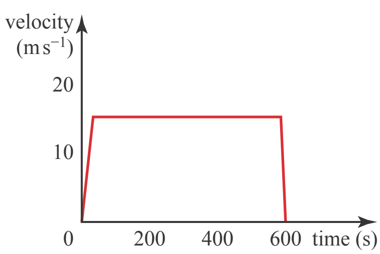


(ii) 60 s

(iii) 6600 m

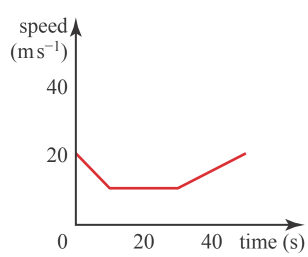
(iv) $v = 20 + 0.5t$,
$0 \leqslant t \leqslant 60$;
$v = 50$, $t \geqslant 60$

6 (i)



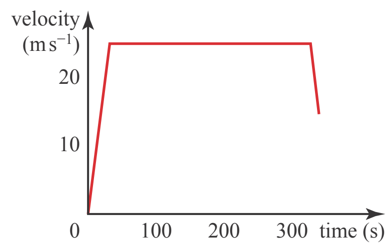
(ii) $15\,m\,s^{-1}$, $-1\,m\,s^{-2}$, 8.66 km

7 (i)



(ii) $13\,m\,s^{-1}$

8 (i)



(ii) 343 s

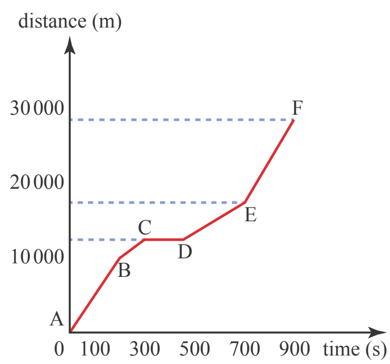
9 (i) BC: decelerates uniformly, CD: stopped, DE: accelerates uniformly

(ii) $a = -0.5\,m\,s^{-2}$, 2500 m

(iii) $0.2\,m\,s^{-2}$, 6250 m

(iv) 325 s

(v)

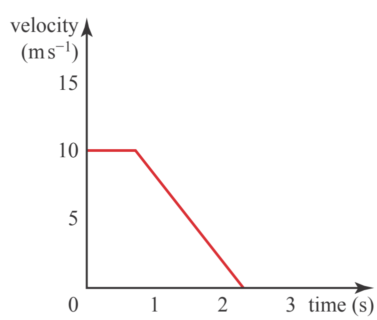


10 462.5 s

11 40 s

12 (i) $10\,m\,s^{-1}$, 0.7 s

(ii)



(iii) $6.25\,m\,s^{-2}$

(iv) 33.9 m

13 (i) 7.5 m

(ii) $1.2\,m\,s^{-1}$

(iii) 10 s

(iv) $a = -0.4\,m\,s^{-2}$

14 (i) A: 17 mins 30 seconds after starting; B: 16 mins 27 seconds after starting

(ii) 16 mins 35 seconds after A starts, 9450 m from the start

15 (i) 11.125 s

(ii) 10 s

Discussion point (page 416)

$s = \frac{1}{2}(2u + at) \times t$

$s = \left(u + \frac{1}{2}at\right) \times t$

$s = ut + \frac{1}{2}at^2$

Discussion point (page 418)

$u = 13.\dot{3}\,m\,s^{-1}$, $v = 26.\dot{6}\,m\,s^{-1}$,
$t = 5$, $s = 100\,m$

Exercise 19.5 (page 419)

1 (i) $22\,m\,s^{-1}$

(ii) 120 m

(iii) 0 m

(iv) $-10\,m\,s^{-2}$

2 (i) $v^2 = u^2 + 2as$

(ii) $v = u + at$

(iii) $s = ut + \frac{1}{2}at^2$

(iv) $s = \frac{1}{2}(u + v)t$

(v) $v^2 = u^2 + 2as$

(vi) $s = ut + \frac{1}{2}at^2$

(vii) $v^2 = u^2 + 2as$

(viii) $s = vt - \frac{1}{2}at^2$

3 (i) $9.8\,m\,s^{-1}$, $98\,m\,s^{-1}$

(ii) 4.9 m, 490 m

(iii) 2 s, speed and distance after 10 s, both over-estimates

4 $2.08\,m\,s^{-2}$, 150 m, assume constant acceleration

5 $4.5\,m\,s^{-2}$, 9 m

6 $-8\,m\,s^{-2}$, 3 s

7 $a = -0.85\,m\,s^{-2}$, 382 m

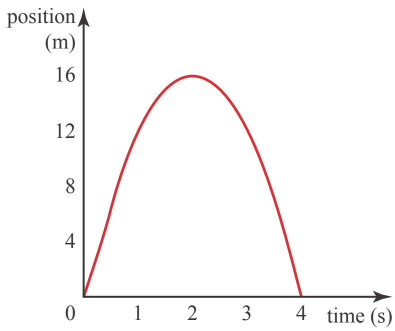
8 3.55 s

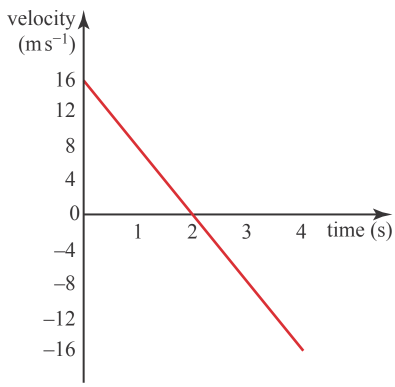
9 (i) $s = 16t - 4t^2$
$v = 16 - 8t$

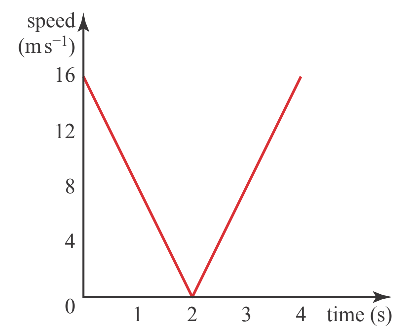
(ii) (a) 2 s

(b) 4 s

(iii)







