



*All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.*

## 1 Number and language

### Exercise 1.1

---

- 1 a 8  
b  $\frac{12}{7}$   
c  $\frac{5}{3}$   
d  $\frac{2}{3}$   
e  $\frac{4}{15}$   
f  $\frac{1}{6}$

### Exercise 1.2

---

- 1 a 1, 2, 3, 6  
b 1, 3, 9  
c 1, 7  
d 1, 3, 5, 15  
e 1, 2, 3, 4, 6, 8, 12, 24  
f 1, 2, 3, 4, 6, 9, 12, 18, 36  
g 1, 5, 7, 35  
h 1, 5, 25  
i 1, 2, 3, 6, 7, 14, 21, 42  
j 1, 2, 4, 5, 10, 20, 25, 50, 100

### Exercise 1.3

---

- 1 Prime numbers are:  
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
- 2 a 3, 5  
b 2, 3  
c 2, 3  
d 2  
e 2, 5  
f 13  
g 3, 11



- h 5, 7
- i 2, 5, 7
- j 2, 7

### Exercise 1.4

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- 1 a  $2^2 \times 3$
- b  $2^5$
- c  $2^2 \times 3^2$
- d  $2^3 \times 5$
- e  $2^2 \times 11$
- f  $2^3 \times 7$
- g  $3^2 \times 5$
- h  $3 \times 13$
- i  $3 \times 7 \times 11$
- j  $3^2 \times 7$

### Exercise 1.5

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- 1 a 4
- b 5
- c 6
- d 3
- e 9
- f 22
- g 8
- h 13
- i 17
- j 12
- 2 a 42
- b 60
- c 70
- d 90
- e 120
- f 105
- g 20
- h 231
- i 240



j 200

## Exercise 1.6

---

- 1
  - a Rational
  - b Rational
  - c Irrational
  - d Rational
  - e Rational
  - f Rational
  - g Irrational
  - h Rational
  - i Rational
- 2
  - a Irrational
  - b Irrational
  - c Rational
  - d Rational
  - e Rational
  - f Rational
- 3
  - a Rational
  - b Irrational
  - c Rational
  - d Rational

## Exercise 1.7

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- 1
  - a 5
  - b 3
  - c 7
  - d 10
  - e 11
  - f 13
  - g 0.1
  - h 0.2
  - i 0.3
  - j 0.5
- 2 Students check their answers.
- 3
  - a  $\frac{1}{3}$

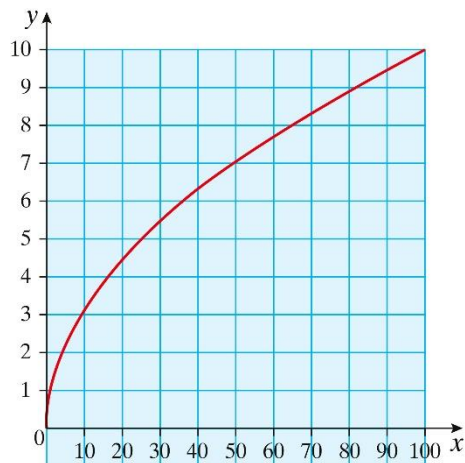


- b  $\frac{1}{4}$
- c  $\frac{1}{5}$
- d  $\frac{1}{7}$
- e  $\frac{1}{10}$
- f  $\frac{2}{3}$
- g  $\frac{3}{10}$
- h  $\frac{7}{9}$
- i  $\frac{5}{3}$
- j  $\frac{5}{2}$

### Exercise 1.8

1

|   |   |   |   |   |    |    |    |    |    |    |     |
|---|---|---|---|---|----|----|----|----|----|----|-----|
| x | 0 | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 | 100 |
| y | 0 | 1 | 2 | 3 | 4  | 5  | 6  | 7  | 8  | 9  | 10  |



The following answers are correct to 1 d.p.:

- a 5.9
- b 6.7
- c 7.4
- d 7.7
- e 1.4

2 Students check their answers

### Exercise 1.9

1 a 2



- b** 5
- c** 3
- d** 0.1
- e** 0.3
- f** 6
- g** 10
- h** 100
- i** -2
- j** -3
- k** -10
- l** -1

### Exercise 1.10

---

- 1 a** 1296
- b** 259
- c** 6561
- d** 100
- e** 7
- f** 2
- g** 27
- h** 36
- i** 64
- j** 64
- k** 5
- l** 9

### Exercise 1.11

---

- 1** 146°C
- 2 a** -\$35
- b** -\$318
- c** -\$88
- d** -\$160
- e** \$90
- 3** 165 m
- 4** 695 m



## Student assessment 1

---

- 1 a Rational  
b Irrational  
c Rational  
d Rational  
e Rational  
f Irrational
- 2 a  $\frac{5}{8}$   
b 3  
c  $\frac{11}{25}$
- 3 a 81  
b 225  
c 0.04  
d 0.49
- 4 a 12.25  
b 16.81  
c 0.0225
- 5 a 15  
b 0.1  
c 0.9  
d  $\frac{3}{5}$   
e  $\frac{7}{3}$   
f  $\frac{11}{7}$
- 6 a 64  
b 0.001  
c  $\frac{8}{27}$
- 7 a 3  
b 100  
c  $\frac{4}{5}$
- 8 a −\$84  
b \$91  
c \$45  
d \$74  
e −\$43



f    -\$15

9 a     $\frac{1}{4}$

b    2048

c    192



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## 2 Accuracy

### Exercise 2.1

---

- 1 a 69 000  
b 74 000  
c 89 000  
d 4000  
e 100 000  
f 1 000 000
- 2 a 78 500  
b 6900  
c 14 100  
d 8100  
e 1000  
f 3000
- 3 a 490  
b 690  
c 8850  
d 80  
e 0  
f 1000

### Exercise 2.2

---

- 1 a 5.6  
b 0.7  
c 11.9  
d 157.4  
e 4.0  
f 15.0  
g 3.0  
h 1.0  
i 12.0
- 2 a 6.47



- b** 9.59
- c** 16.48
- d** 0.09
- e** 0.01
- f** 9.30
- g** 100.00
- h** 0.00
- i** 3.00

### **Exercise 2.3**

---

- 1**
  - a** 50 000
  - b** 48 600
  - c** 7000
  - d** 7500
  - e** 500
  - f** 2.57
  - g** 1000
  - h** 2000
  - i** 15.0
- 2**
  - a** 0.09
  - b** 0.6
  - c** 0.94
  - d** 1
  - e** 0.95
  - f** 0.003
  - g** 0.0031
  - h** 0.0097
  - i** 0.01

### **Exercise 2.4**

---

- 1**
  - a** 419.6
  - b** 5.0
  - c** 166.3
  - d** 23.8
  - e** 57.8
  - f** 4427.1



**g** 1.9

**h** 4.1

**i** 0.6

Answers to Q2–4 may vary slightly from those given below:

**2 a** 1200

**b** 3000

**c** 3600 (using 1 s.f. estimation)

**d** 150 000

**e** 0.8

**f** 100

**3 a** 200

**b** 200

**c** 30

**d** 550

**e** 500

**f** 3000

**4 a** 130

**b** 80

**c** 9

**d** 4

**e** 200

**f** 250

**g**  $\frac{1}{5}$

**h** 800

**i** 5

**5 c, e and f** are incorrect

**c** is incorrect as estimate is  $700 \times 400 = 280\,000$

**e** is incorrect as estimate is  $80 \times 20 = 1600$

**f** is incorrect as estimate is  $= 2000$

**6 a**  $120\text{ m}^2$

**b**  $44\text{ m}^2$

**c**  $400\text{ cm}^2$

**7 a**  $200\text{ cm}^3$

**b**  $4000\text{ cm}^3$

**c**  $1600\text{ cm}^3$



## Exercise 2.5

- 1 a i Lower bound = 5.5; Upper bound = 6.5 ii  $5.5 \leq x < 6.5$   
b i Lower bound = 82.5; Upper bound = 83.5 ii  $82.5 \leq x < 83.5$   
c i Lower bound = 151.5; Upper bound = 152.5 ii  $151.5 \leq x < 152.5$   
d i Lower bound = 999.5; Upper bound = 1000.5 ii  $999.5 \leq x < 1000.5$   
e i Lower bound = 99.5; Upper bound = 100.5 ii  $99.5 \leq x < 100.5$
- 2 a i Lower bound = 3.75; Upper bound = 3.85 ii  $3.75 \leq x < 3.85$   
b i Lower bound = 15.55; Upper bound = 15.65 ii  $15.55 \leq x < 15.65$   
c i Lower bound = 0.95; Upper bound = 1.05 ii  $0.95 \leq x < 1.05$   
d i Lower bound = 9.95; Upper bound = 10.05 ii  $9.95 \leq x < 10.05$   
e i Lower bound = 0.25; Upper bound = 0.35 ii  $0.25 \leq x < 0.35$
- 3 a i Lower bound = 4.15; Upper bound = 4.25 ii  $4.15 \leq x < 4.25$   
b i Lower bound = 0.835; Upper bound = 0.845 ii  $0.835 \leq x < 0.845$   
c i Lower bound = 415; Upper bound = 425 ii  $415 \leq x < 425$   
d i Lower bound = 4950; Upper bound = 5050 ii  $4950 \leq x < 5050$   
e i Lower bound = 0.0445; Upper bound = 0.0455 ii  $0.0445 \leq x < 0.0455$   
f i Lower bound = 24 500; Upper bound = 25 500 ii  $24\,500 \leq x < 25\,500$
- 4 a   
b  $5.35 \leq M < 5.45$
- 5 a   
b  $11.75 \leq T < 11.85$
- 6 a Lower bound =  $615 \text{ m}^3$ ; Upper bound =  $625 \text{ m}^3$   
b  $615 \leq x < 625$
- 7 a Lower bound = 625 m; Upper bound = 635 m  
b  $395 \leq W < 405$

## Exercise 2.6

- 1 a Lower bound = 263.25; Upper bound = 297.25  
b Lower bound = 3295.25; Upper bound = 3455.25  
c Lower bound = 4925.25; Upper bound = 5075.25  
d Lower bound = 3.76 (2 d.p.); Upper bound = 4.26 (2 d.p.)  
e Lower bound = 2.83 (2 d.p.); Upper bound = 3.19 (2 d.p.)  
f Lower bound = 8.03 (2 d.p.); Upper bound = 8.66 (2 d.p.)  
g Lower bound = 44.95 (2 d.p.); Upper bound = 52.82 (2 d.p.)  
h Lower bound = 39.77 (2 d.p.); Upper bound = 42.23 (2 d.p.)  
i Lower bound = 16.14 (2 d.p.); Upper bound = 18.88 (2 d.p.)



- j Lower bound = 3.55 (2 d.p.); Upper bound = 7.12
- k Lower bound = 1.47 (2 d.p.); Upper bound = 1.63 (2 d.p.)
- l Lower bound = 18.51 (2 d.p.); Upper bound = 28.59 (2 d.p.)
- 2 a Lower bound = 6.7; Upper bound = 6.9
- b Lower bound = 29.69 (2 d.p.); Upper bound = 30.80 (2 d.p.)
- c Lower bound = 147.76 (2 d.p.); Upper bound = 150.25 (2 d.p.)
- d Lower bound = 13.3; Upper bound = 13.5
- e Lower bound = 1.75 (2 d.p.); Upper bound = 1.81 (2 d.p.)
- f Lower bound = 0.39 (2 d.p.); Upper bound = 0.46 (2 d.p.)
- g Lower bound = 34.10 (2 d.p.); Upper bound = 40.03 (2 d.p.)
- h Lower bound = 0.98 (2 d.p.); Upper bound = 1.02 (2 d.p.)
- i Lower bound = 0; Upper bound = 0.04
- 3 a Lower bound = 20 002.5; Upper bound = 20 962.5
- b Lower bound = 1.06 (2 d.p.); Upper bound = 1.15 (2 d.p.)
- c Lower bound = 1 116 250; Upper bound = 1 188 250
- d Lower bound = 88.43 (2 d.p.); Upper bound = 91.60 (2 d.p.)
- e Lower bound = 131.75 (2 d.p.); Upper bound = 139.34 (2 d.p.)
- f Lower bound = 18.95 (2 d.p.); Upper bound = 20.39 (2 d.p.)
- g Lower bound = 2.24 (2 d.p.); Upper bound = 2.53 (2 d.p.)
- h Lower bound = 3.61 (2 d.p.); Upper bound = 3.97 (2 d.p.)
- i Lower bound = 60.34 (2 d.p.); Upper bound = 68.52 (2 d.p.)

## Exercise 2.7

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- 1 Lower bound = 3.5 kg; Upper bound = 4.5 kg
- 2 Lower bound = 21.8 cm; Upper bound = 22.2 cm
- 3 Lower bound = 13.7 m; Upper bound = 13.74 m
- 4 Lower bound = 74.13 cm<sup>2</sup> (2 d.p.); Upper bound = 75.88 cm<sup>2</sup> (2 d.p.)
- 5 Lower bound = 68 425 m<sup>2</sup>; Upper bound = 75 625 m<sup>2</sup>
- 6 Lower bound = 13.1 (1 d.p.); Upper bound = 13.5 (1 d.p.)
- 7 Lower bound = 9.48 cm (2 d.p.); Upper bound = 10.55 cm (2 d.p.)
- 8 a Lower bound = 53.1 cm (1 d.p.); Upper bound = 53.7 cm (1 d.p.)
- b Lower bound = 224.3 cm<sup>2</sup> (1 d.p.); Upper bound = 229.7 cm<sup>2</sup> (1 d.p.)
- 9 a Lower bound = 11.2 cm (1 d.p.); Upper bound = 11.4 cm (1 d.p.)
- b Lower bound = 70.5 cm (1 d.p.); Upper bound = 71.3 cm (1 d.p.)
- 10 Lower bound = 11.5 g/cm<sup>3</sup> (1 d.p.); Upper bound = 13.6 g/cm<sup>3</sup> (1 d.p.)
- 11 Least = 93.3 h (1 d.p.); Greatest = 138.5 h (1 d.p.)



## Student assessment 1

---

- 1 a 2800  
b 7090  
c 49 000  
d 1000
- 2 a 3.8  
b 6.8  
c 0.85  
d 1.58  
e 10.0  
f 0.008
- 3 a 4  
b 6.8  
c 0.7  
d 10  
e 830  
f 0.005

Answers to Q4–6 may vary from those given below:

- 4 18 000 yards
- 5  $40 \text{ m}^2$
- 6 a 25  
b 4  
c 4
- 7  $92.3 \text{ cm}^3$  (1 d.p.)

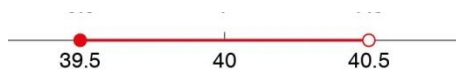
## Student assessment 2

---

- 1 a



- b



- c



- d





- 2 a  $205 \leq x < 215$   
b  $63.5 \leq x < 64.5$   
c  $2.95 \leq x < 3.05$   
d  $0.875 \leq x < 0.885$
- 3 Length:  $349.5 \leq L < 350.5$ ; Width:  $199.5 \leq W < 200.5$

4



- 5  $0.0035 \leq x < 0.0045$
- 6 a  $4.825 \leq x < 4.835$   
b  $5.045 \leq y < 5.055$   
c  $9.95 \leq z < 10.05$   
d  $99.995 \leq p < 100.005$

### Student assessment 3

---

- 1 255 kg
- 2  $31\,575\text{ cm}^3$  (to the nearest whole number)
- 3 a  $124.5 \leq V < 125.5\text{ cm}^3$  (1 d.p.)  
b  $4.99 \leq L < 5.01\text{ cm}$  (2 d.p.)
- 4 a  $25.10 \leq C < 25.16\text{ cm}$  (2 d.p.)  
b  $50.14 \leq A < 50.39\text{ cm}^2$  (2 d.p.)
- 5 22 cups
- 6 a Lower bound = 4.5 cm; Upper bound = 5.5 cm  
b Upper and lower bounds of 100 matches  $\div 100$   
Lower bound = 5.43 cm; Upper bound = 5.44 cm



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### 3 Calculations and order

#### Exercise 3.1

- 1 a  $<$   
b  $=$   
c  $>$   
d  $<$   
e  $=$   
f  $>$

2



- 3 a  $x > 0$   
b  $x \leq 3$   
c  $0 \leq x \leq 4$   
d  $-4 < x \leq -1$
- 4 a  $x \leq 20\,000$   
b  $135 \leq x \leq 180$   
c  $5x + 3 < 20$   
d  $x \leq 25$   
e  $350 \leq x \leq 400$   
f  $11 \leq x \leq 28$



### Exercise 3.2

---

- 1 0.06      0.6      0.606      0.66      6.0      6.6      6.606
- 2  $\frac{4}{5}$        $\frac{1}{2}$        $\frac{6}{13}$        $\frac{7}{18}$        $\frac{1}{3}$        $\frac{2}{19}$
- 3 60 cm      0.75 m      800 mm      180 cm      2 m
- 4 4 kg      3500 g      1 kg       $\frac{3}{4}$  kg      700 g
- 5 150 cm<sup>3</sup>      430 ml      800 cm<sup>3</sup>      1 litre      120 cl

### Exercise 3.3

---

- 1 a 26  
b 10  
c 42  
d 16  
e 8  
f -6
- 2 a 20  
b 34  
c 32  
d 31  
e -36  
f 23
- 3 a 27  
b 64  
c -36  
d 3  
e 144  
f 1.6

### Exercise 3.4

---

- 1 a  $6 \times (2 + 1) = 18$   
b  $1 + 3 \times 5 = 16$   
c  $(8 + 6) \div 2 = 7$   
d  $(9 + 2) \times 4 = 44$   
e  $9 \div 3 \times 4 + 1 = 13$   
f  $(3 + 2) \times (4 - 1) = 15$
- 2 a  $12 \div 4 - 2 + 6 = 7$



- b**  $12 \div (4 - 2) + 6 = 12$
- c**  $12 \div 4 - (2 + 6) = -5$
- d**  $12 \div (4 - 2 + 6) = 1.5$
- e**  $4 + 5 \times 6 - 1 = 33$
- f**  $4 + 5 \times (6 - 1) = 29$
- g**  $(4 + 5) \times 6 - 1 = 53$
- h**  $(4 + 5) \times (6 - 1) = 45$

### Exercise 3.5

---

- 1**
  - a** 2
  - b** 3
  - c** 7
  - d** 4
  - e** 23
  - f** 0
- 2**
  - a** 1
  - b** 5
  - c** 2
  - d** 50
  - e** 7
  - f** -1.5

### Exercise 3.6

---

- 1**
  - a**  $T = 3(48 - 16)$
  - b**  $T = 96^\circ\text{C}$
- 2**
  - a**  $D = -400 \times 2 + 620$
  - b**  $D = -180\text{ m}$  i.e. 180 m below the surface
- 3**
  - a**  $N = \frac{5^2 + 11}{2}$
  - b**  $N = 18$

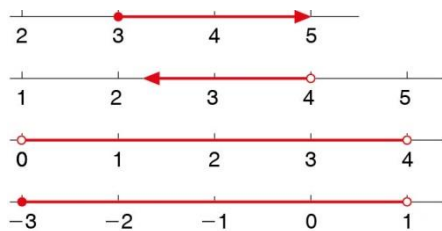
### Student assessment 1

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- 1**
  - a**  $x \geq -1$
  - b**  $x < 2$
  - c**  $-2 \leq x < 2$
  - d**  $-1 \leq x \leq 1$



2



3  $\frac{3}{14}$   $\frac{2}{5}$   $\frac{1}{2}$   $\frac{4}{7}$   $\frac{9}{10}$

## Student assessment 2

- 1 a 44  
b 13  
c 25  
d 19  
e 49  
f 3
- 2 a  $(7 - 4) \times 2 = 6$   
b  $12 + 3 \times (3 + 4) = 33$   
c  $(5 + 5) \times (6 - 4) = 20$   
d  $(5 + 5) \times 6 - 4 = 56$
- 3 a 3.5  
b 10

## Student assessment 3

- 1 a 20  
b 15  
c 11  
d 13  
e 1  
f 4
- 2 a  $(7 - 5) \times 3 = 6$   
b  $16 + 4 \times (2 + 4) = 40$   
c  $(4 + 5) \times (6 - 1) = 45$   
d  $(1 + 5) \times 6 - 6 = 30$
- 3 a 5.5  
b 9



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## 4 Integers, fractions, decimals and percentages

### Exercise 4.1

---

- 1 a 15  
b 16  
c 20  
d 40  
e 18  
f 72  
g 30  
h 48  
i 210  
j 52

### Exercise 4.2

---

- 1 a  $\frac{14}{3}$   
b  $\frac{18}{5}$   
c  $\frac{47}{8}$   
d  $\frac{17}{6}$   
e  $\frac{17}{2}$   
f  $\frac{68}{7}$   
g  $\frac{58}{9}$   
h  $\frac{17}{4}$   
i  $\frac{59}{11}$   
j  $\frac{55}{7}$   
k  $\frac{43}{10}$   
l  $\frac{146}{13}$
- 2 a  $7\frac{1}{4}$   
b  $6\frac{3}{5}$



c  $6\frac{5}{6}$

d  $6\frac{5}{8}$

e  $5\frac{4}{9}$

f  $1\frac{5}{12}$

g  $9\frac{3}{7}$

h  $3\frac{3}{10}$

i  $9\frac{1}{2}$

j  $6\frac{1}{12}$

### Exercise 4.3

---

1 a 4.5

b 6.3

c 17.8

d 3.07

e 9.27

f 11.36

g 4.006

h 5.027

i 4.356

j 9.204

2 a 19.14

b 83.812

c 6.6

d 11.16

e 35.81

f 5.32

g 67.14

h 6.06

i 1.4

j 0.175

### Exercise 4.4

---

1 a  $\frac{29}{50} = \frac{58}{100} = 58\%$



**b**  $\frac{17}{25} = \frac{68}{100} = 68\%$

**c**  $\frac{11}{20} = \frac{55}{100} = 55\%$

**d**  $\frac{3}{10} = \frac{30}{100} = 30\%$

**e**  $\frac{23}{25} = \frac{92}{100} = 92\%$

**f**  $\frac{19}{50} = \frac{38}{100} = 38\%$

**g**  $\frac{3}{4} = \frac{75}{100} = 75\%$

**h**  $\frac{2}{5} = \frac{40}{100} = 40\%$

**2**

|            |                |               |                |                              |               |               |                |               |                |               |               |
|------------|----------------|---------------|----------------|------------------------------|---------------|---------------|----------------|---------------|----------------|---------------|---------------|
| Fraction   | $\frac{1}{10}$ | $\frac{1}{5}$ | $\frac{3}{10}$ | $\frac{4}{10} = \frac{2}{5}$ | $\frac{1}{2}$ | $\frac{3}{5}$ | $\frac{7}{10}$ | $\frac{4}{5}$ | $\frac{9}{10}$ | $\frac{1}{4}$ | $\frac{3}{4}$ |
| Decimal    | 0.1            | 0.2           | 0.3            | 0.4                          | 0.5           | 0.6           | 0.7            | 0.8           | 0.9            | 0.25          | 0.75          |
| Percentage | 10             | 20            | 30             | 40                           | 50            | 60            | 70             | 80            | 90             | 25            | 75            |

## Exercise 4.5

**1 a** 9

**b** 3

**2 a**  $6 \times (4 + 6) \div 3 = 20$

**b**  $(9 - 3) \times (7 + 2) = 54$

**3 a** 29 830

**b** 41 492

**4 a** 2

**b** 7

**5 a** 112

**b** 28

**6 a** 1127.4

**b** 526.1

## Exercise 4.6

**1 a**  $\frac{1}{2}$

**b**  $\frac{1}{3}$

**c**  $\frac{2}{3}$

**d**  $\frac{4}{9}$

**e**  $\frac{3}{4}$



f  $\frac{9}{10}$

### Exercise 4.7

---

1 a  $1\frac{2}{5}$

b  $\frac{10}{11}$

c  $\frac{11}{12}$

d  $1\frac{2}{45}$

e  $1\frac{1}{65}$

f  $1\frac{11}{12}$

2 a  $1\frac{1}{8}$

b  $1\frac{5}{7}$

c  $1\frac{1}{12}$

d  $\frac{47}{60}$

e  $1\frac{29}{40}$

f  $\frac{51}{52}$

3 a  $\frac{1}{7}$

b  $\frac{1}{10}$

c  $\frac{5}{9}$

d  $\frac{1}{12}$

e  $\frac{9}{40}$

f  $1\frac{1}{20}$

4 a  $\frac{17}{60}$

b  $\frac{45}{88}$

c  $\frac{17}{20}$

d  $\frac{44}{195}$

e  $\frac{9}{20}$

f  $\frac{1}{18}$

5 a  $5\frac{3}{4}$

b  $5\frac{3}{10}$



c  $3\frac{1}{10}$

d  $6\frac{7}{24}$

e  $1\frac{1}{8}$

f  $\frac{25}{36}$

6 a  $5\frac{1}{8}$

b  $7\frac{9}{40}$

c  $-\frac{3}{8}$

d  $\frac{7}{20}$

e  $-2\frac{39}{140}$

f  $1\frac{1}{4}$

### Exercise 4.8

---

1 a 8

b  $\frac{12}{7}$

c  $\frac{5}{3}$

d  $\frac{2}{3}$

e  $\frac{4}{15}$

f  $\frac{1}{6}$

2 a  $\frac{1}{6}$

b  $\frac{3}{5}$

c  $\frac{4}{21}$

d  $\frac{2}{3}$

e  $\frac{1}{4}$

f  $\frac{7}{20}$

3 a  $\frac{5}{6}$

b  $2\frac{1}{2}$

c  $1\frac{1}{7}$

d  $4\frac{1}{6}$

e  $\frac{1}{5}$



- f  $\frac{2}{3}$
- 4 a  $\frac{3}{5}$
- b  $\frac{7}{12}$
- c  $\frac{9}{70}$
- d  $\frac{21}{25}$
- e  $\frac{3}{8}$
- f  $1\frac{25}{56}$
- 5 a  $\frac{3}{5}$
- b  $3\frac{107}{120}$
- c  $\frac{8}{15}$
- d  $12\frac{1}{4}$
- 6 a  $\frac{13}{28}$
- b  $-\frac{1}{2}$
- c  $-\frac{1}{24}$
- d 7

### Exercise 4.9

---

- 1 a 0.75
- b 0.8
- c 0.45
- d 0.34
- e  $0.\dot{3}$
- f 0.375
- g 0.4375
- h  $0.\dot{2}$
- i  $0.\dot{6}\dot{3}$
- 2 a 2.75
- b 3.6
- c 4.35
- d 6.22
- e  $5.\dot{6}$
- f 6.875



- g 5.5625  
h 4.2  
i  $5.\dot{4}2857\dot{1}$

### Exercise 4.10

---

- 1 a  $\frac{1}{2}$   
b  $\frac{7}{10}$   
c  $\frac{3}{5}$   
d  $\frac{3}{4}$   
e  $\frac{33}{40}$   
f  $\frac{1}{20}$   
g  $\frac{1}{20}$   
h  $\frac{201}{500}$   
i  $\frac{1}{5000}$
- 2 a  $2\frac{2}{5}$   
b  $6\frac{1}{2}$   
c  $8\frac{1}{5}$   
d  $3\frac{3}{4}$   
e  $10\frac{11}{20}$   
f  $9\frac{51}{250}$   
g  $15\frac{91}{200}$   
h  $30\frac{1}{1000}$   
i  $1\frac{41}{2000}$

### Exercise 4.11

---

- 1 a  $\frac{1}{3}$   
b  $\frac{7}{9}$   
c  $\frac{42}{99} = \frac{14}{33}$   
d  $\frac{65}{99}$



- 2 a  $\frac{55}{990} = \frac{1}{18}$   
b  $\frac{62}{990} = \frac{31}{495}$   
c  $\frac{92}{90} = 1\frac{1}{45}$   
d  $\frac{39638}{9900} = 4\frac{19}{4950}$
- 3  $\frac{1}{5}$
- 4  $\frac{1}{6}$

### Student assessment 1

---

- 1 a 21  
b 27  
c 22  
d 39
- 2 a 60%  
b 49%  
c 25%  
d 90%  
e 150%  
f 327%  
g 5%  
h 35%  
i 77%  
j 3%  
k 290%  
l 400%
- 3 a 0  
b 19
- 4 18 032
- 5 340.7
- 6 a  $1\frac{7}{10}$   
b 2
- 7 a 0.875  
b 1.4  
c  $0.\dot{8}$   
d  $3.\dot{2}8571\dot{4}$



8 a  $6\frac{1}{2}$

b  $\frac{1}{25}$

c  $3\frac{13}{20}$

d  $3\frac{1}{125}$

9 a  $\frac{7}{99}$

b  $\frac{1}{1000}$

c  $3\frac{2}{99}$

10  $\frac{11}{50}$



*All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.*

## **5 Further percentages**

### **Exercise 5.1**

---

- 1**   **a**   60%  
      **b**   40%
- 2**   **a**   87.5%  
      **b**   73.3%  
      **c**   29.16% (2 d.p.)  
      **d**   14.29% (2 d.p.)
- 3**   **a**   0.39  
      **b**   0.47  
      **c**   0.83  
      **d**   0.07  
      **e**   0.02  
      **f**   0.2
- 4**   **a**   31%  
      **b**   67%  
      **c**   9%  
      **d**   5%  
      **e**   20%  
      **f**   75%

### **Exercise 5.2**

---

- 1**   **a**   25%  
      **b**   66.7% (3 s.f.)  
      **c**   62.5%  
      **d**   180%  
      **e**   490%  
      **f**   387.5%
- 2**   **a**   0.75  
      **b**   0.8  
      **c**   0.2  
      **d**   0.07



- e 1.875  
f 0.167 (3 s.f.)
- 3 a 20  
b 100  
c 50  
d 36  
e 4.5  
f 7.5
- 4 a 8.5  
b 8.5  
c 52  
d 52  
e 17.5  
f 17.5
- 5 a 6  
b 3  
c 21
- 6 Apple: 66    Mango: 24    Pineapple: 18    Grapes: 12
- 7 Australian: 143    Pakistani: 44    Greek: 11    Other: 22
- 8 Newspapers: 69    Pens: 36    Books: 18    Other: 27

### Exercise 5.3

---

- 1 a 48%  
b 36.8%  
c 35%  
d 50%  
e 45%  
f 40%  
g  $33\frac{1}{3}\%$   
h 57.1% (3 s.f.)
- 2 Win: 50%    Draw:  $33\frac{1}{3}\%$     Lose:  $16\frac{2}{3}\%$
- 3 A = 34.5% (1 d.p.)    B = 25.6% (1 d.p.)    C = 23.0% (1 d.p.)    D = 16.9% (1 d.p.)
- 4 Red: 35.5%    Blue: 31.0%    White: 17.7%    Silver: 6.6%    Green: 6.0%    Black: 3.2%

### Exercise 5.4

---

- 1 a 187.5



- b** 322
- c** 7140
- d** 245
- e** 90
- f** 121.5
- 2**
  - a** 90
  - b** 38
  - c** 9
  - d** 900
  - e** 50
  - f** 43.5
- 3**
  - a** 20%
  - b** 80%
  - c** 110%
  - d** 5%
  - e** 85%
  - f** 225%
- 4**
  - a** 50%
  - b** 30%
  - c** 5%
  - d** 100%
  - e** 36%
  - f** 5%
- 5** 7475 tonnes
- 6** \$6825
- 7**
  - a** \$75
  - b** \$93.75
- 8**
  - a** 43
  - b** 17.2%
- 9** 1100

## **Exercise 5.5**

---

- 1**
  - a** 600
  - b** 350
  - c** 900
  - d** 250



- e 125
- f 1.5
- 2 a 56
- b 65
- c 90
- d 20
- e 0.25
- f -38
- 3 280 pages
- 4 12 500 families
- 5 22
- 6 12 200 000 m<sup>3</sup>

### **Student assessment 1**

---

- 1 640 m
- 2 \$345.60
- 3 \$10 125
- 4 a 20%
- b 41.7% (3 s.f.)
- c 22.5%
- d 85.7% (3 s.f.)
- e 7%
- f 30%
- 5 16% profit
- 6 a \$36
- b 25%

### **Student assessment 2**

---

- 1 750 m
- 2 \$525
- 3 \$97 200
- 4 a 29.2% (3 s.f.)
- b 21.7% (3 s.f.)
- c 125%
- d 8.33% (3 s.f.)
- e 20%



**f** 10%

**5** 8.33% (3 s.f.)

**6 a** \$650

**b** 61.8% (3 s.f.)

### Student assessment 3

---

**1** \$200      \$25      \$524      \$10

**2** \$462      \$4000      \$4500      \$5500

**3** 15 marks

**4** 35 000 television sets

**5** 25 000 units

**6** 46 500 units



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## 6 Ratio and proportion

### Exercise 6.1

---

- 1 48
- 2 16 h 40 min
- 3 11 units
- 4 a 7500 bricks  
b 53 h
- 5 a 6250 litres  
b 128 km
- 6 1110 km (3 s.f.)
- 7 a 450  
b 75  
c 120
- 8 a 480 km  
b 96 km/h

### Exercise 6.2

---

- 1 a 450 kg  
b 1250 kg
- 2 a 3 : 10 : 1 : 2  
b Butter: 600 g    Flour: 2 kg    Sugar: 200 g    Currants: 400 g  
c 120 cakes
- 3 a 16.8 litres  
b Red: 1.2 litres    White: 14.3 litres
- 4 a 125  
b Red: 216    Yellow: 135  
c 20
- 5 a 42 litres  
b Orange juice: 495 litres    Mango juice: 110 litres

### Exercise 6.3

---

- 1 60 : 90



- 2 16 : 24 : 32
- 3 3.25 kg : 1.75 kg
- 4 18 min : 27 min
- 5 10 min : 50 min
- 6 7 : 1
- 7 Orange: 556 ml (3 s.f.)      Water: 444 ml (3 s.f.)
- 8 a 11 : 9
- b 440 boys      360 girls
- 9 a  $\frac{3}{5}$
- b 32 cm
- 10 4 km and 3 km
- 11 40°, 80°, 120°, 120°
- 12 45°, 75°, 60°
- 13 24 yr old: \$400 000      28 yr old: \$466 667      32 yr old: \$533 333
- 14 Malik \$2000      Zahra \$3500      Ahmet \$2500

## Exercise 6.4

---

1 4

|   |              |    |    |    |     |                |                |    |
|---|--------------|----|----|----|-----|----------------|----------------|----|
| 2 | Speed (km/h) | 60 | 40 | 30 | 120 | 90             | 50             | 10 |
|   | Time (h)     | 2  | 3  | 4  | 1   | $1\frac{1}{3}$ | $2\frac{2}{5}$ | 12 |

3 a i 12 h

ii 4 h

iii 48 h

b i 16

ii 3

iii 48

4 a 30 rows

b 42 chairs

5 6 h 40 min

6 4

7 18 h

## Exercise 6.5

---

1 11600 kg

2 a 7269 seconds



- b** 5.80 m/s
  - c** faster than 5.86 m/s (to 2 d.p.)
- 3**
  - a** Brass
  - b** 7.9 cm
- 4**
  - a** 8281 people/km<sup>2</sup> (to nearest whole number)
  - b** China has a very large area, some of it with very sparse populations e.g. the Gobi desert.
  - c** Answers will vary
- 5**
  - a** 0.48 km<sup>2</sup>
  - b**  $83\frac{1}{3}$  sheep/km<sup>2</sup>
  - c** 86 sheep (note the answer is not given as 86.4)
- 6** 40 cm

### Student assessment 1

---

- 1**
  - a** 15 km
  - b** 6 km/h
- 2** 16 cm and 14 cm
- 3** 1200 g
- 4** 200 g
- 5**
  - a** 2 km
  - b** 48 cm
- 6**
  - a** 26 litres of petrol and 4 litres of oil
  - b** 3250 ml
- 7**
  - a** 1 : 40
  - b** 13.75 cm
- 8** Girl \$1040      Boy \$960
- 9** 24°, 60°, 96°
- 10**
  - a** 15 s
  - b** 8 copiers
- 11** 6 h

### Student assessment 2

---

- 1**
  - a** 30 km
  - b** 30 km/h
- 2**
  - a**  $\frac{7}{10}$
  - b** 45 cm
- 3**
  - a** 375 g



**b** 625 g

**4 a** 450 m

**b** 80 cm

**5 a** 1 : 25

**b** 1.75 m

**6** 300 kg, 750 kg, 1950 kg

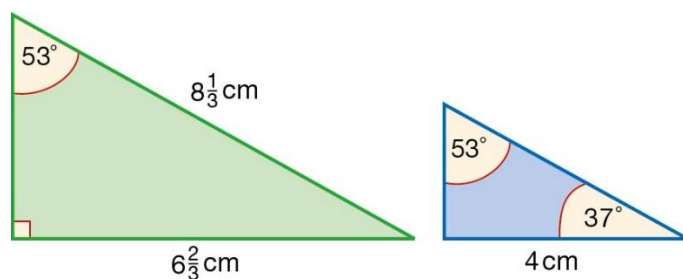
**7**  $60^\circ$ ,  $90^\circ$ ,  $90^\circ$ ,  $120^\circ$

**8**  $150^\circ$

**9 a** 13.5 h

**b** 12 pumps

**10**



**11 a** 4 min 48 s

**b** 1.6 litres/min



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## 7 Indices, standard form and surds

### Exercise 7.1

---

1 a  $3^3$

b  $2^5$

c  $4^2$

d  $6^4$

e  $8^6$

f  $5^1$

2 a  $2^3 \times 3^2$

b  $4^5 \times 5^2$

c  $3^2 \times 4^3 \times 5^2$

d  $2 \times 7^4$

e  $6^2$

f  $3^3 \times 4^2 \times 6^5$

3 a  $4 \times 4$

b  $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

c  $3 \times 3 \times 3 \times 3 \times 3$

d  $4 \times 4 \times 4 \times 6 \times 6 \times 6$

e  $7 \times 7 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

f  $3 \times 3 \times 4 \times 4 \times 4 \times 2 \times 2 \times 2 \times 2$

4 a 32

b 81

c 64

d 216

e 1 000 000

f 256

g 72

h 125 000

### Exercise 7.2

---

1 a  $3^6$

b  $8^7$



c  $5^9$

d  $4^{10}$

e  $2^4$

f  $3^5 \times 6^6$

g  $4^8 \times 5^9 \times 6^2$

h  $2^4 \times 5^{10} \times 6^8$

2 a  $4^4$

b  $5^3$

c 2

d  $6^3$

e  $6^3$

f 8

g  $4^3$

h  $3^7$

3 a  $5^4$

b  $4^{12}$

c  $10^{10}$

d  $3^{15}$

e  $6^8$

f  $8^6$

4 a  $2^3$

b 3

c  $5^3$

d  $4^5$

e  $4^3 \times 2^2 (= 2^8)$

f  $6^4 \times 8^5$

g  $4^3 \times 5^2$

h  $4 \times 6^7$

### Exercise 7.3

---

1 a 8

b 25

c 1

d 1

e 1

f  $\frac{1}{4}$



- 2 a  $\frac{1}{4}$   
b  $\frac{1}{9}$   
c  $\frac{3}{50}$   
d  $\frac{1}{200}$   
e 1  
f  $\frac{1}{1000}$
- 3 a 1  
b 2  
c 4  
d  $\frac{1}{2}$   
e  $\frac{1}{6}$   
f 10
- 4 a 12  
b 32  
c 225  
d 80  
e 7  
f 64

### Exercise 7.4

---

- 1 a 2  
b 4  
c 3  
d 3  
e 4  
f 0
- 2 a 4  
b 1  
c  $\frac{3}{2}$   
d -1  
e 2  
f 0
- 3 a -3  
b -4



c  $-5$

d  $-\frac{2}{3}$

e  $-2$

f  $-\frac{1}{2}$

4 a  $-3$

b  $-7$

c  $-3$

d  $2$

e  $8$

f  $5$

### Exercise 7.5

---

1 d and e

2 a  $6 \times 10^5$

b  $4.8 \times 10^7$

c  $7.84 \times 10^{11}$

d  $5.34 \times 10^5$

e  $7 \times 10^6$

f  $8.5 \times 10^6$

3 a  $6.8 \times 10^6$

b  $7.2 \times 10^8$

c  $8 \times 10^5$

d  $7.5 \times 10^7$

e  $4 \times 10^9$

f  $5 \times 10^7$

4 a 3800

b 4 250 000

c 90 030 000

d 101 000

5 a  $6 \times 10^5$

b  $2.4 \times 10^7$

c  $1.4 \times 10^8$

d  $3 \times 10^9$

e  $1.2 \times 10^{13}$

f  $1.8 \times 10^7$

6  $1.44 \times 10^{11} \text{ m} = 1.44 \times 10^8 \text{ km}$



- 7**   **a**    $8.8 \times 10^8$   
      **b**    $2.04 \times 10^{11}$   
      **c**    $3.32 \times 10^{11}$   
      **d**    $4.2 \times 10^{22}$   
      **e**    $5.1 \times 10^{22}$   
      **f**    $2.5 \times 10^{25}$
- 8**   **a**    $2 \times 10^2$   
      **b**    $3 \times 10^5$   
      **c**    $4 \times 10^6$   
      **d**    $2 \times 10^4$   
      **e**    $2.5 \times 10^6$   
      **f**    $4 \times 10^4$
- 9**   **a**    $4.26 \times 10^5$   
      **b**    $8.48 \times 10^9$   
      **c**    $6.388 \times 10^7$   
      **d**    $3.157 \times 10^9$   
      **e**    $4.5 \times 10^8$   
      **f**    $6.01 \times 10^7$   
      **g**    $8.15 \times 10^{10}$   
      **h**    $3.56 \times 10^7$

### Exercise 7.6

---

- 1**   **a**    $6 \times 10^{-4}$   
      **b**    $5.3 \times 10^{-5}$   
      **c**    $8.64 \times 10^{-4}$   
      **d**    $8.8 \times 10^{-8}$   
      **e**    $7 \times 10^{-7}$   
      **f**    $4.145 \times 10^{-4}$
- 2**   **a**    $6.8 \times 10^{-4}$   
      **b**    $7.5 \times 10^{-7}$   
      **c**    $4.2 \times 10^{-10}$   
      **d**    $8 \times 10^{-9}$   
      **e**    $5.7 \times 10^{-11}$   
      **f**    $4 \times 10^{-11}$
- 3**   **a**   0.008  
      **b**   0.000 42



c  $0.0903$

d  $0.000\,010\,1$

4 a  $-4$

b  $-3$

c  $-8$

d  $-5$

e  $-7$

f  $3$

5  $6.8 \times 10^5$   $6.2 \times 10^3$   $8.414 \times 10^2$   $6.741 \times 10^{-4}$   $3.2 \times 10^{-4}$   $5.8 \times 10^{-7}$   $5.57 \times 10^{-9}$

### Exercise 7.7

---

1 a  $4$

b  $5$

c  $10$

d  $3$

e  $9$

f  $10$

2 a  $2$

b  $3$

c  $2$

d  $2$

e  $6$

f  $4$

3 a  $8$

b  $32$

c  $27$

d  $64$

e  $1$

f  $9$

4 a  $25$

b  $8$

c  $32$

d  $100$

e  $32$

f  $27$

5 a  $\frac{1}{2}$



**b**  $\frac{1}{3}$

**c**  $\frac{1}{2}$

**d**  $\frac{1}{3}$

**e**  $\frac{1}{2}$

**f**  $\frac{1}{6}$

**6 a**  $\frac{1}{3}$

**b**  $\frac{1}{2}$

**c**  $\frac{1}{4}$

**d**  $\frac{1}{3}$

**e**  $\frac{1}{6}$

**f**  $\frac{1}{3}$

### Exercise 7.8

---

**1 a** 1

**b** 7

**c** 2

**d** 1

**e** 81

**f** 6

**2 a** 25

**b** 2

**c** 2

**d** 27

**e** 4

**f** 2

**3 a** 4

**b** 2

**c** 64

**d** 9

**e** 3

**f** 27



### Exercise 7.9

---

1 a 9

b 10

c 12

d 12

e 50

f 54

2 a  $\sqrt{10}$

b 3

c 2

d  $\frac{\sqrt{15}}{15}$

e  $\frac{18}{7}$

f 100

3 a  $2\sqrt{2}$

b  $5\sqrt{2}$

c  $3\sqrt{2}$

d  $3\sqrt{5}$

e  $5\sqrt{3}$

f  $6\sqrt{2}$

g  $10\sqrt{7}$

h  $9\sqrt{2}$

i  $7\sqrt{2}$

j  $11\sqrt{2}$

k  $8\sqrt{3}$

l  $15\sqrt{2}$

4 a  $\frac{\sqrt{6}}{6}$

b  $\frac{3\sqrt{5}}{5}$

c  $\frac{\sqrt{2}}{2}$

d  $\sqrt{6}$

e  $\frac{3\sqrt{2}}{10}$

f  $-\frac{\sqrt{5}}{10}$

g  $\frac{\sqrt{10}}{8}$



- h  $\frac{\sqrt{6}}{16}$
- 5 a  $\frac{5 - \sqrt{3}}{11}$
- b  $2 + 2\sqrt{2}$
- c  $\frac{5\sqrt{5} - 5\sqrt{2}}{3}$
- d  $\frac{35\sqrt{3} - 14\sqrt{6}}{17}$
- e  $-\frac{5 + 4\sqrt{2}}{7}$
- f  $-9 - 4\sqrt{5}$
- 6 a Area =  $6\sqrt{3} \text{ cm}^2$
- b Perimeter =  $(6\sqrt{2} + 2\sqrt{6}) \text{ cm}$
- 7  $(10 + 2\sqrt{5}) \text{ cm}$
- 8  $\sqrt{3} \text{ cm}$
- 9  $3\sqrt{2} \text{ cm}$
- 10 a  $48 \text{ cm}^2$
- b  $12 \text{ cm}^2$
- c  $2\sqrt{3} \text{ cm}$
- d  $20\sqrt{3} \text{ cm}$

### Student assessment 1

---

- 1 a  $2^2 \times 3^5$
- b  $2^{14}$
- 2 a  $6 \times 6 \times 6 \times 6 \times 6$
- b  $\frac{1}{2 \times 2 \times 2 \times 2 \times 2}$
- 3 a 27 000
- b 125
- 4 a  $2^7$
- b  $7^7 \times 3^{12}$
- c  $2^6$
- d  $3^3$
- e  $4^{-1}$
- f  $2^2$
- 5 a 5
- b 16
- c 49



d 48

6 a 2.5

b  $-0.5$

c 0

d  $\frac{2}{9}$

## Student assessment 2

---

1 a  $6 \times 10^6$

b  $4.5 \times 10^{-3}$

c  $3.8 \times 10^9$

d  $3.61 \times 10^{-7}$

e  $4.6 \times 10^8$

f  $3 \times 10^0$

2 a 8 112 000

b 440 000

c 0.000 305

3  $4.05 \times 10^8$     $3.6 \times 10^2$     $9 \times 10^1$     $.5 \times 10^{-2}$     $7.2 \times 10^{-3}$     $2.1 \times 10^{-3}$

4 a  $1.5 \times 10^7$     $4.3 \times 10^5$     $4.35 \times 10^{-4}$     $4.8 \times 10^0$     $8.5 \times 10^{-3}$

b  $4.35 \times 10^{-4}$     $8.5 \times 10^{-3}$     $4.8 \times 10^0$     $4.3 \times 10^5$     $1.5 \times 10^7$

5 a 3

b 9

c  $-3$

d 6

e  $-1$

f 8

6 a  $1.2 \times 10^8$

b  $1.48 \times 10^{11}$

c  $6.73 \times 10^7$

d  $3.88 \times 10^6$

7 43.2 minutes (3 s.f.)

