All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

1 Number and language

Exercise 1.1

1   a   The prime numbers less than 100 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

   b   These are all prime numbers.

2   a   8   b $\frac{12}{7}$ c $\frac{5}{3}$

   d $\frac{2}{3}$ e $\frac{4}{15}$ f $\frac{1}{6}$

Exercise 1.2

The square numbers between 1 and 100 are: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100

Exercise 1.3

a 9^2 b 12^2 c 8^2 d 7^3

e 4^3 f $3^2 \times 2^3$ g $5^3 \times 2^2$ h $4^2 \times 3^2 \times 2^2$

Exercise 1.4

a   1, 2, 3, 6   b   1, 3, 9

c   1, 7   d   1, 3, 5, 15

e   1, 2, 3, 4, 6, 8, 12, 24   f   1, 2, 3, 4, 6, 9, 12, 18, 36

g   1, 5, 7, 35   h   1, 5, 25

i   1, 2, 3, 6, 7, 14, 21, 42   j   1, 2, 4, 5, 10, 20, 25, 50, 100

Exercise 1.5

a   3, 5   b   2, 3   c   2, 3   d   2   e   2, 5

f   13   g   3, 11   h   5, 7   i   2, 5, 7   j   2, 7

Exercise 1.6

a $2^2 \times 3$ b 2^5 c $2^2 \times 3^2$ d $2^3 \times 5$ e $2^2 \times 11$

f $2^3 \times 7$ g $3^2 \times 5$ h 3×13 i $3 \times 7 \times 11$ j $3^2 \times 7$

Exercise 1.7

a	4	b	5	c	6	d	3	e	9
f	22	g	8	h	13	i	17	j	12

Exercise 1.8

1	a	15	b	12	c	14	d	28	e	8
	f	30	g	12	h	12	i	60	j	60
2	a	42	b	60	c	70	d	90	e	120
	f	105	g	20	h	231	i	240	j	200

Exercise 1.9

1	a	Rational	b	Rational	c	Irrational
	d	Rational	e	Rational	f	Rational
	g	Irrational	h	Rational	i	Rational
2	a	Irrational	b	Irrational	c	Irrational
	d	Rational	e	Rational	f	Rational
3	a	Rational	b	Irrational	c	Rational

Exercise 1.10

a	9	b	25	c	64	d	100	e	121
f	144	g	49	h	169	i	225	j	400

Exercise 1.11

1	a	4.41 units^2	b	9.61 units^2	c	1.44 units^2
	d	4.84 units^2	e	6.25 units^2	f	1.96 units^2
2	a	5.76 units^2	b	10.89 units^2	c	7.84 units^2
	d	38.44 units^2	e	21.16 units^2	f	53.29 units^2
	g	0.09 units^2	h	0.64 units^2	i	0.01 units^2
	j	0.81 units^2				

3 Students check their answers using the x^2 key on a calculator

Exercise 1.12

1

x	0	1	2	3	4	5	6	7	8
y	**0**	**1**	**4**	9	**16**	**25**	**36**	49	64
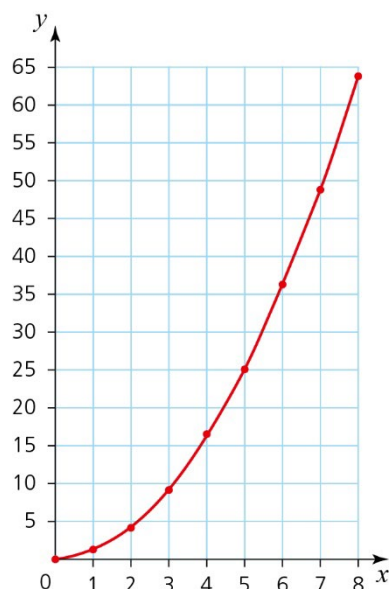
The following are the exact answers:

a	6.25	b	12.25	c	20.25	d	30.25	e	51.84
f	40.96	g	0.64	h	0.04	i	28.09	j	39.69

2 Students check their answers using the x^2 key on a calculator

Exercise 1.13

1 a 5 **b** 3 **c** 7 **d** 10 **e** 11

f 13 **g** 0.1 **h** 0.2 **i** 0.3 **j** 0.5

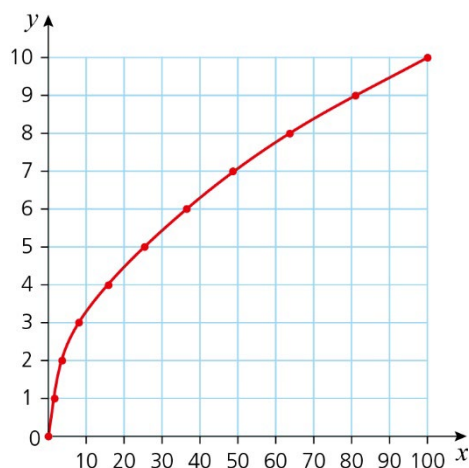**2** Students check their answers using the $\sqrt{\ }$ key on a calculator

3 a $\dfrac{1}{3}$ **b** $\dfrac{1}{4}$ **c** $\dfrac{1}{5}$ **d** $\dfrac{1}{7}$ **e** $\dfrac{1}{10}$

f $\dfrac{2}{3}$ **g** $\dfrac{3}{10}$ **h** $\dfrac{7}{9}$ **i** $\dfrac{5}{3}$ **j** $\dfrac{5}{2}$

Exercise 1.14

1

x	0	1	4	9	16	25	36	49	64	81	100
y	0	1	2	3	4	5	6	7	8	9	10

The following answers are correct to 1 d.p.:

a 8.4	**b** 6.3	**c** 7.1	**d** 9.5
e 5.9	**f** 6.7	**g** 7.4	**h** 7.7
i 1.4	**j** 1.7	**k** 4.5	**l** 5.5
m 3.5	**n** 8.7	**o** 10.7	

2 Students check their answers using the $\sqrt{\ }$ key on a calculator

Exercise 1.15

a 27 cubic units	**b** 125 cubic units	**c** 1000 cubic units
d 64 cubic units	**e** 729 cubic units	**f** 1 000 000 cubic units

Exercise 1.16

1

Number	1	2	3	4	5	6	7	8	9	10
Cube	1	8	27	64	125	216	343	512	729	1000

2 | | | | | |
|---|---|---|---|---|
| **a** 1331 | **b** 0.125 | **c** 3.375 | **d** 15.625 | **e** 8000 |
| **f** 27 000 | **g** 35 | **h** 125 | **i** 370 | **j** 1000 |

Exercise 1.17

a 2	**b** 5	**c** 3	**d** 0.1
e 0.3	**f** 6	**g** 10	**h** 100

i	–2	j	–3	k	–10	l	–1

Exercise 1.18

a	1296	b	259	c	6561	d	100
e	7	f	2	g	27	h	36
i	64	j	64	k	5	l	9

Exercise 1.19

1
a	4 °C	b	–4 °C	c	6 °C	d	–7 °C	e	–18 °C
f	0 °C	g	–9 °C	h	2 °C	i	–6 °C	j	–13 °C

2
a	2 °C	b	10 °C	c	–3 °C	d	–8 °C	e	5 °C

3 146°C

4 44 BCE

5 62 years

6 2206 years

7
a	–$35	b	–$318	c	–$88	d	–$160	e	$90

8
a	+75	b	–80	c	+30	d	–30	e	–65	f	+35

9
a	570 m	b	1080 m

10 44 °C

11 12 °C

12 4 °C

13 1700 m

14 165 m

15 695 m

Student assessment 1

1
a	2, 7	b	2, 19

2
a	30	b	28

3
a Perimeter $= \sqrt{6} + \sqrt{6} + \sqrt{6} + \sqrt{6} = 4\sqrt{6}$. This is irrational

b Area $= \sqrt{6} \times \sqrt{6} = 6$. This is rational

4
a	81	b	225	c	0.04	d	0.49

5 6.25 units^2

6
a	12.25	b	16.81	c	0.0225

7

x	0	1	4	9	16	25	36	49
y	0	1	2	3	4	5	6	7
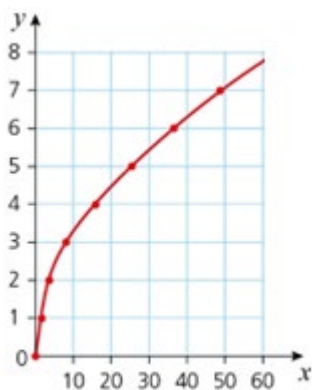
    a 2.6 (1 d.p.)   **b** 5.5 (1 d.p.)   **c** 6.7 (1 d.p.)

8 **a** 15   **b** 0.1   **c** 0.9

    d $\frac{3}{5}$ or 0.6   **e** $\frac{7}{3}$ **f** $\frac{11}{7}$

9 **a** 64   **b** 0.001   **c** $\frac{8}{27}$

10 **a** 3   **b** 100   **c** $\frac{4}{5}$

11 **a** $\frac{1}{9}$ **b** 3125   **c** $\frac{1}{9}$

12 **a** $\frac{1}{6}$ **b** 3   **c** $\frac{7}{2}$

13 **a** 100   **b** 50   **c** 300

Student assessment 2

1 **a** 4800 years

    b 2040 years

    c 1708 years

    d 2200 years

    e 3648 years

    f 4400 years

2 **a** Cerine   **b** East   **c** Akai   **d** Cerine   **e** Bridge

    f Lola   **g** East   **h** James   **i** West   **j** Eyrie

3 **a** –4   **b** –5   **c** –3   **d** +9   **e** –5

    f +4   **g** –5   **h** +1   **i** –2   **j** –5

4 64

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

2 Accuracy

Exercise 2.1

1	a	69 000	b	74 000	c	89 000
	d	4000	e	100 000	f	1 000 000
2	a	78 500	b	6900	c	14 100
	d	8100	e	1000	f	3000
3	a	490	b	690	c	8850
	d	80	e	0	f	1000

Exercise 2.2

1	a	5.6	b	0.7	c	11.9
	d	157.4	e	4.0	f	15.0
	g	3.0	h	1.0	i	12.0
2	a	6.47	b	9.59	c	16.48
	d	0.09	e	0.01	f	9.30
	g	100.00	h	0.00	i	3.00

Exercise 2.3

1	a	50 000	b	48 600	c	7000
	d	7500	e	500	f	2.57
	g	1000	h	2000	i	15.0
2	a	0.09	b	0.6	c	0.94
	d	1	e	0.95	f	0.003
	g	0.0031	h	0.0097	i	0.01

Exercise 2.4

Answers to Q1–3 may vary slightly from those given below:

1	a	1200	b	3000	c	3600
	d	150 000	e	0.8	f	100
2	a	200	b	200	c	30
	d	550	e	500	f	3000
3	a	130	b	80	c	9

d	4	**e**	160	**f**	250
g	$\frac{1}{5}$	**h**	800	**i**	4

4 **c**, **e** and **f** are incorrect.

   c   689 x 413 can be approximated to 700 x 400 = 280000, so 28457 is definitely wrong

   e   77.9 x 22.6 can be approximated to 80 x 20 = 1600. This can still be considered inconclusive proof. so choose two bigger numbers than the ones different i.e 80 x 25 = 2000. As 2000 < 2512.54, the calculation is definitely incorrect.

   f $\frac{8.42 \times 46}{0.2}$ can be approximated to $\frac{9 \times 50}{0.2} = 2250$ (note dividing by 0.2 is the same as multiplying by 5) therefore the calculation is definitely wrong.

Answers to Q5 and 6 may vary slightly from those given below:

5	**a**	120 m^2	**b**	44 m^2	**c**	400 cm^2
6	**a**	200 cm^3	**b**	4000 cm^3	**c**	1600 cm^3
7	**a**	419.6	**b**	5.0	**c**	166.3
	d	23.8	**e**	57.8	**f**	4427.1
	g	1.9	**h**	4.1	**i**	0.6

Exercise 2.5

1   **a**   **i**   Lower bound = 7.5; Upper bound = 8.5

      ii $7.5 \leqslant x < 8.5$

   b   **i**   Lower bound = 70.5; Upper bound = 71.5

      ii $70.5 \leqslant x < 71.5$

   c   **i**   Lower bound = 145.5; Upper bound = 146.5

      ii $145.5 \leqslant x < 146.5$

   d   **i**   Lower bound = 199.5; Upper bound = 200.5

      ii $199.5 \leqslant x < 200.5$

   e   **i**   Lower bound = 0.5; Upper bound = 1.5

      ii $0.5 \leqslant x < 1.5$

2   **a**   **i**   Lower bound = 2.45; Upper bound = 2.55

      ii $2.45 \leqslant x < 2.55$

   b   **i**   Lower bound = 14.05; Upper bound = 14.15

      ii $14.05 \leqslant x < 14.15$

   c   **i**   Lower bound = 1.95; Upper bound = 2.05

      ii $1.95 \leqslant x < 2.05$

   d   **i**   Lower bound = 19.95; Upper bound = 20.05

      ii $19.95 \leqslant x < 20.05$

   e   **i**   Lower bound = 0.45; Upper bound = 0.55

    ii $0.45 \leqslant x < 0.55$

3 **a** **i** Lower bound = 5.35; Upper bound = 5.45

    ii $5.35 \leqslant x < 5.45$

  b **i** Lower bound = 0.745; Upper bound = 0.755

    ii $0.745 \leqslant x < 0.755$

  c **i** Lower bound = 545; Upper bound = 555

    ii $545 \leqslant x < 555$

  d **i** Lower bound = 5950; Upper bound = 6050

    ii $5950 \leqslant x < 6050$

  e **i** Lower bound = 0.0115; Upper bound = 0.0125

    ii $0.0115 \leqslant x < 0.0125$

  f **i** Lower bound = 11 500; Upper bound = 12 500

    ii $11\,500 \leqslant x < 12\,500$

4 **a** 7.7 7.75 7.8 7.85 7.9   **b** $7.75 \leqslant M < 7.85$

5 **a** 12.0 12.05 12.1 12.15 12.20   **b** $12.05 \leqslant T < 12.15$

6 **a** 735 m³, 745 m³   **b** $735 \leqslant x < 745$

7 **a** 565 m, 575 m   **b** $335 \leqslant W < 345$

Student assessment 1

1 **a** 2800

  b 7290

  c 41 000

  d 1000

2 **a** 3.8

  b 6.8

  c .

  d 1.10

  e 10.0

  f 0.008

3 **a** 4

  b 6.8

  c 0.8

  d 10

  e 80

  f 0.005

4 92.3 cm³ to 3 s.f.

5 a 25   **b** 4   **c** 4

6 36000 yards (note other approximations are possible)

Student assessment 2

1 a
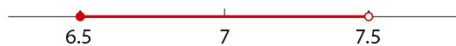
b

c

2 a $205 \leqslant x < 215$ **b** $63.5 \leqslant x < 64.5$ **c** $295 \leqslant x < 305$

3 $349.5 \leqslant L < 350.5$

$199.5 \leqslant W < 201.5$

4

5 $0.0035 \leqslant x < 0.0045$

6 a $4.825 \leqslant x < 4.835$ **b** $5.045 \leqslant x < 5.055$ **c** $9.95 \leqslant x < 10.05$

7 165 cm^2 (note other approximations are possible)

8 a 20 (note other approximations are possible)

   b 1 (note other approximations are possible)

   c 3 (note other approximations are possible)

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3 Calculations and order

Exercise 3.1

1 a a is less than 7   **b** b is greater than 4

  c c is not equal to 8   **d** d is equal to or less than 3

  e e is equal to or greater than 9   **f** f is equal to or less than 11

2 a $a < 4$ **b** $b > 7$ **c** $c \leqslant 9$ **d** $d \geqslant 5$ **e** $e \neq 3$

  f $f \leqslant 6$ **g** $g \geqslant 9$ **h** $h \geqslant 6$ **i** $i \neq 7$ **j** $j \leqslant 20$

3 a n is greater than 5, but less than 10

  b n is equal to or greater than 6, but equal to or less than 15

  c n is equal to or greater than 3, but less than 9

  d n is greater than 8, but equal to or less than 12

  (Other wording possible)

4 a $7 < p < 10$ **b** $3 < q < 12$ **c** $5 \leqslant r < 9$ **d** $8 < s \leqslant 15$

Exercise 3.2

1 a $<$ **b** $=$ **c** $>$

  d $<$ **e** $=$ **f** $>$

2 a

  b

  c

  d

  e

  f

  g

  h

3 a $x > 0$ **b** $x \leqslant 3$ **c** $0 \leqslant x \leqslant 4$ **d** $-4 < x \leqslant -1$

4 a $x \leqslant 20\,000$ **b** $135 \leqslant x \leqslant 180$ **c** $5x + 3 < 20$

  d $x \leqslant 25$ **e** $350 \leqslant x \leqslant 400$ **f** $11 \leqslant x \leqslant 28$

Exercise 3.3

1 4.5, 4.05, 0.45, 0.405, 0.045

2 0.06, 0.6, 0.606, 0.66, 6.0, 6.6, 6.606

3 9.06, 0.96, 0.906, 0.690, 0.609, 0.096

4 $\dfrac{1}{4}, \dfrac{3}{10}, \dfrac{1}{3}, \dfrac{2}{5}, \dfrac{1}{2}, \dfrac{3}{4}$

5 $\dfrac{4}{5}, \dfrac{1}{2}, \dfrac{6}{13}, \dfrac{7}{18}, \dfrac{1}{3}, \dfrac{2}{19}$

6 $\dfrac{1}{2}, \dfrac{5}{9}, \dfrac{4}{7}, \dfrac{3}{5}, \dfrac{2}{3}, \dfrac{3}{4}$

Exercise 3.4

1 15 000 cm $\dfrac{2}{5}$ km 0.5 km 750 m 5000 m

2 60 cm 0.75 m 800 mm 180 cm 2 m

3 4 kg 3500 g 1 kg $\dfrac{3}{4}$ kg 700 g

4 150 cm^3 430 ml 800 cm^3 1 litre 120 cl

Exercise 3.5

1 a 25 **b** 22.68 **c** 17.82

  d 121.24 **e** 134.33 **f** 156.1

2 a 12.3 **b** 11.9 **c** −1.24 **d** −0.01

  e −30.3 **f** −8.4 **g** −13 **h** 0

3 a 80.04 **b** 118.26 **c** 91.02 **d** 3

  e −42.6 **f** −3.1 **g** 4.84 **h** 4.44 (3 s.f.)

Exercise 3.6

1 a 26 **b** 10 **c** 42 **d** 16 **e** 8 **f** −6

2 a 20 **b** 34 **c** 32 **d** 31 **e** −36 **f** 23

3 a 27 **b** 64 **c** −36 **d** 3 **e** 144 **f** 1.6

Exercise 3.7

1 a $6 \times (2 + 1) = 18$ **b** $1 + 3 \times 5 = 16$

  c $(8 + 6) \div 2 = 7$ **d** $(9 + 2) \times 4 = 44$

e $9 \div 3 \times 4 + 1 = 13$ **f** $(3 + 2) \times (4 - 1) = 15$

2 a $12 \div 4 - 2 + 6 = 7$ **b** $12 \div (4 - 2) + 6 = 12$

c $12 \div 4 - (2 + 6) = -5$ **d** $12 \div (4 - 2 + 6) = 1.5$

e $4 + 5 \times 6 - 1 = 33$ **f** $4 + 5 \times (6 - 1) = 29$

g $(4 + 5) \times 6 - 1 = 53$ **h** $(4 + 5) \times (6 - 1) = 45$

Exercise 3.8

1 a 2 **b** 3 **c** 7 **d** 4 **e** 23 **f** 0

2 a 1 **b** 5 **c** 2 **d** 50 **e** 7 **f** -1.5

Exercise 3.9

1 a $T = 3(48 - 16)$

b $T = 96\,°C$

2 a $D = -400 \times 2 + 620$

b $D = -180$ m, i.e. 180 m below the surface

3 a $N = \frac{5^2 + 11}{2}$

b $N = 18$

Student assessment 1

1 a p is not equal to 2

b q is greater than zero

c r is equal to or less than 3

d s is equal to 2

e t is equal to or greater than 1

f u is less than -5

2 a $a < 2$ **b** $b \leqslant 4$ **c** $c = 8$ **d** $d \leqslant 0$
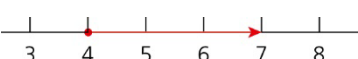
3 a

b

c

d

4 a

b

5 1800 g 1.09 kg 1 kg 900 g 9 g

6 Possible answers: 5012, 5016, 5012, 5021, 5026, 5061, 5062

7 **a** 15.12   **b** −3.6   **c** −2.1

    d 9.4   **e** −14.76   **f** 7.2

8 **a** 20   **b** 15   **c** 11

    d 13   **e** 1   **f** 4

Student assessment 2

1 **a** $4 \times 2 = 2^3$ **b** $6^2 < 2^6$

    c 850 ml > 0.5 litres   **d** Days in May > 30 days

2 **a**

    b

    c

    d

3 **a** $x \geqslant -1$ **b** $x < 2$ **c** $-2 \leqslant x < 2$ **d** $-1 \leqslant x \leqslant 1$

4 **a**

    b

    c

    d
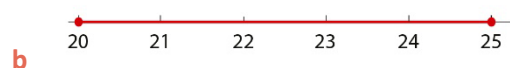
5 $\dfrac{3}{14}\left(=\dfrac{15}{70}\right)$ $\dfrac{2}{5}\left(=\dfrac{28}{70}\right)$ $\dfrac{1}{2}\left(=\dfrac{35}{70}\right)$ $\dfrac{4}{7}\left(=\dfrac{40}{70}\right)$ $\dfrac{9}{10}\left(=\dfrac{63}{70}\right)$

6 **a** $(7-5)\times 3 = 6$ **b** $16+4\times(2+4) = 40$

    c $(4+5)\times(6-1) = 45$ **d** $(1+5)\times 6-6 = 30$

7 **a** 5.5   **b** 9

8 5.60

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

4 Integers, fractions, decimals and percentages

Exercise 4.1

1 a $\frac{5}{6}$ and $\frac{5}{7}$ (note other fractions are possible)

   b $\frac{8}{7}$ and $\frac{9}{7}$ (note other improper fractions are possible)

2

	Proper fraction	Improper fraction	Mixed number
a	$\frac{2}{3}$		
b	$\frac{15}{22}$		
c		$\frac{4}{3}$	
d		$\frac{5}{2}$	
e			$1\frac{1}{2}$
f			$2\frac{3}{4}$
g		$\frac{7}{4}$	
h	$\frac{7}{11}$		
i			$7\frac{1}{4}$
j	$\frac{5}{6}$		
k		$\frac{6}{5}$	
l			$1\frac{1}{5}$

m	$\frac{1}{10}$		
n			$2\frac{7}{8}$
o		$\frac{5}{3}$	

Exercise 4.2

	a		b		c		d		e	
1	a	8	b	24	c	4	d	20	e	9
	f	63	g	5	h	25	i	2	j	6
2	a	9	b	16	c	20	d	40	e	18
	f	72	g	30	h	48	i	210	j	52

Exercise 4.3

a	$\frac{14}{3}$	b	$\frac{18}{5}$	c	$\frac{47}{8}$	d	$\frac{17}{6}$
e	$\frac{17}{2}$	f	$\frac{68}{7}$	g	$\frac{58}{9}$	h	$\frac{23}{5}$
i	$\frac{59}{11}$	j	$\frac{55}{7}$	k	$\frac{43}{10}$	l	$\frac{146}{13}$

Exercise 4.4

a	$7\frac{1}{4}$	b	$6\frac{3}{5}$	c	$6\frac{5}{6}$	d	$6\frac{5}{8}$	e	$5\frac{4}{9}$
f	$1\frac{5}{12}$	g	$9\frac{3}{7}$	h	$3\frac{3}{10}$	i	$9\frac{1}{2}$	j	$6\frac{1}{12}$

Exercise 4.5

1	**H**	**T**	**U**	**.**	$\dfrac{1}{10}$	$\dfrac{1}{100}$	$\dfrac{1}{1000}$
a			6	.	0	2	3
b			5	.	9	4	
c		1	8	.	3		
d			0	.	0	7	1
e			2	.	0	0	1

f			3	.	5	6	

2 **a** 4.5 **b** 6.3 **c** 17.8 **d** 3.07 **e** 9.27

   f 11.36 **g** 4.006 **h** 5.027 **i** 4.356 **j** 9.204

3 **a** 19.14 **b** 83.812 **c** 6.6 **d** 11.16 **e** 35.81

   f 5.32 **g** 67.14 **h** 6.06 **i** 1.4 **j** 0.175

Exercise 4.6

a 39%   **b** 42%   **c** 63%   **d** 5%

Exercise 4.7

1 **a** $\dfrac{29}{50} = \dfrac{58}{100} = 58\%$ **b** $\dfrac{17}{25} = \dfrac{68}{100} = 68\%$

   c $\dfrac{11}{20} = \dfrac{55}{100} = 55\%$ **d** $\dfrac{3}{10} = \dfrac{30}{100} = 30\%$

   e $\dfrac{23}{25} = \dfrac{92}{100} = 92\%$ **f** $\dfrac{19}{50} = \dfrac{38}{100} = 38\%$

   g $\dfrac{3}{4} = \dfrac{75}{100} = 75\%$ **h** $\dfrac{2}{5} = \dfrac{40}{100} = 40\%$

2

Fraction	Decimal	Percentage
$\dfrac{1}{10}$	0.1	10%
$\dfrac{1}{5}$	0.2	20%
$\dfrac{3}{10}$	0.3	30%
$\dfrac{4}{10} = \dfrac{2}{5}$	0.4	40%
$\dfrac{1}{2}$	0.5	50%
$\dfrac{3}{5}$	0.6	60%
$\dfrac{7}{10}$	0.7	70%

$\dfrac{4}{5}$	0.8	80%
$\mathbf{\dfrac{9}{10}}$	0.9	90%
$\dfrac{1}{4}$	0.25	25%
$\mathbf{\dfrac{3}{4}}$	0.75	75%

3 0.614, 61%, 0.4, $\dfrac{3}{8}$, $\dfrac{2}{9}$, 4%

Exercise 4.8

1 **a** 9 **b** 22 **c** 30 **d** 2 **e** 7 **f** 3

2 **a** $6 \times (4 + 6) \div 3 = 20$ **b** $6 \times (4 + 6 \div 3) = 36$ **c** $8 + 2 \times (4 - 2) = 12$

   d $(8 + 2) \times (4 - 2) = 20$ **e** $(9 - 3) \times 7 + 2 = 44$ **f** $(9 - 3) \times (7 + 2) = 54$

3 **a** 1512 **b** 33 984 **c** 29 830 **d** 41 492 **e** 20 736 **f** 40 800

4 **a** 5 **b** 3 **c** 2 **d** 7 **e** 1 **f** 7

5 **a** 7, 9 **b** 100 **c** 11, 16 **d** 7 **e** 112 **f** 28

6 **a** 1127.4 **b** 7603.8 **c** 1181.5 **d** 1141 **e** 526.1 **f** 3328.2

Exercise 4.9

1 **a** $\dfrac{2}{5} = \dfrac{4}{10} = \dfrac{8}{20} = \dfrac{20}{50} = \dfrac{16}{40}$ **b** $\dfrac{3}{8} = \dfrac{6}{16} = \dfrac{9}{24} = \dfrac{15}{40} = \dfrac{27}{72}$

   c $\dfrac{4}{7} = \dfrac{8}{14} = \dfrac{12}{21} = \dfrac{32}{56} = \dfrac{36}{63}$ **d** $\dfrac{5}{9} = \dfrac{15}{27} = \dfrac{20}{36} = \dfrac{50}{90} = \dfrac{55}{99}$

2 **a** $\dfrac{1}{2}$ **b** $\dfrac{1}{3}$ **c** $\dfrac{2}{3}$ **d** $\dfrac{4}{9}$ **e** $\dfrac{3}{4}$ **f** $\dfrac{9}{10}$

3 **a** $4\dfrac{1}{4}$ **b** $4\dfrac{3}{5}$ **c** $2\dfrac{2}{3}$ **d** $6\dfrac{1}{3}$ **e** 4 **f** $3\dfrac{7}{12}$

4 **a** $\dfrac{13}{2}$ **b** $\dfrac{29}{4}$ **c** $\dfrac{27}{8}$ **d** $\dfrac{100}{9}$ **e** $\dfrac{34}{5}$ **f** $\dfrac{97}{11}$

Exercise 4.10

1 **a** $1\dfrac{2}{5}$ **b** $\dfrac{10}{11}$ **c** $\dfrac{11}{12}$ **d** $1\dfrac{2}{45}$ **e** $1\dfrac{1}{65}$ **f** $1\dfrac{11}{12}$

2 **a** $1\dfrac{1}{8}$ **b** $1\dfrac{5}{7}$ **c** $1\dfrac{1}{12}$ **d** $\dfrac{47}{60}$ **e** $1\dfrac{29}{40}$ **f** $\dfrac{51}{52}$

3 **a** $\dfrac{1}{7}$ **b** $\dfrac{1}{10}$ **c** $\dfrac{5}{9}$ **d** $\dfrac{1}{12}$ **e** $\dfrac{9}{40}$ **f** $1\dfrac{1}{20}$

4 **a** $\dfrac{17}{60}$ **b** $\dfrac{45}{88}$ **c** $\dfrac{17}{20}$ **d** $\dfrac{44}{195}$ **e** $\dfrac{9}{20}$ **f** $\dfrac{1}{18}$

5 **a** $6\frac{1}{4}$ **b** $5\frac{3}{10}$ **c** $3\frac{1}{10}$ **d** $6\frac{7}{24}$ **e** $1\frac{1}{8}$ **f** $\frac{25}{36}$

6 **a** $5\frac{1}{8}$ **b** $7\frac{9}{40}$ **c** $-\frac{3}{8}$ **d** $\frac{7}{20}$ **e** $-2\frac{39}{140}$ **f** $1\frac{1}{4}$

Exercise 4.11

1 **a** $\frac{4}{3}$ **b** $\frac{9}{5}$ **c** $\frac{1}{7}$ **d** 9 **e** $\frac{4}{11}$ **f** $\frac{8}{37}$

2 **a** 8 **b** $\frac{12}{7}$ **c** $\frac{5}{3}$ **d** $\frac{2}{3}$ **e** $\frac{8}{27}$ **f** $\frac{1}{6}$

3 **a** $\frac{1}{6}$ **b** $\frac{3}{5}$ **c** $\frac{4}{21}$ **d** $\frac{2}{3}$ **e** $\frac{1}{4}$ **f** $\frac{7}{20}$

4 **a** $\frac{5}{6}$ **b** $2\frac{1}{2}$ **c** $1\frac{1}{7}$ **d** $4\frac{1}{16}$ **e** $\frac{1}{5}$ **f** $\frac{2}{3}$

5 **a** $\frac{3}{5}$ **b** $\frac{7}{12}$ **c** $\frac{9}{70}$ **d** $\frac{21}{25}$ **e** $\frac{3}{8}$ **f** $1\frac{25}{56}$

6 **a** $\frac{3}{5}$ **b** $3\frac{107}{120}$ **c** $\frac{8}{15}$ **d** $12\frac{1}{4}$

7 **a** $\frac{13}{28}$ **b** $-\frac{1}{2}$ **c** $-\frac{1}{24}$ **d** 7

Exercise 4.12

1 **a** 0.75 **b** 0.8 **c** 0.45 **d** 0.34
    e $0.333…$ **f** 0.375 **g** 0.4375 **h** $0.222…$

2 **a** 2.75 **b** 3.6 **c** 4.35 **d** 6.22
    e $5.666…$ **f** 6.875 **g** 5.5625 **h** 4.25

Exercise 4.13

1 **a** $\frac{1}{2}$ **b** $\frac{7}{10}$ **c** $\frac{3}{5}$
    d $\frac{3}{4}$ **e** $\frac{33}{40}$ **f** $\frac{1}{20}$
    g $\frac{1}{20}$ **h** $\frac{201}{500}$ **i** $\frac{1}{5000}$

2 **a** $2\frac{2}{5}$ **b** $6\frac{1}{2}$ **c** $8\frac{1}{5}$
    d $3\frac{3}{4}$ **e** $10\frac{11}{20}$ **f** $9\frac{51}{250}$
    g $15\frac{91}{200}$ **h** $30\frac{1}{1000}$ **i** $1\frac{41}{2000}$

Student assessment 1

1 The improper fraction is **c** $\frac{27}{8}$; the mixed number is **b** $5\frac{3}{4}$

2 **a** 21 **b** 27 **c** 22 **d** 39

3 **a** $\frac{13}{5}$ **b** $\frac{31}{9}$ **c** $\frac{45}{8}$

4 **a** $6\frac{3}{5}$ **b** $5\frac{2}{9}$ **c** $6\frac{1}{11}$

5 $\frac{2}{3} = \frac{4}{6} = \frac{8}{12} = \frac{12}{18} = \frac{18}{27} = \frac{20}{30}$

6 **a** 0.35   **b** 0.275   **c** 6.75   **d** 0.035

7 **a** 60%   **b** 49%   **c** 25%   **d** 90%

     e 150%   **f** 327%   **g** 5%   **h** 35%

     i 77%   **j** 3%   **k** 290%   **l** 400%

8 75 litres

Student assessment 2

1 **a** 0   **b** 19

2 6 and 12

3 294 days

4 18 032

5 340.7

6 $\frac{24}{36} = \frac{8}{12} = \frac{4}{6} = \frac{20}{30} = \frac{60}{90}$

7 **a** $1\frac{7}{10}$ **b** 2

8 **a** 0.875   **b** 1.4

9 **a** $6\frac{1}{2}$ **b** $\frac{1}{25}$ **c** $3\frac{13}{20}$ **d** $3\frac{1}{125}$

10 **a** $\frac{9}{5}$ **b** $\frac{5}{17}$ **c** 10

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

5 Further percentages

Exercise 5.1

1 White = 47%   Blue = 23%   Red = 30%

2 70%

3 a 60%   b 40%

4 a $\frac{73}{100}$ b $\frac{28}{100}$ c $\frac{10}{100}$ d $\frac{25}{100}$

5 a 27%   b 30%   c 14%   d 25%

6 a 0.39   b 0.47   c 0.83

   d 0.07   e 0.02   f 0.2

7 a 31%   b 67%   c 9%

   d 5%   e 20%   f 75%

Exercise 5.2

1 a 25%   b 66.7% (3 s.f.)   c 62.5%

   d 180%   e 490%   f 387.5%

2 a 0.75   b 0.8   c 0.2

   d 0.07   e 1.875   f 0.167 (3 s.f.)