Discussion point (page 422)

$u = -15.0$. No.

Exercise 19.6 (page 423)

1 604.9 s, 9.04 km

2 (i) $v = 2 + 0.4t$

(ii) $s = 2t + 0.2t^2$

(iii) $18\,\text{m s}^{-1}$

3 No, 10 m behind

4 (i) $h = 4 + 2t - 4.9t^2$

(ii) 1.13 s

(iii) $9.08\,\text{m s}^{-1}$

(iv) t greater, v less

5 (i) $11.75\,\text{m s}^{-1}$

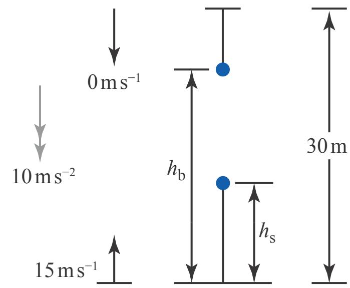
(ii) 8.29 m

(iii) $12.75\,\text{m s}^{-1}$

(iv) 5.31 m

(v) underestimate

6 (i)



(ii) $h_s = 15t - 4.9t^2$

(iii) $h_b = 30 - 4.9t^2$

(iv) $t = 2s$

(v) 10.4 m

7 (i) $5.4\,\text{m s}^{-1}$

(ii) $-4.4\,\text{m s}^{-1}$

(iii) $1\,\text{m s}^{-1}$ gain

(iv) $9\,\text{m s}^{-1}$

(v) too fast

8 2.94 m

9 43.75 m

10 (ii) $15.15 = u + 5a$

(iii) $14.4\,\text{m s}^{-1}$

(iv) No, distance at constant a is 166.5 m

11 $22.7\,\text{m s}^{-1}$

12 $-\frac{5}{12}\,\text{m s}^{-2}$, $40\frac{5}{6}\,\text{m}$

13 2.5 m

14 $3\frac{1}{3}\,\text{m s}^{-1}$, $\frac{1}{9}\,\text{m s}^{-2}$, 9.5 m from the bus

15 107.5 m

Chapter 20

Discussion point (page 431)

The reaction between the chair and the person acts on the chair. The person's weight acts on the person only.

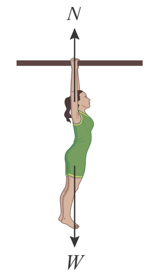
Discussion point (page 432)

Vertically up

Exercise 20.1 (page 433)

In these diagrams, W represents a weight, N a normal reaction with another surface, F a friction force, R air resistance and P another force.

1



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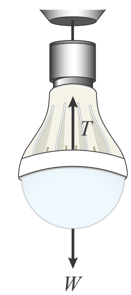

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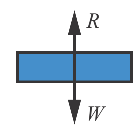

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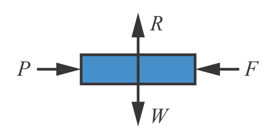

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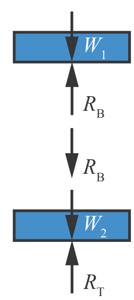

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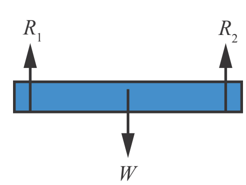
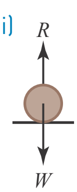
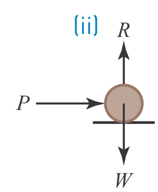


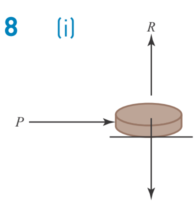
7 (i)



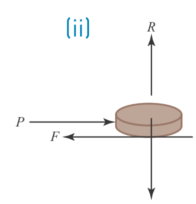
(ii)



8 (i)



(ii)



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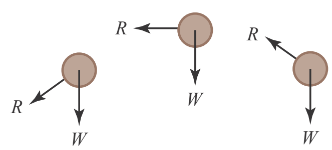
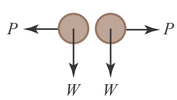
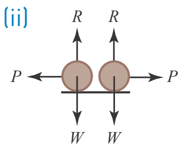


10 (i)



(ii)



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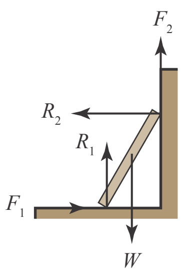
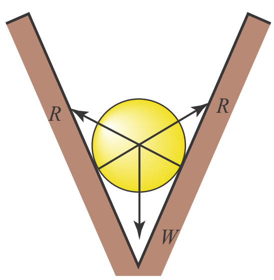


12



Discussion point (page 434 upper)

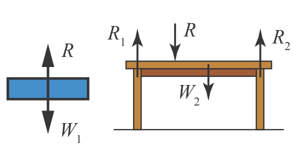
To provide forces when the velocity changes.