8  $2.84 \times 10^{15}$  km (3 s.f.)

## Student assessment 3

---

1 a 9

b 3

c 3



d 125

e 49

f 0.5

g 5

h 16

2 a 1

b 9

c 4

d 25

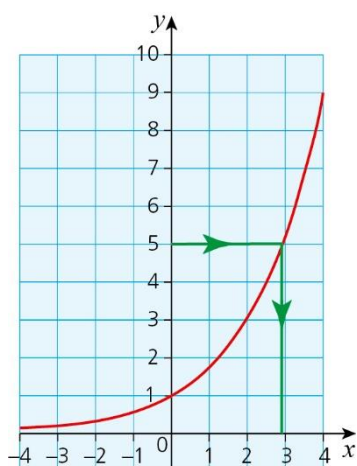
e 2

f 8

g 1

h 45

3 a



b Approx. 2.9

4 a  $5\sqrt{5}$

b  $4\sqrt{5}$

c  $6\sqrt{3}$

5 a  $\frac{5\sqrt{2}}{4}$

b  $-\frac{2+\sqrt{7}}{3}$

6  $45\pi \text{ cm}^2$

## Student assessment 4

1 a 2

b 81



c  $\frac{1}{3}$

d 64

e 3

f 2

g 6

h 32

2 a 15

b 4

c 3

d 3125

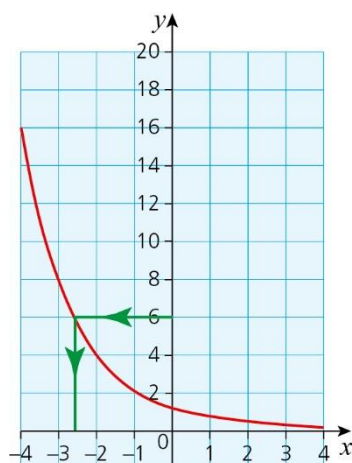
e 4

f 1

g 27

h 10

3 a



b Approx. -2.6

4 a 108

b  $24\sqrt{3}$

5 a  $\frac{7\sqrt{3}}{9}$

b  $\frac{8-2\sqrt{7}}{9}$

6  $(2 + 6\sqrt{3}) \text{ cm}^2$



*All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.*

## **8 Money and finance**

### **Exercise 8.1**

---

- 1
  - a A\$37.50
  - b 3750 rupees
  - c ZIM\$8 256 000
  - d 4500 rand
  - e L520
  - f 5200 yen
  - g 140 dinar
  - h US\$172.50
- 2
  - a €333.33
  - b €2.67
  - c €0.02
  - d €33.33
  - e €187.50
  - f €9.23
  - g €142.86
  - h €130.43

### **Exercise 8.2**

---

- 1 \$154.82
- 2 \$182
- 3 \$131
- 4 \$290.50
- 5
  - a \$195.05
  - b \$132.63
- 6 \$137.50
- 7 \$525
- 8
  - a \$298
  - b \$253.30



### Exercise 8.3

---

- 1 \$1.80 loss
- 2 \$2.88 profit
- 3 \$54.65 profit per seat
- 4 \$240 extra
- 5 \$250 loss

### Exercise 8.4

---

- 1 a 11%  
b 25%
- 2 a 30%  
b 20%
- 3 Type A = 30%  
Type B = 15.4%  
Type C = 33.3%  
Type C makes most percentage profit.
- 4 80%

### Exercise 8.5

---

- 1 a \$72  
b \$420
- 2 a  $t = 5$   
b  $t = 7$
- 3 a  $r = 7$   
b  $r = 4$
- 4 a  $P = 200$   
b  $P = 850$
- 5  $r = 4$
- 6  $t = 2$
- 7  $r = 4.5$
- 8  $r = 9.5$
- 9 \$315
- 10  $r = 6$



### Exercise 8.6

---

- 1 \$11 033 750
- 2 \$52 087.50
- 3 \$10 368
- 4 1331 students
- 5 3 276 800 tonnes
- 6 2 years
- 7 5 years
- 8 3 years

### Student assessment 1

---

- 1 a 3240 rupees  
b HK\$74.07
- 2 a €625  
b €135.14
- 3 \$122.40
- 4 \$26.16 (\$12.96 + \$13.20)
- 5 a \$72  
b 8%  
c 4 years  
d 7.5%  
e \$1250
- 6 \$6000
- 7 30%

### Student assessment 2

---

- 1 a \$82.50  
b \$4800  
c \$2187.50
- 2 3.5%
- 3 5 years
- 4 a Option A: \$4000 interest      Option B: \$3439.16 interest  
Option B is cheapest  
b \$560.84
- 5 a \$24.36  
b \$2969.24



**c** \$9953.45

**6 a i** \$10 625  
**ii** \$9031.25

**b** 16 years



*All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.*

## 9 Time

### Exercise 9.1

---

- 1
  - a  $0^{\circ} 15' 0''$  (15 mins)
  - b  $3^{\circ} 45' 54''$  (3 hrs 45 mins and 54 secs)
  - c  $0^{\circ} 13' 12''$  (13 mins and 12 secs)
- 2 Arrival at stop 1    06 45  
Arrival at stop 2    08 03  
Arrival at stop 3    09 42  
Arrival at stop 4    12 36 and 36 seconds
- 3
  - a 08 40
  - b 18 45
- 4 08 25
- 5
  - a 2 h 18 min
  - b 1 h 24 min
- 6 1st: 30 min    2nd: 32 min 44 s    3rd: 34 min 17 s    4th: 35 min 7s    5th: 36 min
- 7 2.10 a.m. on Wednesday
- 8 21 45
- 9 11 15

### Student assessment 1

---

- 1 09 02
- 2 5 h 54 min
- 3 49 km
- 4 11 h 5 min

### Student assessment 2

---

- 1 08 18
- 2 2 h 15 min
- 3 154 km
- 4
  - a 3 h 30 min or 3.5 h
  - b 90 km/h



*All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.*

## 10 Set notation and Venn diagrams

### Exercise 10.1

---

- 1
  - a
    - i Continents of the world
    - ii Student's own answers
  - b
    - i Even numbers
    - ii Student's own answers
  - c
    - i Days of the week
    - ii Student's own answers
  - d
    - i Months with 31 days
    - ii Student's own answers
  - e
    - i Triangle numbers
    - ii Student's own answers
  - f
    - i Boy's names beginning with the letter M
    - ii Student's own answers
  - g
    - i Prime numbers greater than 7
    - ii Student's own answers
  - h
    - i Vowels
    - ii o, u
  - i
    - i Planets of the solar system
    - ii Student's own answers
  - j
    - i Numbers between 3 and 12 inclusive
    - ii Student's own answers
  - k
    - i Numbers between  $-5$  and  $5$  inclusive
    - ii Student's own answers
- 2
  - a 7
  - c 7
  - d 7
  - f Unquantifiably finite, though theoretically infinite
  - h 5
  - i 8



### Exercise 10.2

---

- 1
  - a  $Q = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28\}$
  - b  $R = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29\}$
  - c  $S = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$
  - d  $T = \{1, 4, 9, 16, 25\}$
  - e  $U = \{1, 3, 6, 10, 15, 21, 28\}$
- 2
  - a  $B = \{55, 60, 65\}$
  - b  $C = \{51, 54, 57, 60, 63, 66, 69\}$
  - c  $D = \{64\}$
- 3
  - a True
  - b True
  - c True
  - d False
  - e True

### Exercise 10.3

---

- 1
  - a  $A \cap B = \{4, 6\}$
  - b  $A \cap B = \{4, 9\}$
  - c  $A \cap B = \{\text{yellow, green}\}$
- 2
  - a  $A \cup B = \{2, 3, 4, 6, 8, 9, 10, 13, 18\}$
  - b  $A \cup B = \{1, 4, 5, 6, 7, 8, 9, 16\}$
  - c  $A \cup B = \{\text{red, orange, blue, indigo, violet, yellow, green, purple, pink}\}$
- 3
  - a  $\mathcal{E} = \{a, b, p, q, r, s, t\}$
  - b  $A' = \{a, b\}$
- 4
  - a  $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8\}$
  - b  $A' = \{1, 4, 6, 8\}$
  - c  $A \cap B = \{2, 3\}$
  - d  $A \cup B = \{1, 2, 3, 4, 5, 7, 8\}$
  - e  $(A \cap B)' = \{1, 4, 5, 6, 7, 8\}$
  - f  $A \cap B' = \{5, 7\}$
- 5
  - a True
  - b True
  - c False
  - d False
  - e False



f True

- 6 a i  $A = \{\text{even numbers from 2 to 14}\}$   
ii  $B = \{\text{multiples of 3 from 3 to 15}\}$   
iii  $C = \{\text{multiples of 4 from 4 to 20}\}$   
b i  $A \cap B = \{6, 12\}$   
ii  $A \cap C = \{4, 8, 12\}$   
iii  $B \cap C = \{12\}$   
iv  $A \cap B \cap C = \{12\}$   
v  $A \cup B = \{2, 3, 4, 6, 8, 9, 10, 12, 14, 15\}$   
vi  $C \cup B = \{3, 4, 6, 8, 9, 12, 15, 16, 20\}$

- 7 a i  $A = \{1, 2, 4, 5, 6, 7\}$   
ii  $B = \{3, 4, 5, 8, 9\}$   
iii  $C' = \{1, 2, 3, 4, 5, 8, 9\}$   
iv  $A \cap B = \{4, 5\}$   
v  $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$   
vi  $(A \cap B)' = \{1, 2, 3, 6, 7, 8, 9\}$

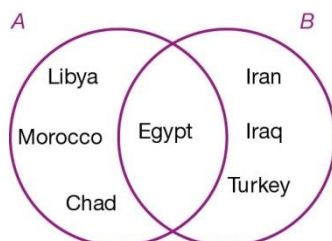
b  $C \subseteq A$

- 8 a i  $W = \{1, 2, 4, 5, 6, 7, 9, 10\}$   
ii  $X = \{2, 3, 6, 7, 8, 9\}$   
iii  $Z' = \{1, 4, 5, 6, 7, 8, 10\}$   
iv  $W \cap Z = \{2, 9\}$   
v  $W \cap X = \{2, 6, 7, 9\}$   
vi  $Y \cap Z = \emptyset$  or  $\{ \}$

b Z

## Exercise 10.4

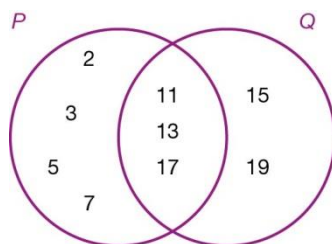
1 a



- b i  $A \cap B = \{\text{Egypt}\}$   
ii  $A \cup B = \{\text{Libya, Morocco, Chad, Egypt, Iran, Iraq, Turkey}\}$



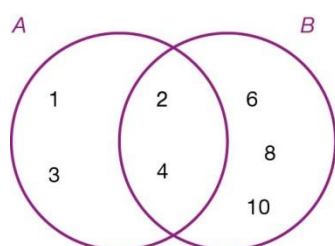
2 a



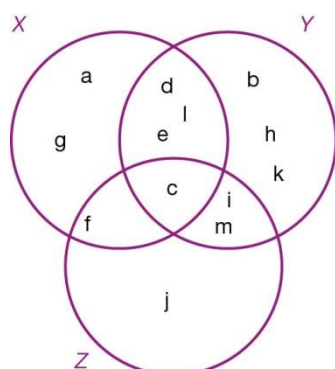
b i  $P \cap Q = \{11, 13, 17\}$

ii  $P \cup Q = \{2, 3, 5, 7, 11, 13, 15, 17, 19\}$

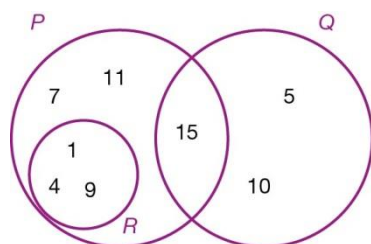
3



4



5



## Exercise 10.5

1 a 5

b 14

c 13

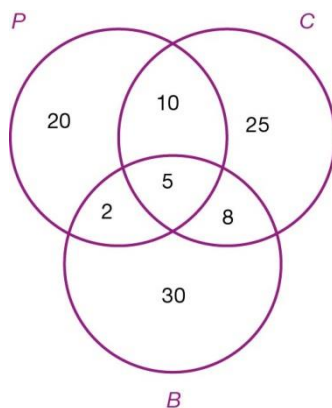
2 45

3 a 10

b 50



4 a



b 100

### Student assessment 1

1 a {even numbers from 2 to 8}

b {even numbers}

c {square numbers}

d {oceans}

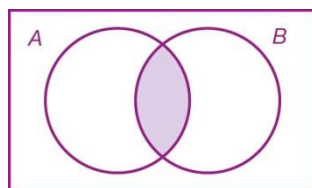
2 a 7

b 2

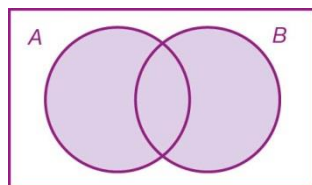
c 6

d 366

3 a



b



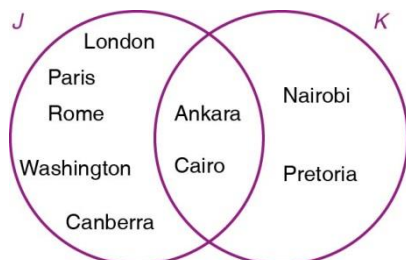
4  $\{a, b\}$ ,  $\{a\}$ ,  $\{b\}$ ,  $\emptyset$

5  $A' = \{m, t, h\}$



## Student assessment 2

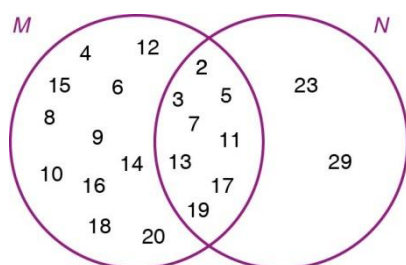
1 a



b {Ankara, Cairo}

c {Nairobi, Pretoria}

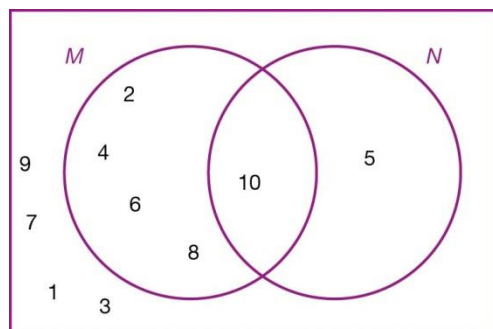
2 a



b {2, 3, 5, 7, 11, 13, 17, 19}

c {4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 23, 29}

3 a

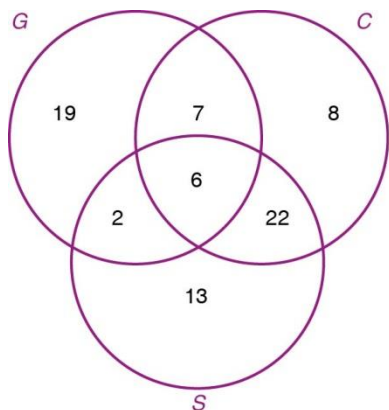


b  $X = \{\text{multiples of } 10\}$

4 {2, 4}, {2, 6}, {2, 8}, {4, 6}, {4, 8}, {6, 8}, {2, 4, 6}, {2, 4, 8}, {2, 6, 8}, {4, 6, 8}, {2, 4, 6, 8}



5 a



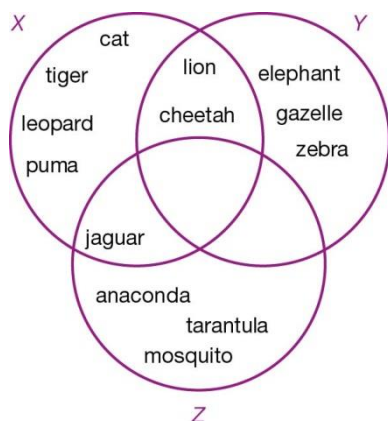
b 6

### Student assessment 3

1 a 32

b {a, e, i, o, u}, {a, e, i, o}, {a, e, i, u}, {a, e, o, u}, {a, i, o, u}, {e, i, o, u}

2 a

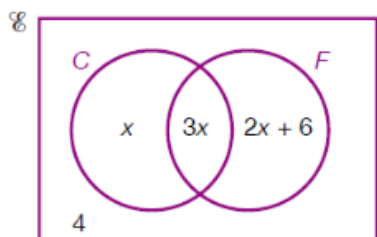


b {lion, cheetah}

c  $\emptyset$

d  $\emptyset$

3 a Let the number liking only cricket be  $x$



b 15

c 5

d 16



- 4**   **a**   5  
      **b**   35  
      **c**   40  
      **d**   50  
      **e**   15  
      **f**   12  
      **g**   10  
      **h**   78  
      **i**   78



## Mathematical investigations and ICT 1

### Primes and squares

---

- 1  $2^2 + 3^2 = 13$   
 $4^2 + 5^2 = 41$
- 2 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
- 3 5, 13, 17, 29, 37, 41, 53, 61, 73, 89, 97
- 4 If a number generated by the rule  $4n + 1$  is a prime, then it can be expressed as the sum of two square numbers. (This was proved by Fermat in the 17th century.) Alternatively, add 1 to the prime number and divide the result by 2. If the answer is even then the prime number cannot be expressed as the sum of two squares.
- 5 The rule works for the numbers shown, but this does not prove that it always works.

### Football leagues

---

- 1  $(17 + 16 + 15 + \dots + 1) \times 2 = 306$
- 2  $n = t(t - 1)$

### ICT activity 1

---

- 2 Student's spreadsheet
- 3 e.g. in cell C4 enter  $=B4/B\$3*100$
- 4 Student's spreadsheet
- 5 Student's report

### ICT activity 2

---

Student's graphs and responses will vary except:

- 6 The gradient of the tangent at a point represents the speed of the runner at that point.



*All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.*

## 11 Algebraic representation and manipulation

### Exercise 11.1

---

- 1
  - a  $4x - 12$
  - b  $10p - 20$
  - c  $-42x + 24y$
  - d  $6a - 9b - 12c$
  - e  $-14m + 21n$
  - f  $-16x + 6y$
- 2
  - a  $3x^2 - 9xy$
  - b  $a^2 + ab + ac$
  - c  $8m^2 - 4mn$
  - d  $-15a^2 + 20ab$
  - e  $4x^2 - 4xy$
  - f  $24p^2 - 8pq$
- 3
  - a  $-2x^2 + 3y^2$
  - b  $a - b$
  - c  $7p - 2q$
  - d  $3x - 4y + 2z$
  - e  $3x - \frac{3}{2}y$
  - f  $2x^2 - 3xy$
- 4
  - a  $12r^3 - 15rs + 6rt$
  - b  $a^3 + a^2b + a^2c$
  - c  $6a^3 - 9a^2b$
  - d  $p^2q + pq^2 - p^2q^2$
  - e  $m^3 - m^2n + m^3n$
  - f  $a^6 + a^5b$

### Exercise 11.2

---

- 1
  - a  $-a - 8$
  - b  $4x - 20$
  - c  $3p - 16$
  - d  $21m - 6n$



e 3

f  $-p - 3p^2$

2 a  $8m^2 + 28m + 2$

b  $x - 4$

c  $2p + 22$

d  $m - 12$

e  $a^2 + 6a + 2$

f  $7ab - 16ac + 3c$

3 a  $4x + 4$

b  $5x - \frac{3}{2}y$

c  $\frac{9}{4}x - \frac{5}{2}y$

d  $\frac{9}{2}x + \frac{1}{2}y$

e  $7x - 4y$

f 0

### Exercise 11.3

---

1 a  $x^2 + 5x + 6$

b  $x^2 + 7x + 12$

c  $x^2 + 7x + 10$

d  $x^2 + 7x + 6$

e  $x^2 + x - 6$

f  $x^2 + 5x - 24$

2 a  $x^2 + 2x - 24$

b  $x^2 - 3x - 28$

c  $x^2 - 2x - 35$

d  $x^2 - 2x - 15$

e  $x^2 - 2x - 3$

f  $x^2 + 2x - 63$

3 a  $2x^2 - 9x + 9$

b  $2x^2 - 9x + 10$

c  $3x^2 - 20x + 32$

d  $5x^2 + 12x - 9$

e  $4x^2 - 12x + 9$

f  $6x^2 - 31x + 35$

4 a  $x^2 - 9$



**b**  $x^2 - 49$

**c**  $x^2 - 64$

**d**  $x^2 - y^2$

**e**  $a^2 - b^2$

**f**  $p^2 - q^2$

### Exercise 11.4

---

**1 a**  $2(2x - 3)$

**b**  $6(3 - 2p)$

**c**  $3(2y - 1)$

**d**  $2(2a + 3b)$

**e**  $3(p - q)$

**f**  $4(2m + 3n + 4r)$

**2 a**  $a(3b + 4c - 5d)$

**b**  $2p(4q + 3r - 2s)$

**c**  $a(a - b)$

**d**  $2x(2x - 3y)$

**e**  $ab(c + d + f)$

**f**  $3m(m + 3)$

**3 a**  $3pq(r - 3s)$

**b**  $5m(m - 2n)$

**c**  $4xy(2x - y)$

**d**  $b^2(2a^2 - 3c^2)$

**e**  $12(p - 3)$

**f**  $6(7x - 9)$

**4 a**  $6(3 + 2y)$

**b**  $7(2a - 3b)$

**c**  $11x(1 + y)$

**d**  $4(s - 4t + 5r)$

**e**  $5q(p - 2r + 3s)$

**f**  $4y(x + 2y)$

**5 a**  $m(m + n)$

**b**  $3p(p - 2q)$

**c**  $qr(p + s)$

**d**  $ab(1 + a + b)$

**e**  $p^3(3 - 4p)$



f  $b^2c(7b + c)$

6 a  $m(m^2 - mn + n^2)$

b  $2r^2(2r - 3 + 4s)$

c  $28xy(2x - y)$

d  $18mn(4m + 2n - mn)$

### Exercise 11.5

---

1 a 0

b 30

c 14

d 20

e -13

f -4

2 a -3

b -30

c 20

d -16

e -40

f 42

3 a -160

b -23

c 42

d -17

e -189

f 113

4 a 48

b -8

c 15

d 16

e -5

f 9

5 a 12

b -5

c -5

d 7

e 7



f 36

### Exercise 11.6

---

- 1 a  $n = r - m$   
b  $m = p - n$   
c  $n = 3p - 2m$   
d  $q = 3x - 2p$   
e  $a = \frac{cd}{b}$   
f  $d = \frac{ab}{c}$
- 2 a  $x = \frac{4m}{3y}$   
b  $r = \frac{7pq}{5}$   
c  $x = \frac{c}{3}$   
d  $x = \frac{y-7}{3}$   
e  $y = \frac{3r+9}{5}$   
f  $x = \frac{5y-9}{3}$
- 3 a  $b = \frac{2a-5}{6}$   
b  $a = \frac{6b+5}{2}$   
c  $z = \frac{3x-7y}{4}$   
d  $x = \frac{4z+7y}{3}$   
e  $y = \frac{3x-4z}{7}$   
f  $p = \frac{8+q}{2r}$
- 4 a  $p = 4r$   
b  $p = \frac{4}{3r}$   
c  $p = \frac{n}{10}$   
d  $n = 10p$   
e  $p = \frac{2t}{q+r}$   
f  $q = \frac{2t}{p} - r$
- 5 a  $r = \frac{3m-n}{t(p+q)}$   
b  $t = \frac{3m-n}{r(p+q)}$   
c  $m = \frac{rt(p+q)+n}{3}$



d  $n = 3m - rt(p + q)$

e  $p = \frac{3m-n}{rt} - q$

f  $q = \frac{3m-n}{rt} - p$

6 a  $d = \frac{ab}{ce}$

b  $a = \frac{dec}{b}$

c  $c = \frac{ab}{de}$

d  $a = cd - b$

e  $b = d - \frac{a}{c}$

f  $c = \frac{a}{d-b}$

### Exercise 11.7

---

1 a  $2y^2 + 7y + 6$

b  $3y^2 + 25y + 28$

c  $2y^2 + 17y + 8$

d  $4y^2 + 6y + 2$

e  $6y^2 + 23y + 20$

f  $18y^2 + 15y + 3$

2 a  $2p^2 + 13p - 24$

b  $4p^2 + 23p - 35$

c  $6p^2 + p - 12$

d  $12p^2 + 13p - 35$

e  $18p^2 - 2$

f  $28p^2 + 44p - 24$

3 a  $4x^2 - 4x + 1$

b  $9x^2 + 6x + 1$

c  $16x^2 - 16x + 4$

d  $25x^2 - 40x + 16$

e  $4x^2 + 24x + 36$

f  $4x^2 - 9$

4 a  $-4x^2 + 9$

b  $16x^2 - 9$

c  $-16x^2 + 9$

d  $-25y^2 + 49$

e  $8y^2 - 18$



f  $25y^2 - 70y + 49$

5 a  $3x^3 + 16x^2 + 23x + 6$

b  $4x^3 + 2x^2 - 16x - 8$

c  $-12x^3 + 7x^2 + 8x - 3$

d  $-2x^3 + 9x^2 + 8x - 15$

e  $-2x^3 + 11x^2 - 13x + 4$

f  $-12x^3 - 9x^2 - 5x + 2$

### Exercise 11.8

---

1 a  $(x + y)(a + b)$

b  $(x - y)(a + b)$

c  $(3 + x)(m + n)$

d  $(m + n)(4 + x)$

e  $(m - n)(3 + x)$

f  $(x + z)(6 + y)$

2 a  $(p + q)(r - s)$

b  $(p + 3)(q - 4)$

c  $(q - 4)(p + 3)$

d  $(r + 2t)(s + t)$

e  $(s + t)(r - 2t)$

f  $(b + c)(a - 4c)$

3 a  $(y + x)(x + 4)$

b  $(x - 2)(x - y)$

c  $(a - 7)(b + 3)$

d  $(b - 1)(a - 1)$

e  $(p - 4)(q - 4)$

f  $(m - 5)(n - 5)$

4 a  $(m - 3)(n - 2)$

b  $(m - 3r)(n - 2r)$

c  $(p - 4q)(r - 4)$

d  $(a - c)(b - 1)$

e  $(x - 2y)(x - 2z)$

f  $(2a + b)(a + b)$

### Exercise 11.9

---

1 a  $(a - b)(a + b)$



**b**  $(m - n)(m + n)$

**c**  $(x - 5)(x + 5)$

**d**  $(m - 7)(m + 7)$

**e**  $(9 - x)(9 + x)$

**f**  $(10 - y)(10 + y)$

**2 a**  $(12 - y)(12 + y)$

**b**  $(q - 13)(q + 13)$

**c**  $(m - 1)(m + 1)$

**d**  $(1 - t)(1 + t)$

**e**  $(2x - y)(2x + y)$

**f**  $(5p - 8q)(5p + 8q)$

**3 a**  $(3x - 2y)(3x + 2y)$

**b**  $(4p - 6q)(4p + 6q)$

**c**  $(8x - y)(8x + y)$

**d**  $(x - 10y)(x + 10y)$

**e**  $(qr - 2p)(qr + 2p)$

**f**  $(ab - cd)(ab + cd)$

**4 a**  $(mn - 3y)(mn + 3y)$

**b**  $\left(\frac{1}{2}x - \frac{1}{3}y\right)\left(\frac{1}{2}x + \frac{1}{3}y\right)$

**c**  $(2x - 9y^2)(2x + 9y^2)$

**d**  $(p^2 - q^2)(p^2 + q^2) = (p - q)(p + q)(p^2 + q^2)$

**e**  $4(m^2 - 3y^2)(m^2 + 3y^2)$

**f**  $(4x^2 - 9y^2)(4x^2 + 9y^2) = (2x - 3y)(2x + 3y)(4x^2 + 9y^2)$

### Exercise 11.10

---

**1 a** 60

**b** 240

**c** 2400

**d** 280

**e** 7600

**f** 9200

**2 a** 2000

**b** 9800

**c** 200

**d** 3200

**e** 998 000



f 161

3 a 68

b 86

c 1780

d 70

e 55

f 5

4 a 72.4

b 0.8

c 65

d 15

e 1 222 000

f 231

### Exercise 11.11

---

1 a  $(x + 4)(x + 3)$

b  $(x + 6)(x + 2)$

c  $(x + 12)(x + 1)$

d  $(x - 3)(x - 4)$

e  $(x - 6)(x - 2)$

f  $(x - 12)(x - 1)$

2 a  $(x + 5)(x + 1)$

b  $(x + 4)(x + 2)$

c  $(x + 3)^2$

d  $(x + 5)^2$

e  $(x + 11)^2$

f  $(x - 6)(x - 7)$

3 a  $(x + 12)(x + 2)$

b  $(x + 8)(x + 3)$

c  $(x - 6)(x - 4)$

d  $(x + 12)(x + 3)$

e  $(x + 18)(x + 2)$

f  $(x - 6)^2$

4 a  $(x + 5)(x - 3)$

b  $(x - 5)(x + 3)$

c  $(x + 4)(x - 3)$



d  $(x-4)(x+3)$

e  $(x+6)(x-2)$

f  $(x-12)(x-3)$

5 a  $(x-4)(x+2)$

b  $(x-5)(x+4)$

c  $(x+6)(x-5)$

d  $(x+6)(x-7)$

e  $(x+7)(x-9)$

f  $(x+9)(x-6)$

6 a  $(2x+1)(x+1)$

b  $(2x+3)(x+2)$

c  $(2x-3)(x+2)$

d  $(2x-3)(x-2)$

e  $(3x+2)(x+2)$

f  $(3x-1)(x+4)$

g  $(2x+3)^2$

h  $(3x-1)^2$

i  $(3x+1)(2x-1)$

### Exercise 11.12

---

1 a  $x = \frac{P}{2m}$

b  $x = \frac{P}{Qr}$

2 a  $t = \frac{v-u}{a}$

b  $s = \frac{v^2-u^2}{2a}$

c  $u = \frac{s}{t} - \frac{1}{2}at$

d  $a = \frac{2(s-ut)}{t^2}$

### Exercise 11.13

---

1 a  $x = \pm\sqrt{\frac{T}{3}}$

b  $x = \pm\sqrt{\frac{y^2}{m}}$

c  $x = \pm\sqrt{p^2 - q^2 - y^2}$

d  $x = \pm\sqrt{y^2 - n^2 - m^2}$



e  $x = \pm \frac{\sqrt{p^2 - q^2 + y^2}}{2}$

2 a  $x = \pm \sqrt{\frac{P}{Qr}}$

b  $x = \pm \sqrt{\frac{Pr}{Q}}$

c  $x = \pm \sqrt{\frac{n}{m}}$

d  $x = \pm \sqrt{\frac{wst}{r}}$

e  $x = \pm \sqrt{\frac{wr}{p+q}}$

3 a  $x = (rp)^2$

b  $x = \left(\frac{mn}{p}\right)^2$

c  $x = \frac{k}{g^2}$

d  $x = \frac{r^2 g}{4\pi^2}$

e  $x = \frac{4m^2 r}{p^2}$

f  $x = \frac{4m^2 r}{p^2}$

4 a  $u = \pm \sqrt{v^2 - 2as}$

b  $t = \pm \sqrt{\frac{2(s-ut)}{a}}$

5 a  $r = \frac{A}{\pi\sqrt{s^2 + t^2}}$

b  $h = \pm \sqrt{\left(\frac{A}{\pi r}\right)^2 - r^2}$

c  $u = \frac{vf}{v-f}$

d  $v = \frac{uf}{u-f}$

e  $l = \left(\frac{t}{2\pi}\right)^2 g$

f  $g = l \left(\frac{2\pi}{t}\right)^2$

6 a  $x = \pm \sqrt{\frac{7(p+2)}{3t}}$

b  $a = \pm \sqrt{4 + (b-3)^2}$

### Exercise 11.14

1 a  $0.53 \text{ m}^3$  (2 d.p.)



**b**  $r = \sqrt{\frac{V}{\pi h}}$

**c** 5.05 cm (3 s.f.)

**2 a** 66 °C (2 s.f.)

**b** -11 °C (2 s.f.)

**c**  $F = \frac{9C}{5} + 32$

**d** 320 °F

**3 a** 15 hours

**b**  $H = 1200(T - k)$

**c** 5000 m

**4 a** \$251.50

**b**  $n = \frac{x-1.50}{0.05}$

**c** 470

**5 a** 524 cm<sup>3</sup> (3 s.f.)

**b**  $r = \sqrt[3]{\frac{3V}{4\pi}}$

**c** 8.42 m (3 s.f.)

### Exercise 11.15

---

**1 a**  $\frac{xp}{yq}$

**b**  $\frac{q}{y}$

**c**  $\frac{p}{r}$

**d**  $\frac{d}{c}$

**e**  $\frac{bd}{c^2}$

**f**  $p$

**2 a**  $m^2$

**b**  $r^5$

**c**  $x^6$

**d**  $xy^2$

**e**  $abc^3$

**f**  $\frac{r^3}{pq}$

**3 a**  $\frac{2x}{y}$

**b**  $4q^2$

**c**  $5n$



d  $3x^3y$

e  $3p$

f  $\frac{2}{3mn}$

4 a  $\frac{2a}{3b}$

b  $\frac{2y}{x}$

c  $2$

d  $3xy$

e  $3$

f  $6xy$

5 a  $\frac{8y}{3}$

b  $\frac{5p}{2}$

c  $\frac{p^3}{s}$

d  $\frac{ef}{c^2}$

6 a  $\frac{20x}{3}$

b  $\frac{9b}{8}$

### Exercise 11.16

---

1 a  $\frac{4}{7}$

b  $\frac{a+b}{7}$

c  $\frac{11}{13}$

d  $\frac{c+d}{13}$

e  $\frac{x+y+z}{3}$

f  $\frac{p^2+q^2}{5}$

2 a  $\frac{3}{11}$

b  $\frac{c-d}{11}$

c  $\frac{4}{a}$

d  $\frac{2a-5b}{3}$

e  $\frac{2x-3y}{7}$

f  $-\frac{1}{2x}$

3 a  $\frac{1}{2}$



b  $\frac{3}{2a}$

c  $\frac{5}{3c}$

d  $\frac{7}{2x}$

e  $\frac{3}{2p}$

f  $-\frac{1}{2w}$

4 a  $\frac{3p-q}{12}$

b  $\frac{x-2y}{4}$

c  $\frac{3m-n}{9}$

d  $\frac{x-2y}{12}$

e  $\frac{5r+m}{10}$

f  $\frac{5s-t}{15}$

5 a  $\frac{7x}{12}$

b  $\frac{9x-2y}{15}$

c  $\frac{m}{2}$

d  $\frac{m}{p}$

e  $\frac{x}{2y}$

f  $\frac{4r}{7s}$

### Exercise 11.17

---

1 a  $\frac{3a+2b}{6}$

b  $\frac{5a+3b}{15}$

c  $\frac{7p+4q}{28}$

d  $\frac{6a+5b}{15}$

e  $\frac{9x+20y}{36}$

f  $\frac{10x+14y}{35}$

2 a  $\frac{a}{6}$

b  $\frac{2a}{15}$

c  $\frac{11p}{28}$

d  $\frac{11a}{15}$



e  $\frac{29x}{36}$

f  $\frac{24x}{35}$

3 a  $\frac{m}{10}$

b  $\frac{r}{10}$

c  $-\frac{x}{4}$

d  $\frac{29x}{28}$

e  $\frac{23x}{6}$

f  $\frac{p}{6}$

4 a  $\frac{p}{2}$

b  $\frac{2c}{3}$

c  $\frac{4x}{5}$

d  $\frac{m}{3}$

e  $\frac{q}{5}$

f  $\frac{w}{4}$

5 a  $\frac{3m}{2}$

b  $\frac{7m}{3}$

c  $-\frac{m}{2}$

d  $\frac{5m}{2}$

e  $\frac{p}{3}$

f  $\frac{36q}{7}$

6 a  $\frac{pr-p}{r}$

b  $\frac{x+xy}{y}$

c  $\frac{mn+m}{n}$

d  $\frac{a+ab}{b}$

e  $\frac{2xy-x}{y}$

f  $\frac{2pq-3p}{q}$

7 a  $\frac{5a+12}{6}$

b  $\frac{11b-20}{15}$

c  $\frac{3c-2}{4}$



d  $\frac{25d-54}{14}$

### Exercise 11.18

---

1 a  $\frac{3x+4}{(x+1)(x+2)}$

b  $\frac{m-7}{(m+2)(m-1)}$

c  $\frac{3p-7}{(p-3)(p-2)}$

d  $\frac{w+11}{(w-1)(w+3)}$

e  $\frac{-12}{(y+4)(y+1)}$

f  $\frac{12-m}{(m-2)(m+3)}$

2 a  $\frac{x}{x+2}$

b  $\frac{y}{y+3}$

c  $\frac{m+2}{m-3}$

d  $\frac{p}{p-5}$

e  $\frac{m}{m+4}$

f  $\frac{m+1}{m+2}$

3 a  $\frac{x}{x+3}$

b  $\frac{x}{x+4}$

c  $\frac{y}{y-3}$

d  $\frac{x}{x+3}$

e  $\frac{x}{x+2}$

f  $\frac{x}{x+1}$

4 a  $\frac{x}{x+1}$

b  $\frac{x}{x+3}$

c  $\frac{x}{x-3}$

d  $\frac{x}{x+2}$

e  $\frac{x}{x-3}$

f  $\frac{x}{x+7}$



### Student assessment 1

---

- 1 a  $10a - 30b + 15c$   
b  $15x^2 - 27x$   
c  $-15xy^2 - 5y^3$   
d  $15x^3y + 9x^2y^2 - 3x^5$   
e  $12 - p$   
f  $14m^2 - 14m$   
g  $4x + 3$   
h  $\frac{13}{2}m^2 - 3m$
- 2 a  $4(3a - b)$   
b  $x(x - 4y)$   
c  $4p^2(2p - q)$   
d  $8xy(3 - 2x + y)$
- 3 a  $-21$   
b  $26$   
c  $43$   
d  $7$   
e  $12$   
f  $12$
- 4 a  $q = x - 3p$   
b  $n = \frac{3m-8r}{5}$   
c  $y = \frac{2mt}{3}$   
d  $w = \frac{2y}{x} - y$   
e  $p = \frac{xyt}{2rs}$   
f  $x = w(m + n) - y$

### Student assessment 2

---

- 1 a  $6x - 9y + 15z$   
b  $8pm - 28p$   
c  $-8m^2n + 4mn^2$   
d  $20p^3q - 8p^2q^2 - 8p^3$   
e  $-2x - 2$   
f  $22x^2 - 14x$   
g  $2$



h  $\frac{5}{2}x^2 - x$

2 a  $8(2p - q)$

b  $p(p - 6q)$

c  $5pq(p - 2q)$

d  $3pq(3 - 2p + 4q)$

3 a 0

b -7

c 29

d 7

e 7

f 35

4 a  $n = p - 4m$

b  $y = \frac{4x-5z}{3}$

c  $y = \frac{10px}{3}$

d  $y = \frac{3w}{m} - x$

e  $r = \frac{pqt}{4mn}$

f  $q = r(m - n) - p$

### Student assessment 3

---

1 a  $x^2 - 2x - 8$

b  $x^2 - 16x + 64$

c  $x^2 + 2xy + y^2$

d  $x^2 - 121$

e  $6x^2 - 13x + 6$

f  $9x^2 - 30x + 25$

2 a  $(q + r)(p - 3r)$

b  $(1 - t^2)(1 + t^2) = (1 - t)(1 + t)(1 + t^2)$

c 750 000

d 50

3 a  $(x - 11)(x + 7)$

b  $(x - 3)^2$

c  $(x - 12)(x + 12)$

d  $3(x + 3)(x - 2)$

e  $(2x - 3)(x + 4)$

f  $(2x - 5)^2$



4 a  $f = \pm \sqrt{\frac{p}{m}}$

b  $t = \pm \sqrt{\frac{m}{5}}$

c  $p = \left(\frac{A}{\pi r}\right)^2 - q$

d  $x = \frac{ty}{y-t}$

5 a  $x^4$

b  $nq$

c  $y^3$

d  $\frac{4}{q}$

e  $\frac{n^3}{2}$

f  $\frac{21bc^2}{4}$

6 a  $\frac{2m}{11}$

b  $-\frac{3p}{16}$

c  $\frac{3x}{4y}$

d  $\frac{6m+13n}{30p}$

e  $\frac{14-y}{3}$

f  $2y + \frac{9}{2}$

7 a  $\frac{7x-23}{(x-5)(x-2)}$

b  $\frac{a-b}{a+b}$

c  $\frac{1}{x+3}$

## Student assessment 4

---

1 a  $0.204 \text{ m}^3$  (3 s.f.)

b  $r = \sqrt{\frac{V}{\pi h}}$

c  $5.40 \text{ cm}$  (3 s.f.)

2 a  $2410 \text{ cm}^2$  (3 s.f.)

b  $h = \frac{A}{2\pi r} - r$

c  $10.9 \text{ cm}$  (3 s.f.)

3 a  $5.39$  (3 s.f.)

b  $3.68 \text{ m}$  (3 s.f.)