3 a 20   b 100   c 50

   d 36   e 4.5   f 7.5

4 a 8.5   b 8.5   c 52

   d 52   e 17.5   f 17.5

5 a 6   b 3   c 21

6 Apple: 66; Mango 24: Pineapple: 18; Grapes: 12

7 Australian: 143   Pakistani: 44   Greek: 11   Other nationalities: 22

8 Newspapers; 69   Pens; 36   Books; 18   Other: 27

Exercise 5.3

1 a 48%   b 36.8%   c 35%   d 50%

   e 45%   f 40%   g $33\frac{1}{3}$%   h 57.1% (3 s.f.)

2 Win 50%   Draw $33\frac{1}{3}$%   Lose $16\frac{2}{3}$%

3 A = 34.5% (1 d.p.)   B = 25.6% (1 d.p.)   C = 23.0% (1 d.p.)   D = 16.9% (1 d.p.)

4   Red 35.5%   Blue 31.0%   White 17.7%   Silver 6.6%   Green 6.0%   Black 3.2%

Exercise 5.4

1   **a**   187.5   **b**   322   **c**   7140

    d   245   **e**   90   **f**   121.5

2   **a**   90   **b**   38   **c**   9

    d   900   **e**   50   **f**   43.5

3   **a**   20%   **b**   80%   **c**   110%

    d   5%   **e**   85%   **f**   225%

4   **a**   50%   **b**   30%   **c**   5%

    d   100%   **e**   36%   **f**   5%

5   7475 tonnes

6 $6825

7   **a** $75   **b** $93.75

8   **a**   43   **b**   17.2%

9   1100

Student assessment 1

1

Fraction	Decimal	Percentage
$\frac{3}{4}$	0.75	75%
$\frac{4}{5}$	0.8	80%
$\frac{5}{8}$	0.625	62.5%
$\frac{3}{2}$	1.5	150%

2   640 m

3 $345.60

4 $10 125

5   **a**   20%   **b**   41.7% to 1 d.p.   **c**   22.5%

    d   85.7% to 1 d.p.   **e**   7%   **f**   30%

6   16%

7   **a** $36   **b**   25%

8   2011

Student assessment 2

1

Fraction	Decimal	Percentage
$\dfrac{1}{4}$	0.25	25%
$\dfrac{3}{5}$	0.6	60%
$\dfrac{5}{8}$	0.625	$62\tfrac{1}{2}\%$
$2\tfrac{1}{4}$	2.25	225%

2 750 m

3 $2100

4 $97 200

5 **a** 29.2% to 1 d.p. **b** 21.7% to 1 d.p. **c** 125%

   d 8.3% to 1 d.p. **e** 20% **f** 10%

6 8.3% to 1 d.p.

7 **a** $325 **b** 61.8% to 1 d.p.

8 8 days

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

6 Ratio and proportion

Exercise 6.1

1 48

2 16 h 40 min

3 11 units

4 **a** 7500 bricks   **b** 53 h

5 **a** 6250 litres   **b** 128 km

6 1111 km (to nearest km)

7 **a** 450   **b** 75   **c** 120

Exercise 6.2

1 **a** 128   **b** 375 hours

2 **a** 120   **b** 1440   **c** 34 560

3 **a** 4   **b** 30 hours

4 **a** 22.5   **b** 36 hours

5 25 hours

6 20 km

7 **a** 270   **b** 10 h 40 min

Exercise 6.3

1 **a** 450 kg   **b** 1250 kg

2 **a** 3 : 10 : 1 : 2

   b Butter 600 g;   Flour 2 kg;   Sugar 200 g;   Currants 400 g

   b 120 cakes

3 **a** 16.8 litres   **b** red 1.2 litres; white 14.3 litres

4 **a** 125   **b** red 216; yellow 135   **c** 20

5 **a** 42 litres

   b orange juice 495 litres; mango juice 110 litres

Exercise 6.4

1 60 : 90

2 16 : 24 : 32

3 3.25 kg : 1.75 kg

4 18 min : 27 min

5 10 min : 50 min

6 7 : 1

7 orange 556 ml (3 s.f.); water 444 ml (3 s.f.)

8 **a** 11 : 9   **b** 440 boys; 360 girls

9 $\frac{3}{5}$

10 32 cm

11 4 km and 3 km

12 40°, 80°, 120°, 120°

13 45°, 75°, 60°

14 24-year-old \$400 000; 28-year-old \$466 667; 32-year-old \$533 333

15 Alex £2000; Maria £3500; Ahmet £2500

Exercise 6.5

1 4

2

Speed (km/h)	60	40	30	120	90	50	10
Time (h)	2	3	4	1	$1\frac{1}{3}$	$2\frac{2}{5}$	12

3 **a** **i** 12 h   **ii** 4 h   **iii** 48 h

    b **i** 16   **ii** 3   **iii** 48

4 **a** 30 rows   **b** 42 chairs

5 6 h 40 min

6 4

7 18 h

Exercise 6.6

1 11 600 kg

2 **a** 7269 seconds   **b** 5.80 m/s   **c** faster than 5.86 m/s (to 2 d.p.)

3 **a** Brass   **b** 7.87 cm

4 **a** 7698 people/km^2 (to nearest person)

    b China has a very large area, some of it with very sparse populations, e.g. the Gobi desert

    c Answers will vary

5 **a** 0.48 km^2

    b $88\frac{1}{3}$ sheep/km^2

   c   86 sheep (note the answer is not given as 86.4)

6   40 cm

Student assessment 1

1 **a**   3 : 7.

    b $\frac{7}{10}$

    c   0.45 m or 45 cm

2 **a**   375 g   **b**   625 g

3 **a**   450 m   **b**   8 km

4 **a**   1 : 25   **b**   1.75 m

5   2 parts = 300 kg

    5 parts = 750 kg

    13 parts = 1950 kg

6   60°, 90°, 90° and 120°

7   150°

8 **a**   13.5 hours   **b**   12 pumps

9
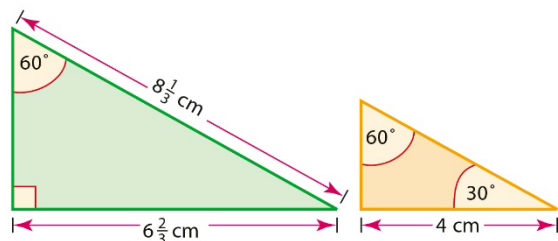
10 **a**   15 km   **b**   6 km/h

11 **a**   4 minutes 48 seconds   **b**   1.6 litres per minute

12 119 people

13 **a**   24.1 km (3 s.f.)

    b   The small car is more efficient as the fuel consumption of the larger car is 20.7 km/l (3 s.f.)

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

7   Indices and standard form

Exercise 7.1

1 a 3^3 **b** 2^5 **c** 4^2

d 6^4 **e** 8^6 **f** 5^1

2 a $2^3 \times 3^2$ **b** $4^5 \times 5^2$ **c** $3^2 \times 4^3 \times 5^2$

d 2×7^4 **e** 6^2 **f** $3^3 \times 4^2 \times 6^5$

3 a 4×4 **b** $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

c $3 \times 3 \times 3 \times 3 \times 3$ **d** $4 \times 4 \times 4 \times 6 \times 6 \times 6$

e $7 \times 7 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ **f** $3 \times 3 \times 4 \times 4 \times 4 \times 2 \times 2 \times 2 \times 2$

4 a 32   **b** 81   **c** 64   **d** 216

e 1 000 000   **f** 256   **g** 72   **h** 125 000

Exercise 7.2

1 a 3^6 **b** 8^7 **c** 5^9 **d** 4^{10}

e 2^4 **f** $3^5 \times 6^6$ **g** $4^8 \times 5^9 \times 6^2$ **h** $2^4 \times 5^{10} \times 6^8$

2 a 4^4 **b** 5^3 **c** 2 **d** 6^3

e 6^3 **f** 8 **g** 4^3 **h** 3^7

3 a 5^4 **b** 4^{12} **c** 10^{10} **d** 3^{15}

e 6^8 **f** 8^6

4 a 2^3 **b** 3 **c** 5^3 **d** 4^5

E 2^8 or 4^4 **f** $6^4 \times 8^5$ **g** $\dfrac{4^3}{5^4}$ **h** 4×6^7

Exercise 7.3

a 8   **b** 25   **c** 1   **d** 1   **e** 1   **f** 0.25

Exercise 7.4

1 a $\dfrac{1}{4}$ **b** $\dfrac{1}{9}$ **c** $\dfrac{3}{50}$ **d** $\dfrac{1}{200}$ **e** 1   **f** $\dfrac{1}{1000}$

2 a 1   **b** 2   **c** 4   **d** $\dfrac{1}{2}$ **e** $\dfrac{1}{6}$ **f** 10

3 a 12   **b** 32   **c** 225   **d** 80   **e** 7   **f** 64

Exercise 7.5

1 **a** 4 **b** 4 **c** 6 **d** 6 **e** 8 **f** 7

2 **a** 6.5×10^4 **b** 4.1×10^4 **c** 7.23×10^5 **d** 1.8×10^7

    e 9.5×10^5 **f** 7.6×10^8 **g** 7.2×10^5 **h** 2.5×10^5

3 **b**, **d**, **f**, **g**, **i** and **j**

4 **a** 3800 **b** 4 250 000 **c** 90 030 000 **d** 101 000

5 **a** 8×10^5 **b** 3×10^7 **c** 1.5×10^7 **d** 1.98×10^8

    e 2×10^9 **f** 2.85×10^8 **g** 1.5×10^7 **h** 4.1×10^8

    i 2.4×10^7 **j** 4.9×10^7

6 **d** and **e**

7 **a** 6×10^5 **b** 4.8×10^7 **c** 7.84×10^{11}

    d 5.34×10^5 **e** 7×10^6 **f** 8.5×10^6

8 **a** 6.8×10^6 **b** 7.2×10^8 **c** 8×10^5

    d 7.5×10^7 **e** 4×10^9 **f** 5×10^7

9 **a** 6×10^5 **b** 2.4×10^7 **c** 1.4×10^8

    d 3×10^9 **e** 1.2×10^{13} **f** 1.8×10^7

10 1.44×10^{11} m or 1.44×10^8 km

Exercise 7.6

1 **a** 8×10^8 **b** 7.5×10^8 **c** 9×10^{13} **d** 8.4×10^{11}

    e 1.4×10^{12} **f** 1.26×10^{10} **g** 4×10^8 **h** 1.6×10^{17}

2 **a** 4×10^3 **b** 4×10^6 **c** 1.9×10^1

    d 5×10^7 **e** $4 \times 10^0 \ (= 4)$ **f** 2×10^8

3 **a** 2×10^4 **b** 2.5×10^8 **c** 5×10^3

    d 7.2×10^4 **e** 5×10^4 **f** 5×10^3

4 **a** 8.8×10^8 **b** 2.04×10^{11} **c** 3.32×10^{11}

    d 4.2×10^{22} **e** 5.1×10^{22} **f** 2.5×10^{25}

5 **a** 2×10^2 **b** 3×10^5 **c** 4×10^6

    d 2×10^4 **e** 2.5×10^6 **f** 4×10^4

Exercise 7.7

a 4.26×10^5 **b** 6.59×10^8 **c** 6.388×10^7 **d** 3.157×10^9

e 4.5×10^8 **f** 6.01×10^7 **g** 8.15×10^{10} **h** 3.56×10^7

Exercise 7.8

1 **a** 10^{-3} **b** 10^{-3} **c** 10^{-4} **d** 10^{-6}
   e 10^{-7} **f** 10^{-9} **g** 10^{-6} **h** 10^{-10}

2 **a** 6×10^{-4} **b** 5.3×10^{-5} **c** 8.64×10^{-4}
   d 8.8×10^{-8} **e** 7×10^{-7} **f** 4.145×10^{-4}

3 **a** 0.008 **b** $0.000\,42$ **c** 0.0903 **d** $0.000\,0101$

4 **a** 6.8×10^{-4} **b** 7.5×10^{-7} **c** 4.2×10^{-10}
   d 8×10^{-9} **e** 5.7×10^{-11} **f** 4×10^{-11}

5 **a** -4 **b** -3 **c** -8 **d** -5 **e** -7 **f** 3

6 6.8×10^5, 6.2×10^3, 8.414×10^2, 6.741×10^{-4}, 3.2×10^{-4}, 5.8×10^{-7}, 5.57×10^{-9}

Student assessment 1

1 **a** $2^3 \times 5^2$ **b** $2^2 \times 3^5$

2 **a** $4 \times 4 \times 4$ **b** $6 \times 6 \times 6 \times 6$

3 **a** 800 **b** 27

4 **a** 3^7 **b** $6^5 \times 3^9$ **c** 2^7 **d** 6 **e** $3^2 \times 4^2$ **f** 1

5 **a** 4 **b** 9 **c** 5 **d** 1

Student assessment 2

1 **a** $2^2 \times 3^5$ **b** 2^{14}

2 **a** $6^5 = 6 \times 6 \times 6 \times 6 \times 6$ **b** $2^{-5} = \frac{1}{2 \times 2 \times 2 \times 2 \times 2}$

3 **a** $27\,000$ **b** 125

4 **a** 2^7 **b** $7^7 \times 3^{12}$ **c** 2^6 **d** 3^3 **e** 4^{-1} **f** 2^2

5 **a** 5 **b** 16 **c** 49 **d** 48

Student assessment 3

1 **a** 6×10^6 **b** 4.5×10^{-3} **c** 3.8×10^9
   d 3.61×10^{-7} **e** 4.6×10^8 **f** 3×10^0

2 **a** $8\,122\,000$ **b** $0.000\,305$

3 4.05×10^8, 3.6×10^2, 9×10^1, 1.5×10^{-2}, 7.2×10^{-3}, 2.1×10^{-3}

4 **a** 1.5×10^7 4.3×10^5 4.35×10^{-4} 4.8×10^0 8.5×10^{-3}
   b $0.000\,435$, 0.0085, 4.8, $430\,000$, 15 million

5 **a** $n = 3$ **b** $n = 9$ **c** $n = -3$
   d $n = 6$ **e** $n = -1$ **f** $n = 8$

6 **a** 1.2×10^8 **b** 1.48×10^{11} **c** 6.73×10^7 **d** 3.88×10^6

7   43.2 minutes to 3 s.f.

8   2.84×10^{15} km to 3 s.f.

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

8   Money and finance

Exercise 8.1

Answers taken from graphs may vary slightly.

1   a   A$30   b   A$45   c   A$7.50
    d   A$37.50   e   A$52.50   f   A$22.50

2   Answers are rounded to the nearest tenth
    a   €13.30   b   €20   c   €26.70
    d   €23.30   e   €16.70   f   €32

3   Student's own graph
    a   750 rupees   b   3000 rupees   c   3750 rupees
    d   6750 rupees   e   1875 rupees   f   75 000 rupees

4   Answers are rounded to the nearest tenth
    a   €1.90   b   €10.30   c   €8.40
    d   €6.50   e   €74.70   f   €36.40

5   Student's own graph
    a   ZIM$8256   b   ZIM$20 640   c   ZIM$30 960
    d   ZIM$12 384   e   ZIM$10 320

6   Student's own graph
    a   €15   b   €28   c   €75.50   d   €7

7   Student's own graph
    L1200 = €300, L400 = €100, L880 = €220

8   Student's own graph
    a   i   33 913 yen   ii   84 783 yen   iii   113 043 yen
    b   i   US$44.23   ii   US$75.19   iii   US$0.88

9   Student's own graph
    a   1500 rand   b   6000 rand   c   1770 rand   d   12 750 rand

Exercise 8.2

1   a $138.50   b $164.14   c $154.82   d $165.06
2   a $229   b $25   c $123   d $55   e $182
3   a $131   b $306   c $72   d $15
4   a $126   b $214.72   c $190.76   d $290.50   e $348

5

	Basic hours worked	Rate per hour ($)	Basic pay ($)	Overtime hours worked	Overtime pay ($)	Total gross pay ($)
a	40	3.60	144.00	8	43.20	187.20
b	35	5.80	203.00	4	34.80	237.80
c	38	4.15	157.70	6	37.35	195.05
d	42	6.10	256.20	5	45.75	301.95
e	44	5.25	231.00	4	31.50	262.50
f	37	4.87	180.19	3	21.92	202.11
g	36	6.68	240.48	6	60.12	300.60
h	45	7.10	319.50	7	74.55	394.05

6   **a** $127.30   **b** $161.70   **c** $132.63   **d** $205.33

      e $178.50   **f** $137.43   **g** $204.41   **h** $267.95

Exercise 8.3

1

	Mon	Tue	Wed	Thur	Fri	Total	Gross pay
Joe	4	5	7	6	6	**28**	**154**
Kirra	3	4	4	5	5	**21**	**115.50**
Delores	5	6	6	5	6	**28**	**154**
Zaffar	3	4	6	6	6	**25**	**137.50**

2

	Mon	Tue	Wed	Thur	Fri	Total	Gross pay
Maria	240	360	288	192	180	1260	525
Chul	168	192	312	180	168	1020	425
Yao	288	156	192	204	180	1020	425
Bianca	228	144	108	180	120	780	325
Erik	192	204	156	228	144	924	385

3   **a** and **b**

	Gross pay ($)	Net pay ($)
Neo	308	261.80
Jing	395	335.75
Luis	338	287.30
Mpho	431	366.35
Saki	453	385.05

4   **a**

Distance walked (km)	Amount per km ($)	Total raised ($)
10	0.80	8
14	0.65	9.10
18	0.38	6.84
10.15	0.72	7.31
12	0.66	7.92
13	1.20	15.60
15	1	15
18	0.88	15.84
18	0.58	10.44
17	0.95	16.15

b $112.20

c   Total of 10 parts, therefore 1 part = $11.22

Charity 1 receives $22.44

Charity 2 receives $33.66

Charity 3 receives $56.10

Exercise 8.4

a $72   **b** $420   **c** $102   **d** $252   **e** $3250   **f** $369.60

Exercise 8.5

a 5 years   **b** 0.41 years   **c** 5 years   **d** 8 years   **e** 6 years   **f** 7 years

Exercise 8.6

a $r = 7$ **b** $r = 4$ **c** $r = 3.5$ **d** $r = 7.5$ **e** $r = 8$ **f** $r = 11$

Exercise 8.7

1   **a** $400   **b** $200   **c** $850
    d $1200   **e** $4000   **f** $1200

2 4%

3 2 years

4 4.5%

5 9.5%

6 $315

7 6%

Exercise 8.8

1 $11 033 750

2 $52 087.50

3 $10 368

4 1331 students

5 3 276 800 tonnes

Exercise 8.9

1 2 years

2 5 years

3 3 years

4 4 years

5 5 years

Exercise 8.10

1 $1.80 loss

2 $2.88 profit

3 $70.38 profit per seat

4 $240 extra

5 $250 loss

6 **a** $4.50 profit **b** $16 profit **c** $80 profit

d $13 profit **e** $2.88 profit **f** $23 loss

Exercise 8.11

1 **a** 11% **b** 25% **c** 50% **d** 20% **e** 30% **f** 66.7% (3 s.f.)

2 **a** 25% **b** 30% **c** 20% **d** 50% **e** 30.8% (3 s.f.) **f** 30%

3 Type A = 30% Type B = 15.4% (3 s.f.) Type C = 33.3% (3 s.f.)

Type D = 30.9% (3 s.f.) Type E = 48.5% (3 s.f.)

Type E makes the largest percentage profit.

4 80%

5 Mirror = 30% Wayfarer = 40% Laser = 45% Fireball = 37.6%

Student assessment 1

Answers taken from graphs may vary slightly.

1 Student's own graph

a A$62.50 **b** A$100 **c** A$87.50

2 **a** €43.20 **b** €64.80 **c** €256

3 Student's own graph

a 23.08 dinars **b** 276.92 dinars **c** 34.62 dinars

4 Student's own graph

a 225.50 nairas **b** 1353.33 nairas **c** 902.22 nairas

5 Student's own graph

a 19.21 ringgits **b** 96.06 ringgits **c** 144.09 ringgits

Student assessment 2

1 $225

2 $64.90

3 **a** $54 **b** 4 years **c** 6.5% **d** $840

4 12.5%

5 11.1%

Student assessment 3

1 $122.40

2 $26.16

3 **a** $72 **b** 8% **c** 4 years **d** 7.5% **e** $1250

4 $6000

5 30%

6 **a** $4680

  b $18 265.43

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

9 Time

Exercise 9.1

1 **a** 9.20 a.m., 09 20   **b** 11.55 a.m., 11 55

2 **a** 4.25 p.m., 16 25   **b** 7.50 p.m., 19 50

3 **a** 14 30   **b** 21 00   **c** 08 45   **d** 06 00   **e** 12 00

     f 22 55   **g** 07 30   **h** 19 30   **i** 01 00   **j** 00 00

4 **a** 07 15   **b** 20 00   **c** 09 10

     d 08 45   **e** 14 45   **f** 19 40

5 **a** 7.20 a.m.   **b** 9.00 a.m.   **c** 2.30 p.m.   **d** 6.25 p.m.

     e 11.40 p.m.   **f** 1.15 a.m.   **g** 12.05 a.m.   **h** 11.35 a.m.

     i 5.50 p.m.   **j** 11.59 p.m.   **k** 4.10 a.m.   **l** 5.45 a.m.

6 **a** 08 05   **b** 08 40   **c** 09 05   **d** 09 30

7 **a** 18 20   **b** 18 45   **c** 19 00   **d** 19 15

8 **a** 08 25   **b** 09 35   **c** 09 50   **d** 10 05

9

Depart	06 15	06 30	09 25	10 20	13 18	14 48	18 54	19 25
Arrive	07 55	08 10	11 05	12 00	14 58	16 28	20 34	21 05

10

Depart	06 00	06 45	08 55	09 09	13 48	14 17	21 25	22 05
Arrive	08 05	08 50	11 00	11 14	15 53	16 22	23 30	00 10

11

Cambridge	04 00	08 35	12 50	19 45	21 10
Stansted	05 15	09 50	14 05	21 00	22 25
Gatwick	06 50	11 25	15 40	22 35	00 00
Heathrow	07 35	12 10	16 25	23 20	00 45

Heathrow	06 25	09 40	14 35	18 10	22 15
Gatwick	08 12	11 27	16 22	19 57	00 02

Stansted	10 03	13 18	18 13	21 48	01 53
Cambridge	11 00	14 15	19 10	22 45	02 50

12

	London	Jo'burg	London	Jo'burg
Sunday	06 15	17 35	14 20	01 40
Monday	07 25	18 45	18 05	05 25
Tuesday	07 20	18 40	15 13	02 33
Wednesday	07 52	19 12	20 10	07 30
Thursday	06 10	17 30	16 27	03 47
Friday	06 05	17 25	20 55	08 15
Saturday	09 55	21 15	18 50	06 10

13

	London	Kuala Lumpur	London	Kuala Lumpur	London	Kuala Lumpur
Sunday	08 28	22 13	14 00	03 45	18 30	08 15
Monday	08 15	22 00	13 30	03 15	20 05	09 50
Tuesday	09 15	23 00	15 25	05 10	17 55	07 40
Wednesday	07 50	21 35	14 15	04 00	18 37	08 22
Thursday	07 00	20 45	13 45	03 30	18 40	08 25
Friday	10 25	00 10	15 00	04 45	17 53	07 38
Saturday	10 12	23 57	14 20	04 05	19 08	08 53

Exercise 9.2

1 a   0° 15′ 0″ (15 mins)

   b   3° 45′ 54″ (3 hrs 45 mins and 54 secs)

   c   0° 13′ 12″ (13 mins and 12 secs)

2 Arrival at stop 1   06 45

   Arrival at stop 2   08 03

   Arrival at stop 3   09 42

   Arrival at stop 4   12 36 and 36 seconds

3 a   4 hours   **b**   8 hours 30 minutes   **c**   3 hours 22.5 minutes

   d   1 hour 40 minutes   **e**   2 hours 20 minutes   **f**   6 hours 13.3 minutes

g 2 hours 18 minutes **h** 1 hour 24 minutes **i** 6 hours

j 7 hours 30 minutes

4 1st: 30 min 2nd: 32 min 44 s

3rd: 34 min 17 s 4th: 35 min 7s

5th: 36 min

5 00 30 the next morning

6 21 45

7 11 15

Student assessment 1

1 a 2.45 p.m. **b** 14 45

2 a 04 35 **b** 18 30 **c** 07 45 **d** 19 30

3 a 8.45 a.m. **b** 6.35 p.m. **c** 9.12 p.m. **d** 12.15 a.m.

4 a 08 09 **b** 08 39 **c** 09 02

5

Alphaville	07 50	11 01	15 53
Betatown	08 27	11 38	16 30
Gammatown	08 45	11 56	16 48
Deltatown	09 27	12 38	17 30

6 a 5 hours **b** 3 hours 45 minutes

c 1 hour 20 minutes **d** 1 hour 40 minutes

e 2 hours 20 minutes **f** 3 hours 45 minutes

Student assessment 2

1 a 12.40 a.m. **b** 00 40

2 a 05 20 **b** 20 15 **c** 08 50 **d** 23 30

3 a 7.15 a.m **b** 4.43 p.m **c** 7.30 p.m **d** 12.35 a.m

4 a 08 10 **b** 08 39 **c** 09 00

5

Apple	10 14	15 42	19 33
Peach	11 52	17 20	21 11
Pear	13 56	19 24	23 15
Plum	15 49	21 17	01 08

6 **a** 5 hours

   b 4 hours 15 minutes

   c 2 hours 40 minutes

   d 3 hours 45 minutes

   e 3 hours 45 minutes

7 Tuesday at 05 30 (Singapore local time)

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

10 Set notation and Venn diagrams

Exercise 10.1

1 a i Continents of the world

     ii Student's own answers

  b i Even numbers

     ii Student's own answers

  c i Days of the week

     ii Student's own answers

  d i Months with 31 days

     ii Student's own answers

  e i Triangle numbers

     ii Student's own answers

  f i Boys' names beginning with the letter 'M'

     ii Student's own answers

  g i Prime numbers greater than 7

     ii Student's own answers

  h i Vowels

     ii o, u

  i i Planets of the solar system

     ii Student's own answers

2 a 7   **c** 7   **d** 7

  f Unquantifiably finite, though theoretically infinite

  h 5   **i** 8 (excluding Pluto)

Exercise 10.2

1 a True   **b** True   **c** False

  d False   **e** False   **f** True

2 a $A \cap B = \{4, 6\}$ **b** $A \cap B = \{4, 9\}$ **c** $A \cap B = \{\text{yellow, green}\}$

3 a $A \cup B = \{2, 3, 4, 6, 8, 9, 10, 13, 18\}$

  b $A \cup B = \{1, 4, 5, 6, 7, 8, 9, 16\}$

  c $A \cup B = \{\text{red, orange, blue, indigo, violet, yellow, green, purple, pink}\}$

4 a $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8\}$

b $A \cap B = \{2, 3\}$

c $A \cup B = \{1, 2, 3, 4, 5, 7, 8\}$

d $A' = \{1, 4, 6, 8\}$

e $B' = \{2, 6, 7\}$

Exercise 10.3

1 a
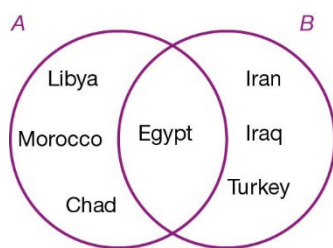
b i $A \cap B = \{Egypt\}$

ii $A \cup B = \{Libya, Morocco, Chad, Egypt, Iran, Iraq, Turkey\}$

2 a
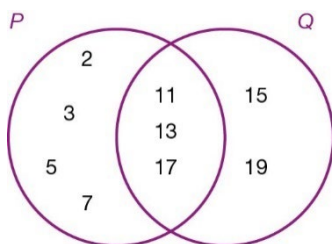
b i $P \cap Q = \{11, 13, 17\}$

ii $P \cup Q = \{2, 3, 5, 7, 11, 13, 15, 17, 19\}$

3
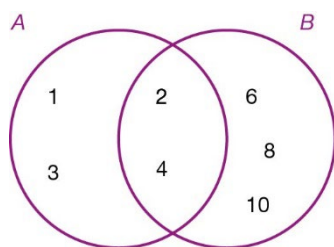
4
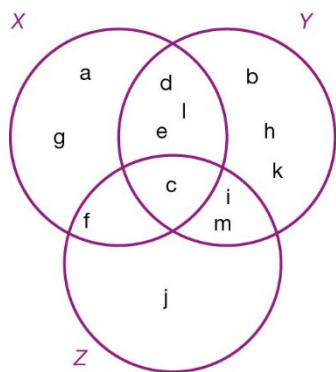
5
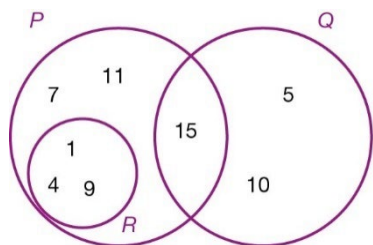
Exercise 10.4

1 **a** 5   **b** 14   **c** 13

2 45

3 **a** 10   **b** 50

Student assessment 1

1 **a** The set is {even numbers from 2 to 8}

   b The set is {even numbers}

   c The set is {square numbers}

   d The set is {oceans}

2 **a** 7

   b 2

   c 6

   d 366
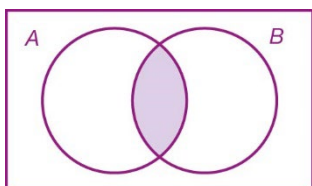
3 **a**   **b**
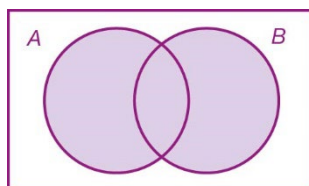
4   **a**
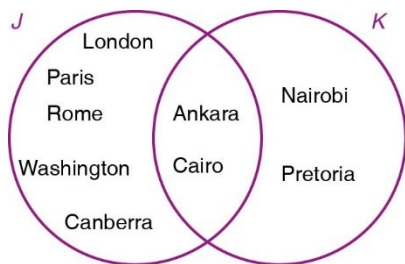
b $J \cap K$ = {Ankara, Cairo}

5   **a**
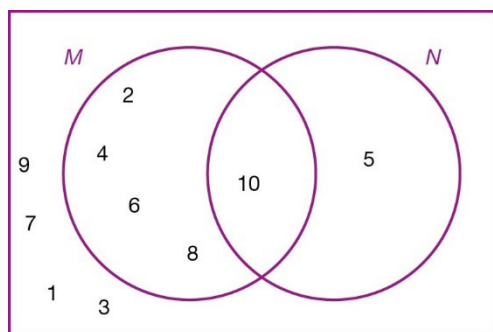
b X = multiples of 10

6   **a**
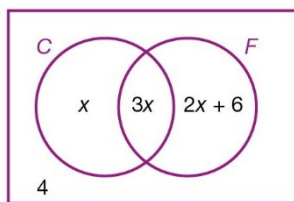
b   15

7 $A' = \{m, t, h\}$

Mathematical investigations and ICT 1

Primes and squares

1 $2^2 + 3^2 = 13$; $4^2 + 5^2 = 41$

2 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

3 5, 13, 17, 29, 37, 41, 53, 61, 73, 89, 97

4 If a number generated by the rule $4n + 1$ is a prime, then it can be expressed as the sum of two square numbers. (This was proved by Fermat in the 17th century.) Alternatively, add 1 to the prime number and divide the result by 2. If the answer is even, then the prime number cannot be expressed as the sum of two squares.