Discussion point (page 434 lower)

The friction force was insufficient to enable his car to change direction at the bend.

Exercise 20.2 (page 436)

1 (i)



(ii) (a) $R = W_1$

(b) $R_1 + R_2 = W_2 + R$

2 (i) $R = W, 0$

(ii) $W > R, W - R$, down

(iii) $R > W, R - W$ up

3 (i) No

(ii) Yes

(iii) Yes

(iv) No

(v) Yes

(vi) Yes

(vii) Yes

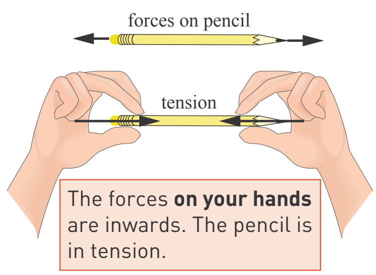
(viii) No

4 Forces are required to give passengers the same acceleration as the car.

(i) A seat belt provides a backwards force.

(ii) The seat provides a forwards force on the body and the head rest is required to make the head move with the body.

Discussion point (page 437)



Exercise 20.3 (page 440)

1 (i) 147 N

(ii) 11760 N = 11.76 kN

(iii) 0.49 N

2 (i) 61.2 kg

(ii) 1122 kg = 1.12 tonne

3 (i) 637 N

(ii) 637 N

4 112 N

5 (i) Both hit the ground together.

(ii) The balls take longer to hit the ground on the moon, but still do so together.

6 *Answers for 60 kg*:

(ii) 588 N

(iii) 96 N

(iv) Its mass is 4 kg.

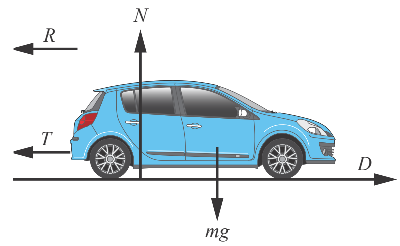
Discussion point (page 440)

No. Scales which measure by balancing an object against fixed masses (weights).

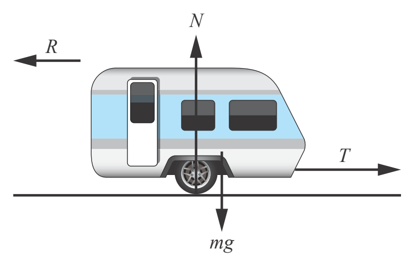
Exercise 20.4 (page 442)

In these diagrams, mg represents a weight, N a normal reaction with another surface, F a friction force, R air resistance, T a tension or thrust, D a driving force and P another force.

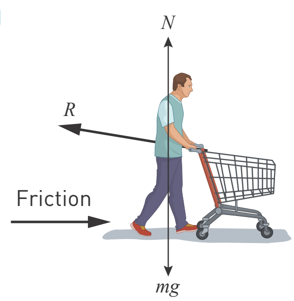
1 (i)



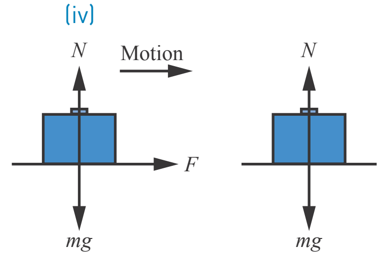
(ii)



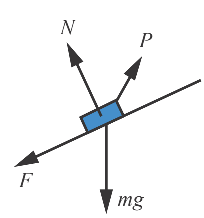
(iii)



(iv)



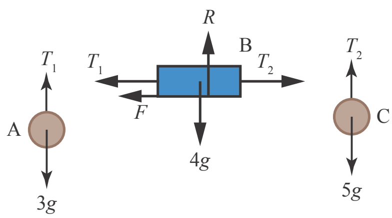
(v)



2 (i) Weight $5g$ down and reaction ($= 5g$) up.
(ii) Weight $5g$ down, reaction with box above ($= 45g$ down) and reaction with ground ($50g$ up).

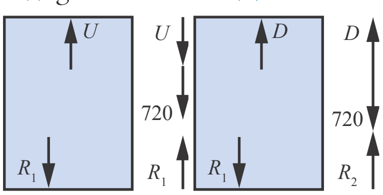
3 (i) $F_1 = 10$ N
(ii) $15 - F_2$ N

4 (i) Towards the left.
(ii)

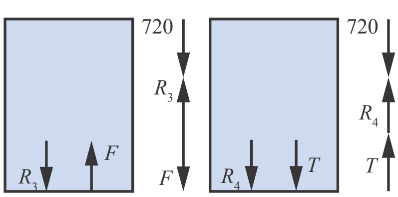


(iii) $3g$ N, $5g$ N
(iv) $2g$ N
(v) $T_1 - 3g$ ↑,
$T_2 - T_1 - F$ →,
$5g - T_2$ ↓

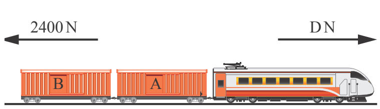
5 All forces are in newtons.
(i) greater (ii) less



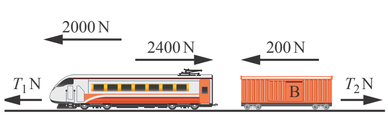
(iii) greater (iv) less



6 (i) 2400 N



(ii)



(iii) $T_1 = 400$ N
(iv) $T_2 = 200$ N

Discussion point (page 444)

The pointer moves up and down as the force on the spring varies. Your weight would seem to change as the speed of the lift changed. You feel the reaction force between your hand and the book which varies as you move the book up and down.