**c**  $x = \sqrt{d^2 - y^2 - z^2}$

**d** 0.713 (3 s.f.)

**4 a** 4.44 (3 s.f.)

**b**  $l = \frac{r^2 g}{4\pi^2}$

**c** 2.28 m (3 s.f.)



*All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.*

## 12 Algebraic indices

### Exercise 12.1

---

- 1 a  $c^8$   
b  $m^2$   
c  $b^9$   
d  $m^3n^6$   
e  $2a^4b$   
f  $3x^3y^2$   
g  $\frac{uv^6}{2y^3}$   
h  $\frac{x^2y^3z^2}{3}$
- 2 a  $12a^5$   
b  $8a^5b^3$   
c  $8p^6$   
d  $16m^4n^6$   
e  $200p^{13}$   
f  $32m^5n^{11}$   
g  $\frac{24x^6}{y^3}$   
h  $ab^{(d+e)}$

### Exercise 12.2

---

- 1 a  $c^3$   
b  $g$   
c  $q^{-2}$   
d  $m^{-1}$  or  $\frac{1}{m}$
- 2 a  $a^2$   
b  $\frac{r^{-6}}{p^{-6}}$  or  $\frac{p^6}{r^6}$   
c  $t^{16}$   
d  $m^9$



### Exercise 12.3

---

- 1 a  $a^{\frac{3}{5}}$   
b  $a^{\frac{1}{3}}$   
c  $a$   
d  $a^{\frac{3}{7}}$
- 2 a  $(\sqrt[7]{b})^2$   
b  $(\sqrt[3]{b})^8$   
c  $(\sqrt[5]{b})^{-2}$   
d  $(\sqrt[3]{b})^{-4}$
- 3 a  $a^{\frac{3}{4}}$   
b  $a^{\frac{3}{20}}$   
c  $a^{\frac{5}{2}}$   
d  $a^{-\frac{2}{3}}$
- 4 a  $(\sqrt[20]{b})^{19}$   
b  $(\sqrt[5]{b})^{-7}$   
c  $(\sqrt[3]{b})^{14}$   
d  $(\sqrt[6]{b})^{-11}$
- 5 a  $\frac{x^{\frac{5}{2}}}{12}$   
b  $2y^{-\frac{1}{3}}$   
c  $8p^{-\frac{5}{2}}$   
d  $\frac{9}{2x^{\frac{1}{3}}}$

### Student assessment 1

---

- 1 a  $a^3b^2$   
b  $d^2e^5$
- 2 a  $m \times m \times m$   
b  $r \times r \times r \times r$
- 3 a  $a^7$   
b  $p^5q^9$   
c  $b^3$   
d  $e^6$



- 4 a  $r^4$   
b  $\frac{1}{b^2}$  or  $b^{-2}$   
c  $\frac{1}{n^{-3}}$  or  $n^3$
- 5 a  $\frac{1}{p^9}$  or  $p^{-9}$   
b  $h^7$

## Student assessment 2

---

- 1 a  $a^{\frac{1}{8}}$   
b  $a^{-\frac{2}{5}}$
- 2 a  $(\sqrt[9]{b^4})$   
b  $(\sqrt[3]{b^{-2}})$
- 3 a  $a^{\frac{11}{6}}$   
b  $a^{\frac{19}{6}}$
- 4 a  $(\sqrt{t})^3$   
b  $(\sqrt[15]{t})^{-19}$



*All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.*

## 13 Equations and inequalities

### Exercise 13.1

---

- 1
  - a  $x = -4$
  - b  $y = 5$
  - c  $y = -5$
  - d  $p = -4$
  - e  $y = 8$
  - f  $x = -5.5$
- 2
  - a  $x = 4\frac{1}{3}$
  - b  $x = 5$
  - c  $x = 6$
  - d  $y = -8$
  - e  $y = 4$
  - f  $m = 10$
- 3
  - a  $m = 1$
  - b  $p = 3$
  - c  $k = -1$
  - d  $x = -21$
  - e  $x = 2$
  - f  $y = 3$
- 4
  - a  $x = 6$
  - b  $y = 14$
  - c  $x = 4$
  - d  $m = 12$
  - e  $x = 35$
  - f  $p = 20$
- 5
  - a  $x = 15$
  - b  $x = -5$
  - c  $x = 7.5$
  - d  $x = 8$
  - e  $x = 2.5$
  - f  $x = 10$



- 6**   **a**    $x = 5$   
      **b**    $x = 14$   
      **c**    $x = 22$   
      **d**    $x = 5$   
      **e**    $x = 8$   
      **f**    $x = 2$
- 7**   **a**    $y = 10$   
      **b**    $x = 17$   
      **c**    $x = 13$   
      **d**    $y = -5$   
      **e**    $x = 4$   
      **f**    $x = 6.5$
- 8**   **a**    $x = 5$   
      **b**    $x = 8$   
      **c**    $x = 6$   
      **d**    $x = -1$   
      **e**    $x = 6$   
      **f**    $x = \frac{1}{2}$

## Exercise 13.2

---

- 1**   **a**   **i**    $(3x + 60)^\circ$   
      **ii**    $3x + 60 = 180$   
      **iii**    $x = 40$   
      **iv**    $40^\circ, 60^\circ, 80^\circ$
- b**   **i**    $3x^\circ$   
      **ii**    $3x = 180$   
      **iii**    $x = 60$   
      **iv**    $20^\circ, 80^\circ, 80^\circ$
- c**   **i**    $18x^\circ$   
      **ii**    $18x = 180$   
      **iii**    $x = 10$   
      **iv**    $20^\circ, 50^\circ, 110^\circ$
- d**   **i**    $6x^\circ$   
      **ii**    $6x = 180$   
      **iii**    $x = 30$   
      **iv**    $30^\circ, 60^\circ, 90^\circ$



- e** i  $(7x - 30)^\circ$   
ii  $7x - 30 = 180$   
iii  $x = 30$   
iv  $10^\circ, 40^\circ, 130^\circ$

- f** i  $(9x - 45)^\circ$   
ii  $9x - 45 = 180$   
iii  $x = 25$   
iv  $25^\circ, 55^\circ, 100^\circ$

- 2 a** i  $12x^\circ$   
ii  $12x = 360$   
iii  $x = 30$   
iv  $90^\circ, 120^\circ, 150^\circ$   
**b** i  $(11x + 30)^\circ$   
ii  $11x + 30 = 360$   
iii  $x = 30$   
iv  $90^\circ, 135^\circ, 135^\circ$   
**c** i  $(12x + 60)^\circ$   
ii  $12x + 60 = 360$   
iii  $x = 25$   
iv  $35^\circ, 80^\circ, 90^\circ, 155^\circ$   
**d** i  $(10x + 30)^\circ$   
ii  $10x + 30 = 360$   
iii  $x = 33$   
iv  $33^\circ, 94^\circ, 114^\circ, 119^\circ$

- 3 a** i  $(11x - 80)^\circ$   
ii  $11x - 80 = 360$   
iii  $x = 40$   
iv  $40^\circ, 80^\circ, 80^\circ, 160^\circ$   
**b** i  $(10x + 60)^\circ$   
ii  $10x + 60 = 360$   
iii  $x = 30$   
iv  $45^\circ, 90^\circ, 95^\circ, 130^\circ$   
**c** i  $(16x + 8)^\circ$   
ii  $16x + 8 = 360$   
iii  $x = 22$   
iv  $44^\circ, 96^\circ, 100^\circ, 120^\circ$   
**d** i  $(9x + 25)^\circ$



- ii  $9x + 25 = 360$
- iii  $x = 37.2$  (to 1 d.p.)
- iv  $37^\circ, 57^\circ, 127^\circ, 139^\circ$  (to the nearest whole number)
- e i  $(12x + 36)^\circ$
- ii  $6x + 18 = 180$
- iii  $x = 27$
- iv  $50^\circ, 50^\circ, 130^\circ, 130^\circ$

- 4 a 20  
b 25  
c 14  
d 25  
e 31  
f 40

- 5 a 50  
b 13  
c 40  
d 40

- 6 a 5  
b 2  
c 7  
d 1.1  
e 25  
f 15

### Exercise 13.3

---

- 1 a  $x = 4$      $y = 2$   
b  $x = 6$      $y = 5$   
c  $x = 6$      $y = -1$   
d  $x = 5$      $y = 2$   
e  $x = 5$      $y = 2$   
f  $x = 4$      $y = 9$
- 2 a  $x = 3$      $y = 2$   
b  $x = 7$      $y = 4$   
c  $x = 1$      $y = 1$   
d  $x = 1$      $y = 5$   
e  $x = 1$      $y = 10$



**f**  $x = 8$      $y = 2$

**3 a**  $x = 5$      $y = 4$

**b**  $x = 4$      $y = 3$

**c**  $x = 10$      $y = 5$

**d**  $x = 6$      $y = 4$

**e**  $x = 4$      $y = 4$

**f**  $x = 10$      $y = -2$

**4 a**  $x = 5$      $y = 4$

**b**  $x = 4$      $y = 2$

**c**  $x = 5$      $y = 3$

**d**  $x = 5$      $y = -2$

**e**  $x = 1$      $y = 5$

**f**  $x = -3$      $y = -3$

**5 a**  $x = -5$      $y = -2$

**b**  $x = -3$      $y = -4$

**c**  $x = 4$      $y = 3\frac{2}{3}$

**d**  $x = 2$      $y = 7$

**e**  $x = 1$      $y = 1$

**f**  $x = 2$      $y = 9$

**6 a**  $x = 2; y = 3$

**b**  $x = 5; y = 10$

**c**  $x = 4; y = 6$

**d**  $x = 4; y = 4$

**e**  $x = 5; y = 1$

**f**  $x = -3; y = -3$

**7 a**  $x = 1; y = -1$

**b**  $x = 11\frac{2}{3}; y = 8$

**c**  $x = 4; y = 0$

**d**  $x = 3; y = 4$

**e**  $x = 2; y = 8$

**f**  $x = 1; y = 1$

### Exercise 13.4

---

**1 a**  $x = 2; y = 3$

**b**  $x = 1; y = 4$

**c**  $x = 5; y = 2$



**d**  $x = 3; y = 3$

**e**  $x = 4; y = 2$

**f**  $x = 6; y = 1$

**2 a**  $x = 1; y = 4$

**b**  $x = 5; y = 2$

**c**  $x = 3; y = 3$

**d**  $x = 6; y = 1$

**e**  $x = 2; y = 3$

**f**  $x = 2; y = 3$

**3 a**  $x = 0; y = 3$

**b**  $x = 5; y = 2$

**c**  $x = 1; y = 7$

**d**  $x = 6; y = 4$

**e**  $x = 2; y = 5$

**f**  $x = 3; y = 0$

**4 a**  $x = 1; y = \frac{1}{2}$

**b**  $x = \frac{5}{2}; y = 4$

**c**  $x = \frac{1}{5}; y = 4$

**d**  $x = \frac{3}{4}; y = \frac{1}{2}$

**e**  $x = 5; y = \frac{1}{3}$

**f**  $x = \frac{1}{2}; y = 1$

### Exercise 13.5

---

**1** 10 and 7

**2** 16 and 9

**3**  $x = 1; y = 4$

**4**  $x = 5; y = 2$

**5** 60 and 20 years old

**6** 60 and 6 years old

### Exercise 13.6

---

**1 a** 7

**b** 3

**c** 6



- d 7
- e 10
- f 15
- 2 a 7
- b 6
- c 6
- d 10
- e 3
- 3 a 8
- b 5
- c 8
- d 6
- e 6
- 4 a 6, 18, 26
- b 160, 214, 246, 246
- c  $50^\circ$ ,  $80^\circ$ ,  $100^\circ$ ,  $140^\circ$ ,  $170^\circ$
- d  $80^\circ$ ,  $80^\circ$ ,  $80^\circ$ ,  $160^\circ$ ,  $160^\circ$ ,  $160^\circ$
- e  $150^\circ$ ,  $150^\circ$ ,  $150^\circ$ ,  $150^\circ$ ,  $120^\circ$ ,  $120^\circ$ ,  $120^\circ$ ,  $120^\circ$

### Exercise 13.7

---

- 1 a -4 and -3
- b -2 and -6
- c -1 and -12
- d 2 and 5
- e 2 and 3
- f 2 and 4
- 2 a -5 and 2
- b -2 and 5
- c -7 and 2
- d -2 and 7
- e -5 and 3
- f -3 and 5
- 3 a -3 and -2
- b -3
- c -8 and -3
- d 4 and 6



e -4 and 3

f -2 and 6

4 a -2 and 4

b -4 and 5

c -6 and 5

d -6 and 7

e -7 and 9

f -9 and 6

### Exercise 13.8

---

1 a -3 and 3

b -4 and 4

c -5 and 5

d -11 and 11

e -12 and 12

f -15 and 15

2 a -2.5 and 2.5

b -2 and 2

c -1.6 and 1.6

d  $-\frac{1}{2}$  and  $\frac{1}{2}$

e  $-\frac{1}{3}$  and  $\frac{1}{3}$

f  $-\frac{1}{20}$  and  $\frac{1}{20}$

3 a -4 and -1

b -5 and -2

c -4 and -2

d 2 and 4

e 2 and 5

f -4 and 2

4 a -2 and 5

b -5 and 2

c -3 and 6

d -6 and 3

e -4 and 6

f -6 and 8

5 a -4 and 3



- b**  $-6$  and  $-2$
- c**  $-9$  and  $4$
- d**  $-1$
- e**  $-2$
- f**  $-9$  and  $-8$
- 6**
  - a**  $0$  and  $8$
  - b**  $0$  and  $7$
  - c**  $-3$  and  $0$
  - d**  $-4$  and  $0$
  - e**  $0$  and  $9$
  - f**  $0$  and  $4$
- 7**
  - a**  $-1.5$  and  $-1$
  - b**  $-1$  and  $2.5$
  - c**  $-1$  and  $\frac{1}{3}$
  - d**  $-5$  and  $-\frac{1}{2}$
  - e**  $1.5$  and  $5$
  - f**  $-1\frac{1}{3}$  and  $\frac{1}{2}$
- 8**
  - a**  $-12$  and  $0$
  - b**  $-9$  and  $-3$
  - c**  $-8$  and  $4$
  - d**  $-7$  and  $2$
  - e**  $-6$  and  $6$
  - f**  $-10$  and  $10$

### **Exercise 13.9**

---

- 1**  $-4$  and  $3$
- 2**  $-6$  and  $7$
- 3**  $2$
- 4**  $4$
- 5** Height =  $3$  cm, base length =  $12$  cm
- 6** Height =  $20$  cm, base length =  $2$  cm
- 7** Base length =  $6$  cm, height =  $5$  cm
- 8**
  - a**  $9x + 14 = 50$
  - b**  $x = 4$
  - c**  $11\text{ m} \times 6\text{ m}$



### Exercise 13.10

- |       |                                    |    |                 |
|-------|------------------------------------|----|-----------------|
| 1 a i | $x = \frac{1 \pm \sqrt{53}}{2}$    | ii | -3.14 and 4.14  |
| b i   | $x = -2 \pm \sqrt{15}$             | ii | -5.87 and 1.87  |
| c i   | $x = \frac{-5 \pm \sqrt{53}}{2}$   | ii | -6.14 and 1.14  |
| d i   | $x = -3 \pm \sqrt{3}$              | ii | -4.73 and -1.27 |
| e i   | $x = \frac{-5 \pm \sqrt{77}}{2}$   | ii | -6.89 and 1.89  |
| f i   | $x = \frac{9 \pm \sqrt{5}}{2}$     | ii | 3.38 and 5.62   |
| 2 a i | $x = \frac{-7 \pm \sqrt{13}}{2}$   | ii | -5.30 and -1.70 |
| b i   | $x = \pm \sqrt{35}$                | ii | -5.92 and 5.92  |
| c i   | $x = \frac{-3 \pm \sqrt{21}}{2}$   | ii | -3.79 and 0.79  |
| d i   | $x = \frac{5 \pm \sqrt{53}}{2}$    | ii | -1.14 and 6.14  |
| e i   | $x = \frac{-1 \pm \sqrt{73}}{2}$   | ii | -4.77 and 3.77  |
| f i   | $x = \pm 2\sqrt{2}$                | ii | -2.83 and 2.83  |
| 3 a i | $x = 1 \pm \sqrt{3}$               | ii | -0.73 and 2.73  |
| b i   | $x = 2 \pm \sqrt{15}$              | ii | -1.87 and 5.87  |
| c i   | $x = \frac{1 \pm \sqrt{21}}{2}$    | ii | -1.79 and 2.79  |
| d i   | $x = 1 \pm 2\sqrt{2}$              | ii | -3.83 and 1.83  |
| e i   | $x = \frac{3 \pm \sqrt{5}}{2}$     | ii | 0.38 and 2.62   |
| f i   | $x = 4 \pm \sqrt{13}$              | ii | 0.39 and 7.61   |
| 4 a i | $x = \frac{3 \pm \sqrt{41}}{4}$    | ii | -0.85 and 2.35  |
| b i   | $x = \frac{-1 \pm \sqrt{21}}{4}$   | ii | -1.40 and 0.90  |
| c i   | $x = \frac{4 \pm \sqrt{11}}{5}$    | ii | 0.14 and 1.46   |
| d i   | No surds in the answer.            | ii | -2 and -0.5     |
| e i   | $x = \frac{2 \pm \sqrt{10}}{3}$    | ii | -0.39 and 1.72  |
| f i   | $x = \frac{-1 \pm \sqrt{421}}{14}$ | ii | -1.54 and 1.39  |

### Exercise 13.11

- 1  $x = 3$  and  $y = 4$   
 2  $x = -6$  and  $y = 9$



$$x = -1 \text{ and } y = -1$$

**3**  $x = -2.5 \text{ and } y = 3.5$

$$x = 2 \text{ and } y = 17$$

**4**  $x = -\frac{1}{3} \text{ and } y = 4\frac{1}{3}$

$$x = 2 \text{ and } y = 16$$

**5**  $x = 0 \text{ and } y = 2$

$$x = 5 \text{ and } y = -8$$

**6**  $x = -3 \text{ and } y = -20$

$$x = \frac{4}{5} \text{ and } y = 2\frac{4}{5}$$

**7**  $x = 5 \text{ and } y = 6$

$$x = 6 \text{ and } y = 5$$

**8**  $x = -0.608 \text{ and } y = -3.29$

$$x = 4.11 \text{ and } y = 0.487 \text{ (all to 3 s.f.)}$$

**9**  $x = 1\frac{1}{3}$

**10**  $p = 1.17 \text{ (3 s.f.)}$

### Exercise 13.12

**1 a**  $x < 4$



**b**  $x > 1$



**c**  $x \leq 3$



**d**  $x \geq 7$



**e**  $x < 2$



**f**  $x > 2$



**2 a**  $x < 7$



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**b**  $x \geq -2$



**c**  $x > -3$



**d**  $x \geq -12$



**e**  $x > -24$



**f**  $x \geq -3$



**3 a**  $x < 2$



**b**  $x \leq 12$



**c**  $x \geq 2$



**d**  $x \geq -2$



**e**  $x > -0.5$



**f**  $x \geq 2$





### Exercise 13.13

1 a  $2 < x \leq 4$



b  $1 \leq x < 5$



c  $3.5 \leq x < 5$



d  $2 \leq x < 4.2$



2 a  $1 < x \leq 5$



b  $-1 \leq x < 1$



c  $2 < x < 3$



d No solution

### Student assessment 1

1 a -6

b 6

c 4

d 2.4

2 a 0.5

b 4

c 9.5

d 5

3 a 6

b 15

c 22

d 6



- 4 a 8.5  
b  $4\frac{1}{3}$   
c 8.5  
d 12  
e 2.5  
f -1
- 5 a  $x = 7$   $y = 4$   
b  $p = 1$   $q = 2$   
c  $x = 7$   $y = 1$   
d  $m = 3$   $n = 5$
- 6  $(\frac{1}{4}, 4\frac{1}{4})$  and  $(-3, 14)$

### Student assessment 2

---

- 1 a  $9x - 90 = 360$   
b  $x = 50$   
c  $50^\circ, 60^\circ, 100^\circ, 150^\circ$
- 2 2
- 3  $85^\circ, 95^\circ, 95^\circ, 130^\circ, 135^\circ$
- 4  $3 < x \leq 5$
- 
- 5 -4 and 5
- 6 2.77 and -1.27 (3 s.f.)
- 7  $x = 1.24$  and  $y = 2.61$  (3 s.f.)  $x = -3.24$  and  $y = 0.382$  (3 s.f.)
- 8  $x > 5$

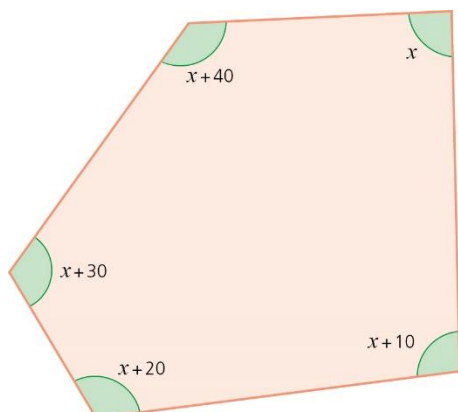
### Student assessment 3

---

- 1 a  $x - y = 30$ ;  $x + y + 40 = 180$   
b  $x = 85$ ;  $y = 55$



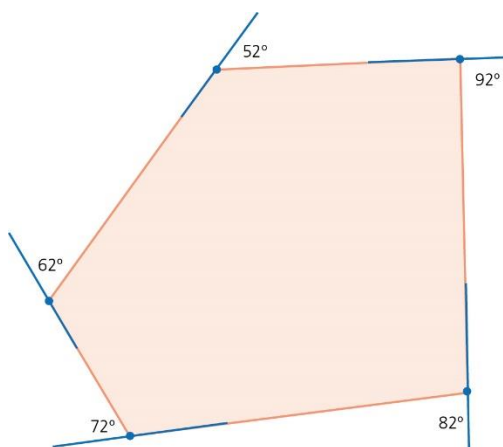
2 a



b  $5x + 100$

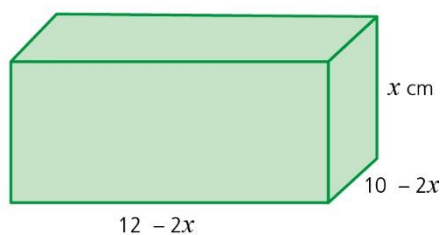
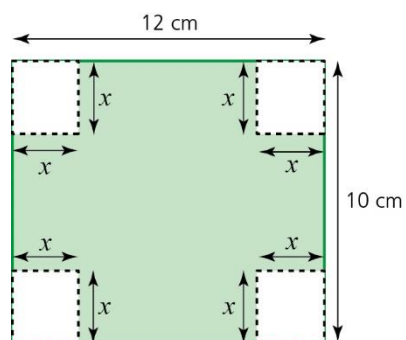
c  $92^\circ$

d



e  $92^\circ + 82^\circ + 72^\circ + 62^\circ + 52^\circ = 360^\circ$

3 a



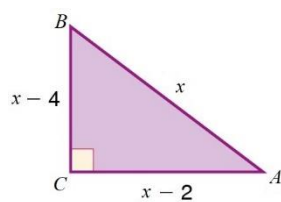
b  $120 - 4x^2$

c  $4x^2 = 64$ ;  $x = 4$

4 b 0.44 and 4.56 (2 d.p.)



5 a



c 10 cm, 8 cm, 6 cm



All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

## 14 Graphing inequalities and regions

### Exercise 14.1

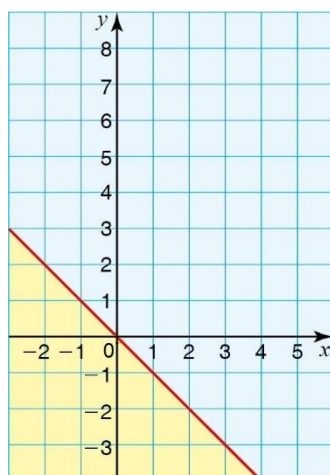
---

- 1 a  $x < 2$   
b  $y \geq 2$   
c  $x \leq -2$   
d  $y \leq 6$   
e  $t > 0$   
f  $p \leq -3$
- 2 a  $2 < y \leq 4$   
b  $1 \leq p < 5$   
c  $5 \leq m < 7$   
d  $3 < x < 4$

### Exercise 14.2

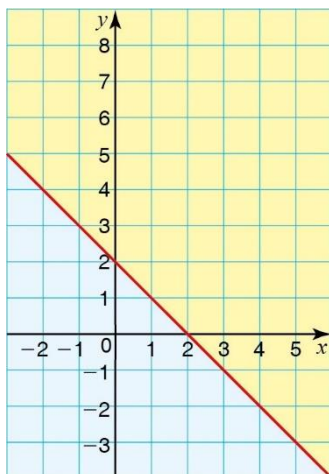
---

- 1 a

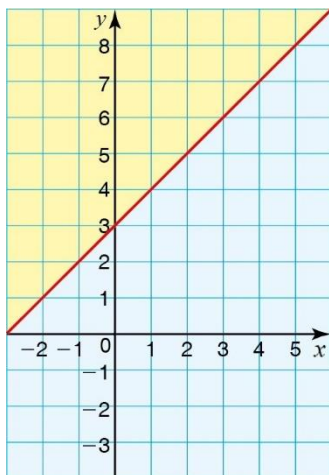




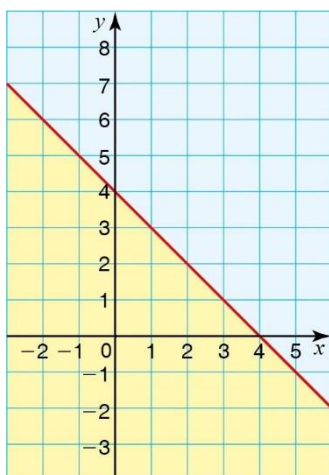
**b**



**c**

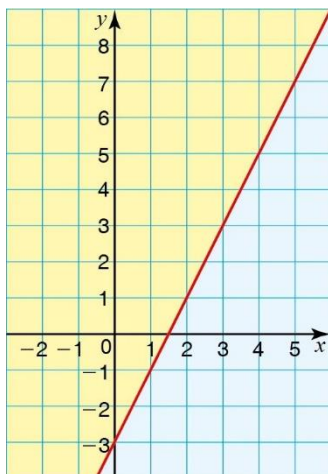


**d**

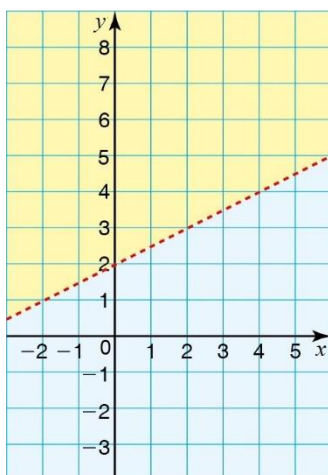




e

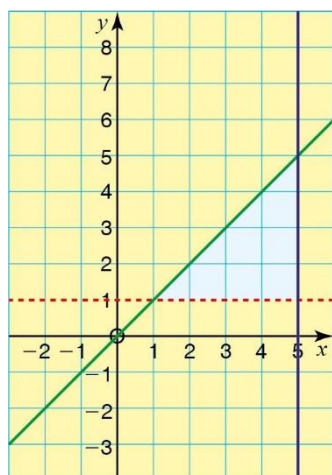


f



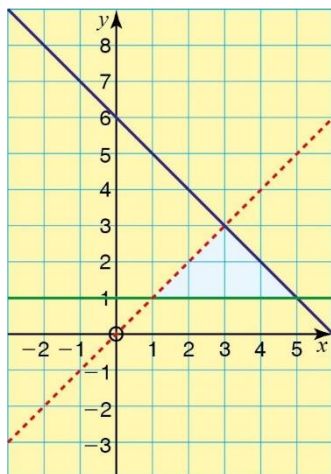
### Exercise 14.3

1

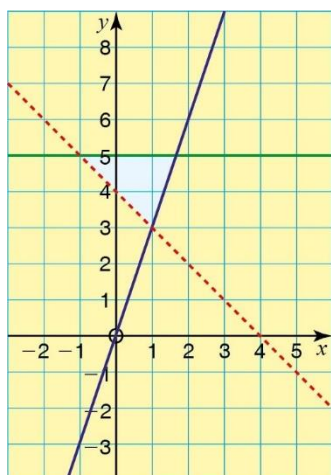




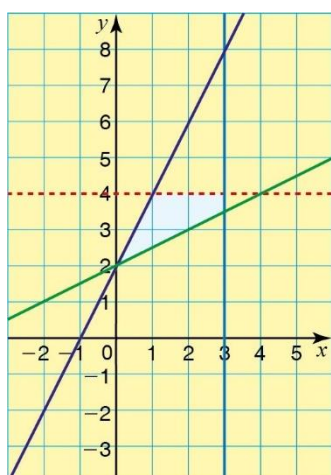
2



3



4



## Student assessment 1

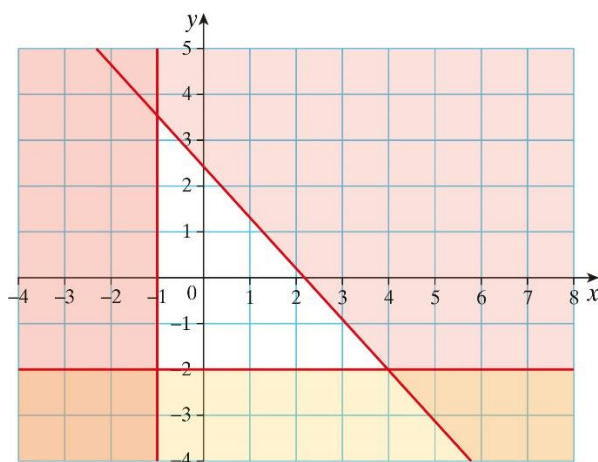
1 a  $x \leq 5$

b  $y \leq 3$



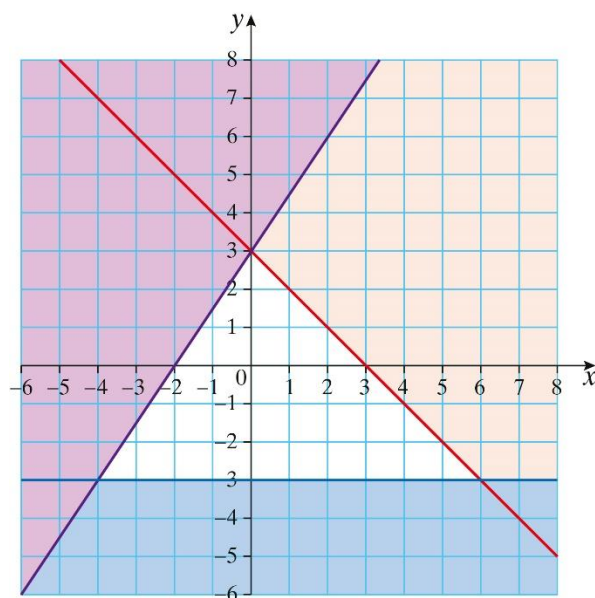
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- 2 a  $x \leq 7$   
b  $y \leq 9$
- 3 a  $2 < y \leq 4$   
b  $4 < p \leq 8$
- 4 a  $1 \leq p < 4$   
b  $4 < x \leq 7$
- 5  $-1 < x \leq 3$
- 6 a



b  $\text{Area} = \frac{1}{2} \times 5 \times 5 \frac{1}{2}$   
 $13 \frac{3}{4} \text{ units}^2$

- 7 a



b 30 units<sup>2</sup>



*All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.*

## 15 Sequences

### Exercise 15.1

---

- 1 a i 20, 23  
ii  $3n + 2$   
b i 25, 29  
ii  $4n + 1$   
c i 29, 34  
ii  $5n - 1$   
d i 18, 20  
ii  $2n + 6$   
e i -6, -13  
ii  $36 - 7n$   
f i 24, 28  
ii  $4n - 4$   
g i 46, 55  
ii  $9n - 8$   
h i 65, 75  
ii  $10n + 5$   
i i 64, 75  
ii  $11n - 2$   
j i 13.5, 15.5  
ii  $2n - \frac{1}{2}$   
k i 5.25, 6.25  
ii  $n - \frac{3}{4}$   
l i 0, -1  
ii  $6 - n$
- 2 a  $5 \times 5 - 3 = 22$ ; so 22 belongs to the sequence  $5n - 3$   
b Neither  $5n - 3 = 51$  or  $77 - 3n = 51$  have whole number solutions, so 51 is not in either sequence  
c One of: 2, 17, 32, 47 or 62



## Exercise 15.2

---

- 1 a  $T_2 = 7, T_3 = 12, T_4 = 17$  linear  
b  $T_2 = 1, T_3 = -2, T_4 = -5$  linear  
c  $T_1 = 10, T_2 = 6, T_4 = -2$  linear  
d  $T_1 = \frac{1}{2}, T_3 = -1, T_4 = -3$  non-linear  
e  $T_2 = 4, T_3 = 5, T_4 = 5.5$  non-linear  
f  $T_1 = 46, T_2 = 22, T_4 = 4$  non-linear
- 2 a i +3, each term is 3 more than the term before  
ii  $T_{n+1} = T_n + 3$   
iii  $3n + 2$   
iv 32  
b i +4, each term is 4 more than the term before  
ii  $T_{n+1} = T_n + 4$   
iii  $4n - 4$   
iv 36  
c i +1, each term is 1 more than the term before  
ii  $T_{n+1} = T_n + 1$   
iii  $n - 0.5$   
iv 9.5  
d i -3, each term is 3 less than the term before  
ii  $T_{n+1} = T_n - 3$   
iii  $-3n + 9$   
iv -21  
e i +3, each term is 3 more than the term before  
ii  $T_{n+1} = T_n + 3$   
iii  $3n - 10$   
iv 20  
f i -4, each term is 4 less than the term before  
ii  $T_{n+1} = T_n - 4$   
iii  $-4n - 5$   
iv -45



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|     |          |      |    |       |     |      |             |
|-----|----------|------|----|-------|-----|------|-------------|
| 3 a | Position | 1    | 2  | 5     | 12  | 50   | n           |
|     | Term     | 1    | 5  | 17    | 45  | 197  | $4n - 3$    |
| b   | Position | 1    | 2  | 5     | 10  | 75   | n           |
|     | Term     | 5    | 11 | 29    | 59  | 449  | $6n - 1$    |
| c   | Position | 1    | 3  | 8     | 50  | 100  | n           |
|     | Term     | 2    | 0  | -5    | -47 | -97  | $-n + 3$    |
| d   | Position | 1    | 2  | 3     | 10  | 100  | n           |
|     | Term     | 3    | 0  | -3    | -24 | -294 | $-3n + 6$   |
| e   | Position | 2    | 5  | 7     | 10  | 50   | n           |
|     | Term     | 1    | 10 | 16    | 25  | 145  | $3n - 5$    |
| f   | Position | 1    | 2  | 5     | 20  | 50   | n           |
|     | Term     | -5.5 | -7 | -11.5 | -34 | -79  | $-1.5n - 4$ |

- 4 a
- i 4
  - ii  $T_{n+1} = T_n + 4$
  - iii  $4n + 1$
  - iv 201
- b
- i +1
  - ii  $u_{n+1} = u_n + 1$
  - iii  $n - 1$
  - iv 49
- c
- i +3
  - ii  $u_{n+1} = u_n + 3$
  - iii  $3n - 13$
  - iv 137
- d
- i +0.5
  - ii  $u_{n+1} = u_n + \frac{1}{2}$
  - iii  $0.5n + 5.5$
  - iv 30.5
- e
- i +4
  - ii  $u_{n+1} = u_n + 4$
  - iii  $4n - 62$
  - iv 138
- f
- i -3
  - ii  $u_{n+1} = u_n - 3$
  - iii  $-3n + 75$



iv -75

- 5 a No, because -8 is a negative number and the sequence is formed of increasing positive numbers.  
b Yes, 67 is the 6th term.  
c Yes, 139 is the 12th term.  
d No, since the 20th term is 235 and the 21st term is 247.

### Exercise 15.3

---

- 1  $T_n = n^2 + 1$   
2  $T_n = n^2 - 1$   
3  $T_n = n^2 + 5$   
4  $T_n = n^2 + 8$   
5  $T_n = n^2 - 3$   
6  $T_n = 2n^2 + 2$   
7  $T_n = 2n^2 - 2$   
8  $T_n = 3n^2 + 2$   
9  $T_n = 4n^2 - 4$   
10  $T_n = 5n^2 - 4$

### Exercise 15.4

---

- 1  $T_n = n^3 + 10$   
2  $T_n = n^3 - n$   
3  $T_n = n^3 - 5$   
4  $T_n = n^3 + n^2$   
5  $T_n = n^3 + n^2 + 5n$   
6  $T_n = n^3 + 3n^2 + 5n - 2$   
7  $T_n = n^3 + n^2 - n$   
8  $T_n = n^3 + 5n + 7$

### Exercise 15.5

---

- 1 a Exponential (Geometric)  
b Exponential (Geometric)  
c Not exponential (Not geometric)  
d Exponential (Geometric)  
e Not exponential (Not geometric)  
f Not exponential (Not geometric)
- 2 a i 3



ii 162, 486

iii  $u_n = 2(3)^{n-1}$

b i  $\frac{1}{5}$

ii  $\frac{1}{25}, \frac{1}{125}$

iii  $u_n = 25\left(\frac{1}{5}\right)^{n-1}$

d i -3

ii -243, 729

iii  $u_n = (-3)^n$

3 a -6, -12, -24

b 8

4 a -4

b  $\frac{1}{4}$

c -65 536

5 \$38 203.20

6 a Because  $0.8^5 \neq 0$

b \$3276.80

c Because  $0.8^n \neq 0$  where  $n$  is the number of years

### Exercise 15.6

---

1 i  $T_n = n^2 + 1$

ii 26, 37

2 i  $T_n = n^3 + 2$

ii 127, 218

3 i  $T_n = \frac{n^2}{2}$

ii  $12\frac{1}{2}, 18$

4 i  $T_n = \frac{1}{2} \times 2^n$  or  $u_n = 2^{(n-1)}$

ii 16, 32

5 i  $T_n = n^2 + n$

ii 30, 42

6 i  $T_n = n^2 + n^3$

ii 150, 252

7 i  $T_n = n^3 - n^2$

ii 100, 180

8 i  $T_n = 3 \times 2^n$



ii 96, 192

## Student assessment 1

---

- 1 a i 45, 54  
ii Each term is 9 more than the term before  
b i 30, 24  
ii Each term is 6 less than the term before  
c i 2.25, 1.125  
ii Each term is half the term before  
d i -12, -18  
ii Each term is 6 less than the term before  
e i 27, 8  
ii Descending order of cube numbers  
f i 81, 243  
ii Each term is 3 times the term before

- 2 a  $4n + 2$   
b  $6n + 7$   
c  $6n - 3$   
d  $n^2 + 3$   
e  $10n - 10$   
f  $n^3 - 1$

- 3 a i  $T_n = 4n - 3$   
ii 37  
b i  $T_n = -3n + 4$   
ii -26

4 a

|                 |    |    |    |     |     |            |
|-----------------|----|----|----|-----|-----|------------|
| <b>Position</b> | 1  | 2  | 3  | 10  | 25  | n          |
| <b>Term</b>     | 17 | 14 | 11 | -10 | -55 | $-3n + 20$ |

b

|                 |    |    |    |    |                    |
|-----------------|----|----|----|----|--------------------|
| <b>Position</b> | 2  | 6  | 10 | 80 | n                  |
| <b>Term</b>     | -4 | -2 | 0  | 35 | $\frac{1}{2}n - 5$ |

## Student assessment 2

---

- 1 a \$515.46  
b 3 years  
2 a \$253.50



**b** \$8.08

**3 a**  $-\frac{1}{3}$

**b** 243

**c** 10

**4**  $T_n = n^3 + 3n^2 + 4$

**5**  $T_n = n^3 + n^2 + 3n + 5$

**6 a**  $T_5 = 27, T_{100} = 597$

**b**  $T_5 = 1.5, T_{100} = -46$



*All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.*

## 16 Proportion

### Exercise 16.1

---

- 1 a 3  
b 21  
c 27  
d 3  
e 10
- 2 a 0.5  
b 8  
c 24.5  
d 8  
e 16
- 3 a 24  
b  $\frac{3}{8}$   
c  $\frac{1}{9}$   
d 1
- 4 a 0.25  
b 25  
c 4  
d  $\frac{1}{16}$

### Exercise 16.2

---

- 1 a i  $y \propto x^3$   
ii  $y = kx^3$
- b i  $y \propto \frac{1}{x^3}$   
ii  $y = \frac{k}{x^3}$
- c i  $t \propto P$   
ii  $t = kP$
- d i  $s \propto \frac{1}{t}$   
ii  $s = \frac{k}{t}$



e i  $A \propto r^2$

ii  $A = kr^2$

f i  $T \propto \frac{1}{\sqrt{g}}$

ii  $T = \frac{k}{\sqrt{g}}$

2 10.5

3 a  $\frac{1}{2}$

b 2

4 32

5 a  $\frac{1}{8}$

b 0.4

6 75

### Exercise 16.3

---

1 a  $h = kt^2$

b 5

c 45 m

d 6 s

2 a  $v = k\sqrt{e}$

b 3

c 49 J

3 a  $l = km^{\frac{1}{3}}$

b 3

c  $l = 6$

4 a  $P = kI^2$

b 5

### Student assessment 1

---

1 a 1.5

b 15

c 3

d 12

2 a 10

b 2.5

c 1



- d 20
- 3 a  $\frac{1}{3}$   
b 72  
c  $\frac{1}{3}$   
d 12
- 4 a 5  
b  $\frac{5}{4}$   
c  $\frac{1}{2}$   
d 1
- 5 a  $\frac{1}{3}$   
b  $\frac{4}{3}$   
c  $\pm 2$   
d  $\pm 1$

## Student assessment 2

- 1 a
- |   |    |    |   |   |    |    |
|---|----|----|---|---|----|----|
| x | 1  | 2  | 4 | 8 | 16 | 32 |
| y | 32 | 16 | 8 | 4 | 2  | 1  |
- b 1.6
- 2 a
- |   |   |    |    |    |    |
|---|---|----|----|----|----|
| x | 1 | 2  | 4  | 5  | 10 |
| y | 5 | 10 | 20 | 25 | 50 |
- b
- |   |    |    |   |   |    |
|---|----|----|---|---|----|
| x | 1  | 2  | 4 | 5 | 10 |
| y | 20 | 10 | 5 | 4 | 2  |
- c
- |   |   |    |    |    |    |
|---|---|----|----|----|----|
| x | 4 | 16 | 25 | 36 | 64 |
| y | 4 | 8  | 10 | 12 | 16 |
- 3 a 0.8  
b 0.8
- 4 a 1.56 (3 s.f.)  
b 2

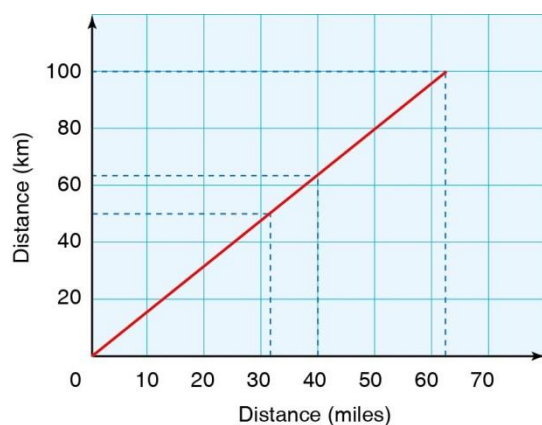


All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

## 17 Graphs in practical situations

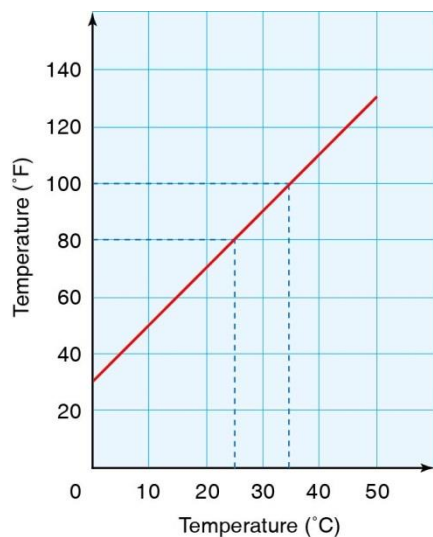
### Exercise 17.1

1



- a 50 km  $\approx$  31 miles
- b 40 miles  $\approx$  64 km, therefore  
80 miles  $\approx$  128 km
- c 100 km/h  $\approx$  62 mph
- d 40 mph  $\approx$  64 km/h

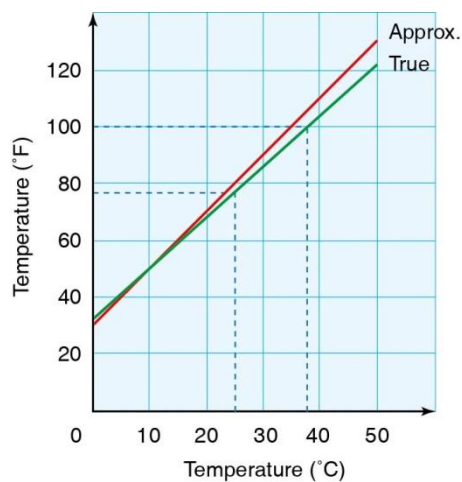
2



- a 25 °C  $\approx$  80 °F
- b 100 °F  $\approx$  35 °C
- c 0 °C  $\approx$  30 °F
- d 120 °F  $\approx$  45 °C

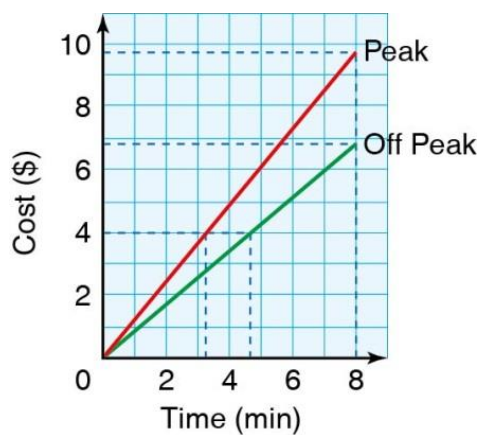


3



- i
- a  $25^{\circ}\text{C} = 77^{\circ}\text{F}$
  - b  $100^{\circ}\text{F} = 38^{\circ}\text{C}$
  - c  $0^{\circ}\text{C} = 32^{\circ}\text{F}$
  - d  $120^{\circ}\text{F} = 49^{\circ}\text{C}$
- ii The rough conversion is most useful at lower temperatures (i.e. between 0 and 20 °C).