5 The rule works for the numbers shown, but this does not prove that it always works.

Football leagues

1 $(17 + 16 + 15 + \ldots + 1) \times 2 = 306$

2 $n = t(t - 1)$

ICT activity 1

1

2

	A	B
1	Pattern	Number of squares
2	1	1
3	2	4
4	3	9
5	4	16
6	5	25
7	10	
8	20	
9	50	

3 The number of squares is the pattern number squared

4

	A	B
1	Pattern	Number of squares
2	1	1
3	2	4
4	3	9
5	4	16
6	5	25
7	10	100
8	20	400
9	50	2500

The formula in cell B7 is =A7^2 or =A7*A7

5 **a** **i**

ii

	A	B
1	Pattern	Number of squares
2	1	2
3	2	6
4	3	12
5	4	20
6	5	30
7	10	
8	20	
9	50	

iii The number of squares is the pattern number squared + the pattern number.

iv

	A	B
1	Pattern	Number of squares
2	1	2
3	2	6
4	3	12
5	4	20
6	5	30
7	10	110
8	20	420
9	50	2550

The formula in cell B7 is =A7^2+A7

b i

ii
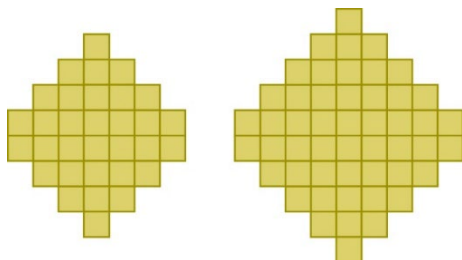
	A	B
	Pattern	Number of squares
1		
2	1	2
3	2	8
4	3	18
5	4	32
6	5	50
7	10	
8	20	
9	50	

iii   The number of squares is the pattern number squared and then multiplied by two.

iv

	A	B
	Pattern	Number of squares
1		
2	1	2
3	2	8
4	3	18
5	4	32
6	5	50
7	10	200
8	20	800
9	50	5000

The formula in cell B7 is =2*A7^2

ICT activity 2

3   The example below shows a possible screen depicting an exchange rate between euros and dollars of $1 = €1.29

	A	B	C	D	E
1	Currency Converter				
2	$		€		
3		$1=€1.29	=A3*1.29		
4					
5					
6	Write in the exchange		Write a formula in this cell to		
7	rate in this cell		convert one currency to the other		
8					

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

11   Algebraic representation and manipulation

Exercise 11.1

1 **a** $2a + 6$ **b** $4b + 28$ **c** $10c + 40$

   d $21d + 63$ **e** $72e - 63$ **f** $24f - 18$

2 **a** $3a^2 + 6ab$ **b** $8ab + 12b^2$ **c** $2ac + 2bc + 2c^2$

   d $6bd + 9cd + 12d^2$ **e** $3ce - 3de - e^2$ **f** $3df - ef - 2f^2$

3 **a** $4a^2 + 6b^2$ **b** $12a^2 + 16b^2$ **c** $-6c - 9d$

   d $-2c - 3d$ **e** $-4c^2 + 8d^2 - 12e^2$ **f** $-10e + 15f^2$

4 **a** $2a^2 + 2ab$ **b** $3ab - 3b^2$ **c** $4b^2c - 4c^3$

   d $3a^2d^2 - 6b^2d^2 + 3c^2d^2$ **e** $-12de^2 + 3e^3$ **f** $-4df + 6e^2f + 4f^2$

Exercise 11.2

1 **a** $4x - 12$ **b** $10p - 20$ **c** $-42x + 24y$

   d $6a - 9b - 12c$ **e** $-14m + 21n$ **f** $-16x + 6y$

2 **a** $3x^2 - 9xy$ **b** $a^2 + ab + ac$ **c** $8m^2 - 4mn$

   d $-15a^2 + 20ab$ **e** $4x^2 - 4xy$ **f** $24p^2 - 8pq$

3 **a** $-2x^2 + 3y^2$ **b** $a - b$ **c** $7p - 2q$

   d $3x - 4y + 2z$ **e** $3x - \frac{3}{2}y$ **f** $2x^2 - 3xy$

4 **a** $12r^3 - 15rs + 6rt$ **b** $a^3 + a^2b + a^2c$ **c** $6a^3 - 9a^2b$

   d $p^2q + pq^2 - p^2q^2$ **e** $m^3 - m^2n + m^3n$ **f** $a^6 + a^5b$

Exercise 11.3

1 **a** $8a + 4$ **b** $7b - 8$ **c** $5c - 6$

   d $d - 8$ **e** $-2e - 1$ **f** $3f + 7$

2 **a** $2a + 3b + 8$ **b** $4a - 6b + 2$ **c** $5c - 7$

   d $d + 2$ **e** $-3e + 7$ **f** $-2f - 2$

3 **a** $2a^2 + 6a + 2b^2 - 2b$ **b** $3a^2 - 2b^2 - 12a + 6b$

   c $2a^2 - 2b^2 + 2ac + 2bc$ **d** $a^2d^2 - c^2d^2$

   e $ac + bc$ **f** $2ad + ae + 2ce$

Exercise 11.4

1 a $-a - 8$ b $4x - 20$ c $3p - 16$

   d $21m - 6n$ e 3 f $-p - 3p^2$

2 a $8m^2 + 28m + 2$ b $x - 4$ c $2p + 22$

   d $m - 12$ e $a^2 + 6a + 2$ f $7ab - 16ac + 3c$

3 a $4x + 4$ b $5x - \frac{3}{2}y$ c $\frac{9}{4}x - \frac{5}{2}y$

   d $\frac{9}{2}x + \frac{1}{2}y$ e $7x - 4y$ f 0

Exercise 11.5

1 a $x^2 + 5x + 6$ b $x^2 + 7x + 12$ c $x^2 + 7x + 10$

   d $x^2 + 7x + 6$ e $x^2 + x - 6$ f $x^2 + 5x - 24$

2 a $x^2 + 2x - 24$ b $x^2 - 3x - 28$ c $x^2 - 2x - 35$

   d $x^2 - 2x - 15$ e $x^2 - 2x - 3$ f $x^2 + 2x - 63$

3 a $x^2 - 5x + 6$ b $x^2 - 7x + 10$ c $x^2 - 12x + 32$

   d $x^2 + 6x + 9$ e $x^2 - 6x + 9$ f $x^2 - 12x + 35$

4 a $x^2 - 9$ b $x^2 - 49$ c $x^2 - 64$

   d $x^2 - y^2$ e $a^2 - b^2$ f $p^2 - q^2$

5 a $2x^2 + 7x + 3$ b $6x^2 + 11x - 10$ c $-3x^2 - 2x + 8$

   d $25y^2 - 70y + 49$ e $-4x^2 + 9$ f $-16x^2 + 9$

Exercise 11.6

1 a $2(2x - 3)$ b $6(3 - 2p)$ c $3(2y - 1)$

   d $2(2a + 3b)$ e $3(p - q)$ f $4(2m + 3n + 4r)$

2 a $a(3b + 4c - 5d)$ b $2p(4q + 3r - 2s)$ c $a(a - b)$

   d $2x(2x - 3y)$ e $ab(c + d + f)$ f $3m(m + 3)$

3 a $3pq(r - 3s)$ b $5m(m - 2n)$ c $4xy(2x - y)$

   d $b^2(2a^2 - 3c^2)$ e $12(p - 3)$ f $6(7x - 9)$

4 a $6(3 + 2y)$ b $7(2a - 3b)$ c $11x(1 + y)$

   d $4(s - 4t + 5r)$ e $5q(p - 2r + 3s)$ f $4y(x + 2y)$

5 a $m(m + n)$ b $3p(p - 2q)$ c $qr(p + s)$

   d $ab(1 + a + b)$ e $p^3(3 - 4p)$ f $b^2c(7b + c)$

6 a $m(m^2 - mn + n^2)$ b $2r^2(2r - 3 + 4s)$

   c $7xy(8x - 4y)$ d $18mn(4m + 2n - mn)$

7 a $a(3a - 2b + 4c)$ b $b(2a - 3b + 4c)$

   c $2c(a^2 - 2b^2 + 3bc)$ d $13cd(3d + 4c)$

8 **a** $4ac(3 - 2c + a)$ **b** $17ab(2a - 3b)$

    c $11c^2(3a + 11c - b^2)$ **d** $19c^2d^2(2c - 3d + 5)$

9 **a** $\dfrac{5}{c}(3a - 5b + 2d)$ **b** $\dfrac{23}{c^2}(2a - b)$

    c $\dfrac{1}{2a}(1 - \dfrac{1}{2}) = \dfrac{1}{4a}$ **d** $\dfrac{1}{5d}\left(3 - \dfrac{1}{2} + \dfrac{4}{3}\right) = \dfrac{23}{30d}$

10 **a** $\dfrac{1}{a}\left(\dfrac{5}{a} - 3\right)$ **b** $\dfrac{3}{b}\left(\dfrac{2}{b} - 1\right)$

    c $\dfrac{1}{3a}\left(2 - \dfrac{3}{a}\right)$ **d** $\dfrac{1}{5d}\left(\dfrac{3}{d} - 4\right)$

Exercise 11.7

	a	**b**	**c**	**d**
1	12	–1	–6	5
2	–15	10	78	24
3	13	34	–19	57
4	–4	4	–27	–27
5	–125	–125	100	–100

Exercise 11.8

	a	**b**	**c**
1	0	30	14
	d 20	**e** –13	**f** –4
2	–3	–30	20
	d –16	**e** –40	**f** 42
3	–160	–23	42
	d –17	**e** –189	**f** 113
4	48	–8	15
	d 16	**e** –5	**f** 9
5	12	–5	–5
	d 7	**e** 7	**f** 36

Exercise 11.9

1 **a** $b = c - a$ **b** $b = d - 2c$ **c** $c = 4a - 2b$ **d** $b = 2a - 3d$

2 **a** $b = \dfrac{c}{a}$ **b** $c = \dfrac{bd}{a}$ **c** $a = \dfrac{c+3}{b}$ **d** $c = \dfrac{b-4}{a}$

3 **a** $n = r - m$ **b** $m = p - n$ **c** $n = 3p - 2m$

    d $q = 3x - 2p$ **e** $a = \dfrac{cd}{b}$ **f** $d = \dfrac{ab}{c}$

4   **a** $x = \frac{4m}{3y}$ **b** $r = \frac{7pq}{5}$ **c** $x = \frac{c}{3}$

     d $x = \frac{y-7}{3}$ **e** $y = \frac{3r+9}{5}$ **f** $x = \frac{5y-9}{3}$

5   **a** $b = \frac{2a-5}{6}$ **b** $a = \frac{6b+5}{2}$ **c** $z = \frac{3x-7y}{4}$

     d $x = \frac{4z+7y}{3}$ **e** $y = \frac{3x-4z}{7}$ **f** $p = \frac{8+q}{2r}$

6   **a** $p = 4r$ **b** $p = \frac{4}{3r}$ **c** $p = \frac{n}{10}$

     d $n = 10p$ **e** $p = \frac{2t}{q+r}$ **f** $q = \frac{2t}{p} - r$

7   **a** $r = \frac{3m-n}{t(p+q)}$ **b** $t = \frac{3m-n}{r(p+q)}$ **c** $m = \frac{rt(p+q)+n}{3}$

     d $n = 3m - rt(p+q)$ **e** $p = \frac{3m-n}{rt} - q$ **f** $q = \frac{3m-n}{rt} - p$

8   **a** $d = \frac{ab}{ce}$ **b** $a = \frac{cde}{b}$ **c** $c = \frac{ab}{de}$

     d $a = cd - b$ **e** $b = d - \frac{a}{c}$ **f** $c = \frac{a}{d-b}$

Student assessment 1

1   **a** $4a + 8$

     b $10b - 15$

     c $2c^2 + 4cd$

     d $6cd - 12d^2$

     e $-15e + 5f$

     f $f - 2g$

2   **a** $7a + 10$

     b $5b - 9$

     c $-2c - 4$

     d $d - 2$

     e $e^2 - 3e + 6$

     f $df - 2ef - f^2 - e^2$

     g $x^2 + x - 56$

     h $3x^2 + 7x + 2$

     i $-4x^2 + 11x - 6$

3   **a** $7(a + 2)$

     b $13b(2b + 3)$

     c $3f(c - 2d + 3g)$

     d $5d^2(1 - 2d)$

4   **a** -6 **b**   1   **c**   20   **d**   49

5 **a** $a = c + b$

    b $b = 2c + 3d$

    c $c = \frac{ad}{b}$

    d $d = \frac{3c+e}{5}$

    e $e = \frac{4a}{f+g}$

    f $f = \frac{4a-eg}{e}$

Student assessment 2

1 **a** $6x - 9y + 15z$

    b $8mp - 28p$

    c $-8m^2n + 4mn^2$

    d $20p^3q - 8p^2q^2 - 8p^3$

    e $-2x - 2$

    f $22x^2 - 14x$

    g 2

    h $\frac{5}{2}x^2 - x$

    i $100 - x^2$

    j $c^2 - d^2$

    k $9x^2 - 30x + 25$

    i $(-2x+1)\left(\frac{1}{2}x+2\right) = -2x\left(\frac{1}{2}x+2\right) + 1\left(\frac{1}{2}x+2\right)$

$$-x^2 - \frac{7}{2}x + 2$$

2 **a** $8(2p - q)$ **b** $p(p - 6q)$

    c $5pq(p - 2q)$ **d** $3pq(3 - 2p + 4q)$

3 **a** 0 **b** -7 **c** 29

    d 7 **e** 7 **f** 35

4 **a** $n = p - 4m$

    b $y = \frac{4x-5z}{3}$

    c $y = \frac{10px}{3}$

    d $y = \frac{3w}{m} - x$

    e $r = \frac{pqt}{4mn}$

    f $q = r(m - n) - p$

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

12 Algebraic indices

Exercise 12.1

1 a c^8 b m^2 c b^9 d m^3n^6

e $2a^4b$ f $3x^3y^2$ g $\dfrac{uv^3}{2}$ h $\dfrac{x^2y^3z^2}{3}$

2 a $12a^5$ b $8a^5b^3$ c $8p^6$ d $16m^4n^6$

e $200p^{13}$ f $32m^5n^{11}$ g $24xy^3$ h $a^{(d+e)}b^{(d+e)}$

Exercise 12.2

a c^3 b g c q^{-2} or $\dfrac{1}{q^2}$ d m^{-1} or $\dfrac{1}{m}$

Exercise 12.3

a a^2 b $\dfrac{r^{-6}}{p^{-6}}$ or $\dfrac{p^6}{r^6}$ c t^{16} d m^9

Exercise 12.4

1 a $x=2$ b $x=4$ c $x=3$

d $x=3$ e $x=4$ f $x=0$

2 a $x=4$ b $x=1$ c $x=\dfrac{3}{2}$

d $x=-1$ e $x=2$ f $x=0$

Student assessment 1

1 a $a^3 \times b^2$ b $d^2 \times e^5$

2 a $m \times m \times m$ b $r \times r \times r \times r$

3 a a^7 b $p^5 \times q^9$ c b^3 d e^6

4 a r^4 b b^{-2} or $\dfrac{1}{b^2}$ c n^3

5 a p^{-9} or $\dfrac{1}{p^9}$ b h^7

6 a $x=\dfrac{5}{2}$ b $x=0$

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

13 Equations

Exercise 13.1

1 **a** $a = 4$ **b** $b = 2$ **c** $c = 8$
      d $d = 8$ **e** $e = 10$ **f** $f = 6$

2 **a** $a = 7$ **b** $b = -9$ **c** $c = 5$ **d** $d = -8$

3 **a** $a = 11$ **b** $b = -12$ **c** $c = 13$ **d** $d = 3$

4 **a** $a = 5$ **b** $b = 5$ **c** $c = 3$ **d** $d = -1$

5 **a** $a = 12$ **b** $b = 8$ **c** $c = 10$
      d $d = 15$ **e** $e = 12$ **f** $f = -16$

6 **a** $a = 9$ **b** $b = 20$ **c** $c = 18$
      d $d = -35$ **e** $e = 6$ **f** $f = -4$

7 **a** $a = 4.5$ **b** $b = \frac{10}{3}$ **c** $c = \frac{5}{2}$
      d $d = 11\frac{1}{5}$ **e** $e = -16$ **f** $f = 14$

8 **a** $a = 5$ **b** $b = 1$ **c** $c = 17$
      d $d = 11$ **e** $e = 8$ **f** $f = 8$

9 **a** $a = 2$ **b** $b = 7$ **c** $c = 7$
      d $d = -0.5$ **e** $e = -2$ **f** $f = 0.5$

10 **a** $a = 13$ **b** $b = -23$ **c** $c = 27$
      d $d = -1$ **e** $e = 7$ **f** $f = -1$

Exercise 13.2

1 **a** $x = -4$ **b** $y = 5$ **c** $y = -5$
      d $p = -4$ **e** $y = 8$ **f** $x = -5.5$

2 **a** $x = 4\frac{1}{3}$ **b** $x = 5$ **c** $x = 6$
      d $y = -8$ **e** $y = 4$ **f** $m = 10$

3 **a** $m = 1$ **b** $p = 3$ **c** $k = -1$
      d $x = -21$ **e** $x = 2$ **f** $y = 3$

4 **a** $x = 6$ **b** $y = 14$ **c** $x = 4$
      d $m = 12$ **e** $x = 35$ **f** $p = 20$

5 **a** $x = 15$ **b** $x = -5$ **c** $x = 7.5$
      d $x = 8$ **e** $x = 2.5$ **f** $x = 10$

6 **a** $x = 5$ **b** $x = 14$ **c** $x = 22$

d $x = 5$ **e** $x = 8$ **f** $x = 2$

7 a $y = 10$ **b** $x = 17$ **c** $x = 13$

   d $y = -5$ **e** $x = 4$ **f** $x = 6.5$

Exercise 13.3

1 a i $(3x + 60)°$ **ii** $3x + 60 = 180$ **iii** $x = 40$ **iv** 40°, 60°, 80°

  b i $3x°$ **ii** $3x = 180$ **iii** $x = 60$ **iv** 20°, 80°, 80°

  c i $18x°$ **ii** $18x = 180$ **iii** $x = 10$ **iv** 20°, 50°, 110°

  d i $6x°$ **ii** $6x = 30$ **iii** $x = 30$ **iv** 30°, 60°, 90°

  e i $(7x - 30)°$ **ii** $7x - 30 = 180$ **iii** $x = 30$ **iv** 10°, 40°, 130°

  f i $(9x - 45)°$ **ii** $9x - 45 = 180$ **iii** $x = 25$ **iv** 25°, 55°, 100°

2 a i $12x°$ **ii** $12x = 360$ **iii** $x = 30$ **iv** 90°, 120°, 150°

  b i $(11x + 30)°$ **ii** $11x + 30 = 360$ **iii** $x = 30$ **iv** 90°, 135°, 135°

  c i $(12x + 60)°$ **ii** $12x + 60 = 360$ **iii** $x = 25$ **iv** 35°, 80°, 90°, 155°

  d i $(10x + 30)°$ **ii** $10x + 30 = 360$ **iii** $x = 33$ **iv** 33°, 94°, 114°, 119°

3 a i $(11x - 80)°$ **ii** $11x - 80 = 360$ **iii** $x = 40$ **iv** 40°, 80°, 80°, 160°

  b i $(10x + 60)°$ **ii** $10x + 60 = 360$ **iii** $x = 30$ **iv** 45°, 90°, 95°, 130°

  c i $(16x + 8)°$ **ii** $16x + 8 = 360$ **iii** $x = 22$ **iv** 44°, 96°, 100°, 120°

  d i $(9x + 45)°$ **ii** $9x + 45 = 360$ **iii** $x = 35$ **iv** 35°, 55°, 120°, 150°

  e i $(12x + 36)°$ **ii** $12x + 36 = 360$ **iii** $x = 27$ **iv** 50°, 50°, 130°, 130°

  f i $(14x - 6)°$ **ii** $14x - 6 = 540$ **iii** $x = 39$ **iv** 77°, 86°, 105°, 119°, 153°

4 a 20   **b** 25   **c** 14   **d** 25   **e** 31   **f** 40

5 a 50   **b** 13   **c** 40   **d** 40

6 a 5   **b** 2   **c** 7   **d** 1.1   **e** 25   **f** 15

Exercise 13.4

1 a $n = 3$ **b** $n = 7$

2 a $n = 10$ **b** $n = 4$

3 a $n = 8$ **b** $n = 14$

4 a $n = 2$ **b** $n = 3$

Exercise 13.5

1 a $x = 4$ $y = 2$ **b** $x = 6$ $y = 5$ **c** $x = 6$ $y = -1$

  d $x = 5$ $y = 2$ **e** $x = 5$ $y = 2$ **f** $x = 4$ $y = 9$

2 a $x = 3$ $y = 2$ **b** $x = 7$ $y = 4$ **c** $x = 1$ $y = 1$

  d $x = 1$ $y = 5$ **e** $x = 1$ $y = 10$ **f** $x = 8$ $y = 2$

3 **a** $x = 5$ $y = 4$ **b** $x = 4$ $y = 3$ **c** $x = 10$ $y = 5$

 d $x = 6$ $y = 4$ **e** $x = 4$ $y = 4$ **f** $x = 10$ $y = -2$

4 **a** $x = 5$ $y = 4$ **b** $x = 4$ $y = 2$ **c** $x = 5$ $y = 3$

 d $x = 5$ $y = -2$ **e** $x = 1$ $y = 5$ **f** $x = -3$ $y = -3$

5 **a** $x = -5$ $y = -2$ **b** $x = -3$ $y = -4$ **c** $x = 4$ $y = 3\frac{2}{3}$

 d $x = 2$ $y = 7$ **e** $x = 1$ $y = 1$ **f** $x = 2$ $y = 9$

6 **a** $x = 2$ $y = 3$ **b** $x = 5$ $y = 10$ **c** $x = 4$ $y = 6$

 d $x = 4$ $y = 4$ **e** $x = 5$ $y = 1$ **f** $x = -3$ $y = -3$

7 **a** $x = 1$ $y = -1$ **b** $x = 11\frac{2}{3}$ $y = 8$ **c** $x = 4$ $y = 0$

 d $x = 3$ $y = 4$ **e** $x = 2$ $y = 8$ **f** $x = 1$ $y = 1$

Exercise 13.6

1 **a** $a = 2$ $b = 1$ **b** $b = 4$ $c = 3$ **c** $c = 5$ $d = 2$

 d $d = 7$ $e = 2$ **e** $e = 3$ $f = 4$ **f** $f = 2$ $g = 1$

2 **a** $a = 2$ $b = 3$ **b** $b = 5$ $c = 1$ **c** $c = 1$ $d = 2$

 d $d = 3$ $e = 3$ **e** $e = 4$ $f = 5$ **f** $f = 2$ $g = 1$

3 **a** $a = -1$ $b = 1$ **b** $b = -1$ $c = 2$ **c** $c = 3$ $d = -1$

 d $d = -2$ $e = 2$ **e** $e = -3$ $f = 2$ **f** $f = -1$ $g = -1$

4 **a** $x = 2$ $y = 3$ **b** $x = 1$ $y = 4$ **c** $x = 5$ $y = 2$

 d $x = 3$ $y = 3$ **e** $x = 4$ $y = 2$ **f** $x = 6$ $y = 1$

5 **a** $x = 1$ $y = 4$ **b** $x = 5$ $y = 2$ **c** $x = 3$ $y = 3$

 d $x = 6$ $y = 1$ **e** $x = 2$ $y = 3$ **f** $x = 2$ $y = 3$

6 **a** $x = 0$ $y = 3$ **b** $x = 5$ $y = 2$ **c** $x = 1$ $y = 7$

 d $x = 6$ $y = 4$ **e** $x = 2$ $y = 5$ **f** $x = 3$ $y = 0$

7 **a** $x = 1$ $y = \frac{1}{2}$ **b** $x = 2.5$ $y = 4$ **c** $x = \frac{1}{5}$ $y = 4$

 d $x = \frac{3}{4}$ $y = \frac{1}{2}$ **e** $x = 5$ $y = \frac{1}{3}$ **f** $x = \frac{1}{2}$ $y = 1$

Exercise 13.7

1 10 and 7

2 16 and 9

3 $x = 1$ $y = 4$

4 $x = 5$ $y = 2$

5 60 and 20 years old

6 60 and 6 years old

Student assessment 1

1 **a** $a = 6$ **b** $b = -7$ **c** $c = -3$ **d** $d = 4$

2 **a** $a = -10$ **b** $b = 4$ **c** $c = -\frac{1}{2}$ **d** $d = 4$

3 **a** $a = 10$ **b** $b = 21$ **c** $c = 6$ **d** $d = 18$

4 **a** $a = 8$ **b** $b = 15$ **c** $c = 24$ **d** $d = 9$

5 **a** $a = -10$ **b** $b = 7$ **c** $c = 23$ **d** $d = 11$

6 **a** $a = 2$ and $b = 1$

   b $b = 4$ and $c = 3$

   c $c = -1$ and $d = 1$

   d $d = -5$ and $e = 4$

7 A ruler costs $0.80 and a pen costs $2.30.

Student assessment 2

1 **a** $x = 9$ **b** $x = 11$ **c** $x = -4$ **d** $x = 6$

2 **a** $m = 1.5$ **b** $m = 7$ **c** $m = 4$ **d** $p = 3$

3 **a** $x = -10$ **b** $x = 12$ **c** $x = 10$ **d** $x = 11\frac{1}{4}$

4 **a** $x = 16$ **b** $x = -8\frac{2}{3}$ **c** $x = 2$ **d** $x = 3.5$

5 **a** $x = 5$ and $y = 2$

   b $x = 3\frac{1}{3}$ and $y = 4\frac{1}{3}$

   c $x = 5$ and $y = 4$

   d $x = 5$ and $y = 1$

6 9.75 and 3.25

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

14 Sequences

Exercise 14.1

a   i   +3 from term to term

   ii   30

b   i   +10 from term to term

   ii   98

c   i   +22 from term to term

   ii   209

d   i   –0.2 from term to term

   ii   –1.1

e   i   +1 to the denominator each time

   ii $\dfrac{1}{11}$

f   i   Denominator and numerator increase by 1 each time

   ii $\dfrac{10}{11}$

g   i   Sequence of square numbers

   ii   100

h   i   Difference between successive terms increases by 2 each time

   ii   103

i   i   Sequence of cube numbers

   ii   1000

j   i   Sequence of powers of 5

   ii 5^{10} or 9 765 625

Exercise 14.2

a	53, 71	b	67, 131	c	39, 63	d	173, 275
e	170, 357	f	127, 221	g	27, 29		

Exercise 14.3

1   a   i   20, 23   ii $3n + 2$

   b   i   25, 29   ii $4n + 1$

   c   i   29, 34   ii $5n - 1$

d   i   18, 20   ii $2n + 6$

e   i   −6, −13   ii   ii) $-7n + 36$ or $36 - 7n$

f   i   24, 28   ii $4n - 4$

g   i   46, 55   ii $9n - 8$

h   i   65, 75   ii $10n + 5$

i   i   64, 75   ii $11n - 2$

j   i   13.5, 15.5   ii $2n - \dfrac{1}{2}$

k   i   5.25, 6.25   ii $n - \dfrac{3}{4}$

l   i   0, −1   ii $-n + 6$ or $6 - n$

3   a   It belongs to the sequence $2n + 8$ as $2 \times 7 + 8 = 22$

   b   It belongs to neither as it must be even to be in the sequence generated by $2n + 8$. $5n - 37 = 51$ when solved gives $n = 17.6$ but n must be an integer.

   c   38 belongs to both sequences when $n = 15$

Exercise 14.4

1   i   50, 72   ii $2n^2$

2   i   30, 42   ii $n(n + 1)$

3   i   123, 214   ii $n^3 - 2$

4   i   29, 40   ii $n^2 + 4$

5   i $62\frac{1}{2}$, 108   ii $\frac{1}{2}n^3$

6   $2n^2 - 1$

7   $n^2 + n - 1$

8   $2n^3 - 4$

Student assessment 1

1   a   i   45, 54

     ii   Each term is 9 more than the term before

   b   i   30, 24

     ii   Each term is 6 less than the term before

   c   i   2.25, 1.125

     ii   Each term is half the term before

   d   i   −12, −18

     ii   Each term is 6 less than the term before

   e   i   27, 8

     ii   The sequence is cube numbers in descending order

f **i**   81, 243

    ii Each term is 3 times the term before

2 **a** **i**   30, 36

    ii Each term is 6 more than the term before

b **i**   12, 9

    ii Each term is 3 less than the term before

c **i**   −5, −10

    ii Each term is 5 less than the term before

d **i**   64, 81

    ii The sequence is square numbers in ascending order

e **i**   1000, 10 000

    ii The sequence is powers of 10 in ascending order

f **i** $\frac{1}{16}$, $\frac{1}{32}$

    ii Each term is half the term before

3 **a** $4n + 2$ **b** $6n + 7$ **c** $6n - 3$

    d $n^2 + 3$ **e** $10n - 10$ **f** $n^3 - 1$

4 **a** $2n + 1$ **b** $6n + 1$ **c** $10n - 2$

    d $8n - 7$ **e** $8n - 12$ **f** $n^2 + 1$

5 **a** 1, 4, 9, 16, 25

    b 1, 8, 27, 64, 125

    c 1, 3, 6, 10, 15

6 **a** **i**   28, 39

    ii nth term is $n^2 + 3$

   b **i**   250, 432

    ii The nth term is $2n^3$

   c **i** $7\frac{1}{2}$, $10\frac{1}{2}$

    ii The nth term is $\frac{1}{4}n(n + 1)$

   d **i**   28, 40

    ii The nth term is $n(n + 1) - 2$

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

15 Graphs in practical situations

Exercise 15.1

1
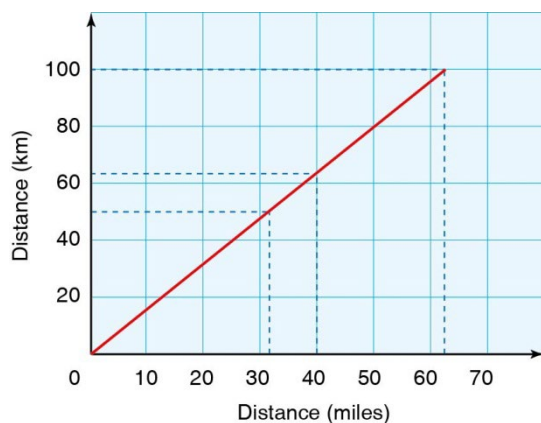
   a $50\,\text{km} \approx 31$ miles

   b 40 miles $\approx 64\,\text{km}$, therefore 80 miles $= 128\,\text{km}$

   c $100\,\text{km/h} \approx 62\,\text{mph}$

   d $40\,\text{mph} \approx 64\,\text{km/h}$

2
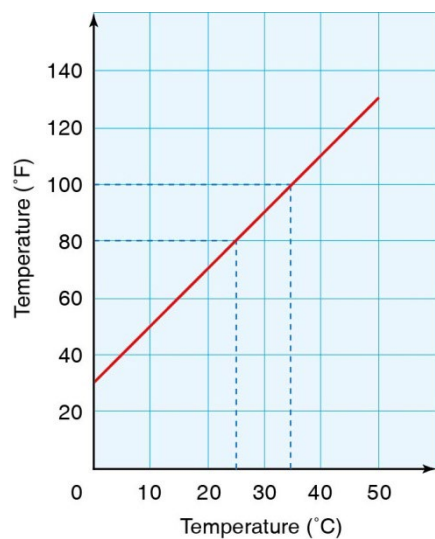
   a $25\,°\text{C} \approx 80\,°\text{F}$

   b $100\,°\text{F} \approx 35\,°\text{C}$

   c $0\,°\text{C} \approx 30\,°\text{F}$

3
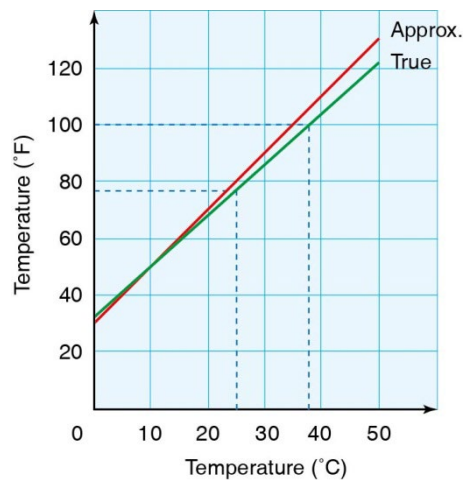
a   25 °C ≈ 77 °F   100 °F ≈ 38 °C   0 °C ≈ 32 °F

b   The rough conversion is most useful at lower temperatures (i.e. between 0 and 20 °C).

4   a
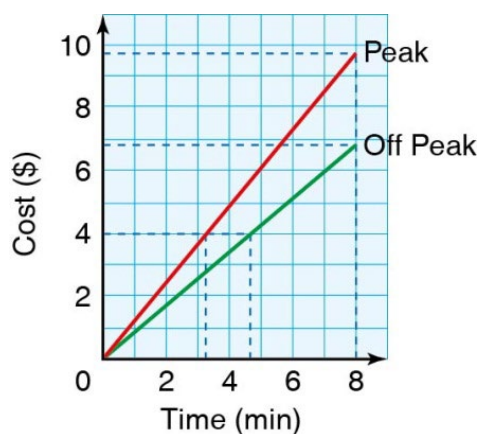
b   8 min ≈ $6.80

c   8 min ≈ $9.60

d   Extra time ≈ 1 min 20 s

5
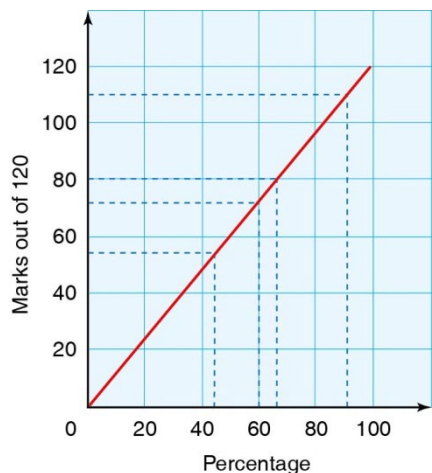
a $80 = 67\%$ **b** $110 = 92\%$ **c** $54 = 45\%$ **d** $72 = 60\%$

Exercise 15.2

1   **a**   6 m/s   **b**   4 m/s   **c**   39 km/h

     d   20 km/h   **e**   160 km/h   **f**   50 km/h

2   **a**   400 m   **b**   182 m   **c**   210 km

     d   255 km   **e**   10 km   **f**   79.2 km

3   **a**   5 s   **b**   50 s   **c**   4 min

     d   71.4 s (1 d.p.)   **e**   5 s   **f**   4 min

Exercise 15.3

1
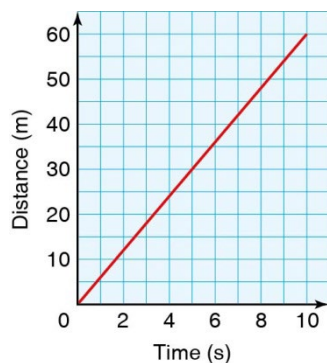
2
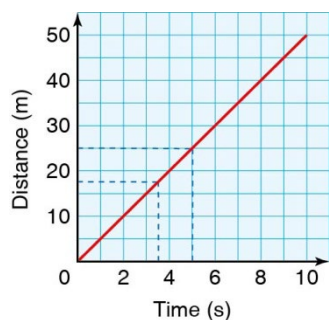
a   25 m = 5 s

b   3.5 s = 17 m

3   a   Speed A = 40 m/s

      Speed B = $13\frac{1}{3}$ m/s

   b   Distance apart = $453\frac{1}{3}$ m

4   a $\frac{2}{3}$ m/s   b   6 m/s, $\frac{2}{3}$ m/s   c   1 m/s   d $\frac{1}{2}$ m   e $7\frac{1}{3}$ m

Exercise 15.4

1   a   45 km/h

   b   20 km/h

   c   Paul has arrived at the restaurant and is waiting for Helena.

2
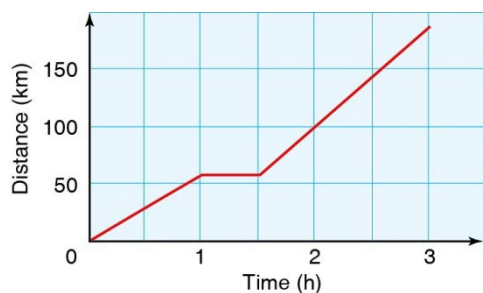
3
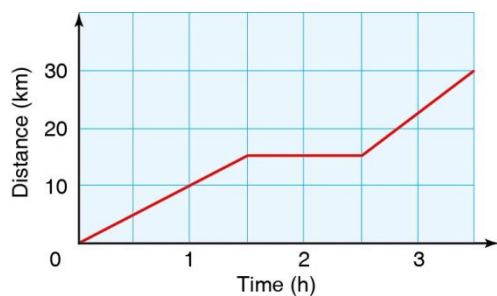
4 a
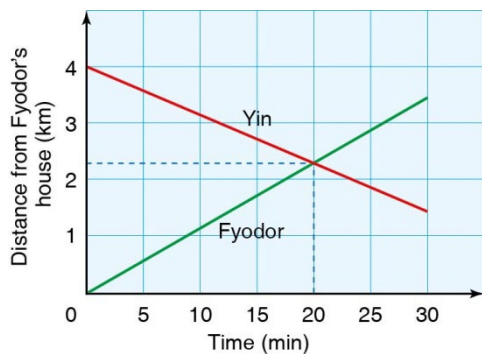
b After 20 min

c Distance $= 2\frac{1}{3}$ km

5 a
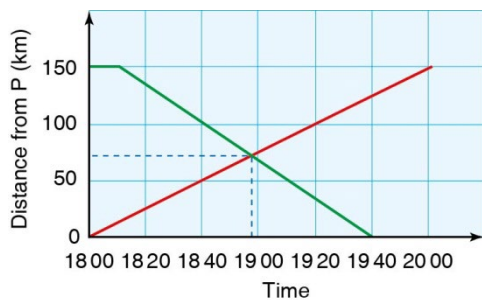
b Time $\approx$ 18 57

c Distance from Q $\approx$ 79 km

d The 18 10 train from station Q arrives first.