Discussion point (page 445)

There is a resultant downward force because the weight is greater than the tension.

Exercise 20.5 (page 447)

1 (i) 800 N
(ii) 88 500 N
(iii) 0.0225 N
(iv) 840 000 N
(v) 8×10^{-20} N
(vi) 548.8 N
(vii) 8.75×10^{-5} N
(viii) 10^{30} N

2 (i) 200 kg
(ii) 50 kg
(iii) 10 000 kg
(iv) 1.02 kg

3 (i) 7.6 N
(ii) 7.84 N

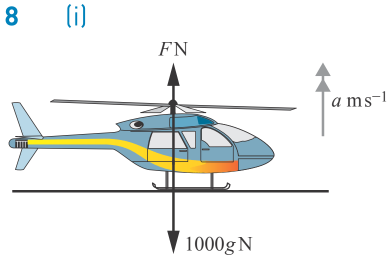
4 (i) $0.5\,\mathrm{m\,s^{-2}}$
(ii) 25 m

5 (i) $1.67\,\mathrm{m\,s^{-2}}$
(ii) 16.2 s

6 (i) 325 N
(ii) 1764 N

7 (i) 13 N
(ii) 90 m
(iii) 13 N

8 (i)



(ii) 11300 N

9 (i) $400 - 250 = 12000a$, $a = 0.0125\ \mathrm{m\,s^{-2}}$
(ii) $0.5\,\mathrm{m\,s^{-1}}$, 40 s
(iii) (a) 15 s
(b) 13.75 m
(c) 55 s

10 (i) $60\,\mathrm{m\,s^{-1}}$
(ii) continues at $60\,\mathrm{m\,s^{-1}}$
(iii) 1.25 N
(iv) the first one by 382 km

11 (i) 6895 N, 6860 N, 6790 N, 1960 N
(ii) 815 kg
(iii) max $T < 9016$ N

12 (i) $7.84\,\mathrm{m\,s^{-2}}$
(ii) $13.7\,\mathrm{m\,s^{-1}}$ which is just over 30 mph.

13 $1\,\mathrm{m\,s^{-2}}$

14 26.2 s

Discussion points (page 450)

Your own weight acts on you and the tensions in the ropes with which you have contact; the other person's weight acts on them. The tension forces acting at the ends of the rope AB are equal and opposite. The accelerations of A and B are equal because they must always travel the same distance in each interval of time, assuming the rope does not stretch.

Discussion point (page 451)

The tension in the rope joining A and B must be greater than B's weight because there must be a resultant force on B to produce an acceleration.

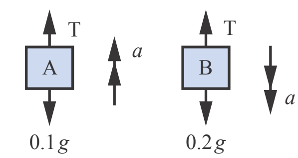
Discussion point (page 453)

Using $v = u + at$ with $u = 0$ and maximum $a = 6.2$, the speed after 1 second would be $6.2\ \mathrm{m\,s^{-1}}$ or about 14 mph. Under the circumstances, a careful driver is unlikely to accelerate at this rate. Alvin and his snowmobile and Bernard are two particles each moving in a straight line, otherwise

Bernard could swing from side to side; contact between the ice and the rope is smooth, otherwise the tensions acting on Alvin and Bernard are different; the rope is light, otherwise its tension could be affected by its weight; the rope is of constant length, otherwise the accelerations would not be equal; there is no air resistance, otherwise the equations of motion would involve a force to allow for it.

Exercise 20.6 (page 455)

1 (i)



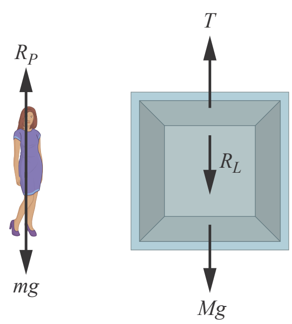
(ii) A: $T - 0.98 = 0.1a$
B: $1.96 - T = 0.2a$

(iii) $3.27\,\text{m s}^{-2}$, 1.31 N

(iv) 1.11 s

2 $2.8\,\text{m s}^{-2}$, 14 N

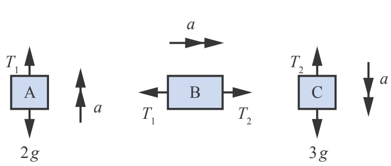
3 (i)



(ii) $R_p = R_L = 490\,\text{N}$, $T = 4900\,\text{N}$

(iii) $R_p = R_L = 530\,\text{N}$, $T = 5300\,\text{N}$

4 (i)