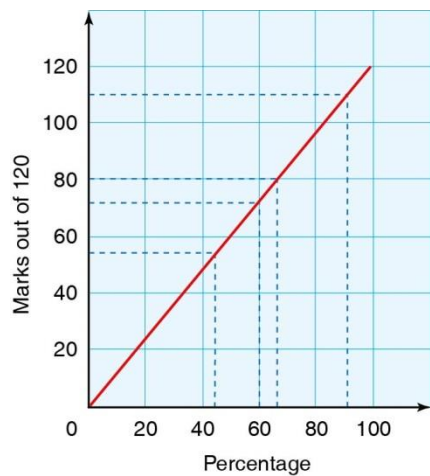
4 a



- b 8 min = \$6.80
- c 8 min = \$9.60
- d Extra time  $\approx 1$  min 20 s



5



- a  $80 = 67\%$
- b  $110 = 92\%$
- c  $54 = 45\%$
- d  $72 = 60\%$

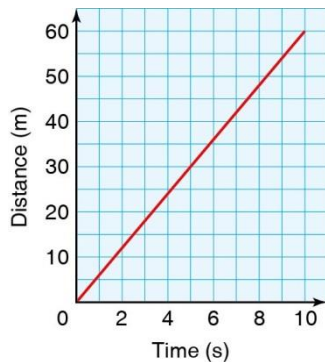
## Exercise 17.2

- 1
  - a 6 m/s
  - b 4 m/s
  - c 39 km/h
  - d 20 km/h
  - e 160 km/h
  - f 50 km/h
- 2
  - a 400 m
  - b 182 m
  - c 210 km
  - d 255 km
  - e 10 km
  - f 79.2 km
- 3
  - a 5 s
  - b 50 s
  - c 4 min
  - d 1 min 11.4 s
  - e 5 s
  - f 4 min

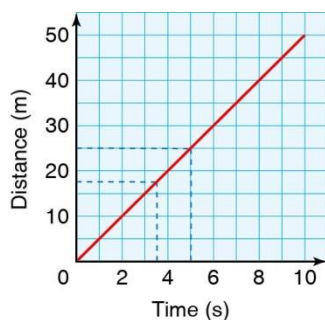


### Exercise 17.3

1



2



a 5 s

b 17.5 m

3 a Speed A = 40 m/s      Speed B =  $13\frac{1}{3}$  m/s

b Distance apart =  $453\frac{1}{3}$  m

4 a  $\frac{2}{3}$  m/s

b 6 m/s,  $\frac{2}{3}$  m/s

c 1 m/s

d  $\frac{1}{2}$  m

e  $7\frac{1}{3}$  m

### Exercise 17.4

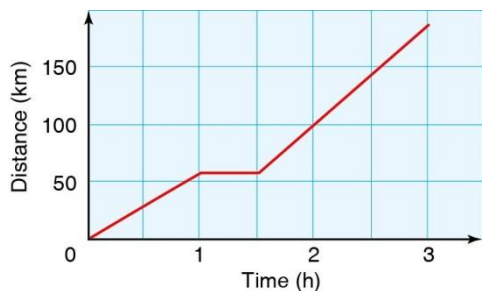
1 a 45 km/h

b 20 km/h

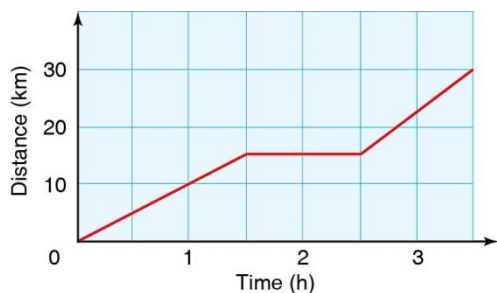
c Paul has arrived at the restaurant and is waiting for Helena.



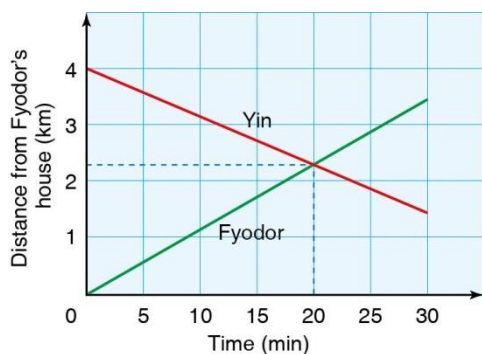
2



3



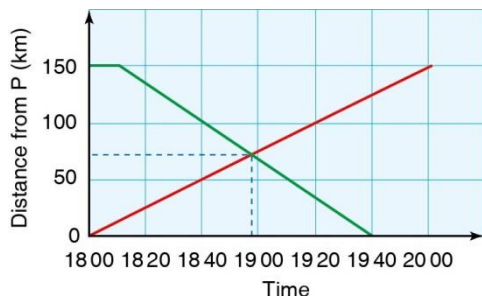
4 a



b After 20 min

c Distance =  $2\frac{1}{3}$  km

5 a



b Time  $\approx 18\ 57$

c Distance from Q  $\approx 79$  km

d The 18 10 train from station Q arrives first.

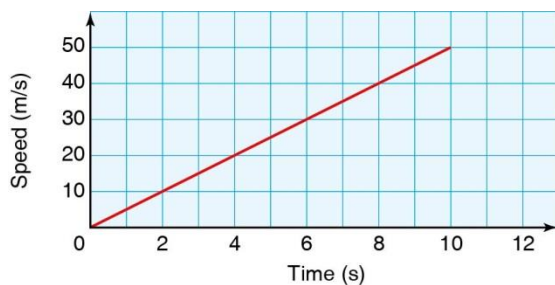
6 a a:  $133\frac{1}{3}$  km/h    b: 0 km/h    c: 200 km/h



b d: 100 km/h e: 200 km/h

### Exercise 17.5

- 1 Acceleration is  $0.375 \text{ m/s}^2$
- 2 Acceleration is  $0.2 \text{ m/s}^2$
- 3 Acceleration is  $7 \text{ m/s}^2$
- 4 Acceleration is  $1.75 \text{ m/s}^2$
- 5 Deceleration is  $0.25 \text{ m/s}^2$
- 6 Deceleration is  $1 \text{ m/s}^2$
- 7



8



### Exercise 17.6

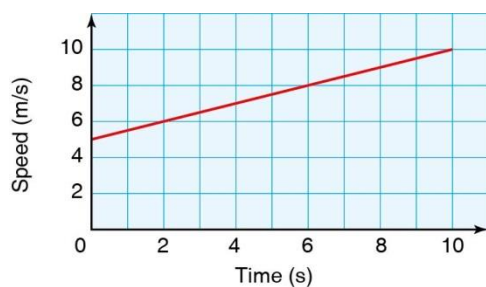
- 1
  - a  $1.5 \text{ m/s}^2$
  - b  $0 \text{ m/s}^2$
  - c  $0.5 \text{ m/s}^2$
- 2
  - a Cheetah
  - b  $7.5 \text{ m/s}^2$
  - c  $5 \text{ m/s}^2$
  - d  $15 \text{ m/s}^2$
- 3
  - a  $0.5 \text{ m/s}^2$
  - b  $0.25 \text{ m/s}^2$
  - c  $0.104 \text{ m/s}^2$  (3 s.f.)



- d Travelling at a constant speed of 30 m/s
- e Stationary

## Exercise 17.7

1 a

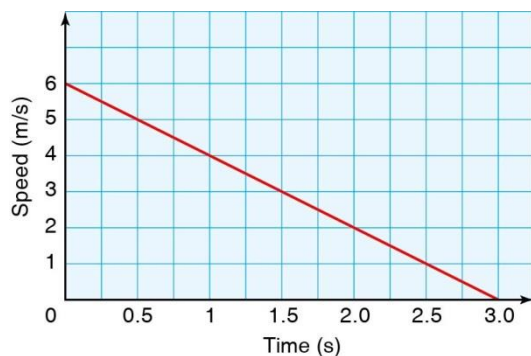


- b  $0.5 \text{ m/s}^2$
- c 75 m

2 a

| Time (s)   | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
|------------|---|-----|---|-----|---|-----|---|
| Speed(m/s) | 6 | 5   | 4 | 3   | 2 | 1   | 0 |

b



- c 9 m

3 a  $1.5 \text{ m/s}^2$

- b 2400 m

- c 40 s

4 a 390 m      b 240 m

5 21.45 km

6 720 m

7 a  $0.37 \text{ m/s}^2$  (2 d.p.)

- b  $2.16 \text{ m/s}^2$  (2 d.p.)

- c 208 m

- d 204 m

- e 4 m

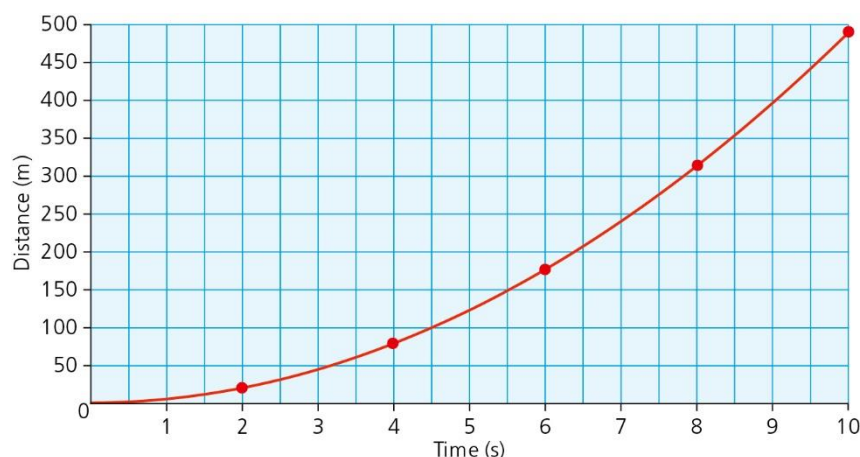


### Exercise 17.8

1 a

|             |   |      |      |       |       |     |
|-------------|---|------|------|-------|-------|-----|
| Time (s)    | 0 | 2    | 4    | 6     | 8     | 10  |
| Speed (m/s) | 0 | 19.6 | 78.4 | 176.4 | 313.6 | 490 |

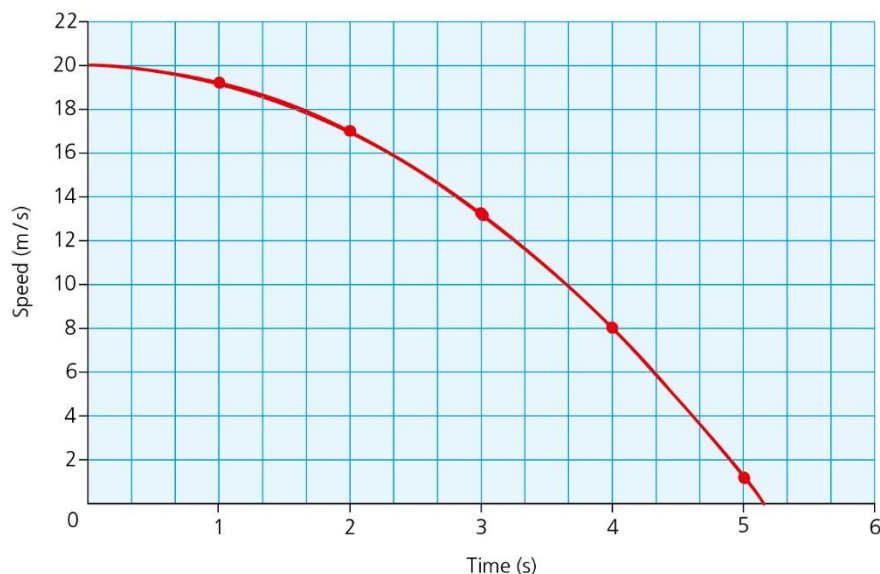
b



c Student's tangent drawn at  $t = 5$  and the gradient calculated.

Speed = 49 m/s (students' answers may vary)

2 a



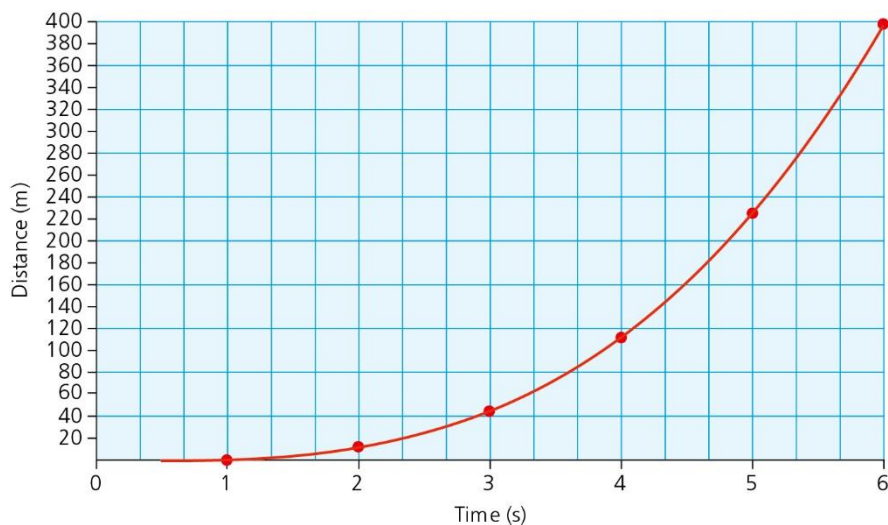
b Approximately 5.2 seconds

c Tangent drawn on graph at  $t = 2$ . Gradient of tangent calculated. Gradient =  $-3$

Deceleration =  $3 \text{ m/s}^2$  (Note: the question asked for deceleration not acceleration)



3 a

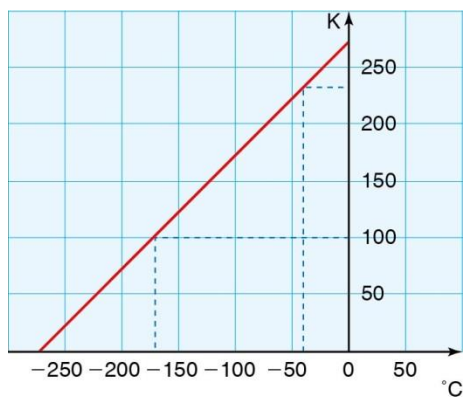


b Student's tangent drawn at  $t = 3.5$  and the gradient calculated.

Speed = 66.5 m/s (students' answers may vary)

## Student assessment 1

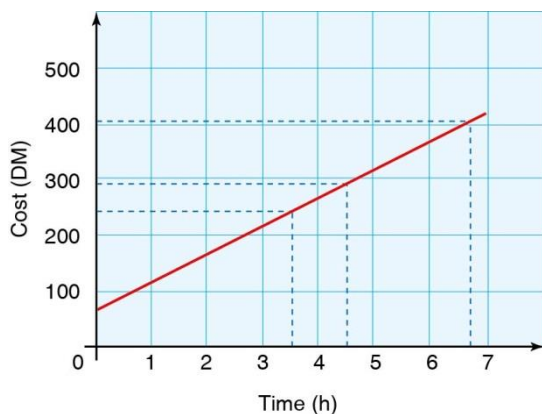
1



a  $-40^{\circ}\text{C} \approx 233\text{ K}$

b  $100\text{ K} \approx -173^{\circ}\text{C}$

2 a



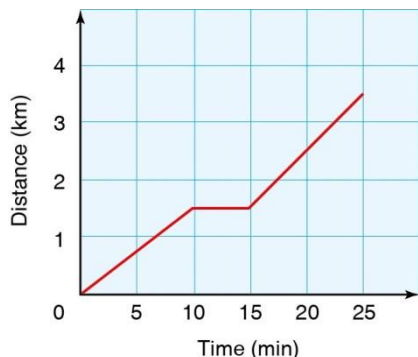
i \$295

ii \$408



b  $3\frac{1}{2}$  h

3 a



b 25 min

4 a B, C

- b B because it illustrates going back in time  
C because it illustrates infinite speed

## Student assessment 2

1 a  $2 \text{ m/s}^2$

b 225 m

c 10.6 s (3 s.f.)

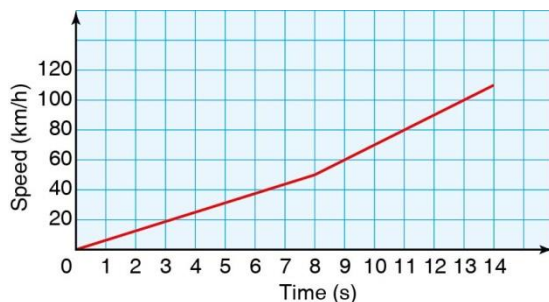
2 a  $4 \text{ m/s}^2$

b  $3 \text{ m/s}^2$

c 102 m

d 9.83 s (3 s.f.)

3 a



b 189 m (3 s.f.)

4 a  $1 \text{ m/s}^2$

b 750 m

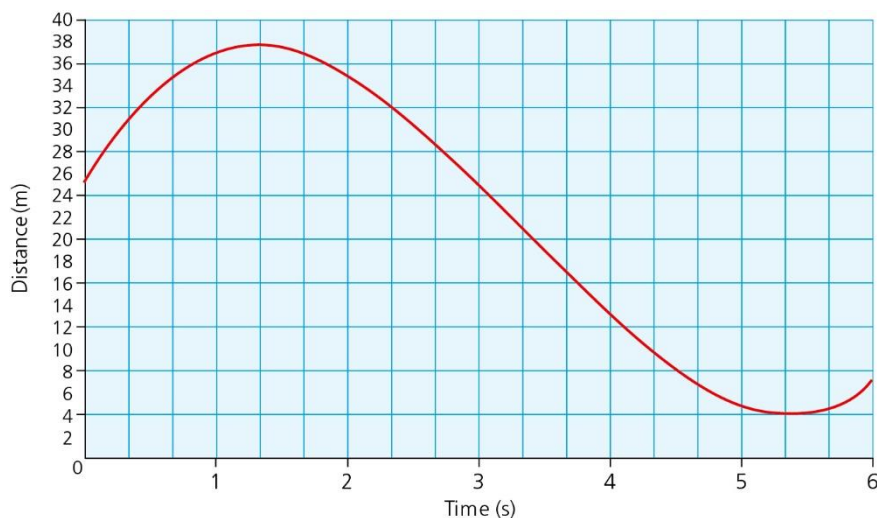
c 1237.5 m

5 a

| Time (s)     | 0  | 1  | 2  | 3  | 4  | 5 | 6 |
|--------------|----|----|----|----|----|---|---|
| Distance (m) | 25 | 37 | 35 | 25 | 13 | 5 | 7 |



**b**



**c** The object is stationary when the distance–time graph is horizontal. When  $t \approx 1.3$  and 5.4 seconds

**d** Student draws a tangent to the graph where  $t = 5$  and calculates its gradient.

Gradient  $\approx -4$ .

Therefore velocity is  $-4$  m/s (students' answers may vary).

### Student assessment 3

**1 a**  $2.5 \text{ m/s}^2$

**b** 180 m

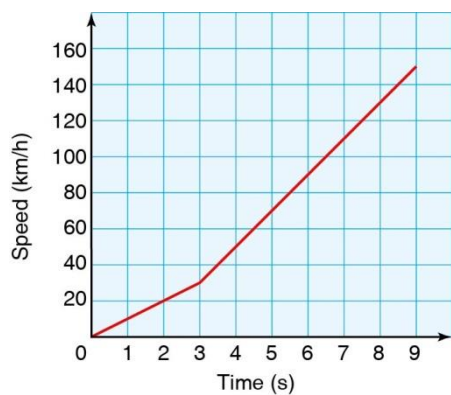
**c** 3.5 s (1 d.p.)

**2 a**  $0.278 \text{ m/s}^2$  (3 s.f.)

**b** 93.8 m (3 s.f.)

**c** 97.2 m (3 s.f.)

**3 a**



**b** 162.5 m

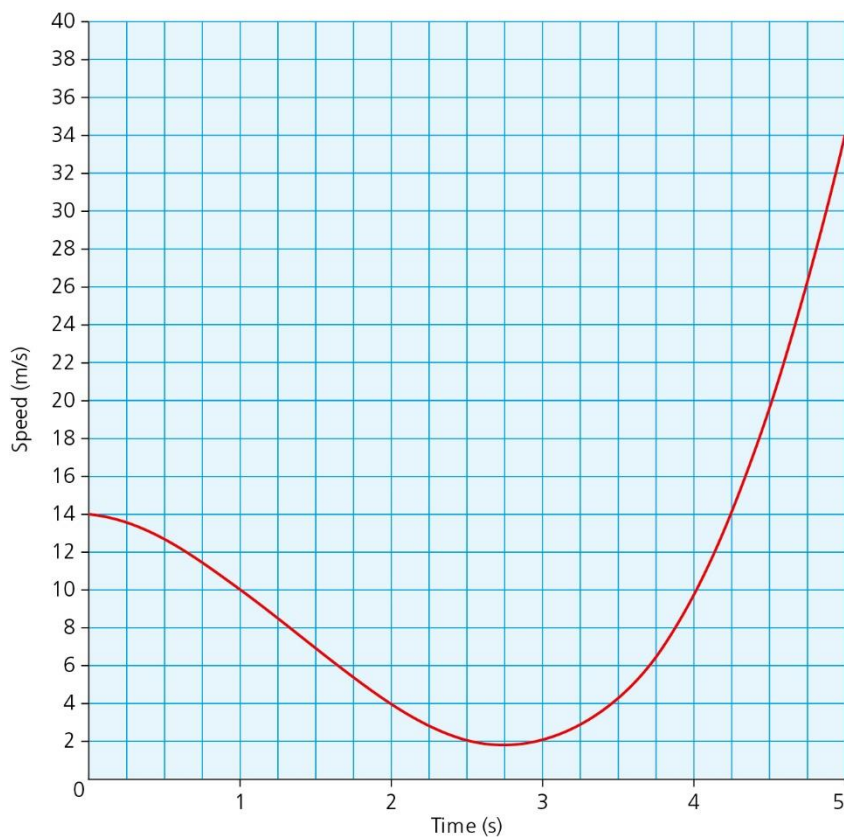
**4 a**  $3.33 \text{ m/s}^2$  (3 s.f.)

**b** 240 m

**c** 212.5 m



5 a



- b** The acceleration of the object at its lowest point is  $0 \text{ m/s}^2$  as the gradient of the graph at this point is zero and the gradient of a speed–time graph represents the acceleration at that point.
- c** Student draws a tangent to the graph where  $t = 2$  and calculates its gradient. Gradient  $\approx -5$ . Therefore acceleration is  $-5 \text{ m/s}^2$  (students' answers may vary).



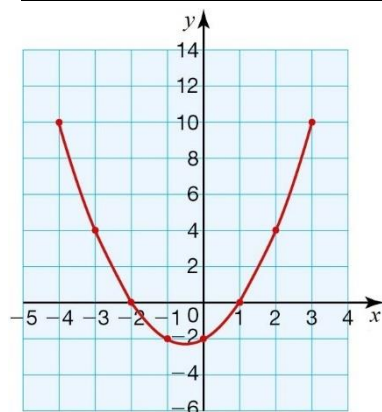
All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

## 18 Graphs of functions

### Exercise 18.1

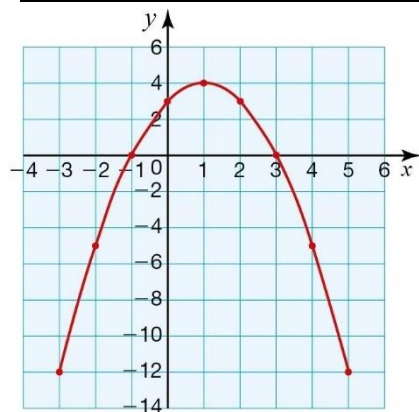
1

|     |    |    |    |    |    |   |   |    |
|-----|----|----|----|----|----|---|---|----|
| $x$ | -4 | -3 | -2 | -1 | 0  | 1 | 2 | 3  |
| $y$ | 10 | 4  | 0  | -2 | -2 | 0 | 4 | 10 |



2

|     |     |    |    |   |   |   |   |    |     |
|-----|-----|----|----|---|---|---|---|----|-----|
| $x$ | -3  | -2 | -1 | 0 | 1 | 2 | 3 | 4  | 5   |
| $y$ | -12 | -5 | 0  | 3 | 4 | 3 | 0 | -5 | -12 |

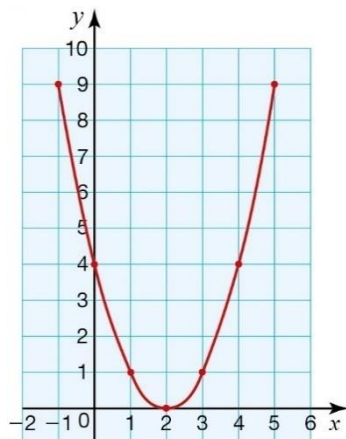




# Cambridge IGCSE™ Mathematics Core and Extended Answers to Student's Book

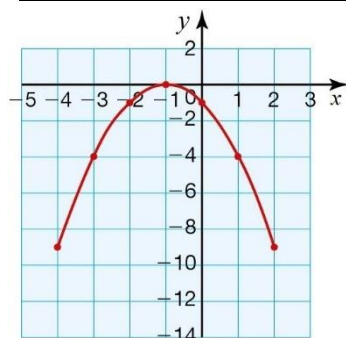
3

|     |    |   |   |   |   |   |   |
|-----|----|---|---|---|---|---|---|
| $x$ | -1 | 0 | 1 | 2 | 3 | 4 | 5 |
| $y$ | 9  | 4 | 1 | 0 | 1 | 4 | 9 |



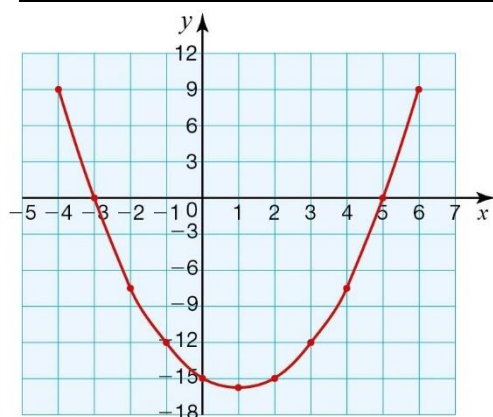
4

|     |    |    |    |    |    |    |    |
|-----|----|----|----|----|----|----|----|
| $x$ | -4 | -3 | -2 | -1 | 0  | 1  | 2  |
| $y$ | -9 | -4 | -1 | 0  | -1 | -4 | -9 |



5

|     |    |    |    |     |     |     |     |     |    |   |   |
|-----|----|----|----|-----|-----|-----|-----|-----|----|---|---|
| $x$ | -4 | -3 | -2 | -1  | 0   | 1   | 2   | 3   | 4  | 5 | 6 |
| $y$ | 9  | 0  | -7 | -12 | -15 | -16 | -15 | -12 | -7 | 0 | 9 |

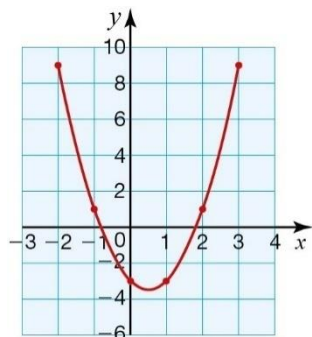




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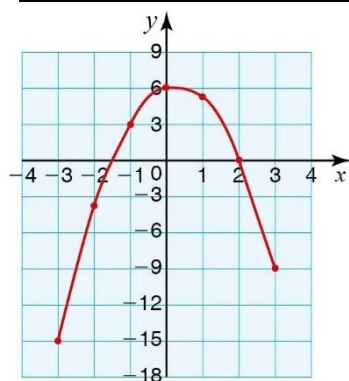
6

|     |    |    |    |    |   |   |
|-----|----|----|----|----|---|---|
| $x$ | -2 | -1 | 0  | 1  | 2 | 3 |
| $y$ | 9  | 1  | -3 | -3 | 1 | 9 |



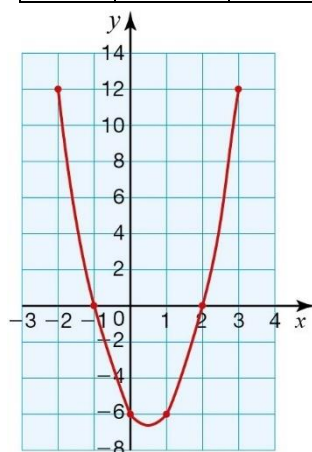
7

|     |     |    |    |   |   |   |    |
|-----|-----|----|----|---|---|---|----|
| $x$ | -3  | -2 | -1 | 0 | 1 | 2 | 3  |
| $y$ | -15 | -4 | 3  | 6 | 5 | 0 | -9 |



8

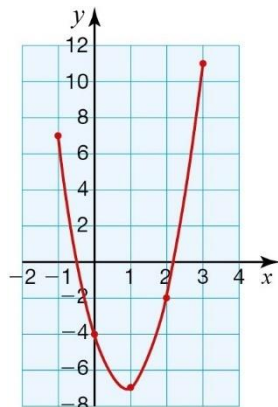
|     |    |    |    |    |   |    |
|-----|----|----|----|----|---|----|
| $x$ | -2 | -1 | 0  | 1  | 2 | 3  |
| $y$ | 12 | 0  | -6 | -6 | 0 | 12 |





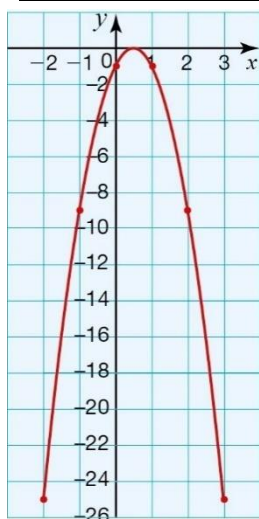
9

|   |    |    |    |    |    |
|---|----|----|----|----|----|
| x | -1 | 0  | 1  | 2  | 3  |
| y | 7  | -4 | -7 | -2 | 11 |



10

|   |     |    |    |    |    |     |
|---|-----|----|----|----|----|-----|
| x | -2  | -1 | 0  | 1  | 2  | 3   |
| y | -25 | -9 | -1 | -1 | -9 | -25 |



## Exercise 18.2

- 1 -2 and 3
- 2 -1 and 1
- 3 3
- 4 -4 and 3
- 5 2
- 6 0.5 and 3
- 7 1
- 8  $-\frac{1}{3}$  and 2

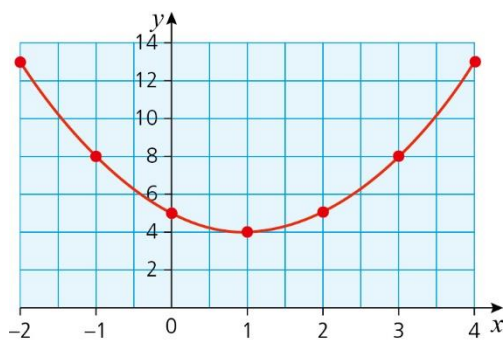


### Exercise 18.3

- 1 -1.6 and 2.6
- 2 No solution
- 3 2 and 4
- 4 -3.5 and 2.5
- 5 0.3 and 3.7
- 6 0 and 3.5
- 7 -0.2 and 2.2
- 8  $-\frac{1}{3}$  and 2

### Exercise 18.4

1 a

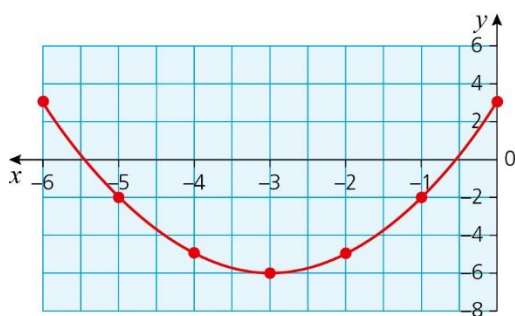


b Minimum

c (1, 4)

d (1, 4)

2 a



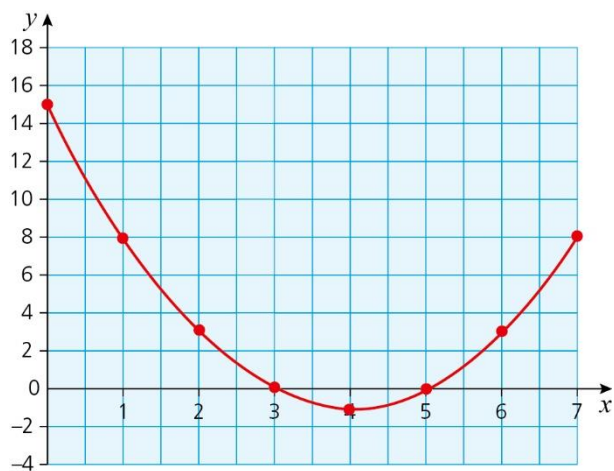
b Minimum

c (-3, -6)

d (-3, -6)



3 a

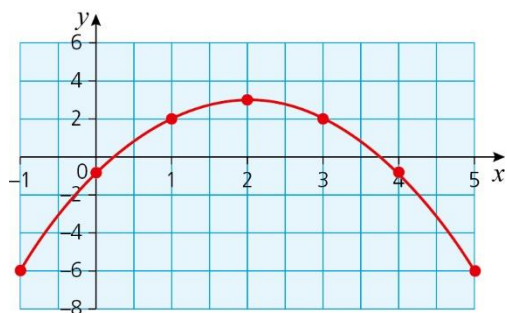


b Minimum

c (4, -1)

d (4, -1)

4 a

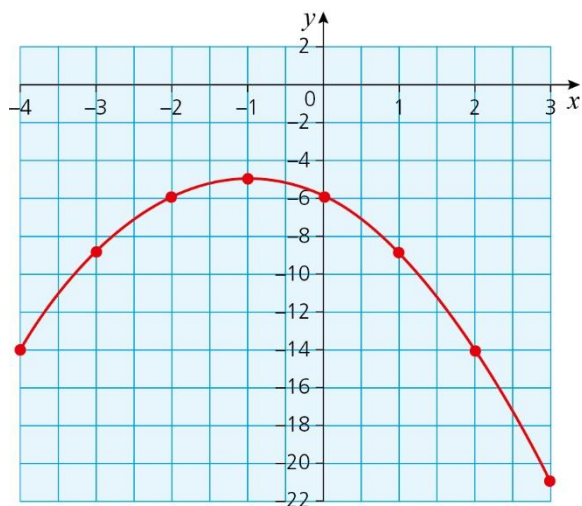


b Maximum

c (2, 3)

d (2, 3)

5 a



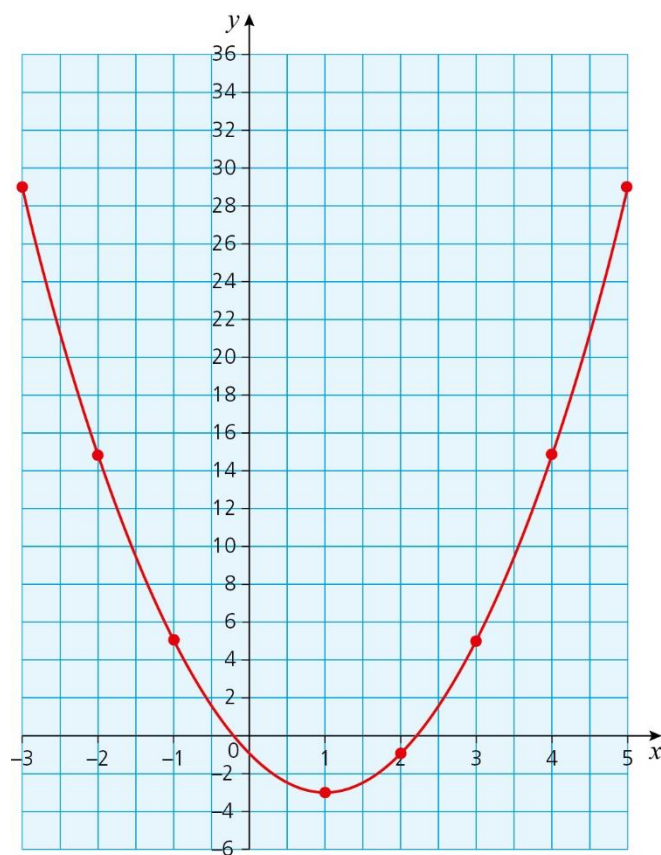


**b** Maximum

**c**  $(-1, -5)$

**d**  $(-1, -5)$

**6 a**



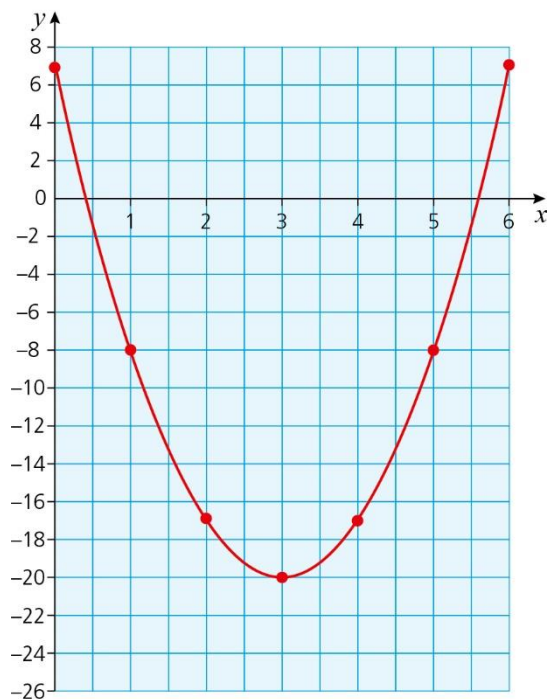
**b** Minimum

**c**  $(1, -3)$

**d**  $(1, -3)$



7 a

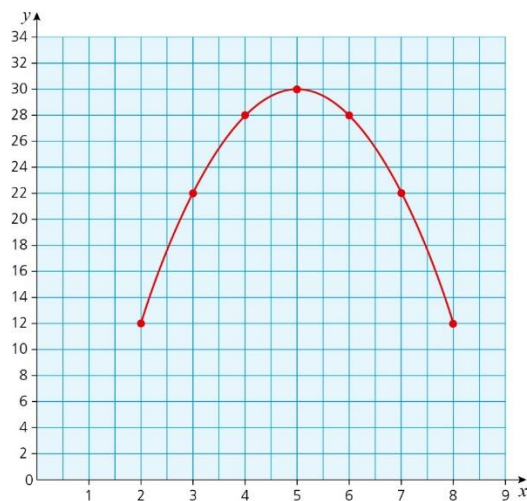


b Minimum

c (3, -20)

d (3, -20)

8 a



b Maximum

c (5, 30)

d (5, 30)

9 a Students' description of pattern.