6 a a: $133\frac{1}{3}$ km/h

b: 0 km/h

c: 200 km/h

b d: 100 km/h

e: 200 km/h

Student assessment 1

1
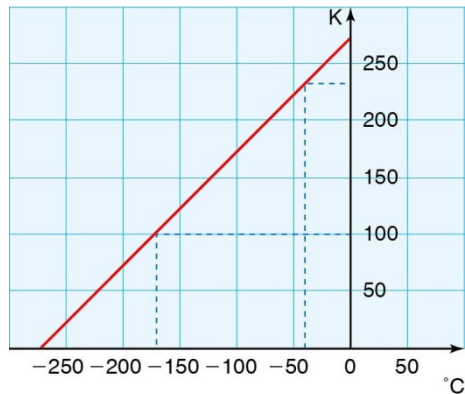
   a $-40\,°\text{C} \approx 233\,\text{K}$

   b $100\,\text{K} \approx -173\,°\text{C}$

2 **a**

Time (hours)	0	1	2	3	4	5	6	7
Cost (€)	70	120	170	220	270	320	370	420
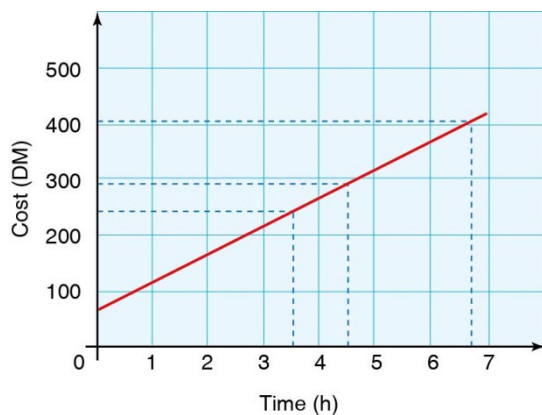
   i $4\frac{1}{2}$ hours costs €295

   ii $6\frac{3}{4}$ hours costs about €408

  b €245 is the price for a $3\frac{1}{2}$ hour job.

3   a
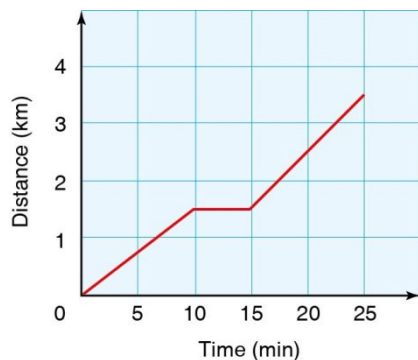
b   The total time to get home is 25 minutes

4   a   i   B and C are impossible.

ii   B is impossible because it illustrates going back in time.

C is impossible because it illustrates infinite speed (in the vertical section).

b   A horizontal line means that there is no motion at that time.

5   a   1 hour, 2 mins and 30 secs.

b   2.7 m/s

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

16   Graphs of functions

Exercise 16.1

1 a

b

c

d
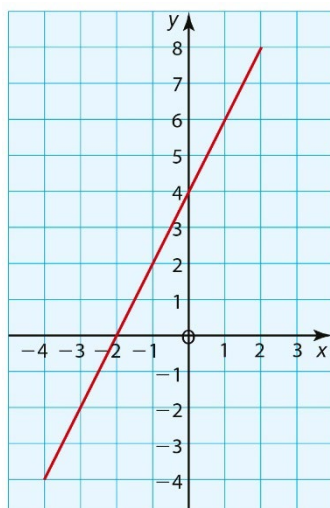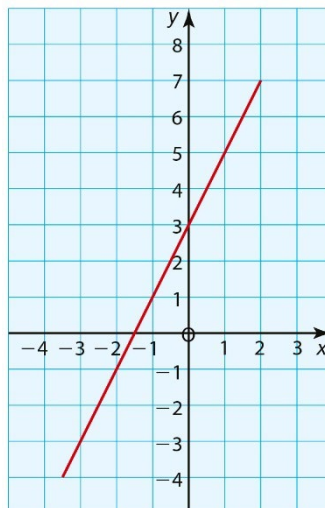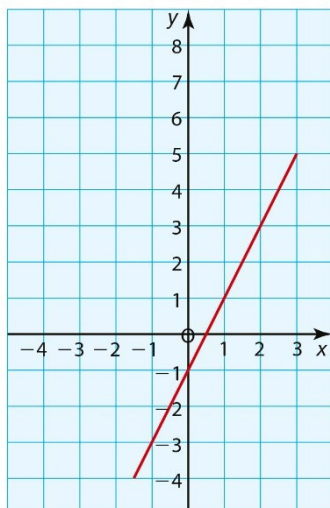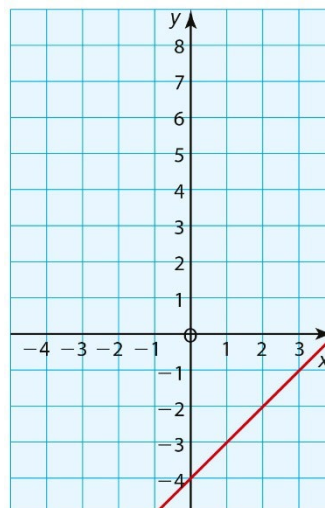
e
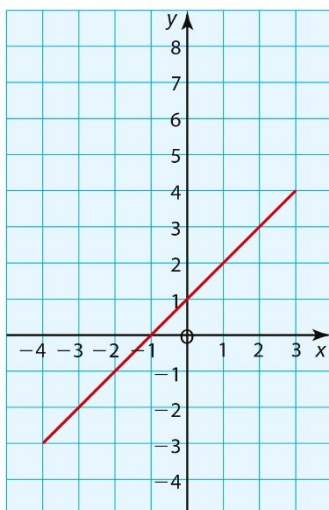
f
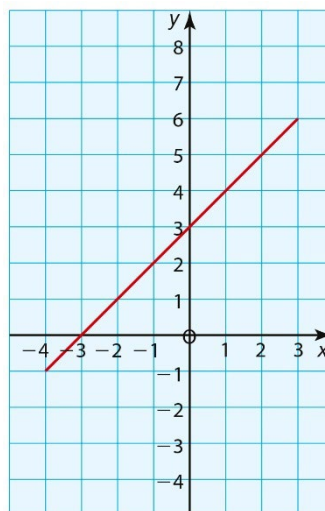
g
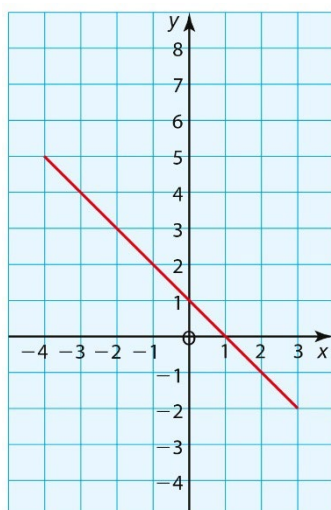
h
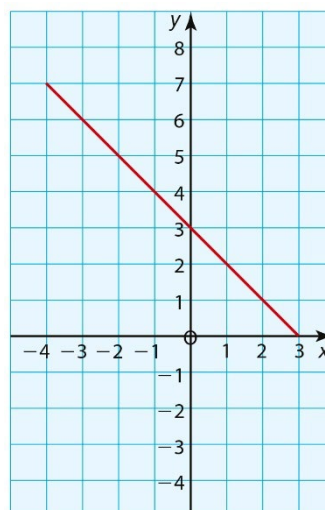
i
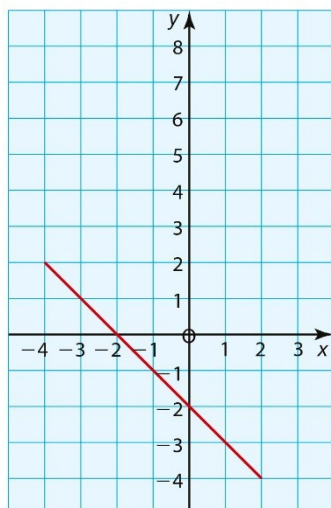
2 a
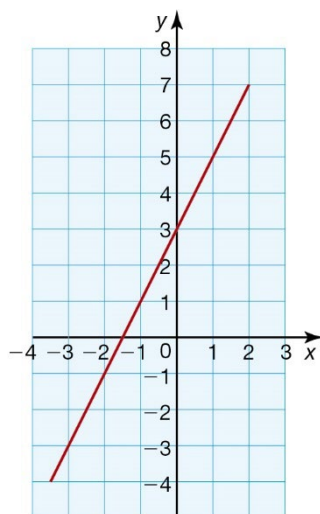
b
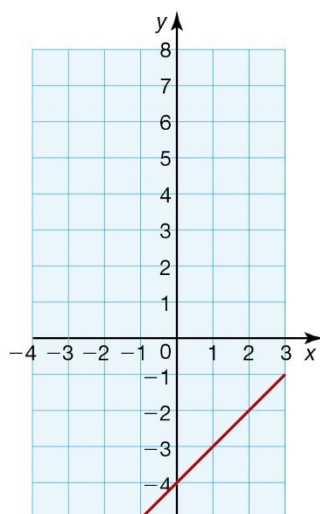
c
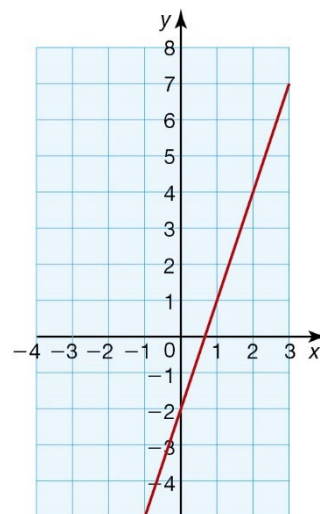
d
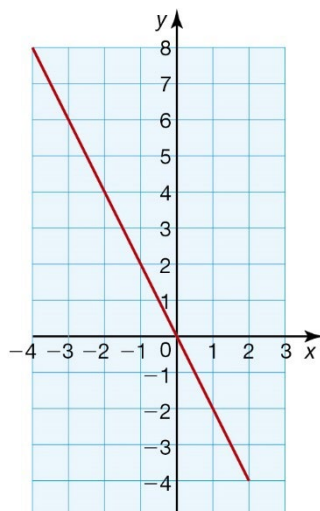
e
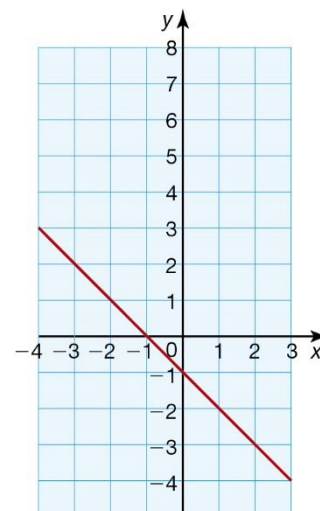
Exercise 16.2

1	**a**	$(3, 2)$	**b**	$(5, 2)$	**c**	$(2, 1)$
	d	$(2, 1)$	**e**	$(-4, 1)$	**f**	$(4, -2)$
2	**a**	$(3, -2)$	**b**	$(-1, -1)$	**c**	$(-2, 3)$
	d	$(-3, -3)$	**e**	Infinite solutions	**f**	No solution

Exercise 16.3

a

x	–4	–3	–2	–1	0	1	2	3
y	10	4	0	–2	–2	0	4	10
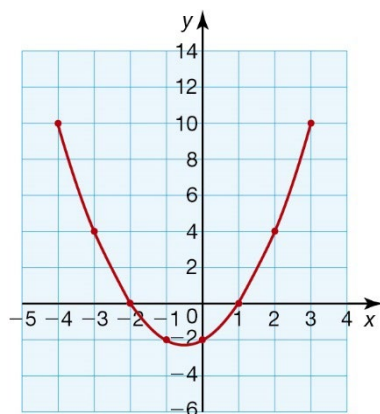
b

x	–3	–2	–1	0	1	2	3	4	5
y	–12	–5	0	3	4	3	0	–5	–12
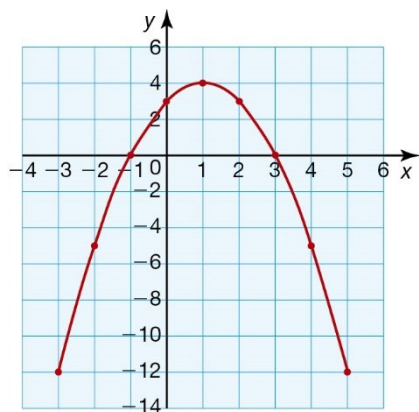
c

x	−1	0	1	2	3	4	5
y	9	4	1	0	1	4	9
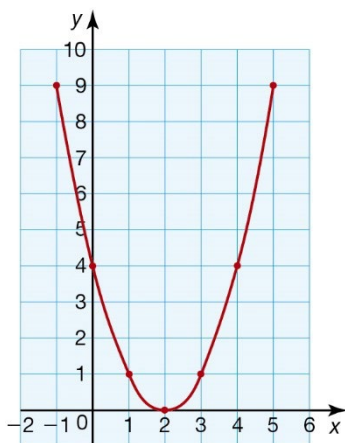
d

x	−4	−3	−2	−1	0	1	2
y	−9	−4	−1	0	−1	−4	−9
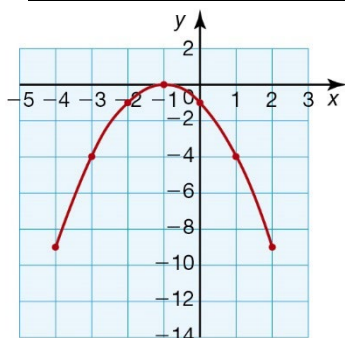
e

x	−4	−3	−2	−1	0	1	2	3	4	5	6
y	9	0	−7	−12	−15	−16	−15	−12	−7	0	9
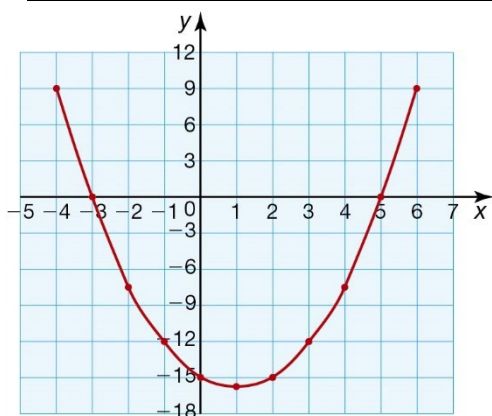
Exercise 16.4

a $x = -2$ and $x = 3$

b $x = -1$ and $x = 1$

c $x = 3$

d $x = -4$ and $x = 3$

e $x = 2$

Exercise 16.5

a $x = -1.6$ and $x = 2.6$ (both to 1 d.p.)

b   No solution

c $x = 2$ and $x = 4$

d $x = -3.5$ and $x = 2.5$ (both to 1 d.p.)

e $x = 0.3$ and $x = 3.7$ (both to 1 d.p.)

Exercise 16.6

1

2
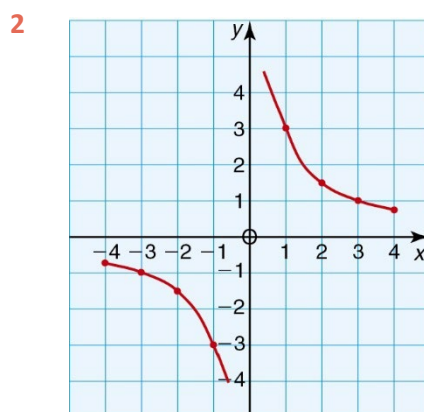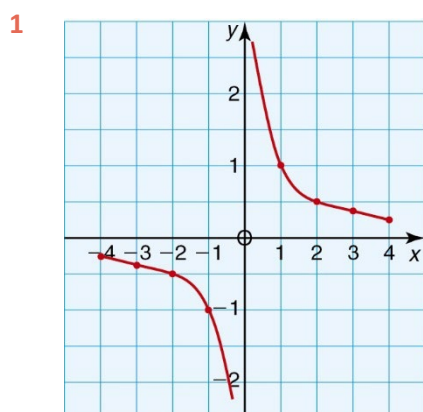
Exercise 16.7

a
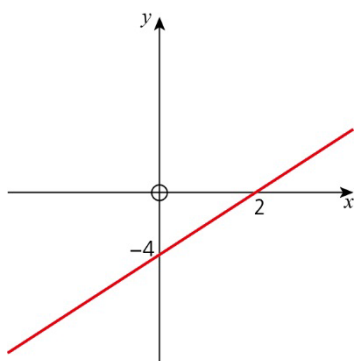
b
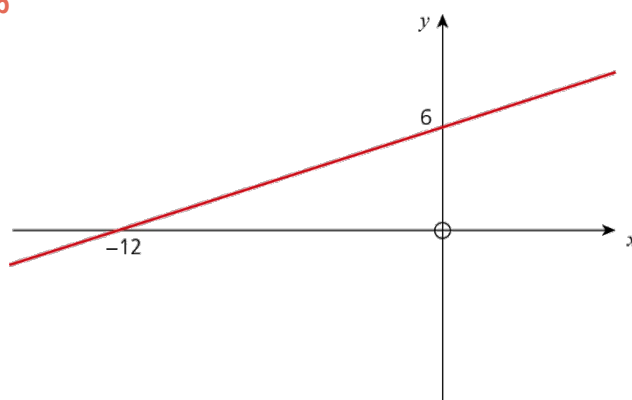
c
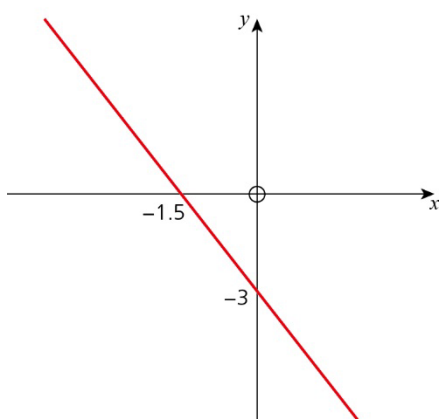
d
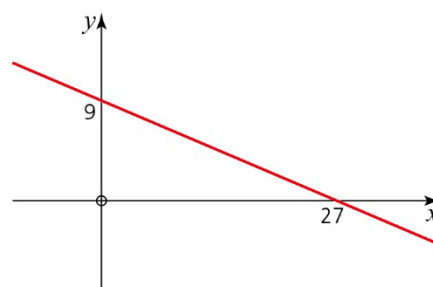
e
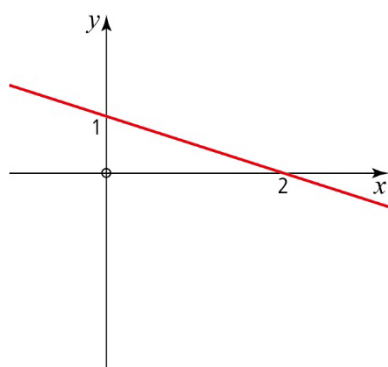
f
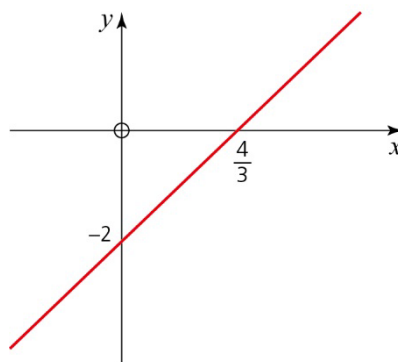
Exercise 16.8

1 a
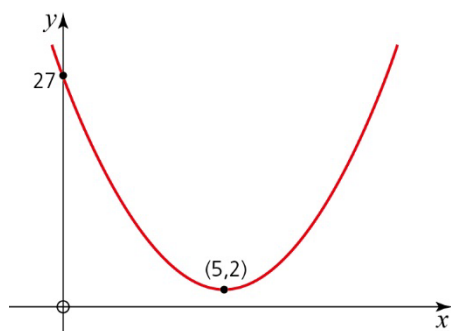
b
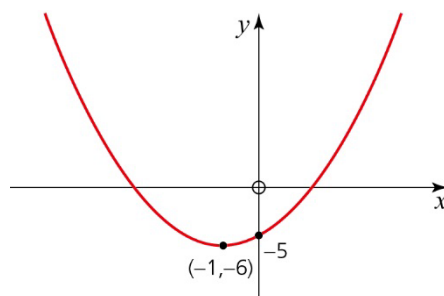
c
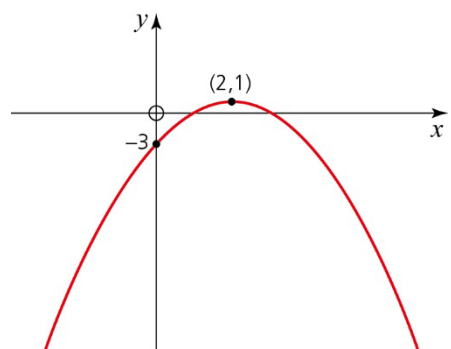
d
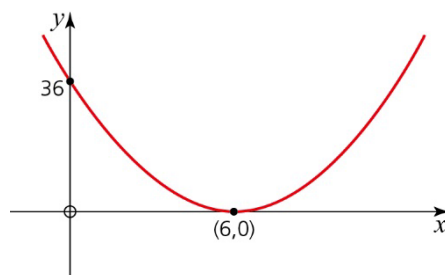
e
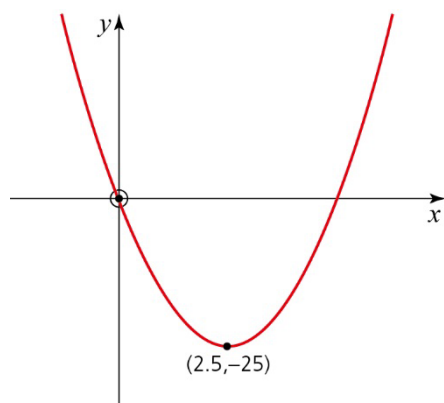
2 a $x^2 - 9$

b
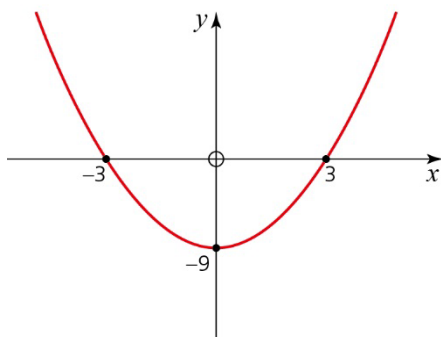
3 a $-x^2 + 6x - 8$

b
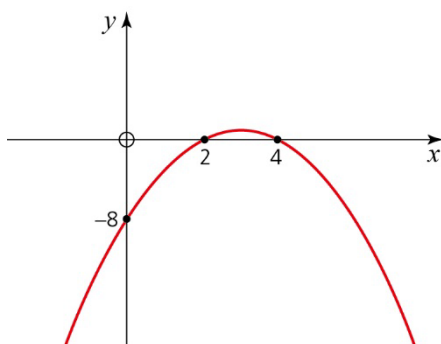
4 a $-3x^2 - 9x - 6$

b
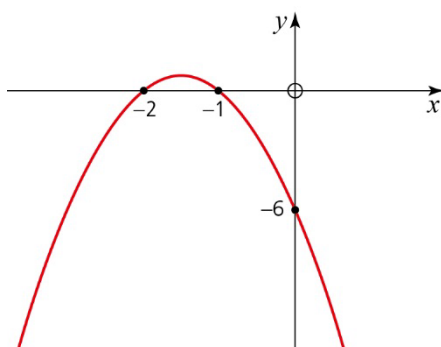
Student assessment 1

1
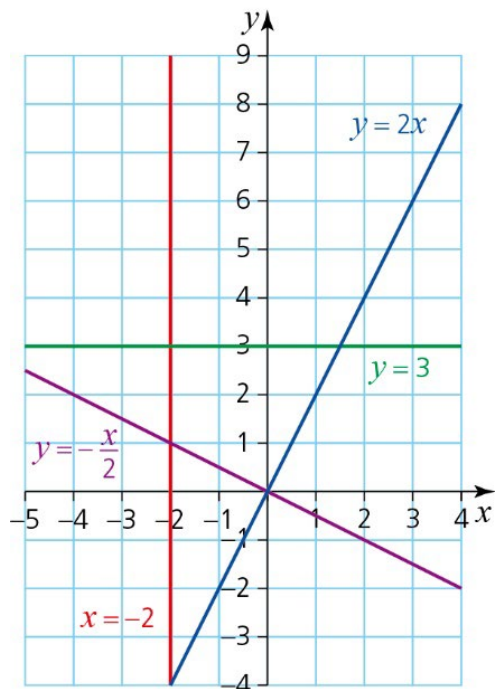
2 a
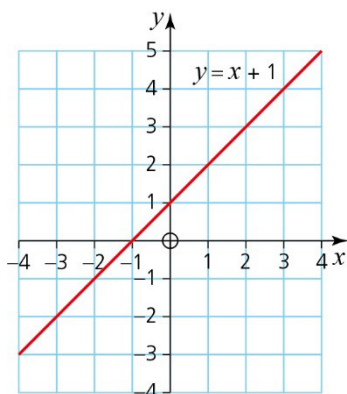
b
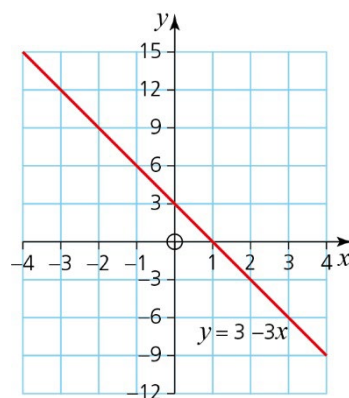
c
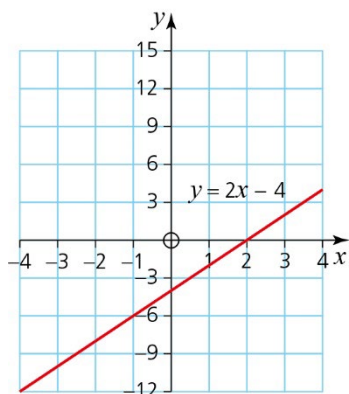
d
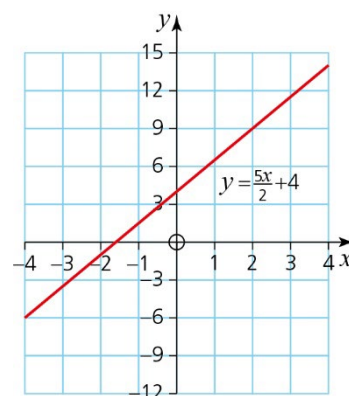
3 **a**
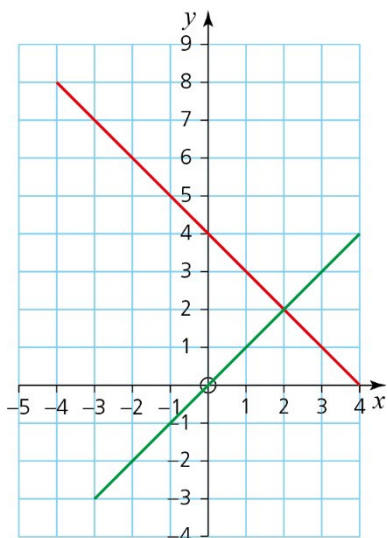
$x = 2$ and $y = 2$

b
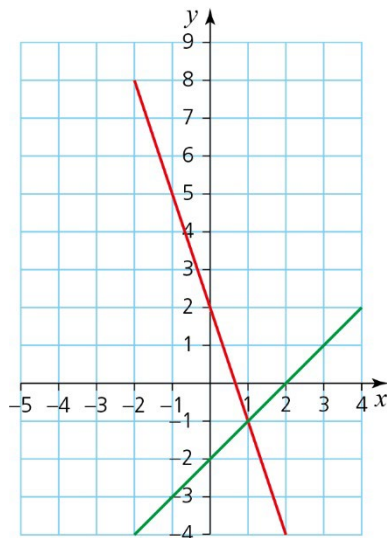
$x = 1$ and $y = -1$

c
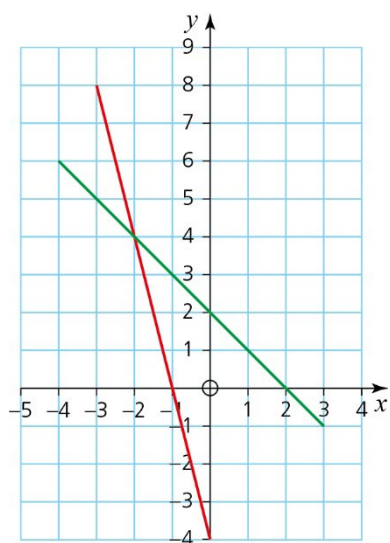
$x = -2$ and $y = 4$

d
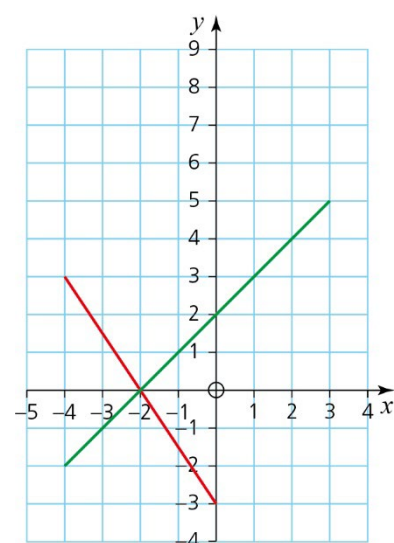
$x = -2$ and $y = 0$

4 a

x	−7	−6	−5	−4	−3	−2	−1	0	1	2
y	8	3	0	−1	0	3	8	15	24	35

b
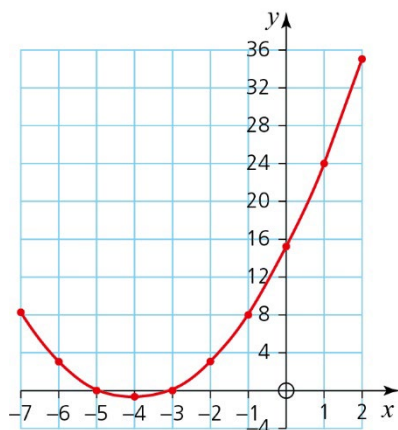
5 a

x	−4	−3	−2	−1	0	1	2	3	4
y	13	6	1	−2	−3	−2	1	6	13

$$y = x^2 - 3$$
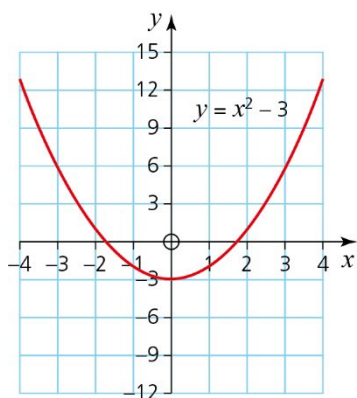
b

x	−4	−3	−2	−1	0	1	2	3	4
y	−13	−6	−1	2	3	2	−1	−6	−13

$y = 3 - x^2$
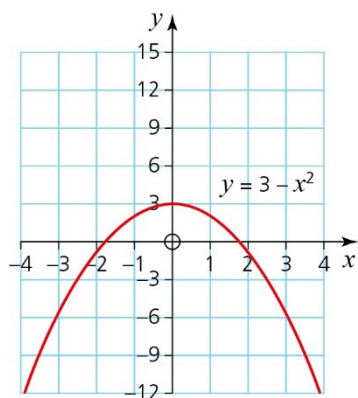
c

x	−3	−2	−1	0	1	2	3
y	−4	−1	0	−1	−4	−9	−16
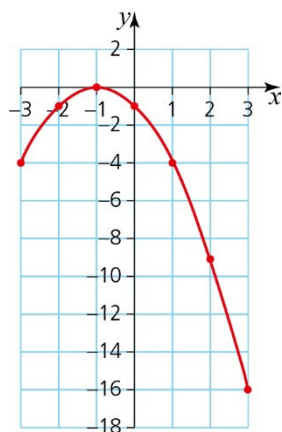
d

x	−5	−4	−3	−2	−1	0	1	2	3
y	8	1	−4	−7	−8	−7	−4	1	8
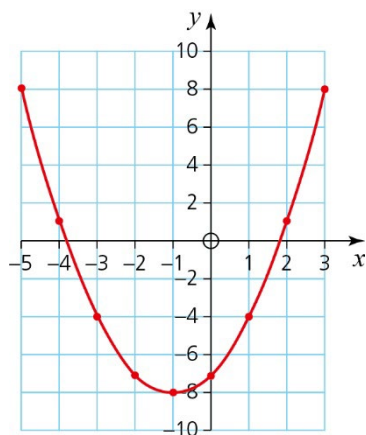
6 a

x	−7	−6	−5	−4	−3	−2
y	6	2	0	0	2	6
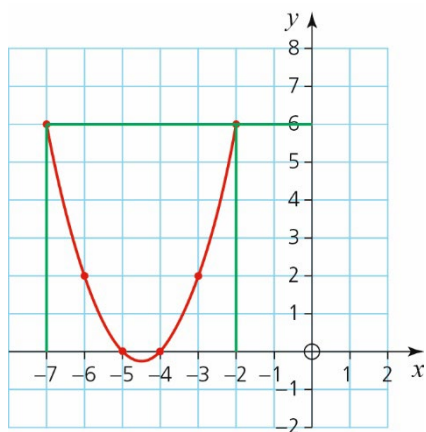
b
$$x^2 = -9x - 14$$

Rearranging: $\quad x^2 + 9x + 14 = 0$

Add 6 to both sides: $\quad x^2 + 9x + 20 = 6$

Read off from the graph where $y = 6$

So $x = -7$ or $x = -2$

7

x	−4	−3	−2	−1	−0.5	0	1	2	3	4
y	−0.25	−0.333	−0.5	−1	−2	2	1	0	0.333	0.25
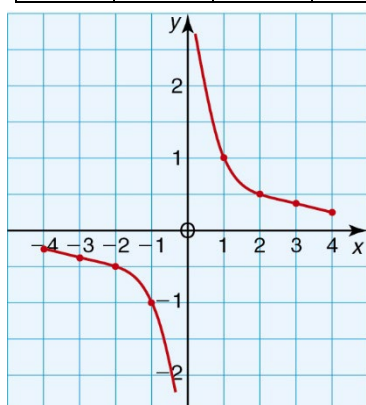
8 When $x = 0$, $y = -5$, so $(0, -5)$ is on the line.

When $y = 0$, $0 = \frac{1}{3}x - 5$. So $(15, 0)$ is on the line.