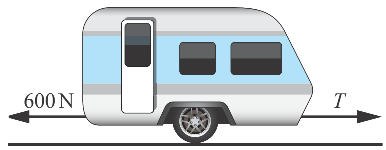


(ii) A: $T_1 - 2g = 2a$
B: $T_2 - T_1 = 5a$
C: $3g - T_2 = 3a$

(iii) $a = 1\,\text{m s}^{-2}$, $T_1 = 22\,\text{N}$, $T_2 = 27\,\text{N}$

(iv) 5 N

5



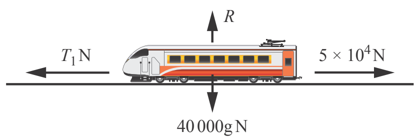
(ii) $T = 750\,\text{N}$

(iii) tension $= 44.\dot{4}\,\text{N}$

(iv) 170 N

6 (i) $0.625\,\text{m s}^{-2}$

(ii) 25 000 N

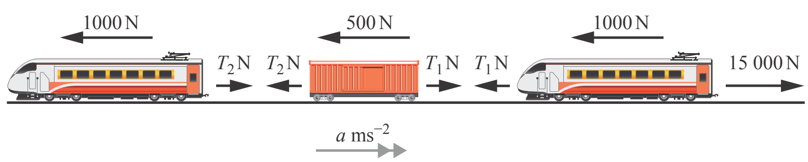


(iii) 12 500 N

(iv) reduced to 10 000 N

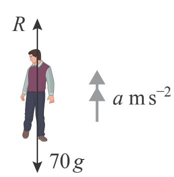
7 (i) $0.25\,\text{m s}^{-2}$

(ii) $T_1 = 9\,\text{kN}$, $T_2 = 6\,\text{kN}$



(iii) $0.25\,\text{m s}^{-2}$, $T_1 = 1.5\,\text{kN}$ in tension, $T_2 = -1.5\,\text{kN}$ in thrust. The second locomotive is now pushing rather than pulling back on the truck.

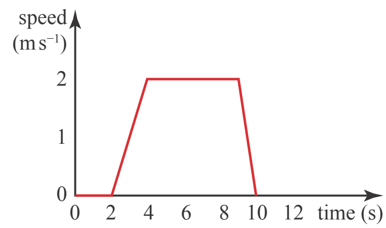
8 (i)



(ii) $1\,\text{m s}^{-2}$

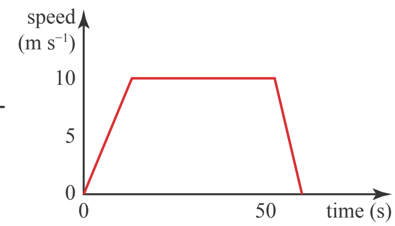
(iii) Stationary for 2 s, accelerating at $1\,\text{m s}^{-2}$ for 2 s, at constant speed for 5 s, decelerating at $2\,\text{m s}^{-2}$ for 1 s, stationary for 2 s.

(iv)



(vi) 13 m

9 (i)



(ii) $0.8\,\text{m s}^{-2}$

(iii) 40 s, $-1.33\,\text{m s}^{-2}$

(iv) 66.4 kN

(v) 90 kg

10 (i) $2.26\,\text{m s}^{-2}$, 60.3 N

(ii) 0.56 m above ground

11 (i) $2\frac{1}{3}\,\text{m s}^{-2}$, 14.9 N

(ii) 1.23 kg

12 (ii) $T = 5200\,\text{N}$

(iii) $-0.186\,\text{m s}^{-2}$

(iv) (a) $T = -1660\,\text{N}$ (compression)

(b) $T = 10340\,\text{N}$ (tension)

13 (i) $0.1\,\text{m s}^{-2}$

(ii) 27.5 kN

(iv) 1.1 kN

14 $\frac{(S_1 - S_n)}{(n-1)}M$

15 (i) 0.81 s

(ii) 1.4 s

Answers

Chapter 21

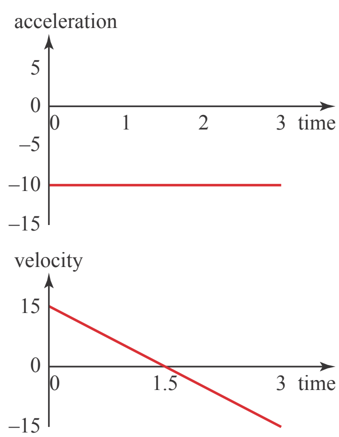
Exercise 21.1 (page 467)

1 (i) (a) $v = 2 - 2t$
(b) 10, 2
(c) 1, 11
(ii) (a) $v = -4 + 2t$
(b) 0, −4
(c) 2, −4
(iii) (a) $v = 3t^2 - 10t$
(b) 4, 0
(c) 0, 4 and $3\frac{1}{3}$, −14.5

2 (i) (a) $a = 4$
(b) 3, 4
(ii) (a) $a = 12t - 2$
(b) 1, −2
(iii) (a) $a = 7$
(b) −5, 7

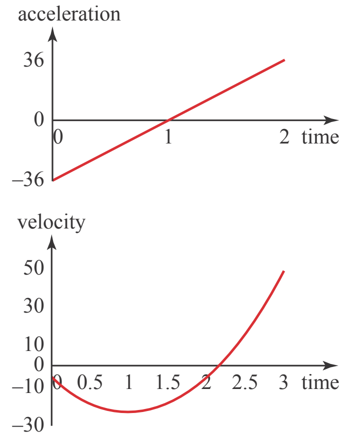
3 $v = 4 + t \quad a = 1$

4 (i) $v = 15 - 10t \quad a = -10$
(ii)



(iii) the acceleration is the gradient of the velocity–time graph
(iv) the acceleration is constant, the velocity decreases at a constant rate

5 (i) $v = 18t^2 - 36t - 6$
$a = 36t - 36$
(ii)



(iii) The acceleration is the gradient of the velocity–time graph, the velocity is at a minimum when the acceleration is zero.
(iv) It starts in the negative direction, v is initially −6 and decreases to −24 when $t = 1$, before increasing rapidly to 0, where the object turns round to move in the positive direction.