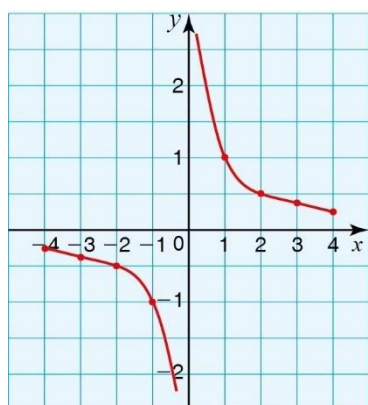
b (b, c)



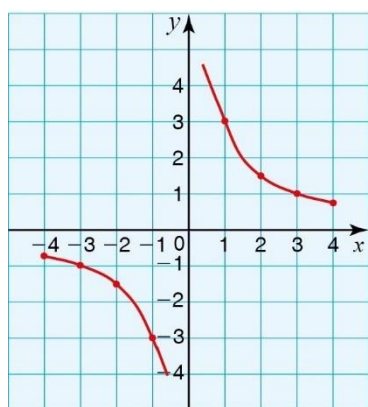
- 10 a (6, 5)  
b (-8, -6)  
c (-4, 12)  
d (12, 1)

### Exercise 18.5

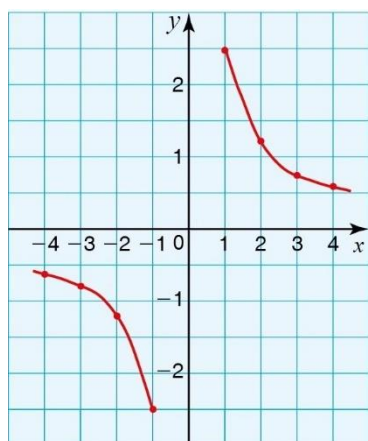
1



2



3



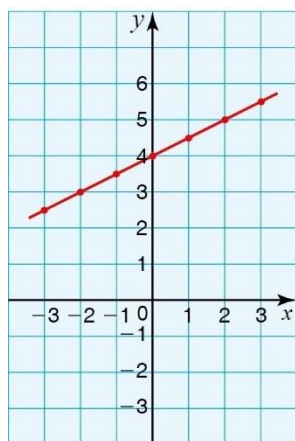


### Exercise 18.6

1 i

|        |     |    |     |   |     |   |     |
|--------|-----|----|-----|---|-----|---|-----|
| $x$    | -3  | -2 | -1  | 0 | 1   | 2 | 3   |
| $f(x)$ | 2.5 | 3  | 3.5 | 4 | 4.5 | 5 | 5.5 |

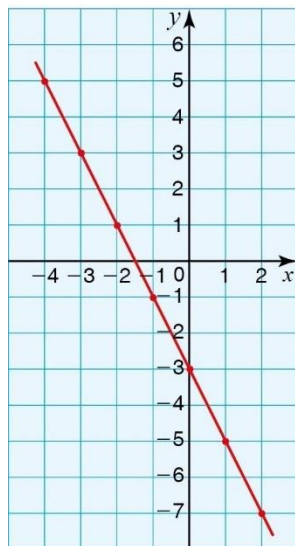
ii



2 i

|        |    |    |    |    |    |    |    |
|--------|----|----|----|----|----|----|----|
| $x$    | -4 | -3 | -2 | -1 | 0  | 1  | 2  |
| $f(x)$ | 5  | 3  | 1  | -1 | -3 | -5 | -7 |

ii



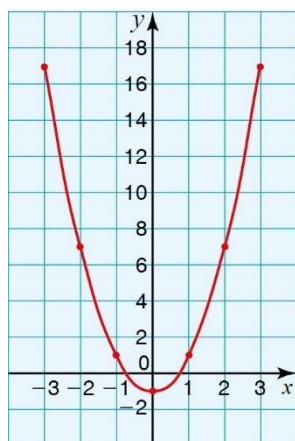


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3 i

|        |    |    |    |    |   |   |    |
|--------|----|----|----|----|---|---|----|
| $x$    | -3 | -2 | -1 | 0  | 1 | 2 | 3  |
| $f(x)$ | 17 | 7  | 1  | -1 | 1 | 7 | 17 |

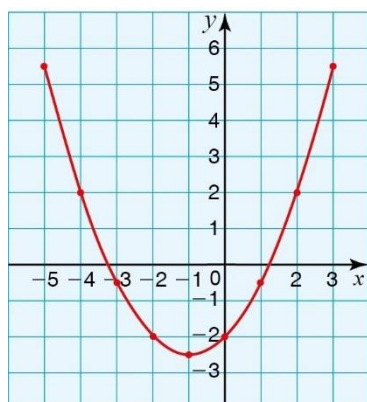
ii



4 i

|        |     |    |      |    |      |    |      |   |     |
|--------|-----|----|------|----|------|----|------|---|-----|
| $x$    | -5  | -4 | -3   | -2 | -1   | 0  | 1    | 2 | 3   |
| $f(x)$ | 5.5 | 2  | -0.5 | -2 | -2.5 | -2 | -0.5 | 2 | 5.5 |

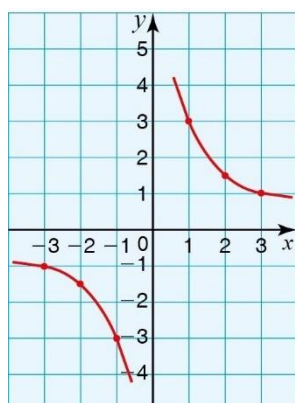
ii



5 i

|        |    |      |    |   |   |     |   |
|--------|----|------|----|---|---|-----|---|
| $x$    | -3 | -2   | -1 | 0 | 1 | 2   | 3 |
| $f(x)$ | -1 | -1.5 | -3 | - | 3 | 1.5 | 1 |

ii



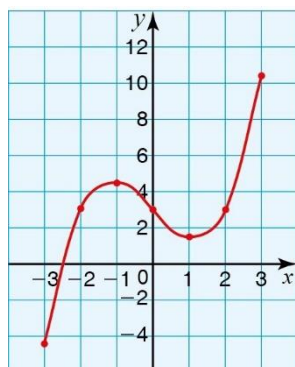


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6 i

|        |      |    |     |   |     |   |      |
|--------|------|----|-----|---|-----|---|------|
| $x$    | -3   | -2 | -1  | 0 | 1   | 2 | 3    |
| $f(x)$ | -4.5 | 3  | 4.5 | 3 | 1.5 | 3 | 10.5 |

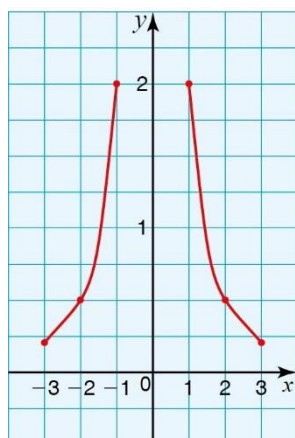
ii



7 i

|        |      |     |    |   |   |     |      |
|--------|------|-----|----|---|---|-----|------|
| $x$    | -3   | -2  | -1 | 0 | 1 | 2   | 3    |
| $f(x)$ | 0.22 | 0.5 | 2  | — | 2 | 0.5 | 0.22 |

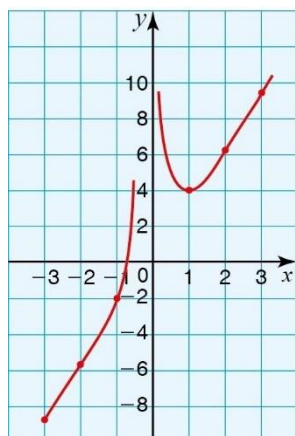
ii



8 i

|        |       |       |    |   |   |      |      |
|--------|-------|-------|----|---|---|------|------|
| $x$    | -3    | -2    | -1 | 0 | 1 | 2    | 3    |
| $f(x)$ | -8.89 | -5.75 | -2 | — | 4 | 6.25 | 9.11 |

ii

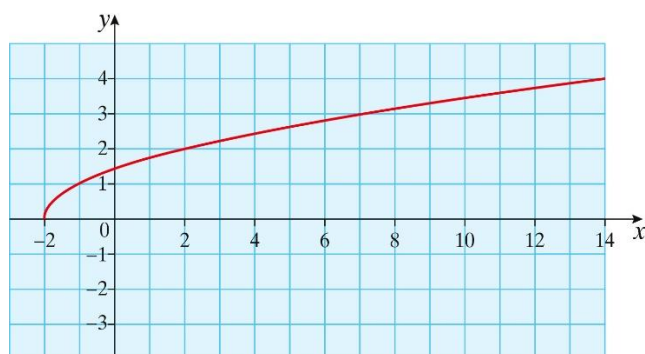




9 i

|        |    |   |   |    |
|--------|----|---|---|----|
| $x$    | -2 | 2 | 7 | 14 |
| $f(x)$ | 0  | 2 | 3 | 4  |

ii



10 i

|        |                |               |    |                |                 |
|--------|----------------|---------------|----|----------------|-----------------|
| $x$    | $\frac{1}{16}$ | $\frac{1}{4}$ | 1  | 4              | 9               |
| $f(x)$ | 0              | -2            | -3 | $-\frac{7}{2}$ | $-\frac{11}{3}$ |

ii





### Exercise 18.7

1 i

|        |      |      |      |   |   |   |    |
|--------|------|------|------|---|---|---|----|
| $x$    | -3   | -2   | -1   | 0 | 1 | 2 | 3  |
| $f(x)$ | 0.04 | 0.11 | 0.33 | 1 | 3 | 9 | 27 |

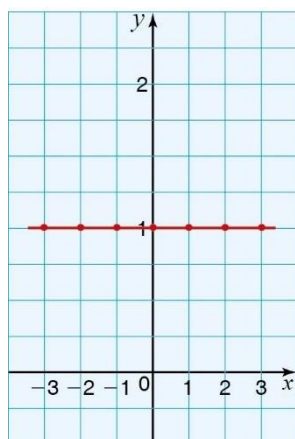
ii



2 i

|        |    |    |    |   |   |   |   |
|--------|----|----|----|---|---|---|---|
| $x$    | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| $f(x)$ | 1  | 1  | 1  | 1 | 1 | 1 | 1 |

ii



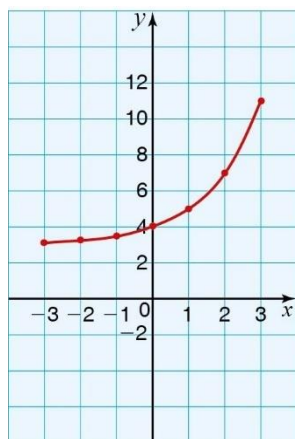


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3 i

| $x$    | -3    | -2   | -1  | 0 | 1 | 2 | 3  |
|--------|-------|------|-----|---|---|---|----|
| $f(x)$ | 3.125 | 3.25 | 3.5 | 4 | 5 | 7 | 11 |

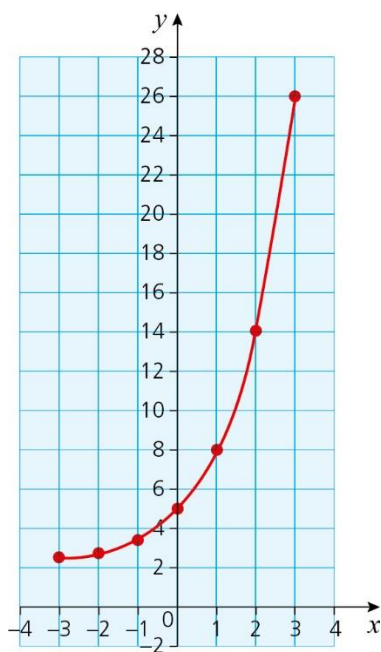
ii



4 i

| $x$    | -3    | -2   | -1  | 0 | 1 | 2  | 3  |
|--------|-------|------|-----|---|---|----|----|
| $f(x)$ | 2.375 | 2.75 | 3.5 | 5 | 8 | 14 | 26 |

ii



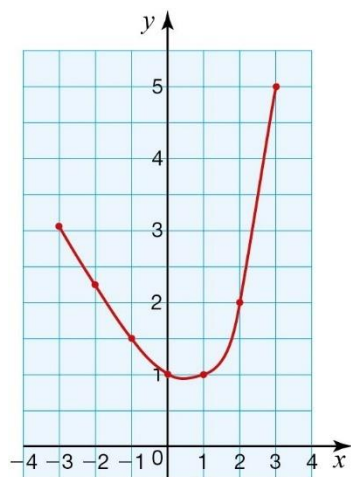


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5 i

| $x$    | -3    | -2   | -1  | 0 | 1 | 2 | 3 |
|--------|-------|------|-----|---|---|---|---|
| $f(x)$ | 3.125 | 2.25 | 1.5 | 1 | 1 | 2 | 5 |

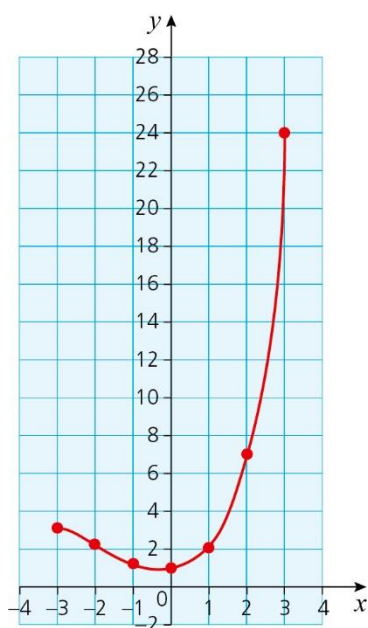
ii



6 i

| $x$    | -3   | -2   | -1   | 0 | 1 | 2 | 3  |
|--------|------|------|------|---|---|---|----|
| $f(x)$ | 3.04 | 2.11 | 1.33 | 1 | 2 | 7 | 24 |

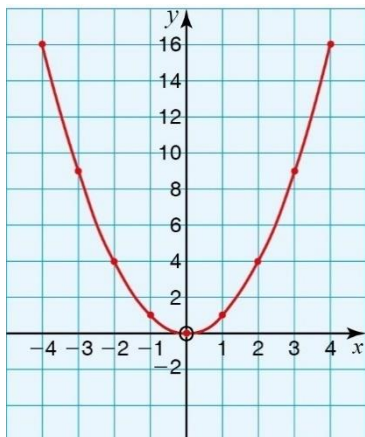
ii





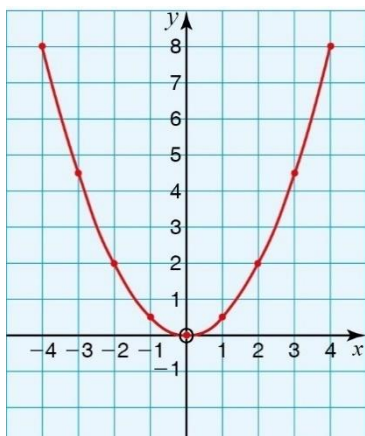
### Exercise 18.8

1 i



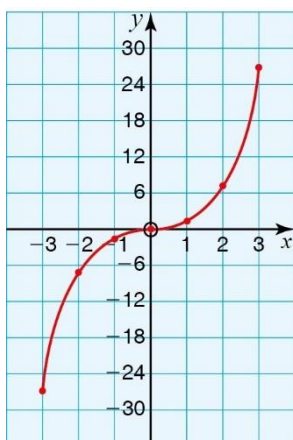
ii 2

2 i



ii -2

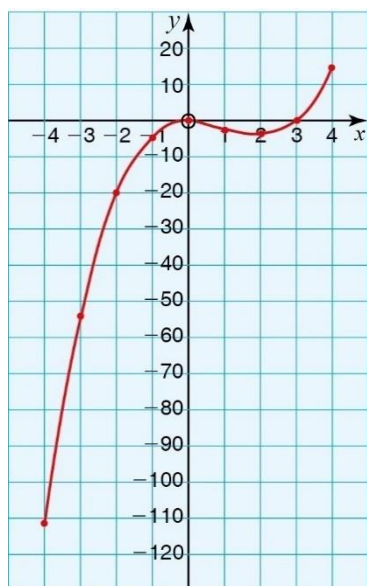
3 i



ii 3

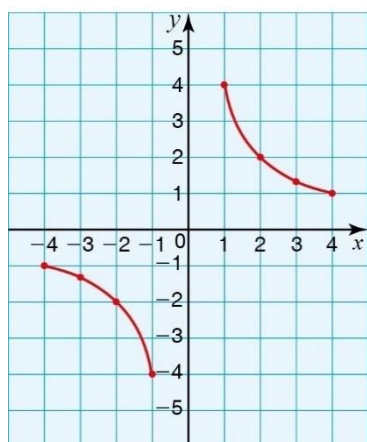


4 i



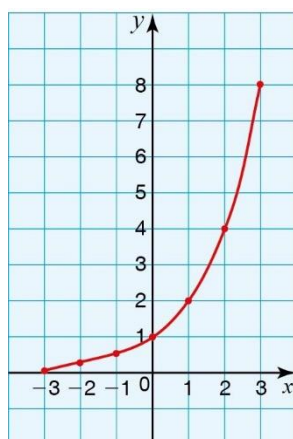
ii 24

5 i



ii -4

6 i

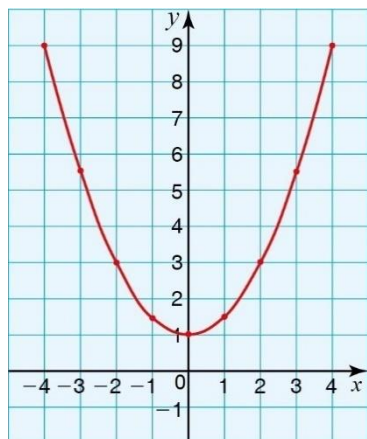


ii 0.7



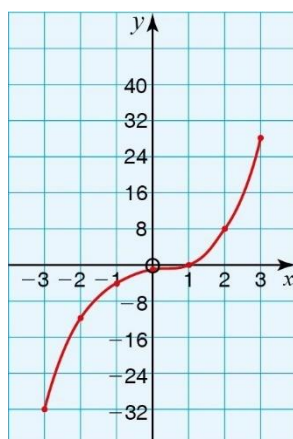
### Exercise 18.9

1 a



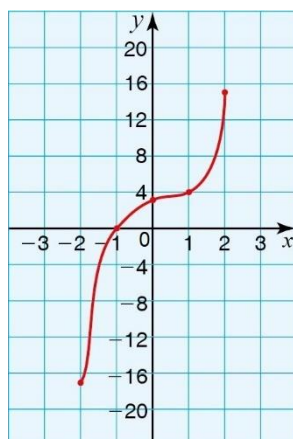
b  $x = \pm 2.8$

2 a



b  $x = 1.7$

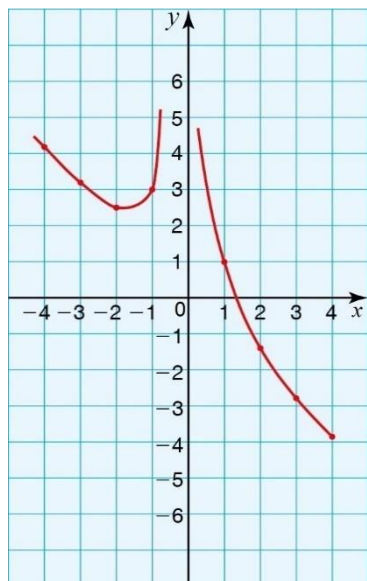
3 a



b  $x = 1.5$



4 a



b  $x = -0.4, 0.5, 2.4$

5 a  $(x - 10)$  cm

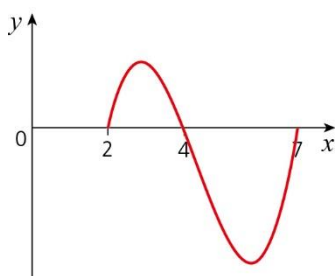
b  $5(x - 10)^2 = V$

c 6 cm by 6 cm or 14 cm by 14 cm

6 10 cm

7 a  $V = (x - 2)(x - 4)(x - 7)$

b



c The y-value on the graph is positive and the volume must be a positive value.

d  $x = 2.9$



8 a



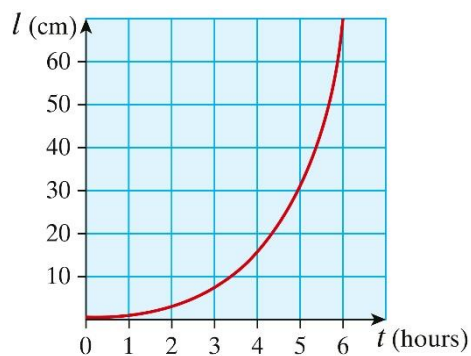
b  $x = -0.7, 3$

9 a 7 cm

b 0 cm

c 5 hours

d



e Approx.  $5\frac{1}{2}$  hours

10 a Approx. 2.5

b  $-\frac{1}{2}$

11 a Approx. 4.3

b Approx. 3.3

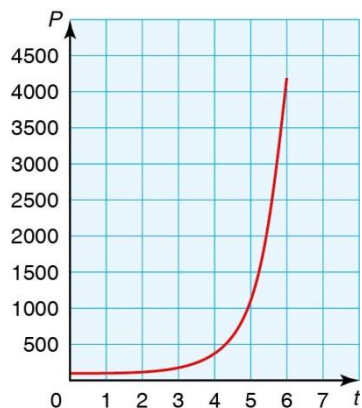
c Approx. 2.3

12 a 101

b 1120



c

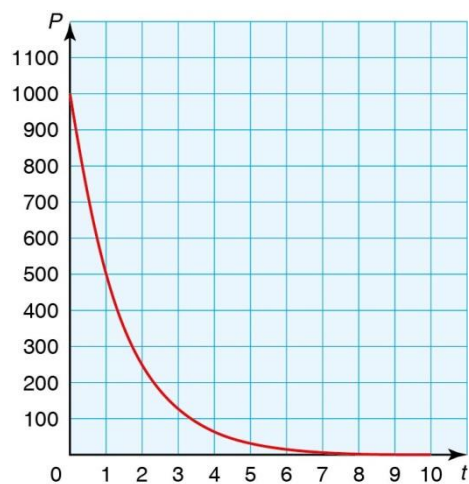


d Approx. 4.5 hours

13 a

| $t$ | 0    | 1   | 2   | 3   | 4  | 5  | 6  | 7 | 8 | 9 | 10 |
|-----|------|-----|-----|-----|----|----|----|---|---|---|----|
| $P$ | 1000 | 500 | 250 | 125 | 63 | 31 | 16 | 8 | 4 | 2 | 1  |

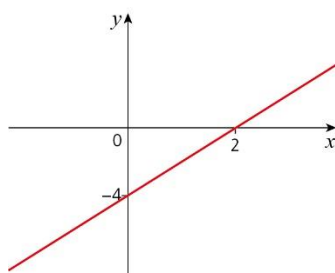
b



c Approx. 90 insects

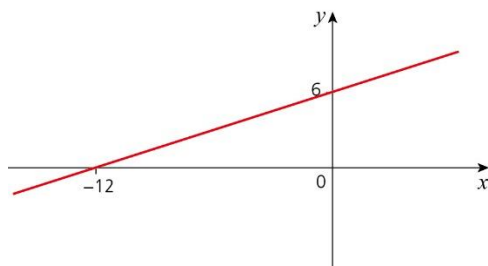
## Exercise 18.10

1 a

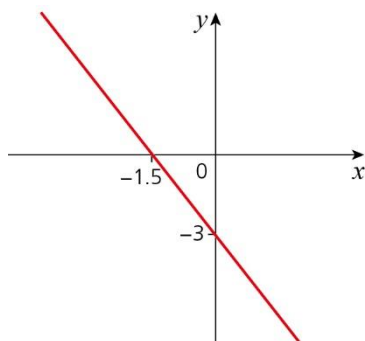




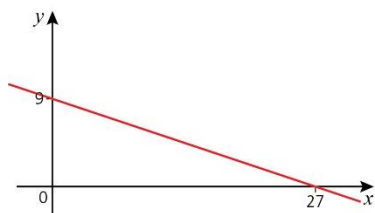
**b**



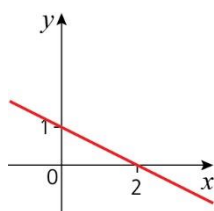
**c**



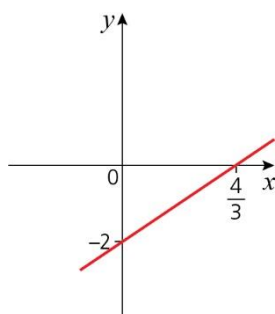
**d**



**e**

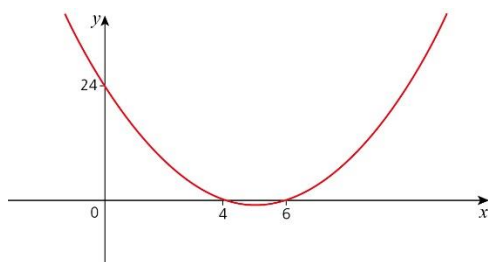


**f**



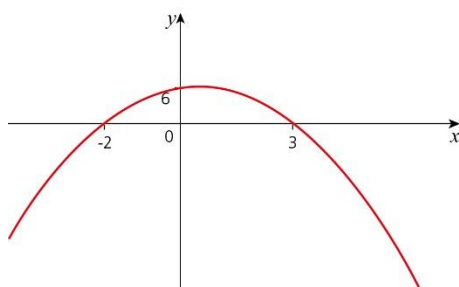


2 a



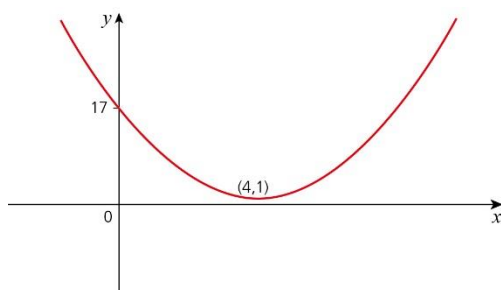
Turning point (5, -1)

b



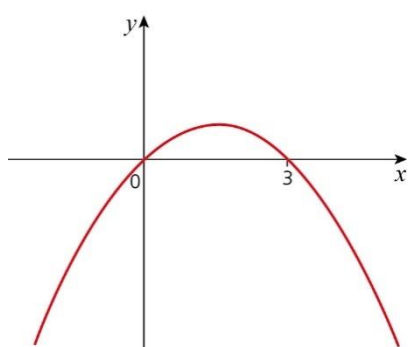
Turning point  $(\frac{1}{2}, 6\frac{1}{4})$

c



Turning point (4, 1)

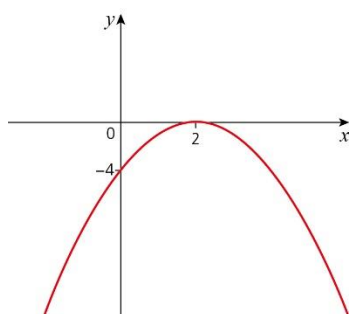
d



Turning point  $(\frac{3}{2}, \frac{9}{4})$

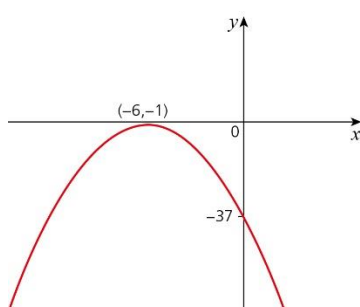


e



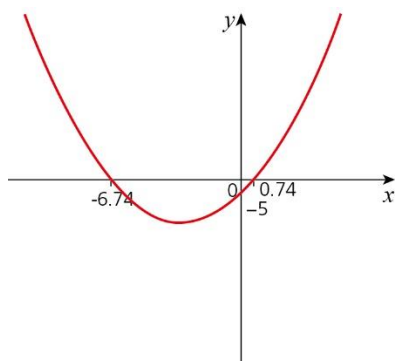
Turning point (2, 0)

f



Turning point (-6, -1)

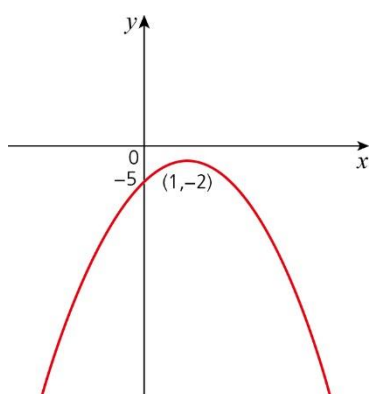
g



Roots given to 2 d.p. Exact values are  $x = -3 \pm \sqrt{14}$

Turning point (-3, -14)

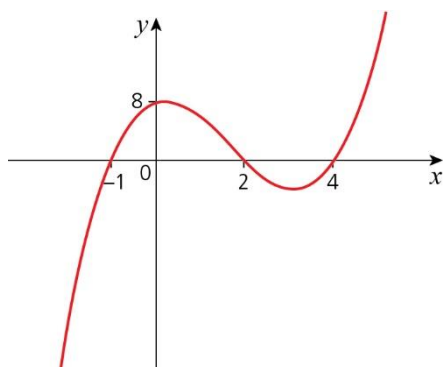
h



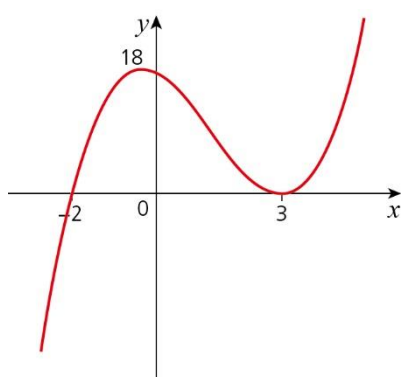
Turning point (1, -2)



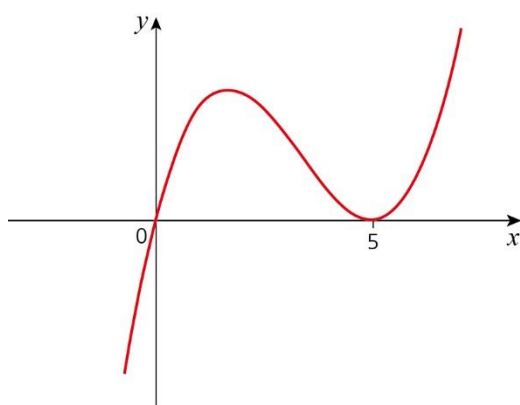
3 a



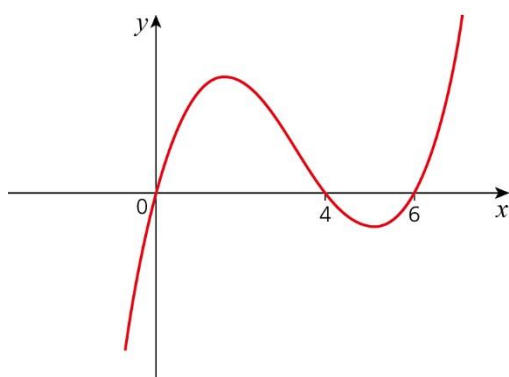
b



c

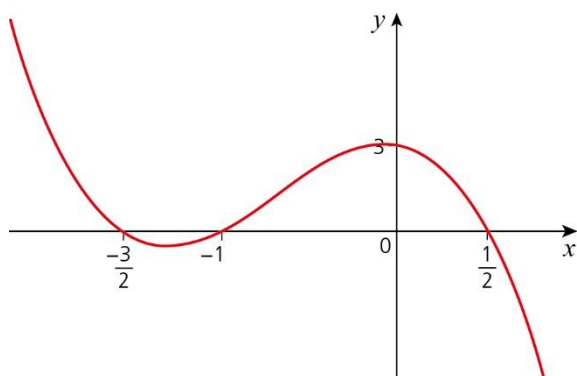


d





e

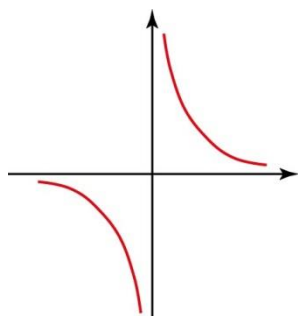


### Exercise 18.11

- 1 A  $y = 2^x$   
B  $y = \frac{3}{x}$   
C  $y = \frac{1}{x}$   
D  $y = -\frac{2}{x}$   
E  $y = 2^{-x}$
- 2 A  $y = -\frac{1}{x}$   
B  $y = -3^x$   
C  $y = -5^x$   
D  $y = \frac{2}{x}$   
E  $y = \frac{1}{x}$

### Student assessment 1

1



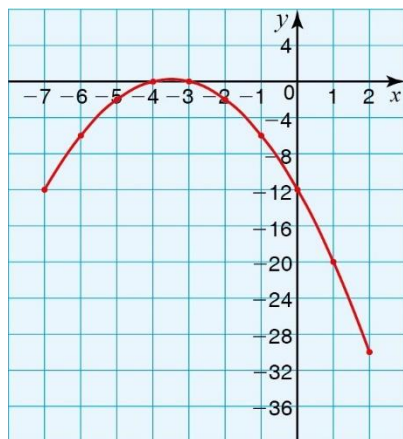


# Cambridge IGCSE™ Mathematics Core and Extended Answers to Student's Book

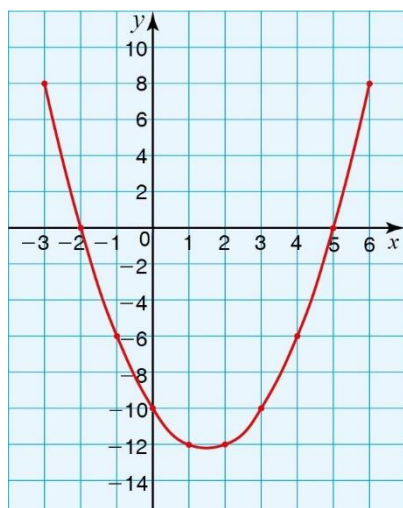
2 a

| x | -7  | -6 | -5 | -4 | -3 | -2 | -1 | 0   | 1   | 2   |
|---|-----|----|----|----|----|----|----|-----|-----|-----|
| y | -12 | -6 | -2 | 0  | 0  | -2 | -6 | -12 | -20 | -30 |

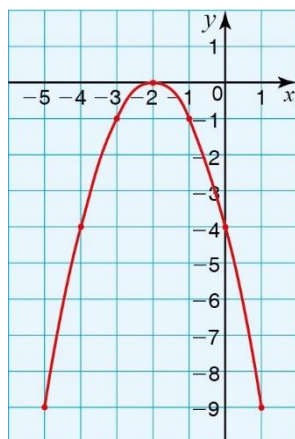
b



3 a

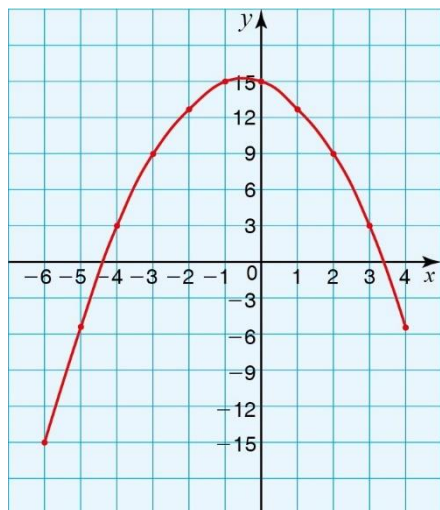


b





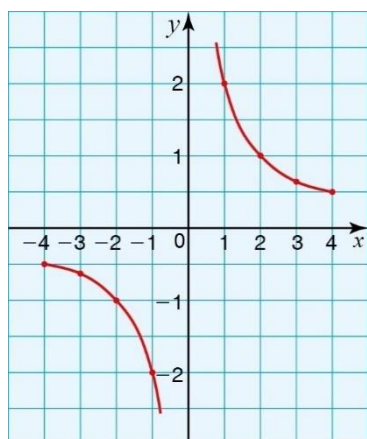
4 a



b i  $x = -3.7$  or  $2.7$

ii  $x = -1.8$  or  $2.8$

5 a



b  $x = -2$  or  $1$

6 a i maximum as the  $x^2$  term is negative

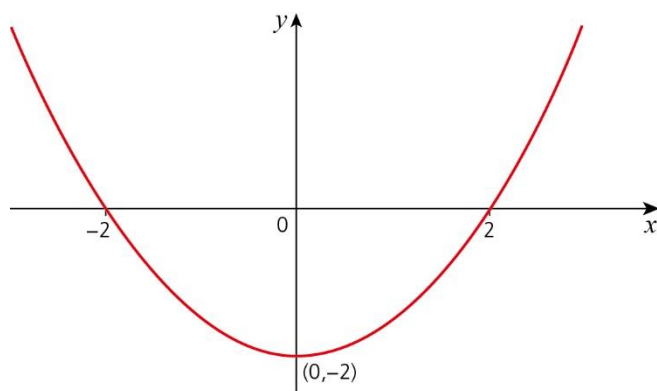
ii  $(-3, -5)$

b i minimum as the  $x^2$  term is positive

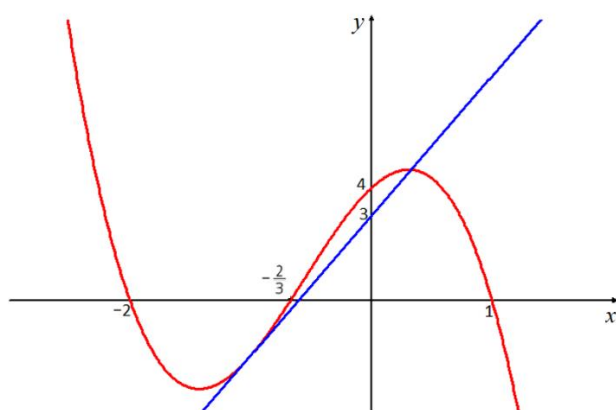
ii  $(6, 2)$



7



8 a, b i



b ii  $(-\frac{3}{5}, 0)$

## Student assessment 2

1 a i Linear

ii Quadratic

b i Any answer in the form  $y = k - cx$  where  $k$  and  $c$  are positive constants,  
e.g.  $y = 5 - 2x$

ii Any answer in the form  $y = k - cx^2$  where  $k$  and  $c$  are positive constants,  
e.g.  $y = 4 - x^2$

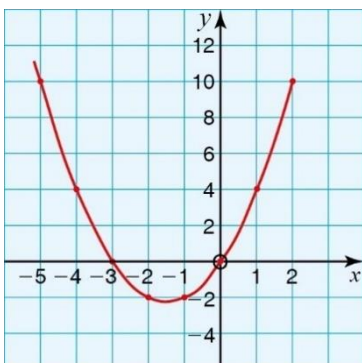


# Cambridge IGCSE™ Mathematics Core and Extended Answers to Student's Book

2 a i

|        |    |    |    |    |    |   |   |    |
|--------|----|----|----|----|----|---|---|----|
| $x$    | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2  |
| $f(x)$ | 10 | 4  | 0  | -2 | -2 | 0 | 4 | 10 |

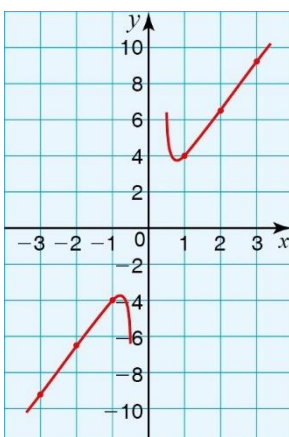
ii



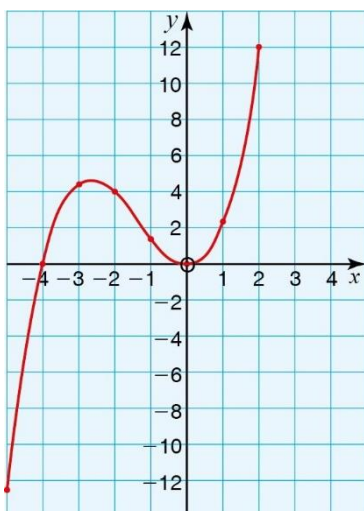
b i

|        |      |      |    |       |   |      |   |     |     |
|--------|------|------|----|-------|---|------|---|-----|-----|
| $x$    | -3   | -2   | -1 | -0.1  | 0 | 0.1  | 1 | 2   | 3   |
| $f(x)$ | -9.3 | -6.5 | -4 | -10.3 | - | 10.3 | 4 | 6.5 | 9.3 |

ii



3 a

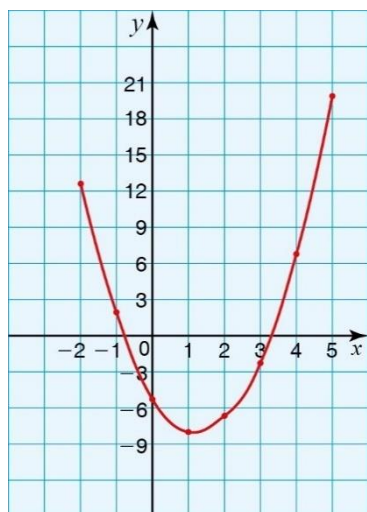


b i 5.5

ii 2.5



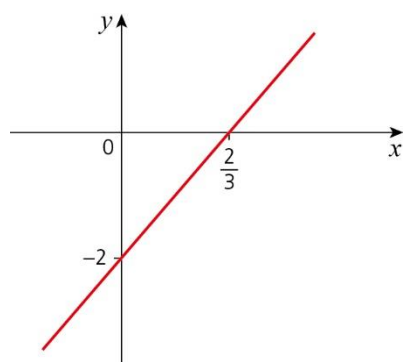
4 a



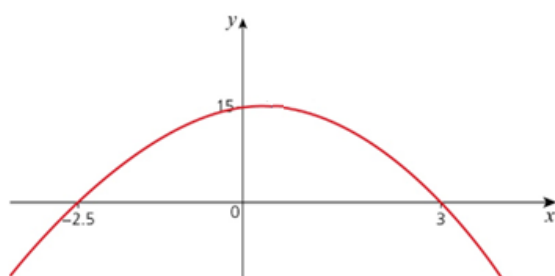
b  $x = -0.8$  or  $3.3$

c  $x = -1.6$  or  $3.1$

5



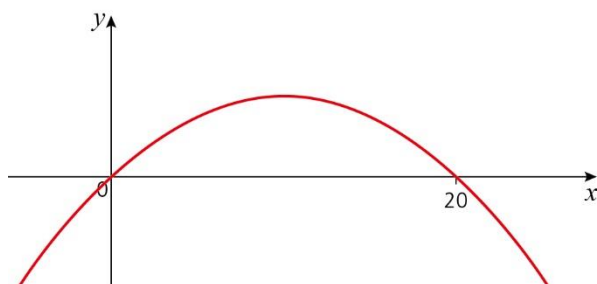
6



Turning point is at  $(\frac{1}{4}, \frac{121}{8})$



- 7 a  $58 - 3x$   
b  $A = x(60 - 3x)$   
c



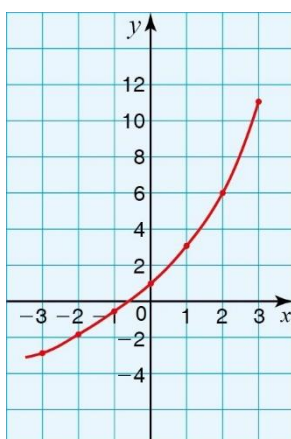
- d  $x = 10$   
e  $300 \text{ m}^2$

## Student assessment 3

- 1 a i Reciprocal  
ii Exponential  
b i For example,  $y = \frac{1}{x}$   
ii For example,  $y = 2^x$

- 2 a i  
ii

| $x$    | -3   | -2   | -1   | 0 | 1 | 2 | 3  |
|--------|------|------|------|---|---|---|----|
| $f(x)$ | -2.9 | -1.8 | -0.5 | 1 | 3 | 6 | 11 |

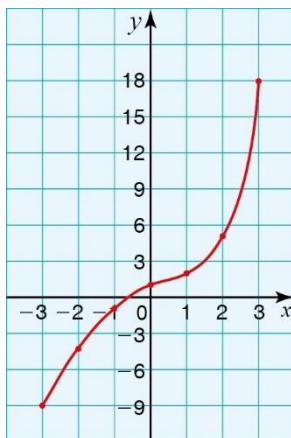


- b i

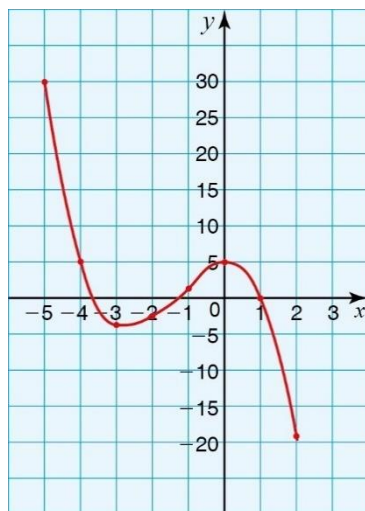
| $x$    | -3   | -2   | -1   | 0 | 1 | 2 | 3  |
|--------|------|------|------|---|---|---|----|
| $f(x)$ | -9.0 | -3.9 | -0.7 | 1 | 2 | 5 | 18 |



ii



3 a



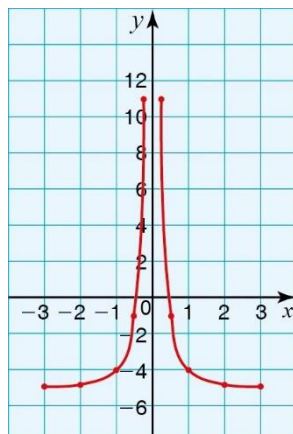
b i 0

ii 4

4 a

| x | -3   | -2   | -1 | -0.5 | -0.25 | 0 | 0.25 | 0.5 | 1  | 2    | 3    |
|---|------|------|----|------|-------|---|------|-----|----|------|------|
| y | -4.9 | -4.8 | -4 | -1   | 11    | - | 11   | -1  | -4 | -4.8 | -4.9 |

b

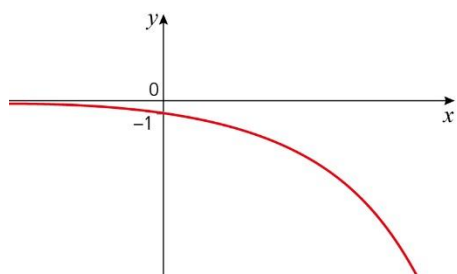




**c**  $x = \pm 0.4$

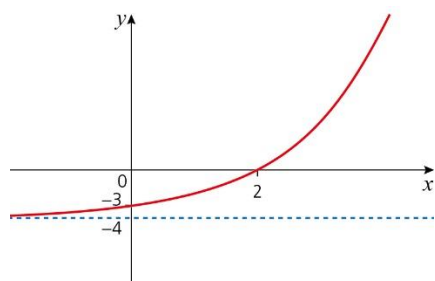
**d**  $x = \pm 0.4$  and  $\pm 2.6$

**5 a**



**b**  $y = 0$

**6 a**

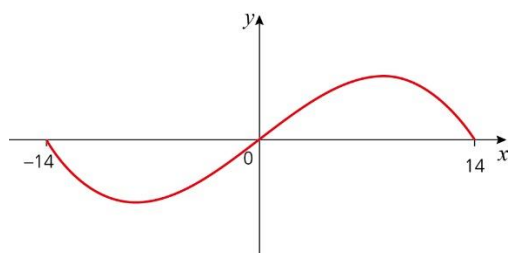


**b**  $y = -4$

**7 a** Students' working

**b**  $V = 98x - \frac{1}{2}x^3$

**c**



**d**  $x = 8$

**e**  $528 \text{ cm}^3$



All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

## 19 Differentiation and the gradient function

### Exercise 19.1

---

- 1 a i 4  
ii 6  
iii -2

b

|          |   |   |   |
|----------|---|---|---|
| x        | 0 | 1 | 2 |
| Gradient | 0 | 2 | 4 |

- c  $2x$

- 2 a i 4  
ii 8  
iii -8

- b  $4x$

- 3 a i 1  
ii 2  
iii 3

- b  $x$

### Exercise 19.2

---

- 1 a  $\frac{dy}{dx} = 3x^2$   
b  $\frac{dy}{dx} = 6x$   
c  $\frac{dy}{dx} = 2x + 2$   
d  $\frac{dy}{dx} = 2x$   
e  $\frac{dy}{dx} = 3$   
f  $\frac{dy}{dx} = 4x - 1$



| 2 | Function $f(x)$  | Gradient function $f'(x)$ |
|---|------------------|---------------------------|
|   | $x^2$            | $2x$                      |
|   | $2x^2$           | $4x$                      |
|   | $\frac{1}{2}x^2$ | $x$                       |
|   | $x^2 + x$        | $2x + 1$                  |
|   | $x^3$            | $3x^2$                    |
|   | $3x^2$           | $6x$                      |
|   | $x^2 + 2x$       | $2x + 2$                  |
|   | $x^2 - 2$        | $2x$                      |
|   | $3x - 3$         | $3$                       |
|   | $2x^2 - x + 1$   | $4x - 1$                  |

3 Student's observations (e.g. multiply the coefficient by the power and then reduce the power by one)

### Exercise 19.3

---

- 1 a  $f'(x) = 4x^3$   
b  $f'(x) = 1$   
c  $f'(x) = 6x$   
d  $f'(x) = 15x^2$   
e  $f'(x) = 18x^2$   
f  $f'(x) = 56x^6$
- 2 a  $f'(x) = x^2$   
b  $f'(x) = x^3$   
c  $f'(x) = \frac{1}{2}x$   
d  $f'(x) = 2x^3$   
e  $f'(x) = \frac{6}{5}x^2$   
f  $f'(x) = \frac{2}{3}x^2$

### Exercise 19.4

---

- 1 a  $\frac{dy}{dx} = 15x^2$   
b  $\frac{dy}{dx} = 14x$   
c  $\frac{dy}{dx} = 24x^5$   
d  $\frac{dy}{dx} = \frac{1}{2}x$



e  $\frac{dy}{dx} = 4x^5$

f  $\frac{dy}{dx} = \frac{15}{4}x^4$

g  $\frac{dy}{dx} = 0$

h  $\frac{dy}{dx} = 6$

i  $\frac{dy}{dx} = 0$

2 a  $\frac{dy}{dx} = 6x + 4$

b  $\frac{dy}{dx} = 15x^2 - 4x$

c  $\frac{dy}{dx} = 30x^2 - x$

d  $\frac{dy}{dx} = 18x^2 - 6x + 1$

e  $\frac{dy}{dx} = 48x^3 - 4x$

f  $\frac{dy}{dx} = x^2 - x + 1$

g  $\frac{dy}{dx} = -12x^3 + 8x$

h  $\frac{dy}{dx} = -30x^4 + 12x^3 - 1$

i  $\frac{dy}{dx} = -\frac{9}{2}x^5 + 2x^2$

3 a  $\frac{dy}{dx} = 2x + 1$

b  $\frac{dy}{dx} = 4$

c  $\frac{dy}{dx} = 6x + 1$

d  $\frac{dy}{dx} = \frac{1}{2}x + \frac{1}{2}$

e  $\frac{dy}{dx} = 6x + 3$

f  $\frac{dy}{dx} = 6x^2 - 8x$

g  $\frac{dy}{dx} = 2x + 10$

h  $\frac{dy}{dx} = 4x + 7$

i  $\frac{dy}{dx} = 3x^2 - 4x - 3$

### Exercise 19.5

---

1 a  $\frac{dy}{dt} = 6t + 1$

b  $\frac{dv}{dt} = 6t^2 - 2t$

c  $\frac{dm}{dt} = 15t^2 - 2t$



- 2 a  $\frac{dy}{dx} = 2x + 4$   
b  $\frac{dr}{dt} = 1 - 2t$   
c  $\frac{dv}{dt} = 3t^2$   
d  $\frac{dp}{dr} = 2 - 6r$
- 3 a  $\frac{dy}{dt} = 2t$   
b  $\frac{dr}{dt} = 4t$   
c  $\frac{dv}{dt} = 2t^2 - \frac{4}{3}t + 1$

### Exercise 19.6

---

- 1 a  $\frac{d^2y}{dx^2} = 12x$   
b  $\frac{d^2y}{dx^2} = 12x^2 - 1$   
c  $\frac{d^2y}{dx^2} = 10x^4$   
d  $\frac{d^2y}{dx^2} = 6$   
e  $\frac{d^2y}{dx^2} = \frac{1}{2}$   
f  $\frac{d^2y}{dx^2} = 0$
- 2 a  $\frac{d^2v}{dx^2} = 6x - 6$   
b  $\frac{d^2P}{dx^2} = 6x^2 + 3x$   
c  $\frac{d^2t}{dx^2} = 2$   
d  $\frac{d^2y}{dx^2} = 20x^3$

### Exercise 19.7

---

- 1 a 6  
b -3  
c 0  
d 0  
e  $-\frac{1}{2}$  and -5  
f 6
- 2 a i  $\frac{9}{2}$   
ii  $31\frac{1}{2}$



iii 72

iv 0

b  $\frac{dN}{dt} = 10t - \frac{3}{2}t^2$

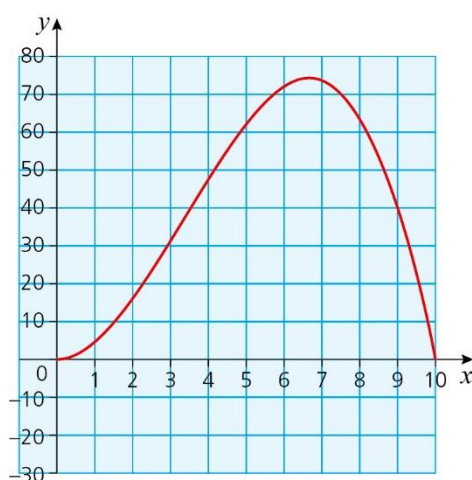
c i  $8\frac{1}{2}$

ii  $16\frac{1}{2}$

iii 6

iv -50

d



e The graph increases during the first 6–7 days, hence the number of infections increases.