9
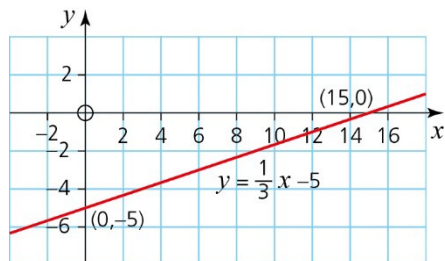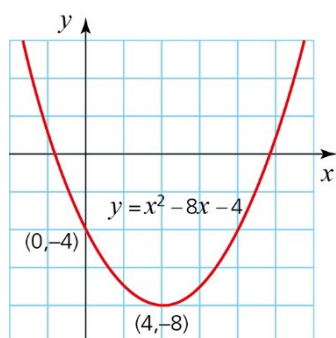
Student assessment 2 page 152

1

2 a

$y = 2x + 3$

b

$y = 4 - x$

c

$y = 2x - 3$

d

$y = \frac{3}{2}x + \frac{5}{2}$

3 a

$x = 3$ and $y = 3$

b

(8,0)
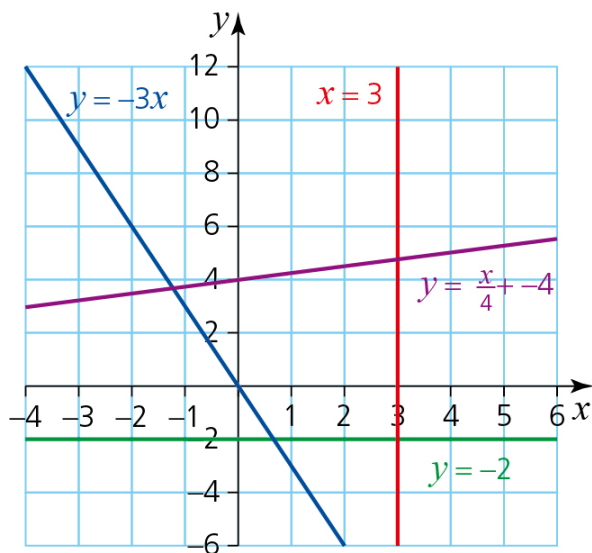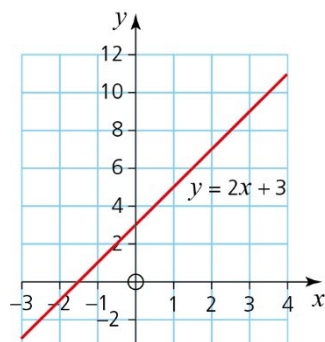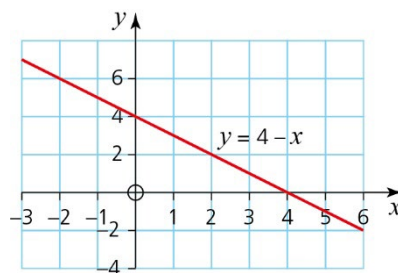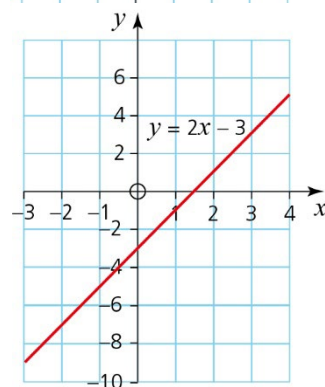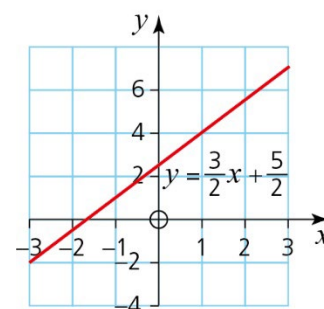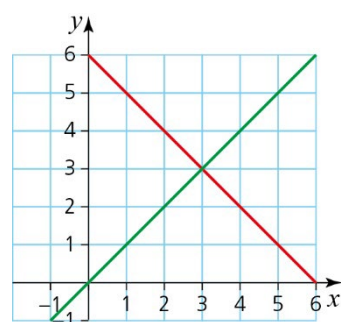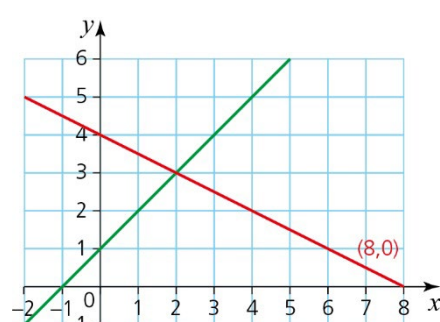
$x = 2$ and $y = 3$

c

$x = -3$ and $y = -1$

d
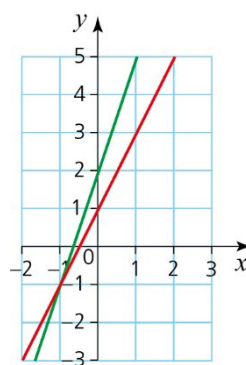
$x = -1$ and $y = -1$

4 a

x	−7	−6	−5	−4	−3	−2	−1	0	1	2
y	−12	−6	−2	0	0	−2	−6	−12	−20	−30

b
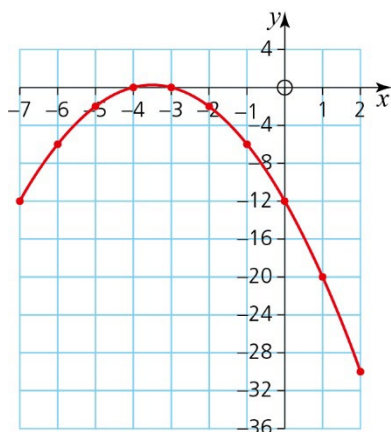
5 a

x	−4	−3	−2	−1	0	1	2	3	4
y	11	4	−1	−4	−5	−4	−1	4	11
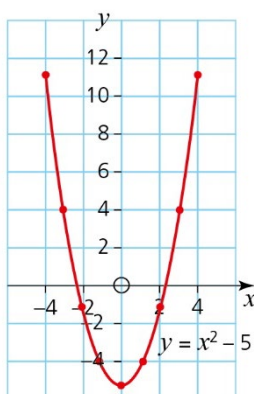
b

x	−4	−3	−2	−1	0	1	2	3	4
y	−15	−8	−3	0	1	0	−3	−8	−15

$y = 1 - x^2$
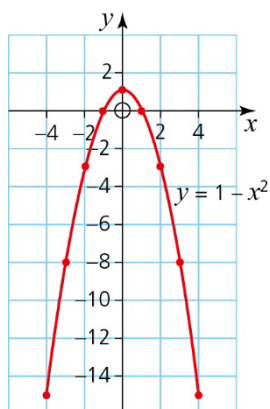
c

x	−3	−2	−1	0	1	2	3	4	5	6
y	8	0	−6	−10	−12	−12	−10	−6	0	8

$y = x^2 - 3x - 10$
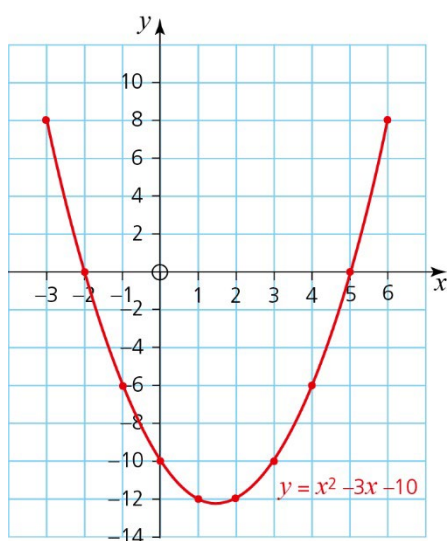
d

x	−5	−4	−3	−2	−1	0	1
y	−9	−4	−1	0	−1	−4	−9

$y = -x^2 - 4x - 4$
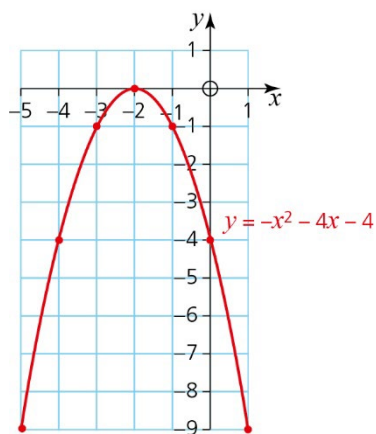
6 a

x	−6	−5	−4	−3	−2	−1	0	1	2	3	4
y	−15	−5	3	9	13	15	15	13	9	5	−5
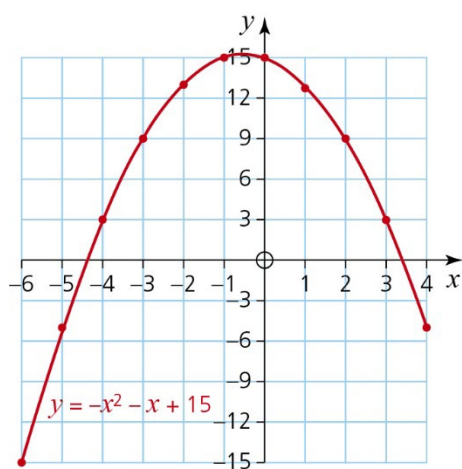
$y = -x^2 - x + 15$

b i $10 = x^2 + x$

Rearranging: $-x^2 - x + 10 = 0$

Add 5 to both sides: $-x^2 - x + 15 = 5$

Read off from the graph where $y = 5$

So $x = -3.7$ or $x = 2.7$

ii $x^2 = -x + 5$

Rearranging: $-x^2 - x + 5 = 0$

Add 10 to both sides: $-x^2 - x + 15 = 10$

Read off from the graph where $y = 10$

So $x = -2.8$ or $x = 1.8$ (both to 1 d.p.)
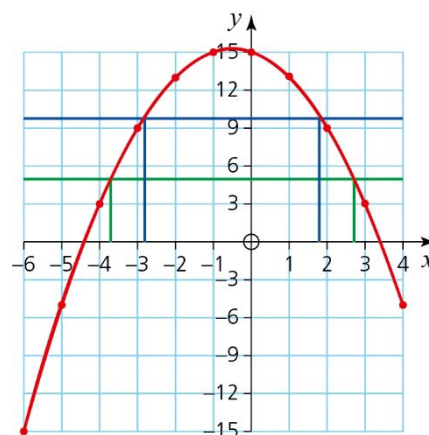
7

x	−4	−3	−2	−1	−0.5	0.5	1	2	3	4
y	−0.5	−0.666	−1	−2	−4	4	2	1	0.666	0.5
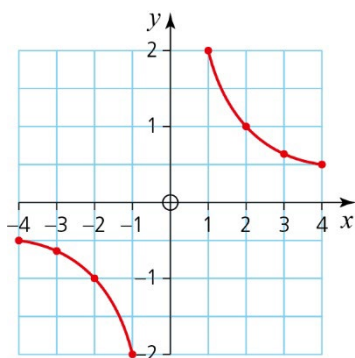
8 When $x = 0$, $y = 10$, so $(0, 10)$ is on the line.

When $y = 0$, $0 = -\dfrac{5}{2}x + 10$. So $(4, 0)$ is on the line.
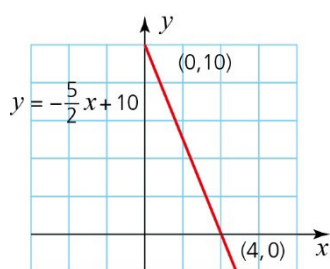
9 When $y = 0$, either $2x - 2 = 0$ or $x - 7 = 0$.

If $2x - 2 = 0$ then $x = 1$, so $(1, 0)$ is on the curve.

If $x - 7 = 0$ then $x = 7$, so $(7, 0)$ is on the curve.

When $x = 0$, $y = -2(0)^2 + 16(0) - 14 = -14$.
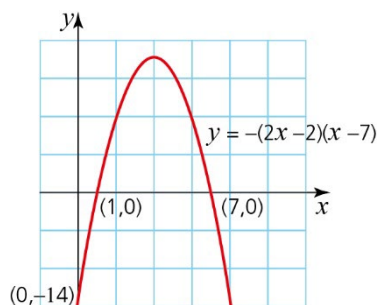
Mathematical investigations and ICT 2

House of cards

1 155

2 8475

3 The formula can be proved using the method of differences explained in Chapter 13 as follows:

Height of house	1	2	3	4	5
Number of cards	2	7	15	26	40
1st difference		5	8	11	14	
2nd difference			3	3	3	

Comparing with the algebraic table below:

Position	1	2	3	4	5
Term	$a + b + c$	$4a + 2b + c$	$9a + 3b + c$	$16a + 4b + c$	$25a + 5b + c$
1st difference		$3a + b$	$5a + b$	$7a + b$	$9a + b$	
2nd difference			$2a$	$2a$	$2a$	

It can be deduced that:

$2a = 3$ therefore $a = \dfrac{3}{2}$

$3a + b = 5$ therefore $b = \dfrac{1}{2}$

$a + b + c = 2$ therefore $c = 0$

This produces the rule $c = \dfrac{3}{2}n^2 + \dfrac{1}{2}n$

which factorises to $c = \dfrac{1}{2}n(3n + 1)$

Chequered boards

1   Student's own diagrams and results, presented in a logical table.

2   Where either m or n is even, the number of black and white squares is given by $\dfrac{mn}{2}$.

Where both m and n are odd, the numbers of black and white squares differ by one. The number of black squares is $\dfrac{mn-1}{2}$ and the number of white squares is $\dfrac{mn+1}{2}$, assuming that the bottom right-hand corner is white.

Modelling: Stretching a spring

1
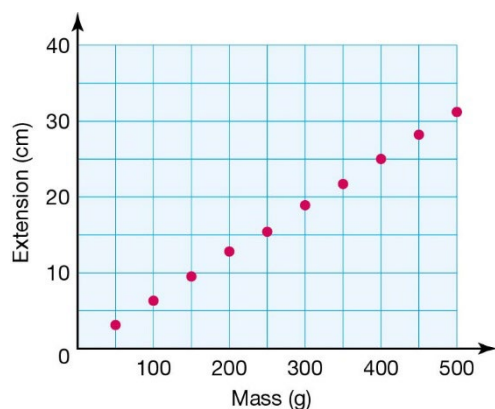
2   Linear

3
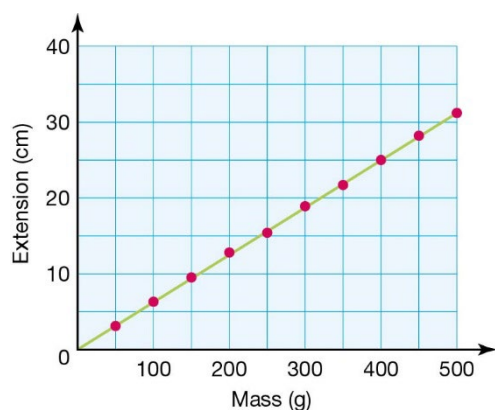
4   $y \approx 0.06x$

5   16.5 cm

6   The spring is likely to snap (or exceed its elastic limit).

ICT activity 1

1 **a** (2, 2)

   b (1, 2)

   c $(1\frac{1}{2}, 3)$

   d (2, 4)

   e (4, 1)

2 Student's solutions to the simultaneous equations, giving the same answers as in Q.1 above.

ICT activity 2

a $x = -2$ and $x = 1$

b $x = 1$ and $x = 6$

c $x = -4$ and $x = 3$

d $x = -3$ and $x = 0.5$

e $x = 1$

f $x = -2$

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

17 Coordinates and straight line graphs

Exercise 17.1

1
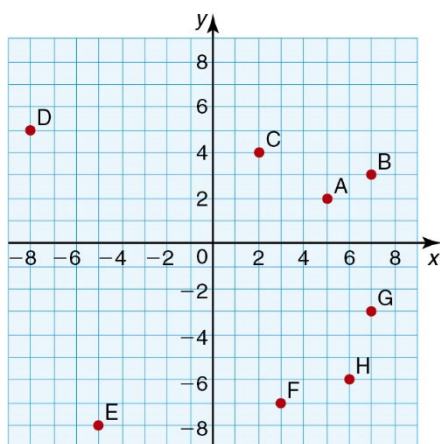
2
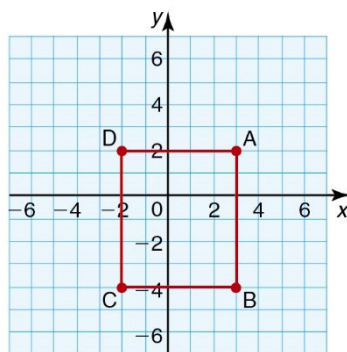
Rectangle

3
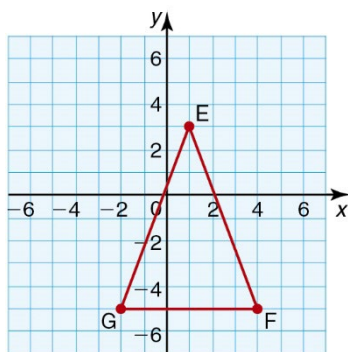
Isosceles triangle

4
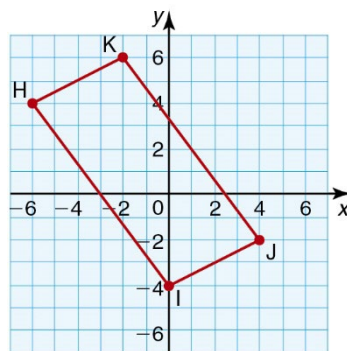
Parallelogram

Exercise 17.2
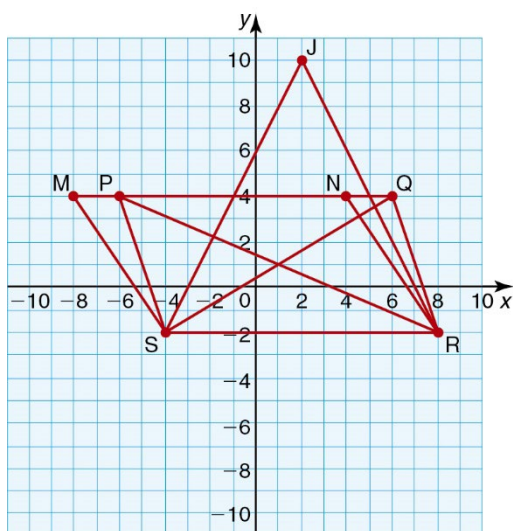
1 **a** (1, 1)

    b 72 units^2

2 **a** Parallelogram

    b 72 units^2

    c The base length and perpendicular height of MNRS are the same as the base length and perpendicular height of PQRS, so their areas are the same.

3 2

Exercise 17.3

1 **a**
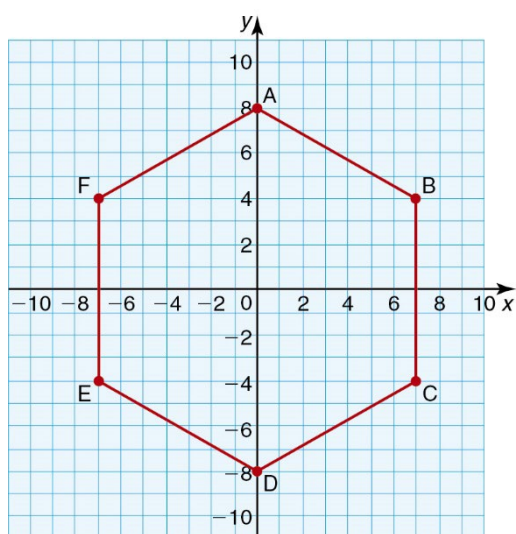
    b C(7, –4), D(0, –8), E(–7, –4), F(–7, 4)

2 a
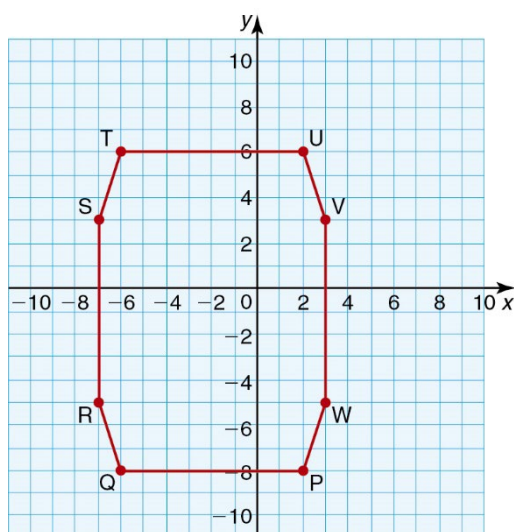
b   S(–7, 3), T(–6, 6), U(2, 6), V(3, 3), W(3, –5)

c   (–2, –1)

Exercise 17.4

1   B at 1.5, C at 2.4, D at 4.8

2   I at 4.4, J at 5.2, K at 5.9, L at 6.3, M at 6.8

3   F at 1.8, G at 3, H at 3.5

4   Q at 2.4, R at 4.6, S at 5.8, T at 6.4, U at 7.8, V at 8.8

Exercise 17.5

A(1, 1.5); B(1.2, –1.5); C(–0.9, –1.6); D(–1.8, 0.7); E(1, 1.8); F(3, –2.4); G(–3.6, –1.6); H(–1.6, 3.6)

Exercise 17.6

1 a 1   **b** $\frac{3}{2}$ **c** –2   **d** $-\frac{1}{4}$ **e** 0   **f** undefined

2   A horizontal line has a zero gradient.

3   A vertical line has an undefined gradient as you cannot divide by 0.

4   Gradient of A = 2

Gradient of B = 0

Gradient of C = –3

Gradient of D = $-\frac{1}{2}$

Gradient of E = $\frac{1}{2}$

Gradient of F is undefined.

Exercise 17.7

a $y = 7$ **b** $y = 2$ **c** $x = 7$ **d** $x = 3$

e $y = x$ **f** $y = \frac{1}{2}x$ **g** $y = -x$ **h** $y = -2x$

Exercise 17.8

1 a $y = x + 1$ **b** $y = x + 3$ **c** $y = x - 2$

  d $y = 2x + 2$ **e** $y = \frac{1}{2}x + 5$ **f** $y = \frac{1}{2}x - 1$

2 a $y = -x + 4$ **b** $y = -x - 2$ **c** $y = -2x - 2$

  d $y = -\frac{1}{2}x + 3$ **e** $y = -\frac{3}{2}x + 2$ **f** $y = -4x + 1$

3 a For graphs in question 1:

  1 **b** 1 **c** 1 **d** 2 **e** $\frac{1}{2}$ **f** $\frac{1}{2}$

  For graphs in question 2:

  −1 **b** −1 **c** −2 **d** $-\frac{1}{2}$ **e** $-\frac{3}{2}$ **f** −4

  b The gradient is equal to the coefficient of x.

  c The constant being added/subtracted indicates where the line intersects the y-axis.

4 a Student's own diagram.

  b Only the intercept c is different.

5 The lines are parallel.

Exercise 17.9

1 a $m = 2,\ c = 1$ **b** $m = 3,\ c = 5$ **c** $m = 1,\ c = -2$

  d $m = \frac{1}{2},\ c = 4$ **e** $m = -3,\ c = 6$ **f** $m = -\frac{2}{3},\ c = 1$

  g $m = -1,\ c = 0$ **h** $m = -1,\ c = -2$ **i** $m = -2,\ c = 2$

Exercise 17.10

1 Any line with a gradient of 1

2 **a**, **b** and **d** are parallel to $y = -x + 6$.

Exercise 17.11

1 $y = 4x$

2 a $y = -3x + 4$ **b** $y = -3x - 2$ **c** $y = -3x - \frac{7}{2}$

3 a $y = \frac{1}{2}x + 3$ **b** $y = \frac{1}{2}x - \frac{1}{4}$

Student assessment 1

1 P(37.5, 25)   Q(25, −37.5)   R(−37.5, −7.5)   S(−42.5, 45)

2 **a** Student's own points; gradient = 2

   b Student's own points; gradient = −1

3 **a** $.y = 2x − 3$

   b $y = \frac{3}{2}x + \frac{5}{2}$

4 **a** $m = \frac{1}{2}$ and $c = 0$

   b $m = 4$ and $c = 6$

   c $m = -\frac{3}{2}$ and $c = \frac{5}{2}$

5 $y = 5x$

Mathematical investigations and ICT 3

Plane trails

1 Student's own investigation

2 Student's ordered table similar to the one shown.

Number of planes (p)	Maximum number of crossing points (n)
1	0
2	1
3	3
4	6
5	10
…	…

3 The sequence of the number of crossing points is the sequence of triangular numbers.

$$n = \frac{1}{2}p(p-1)$$

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

18   Geometrical vocabulary

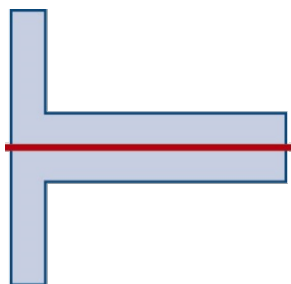## Exercise 18.1

1

right angle   acute   obtuse   reflex
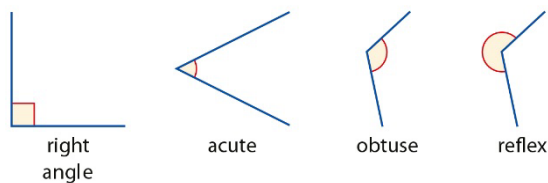
2 a acute   **b** right angle   **c** obtuse   **d** reflex

   e acute   **f** obtuse   **g** reflex   **h** right angle

3 a complementary   **b** supplementary

   c complementary   **d** neither

   e complementary   **f** supplementary

   g neither   **h** neither

   i complementary   **j** supplementary

Exercise 18.2

a a = BAC

b b = ABC

c c = ECD d = CDE e = CED

d f = IFG g = FGH h = GHI i = FIH

Exercise 18.3

a RQ and PU, ST and PU

b QT and AB, AT and BP, BT and AR

Exercise 18.4

1 Student's own diagrams

2
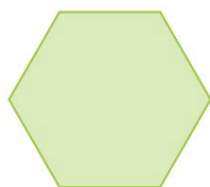
regular
hexagon
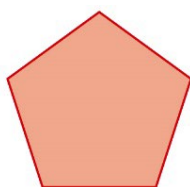
regular
pentagon
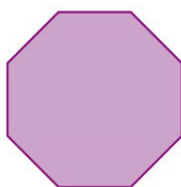
regular
octagon

Exercise 18.5

1 **a** The interior angles are the same, i.e. 60°, 30° and 90°.

   b $\frac{5}{8}$

   c $x = 6.25$, $y = 3.75$

2 A, C and F are similar. B and D are similar.

3 **a** 6 cm

   b 9 cm

4 p: 4.8 cm; q: 4.5 cm; r: 7.5 cm

5 e: 10 cm, f: $2\frac{2}{3}$ cm

Exercise 18.6

1 P and Q

2 10°

3 (2, 5)

4 (10, 5), (9, 5), (10, –3) or (9, –3)

Student assessment 1

1 **a** acute   **b** obtuse   **c** reflex   **d** right angle

2 **a**, **b**, **c**
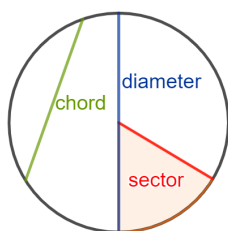
3 Congruent shapes are exactly the same shape and size.
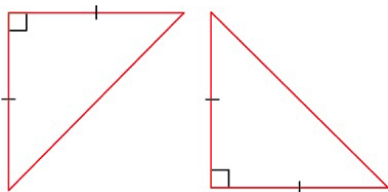
4 The missing vertex could be at (3, 4), (3, 1), (9, 4) or (9, 1).

Student assessment 2

1 Student's own diagram

2 The triangles are congruent and right-angled

3 **a** CMNA or ABML

   b CLM, NMB or ANL.

4 **a** True

   b True

   c False

   d False

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

19 Geometrical constructions and scale drawings

Exercise 19.1

1 **a** 60 mm   **b** 20 mm   **c** 83 mm

    d 91 mm   **e** 58 mm   **f** 98 mm

2 Student's own lines

Exercise 19.2

1 **a** 47°   **b** 24°   **c** 100°

    d 150°   **e** 110°   **f** 158°

2 **a** $a = 90°$ $b = 166°$ $c = 104°$

    b $d = 24°$ $e = 55°$ $f = 125°$ $g = 156°$

    c $h = 44°$ $i = 316°$

    d $j = 35°$ $k = 85°$ $l = 240°$

    e $m = 30°$ $n = 140°$ $o = 60°$ $p = 330°$

    f $q = 112°$ $r = 339°$ $s = 336°$ $t = 44°$

3 Student's own angles

Exercise 19.3

a Student's own construction

b Student's own construction

c Student's own construction

d **i** Student's own construction attempt

    ii It is not possible as AC + BC < AB

Exercise 19.4

1 **a** 300 m   **b** 250 m   **c** 300 m   **d** 416 m

2 **a** 10 cm   **b** 8 cm   **c** 6 cm   **d** 6.8 cm

3 **a** Student's own construction

    b 41.2 m

4 **a** Student's own construction

    b 24 cm^2

5
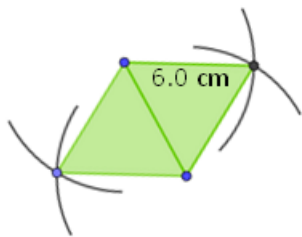
Exercise 19.5

1 a i 2 cm   **ii** 4cm   **iii** 2 cm

  b i B

    ii The two flaps at the top will overlap when folded.

  c Student's different net of the cuboid.

2
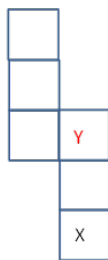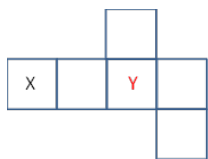
3 a Square-based pyramid

  b Student's construction

4 Several different nets are possible. One possibility is given below. Students should leave construction lines.
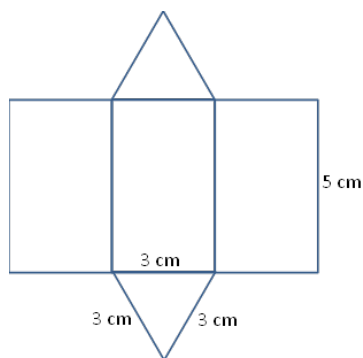
Student assessment 1

1 a The line is 7.2 cm

  b Student's own drawing; 4.7 cm long

2 a 145°

b $360° - 300° = 60°$, student's own drawing

3
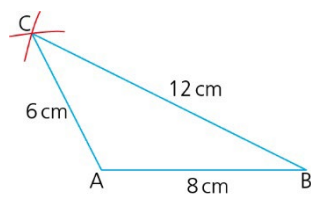
4   a
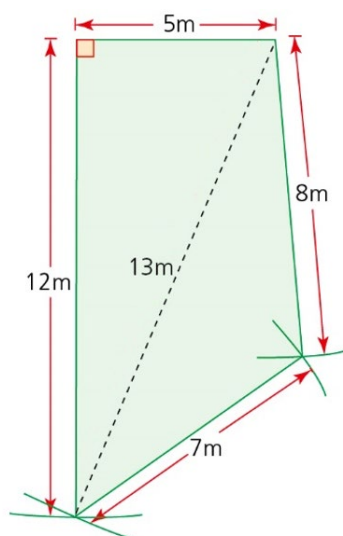
b   Area is approximately $53\,\text{m}^2$

5   47°, 130°, 60°, 25°, 278°

6
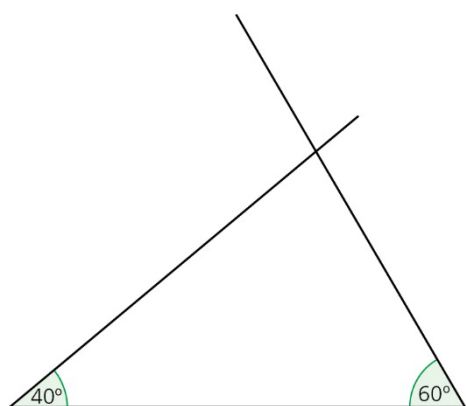
7   a   22 cm

b   24.4 cm

8   Student's own diagram

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

20 Symmetry

Exercise 20.1

1 a   b   c

d   e   f

g   h   i
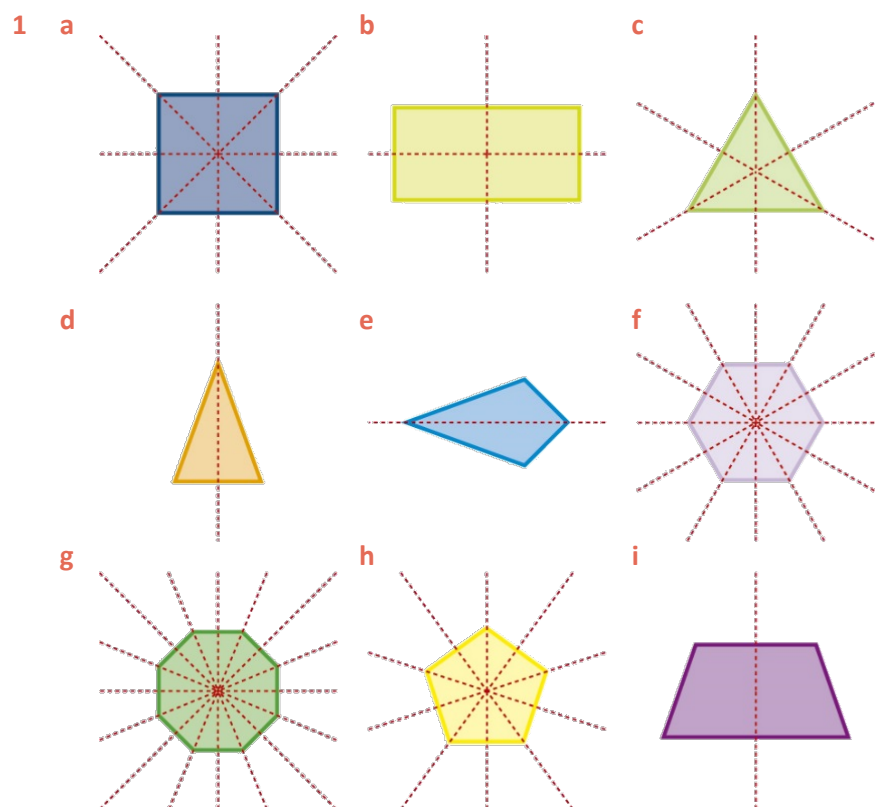
j   It is not possible to show all the lines of symmetry as there is an infinite number.