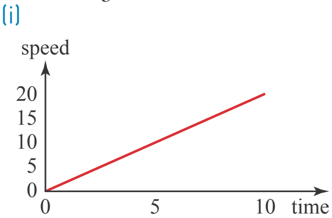
6 (i) $-16\,\text{m s}^{-1}$, $4\,\text{m s}^{-2}$
(ii) 25.41 m

7 $12\,\text{m s}^{-2}$ when $t = 1$,
$-12\,\text{m s}^{-2}$ when $t = 5$

Exercise 21.2 (page 472)

1 (i) $s = 2t^2 + 3t$
(ii) $s = \frac{3}{2}t^4 - \frac{2}{3}t^3 + t + 1$
(iii) $s = \frac{7}{3}t^3 - 5t + 2$

2 (i)



(ii) 85 m

3 (i) when $t = 6$
(ii) 972 m

4 (i) 4.47 s
(ii) 119 m

5 (i) $v = 10t + \frac{3}{2}t^2 - \frac{1}{3}t^3$,
$x = 5t^2 + \frac{1}{2}t^3 - \frac{1}{12}t^4$
(ii) $v = 2 + 2t^2 - \frac{2}{3}t^3$,
$x = 1 + 2t + \frac{2}{3}t^3 - \frac{1}{6}t^4$
(iii) $v = -12 + 10t - 3t^2$,
$x = 8 - 12t + 5t^2 - t^3$

Discussion points (page 472)

Case (i) $s = ut + \frac{1}{2}at^2$; $v = u + at$;
$a = 4$, $u = 3$.

In the other two cases, the acceleration is not constant.

Activity 21.1 (page 472)

Substituting in $at = v - u$ ② gives

$s = ut + \frac{1}{2}(v - u)t + s_0$

$s = \frac{1}{2}(u + v)t + s_0$ ③

Substituting $v - u = at$ and
$v + u = \frac{2}{t}(s - s_0)$

$\Rightarrow (v - u)(v + u) = at \times \frac{2}{t}(s - s_0)$

$v^2 - u^2 = 2a(s - s_0)$ ④

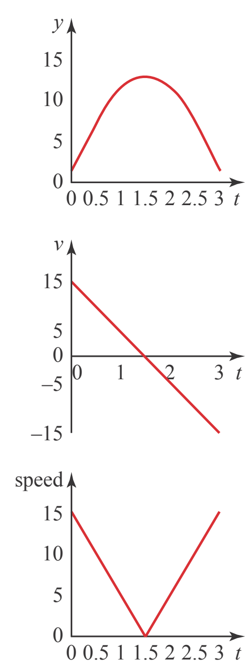
Substituting $u = v - at$ in ② gives

$s = (v - at)t + \frac{1}{2}at^2 + s_0$

$s = vt - \frac{1}{2}at^2 + s_0$ ⑤

Exercise 21.3 (page 473)

1 (i) $v = 15 - 10t$
(ii) 11.5 m, $5\,\text{m s}^{-1}$, $5\,\text{m s}^{-1}$;
11.5 m, $-5\,\text{m s}^{-1}$, $5\,\text{m s}^{-1}$
(iii)



(iv) 3 s

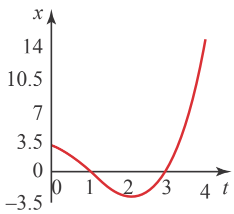
(v) this gives the displacement of the ball, which is 0

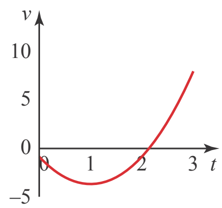
2 (i) $-3\,\text{m}, -1\,\text{m}\,\text{s}^{-1}, 1\,\text{m}\,\text{s}^{-1}$

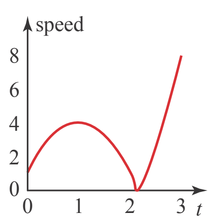
(ii) (a) 1 s

(b) 2.15 s

(iii)





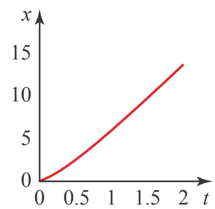


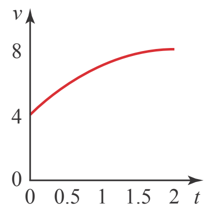
(iv) The object starts at 3 and moves in the negative direction from 3 to −3, which is reached when $t = 2.15$. It then moves in the positive direction with increasing speed.

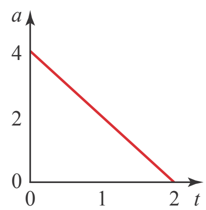
3 (i) $v = 4 + 4t - t^2$

$x = 4t + 2t^2 - \frac{1}{3}t^3$

(ii)







(iii) The object starts at the origin and moves in the positive direction with increasing speed reaching a maximum speed of $8\,\text{m}\,\text{s}^{-1}$ after 2 s.

4 (i) 0, 10.5, 18, 22.5, 24

(ii) The ball reaches the hole at 4 s.