When  $t = 10$  the graph is at zero, hence the number of new infections is zero.

f The rate is initially increasing, i.e. the gradient of the graph is increasing. After approximately 4 days, the rate of increase starts to decrease, i.e. the gradient of the curve is less steep. After 7 days the gradient is negative, hence the rate of increase is negative too.

4 a i 243 m

ii 2000 m

b  $\frac{dh}{dt} = 60t - 3t^2$

c i  $108 \text{ m h}^{-1}$

ii  $225 \text{ m h}^{-1}$

iii  $0 \text{ m h}^{-1}$

d The gradient of the graph initially is quite low getting to its steepest point after approximately 10 hours. Hence the answers in part c show the rate of ascent being greater when  $t = 5$  compared with  $t = 2$ . After 20 hours the graph is flat therefore the rate of ascent is zero.

e The steepest part of the graph occurs when  $t = 10$  hours. Therefore, this represents when the balloon is climbing at its fastest rate.



### Exercise 19.8

---

- 1 a (3, 6)  
b (2.5, 19.75)  
c (2, 10)  
d (0, -1)  
e (3, 15)  
f (-5, -14)
- 2 a  $\left(1, 4\frac{5}{6}\right)$  and  $\left(-2, -8\frac{2}{3}\right)$   
b  $\left(-1, -4\frac{1}{3}\right)$  and  $(-3, -9)$   
c  $\left(2, -5\frac{1}{3}\right)$   
d (1, 4) and  $\left(-\frac{1}{3}, -1\frac{13}{27}\right)$
- 3 a  $\frac{ds}{dt} = 4 + 10t$   
b  $\frac{1}{2}$  second  
c 3 seconds  
d 57 m
- 4 a 20 °C  
b  $\frac{dT}{dt} = 24t - 3t^2$   
c i 21 °C/min  
ii 48 °C/min  
iii 0 °C/min  
d  $t = 2$  or 6 minutes  
e 236 °C

### Exercise 19.9

---

- 1 a  $f'(x) = 2x - 3$   
b 1  
c 1  
d  $y = x - 3$
- 2 a 4  
b  $y = 4x - 10$
- 3 a -4  
b  $y = -4x - 2$
- 4 a -4  
b  $y = -4x + 17$



c  $y = 5$

5 a  $y = -2x + 2$

b  $(-6, 10)$

c  $(-4, 10)$

6 a  $f'(x) = -x - 1$

b  $-3$

c  $(2, -8)$

### Exercise 19.10

---

1 a i  $f'(x) = 2x - 6$

ii  $(3, 4)$

b i  $f'(x) = 2x + 12$

ii  $(-6, -1)$

c i  $f'(x) = -2x + 8$

ii  $(4, 3)$

d i  $f'(x) = -6$

ii No stationary points

2 a i  $f'(x) = 3x^2 - 24x + 48$

ii  $(4, 6)$

b i  $f'(x) = 3x^2 - 12$

ii  $(-2, 16)$  and  $(2, -16)$

c i  $f'(x) = 3x^2 - 6x - 45$

ii  $(-3, 89)$  and  $(5, -167)$

d i  $f'(x) = x^2 + 3x - 4$

ii  $\left(-4, 13\frac{2}{3}\right)$  and  $\left(1, -7\frac{1}{6}\right)$

3 a  $f'(x) = -4 - 2x$

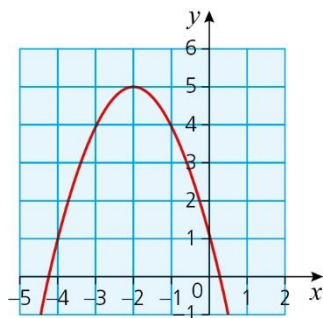
b  $(-2, 5)$

c a maximum

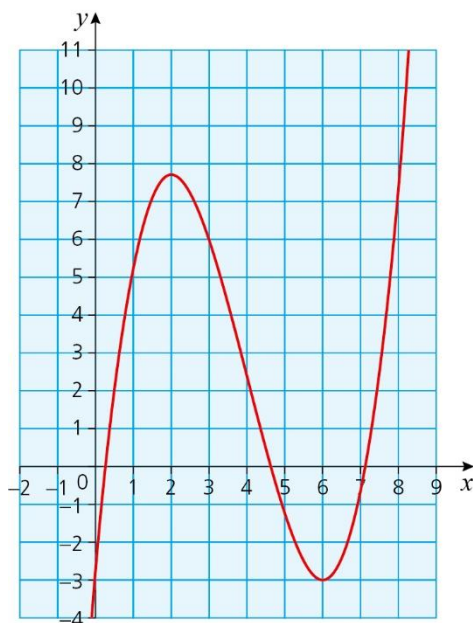
d  $(0, 1)$



e



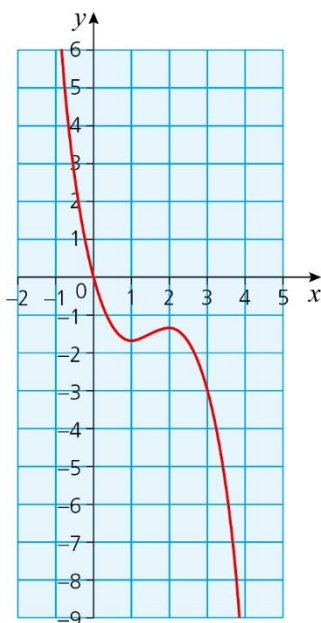
- 4 a  $f'(x) = x^2 - 8x + 12$   
b  $(2, 7\frac{2}{3})$  and  $(6, -3)$   
c  $(2, 7\frac{2}{3})$  is a maximum and  $(6, -3)$  is a minimum  
d  $(0, -3)$   
e



- 5 a  $f'(x) = -2x^2 + 6x - 4$   
b  $(1, -1\frac{2}{3})$  and  $(2, -1\frac{1}{3})$   
c  $(1, -1\frac{2}{3})$  is a minimum and  $(2, -1\frac{1}{3})$  is a maximum point  
d  $(0, 0)$



e



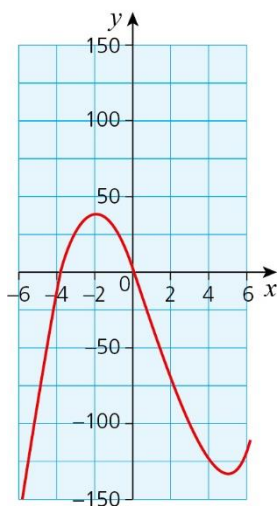
6 a  $f'(x) = 3x^2 - 9x - 30$

b  $(-2, 38)$  and  $(5, -133\frac{1}{2})$

c  $(-2, 38)$  is a maximum and  $(5, -133\frac{1}{2})$  is a minimum point

d  $(0, 4)$

e



## Student assessment 1

1 a  $\frac{dy}{dx} = 3x^2$

b  $\frac{dy}{dx} = 4x - 1$

c  $\frac{dy}{dx} = -x + 2$



- d  $\frac{dy}{dx} = 2x^2 + 8x - 1$
- 2 a  $f'(x) = 2x + 2$   
b  $f'(x) = 2x - 1$   
c  $f'(x) = 2x$   
d  $f'(x) = x + 1$
- 3  $p = 4, q = 2$
- 4 a  $\frac{d^2y}{dx^2} = 12x^2 - 6$   
b  $\frac{d^2s}{dt^2} = 40t^3 - 6t$
- 5 a  $f'(1) = 2$   
b  $f'(0) = 1$   
c  $f'\left(\frac{1}{4}\right) = 5\frac{1}{2}$
- 6 a  $v = \frac{ds}{dt} = 10t$   
b  $v = 30 \text{ m/s}$   
c i 4.2 s  
ii 88.2 m

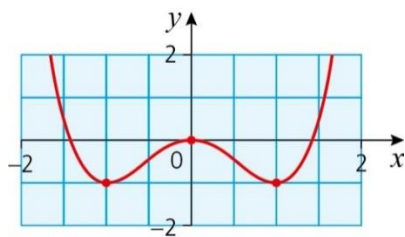
## Student assessment 2

---

- 1 a  $f'(x) = 3x^2 + 2x$   
b  $P\left(-\frac{2}{3}, -\frac{23}{27}\right)$   
c  $Q(0, -1)$   
d P is a maximum and Q a minimum
- 2 a Substituting (1, 1) into the equation gives  $1 = 1^3 - 1^2 + 1 \rightarrow 1 = 1$   
b 2  
c  $y = 2x - 1$   
d  $y = -\frac{1}{2}x + \frac{3}{2}$
- 3 a  $f'(x) = 2x - 4$   
b (2, 3)
- 4 a  $f'(x) = 4x^3 - 4x$   
b (0, 0), (1, -1) and (-1, -1)  
c (0, 0) is a maximum, (1, -1) and (-1, -1) are both minimum points  
d i (0, 0)  
ii (0, 0), ( $\sqrt{2}$ , 0) and ( $-\sqrt{2}$ , 0)



e





*All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.*

## 20 Functions

### Exercise 20.1

---

- 1 a, b and c are functions
- 2 Domain:  $-1 \leq x \leq 3$  Range:  $-3 \leq f(x) \leq 5$
- 3 Domain:  $-4 \leq x \leq 0$  Range:  $-10 \leq f(x) \leq 2$
- 4 Domain:  $-4 \leq x \leq 4$  Range:  $0 \leq f(x) \leq 8$
- 5 Domain:  $-3 \leq x \leq 3$  Range:  $2 \leq f(x) \leq 11$
- 6 Domain:  $x \in \mathbb{R}$  Range:  $f(x) \geq 2$
- 7 Domain:  $0 \leq x \leq 4$  Range:  $-14 \leq f(x) \leq 2$
- 8 Domain:  $-3 \leq x \leq 1$  Range:  $-29 \leq f(x) \leq -1$

### Exercise 20.2

---

- 1 a 6  
b 10  
c 3  
d 5  
e 2  
f -2  
g -10  
h 1
- 2 a 10  
b 22  
c 8  
d -4  
e -5  
f -18  
g -23  
h -6
- 3 a 2  
b -28  
c -20.5  
d -14



e 1.5

f 12

g 34.5

h 13.5

4 a -19

b -26.5

c -7

d -8.2

e 20

f 8

g -1

h 3.5

### Exercise 20.3

---

1 a 2

b 6.5

c 2.375

d 0.5

e 0.125

f -4

g -2.5

h -0.7

2 a 4

b 9

c -1

d -6

e -3.5

f -16

g  $-\frac{4}{3}$

h  $-\frac{7}{6}$

3 a 0.5

b 2

c -4

d -0.25

e 5

f 2.75



- g 35  
h 4.25  
4 a 4  
b 1.5  
c 2.75  
d 0.25  
e -3.5  
f 0.5  
g 0.375  
h 0.875

### Exercise 20.4

---

- 1 a 19  
b 52  
c 4  
d 3  
e 4  
f 3.25  
g 12  
h 5  
2 a 70  
b 187  
c -2  
d -5  
e 7  
f 4  
g -4.25  
h  $-4\frac{2}{3}$   
3 a -14  
b 3.5  
c 4  
d -0.5  
e -28  
f 2  
g -6  
h -68



- 4 a  $-5$   
b  $32.5$   
c  $0$   
d  $-6.875$   
e  $-7.5$   
f  $15$   
g  $-6.25$   
h  $8.125$
- 5 a  $0$   
b  $24$   
c  $0$   
d  $10.5$   
e  $-13.5$   
f  $-\frac{25}{6}$   
g  $-97.5$   
h  $21.9$  (3 s.f.)
- 6 a  $9$   
b  $0$   
c  $-5$   
d undefined  
e  $0$   
f  $11.2$   
g  $-\frac{11}{6}$   
h  $9$

### Exercise 20.5

---

- 1 a  $2x + 3$   
b  $4x - 5$   
c  $2x^2 + 1$   
d  $x + 1$   
e  $\frac{x}{2} + 3$   
f  $x + 1$
- 2 a  $12x^2 - 4$   
b  $\frac{3x^2}{16} - 4$



c  $6x - 4$

d  $27x^2$

e  $3x^2 - 6x - 1$

f  $12x^2 + 24x + 8$

3 a  $4x^2 + 3x + 2$

b  $16x^2 + 6x$

c  $4x^2 + 19x$

d  $4x^2 - 5x$

e  $\frac{2x^2 + 3x - 4}{2}$

f  $36x^2 + 57x + 20$

## Exercise 20.6

---

1 a  $x - 3$

b  $x - 6$

c  $x + 5$

d  $x$

e  $\frac{x}{2}$

f  $3x$

2 a  $\frac{x}{4}$

b  $\frac{x-5}{2}$

c  $\frac{x+6}{3}$

d  $2x - 4$

e  $\frac{4x+2}{3}$

f  $\frac{5x-7}{8}$

3 a  $2(x - 3)$

b  $4(x + 2)$

c  $\frac{x+24}{12}$

d  $\frac{x-18}{6}$

e  $\frac{x+4}{6}$

f  $\frac{3x+10}{8}$



### Exercise 20.7

---

- 1 a 6  
b 4  
c -1
- 2 a 2  
b -0.5  
c -6
- 3 a 3  
b 1.5  
c 2
- 4 a 4  
b -2  
c -11
- 5 a 4.5  
b 6  
c 0
- 6 a 8  
b -2  
c 0.5

### Exercise 20.8

---

- 1 a  $x + 2$   
b  $x + 3$   
c  $2x$   
d  $x$
- 2 a  $2x + 8$   
b  $6x + 1$   
c  $16x^2 - 16x + 10$   
d  $-x^2 - 4x$
- 3 a  $\frac{2x-1}{2}$   
b  $\frac{3x-5}{2}$   
c  $\frac{x+7}{3}$   
d  $\frac{8x-13}{20}$
- 4 a 1  
b 8



c 39

d -2

5 a 50

b -2.9

c 10

d -1.6

## Student assessment 1

---

1 a 9

b -1

c -16

2 a 5

b -1

c -5.5

3 a -6

b 0

c 15

4 a  $-x + 4$

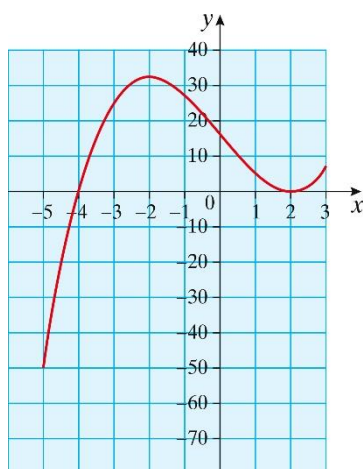
b  $\frac{2x+18}{3}$

5 a 5

b 2

6  $-4x + 14$

7 a



b  $-49 \leq f(x) \leq 32$



## Student assessment 2

---

1 a 13

b -2

c 1

2 a -2

b 1

c -1

3 a -2

b 0

c 18

4 a  $\frac{9-x}{3}$

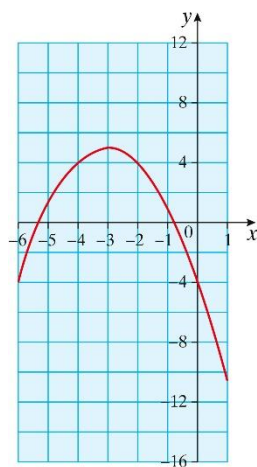
b  $4x + 2$

5 a 1

b 2

6  $32x - 6$

7 a



b  $-11 \leq g(x) \leq 5$



All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

## Mathematical investigations and ICT 2

### House of cards

- 155
- 8475
- The formula can be proved using the method of differences (see Chapter 15) as follows:

|                 |   |   |    |    |    |
|-----------------|---|---|----|----|----|
| Height of house | 1 | 2 | 3  | 4  | 5  |
| Number of cards | 2 | 7 | 15 | 26 | 40 |
| 1st difference  |   | 5 | 8  | 11 | 14 |
| 2nd difference  |   |   | 3  | 3  | 3  |

Comparing with the algebraic table below:

|                |             |               |               |                |                |
|----------------|-------------|---------------|---------------|----------------|----------------|
| Position       | 1           | 2             | 3             | 4              | 5              |
| Term           | $a + b + c$ | $4a + 2b + c$ | $9a + 3b + c$ | $16a + 4b + c$ | $25a + 5b + c$ |
| 1st difference |             | $3a + b$      | $5a + b$      | $7a + b$       | $9a + b$       |
| 2nd difference |             |               | $2a$          | $2a$           | $2a$           |

It can be deduced that:

$$2a = 3 \quad \text{therefore } a = \frac{3}{2}$$

$$3a + b = 5 \quad \text{therefore } b = \frac{1}{2}$$

$$a + b + c = 2 \quad \text{therefore } c = 0.$$

This produces the rule  $c = \frac{3}{2}n^2 + \frac{1}{2}n$  which factorises to  $c = \frac{1}{2}n(3n + 1)$ .

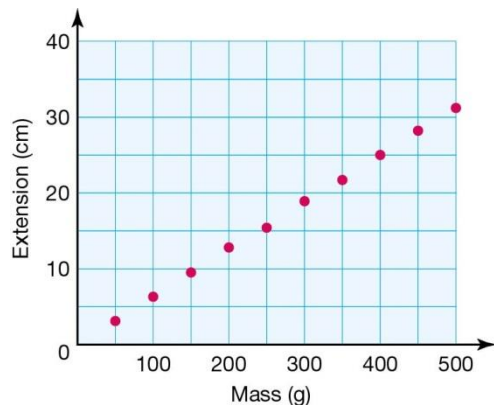
### Chequered boards

- Student's own diagrams and results, presented in a logical table.
- Where either  $m$  or  $n$  is even, the number of black and white squares is given by  $\frac{mn}{2}$ . Where both  $m$  and  $n$  are odd, the number of black and white squares differ by one. The number of black squares is  $\frac{mn-1}{2}$  and the number of white squares is  $\frac{mn+1}{2}$ , assuming that the bottom right-hand corner is white.



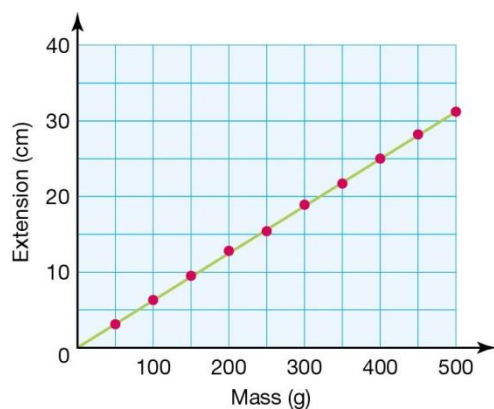
## Modelling: Stretching a spring

1



2 Linear

3



4  $y \approx 0.06x$

5 16.5 cm

6 The spring is likely to snap (or exceed its elastic limit).

## ICT activity

1  $x \approx 2.7$

2  $x \approx 2.6$

3  $x \approx 2.1$

4  $x \approx 0.6$

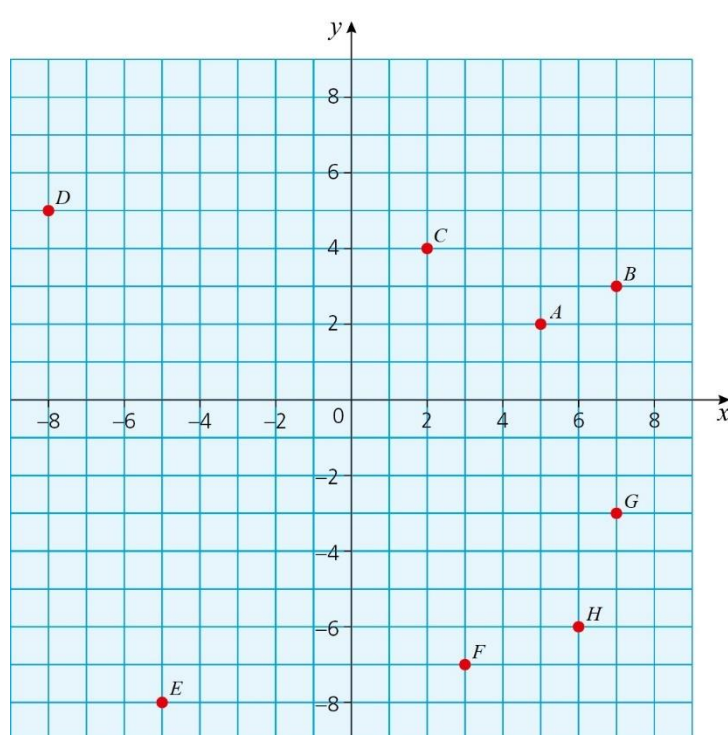


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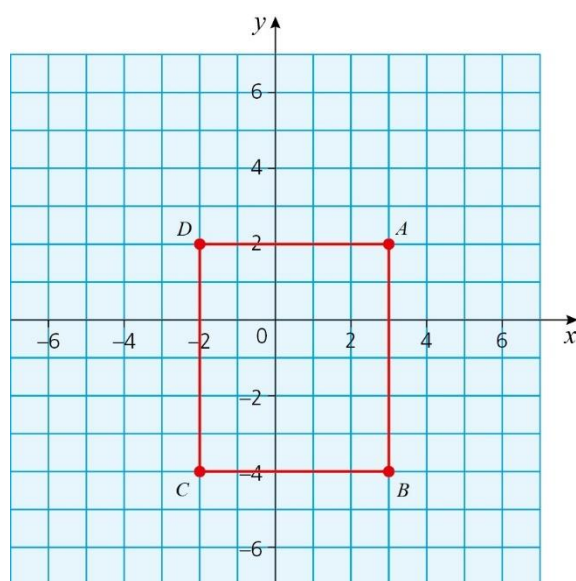
## 21 Straight-line graphs

### Exercise 21.1

1



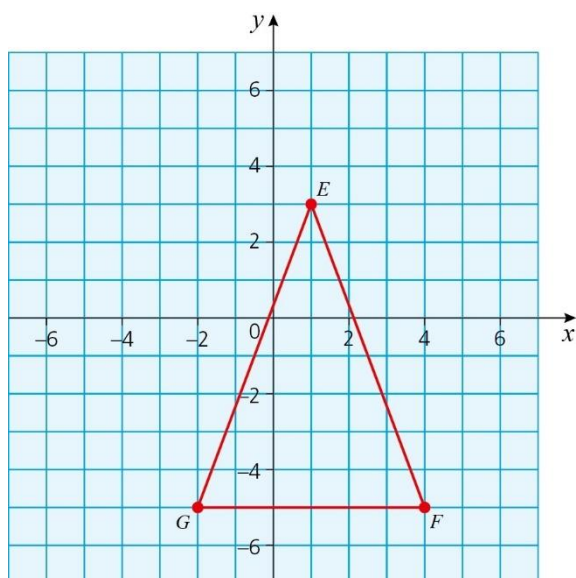
2



Rectangle

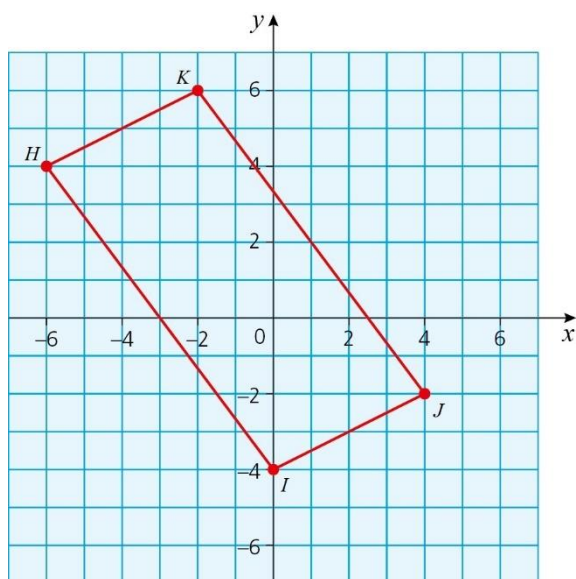


3



Isosceles triangle

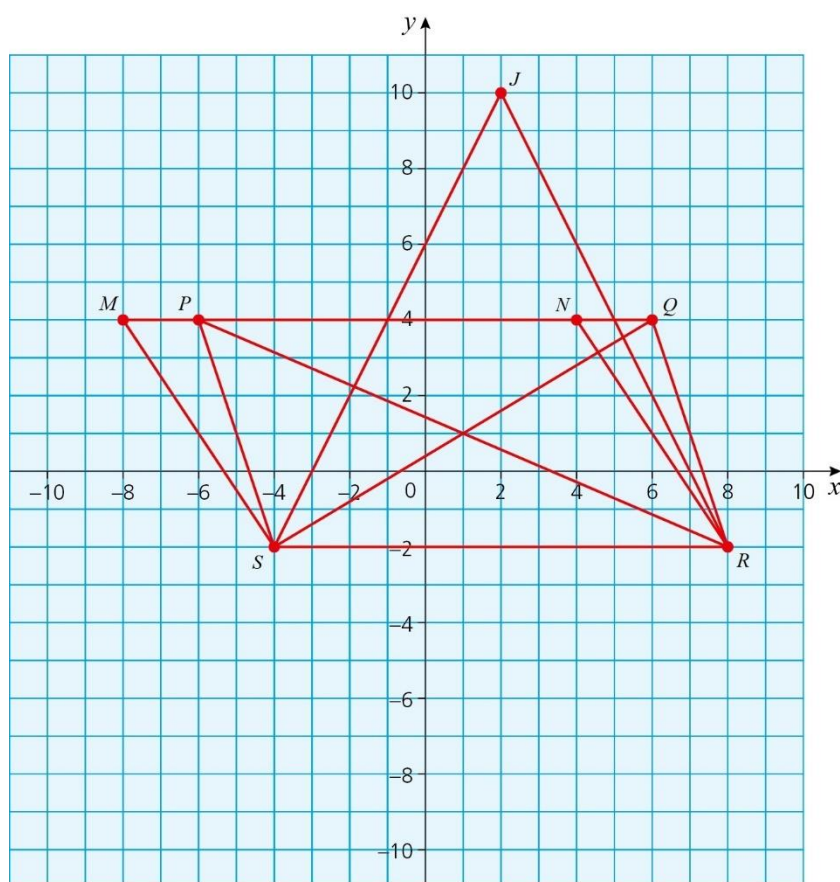
4



Parallelogram



## Exercise 21.2

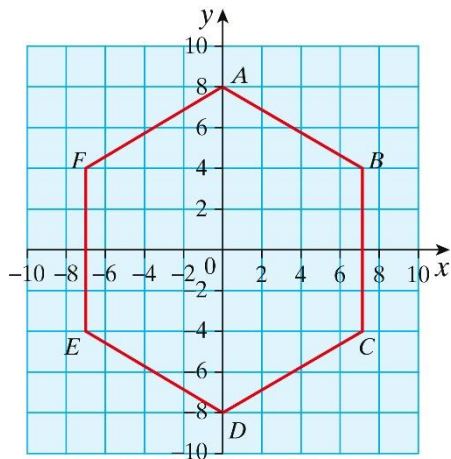


- 1 a (1, 1)  
b 72 units<sup>2</sup>
- 2 a Parallelogram  
b 72 units<sup>2</sup>  
c The base length and perpendicular height of MNRS are the same as the base length and perpendicular height of PQRS, so their areas are the same.
- 3 2



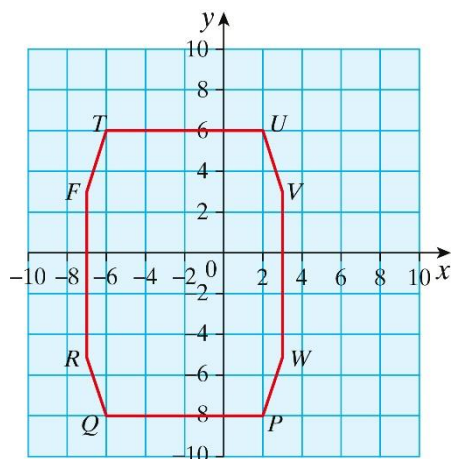
### Exercise 21.3

1 a



b  $C(7, -4), D(0, -8), E(-7, -4), F(-7, 4)$

2 a



b  $S(-7, 3), T(-6, 6), U(2, 6), V(3, 3), W(3, -5)$

c  $(-2, -1)$

### Exercise 21.4

1 B at 1.5, C at 2.4, D at 4.8

2 F at 0.9, G at 1.5, H at 1.75

3 I at 4.4, J at 5.2, K at 5.9, L at 6.3, M at 6.8

4 Q at 2.4, R at 4.6, S at 5.8, T at 6.4, U at 7.8, V at 8.8

### Exercise 21.5

$A(1, 1.5), B(1.2, -1.5), C(-0.9, -1.6), D(-1.8, 0.7), E(1, 1.8), F(3, -2.4), G(-3.6, -1.6), H(-1.6, 3.6)$



### Exercise 21.6

---

- 1 a 1  
b  $\frac{3}{2}$   
c  $-2$   
d  $-\frac{1}{4}$   
e 0  
f undefined
- 2 A horizontal line has a zero gradient
- 3 A vertical line has an undefined gradient as you cannot divide by 0
- 4 Gradient of A = 2  
Gradient of B = 0  
Gradient of C =  $-3$   
Gradient of D =  $-\frac{1}{2}$   
Gradient of E =  $\frac{1}{2}$   
Gradient of F is undefined

### Exercise 21.7

---

- 1 a  $y = 7$   
b  $y = 2$   
c  $x = 7$   
d  $x = 3$   
e  $y = x$   
f  $y = \frac{1}{2}x$   
g  $y = -x$   
h  $y = -2x$

### Exercise 21.8

---

- 1 a  $y = x + 1$   
b  $y = x + 3$   
c  $y = x - 2$   
d  $y = 2x + 2$   
e  $y = \frac{1}{2}x + 5$   
f  $y = \frac{1}{2}x - 1$
- 2 a  $y = -x + 4$