2 a   b   c

d   e   f   g
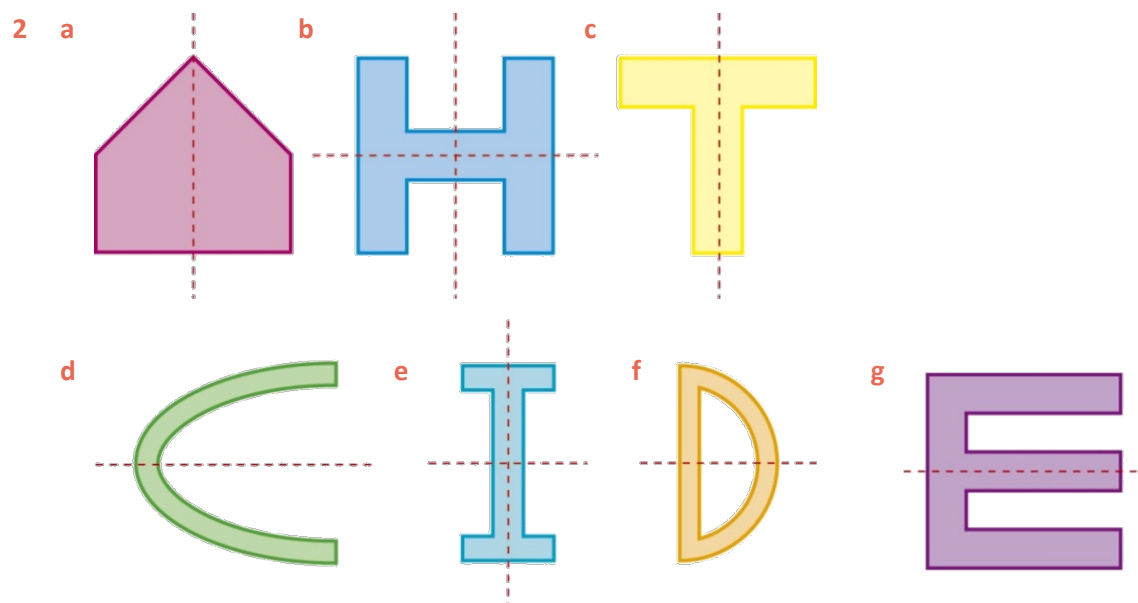
h It is not possible to show all the lines of symmetry as there is an infinite number.

3 a

b
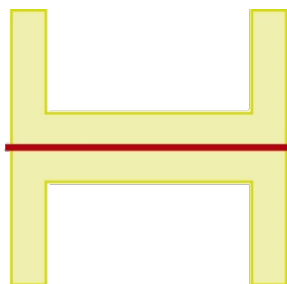
c
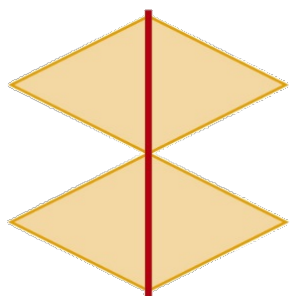
d
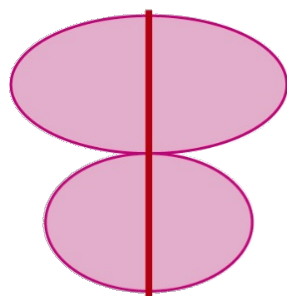
e
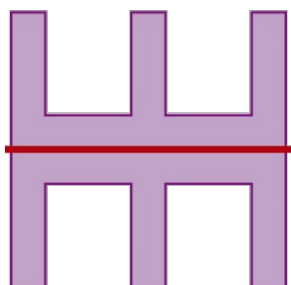
f
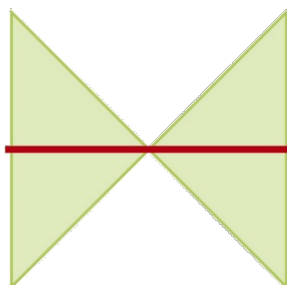
4 a
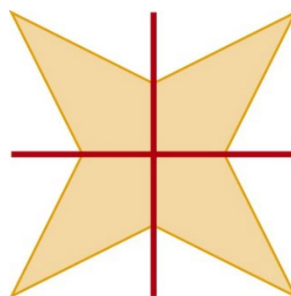
b
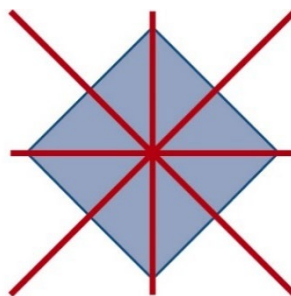
c
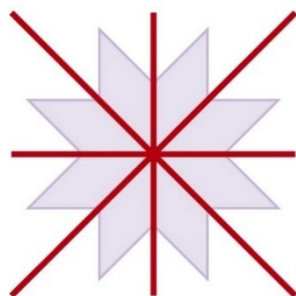
d
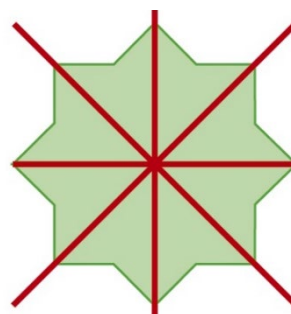
Exercise 20.2
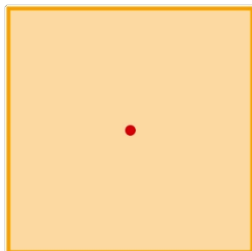
1 a
order 4
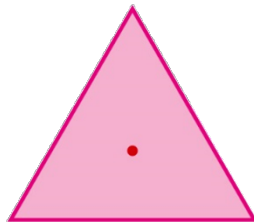
b
order 3
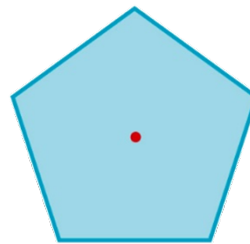
c
order 5
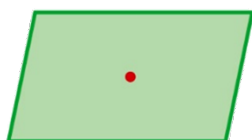
d
order 2

e
order 2
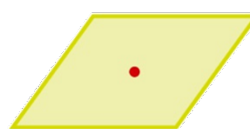
f
order 2
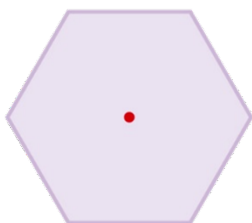
g
order 6
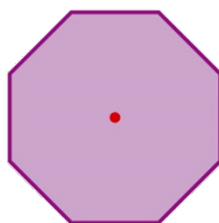
h
order 8
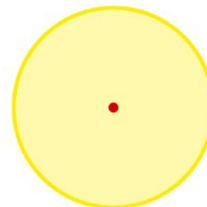
i
order infinite
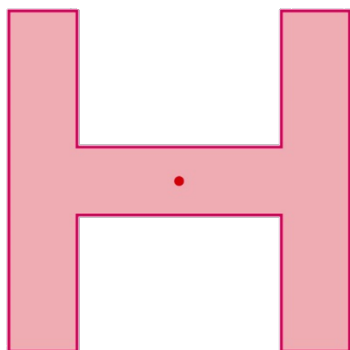
2 a
order 2
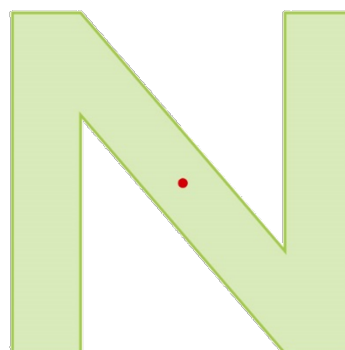
b
order 2

c
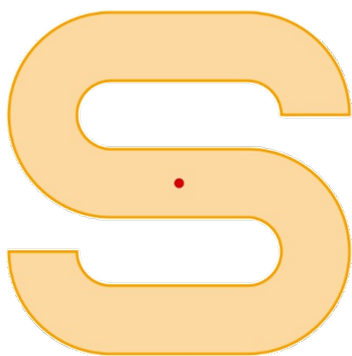
order 2

d
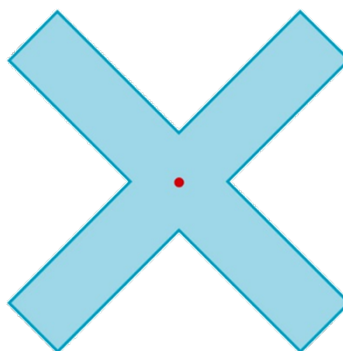
order 4

e
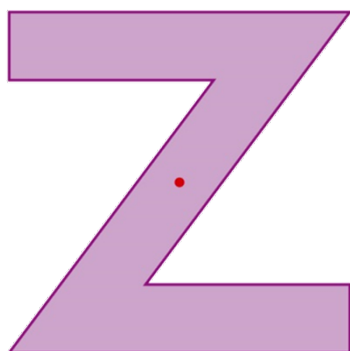
order 2

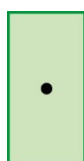## Student assessment 1

1 Other answers possible.
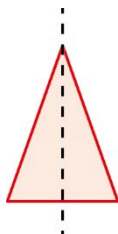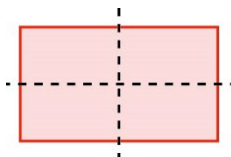
   a   **b**   **c**
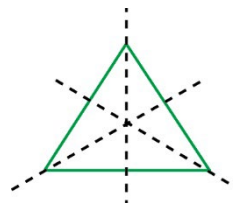
2 **Other answers possible.**

   a A rectangle   **b** An equilateral triangle
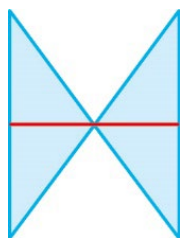
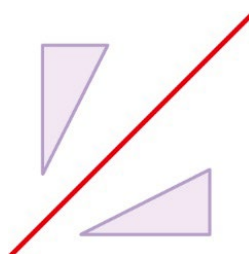**3** **a**   **b**
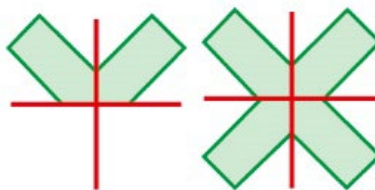
   c   **d**

4 **a** The shape has rotational symmetry of order 2.

   b The shape has rotational symmetry of order 4.

   c The shape has rotational symmetry of order 4.

   d The shape has no rotational symmetry.

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

21   Angle properties

Exercise 21.1

1 **a** 75°   **b** 96°   **c** 96°   **d** 146°

2 **a** 111°   **b** 37°   **c** 22°   **d** 62°

3 **a** 40°   **b** 45°   **c** 42°   **d** 30°

Exercise 21.2

1 Student's own diagram and measurements; finding $a° = b°$ and $c° = d°$

2 Student's own diagram and measurements; finding $p° = q°$ and $r° = s°$

3 Student's own diagram and measurements; finding $w° = x°$ and $y° = z°$

4 Student's own observations. For example, angles opposite each other are the same size.

Exercise 21.3

1 Student's own diagram and measurements; finding $p° = r°$ and $q° = s°$

2 Student's own diagram and measurements; finding $l° = n°$ and $m° = o°$

3 Student's own diagram and measurements; finding $w° = y°$ and $x° = z°$

4 Student's own observations. For example, pairs of equal angles are formed.

Exercise 21.4

Other reasons are possible.

a $p° = 54°$; angles on a straight line sum to 180°

  $q° = 63°$; angles on a straight line sum to 180°

  $r° = 54°$; vertically opposite angles are equal

b $a° = 55°$; angles on a straight line sum to 180°

  $b° = 80°$; angles on a straight line sum to 180°

  $c° = 100°$; angles on a straight line sum to 180°

c $v° = 120°$; angles on a straight line sum to 180°

  $w° = 60°$; vertically opposite angles are equal

  $x° = 120°$; corresponding angles are equal

  $y° = 60°$; corresponding angles are equal

  $z° = 60°$; alternate angles are equal

d $a° = 50°$; corresponding angles are equal

$b° = 130°$; angles on a straight line sum to $180°$

$c° = 45°$; corresponding angles are equal

$d° = 135°$; angles on a straight line sum to $180°$

e $p° = 45°$; alternate angles are equal

$q° = 135°$; angles on a straight line sum to $180°$

$r° = 45°$; corresponding angles are equal

$s° = 45°$; alternate angles are equal

$t° = 135°$; alternate angles are equal

f $d° = 70°$; alternate angles are equal

$e° = 30°$; alternate angles are equal

g $a° = 37°$; angles rounda point sum to $360°$

h $a° = 36°$; co-interior angles sum to $180°$

Exercise 21.5

1   **a**   Obtuse-angled isosceles triangle

b   Right-angled isosceles triangle

c   Obtuse-angled scalene triangle

d   Acute-angled scalene triangle

e   Right-angled scalene triangle

f   Equilateral, therefore acute-angled, triangle

2   Student's own diagrams

Exercise 21.6

1   **a**   Student's own diagrams and measurements

b   The angles have a sum of $180°$

2   **a**   Student's arrangement of angles a, b and c

b   The nagles lie along a straight line

Exercise 21.7

1   **a** $70°$ **b** $55°$ **c** $60°$ **d** $73°$ **e** $45°$ **f** $110°$

2   **a** $a° = 30°$ $b° = 45°$

b $x° = 50°$ $y° = 80°$ $z° = 70°$

c $p° = 130°$ $q° = 15°$ $r° = 60°$

d $d° = 35°$ $e° = 55°$ $f° = 55°$

e $a° = 27.5°$ $b° = 27.5°$ $c° = 55°$ $d° = 27.5°$ $e° = 97.5°$

f $p° = 45°$ $q° = 45°$ $r° = 67.5°$ $s° = 112.5°$

Exercise 21.8

1 **a** Parallelogram **b** Isosceles trapezium **c** Trapezium

    d Quadrilateral **e** Kit**e** **f** Rhombus

2

	Rectangle	**Square**	**Parallelogram**	**Kite**	**Rhombus**	**Equilateral triangle**
Opposite sides equal in length	Yes	Yes	Yes	No	Yes	–
All sides equal in length	No	Yes	No	No	Yes	Yes
All angles right angles	Yes	Yes	No	No	No	No
Both pairs of opposite sides parallel	Yes	Yes	Yes	No	Yes	–
Diagonals equal in length	Yes	Yes	No	No	No	–
Diagonals intersect at right angles	No	Yes	No	Yes	Yes	–
All angles equal	Yes	Yes	No	No	No	Yes

Exercise 21.9

a $a° = 115°$

b $x° = 40°$ $y° = 140°$ $z° = 140°$

c $m° = 75°$ $n° = 75°$

d $s° = 65°$ $t° = 115°$ $u° = 115°$

e $h° = 120°$ $i° = 60°$ $j° = 120°$ $k° = 60°$

f $a° = 80°$ $b° = 20°$ $c° = 20°$ $d° = 20°$ $e° = 140°$

g $p° = 40°$ $q° = 130°$ $r° = 50°$

h $p° = 75°$ $q° = 30°$ $r° = 50°$ $s° = 80°$

    $t° = 70°$ $u° = 70°$ $v° = 40°$

Exercise 21.10

1 **a** 720° **b** 1260° **c** 900°

2 **a** 135° **b** 90° **c** 144° **d** 150°

3 **a** 72° **b** 30° **c** 51.4° (1 d.p.)

4 **a** 18   **b** 10   **c** 36
    d 8   **e** 20   **f** 120

5 **a** 5   **b** 12   **c** 20
    d 15   **e** 40   **f** 360

6 12

7

Number of sides	Name	Sum of exterior angles	Size of an exterior angle	Sum of interior angles	Size of an interior angle
3	Equilateral triangle	360°	120°	180°	60°
4	Square	360°	90°	360°	90°
5	Pentagon	360°	72°	540°	108°
6	Hexagon	360°	60°	720°	120°
7	Heptagon	360°	51.4°	900°	128.6°
8	Octagon	360°	45°	1080°	135°
9	Nonagon	360°	40°	1260°	140°
10	Decagon	360°	36°	1440°	144°
12	Dodecagon	360°	30°	1800°	150°

6 The number of exterior angles must be an integer value, therefore the size of the exterior angles must be a factor of 360°. Kwasi is definitely wrong as 50 is not a factor of 360.

Exercise 21.11

a 60°   **b** 135°   **c** 20°   **d** 32°   **e** 110°   **f** 22.5°

Exercise 21.12

1 **a** 60°   **b** 40°   **c** 24°

Student assessment 1

1 **a** $a° = 80°$ $b° = 100°$
    b $x° = 65°$ $y° = 65°$ $z° = 20°$
    c $a° = 45°$

    d $p° = 73°$

2 **a** $p° = 135°$ $q° = 135°$ $r° = 45°$

    b $a° = 120°$ $b° = 60°$ $c° = 60°$ $d° = 60°$

3 **a** $m° = 50°$ $n° = 90°$ $p° = 40°$ $q° = 140°$

    b $w° = 55°$ $x° = 70°$ $y° = 55°$ $z° = 55°$

    c $a° = 70°$ $b° = 110°$ $c° = 110°$ $d° = 70°$ $e° = 30°$

    d $a° = 30°$ $b° = 120°$ $c° = 80°$ $d° = 80°$ $e° = 80°$

Student assessment 2

1
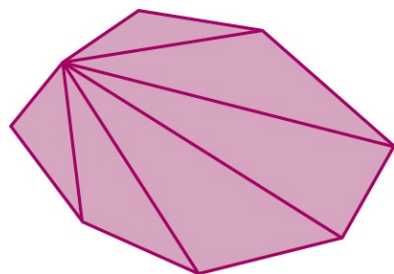
The number of triangles is 2 less than the number of sides.

An octagon has 8 sides, so there are $8 - 2 = 6$ triangles.

Since the angles in a triangle add up to 180°, the sum of the interior angles in an octagon is $(8 - 2) × 180°$ or $180 × (8 - 2)°$ as required.

2 162°

3 1260°

4 360°

5 72°

6 **a**   Angle ACB = 90°   **b**   Radius = 6.5 cm

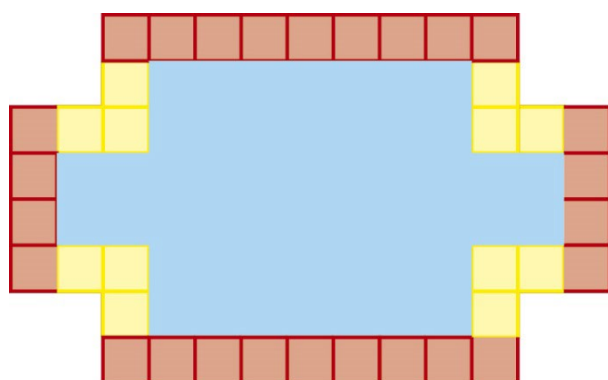**7** $x° = 58°$

8 $x° = 30°$

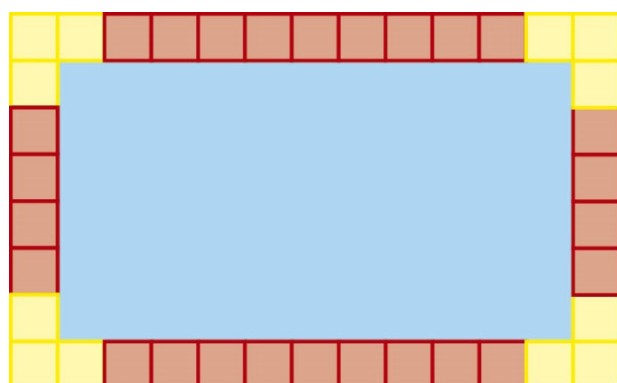**9** $x° = 25°$

10 $x° = 152°$

Mathematical investigations and ICT 4

Fountain borders

1 Student's results

2 $T = 2(m + n + 2)$

3 There are many ways to prove the algebraic rule, for example:
The original pool considered:

has the same number of tiles as a rectangular pool of dimensions 11×6 units:

In the diagram below it can be seen that the number of tiles along the length and width of the pool is twice the length and width. This leaves the four tiles needed for the corners.
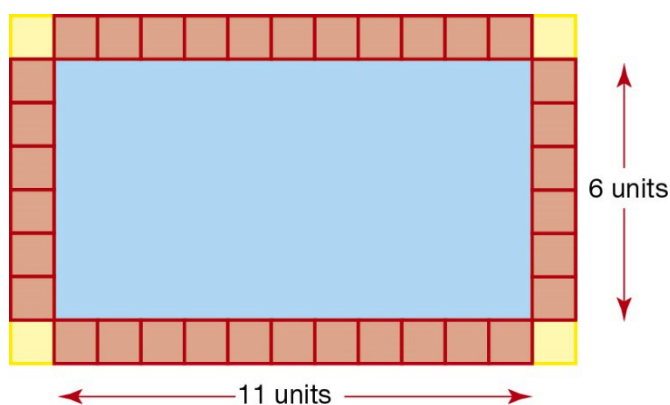
Hence $T = 2m + 2n + 4$ which factorises to $T = 2(m + n + 2)$.

Tiled walls

1 Student's diagrams

2 Student's ordered table of results

3 $c = (l - 1)(w - 1)$

4 $t = 2(l - 1) + 2(w - 1)$

ICT activity page 224

1

	A	B	C	D	E	F
1			Regular Polygons			
2	Number of sides	Name	Sum of exterior angles	Size of an exterior angle	Size of an interior angle	Sum of interior angles
3	3	Equilateral triangle	360	120	60	180
4	4	Square	360	90	90	360
5	5	Pentagon	360	72	108	540
6	6	Hexagon	360	60	120	720
7	7	Heptagon	360	51.428571	128.571429	900
8	8	Octagon	360	45	135	1080
9	9	Nonagon	360	40	140	1260
10	10	Decagon	360	36	144	1440
11	12	Dodecagon	360	30	150	1800
12	20	Icosagon	360	18	162	3240
13						
14		Formulae in row 3	=360	=C3/A3	=180-D3	=E3*A3

2 Size of an exterior angle $= \dfrac{360}{n}$

Size of an interior angle $= 180 - \dfrac{360}{n}$

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

22 Measures

Exercise 22.1

1 a 100 b 1000 c $\frac{1}{1000}$ d 1000

   e 1000 f $\frac{1}{1000}$ g tonne h 1 milligram

   i 1 litre j millilitre

2 a m, cm b cm, mm c g d ml

   e cm, m f tonnes g litres h km

   i tonnes j litres

3 Student's own lines

4 Students draw, estimate and measure the length of their lines A, B, C and D

5 a m b km c m d kg e l

   f m g tonnes h m i g j l

Exercise 22.2

1 a mm b mm c km d m e mm

   f m g mm h m i km j m

2 a 20 b 85 c 230 d 1200 e 830

   f 50 g 625 h 87 i 4

3 a 3000 b 4.7 c 5.6 d 6400 e 800

   f 0.96 g 0.625 h 87 i 4

4 a 5 b 6.3 c 1.15 d 2.535 e 250

   f 0.5 g 0.07 h 0.008 i 1000 j 700 000

Exercise 22.3

1 a 3800 b 28.5 c 4280 d 0.32 e 500

2 1.35 kg

3 a 3530 kg b 3.77 g c 9.3 kg d 5.15 kg e 14 kg

Exercise 22.4

1 a 8.4 b 0.65 c 87.5 d 0.05 e 2.5

2 a 3200 b 750 c 87

	d	8000	**e**	8	**f**	300		
3	**a**	4500 ml	**b**	1530 ml	**c**	7050 ml	**d**	1000 ml
4	**a**	1.2 litres	**b**	1.34 litres	**c**	1.4 litres	**d**	1.4 litres
5	a	1200 cm³	b	500 litres	c	4250 cm³	d	0.22 m³
6		360 litres						

Exercise 22.5

1 **a** $100\,000 \text{ cm}^2$

   b $2\,000\,000 \text{ mm}^2$

   c $5\,000\,000 \text{ m}^2$

   d $3\,200\,000 \text{ m}^2$

   e 830 mm^2

2 **a** 0.05 m^2

   b 150 cm^2

   c 0.001 km^2

   d 0.04 m^2

   e $0.000\,25 \text{ km}^2$

3 **a** $2\,500\,000 \text{ cm}^3$

   b 3400 mm^3

   c $2\,000\,000\,000 \text{ m}^3$

   d $200\,000 \text{ cm}^3$

   e $30\,000\,000 \text{ mm}^3$

4 **a** 0.15 m^3

   b 24 cm^3

   c $0.000\,85 \text{ km}^3$

   d 0.3 cm^3

   e $0.000\,015 \text{ m}^3$

5 **a** 1200 cm^3

   b 500 litres

   c 4250 cm^3

   d 0.22 m^3

6 360 litres

Student assessment 1

1	**a**	26 mm	**b**	625 mm	**c**	88 cm	**d**	7 mm
	e	4.8 m	**f**	7810 m	**g**	6.8 km	**h**	875 m

	i	2000 mm	**j**	85 mm				
2	**a**	4200 mg	**b**	0.75 g	**c**	3.94 kg	**d**	4100 g
	e	720 kg	**f**	4.1 tonnes	**g**	6.28 kg	**h**	830 000 g
	i	47 000 tonnes	**j**	1 kg = 1 million mg				
3	8330 kg							
4	**a**	1.8 litres	**b**	3200 ml	**c**	83 ml	**d**	250 litres
5	**a**	5600 mm^2	**b**	20 500 cm^2				
6	**a**	0.008 67 m^3	**b**	0.444 m^3				

Student assessment 2

1	**a**	47 mm	**b**	3 mm	**c**	310 cm	**d**	6400 m
	e	49 km	**f**	4000 mm	**g**	4 mm	**h**	34 mm
	i	46 cm	**j**	50 km				
2	**a**	0.0036 g	**b**	0.55 g	**c**	6.5 kg	**d**	6700 g
	e	370 kg	**f**	1.51 tonnes	**g**	380 tonnes	**h**	77 g
	i	6 kg	**j**	2 million mg				
3	430 kg							
4	**a**	3.4 litres	**b**	6700 ml	**c**	730 ml	**d**	300 litres
5	**a**	30 000 mm^2	**b**	5000 m^2				
6	**a**	0.1004 m^3	**b**	0.000 005 005 km^3				

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

23   Perimeter, area and volume

Exercise 23.1

a $A = 24$ cm^2 $P = 20$ cm

b $A = 45$ cm^2 $P = 28$ cm

c $A = 27$ cm^2 $P = 21$ cm

d $A = 38$ m^2 $P = 27.6$ m

e $A = 21$ m^2 $P = 18.4$ m

f $A = 22.5$ cm^2 $P = 19.5$ cm

g $A = 15.04$ cm^2 $P = 15.8$ cm

h $A = 1.0285$ m^2 $P = 37.51$ m

i $A = 1.02$ m^2 $P = 4.1$ m

j $A = 2.475$ m^2 $P = 8.1$ m

Exercise 23.2

1   a $A = 61.2$ b $b = 10$ c $l = 16$ d $b = 10$

   e $l = 12$ f $b = 15$ g $l = 25$

2   a   Area = 60 m^2 Perimeter = 31 m

   b   Area = 875 mm^2 Perimeter = 120 mm

   c   Area = 20.5 m^2 Perimeter = 21.4 m

   d   Area = 11 050 m^2 Perimeter = 430 m

   e   Area = 2025 cm^2 Perimeter = 180 cm

   f   Area = 875 cm^2 Perimeter = 120 cm

   g   Area = 2.8 km^2 Perimeter = 8.6 km

   h   Area = 0.03 km^2 Perimeter = 2.45 km

   i   Area = 2925 m^2 Perimeter = 220 m

   j   Area = 15 625 mm^2 Perimeter = 500 mm

Exercise 23.3

a   6 cm^2 b   32.5 cm^2 c   60 cm^2

d   20 cm^2 e   132 mm^2

Exercise 23.4

a 90 cm² **b** 104 cm² **c** 64 cm²

d 1168 mm² **e** 300 cm² **f** 937.5 mm²

Exercise 23.5

a 58.5 cm² **b** 84 cm² **c** 157.5 cm²

Exercise 23.6

1 4

2 3

3 23.5 m²

4 **a** 16 m², 24 m² **b** 100 m² **c** 15

Exercise 23.7

1 **a** 25.13 cm **b** 21.99 cm **c** 28.90 cm **d** 1.57 m

2 **a** 50.27 cm² **b** 38.48 cm² **c** 66.48 cm² **d** 0.20 m²

3 **a** 2.39 cm (3 s.f.) **b** 0.5 cm (exact) **c** 0.637 m (3 s.f.) **d** 1.27 mm (3 s.f.)

4 **a** 4.51 cm (3 s.f.) **b** 6 cm (exact) **c** 3.23 m (3 s.f.) **d** 4.31 mm (3 s.f.)

Exercise 23.8

1 **a** 1.57 m (3 s.f.) **b** 637 times (3 s.f.)

2 188.5 m (1 d.p.)

3 84π mm²

4 $(144 - 36\pi)$ cm²

5 **a** $(32 + 8\pi)$ m **b** D128 + 16π) m²

Exercise 23.9

1 **a** 460 cm² **b** 208 cm² **c** 147.78 cm² **d** 33.52 cm²

2 **a** 2 cm **b** 4 cm **c** 6 cm **d** 5 cm

3 **a** 100.5 cm² (1 d.p.) **b** 276.5 cm² (1 d.p.)

  c 279.3 cm² (1 d.p.) **d** 25.6 cm² (1 d.p.)

4 **a** 1.2 cm **b** 0.5 cm **c** 1.7 cm **d** 7.0 cm

Exercise 23.10

1 **a** 24 cm² **b** 2 cm

2   4.4 cm

3   **a**   216 cm^2 **b**   15.2 cm (1 d.p.)

4   **a**   30π cm^2 **b**   14 cm

Exercise 23.11

1   **a**   24 cm^3 **b**   18 cm^3 **c**   27.6 cm^3 **d**   8.82 cm^3

2   **a**   452 cm^3 (3 s.f.)   **b**   277 cm^3 (3 s.f.)

    c   196 cm^3 (3 s.f.)

    d   0.481 cm^3 (3 s.f.)

3   **a**   Volume = 48 cm^3 Total surface area = 108 cm^2

    b   Volume = 102.5 cm^3 Total surface area = 165.22 cm^2

4.   **a**   264 cm^2

    b   216 cm^3

5   **a**   70 cm^3 **b**   96 cm^3 **c**   380 cm^3 **d**   137.5 cm^3

Exercise 23.12

1   21.5% (1 d.p.)

2   **a**   42 cm^2 **b**   840 cm^3

3   6.3 cm

4   0.923π m^3 or 923 000π cm^3

Exercise 23.13

1   **a**   6.28 cm   **b**   2.09 cm   **c**   12.6 cm   **d**   23.6 cm

2   **a**   36°   **b**   90°   **c**   60°   **d**   40°

3   **a**   10.2 cm (3 s.f.)   **b**   38.2 cm (3 s.f.)

    c   18.6 cm (3 s.f.)   **d**   13.5 cm (3 s.f.)

Exercise 23.14

1   **a** $\left(34 + \dfrac{34}{9}\pi\right)$ cm   **b** $(48 + 18\pi)$ cm

2   **a**   12 cm   **b**   54 cm   **c**   47.7° (1 d.p.)

Exercise 23.15

1   **a**   33.5 cm^2 (3 s.f.)   **b**   205 cm^2 (3 s.f.)

    c   5.65 cm^2 (3 s.f.)   **d**   123 cm^2 (3 s.f.)

2 **a** 18.5 cm (3 s.f.) **b** 20.0 cm (3 s.f.)

  c 2.52 cm (3 s.f.) **d** 50.5 cm (3 s.f.)

3 **a** 20° **b** 36° **c** 45° **d** 24°

Exercise 23.16

1 $\left(96 - \frac{16}{3}\pi\right)$ m^2

2 **a** 118 cm^2 (3 s.f.) **b** 39.3 cm^2 (3 s.f.) **c** 8.66 cm (3 s.f.)

3 **a** 4.19 cm (3 s.f.) **b** 114 cm^2 (3 s.f.) **c** 62.8 cm^3 (3 s.f.)

Exercise 23.17

1 **a** 905 cm^3 (3 s.f.) **b** 3590 cm^3 (3 s.f.)

  c 2310 cm^3 (3 s.f.) **d** 1.44 cm^3 (3 s.f.)

2 **a** 3.14 cm **b** 5.56 cm **c** 36.3 cm **d** 0.620 cm

Exercise 23.18

1 6.30 cm

2 $\frac{331}{12}\pi$ cm^3

3 11.9 cm

4 48% (to nearest whole number)

Exercise 23.19

1 **a** 452 cm^2 (3 s.f.) **b** 254 cm^2 (3 s.f.)

  c 1890 cm^2 (3 s.f.) **d** 1.40 cm^2 (3 s.f.)

2 **a** 1.99 cm (3 s.f.) **b** 1.15 cm (3 s.f.)

  c 3.09 mm (3 s.f.) **d** 0.5 cm

3 1 : 4

4 707 cm^2 (3 s.f.)

Exercise 23.20

a 40 cm^3 **b** 133 cm^3 (3 s.f.) **c** 64 cm^3 **d** 70 cm^3

Exercise 23.21

1 6.93 cm^2 (3 s.f.)

2 189 cm^2 (3 s.f.)

3   73.3 cm² (3 s.f.)

Exercise 23.22

1   **a**   56.5 cm³ (3 s.f.)   **b**   264 cm³ (3 s.f.)   **c**   1.34 cm³ (3 s.f.)   **d**   166 cm³ (3 s.f.)

2   **a**   6.91 cm (3 s.f.)   **b**   10.9 cm (3 s.f.)   **c**   0.818 cm (3 s.f.)   **d**   51.3 cm (3 s.f.)

3   **a**   **i**   7.96 cm   **ii**   12.7 cm   **iii**   843 cm³

    b   **i**   15.9 cm   **ii**   8.41 cm   **iii**   2230 cm³

    c   **i**   6.37 cm   **ii**   3.97 cm   **iii**   168 cm³

    d   **i**   3.82 cm   **ii**   4.63 cm   **iii**   70.7 cm³

Exercise 23.23

1   3.88 cm (3 s.f.)

2   1700 cm³ (3 s.f.)

3 $\dfrac{625}{24}\pi$ cm³

4   771 cm³ (3 s.f.)

5   **a**   6 cm   **b** 25.1 cm³ (3 s.f.)   **c**   63

6   **a**   654 cm³   **b**   12.5 cm   **c**   2950 cm³

Exercise 23.24

1   **a** $132\,\boldsymbol{\pi}$ cm²   **b**   525π cm²

2   352π cm²

Student assessment 1

1   1.25 km²

2   **a**   1200   **b**   67 boxes needed, therefore cost = $2010

3
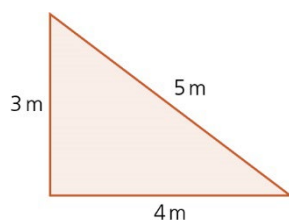
Area = 6 m²

Perimeter = 12 m

4   600 cm²

5   3600 cm²

Student assessment 2

1 **a** Circumference = 34.6 cm   Area = 95.0 cm^2

    b Circumference = 50.3 mm   Area = 201.1 mm^2

2 9.86 cm^2

3 **a** 39.3 cm^2 to 1 d.p.   **b** 34 cm^2 **c** 101.3 cm^2 to 1 d.p.