(iii) $v = 12 - 3t$

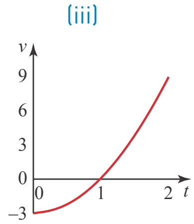
(iv) $0\,\text{m}\,\text{s}^{-1}$

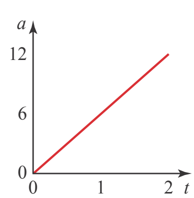
(v) $a = -3\,\text{m}\,\text{s}^{-2}$

5 (i) $v = 3t^2 - 3$, $a = 6t$

(ii) $t = 1$

(iii)



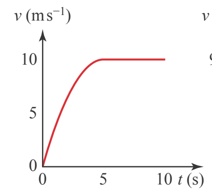


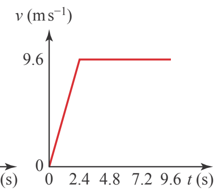
(iv) Object starts at O and moves towards A, slowing down to 0 when $t = 1$, then accelerates back towards B reaching a speed of $9\,\text{m}\,\text{s}^{-1}$ when $t = 2$.

(v) 6 m

6 (i) Andrew: $10\,\text{m}\,\text{s}^{-1}$, Elizabeth: $9.6\,\text{m}\,\text{s}^{-1}$

(ii)





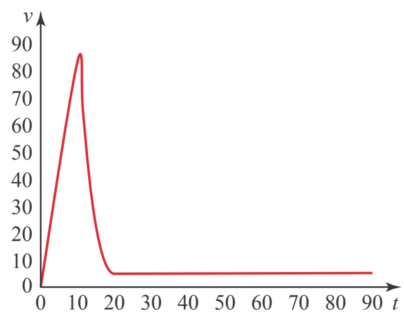
(iii) 11.52 m

(iv) 11.62 s

(v) Elizabeth by 0.05 s and 0.5 m

(vi) Andrew wins

7 (i)



Cara is in free fall until $t = 10\,\text{s}$, then the parachute opens and she slows down to terminal velocity of $5\,\text{m}\,\text{s}^{-1}$

(ii) 1092 m

(iii) $8.5\,\text{m}\,\text{s}^{-2}$, $1.6t - 32$, $0\,\text{m}\,\text{s}^{-2}$, $16\,\text{m}\,\text{s}^{-2}$

8 2 s

9 (i) 40 m

(ii) $s = 0$ when $t = 0$ and 10

(iii) $25 - 5t$

(iv) 62.5 m

(v) $s = 0$ for $t = 0$ and $t = 10$ for both models. In Michelle's model, the velocity starts at $25\,\text{m}\,\text{s}^{-1}$ and decreases to $-25\,\text{m}\,\text{s}^{-1}$ at $t = 10$. The teacher's model is better as the velocity starts and ends at 0.

10 (i) (a) 112 cm

(b) 68 cm

(ii) $4t$, 16

(iii) $2t^2$, $16t - 32$

(iv) $\frac{8}{9}$ cm less

11 (i) PQ: train speeds up with gradually decreasing acceleration
QR: train travels at constant speed
ST: train slows down with constant deceleration
(ii) $a = -0.000025t + 0.05$
(iii) $50\,\text{m s}^{-1}$
(iv) $0\,\text{m s}^{-1}$
(v) $111\frac{2}{3}$ km

12 (i) $-24 + 18t - 3t^2$, 2, 4
(ii) $-2, 2$
(iii) 28

13 $4\frac{4}{15}$ m, $4\,\text{m s}^{-1}$

14 (i) 12.15 m
(ii) 13.85 s
(iii) 26.33 s

15 $s(2) = 2$, 0 and $\frac{2}{3}$ s,
$s(0) = 0 \quad s\left(\frac{2}{3}\right) = -\frac{2}{27}$,
distance $= 2\frac{4}{27}$ m

16 (i) $5\,\text{m s}^{-1}$, $0.5\,\text{m s}^{-2}$
(ii) 15 s
(iii) $41\frac{2}{3}$ m

Practice questions Mechanics (page 478)

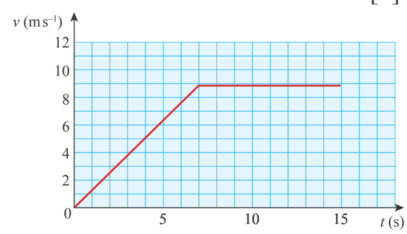
1 $s = 2800\,\text{m}$, $u = 0$,
$v = 70\,\text{m s}^{-1}$, $a = $, $t = ?$
$s = \frac{1}{2}(u + v)t$
$2800 = \frac{1}{2}(0 + 70)t$
$t = \frac{2800}{35} = 80\,\text{s}$

2 $x = 5 + 2.1t^2 - 0.07t^3$
$v = \frac{dx}{dt} = 4.2t - 0.21t^2$ [1]
When $t = 7$,
$v = 4.2 \times 7 - 0.21 \times 7^2 =$
$19.11\,\text{m s}^{-1}$ ($19.1\,\text{m s}^{-1}$ to 3 s.f.) [1, 1]

3 $2\mathbf{F}_1 - 3\mathbf{F}_2 + \mathbf{F}_3 = 0$ [1]
$2(7\mathbf{i} - 2\mathbf{j}) - 3(9\mathbf{i} - 3\mathbf{j}) + \mathbf{F}_3 = 0$ [1]
$-13\mathbf{i} + 5\mathbf{j} + \mathbf{F}_3 = 0$
$\mathbf{F}_3 = 13\mathbf{i} - 5\mathbf{j}$ [1]