**b**  $y = -x - 2$

**c**  $y = -2x - 2$

**d**  $y = -\frac{1}{2}x + 3$

**e**  $y = -\frac{3}{2}x + 2$

**f**  $y = -4x + 1$

**3 a 1a** 1

**1b** 1

**1c** 1

**1d** 2

**1e**  $\frac{1}{2}$

**1f**  $\frac{1}{2}$

**2a** -1

**2b** -1

**2c** -2

**2d**  $-\frac{1}{2}$

**2e**  $-\frac{3}{2}$

**2f** -4

**b** The gradient is equal to the coefficient of  $x$

**c** The constant being added/subtracted indicates where the line intersects the  $y$ -axis

**4 a** Students' own answers

**b** Only the intercept  $c$  is different

**5** The lines are parallel

### Exercise 21.9

---

**1 a**  $m = 2$   $c = 1$

**b**  $m = 3$   $c = 5$

**c**  $m = 1$   $c = -2$

**d**  $m = \frac{1}{2}$   $c = 4$

**e**  $m = -3$   $c = 6$

**f**  $m = -\frac{2}{3}$   $c = 1$

**g**  $m = -1$   $c = 0$

**h**  $m = -1$   $c = -2$

**i**  $m = -2$   $c = 2$

**2 a**  $m = 3$   $c = 1$



**b**  $m = -\frac{1}{2}$   $c = 2$

**c**  $m = -2$   $c = -3$

**d**  $m = -2$   $c = -4$

**e**  $m = \frac{1}{4}$   $c = 6$

**f**  $m = 3$   $c = 2$

**g**  $m = 1$   $c = -2$

**h**  $m = -8$   $c = 6$

**i**  $m = 3$   $c = 1$

**3 a**  $m = 2$   $c = -3$

**b**  $m = \frac{1}{2}$   $c = 4$

**c**  $m = 2$   $c = -4$

**d**  $m = -8$   $c = 12$

**e**  $m = 2$   $c = 0$

**f**  $m = -3$   $c = 3$

**g**  $m = 2$   $c = 1$

**h**  $m = -\frac{1}{2}$   $c = 2$

**i**  $m = 2$   $c = -\frac{1}{2}$

**4 a**  $m = 2$   $c = -4$

**b**  $m = 1$   $c = 6$

**c**  $m = -3$   $c = -1$

**d**  $m = -1$   $c = 4$

**e**  $m = 10$   $c = -2$

**f**  $m = -3$   $c = \frac{3}{2}$

**g**  $m = -9$   $c = 2$

**h**  $m = 6$   $c = -14$

**i**  $m = 2$   $c = -\frac{3}{2}$

**5 a**  $m = 2$   $c = -2$

**b**  $m = 2$   $c = 3$

**c**  $m = 1$   $c = 0$

**d**  $m = \frac{3}{2}$   $c = 6$

**e**  $m = -1$   $c = \frac{2}{3}$

**f**  $m = -4$   $c = 2$

**g**  $m = 3$   $c = -12$



**h**  $m = 0$   $c = 0$

**i**  $m = -3$   $c = 0$

**6 a**  $m = 1$   $c = 0$

**b**  $m = -\frac{1}{2}$   $c = -2$

**c**  $m = -3$   $c = 0$

**d**  $m = 1$   $c = 0$

**e**  $m = -2$   $c = -\frac{2}{3}$

**f**  $m = \frac{2}{3}$   $c = -4$

**7 a**  $m = -\frac{2}{5}$   $c = 0$

**b**  $m = \frac{1}{3}$   $c = -\frac{5}{6}$

**c**  $m = 3$   $c = 0$

**d**  $m = -1$   $c = -4$

**8 a**  $x - 3y + 3 = 0$

**b**  $2x - 5y - 10 = 0$

**c**  $3x - 6y - 1 = 0$

**d**  $6x + 3y + 2 = 0$

**e**  $3x - 2y - 6 = 0$

**f**  $5x + 3y + 20 = 0$

**9 a**  $3x - 2y + 1 = 0$

**b**  $x + 5y = 0$

**c**  $15x + 2y = 0$

**d**  $12x + 9y = 0$

**e**  $x + 6y = 0$

**f**  $3x - 5y = 0$

### Exercise 21.10

---

**1** Any line with a gradient of 1

**2** a, b and d are parallel to the line

**3** Any line with a gradient of 1

**5** b and d are parallel to the line

**5**  $y = 4x$

**6 a**  $y = -3x + 4$

**b**  $y = -3x - 2$

**c**  $y = -3x - \frac{7}{2}$

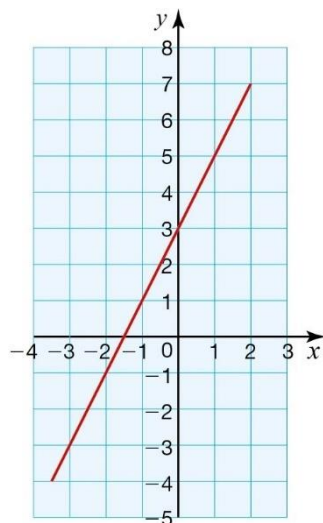


7 a  $y = \frac{1}{2}x + 3$

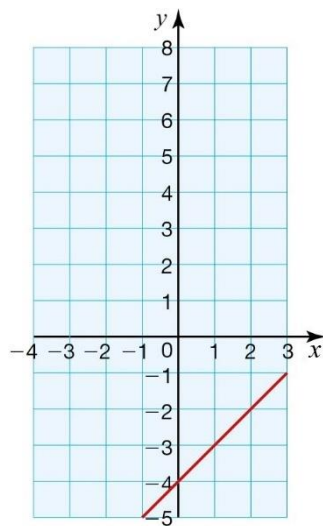
b  $y = \frac{1}{2}x - \frac{1}{4}$

### Exercise 21.11

1 a

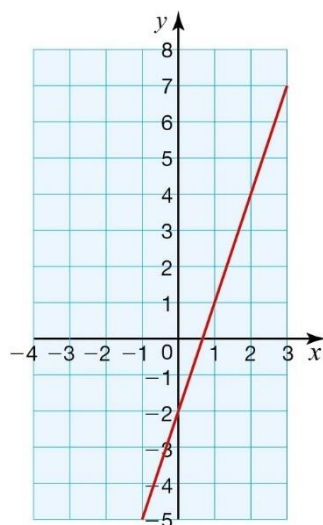


b

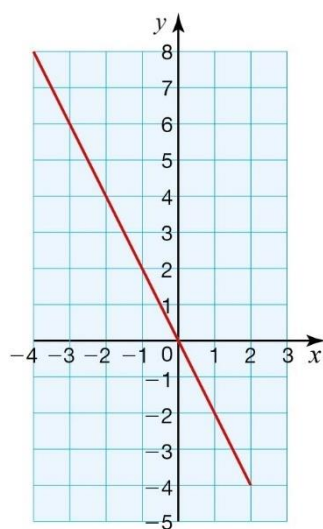




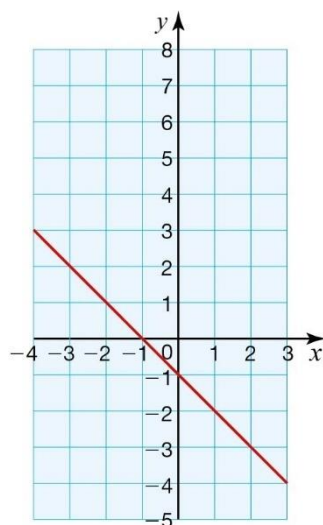
c



d

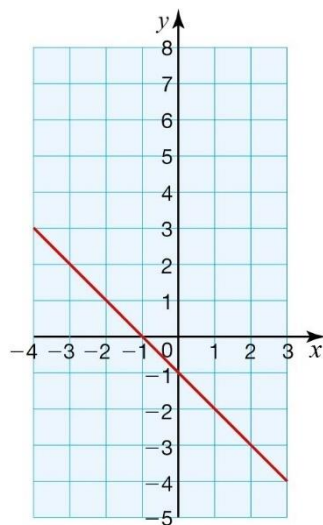


e

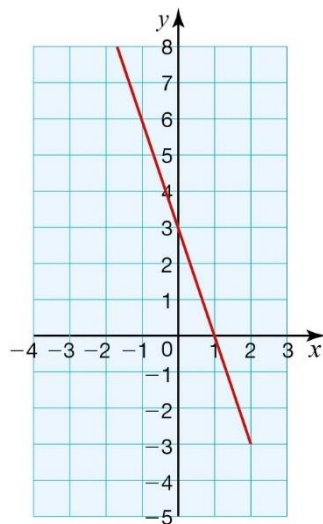




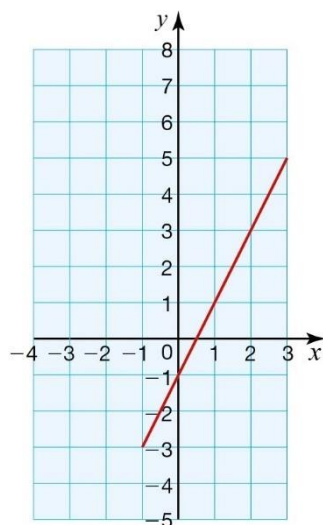
f



g

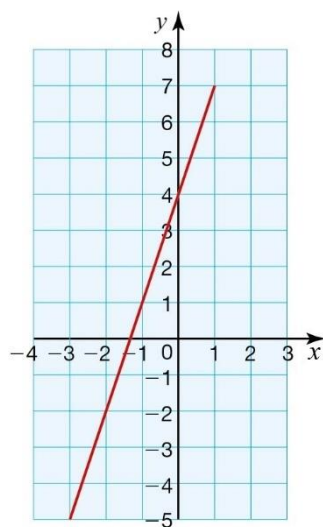


h

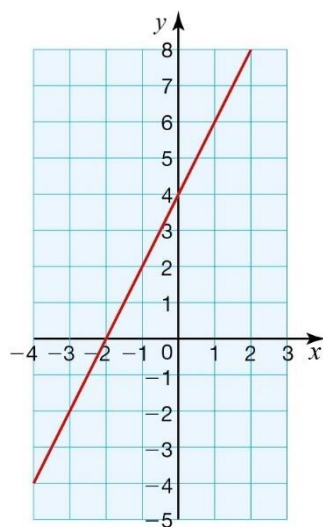




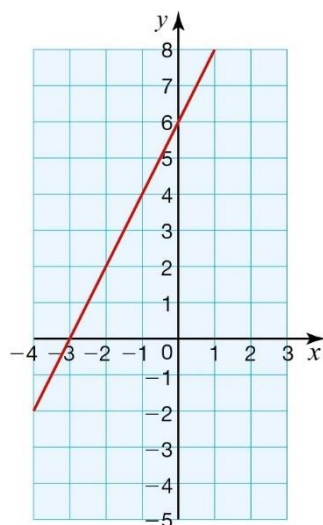
i



2 a

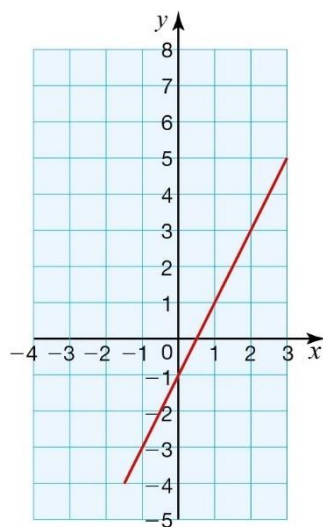


b

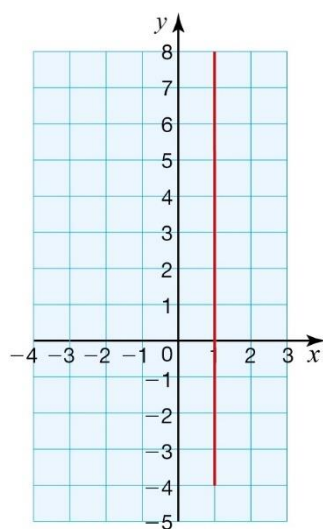




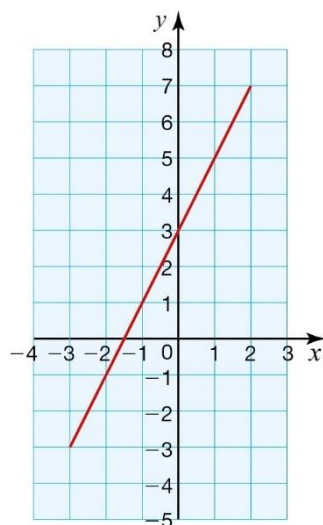
c



d

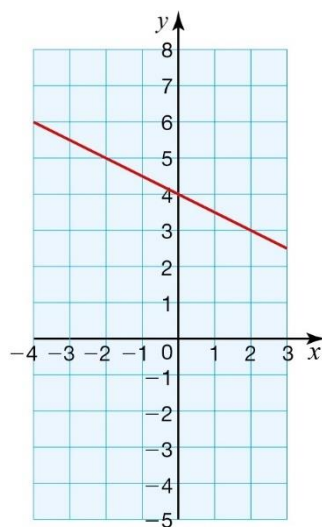


e

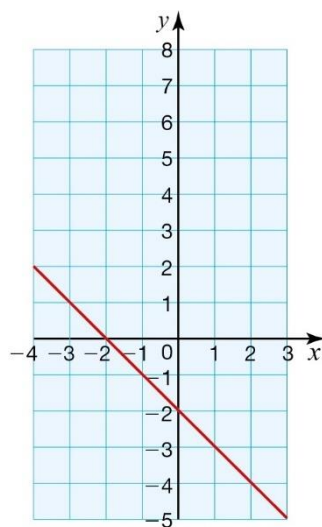




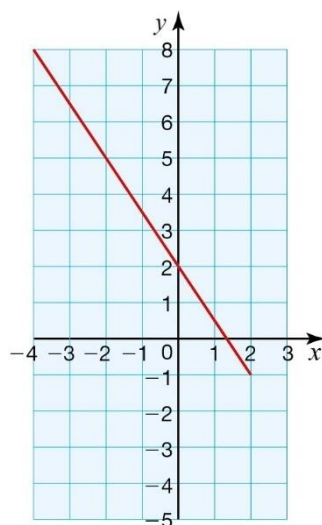
f



g

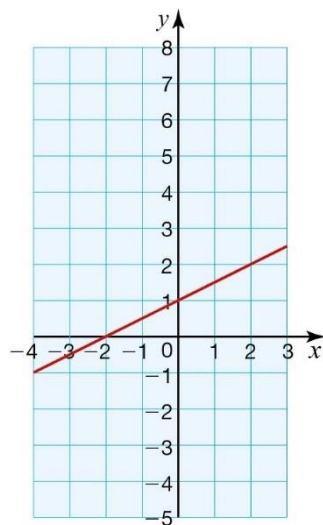


h

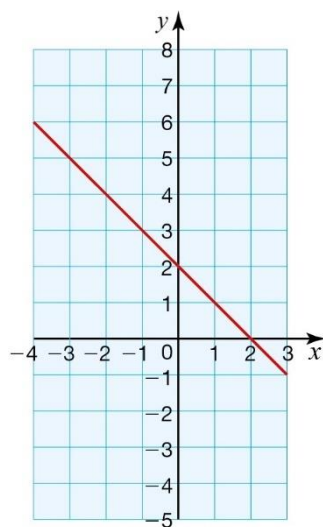




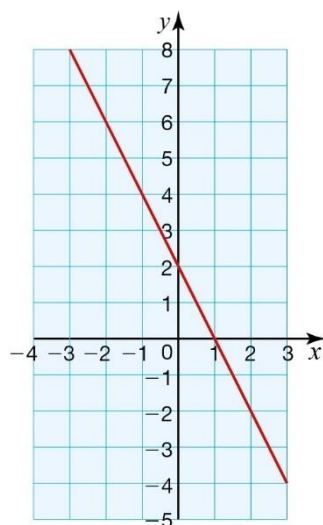
i



3 a

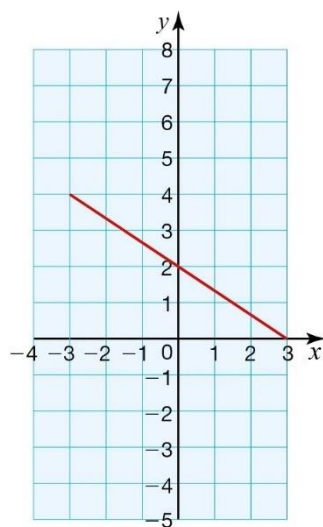


b

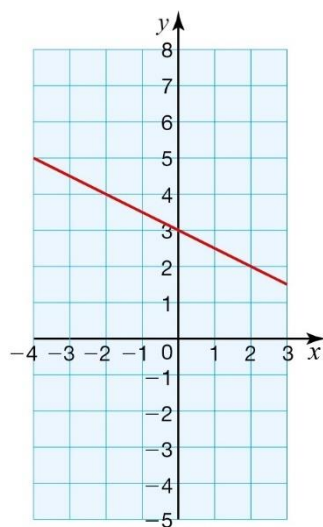




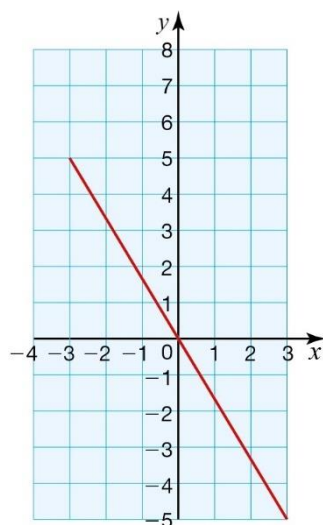
c



d

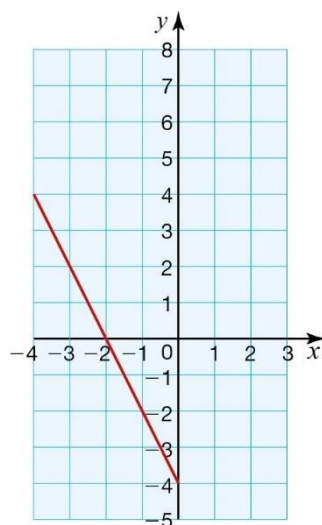


e

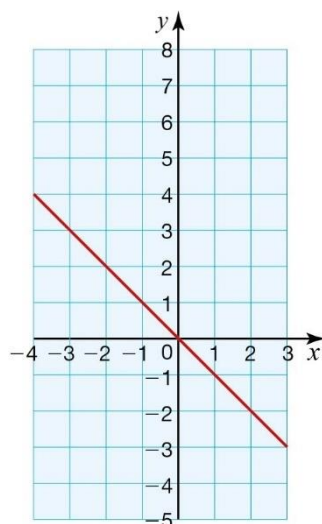




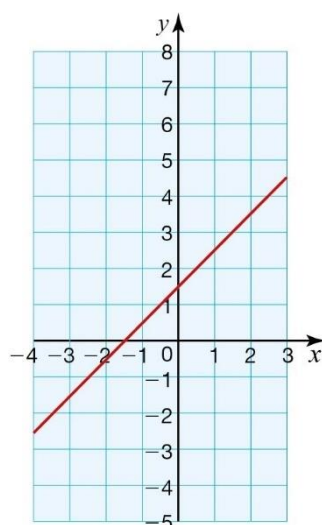
f



g

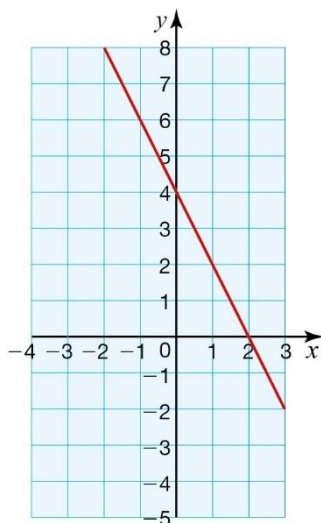


h





i



### Exercise 21.12

- 1 a (3, 2)  $x = 3, y = 2$   
b (5, 2)  $x = 5, y = 2$   
c (2, 1)  $x = 2, y = 1$   
d (2, 1)  $x = 2, y = 1$   
e (-4, 1)  $x = -4, y = 1$   
f (4, -2)  $x = 4, y = -2$
- 2 a (3, -2)  $x = 3, y = -2$   
b (-1, -1)  $x = -1, y = -1$   
c (-2, 3)  $x = -2, y = 3$   
d (-3, -3)  $x = -3, y = -3$   
e Infinite solutions  
f No solution

### Exercise 21.13

- 1 a ii 5.66 units (3 s.f.)  
iii (3, 4)  
b ii 4.24 units (3 s.f.)  
iii (4.5, 2.5)  
c ii 5.66 units (3 s.f.)  
iii (3, 6)  
d ii 8.94 units (3 s.f.)  
iii (2, 4)



- e** **ii** 6.32 units (3 s.f.)  
**iii** (3, 4)
- f** **ii** 6.71 units (3 s.f.)  
**iii** (−1.5, 4)
- g** **ii** 8.25 units (3 s.f.)  
**iii** (−2, 1)
- h** **ii** 8.94 units (3 s.f.)  
**iii** (0, 0)
- i** **ii** 7 units  
**iii** (0.5, 5)
- j** **ii** 6 units  
**iii** (2, 3)
- k** **ii** 8.25 units (3 s.f.)  
**iii** (0, 4)
- l** **ii** 10.8 units (3 s.f.)  
**iii** (0, 1.5)
- 2 a** **i** 4.24 units (3 s.f.)  
**ii** (2.5, 2.5)
- b** **i** 5.66 units (3 s.f.)  
**ii** (5, 4)
- c** **i** 8.94 units (3 s.f.)  
**ii** (4, 2)
- d** **i** 8.94 units (3 s.f.)  
**ii** (5, 0)
- e** **i** 4.24 units (3 s.f.)  
**ii** (−1.5, 4.5)
- f** **i** 4.47 units (3 s.f.)  
**ii** (−4, −3)
- g** **i** 7.21 units (3 s.f.)  
**ii** (0, 3)
- h** **i** 7.21 units (3 s.f.)  
**ii** (5, −1)
- i** **i** 12.4 units (3 s.f.)  
**ii** (0, 2.5)
- j** **i** 8.49 units (3 s.f.)  
**ii** (1, −1)
- k** **i** 11 units



- ii (0.5, -3)
- I i 8.25 units (3 s.f.)
- ii (4, 2)

### Exercise 21.14

---

- 1 a  $y = 2x - 1$
- b  $y = 3x + 1$
- c  $y = 2x + 3$
- d  $y = x - 4$
- e  $y = 4x + 2$
- f  $y = -x + 4$
- g  $y = -2x + 2$
- h  $y = -3x - 1$
- i  $y = \frac{1}{2}x$
- 2 a  $y = \frac{1}{7}x + \frac{26}{7}$
- b  $y = \frac{6}{7}x + \frac{4}{7}$
- c  $y = \frac{3}{2}x + \frac{15}{2}$
- d  $y = 9x - 13$
- e  $y = -\frac{1}{2}x + \frac{5}{2}$
- f  $y = -\frac{3}{13}x + \frac{70}{13}$
- g  $y = 2$
- h  $y = -3x$
- i  $x = 6$

### Exercise 21.15

---

- 1 a i -1
- ii 1
- iii  $y = x - 3$
- b i -1
- ii 1
- iii  $y = x - 5$
- c i -2
- ii  $\frac{1}{2}$
- iii  $y = \frac{1}{2}x - 5$



d i  $-\frac{1}{2}$

ii 2

iii  $y = 2x - 20$

e i -1

ii 1

iii  $y = x + 9$

f i -2

ii  $\frac{1}{2}$

iii  $y = \frac{1}{2}x + \frac{3}{2}$

g i  $-\frac{3}{2}$

ii  $\frac{2}{3}$

iii  $y = \frac{2}{3}x - \frac{4}{3}$

h i  $\frac{2}{3}$

ii  $-\frac{3}{2}$

iii  $y = -\frac{3}{2}x + 13$

i i  $-\frac{1}{4}$

ii 4

iii  $y = 4x + 28$

j i -1

ii 1

iii  $y = x - 8$

k i 0

ii undefined

iii  $x = 6$

l i -4

ii  $\frac{1}{4}$

iii  $y = \frac{1}{4}x - \frac{13}{4}$

2 a  $-\frac{2}{3}$

b  $-\frac{5}{4}$

c  $\frac{1}{8}$

d  $-\frac{4}{3}$

3 a  $\frac{1}{3}$



**b**  $y = \frac{1}{3}x + \frac{5}{3}$

**c**  $-3$

**d**  $y = -3x + 15$

**e**  $y = -3x + 35$

**f**  $(8, 11)$

**g**  $(2, 9)$

**h**  $y = \frac{1}{3}x + \frac{25}{3}$

**i**  $6.3$  units

**j**  $(6, 7)$

**4 a**  $y = \frac{2}{5}x - \frac{8}{5}$

**b**  $y = -\frac{5}{2}x - \frac{9}{2}$

**c**  $15.2$  units

**d** Midpoint AB =  $(4, 0)$

Midpoint AC =  $(-3, 3)$

Midpoint BC =  $(2, 5)$

**e**  $y = -\frac{5}{2}x + 10$

## Student assessment 1

---

**1 a**  $1$

**b**  $-3$

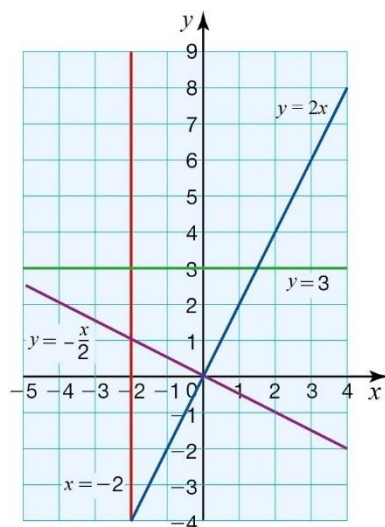
**2 a**  $y = 2x + 4$

**b**  $y = \frac{5}{2}x + 4$

**3**  $y = -\frac{2}{3}x + 6$



4 a, b, c, d

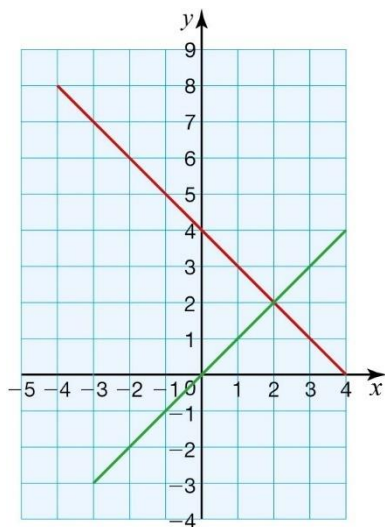


5 a  $m = -3$ ;  $c = 4$

b  $m = 3$ ;  $c = 6$

c  $m = -\frac{1}{2}$ ;  $c = \frac{3}{2}$

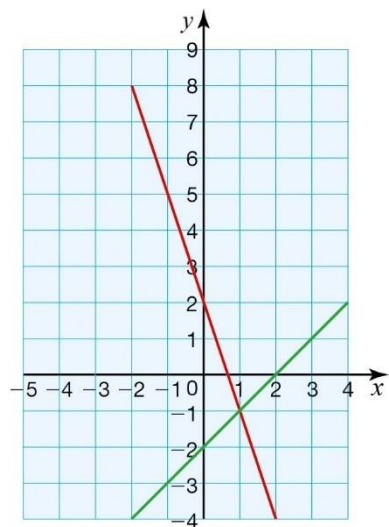
6 a



Solution is (2, 2), i.e.  $x = 2$  and  $y = 2$

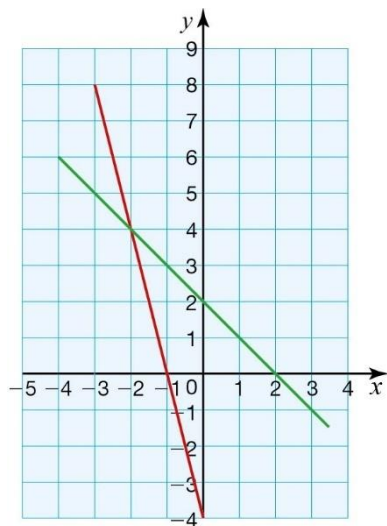


b



Solution is  $(1, -1)$ , i.e.  $x = 1$  and  $y = -1$

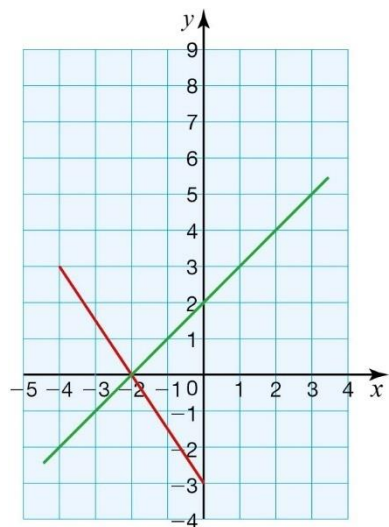
c



Solution is  $(-2, 4)$ , i.e.  $x = -2$  and  $y = 4$



d



Solution is  $(-2, 0)$ , i.e.  $x = -2$  and  $y = 0$

7 a i 13 units

ii  $(0, 1.5)$

b i 10 units

ii  $(4, 6)$

8 a i  $y = 3x - 4$

ii  $3x - y - 4 = 0$

b i  $y = -2x + 7$

ii  $2x + y - 7 = 0$

9 a  $y = -\frac{2}{7}x + \frac{31}{7}$

b  $y = \frac{7}{2}x + 12$

10 a i  $(5, 6)$

ii  $y = 3x - 9$

iii  $y = -\frac{1}{3}x + 6$

b The diagonals are perpendicular as the product of their gradients is  $-1$



## Mathematical investigations and ICT 3

### Plane trails

---

- 1 Student's investigation
- 2 Student's ordered table similar to the one shown

| Number of planes ( $p$ ) | Maximum number of crossing points ( $n$ ) |
|--------------------------|-------------------------------------------|
| 1                        | 0                                         |
| 2                        | 1                                         |
| 3                        | 3                                         |
| 4                        | 6                                         |
| 5                        | 10                                        |
| ...                      | ...                                       |

- 3 The sequence of the number of crossing points is the sequence of triangular numbers

$$n = \frac{1}{2}p(p - 1)$$



## Hidden treasure

1, 2 The results for up to 20 contestants are given in the table below

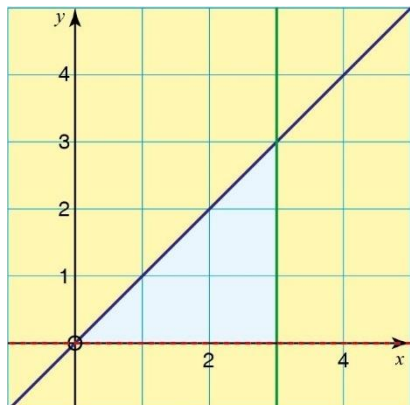
| Number of contestants ( $n$ ) | Winning chest ( $x$ ) |
|-------------------------------|-----------------------|
| 1                             | 1                     |
| 2                             | 2                     |
| 3                             | 2                     |
| 4                             | 4                     |
| 5                             | 2                     |
| 6                             | 4                     |
| 7                             | 6                     |
| 8                             | 8                     |
| 9                             | 2                     |
| 10                            | 4                     |
| 11                            | 6                     |
| 12                            | 8                     |
| 13                            | 10                    |
| 14                            | 12                    |
| 15                            | 14                    |
| 16                            | 16                    |
| 17                            | 2                     |
| 18                            | 4                     |
| 19                            | 6                     |
| 20                            | 8                     |

- 3 Student's observed pattern: key pattern is that  $x = n$  when  $n$  is a power of 2.
- 4 31 contestants, winning chest is 30.  
32 contestants, winning chest is 32.  
33 contestants, winning chest is 2.
- 5  $x = 2(n - T)$ , where  $x$  = the winning chest,  $n$  = number of contestants and  $T$  = the nearest power of 2 below  $n$ .

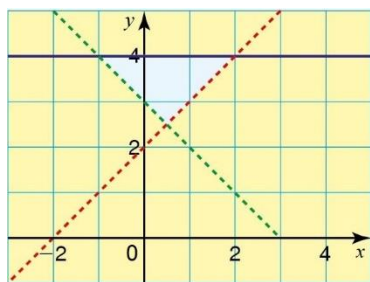


## ICT activity

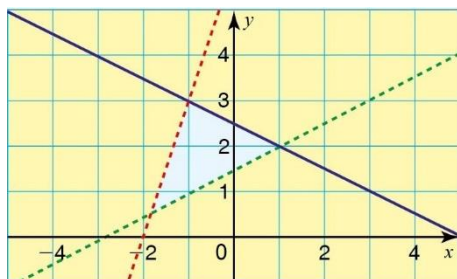
1



2



3





All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

## 22 Geometrical vocabulary and construction

### Exercise 22.1

- 1 a and f  
b and c  
d and e  
g and h

### Exercise 22.2

| 1 |                                       | Rectangle | Square | Parallelogram | Kite | Rhombus | Equilateral triangle |
|---|---------------------------------------|-----------|--------|---------------|------|---------|----------------------|
|   | Opposite sides equal in length        | Yes       | Yes    | Yes           | No   | Yes     | n/a                  |
|   | All sides equal in length             | No        | Yes    | No            | No   | Yes     | Yes                  |
|   | All angles right angles               | Yes       | Yes    | No            | No   | No      | No                   |
|   | Both pairs of opposite sides parallel | Yes       | Yes    | Yes           | No   | Yes     | —                    |
|   | Diagonals equal in length             | Yes       | Yes    | No            | No   | No      | —                    |
|   | Diagonals intersect at right angles   | No        | Yes    | No            | Yes  | Yes     | —                    |
|   | All angles equal                      | Yes       | Yes    | No            | No   | No      | Yes                  |

### Exercise 22.3

Student's own diagrams

### Exercise 22.4

- 1 Student's own construction
- 2 Student's own construction
- 3 Student's own construction
- 4 a Student's own construction attempt  
b It is not possible as  $AC + BC < AB$



### **Exercise 22.5**

---

- 1   a   300 m  
     b   250 m  
     c   300 m  
     d   416 m
- 2   a   10 cm  
     b   8 cm  
     c   6 cm  
     d   6.8 cm
- 3   Student's own construction
- 4   a   Student's own construction  
     b   24 m<sup>2</sup>

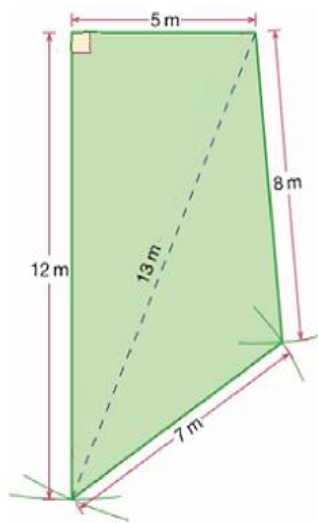
### **Student assessment 1**

---

- 1   a   Acute  
     b   Obtuse  
     c   Reflex  
     d   Right angle
- 2   Student's own diagram
- 3   a   Obtuse-angled scalene triangle  
     b   Right-angled scalene
- 4   Student's own diagram
- 5   Any three from:  
     All sides equal in length  
     Both pairs of opposite sides are parallel  
     Diagonals intersect at right angles  
     Opposite angles are equal
- 6   Student's own diagram
- 7   Student's own construction



- 8 a Student's own construction



- b Approx.  $54 \text{ m}^2$
- 9 a 22 cm
- b 24.4 cm



*All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.*

## 23 Similarity and congruence

### Exercise 23.1

---

- 1
  - a Interior angles are the same, i.e.  $60^\circ$ ,  $30^\circ$  and  $90^\circ$
  - b  $\frac{5}{8}$
  - c  $x = 6.25$        $y = 3.75$
- 2 A, C and F are similar, B and D are similar
- 3
  - a 6 cm
  - b 9 cm
- 4  $p$ : 4.8 cm       $q$ : 4.5 cm       $r$ : 7.5 cm
- 5  $e$ : 10 cm       $f$ :  $2\frac{2}{3}$  cm
- 6
  - a  $10 \text{ cm}^2$
  - b 1.6
  - c  $25.6 \text{ cm}^2$
- 7
  - a 10 cm
  - b 2.5
  - c  $150 \text{ cm}^2$
- 8
  - a  $33\frac{1}{3} \text{ cm}^2$
  - b  $6\frac{2}{3}$  cm
- 9 9.6 cm
- 10 No, as the corresponding angles may not be the same
- 11 No, as, despite the corresponding angles being the same, the slanting side lengths may not be in the same ratio as the horizontal sides

### Exercise 23.2

---

- 1  $50 \text{ cm}^2$
- 2  $10 \text{ cm}^2$
- 3
  - a
    - i  $456 \text{ cm}^2$  (3 s.f.)
    - ii  $90 \text{ cm}^2$
    - iii  $40 \text{ cm}^2$
  - b Triangle I
- 4  $43.56 \text{ cm}^2$



- 5  $56.25 \text{ cm}^2$   
6  $18.1 \text{ cm}^2$  (3 s.f.)

### Exercise 23.3

---

- 1 a i 8 m  
ii 4.8 m  
b  $2\sqrt{2} \text{ m}$   
c  $25\sqrt{2} \text{ cm}^2$   
2 a  $220 \text{ cm}^2$   
b  $1375 \text{ cm}^2$   
c  $200 \text{ cm}^3$   
d  $3125 \text{ cm}^3$   
3 a  $54 \text{ cm}^2$   
b 3  
c  $729 \text{ cm}^3$   
4 a  $1 : n^2$   
b  $1 : n^3$   
5  $112 \text{ cm}^3$   
6 0.64 litres

### Exercise 23.4

---

- 1 20 cm  
2 a 1 : 8  
b 1 : 7  
3  $16 \text{ cm}^3$   
4 a Not similar. Student's own reasons  
b 1 : 3  
5 a  $16 \text{ cm}^2$   
b  $64 \text{ cm}^2$   
c  $144 \text{ cm}^2$   
6 a  $30 \text{ km}^2$   
b  $6 \text{ cm}^2$   
7 a  $10 \text{ cm} \times 20 \text{ cm} \times 30 \text{ cm}$   
b 100 g



### Exercise 23.5

---

- 1 P and Q
- 2 10
- 3 (2, 5)
- 4 (10, 5), (9, 5), (10, -3) or (9, -3)
- 5 **a** Any two of: *BDE*, *CEF*, *DFA*, *EAB*, *FBC*  
**b** Any two of: *ABC*, *BCD*, *CDE*, *DEF*, *EFA*
- 6 Yes; sides  $SR = PQ$ ,  $PS = QR$  and  $SQ$  is common to both triangles.

### Student assessment 1

---

- 1 **a** 5  
**b**  $x = 4.5$       $y = 7.5$
- 2 **a**  $h^2 : H^2$   
**b**  $h^3 : H^3$
- 3  $x = \sqrt{41} = 6.40$       $y = \frac{3\sqrt{41}}{5} = 3.84$  (both to 3 s.f.)      $z = 6.4$
- 4  $156 \text{ cm}^2$  (3 s.f.)
- 5 **a**  $1000 \text{ cm}^3$   
**b**  $600 \text{ cm}^2$
- 6  $18.75 \text{ cm}^3$
- 7 **a** 4 m by 3 m  
**b**  $12 \text{ m}^2$
- 8  $3200 \text{ cm}^3$
- 9 **a** *ANMC* or *ABML*  
**b**  $\triangle ABC$  or  $\triangle CLM$  or  $\triangle NMB$  or  $\triangle ANL$
- 10 **a** True  
**b** True  
**c** False  
**d** False



All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

## 24 Symmetry

### Exercise 24.1

1

| Regular polygon      | Number of lines of symmetry | Order of rotational symmetry |
|----------------------|-----------------------------|------------------------------|
| Equilateral triangle | 3                           | 3                            |
| Square               | 4                           | 4                            |
| Pentagon             | 5                           | 5                            |
| Hexagon              | 6                           | 6                            |
| Heptagon             | 7                           | 7                            |
| Octagon              | 8                           | 8                            |
| Nonagon              | 9                           | 9                            |
| Decagon              | 10                          | 10                           |

2 a i Student's planes

ii 3

b i Student's planes

ii 2

c i Student's planes

ii 4

d i Student's planes

ii 4

e i Student's planes

ii Infinite

f i Student's planes

ii Infinite

g i Student's planes

ii Infinite

h i Student's planes

ii 9

3 a 2

b 2

c 3



- d 4
- e Infinite
- f Infinite
- g Infinite
- h 4

## **Exercise 24.2**

---

- 1
  - a Isosceles
  - b Perpendicular bisector
  - c  $50^\circ$
  - d  $50^\circ$
  - e 3.58 cm (3 s.f.)
  - f 9.33 cm (3 s.f.)
- 2
  - a True
  - b True
  - c False
  - d True
- 3
  - a False
  - b True

## **Exercise 24.3**

---

- 1
  - a  $70^\circ$
  - b  $72^\circ$
  - c  $21^\circ$
- 2
  - a 10.9 cm (3 s.f.)
  - b 6.23 cm (3 s.f.)
  - c 3.48 cm (3 s.f.)

## **Student assessment 1**

---

- 1 a, b, c Student's own diagrams
- 2 a, b, c Student's own diagrams
- 3 Student's own proof
- 4
  - a  $24^\circ$
  - b 9.14 cm (3 s.f.)
- 5 88.0 cm (3 s.f.)



All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

## 25 Angle properties

### Exercise 25.1

---

- 1  $p^\circ = 54^\circ$        $q^\circ = 63^\circ$
- 2  $a^\circ = 55^\circ$        $b^\circ = 80^\circ$        $c^\circ = 100^\circ$
- 3  $v^\circ = 120^\circ$        $w^\circ = 60^\circ$        $x^\circ = 120^\circ$        $y^\circ = 60^\circ$        $z^\circ = 60^\circ$
- 4  $a^\circ = 50^\circ$        $b^\circ = 130^\circ$        $c^\circ = 45^\circ$        $d^\circ = 135^\circ$
- 5  $p^\circ = 45^\circ$        $q^\circ = 135^\circ$        $r^\circ = 45^\circ$        $s^\circ = 45^\circ$        $t^\circ = 135^\circ$
- 6  $d^\circ = 70^\circ$        $e^\circ = 30^\circ$
- 7  $a^\circ = 37^\circ$
- 8  $a^\circ = 36^\circ$

### Exercise 25.2

---

- 1 a  $70^\circ$   
b  $55^\circ$   
c  $60^\circ$   
d  $73^\circ$   
e  $45^\circ$   
f  $110^\circ$
- 2 a  $a^\circ = 30^\circ$        $b^\circ = 45^\circ$   
b  $x^\circ = 50^\circ$        $y^\circ = 80^\circ$        $z^\circ = 70^\circ$   
c  $p^\circ = 130^\circ$        $q^\circ = 15^\circ$        $r^\circ = 60^\circ$   
d  $d^\circ = 35^\circ$        $e^\circ = 55^\circ$        $f^\circ = 55^\circ$   
e  $a^\circ = 27.5^\circ$        $b^\circ = 27.5^\circ$        $c^\circ = 55^\circ$        $d^\circ = 27.5^\circ$        $e^\circ = 97.5^\circ$   
f  $p^\circ = 45^\circ$        $q^\circ = 45^\circ$        $r^\circ = 67.5^\circ$        $s^\circ = 112.5^\circ$

### Exercise 25.3

---

- 1  $a^\circ = 115^\circ$
- 2  $x^\circ = 40^\circ$        $y^\circ = 140^\circ$        $z^\circ = 140^\circ$
- 3  $m^\circ = 75^\circ$        $n^\circ = 75^\circ$
- 4  $s^\circ = 65^\circ$        $t^\circ = 115^\circ$        $u^\circ = 115^\circ$
- 5  $h^\circ = 120^\circ$        $i^\circ = 60^\circ$        $j^\circ = 120^\circ$        $k^\circ = 60^\circ$
- 6  $a^\circ = 80^\circ$        $b^\circ = 20^\circ$        $c^\circ = 20^\circ$        $d^\circ = 20^\circ$        $e^\circ = 140^\circ$



7  $p^\circ = 40^\circ$      $q^\circ = 130^\circ$      $r^\circ = 50^\circ$

8  $p^\circ = 75^\circ$      $q^\circ = 30^\circ$      $r^\circ = 50^\circ$      $s^\circ = 80^\circ$      $t^\circ = 70^\circ$      $u^\circ = 70^\circ$      $v^\circ = 40^\circ$

### Exercise 25.4

---

- 1 a  $720^\circ$   
b  $1260^\circ$   
c  $900^\circ$
- 2 a  $135^\circ$   
b  $90^\circ$   
c  $144^\circ$   
d  $150^\circ$
- 3 a  $72^\circ$   
b  $30^\circ$   
c  $51.4^\circ$  (1 d.p.)
- 4 a 18  
b 10  
c 36  
d 8  
e 20  
f 120
- 5 a 5  
b 12  
c 20  
d 15  
e 40  
f 360
- 6 12

### Exercise 25.5

---

- 1 60  
2 135  
3 20  
4 32  
5 110  
6 22.5



### Exercise 25.6

---

- 1 35
- 2 60
- 3 40
- 4 45
- 5 24
- 6 26
- 7 13
- 8 8
- 9 17.7 (1 d.p.)

### Exercise 25.7

---

- 1 **a**  $x = 54$   
**b**  $54^\circ, 108^\circ, 162^\circ, 54^\circ, 162^\circ$
- 2  $125^\circ, 145^\circ$
- 3 64
- 4  $a^\circ = 135^\circ, b^\circ = 125^\circ, c^\circ = 130^\circ, d^\circ = 110^\circ, e^\circ = 85^\circ$

### Exercise 25.8

---

- 1  $55^\circ$
- 2  $80^\circ$
- 3  $90^\circ$
- 4  $100^\circ$
- 5  $80^\circ$
- 6  $20^\circ$
- 7  $x^\circ = 54^\circ$        $y^\circ = 18^\circ$
- 8  $x^\circ = 50^\circ$        $y^\circ = 25^\circ$

### Exercise 25.9

---

- 1  $a^\circ = 72^\circ$
- 2  $b^\circ = 33^\circ$        $c^\circ = 66^\circ$
- 3  $d^\circ = 48^\circ$        $e^\circ = 32^\circ$
- 4  $f^\circ = 30^\circ$        $g^\circ = 120^\circ$        $h^\circ = 120^\circ$        $i^\circ = 30^\circ$        $j^\circ = 30^\circ$
- 5  $k^\circ = 55^\circ$        $l^\circ = 55^\circ$        $m^\circ = 55^\circ$        $n^\circ = 55^\circ$
- 6  $p^\circ = 65^\circ$        $q^\circ = 40^\circ$



### Exercise 25.10

---

- 1  $a^\circ = 80^\circ$        $b^\circ = 65^\circ$   
2  $c^\circ = 110^\circ$        $d^\circ = 98^\circ$        $e^\circ = 70^\circ$   
3  $f^\circ = 115^\circ$        $g^\circ = 75^\circ$   
4  $i^\circ = 98^\circ$        $j^\circ = 90^\circ$        $k^\circ = 90^\circ$   
5  $l^\circ = 95^\circ$        $m^\circ = 55^\circ$        $n^\circ = 85^\circ$        $p^\circ = 95^\circ$        $q^\circ = 55^\circ$   
6  $r^\circ = 70^\circ$        $s^\circ = 120^\circ$        $t^\circ = 60^\circ$        $u^\circ = 110^\circ$   
7  $v^\circ = 80^\circ$        $w^\circ = 45^\circ$        $x^\circ = 55^\circ$        $y^\circ = 100^\circ$   
8  $\alpha^\circ = 63^\circ$        $\beta^\circ = 63^\circ$        $\phi^\circ = 54^\circ$

### Student assessment 1

---

- 1 a  $p^\circ = 135^\circ$        $q^\circ = 135^\circ$        $r^\circ = 45^\circ$   
b  $a^\circ = 120^\circ$        $b^\circ = 60^\circ$        $c^\circ = 60^\circ$        $d^\circ = 60^\circ$   
2 a  $m^\circ = 50^\circ$        $n^\circ = 90^\circ$        $p^\circ = 40^\circ$        $q^\circ = 140^\circ$   
b  $w^\circ = 55^\circ$        $x^\circ = 70^\circ$        $y^\circ = 55^\circ$        $z^\circ = 55^\circ$   
c  $a^\circ = 70^\circ$        $b^\circ = 110^\circ$        $c^\circ = 110^\circ$        $d^\circ = 70^\circ$        $e^\circ = 30^\circ$   
3  $162^\circ$   
4  $1260^\circ$   
5  $360^\circ$   
6  $72^\circ$   
7 15  
8 a  $90^\circ$   
b 6.5 cm  
9  $58^\circ$   
10  $30^\circ$   
11  $25^\circ$   
12  $152^\circ$   
13 a  $67^\circ$ . Alternate segment theorem  
b  $226^\circ$ . Obtuse angle  $AOB = 134^\circ$ , angle subtended at centre by an arc is twice the angle on the circumference subtended by same arc.  
Reflex angle  $AOB = 360^\circ - 134^\circ = 226^\circ$ , angle around a point is  $360^\circ$ .  
c  $23^\circ$ .  $\triangle AOB$  is isosceles.  $\frac{180-134}{2} = 23$

### Student assessment 2

---

- 1 a ii angle  $OBA$       angle  $OBC$   
b i angle  $DAB$  + angle  $DCB$       angle  $ADC$  + angle  $CBA$



- c**   **iii**  $\angle DAC = \angle DBC$        $\angle ADB = \angle ACB$
- d**   **ii**  $\angle CAB$   
     **iii**  $\angle ACB = \angle ABC$
- 2**   **a**  $42^\circ$   
     **b**  $21^\circ$
- 3**  $\angle DAB = 117^\circ$        $\angle ABC = 92^\circ$
- 4**  $\angle BDC = 25^\circ$        $\angle DAB = 115^\circ$
- 5**  $\angle OQR = 15^\circ$        $\angle ORQ = \angle OPQ = 90^\circ$   
 $\angle RPQ = \angle PRQ = 75^\circ$   
 $\angle OPR = \angle ORP = 15^\circ$        $\angle ROP = 150^\circ$
- 6**  $35^\circ, 54^\circ, 91^\circ$  respectively
- 7**  $95^\circ, 85^\circ, 85^\circ, 92^\circ$  respectively
- 8**  $\angle BAC = 51^\circ$ .  $\angle ADC = 124^\circ$ , alternate segment theorem.  $\angle ABC = 56^\circ$  opposite angles of a cyclic quadrilateral add up to  $180^\circ$ . Therefore,  $\angle BAC = 51^\circ$ , angles of a triangle equal  $180^\circ$ .

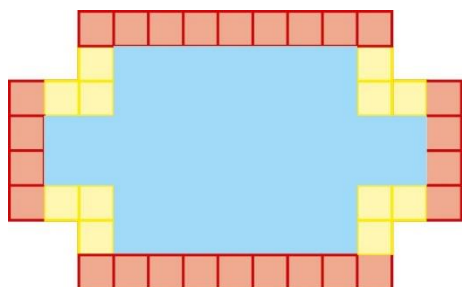


## Mathematical investigations and ICT 4

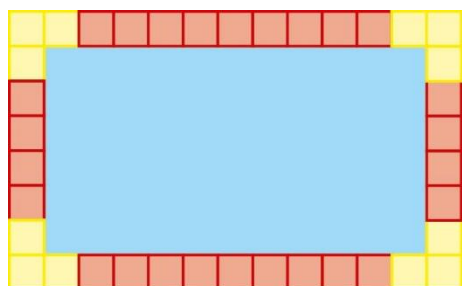
### Fountain borders

- 1 Student's results
- 2  $T = 2(m + n + 2)$
- 3 There are many ways to prove the algebraic rule, for example:

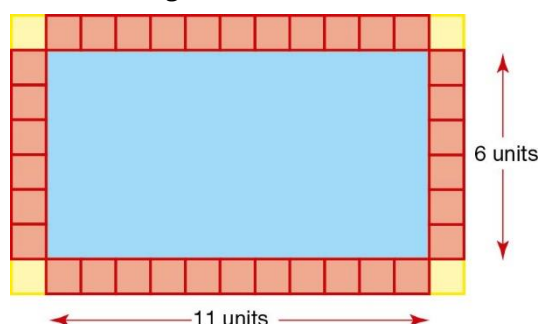
The original pool considered:



has the same number of tiles as a rectangular pool of dimensions  $11 \times 6$  units:



In the diagram below it can be seen that the number of tiles along the length and width of the pool is twice the length and width. This leaves the four tiles needed for the corners.



Hence  $T = 2m + 2n + 4$  which factorises to  $T = 2(m + n + 2)$ .

### Tiled walls

- 1 Student's diagrams
- 2 Student's ordered table of results
- 3  $c = (l - 1)(w - 1)$
- 4  $t = 2(l - 1) + 2(w - 1)$



## ICT activity 1

---

- 2 The ratios are the same.
- 3
  - a The ratios remain the same.
  - b Ratios are still equal to each other (but probably of a different value from 1d).
- 4 The ratios change as  $ED$  no longer parallel to  $AB$ .

## ICT activity 2

---

Student's own demonstrations



*All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.*

## 26 Measures

### Exercise 26.1

---

- 1 a 100  
b 1000  
c  $\frac{1}{1000}$   
d  $\frac{1}{1000}$   
e Millilitre
- 2 a m, cm  
b cm  
c g  
d ml  
e m, cm  
f tonne  
g litres  
h km  
i tonne  
j litres

### Exercise 26.2

---

- 1 a mm  
b m  
c mm  
d m  
e m
- 2 a 85  
b 230  
c 830  
d 50  
e 4
- 3 a 5.6  
b 6400  
c 0.96



**d** 4

**e** 0.012

**4 a** 1.15

**b** 250

**c** 0.5

**d** 0.07

**e** 0.008

### **Exercise 26.3**

---

**1 a** 3800

**b** 28.5

**c** 4280

**d** 0.32

**e** 500

### **Exercise 26.4**

---

**1 a** 4500 ml

**b** 1530 ml

**c** 7050 ml

**d** 1000 ml

**2 a** 1.2 litres

**b** 1.34 litres

**c** 1.4 litres

**d** 1.4 litres

### **Exercise 26.5**

---

**1 a** 100 000 cm<sup>2</sup>

**b** 2 000 000 mm<sup>2</sup>

**c** 5 000 000 m<sup>2</sup>

**d** 3 200 000 m<sup>2</sup>

**e** 830 mm<sup>2</sup>

**2 a** 0.05 m<sup>2</sup>

**b** 150 cm<sup>2</sup>

**c** 0.001 km<sup>2</sup>

**d** 0.04 m<sup>2</sup>

**e** 0.000 25 km<sup>2</sup>



- 3**   **a**    $2\,500\,000\text{ cm}^3$   
      **b**    $3400\text{ mm}^3$   
      **c**    $2\,000\,000\,000\text{ m}^3$   
      **d**    $200\,000\text{ cm}^3$   
      **e**    $30\,000\,000\text{ mm}^3$
- 4**   **a**    $0.15\text{ m}^3$   
      **b**    $24\text{ cm}^3$   
      **c**    $0.000\,85\text{ km}^3$   
      **d**    $0.3\text{ cm}^3$   
      **e**    $0.000\,015\text{ m}^3$
- 5**   **a**    $1200\text{ cm}^3$   
      **b**   500 litres  
      **c**    $4250\text{ cm}^3$   
      **d**    $0.22\text{ m}^3$
- 6**   360 litres

### **Student assessment 1**

---

- 1**   **a**   26 mm  
      **b**   88 cm  
      **c**   6.8 km  
      **d**   875 m
- 2**   **a**   4200 mg  
      **b**   3.94 kg  
      **c**   4100 g  
      **d**   720 kg
- 3**   **a**   1.8 L  
      **b**   3200 ml  
      **c**   83 ml  
      **d**   250 L
- 4**   **a**    $5600\text{ mm}^2$   
      **b**    $2\,050\,000\text{ cm}^2$
- 5**   **a**    $0.008\,67\text{ m}^3$   
      **b**    $0.444\text{ m}^3$

### **Student assessment 2**

---

- 1**   **a**   310 cm



- b** 6400 m
  - c** 4 mm
  - d** 46 cm
- 2**
- a** 0.0036 g
  - b** 0.55 g
  - c** 6.5 kg
  - d** 1.51 tonnes
- 3**
- a** 3.4 L
  - b** 6700 ml
  - c** 730 ml
  - d** 300 L
- 4**
- a** 30 000 mm<sup>2</sup>
  - b** 5000 m<sup>2</sup>
- 5**
- a** 0.1004 m<sup>3</sup>
  - b** 0.000 005 005 km<sup>3</sup>



*All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.*

## **27 Perimeter, area and volume**

### **Exercise 27.1**

---

- 1 a  $6 \text{ cm}^2$   
b  $32.5 \text{ cm}^2$   
c  $20 \text{ cm}^2$   
d  $60 \text{ cm}^2$   
e  $108 \text{ cm}^2$   
f  $55 \text{ cm}^2$
- 2 a  $64 \text{ cm}^2$   
b  $1168 \text{ mm}^2$   
c  $300 \text{ cm}^2$   
d  $937.5 \text{ mm}^2$

### **Exercise 27.2**

---

- 1  $58.5 \text{ cm}^2$
- 2  $84 \text{ cm}^2$
- 3  $118.7 \text{ cm}^2$
- 4  $157.5 \text{ cm}^2$

### **Exercise 27.3**

---

- 1 4
- 2 3
- 3  $23.5 \text{ m}^2$
- 4 a  $16 \text{ m}^2, 24 \text{ m}^2$   
b  $100 \text{ m}^2$   
c 15

### **Exercise 27.4**

---

- 1 a 25.1 cm  
b 22.0 cm  
c 28.9 cm  
d 1.57 m



- 2 a  $50.3 \text{ cm}^2$   
b  $38.5 \text{ cm}^2$   
c  $66.5 \text{ cm}^2$   
d  $0.196 \text{ m}^2$
- 3 a  $2.39 \text{ cm}$  (3 s.f.)  
b  $0.5 \text{ cm}$   
c  $0.637 \text{ m}$  (3 s.f.)  
d  $1.27 \text{ mm}$  (3 s.f.)
- 4 a  $4.51 \text{ cm}$  (3 s.f.)  
b  $6 \text{ cm}$   
c  $3.23 \text{ m}$  (3 s.f.)  
d  $4.31 \text{ mm}$  (3 s.f.)