4 **a** 10.2 cm^2 **b** 283 cm^2 **c** 633cm^2

5 **a** 339 mm^3 to 3 s.f   **b** 9.82 cm^3 to 3 s.f

6 **a** 96 cm^2

    b 42 cm^3

Student assessment 3

1 95.5 cm^2 (3 s.f.)

2 **a** 603.2 cm^2 **b** 1072.3 cm^3

3 **a** $\frac{h}{4} = \frac{h+15}{12} \rightarrow 3h = h + 15 \rightarrow 2h = 15$ therefore $h = 7.5$

    b 3270 cm^3 to 3 s.f

4 **a** 38.2 cm to 3 s.f.   **b** 382 cm^2 to 3 s.f

    c 1150 cm^2 to 3 s.f.   **d** 3060 cm^3 to 3 s.f.

5 **a** 3620 cm^3 to 3 s.f   **b** 3620 cm^3 to 3 s.f

    c 905 cm^2 to 3 s.f   **d** 1920 cm^2 to 3 s.f.

Mathematical investigations and ICT 5

Metal trays

1 a length = 38 cm   width = 28 cm   height = 1 cm

  b 1064 cm^3

2 a length = 36 cm   width = 26 cm   height = 2 cm

  b 1872 cm^3

3 Student's investigation and ordered table of results similar to the one shown.

Square length (cm)	Tray dimensions (cm)			Volume (cm^3)
	Length	**Width**	**Height**	
1	38	28	1	1064
2	36	26	2	1872
3	34	24	3	2448
4	32	22	4	2816
5	30	20	5	3000
6	28	18	6	3024
7	26	16	7	2912
8	24	14	8	2688
9	22	12	9	2376
10	20	10	10	2000
11	18	8	11	1584
12	16	6	12	1152
13	14	4	13	728
14	12	2	14	336
15	10	0	15	0

4 x = 5.7 cm

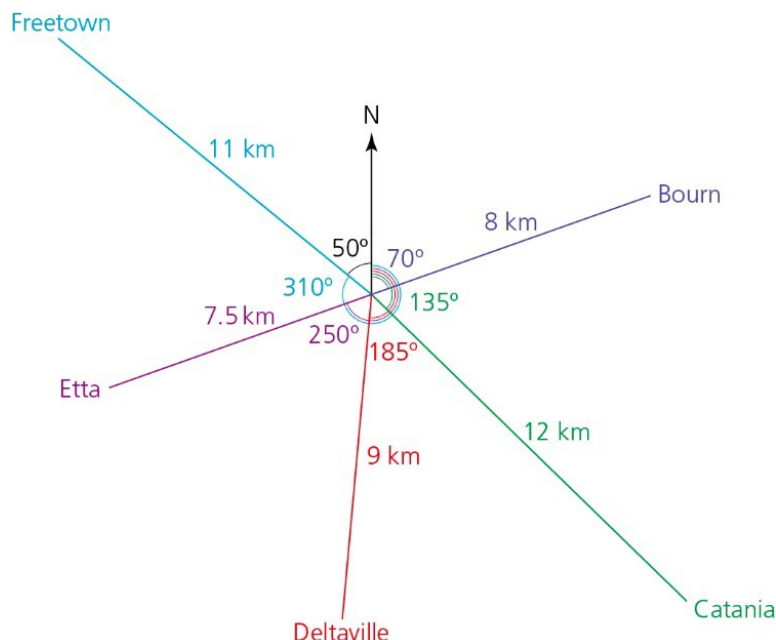**5** Maximum volume = 3032 cm^3

24 Bearings

Exercise 24.1

1 Student's own diagram

2 Student's own diagrams leading to:

    a 11.0 km, 035°   **b** 13.2 km, 300°   **c** 11.7 km, 061°

Exercise 24.2
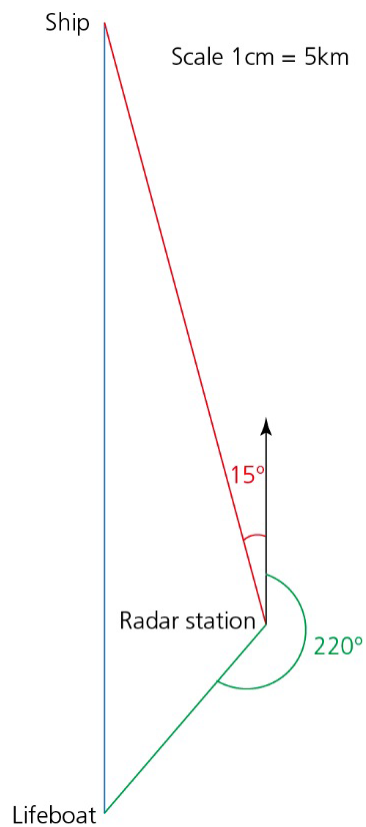
1 Student's own diagrams leading to:

    a 310°   **b** 325°   **c** 040°   **d** 020°

    e 332°   **f** 054°   **g** 343°   **h** 034°

2 Student's own diagrams leading to:

    a 120°   **b** 140°   **c** 110°   **d** 102°

Student assessment 1

1 **a** 14.3 km on a bearing of 035°

    b 16.6 km on a bearing of 291°

2 The ship is due North of the lifeboat and is 64 km away.
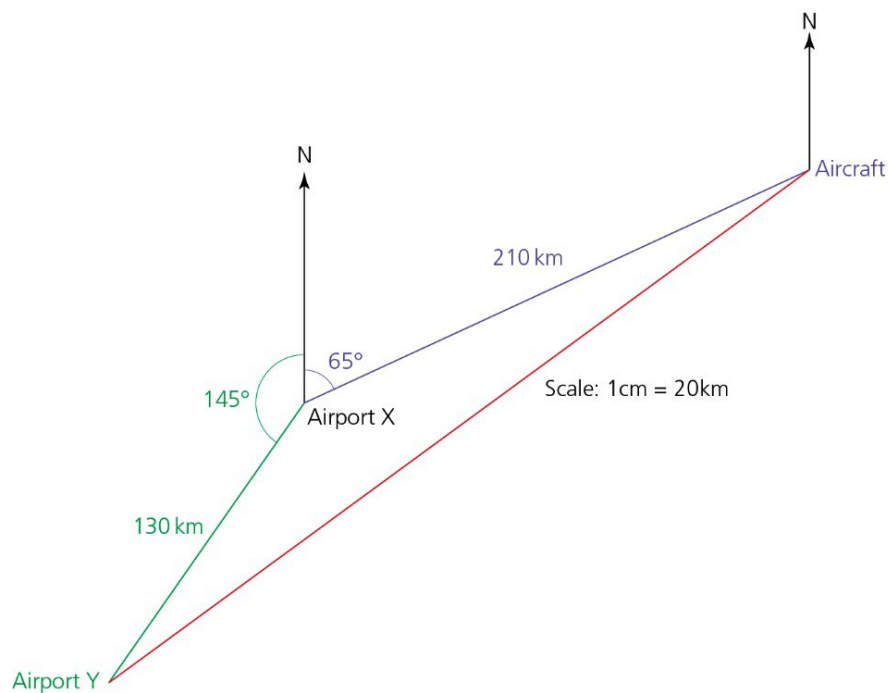
Ship

Scale 1cm = 5km

15°

Radar station

220°

Lifeboat

3 **a** 66 km, 047°

b 75 km, 313°

Meribel

Scale 1 cm = 5 km

N

35 km

40°

82°

30 km   Val d'Isere

155°

135°

40 km

45 km

Les Arcs

La Plagne
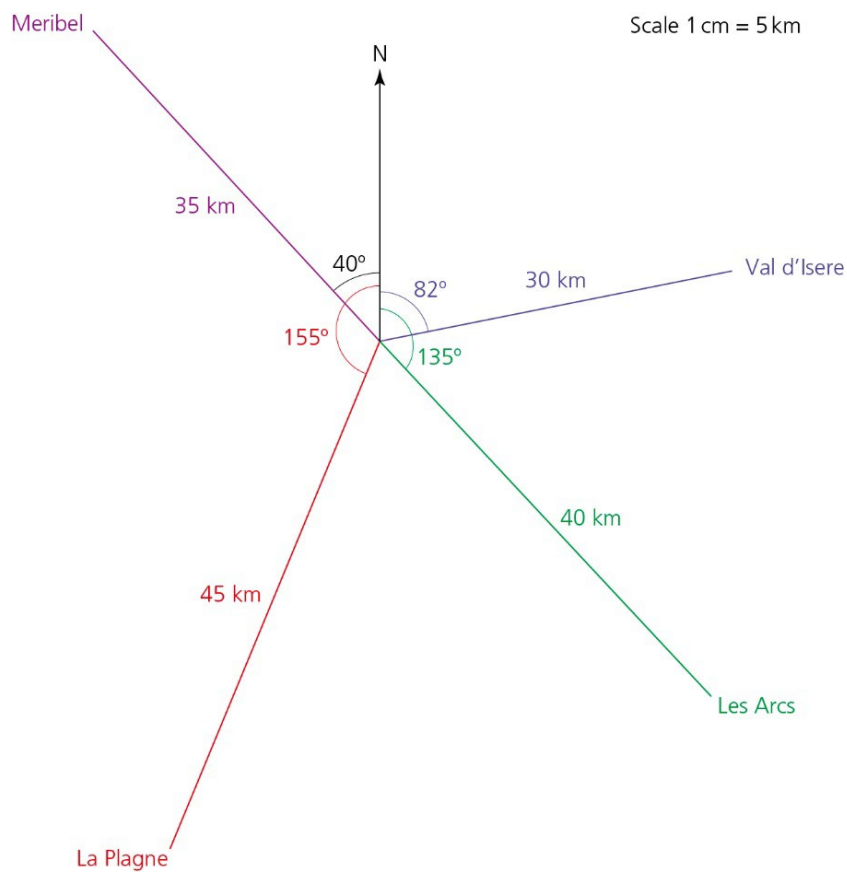
4 Airport Y is 329 km from the aircraft on a bearing of 234°.

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

25   Right-angled triangles

Exercise 25.1

1	**a** 1.8 cm	**b** 4.0 cm	**c** 19.2 cm		
	d 4.9 cm	**e** 37.3 cm	**f** 13.9 cm		
2	**a** 14.3 cm	**b** 9.0 cm	**c** 9.3 cm		
	d 4.1 cm	**e** 13.9 cm	**f** 6.2 cm		
3	**a** 49.4°	**b** 51.1°	**c** 51.3°		
	d 63.4°	**e** 50.4°	**f** 71.6°		

Exercise 25.2

1	**a** 2.4 cm	**b** 18.5 cm	**c** 6.2 cm		
	d 2.4 cm	**e** 43.8 cm	**f** 31.8 cm		
2	**a** 38.7°	**b** 48.6°	**c** 38.1°		
	d 49.8°	**e** 32.6°	**f** 14.5°		

Exercise 25.3

a 36.0 cm	**b** 15.1 cm	**c** 48.2°	**d** 81.1°
e 6.7 cm	**f** 16.8 cm	**g** 70.5°	**h** 2.1 cm

Exercise 25.4

1	**a** 5 cm	**b** 11.4 cm (1 d.p.)	
	c 12 cm	**d** 13.2 cm (1 d.p.)	
2	**a** 11.0 cm (1 d.p.)	**b** 14.8 cm (1 d.p.)	**c** 7.9 cm (1 d.p.)
	d 7.3 cm (1 d.p.)	**e** 3 cm	**f** 13.9 cm (1 d.p.)
3	71.6 km		
4	67 km		
5	**a** 225° − 135° = 90°	**b** 73.8 km	
6	57 km		
7	**a** 8.5 km	**b** 15.5 km (1 d.p.)	
8	15.0 m (1 d.p.)		

Exercise 25.5

1 **a** 43.6° (1 d.p.)   **b** 19.5 cm (1 d.p.)

    c 16.7 cm (1 d.p.)   **d** 42.5° (1 d.p.)

2 **a** **i** $25^2 + 60^2 = 65^2$, therefore a right-angled triangle

       ii 67.4° (1 d.p)

    b **i** $21^2 + 28^2 \neq 34^2$, therefore not a right-angled triangle

3 **a** 20.8 km (1 d.p.)   **b** 215.2° (1 d.p.)

4 **a** 228.4 km (1 d.p.)   **b** 101.7 km (1 d.p.)   **c** 103.2 km (1 d.p.)

    d 147.4 km (1 d.p.)   **e** 414.8 km (1 d.p.)   **f** 216.9° (1 d.p.)

5 15.3 m (1 d.p.)

6 **a** 48.2° (1 d.p.)   **b** 41.8° (1 d.p.)   **c** 8.0 cm (1 d.p.)

    d 8.9 cm (1 d.p.)   **e** 76.0 cm^2 (1 d.p.)

Student assessment 1

1 **a** 20 cm   **b** 32.5 cm   **c** 0.9 cm

    d 18 cm   **e** 13.0 cm to 1 d.p.

2 **a** 37°

    b 56°

    c 31°

    d 34°

3 **a** 5 cm   **b** 6.6 cm to 1 d.p.

    c 9.3 cm to 1 d.p.   **d** q = 28.5 cm to 1 d.p.

4 4.6 m

Student assessment 2

1 **a** AB = 160.8 km

    b BC = 177.5 km

2 l = 32   **e** $x°$ = 8.9° to 1 d.p.

3 **a** BY = 285 m to 3 s.f.

    b 117° to 3 s.f.

    c 297° to 3 s.f.

Mathematical investigations and ICT 6

Pythagoras and circles

1–3 Student's own drawings and calculations

4 Yes

5 Yes

6 Student's own investigation

Towers of Hanoi

1 3

2 15

3 Student's investigation

4 The results for up to 8 discs are given below:

Number of discs	1	2	3	4	5	6	7	8
Smallest number of moves	1	3	7	15	31	63	127	255

5 The numbers of moves are 1 less than the powers of 2.

6 1023

7 Number of moves = $2^n - 1$, where n = number of discs.

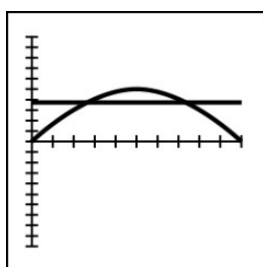**8** Time taken to move 64 discs is $2^{64} - 1$ seconds

This equates to 5.85×10^{11} years, i.e. 585 billion years.

Therefore, according to the legend, we needn't be too worried!
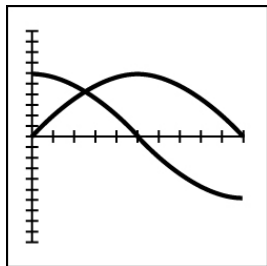
ICT activity

1 **b** 0.940 (3 d.p.)

c

The graph of $y = \sin x$ intersects the line $y = 0.7$ in two places as shown.
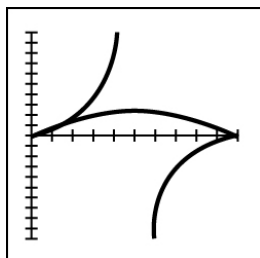
d $x = 30°$ and $x = 150°$

2　**a**

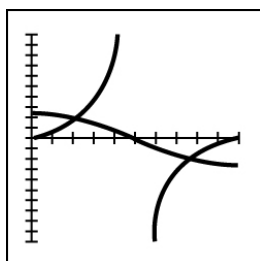　**b**　One solution

　c　$x = 45°$

3　**a**

　　$x = 0°$ and $x = 180°$

　b

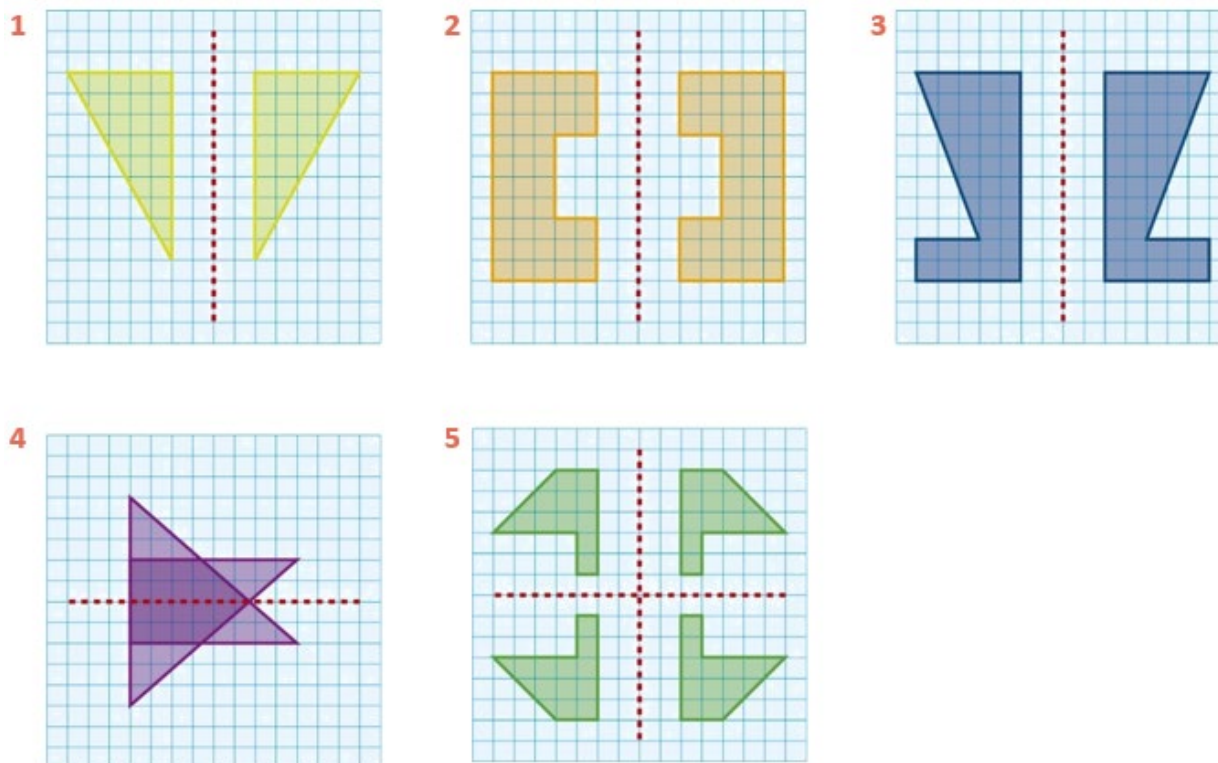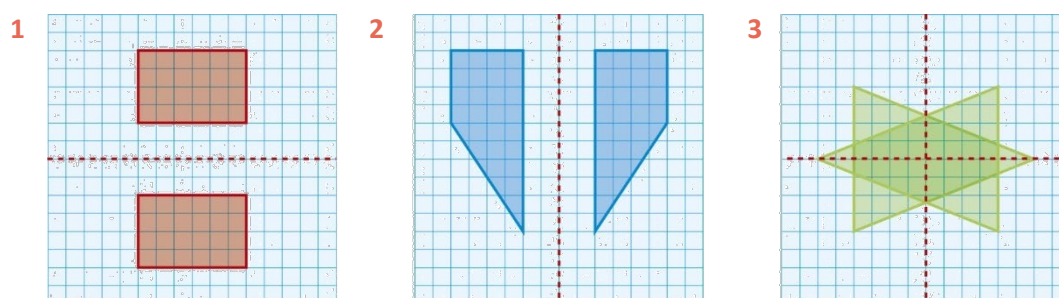　　$x = 38.2°$ and $x = 141.8°$ (1 d.p.)

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

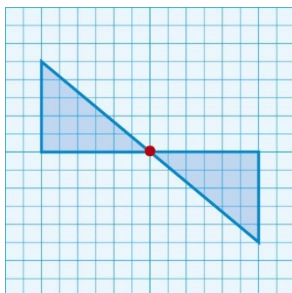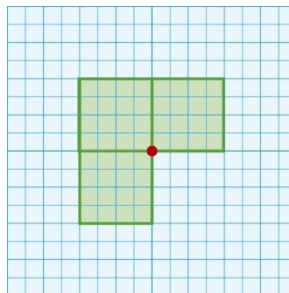# 26 Transformations

Exercise 26.1

Exercise 26.2

Exercise 26.3

1

2

3

4

5

6

Exercise 26.4

1 a

b 180°

2 a

b 90° anti-clockwise

3 a

b 90° anti-clockwise

4 a
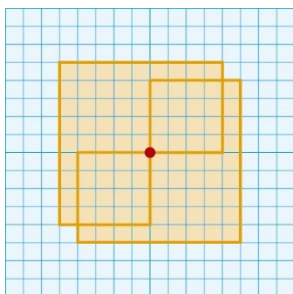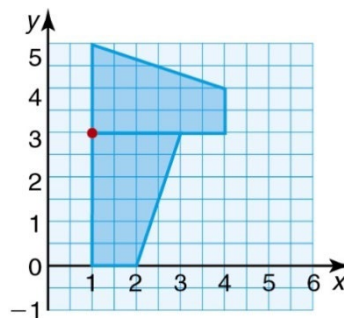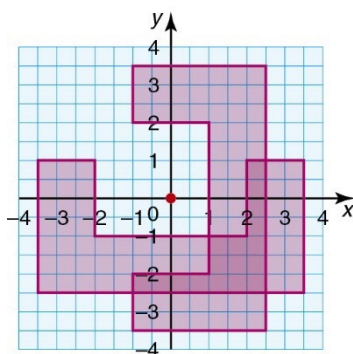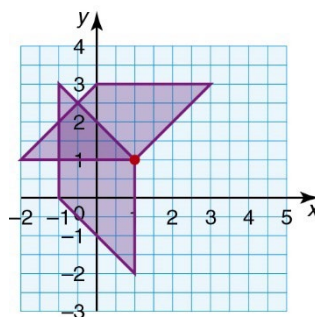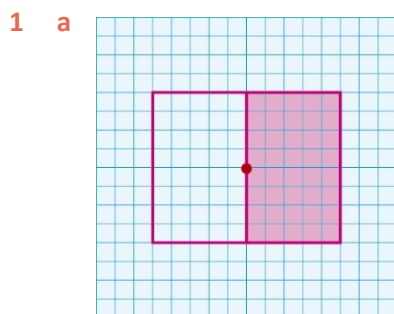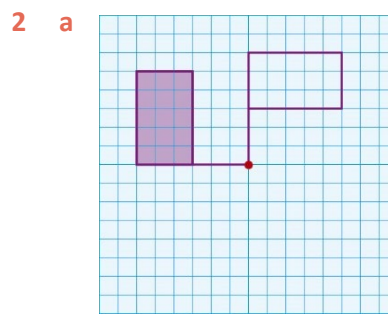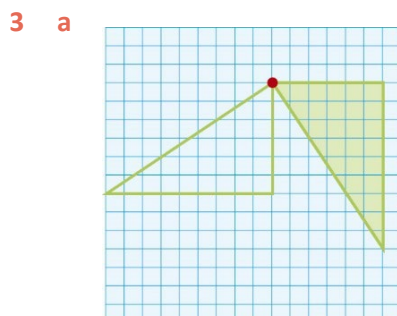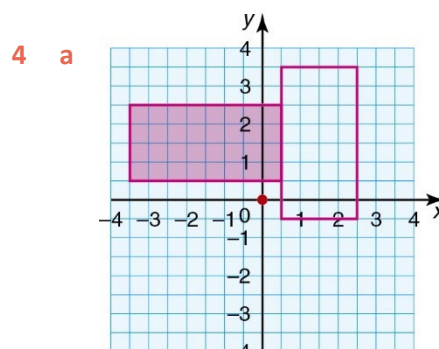
b 90° anti-clockwise about (0, 0)

5 a
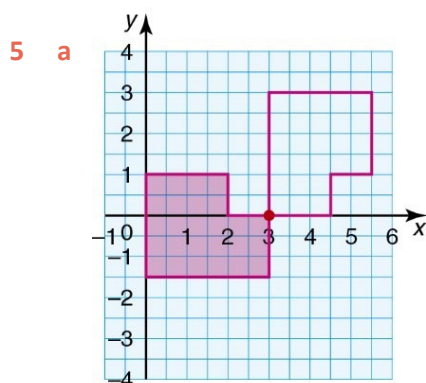
b 90° anti-clockwise about (3.75, −0.75)

6 a
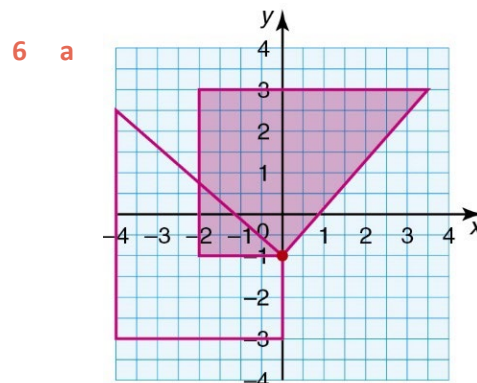
b 90° clockwise about (0, −1)

Exercise 26.5

1 $A \to B = \begin{pmatrix} -6 \\ 0 \end{pmatrix}$ $A \to C = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$

2 $A \to B = \begin{pmatrix} 0 \\ -7 \end{pmatrix}$ $A \to C = \begin{pmatrix} -6 \\ 1 \end{pmatrix}$

3 $A \to B = \begin{pmatrix} 0 \\ 6 \end{pmatrix}$ $A \to C = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$

4 $A \to B = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$ $A \to C = \begin{pmatrix} -3 \\ -6 \end{pmatrix}$

Exercise 26.6

1

2

3

4

5

6

Exercise 26.7

1

Scale factor is 2

2

Scale factor is 1.5

3

Scale factor is 2

4

Scale factor is 3
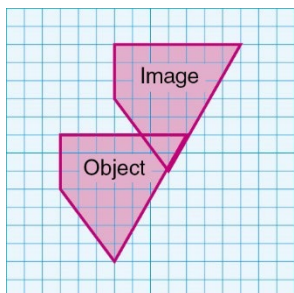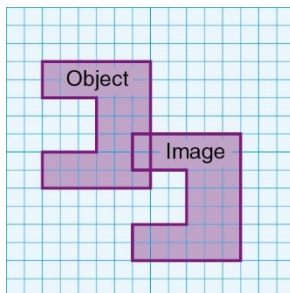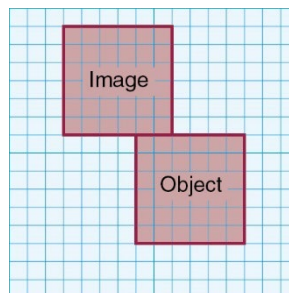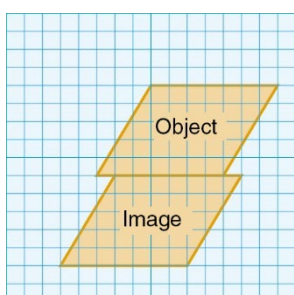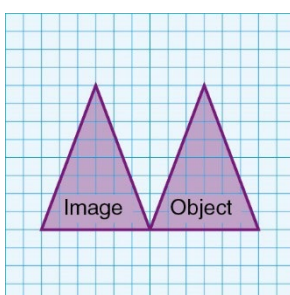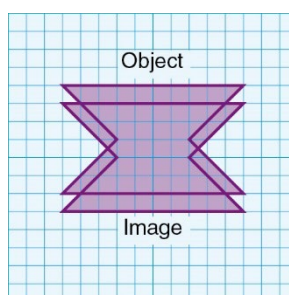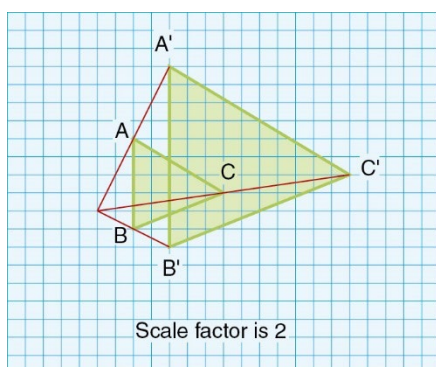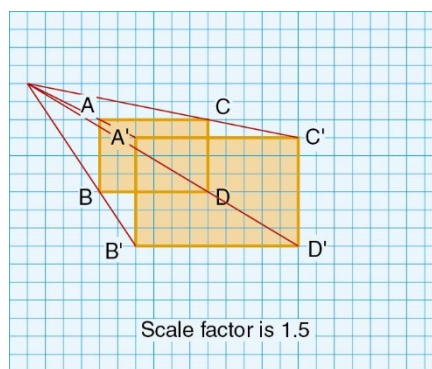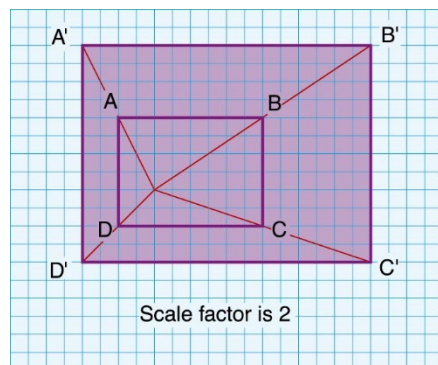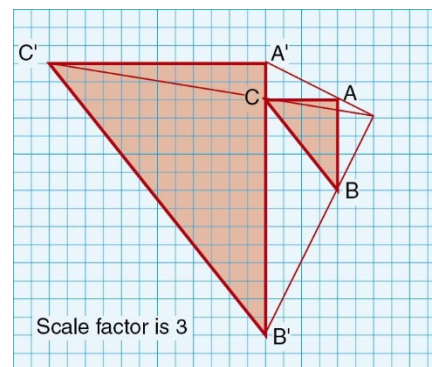
5
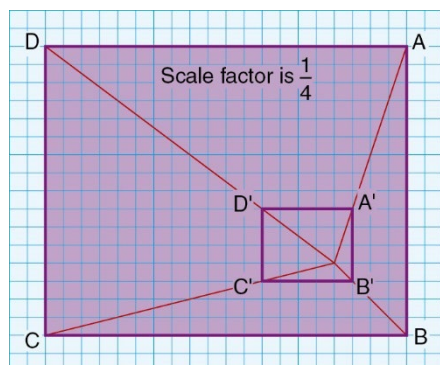
Exercise 26.8

1
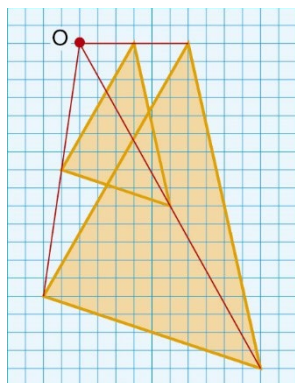
2

3

4
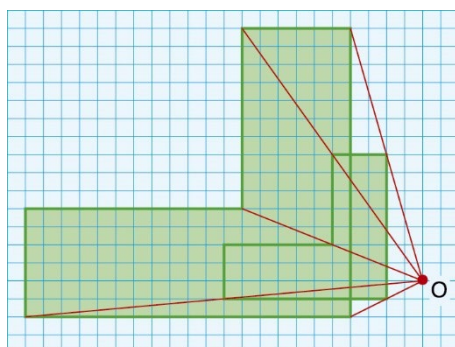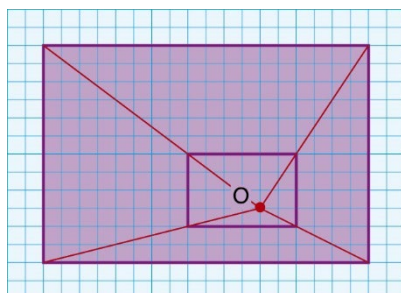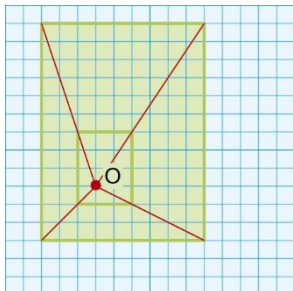
Student assessment 1

1
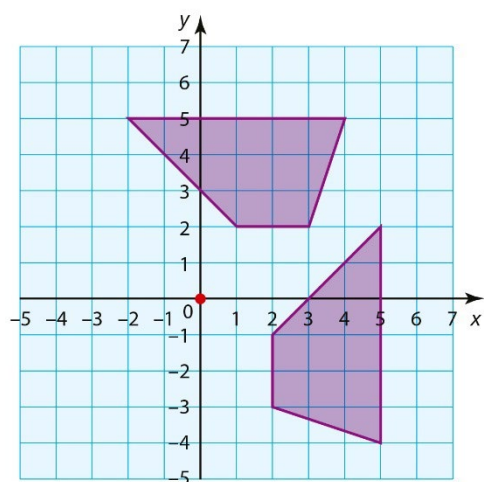
2 **a** $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$ **b** $\begin{pmatrix} -3 \\ -5 \end{pmatrix}$

3

4
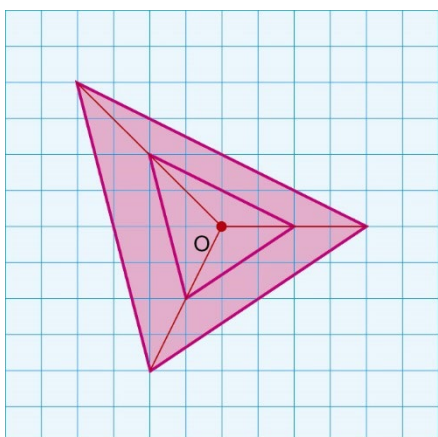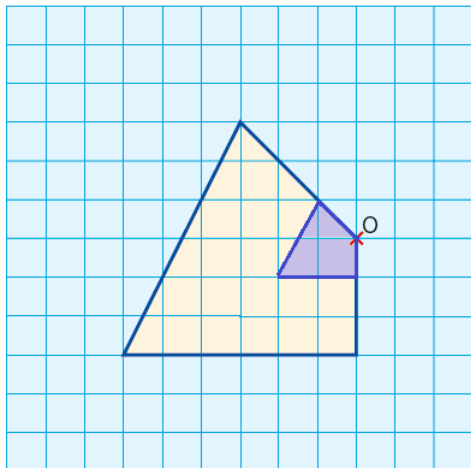
Mathematical investigations and ICT 7