4 (i) Diagram showing weight for both objects [1]
Tension with arrows and labels [1]
Normal reaction and friction in correct directions [1]
(ii) $T = F$ and $T = 1.25g$ [1]
$F = 1.25g$ (= 12.3 N to 3 s.f.) [1]
(iii) inextensible string [1]
(iv) N2L for block:
$T - F = 5a$ [1]
N2L for hanging mass:
$2g - T = 2a$ [1]
Solving simultaneous equations $2g - 1.25g = 7a$ [1]
$a = 1.05\,\text{m s}^{-2}$ [1]

5 (i) Displacement = area under graph [1]
$d = \frac{1}{2} \times 7 \times 9 = 31.5\,\text{m}$ [1]
(ii) Distance to go =
$100 - 31.5 = 68.5\,\text{m}$ [1]
Time at maximum velocity
$= \frac{68.5}{9} = 7.61$ s [1]
Total time $= 7 + 7.61$
$= 14.61$ s
(14.6 to 3 s.f.) [1]
(iii) Displacement =
$\int_0^6 0.9t^2 - 0.1t^3\,dt = \left[0.9\frac{t^3}{3} - 0.1\frac{t^4}{4}\right]_0^6$ [1, 1]
$= 32.4\,\text{m}$ [1]
(iv) EITHER distance to cover $= 100 - 32.4$
$= 67.6\,\text{m}$ [1]
Time $= \frac{67.6}{10.8} = 6.26$ s [1]
Total time $= 6 + 6.26$
$= 12.26$ s
(12.3 to 3 s.f.) which is less than 14.61 s for Sunil, [1]
so Mo will win [1]
OR distance Mo covers by the time Sunil finishes [1, 1]
$= 32.4 + 10.8 \times (14.6 - 6) = 125\,\text{m}$
which is beyond the finish, [1]
so Mo will win [1]
(v) Distance = 100 m [1]
$100 = \frac{1}{2}(12.26 + (12.26 - 7))v$ [1]
$v = \frac{200}{17.52} = 11.4\,\text{m s}^{-1}$ [1]



6 (i) Returns when $s = 0$
$u = 25$, $v =$,
$a = -g$, $t = ?$
$s = ut + \frac{1}{2}at^2$ [1]
$s = 25t - 4.9t^2 = 0$ [1]
$t(25 - 4.9t) = 0$
$t = 0, 5.1$ s
Assume that air resistance is negligible, assume stone is a particle, assume gravity is constant, assume no horizontal motion [any two]. [1, 1]
(ii) Max. height when $v = 0$ [1]
$s = h$, $u = u$, $v = 0$,
$a = -g$, $t =$
$v^2 = u^2 + 2as$ [1]
$h = \frac{u^2}{2g}$ [1]
(iii) Height above point of projection $= h$ + distance travelled upwards in t seconds [1]

height $= h + \left(ut + \frac{1}{2}at^2\right)$

$= h + \left(0 - \frac{1}{2}gt^2\right)$

$= h - \frac{1}{2}gt^2$

$= \frac{u^2}{2g} - \frac{1}{2}gt^2$ [1, 1]

(iv) Height of second stone t seconds after launch

$s = ut - \frac{1}{2}gt^2$

Stones cross when equal heights [1]

$h - \frac{1}{2}gt^2 = ut - \frac{1}{2}gt^2$

$\frac{u^2}{2g} = ut$

$t = \frac{u}{2g}$ [1]

Height

$s = ut - \frac{1}{2}gt^2$

$= u\frac{u}{2g} - \frac{1}{2}g\left(\frac{u}{2g}\right)^2$ [1]

$\frac{u^2}{2g} - \frac{u^2}{8g} = \frac{3u^2}{8g}$

$= \frac{3}{4}\left(\frac{u^2}{2g}\right) = \frac{3}{4}h$ [1]

7 (i) In cell C3, we need acceleration during the 200 m journey

$s = 200$ m, $u = 22\,\text{m s}^{-1}$, $v = 18\,\text{m s}^{-1}$, $a = ?$, $t =$

$v^2 = u^2 + 2as$ [1]

$18^2 = 22^2 + 2 \times 200a$

$a = \frac{18^2 - 22^2}{400} = -0.4$ [1]

In cell D3, we need magnitude of the resistance force [1]

$= ma = 800 \times 0.4$

$= 320$ N [1]

(ii) $s = 200$ m, $u = 18\,\text{m s}^{-1}$, $v = ?$, $a = -0.4\,\text{m s}^{-2}$, $t =$

$v^2 = u^2 + 2as$ [1]

$v^2 = 18^2 - 2 \times 0.4 \times 200$

$v = \sqrt{164} = 12.8\,\text{m s}^{-1}$ [1]

(iii) $s =$, $u = 22\,\text{m s}^{-1}$, $v = 0$, $a = -0.4\,\text{m s}^{-2}$, $t =$

$v = u +$ at [1]

$0 = 22 - 0.4t$

$t = \frac{22}{0.4} = 55$ s [1]

(iv) Predicted time (55 s) is much shorter than the actual time (74.2 s) [1] so model A is not a suitable model. [1]

(v) Calculation – for example

To come to rest in 74.2 s, constant acceleration model gives

$v = u + at$ [1]

$0 = 22 + 74.2a$

$a = -\frac{22}{74.2}$

$= -0.296$ [1]

Resistance = 800 × 0.296 = 237 N, which is less than the force in the first 200 m, so the force must decrease as the car slows down. [1]