A painted cube

1 **a** 8   **b** 12   **c** 6   **d** 1

2 A: 8, B: 24, C: 24, D: 8

3 A: 8, B: 96, C: 384, D: 512

4 When $(n - 2) \geq 0$:

A: 8   B: $12(n - 2)$ C: $6(n - 2)^2$ D: $(n - 2)^3$

5 When $(l - 2) \geq 0$, $(w - 2) \geq 0$ and $(h - 2) \geq 0$:

A: 8

B: $4(l - 2) + 4(w - 2) + 4(h - 2)$

C: $2(l - 2)(w - 2) + 2(l - 2)(h - 2) + 2(w - 2)(h - 2)$

D: $(l - 2)(w - 2)(h - 2)$

Triangle count

1 9

2 Student's investigation and ordered table

Number of horizontal lines, h	Number of triangles, n
0	3
1	6
2	9
3	12
…	…

3 $n = 3(h + 1)$

ICT activity

2 a Centre of enlargement

b Centre of enlargement

c Centre of enlargement

d Centre of enlargement
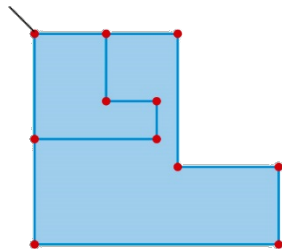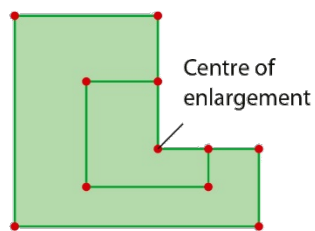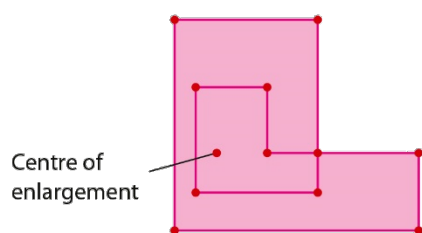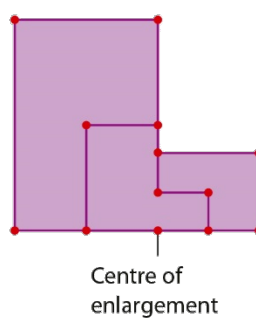
3 Student's own observations

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

27 Probability

Exercise 27.1

Student's own drawing

Exercise 27.2

1 Student's own drawing

2 Student's own answers

Exercise 27.3

1 a $\frac{1}{6}$ **b** $\frac{5}{6}$ **c** $\frac{1}{2}$ **d** $\frac{5}{6}$ **e** 0 **f** 1

2 a i $\frac{1}{7}$ **ii** $\frac{6}{7}$ **b** Total = 1

3 a $\frac{1}{250}$ **b** $\frac{1}{2}$ **c** $\frac{151}{250}$

   d 1; it is certain that a person is either born on a Wednesday or they are not born on a Wednesday

4 a $\frac{5}{8}$ **b** $\frac{3}{8}$

5 a $\frac{1}{13}$ **b** $\frac{5}{26}$ **c** $\frac{21}{26}$ **d** $\frac{3}{26}$ **e** Student's own answer

6 $\frac{1}{6}$

7 a i $\frac{1}{10}$ **ii** $\frac{1}{4}$

   b i $\frac{1}{19}$ **ii** $\frac{3}{19}$

8 a $\frac{1}{37}$ **b** $\frac{36}{37}$ **c** $\frac{18}{37}$ **d** $\frac{1}{37}$

   e $\frac{21}{37}$ **f** $\frac{12}{37}$ **g** $\frac{17}{37}$ **h** $\frac{11}{37}$

9 a RCA   RAC   CRA   CAR   ARC   ACR

   b $\frac{1}{6}$ **c** $\frac{1}{3}$ **d** $\frac{1}{2}$ **e** $\frac{1}{6}$

10 a $\frac{1}{4}$ **b** $\frac{3}{4}$ **c** $\frac{1}{13}$ **d** $\frac{1}{26}$

   e $\frac{3}{13}$ **f** $\frac{1}{52}$ **g** $\frac{5}{13}$ **h** $\frac{4}{13}$

Exercise 27.4

1 a 140 **b i** $\frac{1}{5}$ **ii** $\frac{3}{70}$ **iii** $\frac{6}{7}$

2 a 32 **b i** $\frac{9}{16}$ **ii** $\frac{5}{32}$ **iii** $\frac{3}{8}$ **iv** $\frac{5}{8}$

3 **a** 70 **b** **i** $\frac{11}{35}$ **ii** $\frac{2}{7}$ **iii** $\frac{5}{7}$ **iv** $\frac{33}{35}$

Exercise 27.5

1 **a**

```
              1  111
          1 < 2  112
              3  113
              1  121
      1 <   2 < 2  122
              3  123
              1  131
          3 < 2  132
              3  133
              1  211
          1 < 2  212
              3  213
              1  221
  2 <   2 < 2  222
              3  223
              1  231
          3 < 2  232
              3  233
              1  311
          1 < 2  312
              3  313
              1  321
      3 <   2 < 2  322
              3  323
              1  331
          3 < 2  332
              3  333
```
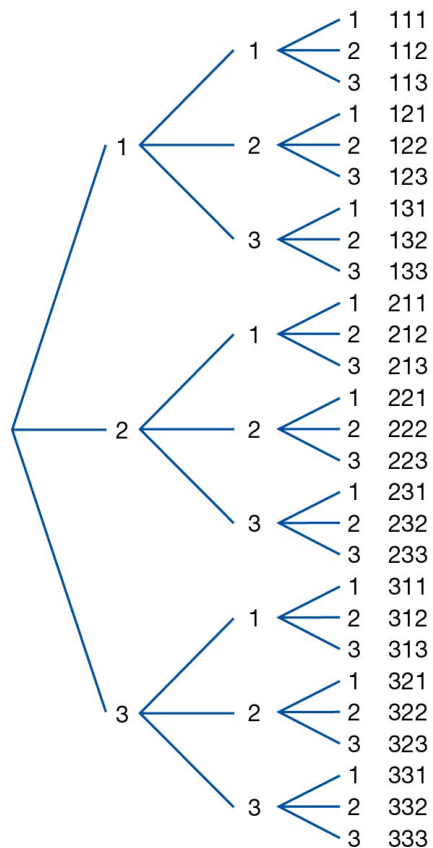
b **i** $\dfrac{1}{27}$ **ii** $\dfrac{1}{3}$ **iii** $\dfrac{1}{9}$ **iv** $\dfrac{1}{3}$ **v** $\dfrac{5}{9}$ **vi** $\dfrac{1}{3}$

2 a
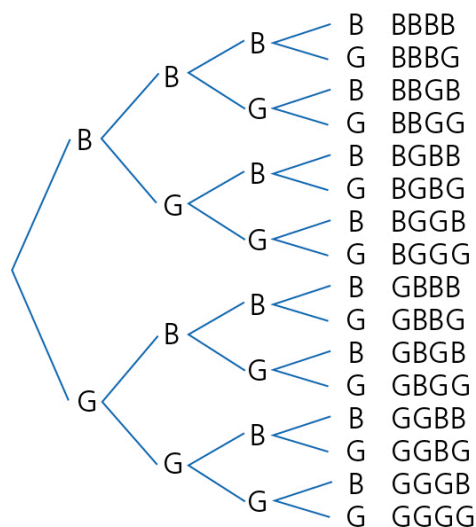
b i $\dfrac{1}{16}$ **ii** $\dfrac{3}{8}$ **iii** $\dfrac{15}{16}$ **iv** $\dfrac{5}{16}$

3 a
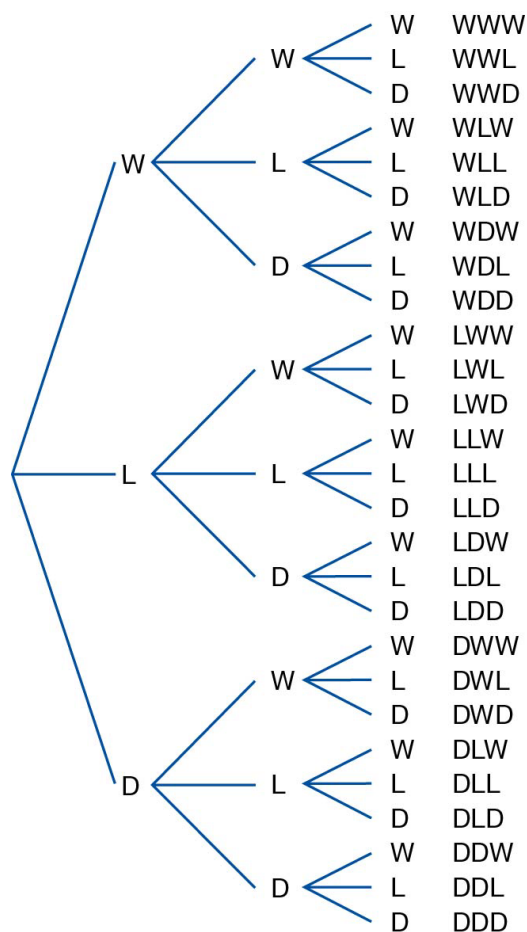
b i $\dfrac{1}{27}$ **ii** $\dfrac{10}{27}$ **iii** $\dfrac{19}{27}$ **iv** $\dfrac{8}{27}$

c Equally likely outcomes imply that the result of a game is a matter of chance, but in reality skill and other factors such as the weather will play a part.

4 a


```
        G   GG
        W   GW
    G   B   GB
        O   GO
        G   WG
        W   WW
    W   B   WB
        O   WO
        G   BG
        W   BW
    B   B   BB
        O   BO
        G   OG
        W   OW
    O   B   OB
        O   OO
```
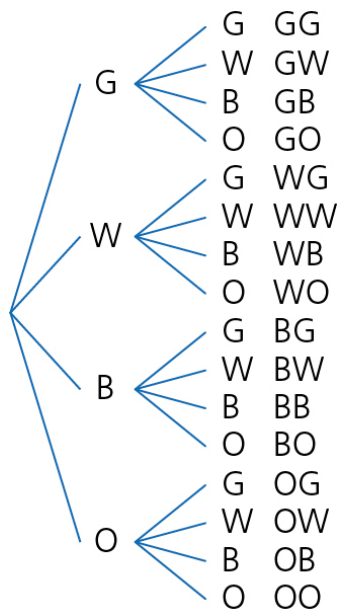
b i $\frac{1}{16}$ **ii** $\frac{1}{8}$ **iii** $\frac{9}{16}$

Exercise 27.6

1 a

Roll 1	Roll 2	Outcomes	Probability
$\frac{1}{6}$ Six	$\frac{1}{6}$ Six	Six, Six	$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$
	$\frac{5}{6}$ Not six	Six, Not six	$\frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$
$\frac{5}{6}$ Not six	$\frac{1}{6}$ Six	Not six, Six	$\frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$
	$\frac{5}{6}$ Not six	Not six, Not six	$\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$
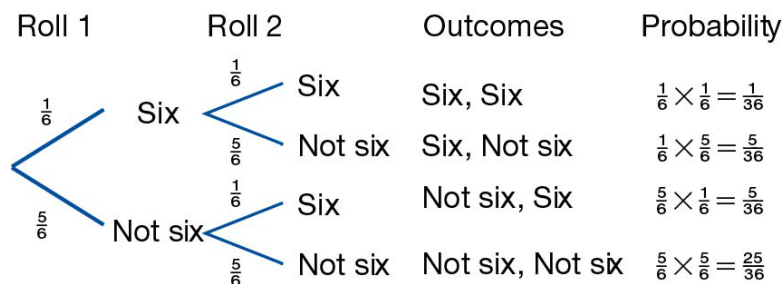
b i $\frac{1}{6}$ **ii** $\frac{11}{36}$ **iii** $\frac{5}{36}$ **iv** $\frac{125}{216}$ **v** $\frac{91}{216}$

c They total 1; you are certain to either start the game within 3 rolls or to not start the game within 3 rolls.

2 a $\frac{4}{25}$ **b** $\frac{54}{125}$ **c** $\frac{98}{125}$

3   a
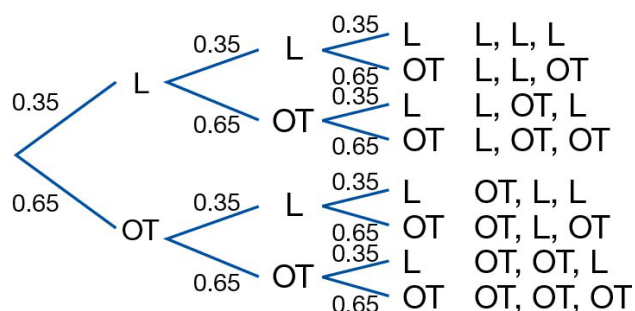
b   i   0.275 (3 s.f.)   ii   0.123 (3 s.f.)   iii   0.444 (3 s.f.)   iv   0.718 (3 s.f.)

4   0.027

Exercise 27.7

1   a
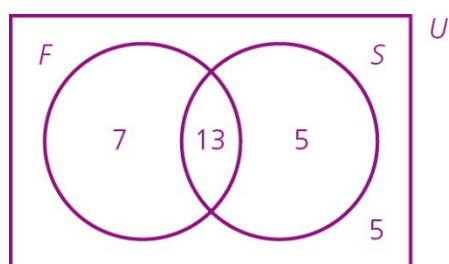
b $\frac{13}{30}$

2   a $\frac{5}{35} = \frac{1}{7}$ b $\frac{14}{35} = \frac{2}{5}$ c $\frac{13}{35}$

3 $\frac{45}{108} = \frac{5}{12}$

Exercise 27.8

1   Student's graph. The relative frequency approaches 0.5.

2   Student's graph

3   Student's graph

4   Student's explanation

Exercise 27.9

1   2

2   25

3   a $\frac{1}{16}$ b $\frac{7}{16}$ c $\frac{1}{2}$ d $\frac{15}{16}$ e $\frac{11}{16}$

4   a $\frac{14}{45}$ b $\frac{7}{45}$ c $\frac{13}{30}$ d $\frac{1}{15}$ e $\frac{14}{15}$

5   35 blue, 28 red, 21 yellow, 49 green, 7 white

6 300

7 14

8 200

9 2

Student assessment 1

1 **a** $\frac{1}{8}$ **b** $\frac{7}{8}$ **c** $\frac{5}{8}$ **d** 0 **e** 160

2 **a** 32

    b **i** $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{1}{4}$ **iv** $\frac{1}{8}$ **v** $\frac{11}{32}$

3 **a** $\frac{1}{180}$ **b** $\frac{1}{20}$ **c** $\frac{1}{12}$ **d** $\frac{2}{9}$ **e** 1

4 32

    a $\frac{9}{32}$ **b** $\frac{3}{8}$ **c** $\frac{21}{32}$

5 **a** 0.35

    b Red 128 marbles; Blue 80 marbles; Green 112 marbles

7 **a** $\frac{3}{20}$ **b** $\frac{19}{20}$ **c** $\frac{7}{10}$

7 **a** 50 000

    b The assumption is that each entry has an equal chance of winning.

8 **a** 240

    b 400

9
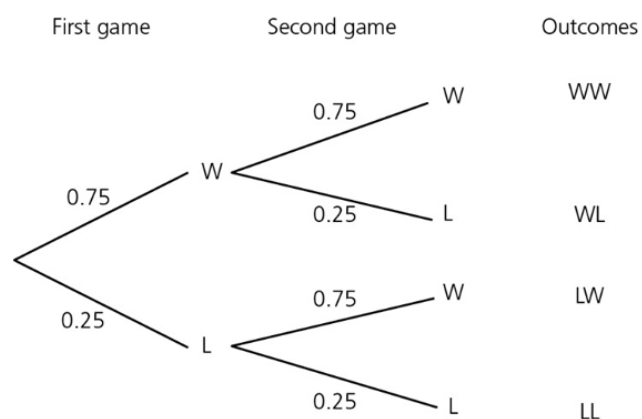
    a 0.5625

    b 0.4219 (4 d.p.)

    c 0.0563 (4 d.p.)

Mathematical investigations and ICT 8

Probability drop

1   Tray 1: LLL

   Tray 2: LLR   LRL   RLL

   Tray 3: LRR   RLR   RRL

   Tray 4: RRR

2   Tray 1: LLLL

   Tray 2: LLLR   LLRL LRLL RLLL

   Tray 3: LLRR   LRLR LRRL RLLR RLRL RRLL

   Tray 4: RRRL   RRLR RLRR LRRR

   Tray 5: RRRR

3   $\frac{1}{16}$ because there are 16 possible routes and only one results in the marble landing in tray 1.

4   Tray 2: $\frac{4}{16} = \frac{1}{4}$

   Tray 3: $\frac{6}{16} = \frac{3}{8}$

   Tray 4: $\frac{4}{16} = \frac{1}{4}$

   Tray 5: $\frac{1}{16}$

5   Student's investigation

6   $\frac{210}{1024} = \frac{105}{512}$

.

Dice sum

1

		Dice 1					
		1	**2**	**3**	**4**	**5**	**6**
Dice 2	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

2 36

3 7

4 $\frac{3}{36} = \frac{1}{12}$

5 $\frac{6}{36} = \frac{1}{6}$

6 You are four times more likely to get a 5 than a 2.

7

		Dice 2			
		1	**2**	**3**	**4**
Dice 1	**1**	2	3	4	5
	2	3	4	5	6
	3	4	5	6	7
	4	5	6	7	8

8 16

9 5

10 $\frac{4}{16} = \frac{1}{4}$

11 Student's investigation

12 a m^2 **b** $m + 1$ **c** $\frac{m}{m^2} = \frac{1}{m}$

13 a $m \times n$

   b All totals, x, in the range $n + 1 \leq x \leq m + 1$ are most likely.

   c $\dfrac{n}{nm} = \dfrac{1}{m}$

ICT activity

4 Theoretical probability is $\dfrac{n}{6}$, where n is the number of times a colour is used.

8 The fairness of a spinner will *generally* depend on how close experimental probability is to the theoretical probability. (Note: 60 spins is probably not sufficient to reliably determine whether a spinner is fair or not.)

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

28 Mean, median, mode and range

Exercise 28.1

1 a Mean = 9.57 (3s.f.)   Range = 7   **b** Mean = 6.375   Range = 6

  c Mean = 44.1 (3 s.f.) Range = 19   **d** Mean = 8   Range = 8

  e Mean = 36   Range = 48   **f** Mean = 20.6 (3s.f.)   Range = 11.5

2 a Median = 5   Range = 4   **b** Median = 9   Range = 8

  c Median = 9   Range = 2   **d** Median = 6   Range = 6

  e Median = 5   Range = 6   **f** Median = 9.5   Range = 7

  g Median = 5.85   Range = 6.1   **h** Median = 46   Range = 81

3 a Mode = 8   Range = 5   **b** Mode = 6   Range = 5

  c Mode = 3   Range = 5   **d** Mode = 4   Range = 2

  e Mode = 75   Range = 20   **f** Mode = 8   Range = 3

Exercise 28.2

1 Mean = 1.67 (3 s.f.)   Median = 1   Mode = 1   Range = 5

2 Mean = 6.2   Median = 6.5   Mode = 7   Range = 9

3 Mean = 14 years 3 months

  Median = 14 years 3 months

  Mode = 14 years 3 months

  Range = 8 months

4 Mean = 26.4   Median = 27   Mode = 28   Range = 5

5 Mean = 13.9 s (3 s.f.)   Median = 13.9 s   Mode = 13.8 s   Range = 0.6 s

6 91.1 kg

7 103 points

Exercise 28.3

1 Mean = 7.03

2 Mean = 3.35   Median = 3   Mode = 1 and 4   Range = 5

Exercise 28.4

1 a It is a mathematically accurate statement as the mode is a type of average and the mode is 2.

   b As 5 of the 7 students scored more than 2, the mode isn't a good choice of average.

      The mean = $44 \div 7 = 6.3$ (1 d.p.)

      The median = 7 as it is the middle score.

      Either value is better than the mode. However, the median gave a whole number answer, which in the context of the question is preferable.

2 Mean = 12.21 s (2 d.p.)

   Median = 12.2 s

   Mode = 12.2 s and 12.3 s

   The mean is 12.21 seconds which is a slower running time than the 12.2 cut-off time. The mode is not a useful average here as there are two modes and the data set is small. However, the median is 12.2 seconds, which is within the acceptable limits.

3 a Mean = 41.0 (1 d.p.)

      Median = 41

      Mode = 38 & 44

   b Because only one pair of size 41 shoes were sold, the mean is not a good choice of average.

   c The modal size as it represents the shoe sizes that have sold the most.

Student assessment 1

1 a Mean = 75.4   Median = 72   Mode = 72   Range = 23

   b Mean = 12   Median = 9   Mode = 6   Range = 18

   c Mean = 11.5   Median = 10   Mode = 5   Range = 20

2 61 kg

3 Mean = 3.52   Median = 4   Mode = 2

4 a 26

   b i 7.96 (3.s.f.)   **ii** 8   **iii** 8

5 Mean = 86.8 m   Median = 90.5 m   Mode = 93 m   Range = 18 m

All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

29 Collecting, displaying and interpreting data

Exercise 29.1

1

	Sleep	**Meals**	**Sport**	**TV**	**School**
Ayse	8 h 20	2 h	5 h	2 h	6 h 40
Ahmet	8 h 40	2 h	5 h 20	2 h	6 h

2
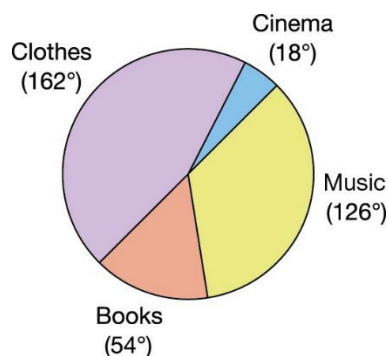
3

	Fraction	**$**	**Degrees**
Clothes	$\frac{1}{3}$	800	120
Transport	$\frac{1}{5}$	480	72
Entertainment	$\frac{1}{4}$	600	90
Saved	$\frac{13}{60}$	520	78

Saved (78°), Ents. (90°), Clothes (120°), Transport (72°)

4

University A

Industrial 8% (29°), Aerospace 18% (65°), Electrical 12% (43°), Civil 22% (79°), Chemical 24% (86°), Mechanical 16% (58°)

University B

Industrial 10% (36°), Electrical 24% (86°), Aerospace 4% (14°), Chemical 16% (58°), Civil 38% (137°), Mechanical 8% (29°)

  a   Student's own two statements

  b   360

5 **a**

Ages in Club 1												Ages in Club 2							
						9	8	8	**0**										
8	6	5	5	5	4	4	4	2	2	1	**1**	0	2	8					
						7	4	3	1	**2**	2								
							8	2	1	**3**	5	8							
							8	6	4	**4**	2	3	4	8					
							9	5	**5**	0	7	7	9						
						7	2	**6**	0	2	2	4	6	6	6	8			
							1	0	**7**	1	4	7							
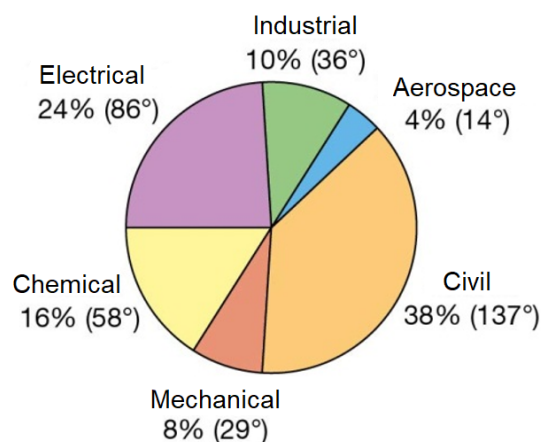
Key: 1 | 1 | 0 represents 11 yrs (club 1) and 10 yrs (club 2)

  b **i**   Club 1: range = 63   Club 2: range = 67

      ii Club: 1 median = 22   Club 2: median = 57

c Club 1 is likely to be the computer gaming club as although it has a similar range of ages to club 2, the median is considerably lower. The back-to-back Stem & Leaf diagram could be reversed, with Club 2 on the left.

6 a 14

b 3

c The boys tend to be born later in the year than the girls as their bars extend to later months.

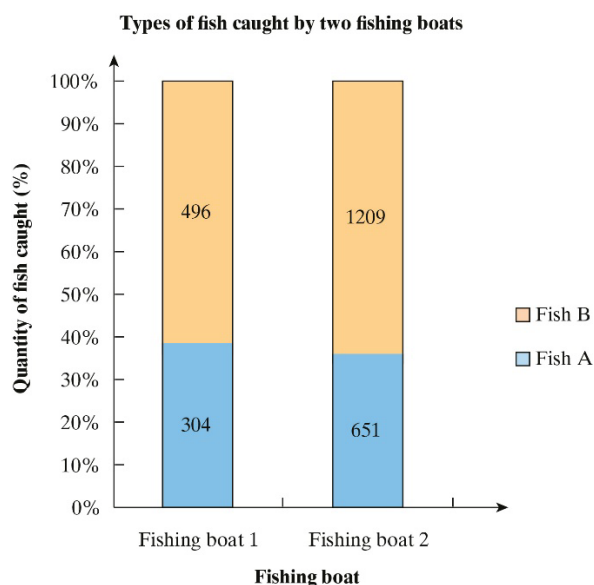**d** Student's own comparative bar chart

7 a Fishing boat 2 as it caught 651 kg of fish type A, compared to boat 1's 304 kg.

b Fishing boat 1, percentage of fish type A $= \frac{304}{800} = 38\%$

Fishing boat 2, percentage of fish type A $= \frac{651}{1860} = 35\%$

Therefore boat 1 had a higher percentage of fish type A in its catch.

c

Types of fish caught by two fishing boats

Exercise 29.2

1 Student's answers may differ from those given below.

a Possible positive correlation (strength depending on topics tested)

b No correlation

c Positive correlation (likely to be quite strong)

d Negative correlation (likely to be strong). Assume that motorcycles are not rare/vintage.

e Factors such as social class, religion and income are likely to affect results.
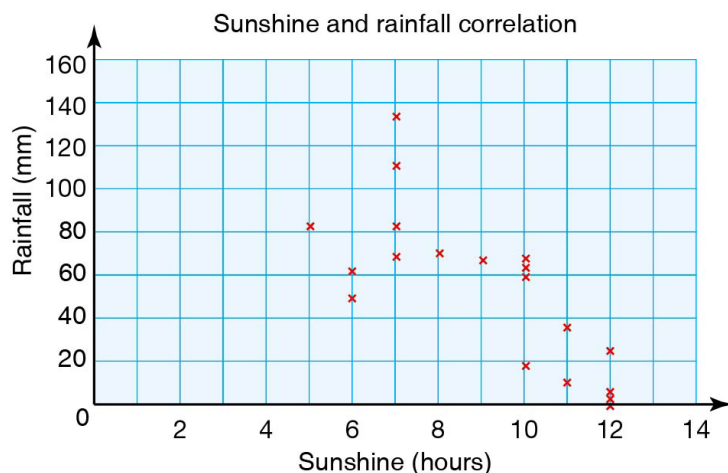
Therefore little correlation is likely.

f Negative correlation (likely to be strong)

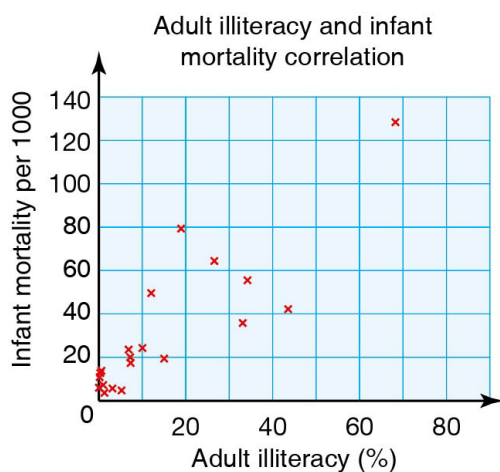**g** 0–16 years likely to be a positive correlation

h Strong positive correlation

2 a

Sunshine and rainfall correlation

b Graph shows a very weak negative correlation.

3 a

Adult illiteracy and infant mortality correlation

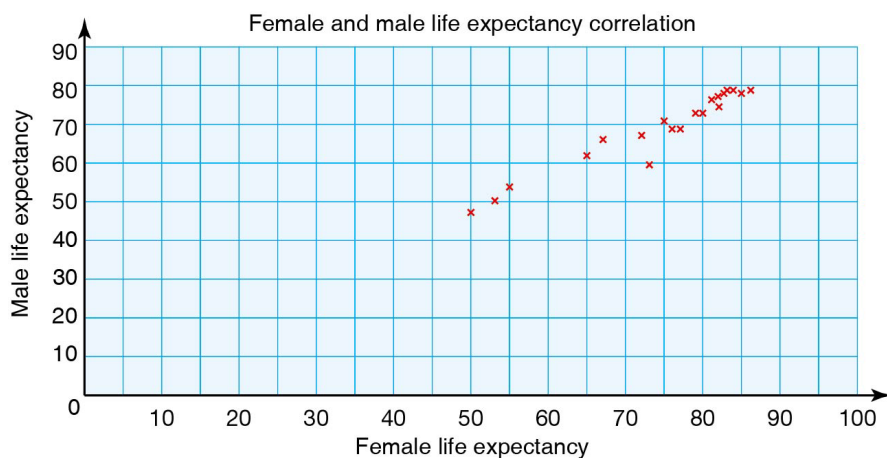**b** Positive correlation

c Student's answer. However, although there is a correlation, it doesn't imply that one variable affects the other.
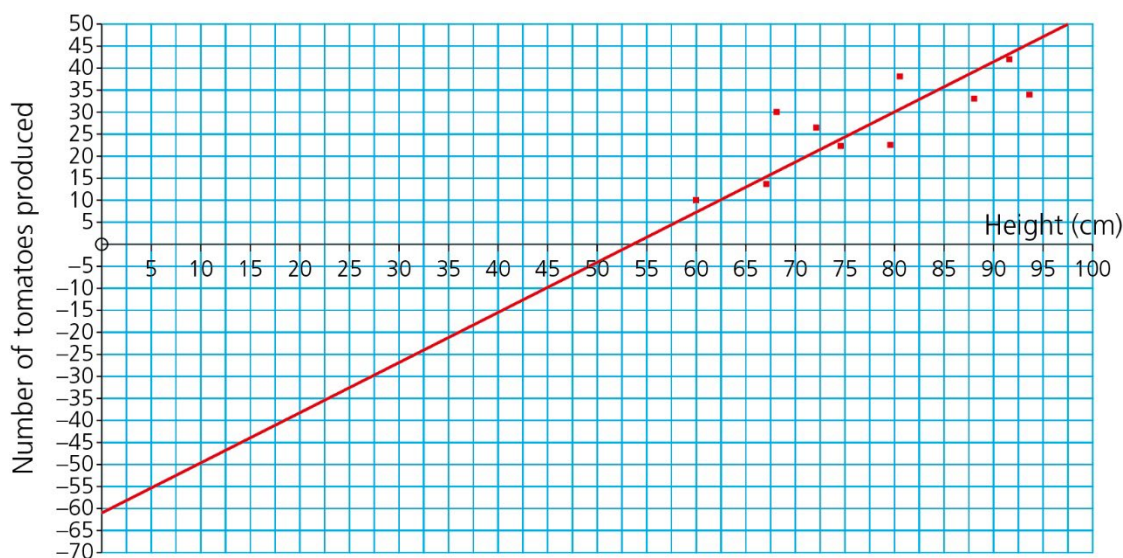
d

Female and male life expectancy correlation

4 a Moderate/strong positive correlation

b   Approx. 32 tomatoes

c   Approx. 60 cm

d

Extending the height axis to 0 cm and the line of best fit does not make sense in this case, as it implies that if a plant has a height of less than 53cm, then the plants will produce a negative number of tomatoes.

5   **a**   Moderate positive correlation

b   There is no evidence that the gold medal winners are part of the overweight population. A wealthy country is more likely to have an overweight population but also spend more on sports development. This is an example of winning gold medals (the effect) not likely a result of being overweight (the cause).
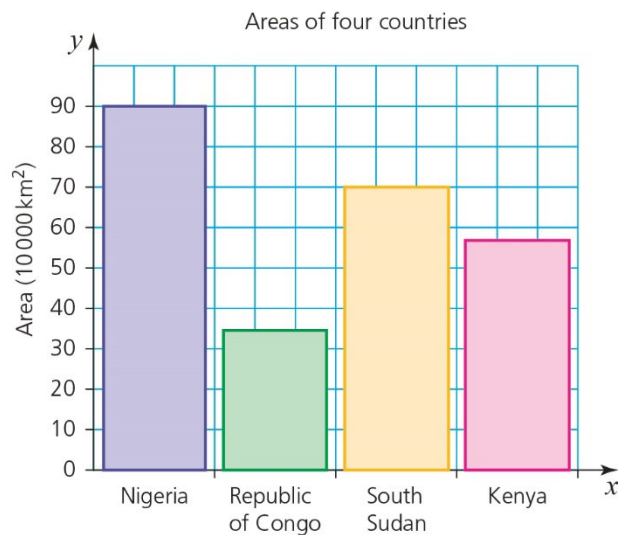
c   Student analysis

Exercise 29.3

1   Student's own survey results and pie charts

2   Student's report
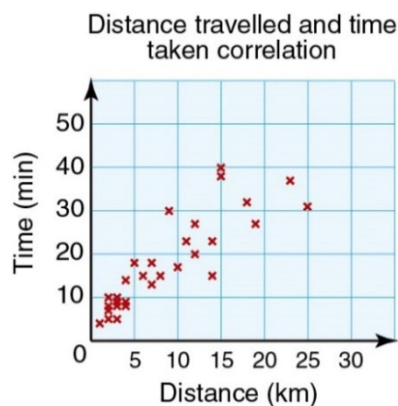
Student assessment 1

1

Areas of four countries

2 a

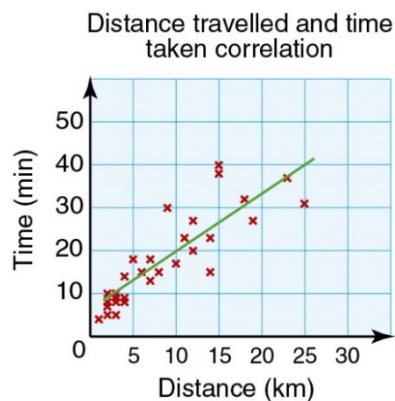**b** The scatter graph shows strong positive correlation.

c Some students who live further away might get to school more quickly than some of those that live closer because they might be driven to school. Journey time depends on both distance and the mode of transport.

d

e A 19-minute journey time corresponds with a distance of approximately 9.5 km.

3 a

	Length of line to nearest mm								
5	4	8	9	9					
6	1	2	2	5	8				
7	2	3	3	7					
8	1	1	1	2	3	3	3	8	8	9
9	4	7							
10	0	4							

Key: 6 | 1 represents 6.1 cm

b 5 cm

c Median = 8.1 cm

d Mean = 7.69 to 3 s.f.

e The median measurement was greater than the target, so at least half the students overestimated the length of the line.

The mean measurement is smaller than the target, which indicates that there is a greater spread in the measurements of those who underestimated compared to those who overestimated.

Mathematical investigations and ICT 9

Reading age

1   Possible answers include: length of sentences, number of words with 3 or more syllables, size of type, etc.

2   Student's choices

3   Student's calculations

4   Students should choose articles on a similar topic. Ignore proper nouns. Choose more than one article from each paper.

ICT activity

1–5 Student's timelines, tables and fully labelled pie charts.

6   Student's comments on the differences between the time spent on activities on a weekday compared with a day at the weekend.