### Exercise 27.5

---

- 1 a  $1.57 \text{ m}$  (3 s.f.)  
b 637 times (3 s.f.)
- 2  $188 \text{ m}$  (3 s.f.)
- 3  $84\pi \text{ mm}^2$
- 4  $36(4 - \pi) \text{ cm}^2$
- 5 a  $8(4 + \pi) \text{ m}$   
b  $16(8 + \pi) \text{ m}^2$

### Exercise 27.6

---

- 1 a  $460 \text{ cm}^2$   
b  $208 \text{ cm}^2$   
c  $147.78 \text{ cm}^2$   
d  $33.52 \text{ cm}^2$
- 2 a  $2 \text{ cm}$   
b  $4 \text{ cm}$   
c  $6 \text{ cm}$   
d  $5 \text{ cm}$
- 3 a i  $32\pi \text{ cm}^2$  ii  $101 \text{ cm}^2$   
b i  $88\pi \text{ cm}^2$  ii  $276 \text{ cm}^2$   
c i  $\frac{889}{10}\pi \text{ cm}^2$  ii  $279 \text{ cm}^2$   
d i  $\frac{204}{25}\pi \text{ cm}^2$  ii  $26.5 \text{ cm}^2$
- 4 a  $1.2 \text{ cm}$



- b** 0.5 cm
- c** 1.7 cm
- d** 7.0 cm

### **Exercise 27.7**

---

- 1 a**  $24 \text{ cm}^2$   
**b** 2 cm
- 2 a**  $216 \text{ cm}^2$   
**b** 15.2 cm (3 s.f.)
- 3 a**  $30\pi \text{ cm}^2$   
**b** 14 cm
- 4** 4.4 cm

### **Exercise 27.8**

---

- 1 a**  $24 \text{ cm}^3$   
**b**  $18 \text{ cm}^3$   
**c**  $27.6 \text{ cm}^3$   
**d**  $8.82 \text{ cm}^3$
- 2 a**  $452 \text{ cm}^3$  (3 s.f.)  
**b**  $277 \text{ cm}^3$  (3 s.f.)  
**c**  $196 \text{ cm}^3$  (3 s.f.)  
**d**  $0.481 \text{ cm}^3$  (3 s.f.)
- 3 a**  $48 \text{ cm}^3$ ,  $108 \text{ cm}^2$   
**b**  $102.5 \text{ cm}^3$ ,  $165.22 \text{ cm}^2$
- 4 a**  $264 \text{ cm}^2$   
**b**  $216 \text{ cm}^3$
- 5 a**  $70 \text{ cm}^3$   
**b**  $96 \text{ cm}^3$   
**c**  $380 \text{ cm}^3$   
**d**  $137.5 \text{ cm}^3$

### **Exercise 27.9**

---

- 1** 21.5% (3 s.f.)
- 2 a**  $42 \text{ cm}^2$   
**b**  $840 \text{ cm}^3$
- 3** 6.3 cm



4  $923\,000\pi\text{ cm}^3$

### Exercise 27.10

---

- 1 a 6.28 cm  
b 2.09 cm  
c 11.5 cm  
d 23.6 cm
- 2 a  $32.7^\circ$  (1 d.p.)  
b  $229.2^\circ$  (1 d.p.)  
c  $57.3^\circ$  (1 d.p.)  
d  $114.6^\circ$  (1 d.p.)
- 3 a 12.2 cm (3 s.f.)  
b 4.58 cm (3 s.f.)  
c 18.6 cm (3 s.f.)  
d 4.01 cm (3 s.f.)

### Exercise 27.11

---

- 1 a  $\left(\frac{85}{18}\pi + 34\right)\text{ cm}$   
b  $(18\pi + 48)\text{ cm}$
- 2 a 3.67 cm (3 s.f.)  
b 49.7 cm (3 s.f.)  
c  $68.8^\circ$  (1 d.p.)
- 3 a 12 cm  
b 54 cm  
c  $47.7^\circ$  (1 d.p.)

### Exercise 27.12

---

- 1 a  $33.5\text{ cm}^2$  (3 s.f.)  
b  $205\text{ cm}^2$  (3 s.f.)  
c  $5.65\text{ cm}^2$  (3 s.f.)  
d  $44.7\text{ cm}^2$  (3 s.f.)
- 2 a 18.5 cm (3 s.f.)  
b 20.0 cm (3 s.f.)  
c 1.75 cm (3 s.f.)  
d 12.4 cm (3 s.f.)



- 3 a  $48^\circ$   
b  $34^\circ$   
c  $20^\circ$   
d  $127^\circ$

### Exercise 27.13

---

- 1  $(96 - \frac{16}{3}\pi) \text{ m}^2$
- 2 a  $118 \text{ cm}^2$  (3 s.f.)  
b  $39.3 \text{ cm}^2$  (3 s.f.)  
c  $8.66 \text{ cm}$  (3 s.f.)
- 3 a  $4.19 \text{ cm}$  (3 s.f.)  
b  $114 \text{ cm}^2$  (3 s.f.)  
c  $62.8 \text{ cm}^3$  (3 s.f.)
- 4 a  $9.40 \text{ cm}$  (3 s.f.)  
b  $0.60 \text{ cm}$  (2 s.f.)  
c  $34.9 \text{ cm}^2$  (3 s.f.)  
d Length =  $17.1 \text{ cm}$  (3 s.f.)  
Width =  $10.6 \text{ cm}$  (3 s.f.)  
e  $41.7 \text{ cm}^2$  (3 s.f.)
- 5 a  $20^\circ$   
b  $0.64 \text{ cm}$  (2 s.f.)  
c Length =  $13.7 \text{ cm}$  (3 s.f.)  
Width =  $11.3 \text{ cm}$  (3 s.f.)  
d  $5.43 \text{ cm}^2$  (3 s.f.)

### Exercise 27.14

---

- 1 a  $905 \text{ cm}^3$  (3 s.f.)  
b  $3590 \text{ cm}^3$  (3 s.f.)  
c  $2310 \text{ cm}^3$  (3 s.f.)  
d  $1.44 \text{ cm}^3$  (3 s.f.)
- 2 a  $3.1 \text{ cm}$   
b  $5.6 \text{ cm}$   
c  $36.3 \text{ cm}$   
d  $0.6 \text{ cm}$



### Exercise 27.15

---

- 1 6.3 cm
- 2  $\frac{331}{12}\pi \text{ cm}^3$
- 3 11.9 cm
- 4 10.0 cm
- 5  $A = 4.1 \text{ cm}$ ,  $B = 3.6 \text{ cm}$ ,  $C = 3.1 \text{ cm}$
- 6 3 : 2

### Exercise 27.16

---

- 1
  - a  $452 \text{ cm}^2$  (3 s.f.)
  - b  $254 \text{ cm}^2$  (3 s.f.)
  - c  $1890 \text{ cm}^2$  (3 s.f.)
  - d  $4 \text{ cm}^2$
- 2
  - a 1.99 cm (3 s.f.)
  - b 1.15 cm (3 s.f.)
  - c 3.09 mm (3 s.f.)
  - d 0.5 cm
- 3 1 : 4
- 4  $707 \text{ cm}^2$  (3 s.f.)
- 5 5.9 cm (1 d.p.)

### Exercise 27.17

---

- 1  $40 \text{ cm}^3$
- 2  $133 \text{ cm}^3$  (3 s.f.)
- 3  $64 \text{ cm}^3$
- 4  $70 \text{ cm}^3$

### Exercise 27.18

---

- 1 7 cm
- 2 5 cm
- 3
  - a 8 cm
  - b  $384 \text{ cm}^3$
  - c  $378 \text{ cm}^3$
- 4
  - a  $\sqrt{6} \text{ cm}$
  - b  $14.7 \text{ cm}^3$  (3 s.f.)



c  $60.3 \text{ cm}^3$  (3 s.f.)

### Exercise 27.19

---

1  $6.93 \text{ cm}^2$  (3 s.f.)

2  $189 \text{ cm}^2$  (3 s.f.)

3  $73.3 \text{ cm}^2$  (3 s.f.)

4 a  $693 \text{ cm}^2$  (3 s.f.)

b  $137 \text{ cm}^2$  (3 s.f.)

c  $23.6 \text{ cm}$  (3 s.f.)

5  $1120 \text{ cm}^2$  (3 s.f.)

6 a  $211 \text{ cm}^2$

b  $176 \text{ cm}^2$

### Exercise 27.20

---

1 a  $56.5 \text{ cm}^3$  (3 s.f.)

b  $264 \text{ cm}^3$  (3 s.f.)

c  $1.34 \text{ cm}^3$  (3 s.f.)

d  $166 \text{ cm}^3$  (3 s.f.)

2 a  $6.91 \text{ cm}$  (3 s.f.)

b  $10.9 \text{ cm}$  (3 s.f.)

c  $0.818 \text{ cm}$  (3 s.f.)

d  $51.3 \text{ cm}$  (3 s.f.)

3 a i  $7.96 \text{ cm}$

ii  $12.7 \text{ cm}$

iii  $843 \text{ cm}^3$

b i  $15.9 \text{ cm}$

ii  $8.41 \text{ cm}$

iii  $2230 \text{ cm}^3$

c i  $6.37 \text{ cm}$

ii  $3.97 \text{ cm}$

iii  $168 \text{ cm}^3$

d i  $3.82 \text{ cm}$

ii  $4.63 \text{ cm}$

iii  $70.7 \text{ cm}^3$



### Exercise 27.21

---

- 1 3.88 cm (3 s.f.)
- 2  $1700 \text{ cm}^3$  (3 s.f.)
- 3 a  $\frac{21}{2}\pi \text{ cm}$   
b  $\frac{21}{4} \text{ cm}$   
c  $\frac{3\sqrt{95}}{4} \text{ cm}$   
d  $211 \text{ cm}^3$
- 4 8 : 27

### Exercise 27.22

---

- 1  $\frac{625}{24}\pi \text{ cm}^3$
- 2  $771 \text{ cm}^3$  (3 s.f.)
- 3  $3170 \text{ cm}^3$  (3 s.f.)
- 4 a  $654 \text{ cm}^3$   
b 12.5 cm  
c  $2950 \text{ cm}^3$

### Exercise 27.23

---

- 1 a  $415 \text{ cm}^2$  (3 s.f.)  
b  $1650 \text{ cm}^2$  (3 s.f.)
- 2  $1130 \text{ cm}^2$  (3 s.f.)

### Student assessment 1

---

- 1  $90 \text{ cm}^2$
- 2 a Circumference = 34.6 cm      Area =  $95.0 \text{ cm}^2$   
b Circumference = 50.3 mm      Area =  $201 \text{ mm}^2$
- 3  $3\left(8 - \frac{3}{2}\pi\right) \text{ cm}^2$
- 4 a  $39.3 \text{ cm}^2$   
b  $34 \text{ cm}^2$   
c  $101.3 \text{ cm}^2$
- 5 a  $10.2 \text{ cm}^2$   
b  $283 \text{ cm}^2$   
c  $633 \text{ cm}^2$
- 6 a  $108\pi \text{ mm}^3$



**b**  $\frac{25}{8}\pi \text{ cm}^3$

## Student assessment 2

---

**1 a** 11.8 cm (3 s.f.)

**b** 35.3 cm (3 s.f.)

**2**  $\frac{729}{\pi} \text{ cm}^2$

**3 a** 531 cm<sup>2</sup>

**b** 1150 cm<sup>3</sup>

**4 a** 272.2° (1 d.p.)

**b** 5.7° (1 d.p.)

## Student assessment 3

---

**1** 48%

**2 a** 22.9 cm (3 s.f.)

**b** 229 cm<sup>2</sup> (3 s.f.)

**c** 985 cm<sup>2</sup> (3 s.f.)

**d** 1830 cm<sup>3</sup> (3 s.f.)

**3 a** 288π cm<sup>3</sup>

**b** 12 cm

**c** 6√5 cm

**d** 72π(2 + √5) cm<sup>2</sup>

**4 a** 10 cm

**b** 82.1 cm<sup>3</sup> (3 s.f.)

**c** 71.8 cm<sup>3</sup> (3 s.f.)

**d** 30.8 cm<sup>3</sup> (3 s.f.)

**e** 41.1 cm<sup>3</sup> (3 s.f.)



## Mathematical investigations and ICT 5

### Metal trays

---

- 1 a length = 38 cm  
width = 28 cm  
height = 1 cm  
b  $1064 \text{ cm}^3$
- 2 a length = 36 cm  
width = 26 cm  
height = 2 cm  
b  $1872 \text{ cm}^3$
- 3 Student's investigation and ordered table of results similar to this one.

| Square length (cm) | Tray dimensions (cm) |       |        | Volume |
|--------------------|----------------------|-------|--------|--------|
|                    | Length               | Width | Height |        |
| 1                  | 38                   | 28    | 1      | 1064   |
| 2                  | 36                   | 26    | 2      | 1872   |
| 3                  | 34                   | 24    | 3      | 2448   |
| 4                  | 32                   | 22    | 4      | 2816   |
| 5                  | 30                   | 20    | 5      | 3000   |
| 6                  | 28                   | 18    | 6      | 3024   |
| 7                  | 26                   | 16    | 7      | 2912   |
| 8                  | 24                   | 14    | 8      | 2688   |
| 9                  | 22                   | 12    | 9      | 2376   |
| 10                 | 20                   | 10    | 10     | 2000   |
| 11                 | 18                   | 8     | 11     | 1584   |
| 12                 | 16                   | 6     | 12     | 1152   |
| 13                 | 14                   | 4     | 13     | 728    |
| 14                 | 12                   | 2     | 14     | 336    |
| 15                 | 10                   | 0     | 15     | 0      |

- 4  $x = 5.7 \text{ cm}$
- 5 Maximum volume =  $3032 \text{ cm}^3$



## Tennis balls

- 1 Cuboids of the following dimensions should be considered. Note each unit represents the diameter of one tennis ball and only different combinations are considered.

$$1 \times 1 \times 12 \quad 1 \times 2 \times 6 \quad 1 \times 3 \times 4 \quad 2 \times 2 \times 3$$

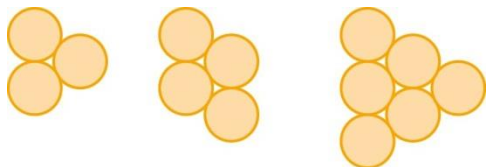
- 2 Total surface area of a cuboid is given by the formula:  $A = 2(lw + lh + wh)$

Total surface area of the four options are as shown (to nearest whole number):

| Dimensions (units)     | Length (cm) | Width (cm) | Height (cm) | Surface area (cm <sup>2</sup> ) |
|------------------------|-------------|------------|-------------|---------------------------------|
| $1 \times 1 \times 12$ | 6.6         | 6.6        | 79.2        | 2178                            |
| $1 \times 2 \times 6$  | 6.6         | 13.2       | 39.6        | 1742                            |
| $1 \times 3 \times 4$  | 6.6         | 19.8       | 26.4        | 1655                            |
| $2 \times 2 \times 3$  | 13.2        | 13.2       | 19.8        | 1394                            |

The optimum dimensions of the box are 13.2 cm × 13.2 cm × 19.8 cm.

- 3 Cross-sections of possible designs are shown below:



- 4 Student's investigations  
5 Student's conclusion based on their calculations

## ICT activity

- 1 Possible formulae are given:

In cell

$$B2: =A2/360*2*PI()*10$$

$$C2: =B2$$

$$D2: =C2/(2*PI())$$

$$E2: =SQRT((100-D2^2))$$

$$F2: =1/3*PI()*D2^2*E2$$



*All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.*

## 28 Bearings

### Exercise 28.1

---

- 1 Student's own diagram leading to:  
Distance  $\approx 11$  km  
Bearing  $\approx 035^\circ$
- 2 Student's own diagrams leading to:
  - a  $343^\circ$
  - b  $034^\circ$
- 3 Student's own diagrams leading to:
  - a  $120^\circ$
  - b  $102^\circ$

### Student assessment 1

---

- 1 Student's own diagram leading to:
  - a 66 km,  $047^\circ$
  - b 75 km,  $313^\circ$
- 2 Student's own diagram:  
The ship is due North of the lifeboat and is 64 km away.
- 3 Student's own diagram:  
Airport B is 329 km from the aircraft on a bearing of  $234^\circ$ .



*All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.*

## 29 Trigonometry

### Exercise 29.1

---

- 1   a   1.82 cm (3 s.f.)  
     b   4.04 cm (3 s.f.)  
     c   19.2 cm (3 s.f.)  
     d   4.87 cm (3 s.f.)  
     e   37.3 cm (3 s.f.)  
     f   13.9 cm (3 s.f.)
- 2   a   14.3 cm (3 s.f.)  
     b   8.96 cm (3 s.f.)  
     c   9.33 cm (3 s.f.)  
     d   4.10 cm (3 s.f.)  
     e   13.9 cm (3 s.f.)  
     f   6.21 cm (3 s.f.)
- 3   a    $49.4^\circ$  (1 d.p.)  
     b    $51.1^\circ$  (1 d.p.)  
     c    $51.3^\circ$  (1 d.p.)  
     d    $63.4^\circ$  (1 d.p.)  
     e    $50.4^\circ$  (1 d.p.)  
     f    $71.6^\circ$  (1 d.p.)

### Exercise 29.2

---

- 1   a   2.44 cm (3 s.f.)  
     b   18.5 cm (3 s.f.)  
     c   6.19 cm (3 s.f.)  
     d   2.44 cm (3 s.f.)  
     e   43.8 cm (3 s.f.)  
     f   31.8 cm (3 s.f.)
- 2   a    $38.7^\circ$  (1 d.p.)  
     b    $48.6^\circ$  (1 d.p.)  
     c    $38.1^\circ$  (1 d.p.)  
     d    $49.8^\circ$  (1 d.p.)



e  $32.6^\circ$  (1 d.p.)

f  $14.5^\circ$  (1 d.p.)

### Exercise 29.3

---

1 a 36.0 cm (3 s.f.)

b 15.1 cm (3 s.f.)

c  $48.2^\circ$  (1 d.p.)

d  $81.1^\circ$  (1 d.p.)

e 6.71 cm (3 s.f.)

f 16.8 cm (3 s.f.)

g  $70.5^\circ$  (1 d.p.)

h 2.14 cm (3 s.f.)

### Exercise 29.4

---

1 a 5 cm

b 12 cm

c  $\sqrt{130}$  mm

d  $5\sqrt{7}$  cm

2 a  $\sqrt{122}$  cm

b  $\sqrt{218}$  cm

c  $(8\sqrt{3} - 6)$  cm

d  $3\sqrt{6}$  cm

e 3 cm

f  $8\sqrt{3}$  cm

3 71.6 km

4 67 km

5 a  $225^\circ - 135^\circ = 90^\circ$

b 73.8 km

6 57 009 m

7 a 8.5 km

b 15.5 km (3 s.f.)

8 a 13.3 m (3 s.f.)

b 15.0 m (3 s.f.)



### **Exercise 29.5**

---

- 1 a  $43.6^\circ$   
b 19.5 cm  
c 16.7 cm  
d  $42.5^\circ$
- 2 a 20.8 km (3 s.f.)  
b  $215.2^\circ$  (1 d.p.)
- 3 a 228 km (3 s.f.)  
b 102 km (3 s.f.)  
c 103 km (3 s.f.)  
d 147 km (3 s.f.)  
e 415 km (3 s.f.)  
f  $217^\circ$  (3 s.f.)
- 4 a  $48.2^\circ$  (3 s.f.)  
b  $41.8^\circ$  (3 s.f.)  
c 8 cm  
d 8.94 cm (3 s.f.)  
e  $76.0 \text{ cm}^2$  (3 s.f.)

### **Exercise 29.6**

---

- 1 a 12.2 km (3 s.f.)  
b  $9.5^\circ$  (1 d.p.)
- 2 a 10.1 km (3 s.f.)  
b 1.23 km (3 s.f.)
- 3 a  $22.6^\circ$  (1 d.p.)  
b 130 m
- 4 a 0.342 km (3 s.f.)  
b 0.940 km (3 s.f.)
- 5 a 64.0 m (3 s.f.)  
b 30.2 m (3 s.f.)
- 6 6.93 km (3 s.f.)
- 7 a 7.46 km (3 s.f.)  
b 3.18 km (3 s.f.)
- 8 a 2.9 km  
b 6.9 km  
c  $11.4^\circ$



- d 20.4 km (1 d.p.)
- 9 a 2.68 km (3 s.f.)  
b 1.02 km (3 s.f.)  
c  $3.5^\circ$  (1 d.p.)  
d 16.83 km (2 d.p.)
- 10 a 225 m  
b  $48.4^\circ$  (1 d.p.)
- 11 a 6.71 m (3 s.f.)  
b 19.6 m (3 s.f.)  
c 15.3 m (3 s.f.)

### Exercise 29.7

---

- 1 a  $\sin 120^\circ$   
b  $\sin 100^\circ$   
c  $\sin 65^\circ$   
d  $\sin 340^\circ$   
e  $\sin 240^\circ$   
f  $\sin 275^\circ$
- 2 a  $\sin 145^\circ$   
b  $\sin 130^\circ$   
c  $\sin 150^\circ$   
d  $\sin 292^\circ$   
e  $\sin 236^\circ$   
f  $\sin 213^\circ$
- 3 a  $19^\circ, 161^\circ$   
b  $82^\circ, 98^\circ$   
c  $5^\circ, 175^\circ$   
d  $210^\circ, 330^\circ$   
e  $240^\circ, 300^\circ$   
f  $225^\circ, 315^\circ$
- 4 a  $70^\circ, 110^\circ$   
b  $9^\circ, 171^\circ$   
c  $53^\circ, 127^\circ$   
d  $214^\circ, 326^\circ$   
e  $196^\circ, 344^\circ$   
f  $199^\circ, 341^\circ$



### Exercise 29.8

---

- 1 a  $\cos 340^\circ$   
b  $\cos 275^\circ$   
c  $\cos 328^\circ$   
d  $\cos 265^\circ$   
e  $\cos 213^\circ$   
f  $\cos 254^\circ$
- 2 a  $\cos 262^\circ$   
b  $\cos 216^\circ$   
c  $\cos 200^\circ$   
d  $\cos 177^\circ$   
e  $\cos 149^\circ$   
f  $\cos 126^\circ$
- 3 a  $\cos 80^\circ$   
b  $-\cos 90^\circ$   
c  $\cos 70^\circ$   
d  $\cos 135^\circ$   
e  $\cos 58^\circ$   
f  $\cos 155^\circ$
- 4 a  $-\cos 55^\circ$   
b  $-\cos 73^\circ$   
c  $\cos 60^\circ$   
d  $\cos 82^\circ$   
e  $\cos 88^\circ$   
f  $\cos 70^\circ$
- 5 In a right-angled triangle  $\tan x = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$ , as  $x$  approaches  $90^\circ$  the length of the adjacent side approaches zero. At  $x = 90^\circ$  there is no triangle as in theory it would have two right-angles.

### Exercise 29.9

---

- 1 a  $14.5^\circ, 165.5^\circ$  (1 d.p.)  
b  $45^\circ, 315^\circ$   
c  $210^\circ, 330^\circ$   
d  $120^\circ, 300^\circ$   
e  $78.5^\circ, 281.5^\circ$  (1 d.p.)  
f  $116.6^\circ, 296.6^\circ$  (1 d.p.)



2 a  $\sin x = \frac{3}{5}$

b  $\cos x = \frac{4}{5}$

3 a  $\cos x = \frac{\sqrt{91}}{10}$

b  $\tan x = \frac{3}{\sqrt{91}}$

4  $\sin x = \frac{\text{opposite}}{\text{hypotenuse}}, \cos x = \frac{\text{adjacent}}{\text{hypotenuse}}$

When  $\sin x = \cos x$ ,  $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{\text{adjacent}}{\text{hypotenuse}}$  therefore opposite = adjacent.

If opposite side length = adjacent side length then  $x = 45^\circ$

### Student assessment 1

---

1 a 4 cm

b 43.9 cm (3 s.f.)

c 20.8 cm

d 3.92 cm (3 s.f.)

2 a  $36.9^\circ$  (1 d.p.)

b  $56.3^\circ$  (1 d.p.)

c  $31.0^\circ$  (1 d.p.)

d  $33.8^\circ$  (1 d.p.)

3 a 5 cm

b 6.63 cm (3 s.f.)

c 9.29 cm (3 s.f.)

d 28.5 cm (3 s.f.)

4 a  $\sin x = \frac{\sqrt{11}}{6}$

b  $\tan x = \frac{\sqrt{11}}{5}$

### Student assessment 2

---

1 a 160.8 km

b 177.5 km

2 a 285 m (3 s.f.)

b  $117^\circ$  (3 s.f.)

c  $297^\circ$  (3 s.f.)

3 a  $z = \tan^{-1}\left(\frac{5}{x}\right)$

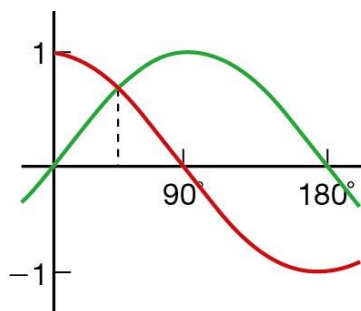
b  $\tan z = \frac{7.5}{x+16} \quad z = \tan^{-1}\left(\frac{7.5}{x+16}\right)$



- c  $\frac{5}{x} = \frac{7.5}{(x+16)}$   
d 32 m  
e  $8.9^\circ$  (1 d.p.)  
4 a 1.96 km (3 s.f.)  
b 3.42 km (3 s.f.)  
c 3.57 km (3 s.f.)  
5 a  $23.6^\circ$ ,  $156.4^\circ$  (1 d.p.)  
b  $150^\circ$ ,  $330^\circ$   
c  $95.7^\circ$ ,  $264.3^\circ$  (1 d.p.)  
d  $90^\circ$

### Student assessment 3

- 1 By drawing the graph of cosine, you can see that the maximum value of cosine is 1, so  $\cos x = \frac{3}{2}$  has no solutions.  
2 a  $-\cos 52^\circ$   
b  $\cos 100^\circ$   
3 a Angle ACB =  $90^\circ$  due to angle in a semi-circle = right-angle  
b  $\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$  or  $\left(\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$   
4 a 678 m (3 s.f.)  
b  $11.6^\circ$  (1 d.p.)  
c 718 m (3 s.f.)  
5 a  $21.8^\circ$  (1 d.p.)  
b  $8.5^\circ$  (1 d.p.)  
c 2.2 km  
d  $10.5^\circ$  (1 d.p.)  
e  $15.3^\circ$  (1 d.p.)  
f 1.76 km (3 s.f.)  
6 a



- b  $x = 45$

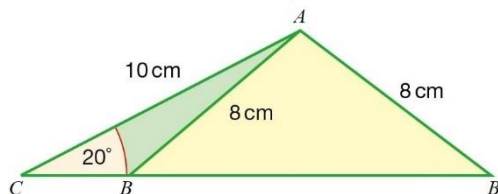


All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

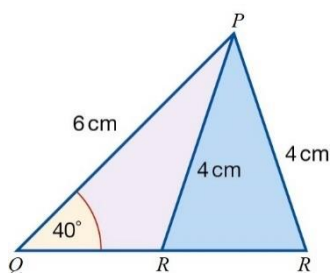
## 30 Further trigonometry

### Exercise 30.1

- 1 a 8.91 cm (3 s.f.)  
b 8.93 cm (3 s.f.)  
c 5.96 mm (3 s.f.)  
d 8.64 cm (3 s.f.)
- 2 a  $33.2^\circ$ ,  $x = 33.2^\circ$ ,  $146.8^\circ$  (1 d.p.)  
b  $52.7^\circ$ ,  $x = 52.7^\circ$ ,  $127.3^\circ$  (1 d.p.)  
c  $77.0^\circ$ ,  $x = 77.0^\circ$ ,  $103.0^\circ$  (1 d.p.)  
d  $44.0^\circ$ ,  $x = 44.0^\circ$ ,  $136.0^\circ$  (1 d.p.)
- 3 a  $25^\circ$ ,  $155^\circ$  (nearest degree)  
b



- 4 a  $75^\circ$ ,  $105^\circ$  (nearest degree)  
b



### Exercise 30.2

- 1 a 4.71 m (3 s.f.)  
b 12.1 cm (3 s.f.)  
c 9.15 cm (3 s.f.)  
d 3.06 cm (3 s.f.)  
e 10.7 cm (3 s.f.)
- 2 a  $125.1^\circ$  (1 d.p.)



- b**  $108.2^\circ$  (1 d.p.)
- c**  $33.6^\circ$  (1 d.p.)
- d**  $37.0^\circ$  (1 d.p.)
- e**  $122.9^\circ$  (1 d.p.)

### **Exercise 30.3**

---

- 1**
  - a** 42.9 m (3 s.f.)
  - b**  $116.9^\circ$  (1 d.p.)
  - c**  $24.6^\circ$  (1 d.p.)
  - d**  $33.4^\circ$  (1 d.p.)
  - e** 35.0 m (3 s.f.)
- 2** 370 m
- 3** 73.9 m (3 s.f.)

### **Exercise 30.4**

---

- 1**
  - a**  $70. \text{ cm}^2$
  - b**  $70.9 \text{ mm}^2$  (3 s.f.)
  - c**  $122 \text{ cm}^2$  (3 s.f.)
  - d**  $17.0 \text{ cm}^2$  (3 s.f.)
- 2**
  - a**  $24.6^\circ$  or  $155.4^\circ$  (1 d.p.)
  - b** 13.0 cm (3 s.f.)
  - c** 23.1 cm (3 s.f.)
  - d**  $63.2^\circ$  or  $116.8^\circ$  (1 d.p.)
- 3** 465 m
- 4** 1506 km<sup>2</sup>
- 5** 16 800 m<sup>2</sup>
- 6**
  - a**  $3.90 \text{ m}^2$  (3 s.f.)
  - b**  $222 \text{ m}^3$  (3 s.f.)

### **Exercise 30.5**

---

- 1**
  - a** 5.66 cm (3 s.f.)
  - b** 6.93 cm (3 s.f.)
  - c**  $54.7^\circ$  (1 d.p.)
- 2**
  - a** 5.83 cm (3 s.f.)
  - b** 6.16 cm (3 s.f.)
  - c**  $18.9^\circ$  (1 d.p.)



- 3 a 6.40 cm (3 s.f.)  
b 13.6 cm (3 s.f.)  
c  $61.9^\circ$  (1 d.p.)
- 4 a  $75.3^\circ$  (1 d.p.)  
b  $56.3^\circ$  (1 d.p.)
- 5 a i 7.21 cm (3 s.f.)  
ii  $21.1^\circ$  (1 d.p.)  
b i  $33.7^\circ$  (1 d.p.)  
ii  $68.9^\circ$  (1 d.p.)
- 6 a i 8.54 cm (3 s.f.)  
ii  $28.3^\circ$  (1 d.p.)  
b i  $20.6^\circ$  (1 d.p.)  
ii  $61.7^\circ$  (1 d.p.)
- 7 a 6.5 cm  
b 11.3 cm (3 s.f.)  
c 70.7 cm (3 s.f.)
- 8 a 11.7 cm (3 s.f.)  
b 7.55 cm (3 s.f.)
- 9 a  $TU = TQ = 10$  cm  
 $QU = 8.49$  (3 s.f.)  
b  $90^\circ, 36.9^\circ, 53.1^\circ$   
c  $24 \text{ cm}^2$

### Exercise 30.6

---

- 1 a  $RW$   
b  $TQ$   
c  $SQ$   
d  $WU$   
e  $QV$   
f  $SV$
- 2 a  $JM$   
b  $KN$   
c  $HM$   
d  $HO$   
e  $JO$   
f  $MO$



- 3**   **a**   Angle *TPS*  
      **b**   Angle *UPQ*  
      **c**   Angle *VSW*  
      **d**   Angle *RTV*  
      **e**   Angle *SUR*  
      **f**   Angle *VPW*
- 4**   **a**   5.83 cm (3 s.f.)  
      **b**   31.0° (1 d.p.)
- 5**   **a**   10.2 cm (3 s.f.)  
      **b**   29.2° (1 d.p.)  
      **c**   51.3° (1 d.p.)
- 6**   **a**   6.71 cm (3 s.f.)  
      **b**   61.4° (1 d.p.)
- 7**   **a**   7.81 cm (3 s.f.)  
      **b**   11.3 cm (3 s.f.)  
      **c**   12.4° (1 d.p.)
- 8**   **a**   14.1 cm (3 s.f.)  
      **b**   8.49 cm (3 s.f.)  
      **c**   7.48 cm (3 s.f.)  
      **d**   69.3° (1 d.p.)
- 9**   **a**   17.0 cm (3 s.f.)  
      **b**   5.66 cm (3 s.f.)  
      **c**   7 cm  
      **d**   51.1° (1 d.p.)

### **Student assessment 1**

---

- 1**   **a**   47.3°, 132.7° (1 d.p.)  
      **b**   0.886 cm (3 s.f.)
- 2**   134° (3 s.f.)
- 3**   **a**    $\frac{1}{2}$   
      **b**   150°
- 4**   **a**   11.7 cm (3 s.f.)  
      **b**   12.3 cm (3 s.f.)  
      **c**   29.1° (1 d.p.)
- 5**   **a**   18.0 m (3 s.f.)  
      **b**   26.5° (1 d.p.)



- c** 28.8 m (3 s.f.)
- d**  $278 \text{ m}^2$  (3 s.f.)
- 6 a** 12.7 cm (3 s.f.)
- b**  $66.6^\circ$  (1 d.p.)
- c**  $93.4 \text{ cm}^2$  (3 s.f.)
- d** 14.7 cm (1 d.p.)

## **Student assessment 2**

---

- 1 a** 528 m
- b** 334 m
- 2 a** 10.8 cm (3 s.f.)
- b** 11.9 cm (3 s.f.)
- c**  $30.2^\circ$  (1 d.p.)
- d**  $49.0^\circ$  (1 d.p.)
- 3 a** 9.81 cm (3 s.f.)
- b**  $30^\circ$
- c** 19.6 cm (3 s.f.)
- 4 a** 5.83 cm (3 s.f.)
- b** 6.71 cm (3 s.f.)
- c** 7.81 cm (3 s.f.)
- d**  $46.6^\circ$  (1 d.p.)
- e**  $19.0 \text{ cm}^2$  (3 s.f.)
- f**  $36.7^\circ$  (1 d.p.)



## Mathematical investigations and ICT 6

### Numbered balls

---

- 1 If a ball is odd ( $n$ ), the next ball is  $n + 1$   
If the ball is even ( $n$ ), the next ball is  $\frac{n}{2}$
- 2 65, 66, 33, 34, 17, 18, 9, 10, 5, 6, 3, 4, 2, 1
- 3 The first ball must be odd. Start at 1 and work backwards.
- 4 513, 514, 257, 258, 129, 130, 65, 66, 33, 34, 17, 18, 9, 10, 5, 6, 3, 4, 2, 1

### Towers of Hanoi

---

- 1 3
- 2 15
- 3 Student's investigation
- 4 The results up to 8 discs are given below:

| Number of discs | Smallest number of moves |
|-----------------|--------------------------|
| 1               | 1                        |
| 2               | 3                        |
| 3               | 7                        |
| 4               | 15                       |
| 5               | 31                       |
| 6               | 63                       |
| 7               | 127                      |
| 8               | 255                      |

- 5 The number of moves are 1 less than the powers of 2.
- 6 1023
- 7 Number of moves =  $2^n - 1$ , where  $n$  = number of discs.
- 8 Time taken to move 64 discs is  $2^{64} - 1$  seconds  
This equates to  $5.85 \times 10^{11}$  years, i.e. 585 billion years.  
Therefore according to the legend we needn't be too worried!

### ICT activity

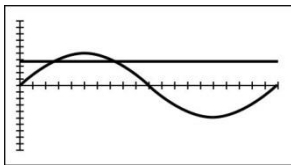
---

- 1 **b i** 0.940 (3 d.p.)  
**ii** 0.819 (3 d.p.)



iii  $-0.866$  (3 d.p.) or  $-\frac{\sqrt{3}}{2}$

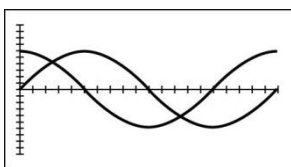
c



The graph of  $y = \sin x$  intersects the line  $y = 0.7$  in two places as shown.

d  $30^\circ$  and  $150^\circ$

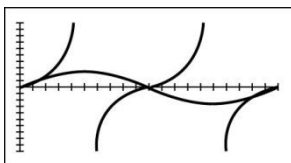
2 a



b Two solutions

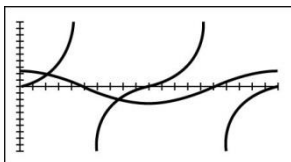
c  $225^\circ$

3 a



Solutions are  $0^\circ$ ,  $180^\circ$  and  $360^\circ$

b



Solutions are  $38.2^\circ$  and  $141.8^\circ$  (1 d.p.)



All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

## 31 Vectors

### Exercise 31.1

1 a  $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$

b  $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

c  $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$

d  $\begin{pmatrix} -2 \\ 6 \end{pmatrix}$

e  $\begin{pmatrix} -4 \\ -2 \end{pmatrix}$

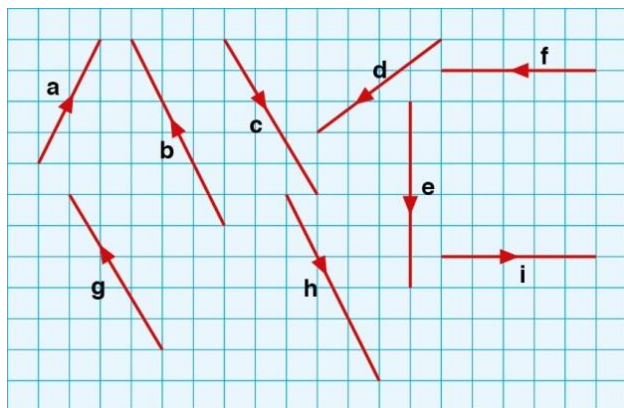
f  $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$

g  $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$

h  $\begin{pmatrix} 2 \\ -6 \end{pmatrix}$

i  $\begin{pmatrix} -4 \\ -4 \end{pmatrix}$

2



3 a  $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$

b  $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

c  $\begin{pmatrix} -2 \\ -4 \end{pmatrix}$

d  $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

e  $\begin{pmatrix} -6 \\ 1 \end{pmatrix}$



f  $\begin{pmatrix} 6 \\ -1 \end{pmatrix}$

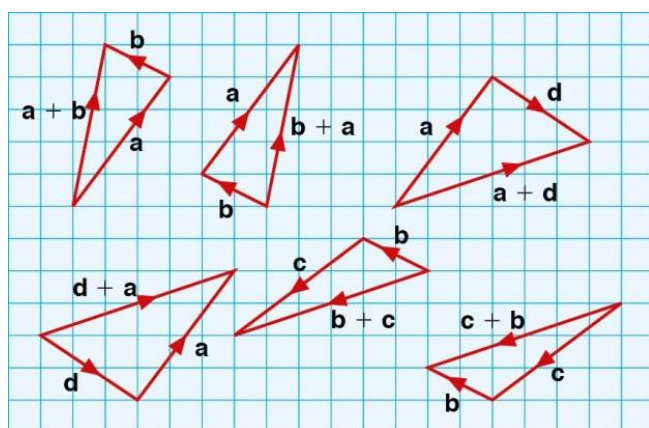
g  $\begin{pmatrix} -3 \\ -1 \end{pmatrix}$

h  $\begin{pmatrix} -5 \\ -5 \end{pmatrix}$

i  $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

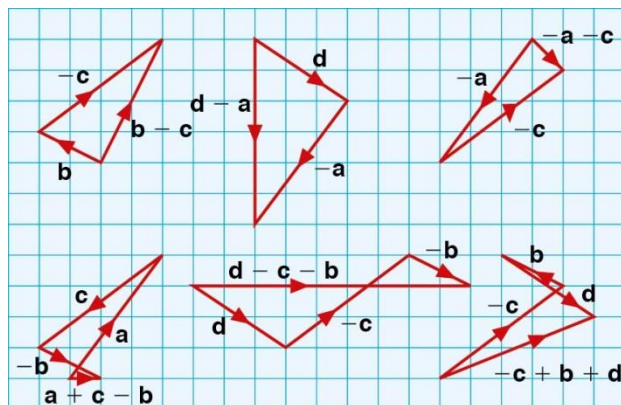
## Exercise 31.2

1 a–f



2  $a + b = b + a$ ,  $a + d = d + a$ ,  $b + c = c + b$

3 a–f



4 a  $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$

b  $\begin{pmatrix} 0 \\ -6 \end{pmatrix}$

c  $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

d  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

e  $\begin{pmatrix} 9 \\ 0 \end{pmatrix}$



$$f \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

### Exercise 31.3

---

1  $d = -c$

$$e = -a$$

$$f = 2a$$

$$g = \frac{1}{2}c$$

$$h = \frac{1}{2}b$$

$$i = -\frac{1}{2}b$$

$$j = \frac{3}{2}b$$

$$k = -\frac{3}{2}a$$

2 a  $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$

$$b \begin{pmatrix} -12 \\ -3 \end{pmatrix}$$

$$c \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$$d \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

$$e \begin{pmatrix} -2 \\ -5 \end{pmatrix}$$

$$f \begin{pmatrix} -8 \\ 9 \end{pmatrix}$$

$$g \begin{pmatrix} -10 \\ -5 \end{pmatrix}$$

$$h \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$i \begin{pmatrix} 10 \\ -6 \end{pmatrix}$$

3 a  $2a$

$$b -b$$

$$c b + c$$

$$d a - b$$

$$e 2c$$

$$f 2c - a$$



### Exercise 31.4

---

- 1  $|a| = 5$  units  
 $|b| = 4.1$  units  
 $|c| = 4.5$  units  
 $|d| = 7$  units  
 $|e| = 7.3$  units  
 $|f| = 6.4$  units
- 2 a  $|\overline{AB}| = 4$  units  
b  $|\overline{BC}| = 5.4$  units  
c  $|\overline{CD}| = 7.2$  units  
d  $|\overline{DE}| = 13$  units  
e  $|\overline{2AB}| = 8$  units  
f  $|\overline{2CD}| = 14.4$  units
- 3 a 4.1 units  
b 18.4 units  
c 15.5 units  
d 17.7 units  
e 31.8 units  
f 19.6 units

### Exercise 31.5

---

- 1  $A = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$   
 $B = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$   
 $C = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$   
 $D = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$   
 $E = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$   
 $F = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$   
 $G = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$   
 $H = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$



### Exercise 31.6

---

- 1**
- a**  $a$
  - b**  $a$
  - c**  $b$
  - d**  $-b$
  - e**  $2b$
  - f**  $-2a$
  - g**  $a + b$
  - h**  $b - a$
  - i**  $b - 2a$
- 2**
- a**  $2a$
  - b**  $b$
  - c**  $b - a$
  - d**  $b - a$
  - e**  $-a$
  - f**  $a - 2b$
- 3**
- a**  $-2a$
  - b**  $-a$
  - c**  $b$
  - d**  $b - a$
  - e**  $2(b - a)$
  - f**  $2b - a$
  - g**  $-2b$
  - h**  $b - 2a$
  - i**  $-a - b$
- 4**
- a**  $5a$
  - b**  $\frac{8}{3}b$
  - c**  $\frac{1}{3}(8b - 15a)$
  - d**  $\frac{1}{15}(8b - 15a)$
  - e**  $b - 2a$
  - f**  $5a - 2b$
  - g**  $\frac{8}{5}b$
  - h**  $\frac{1}{5}(10a + 8b)$
  - i**  $\frac{1}{5}(8b - 5a)$



### Exercise 31.7

---

1 a i  $2a$

ii  $2a - b$

iii  $\frac{1}{4}(2a + 3b)$

b Proof

2 a, b Proofs

3 a i  $4a$

ii  $2a$

iii  $2(a + b)$

iv  $\frac{3}{2}a$

b Proof

4 a i  $\frac{1}{2}(2q - p)$

ii  $\frac{2}{7}(p - q)$

b Proof

5 As  $\overrightarrow{OM} = a + \frac{1}{2}c$  and  $\overrightarrow{OX} = \frac{1}{2}a + \frac{1}{4}c$  they must be parallel as one is a scalar multiple of the other. As they share the common point O, they must lie on the same line and therefore are collinear.

### Student assessment 1

---

1 a  $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$

b  $\begin{pmatrix} 7 \\ 2 \end{pmatrix}$

c  $\begin{pmatrix} 6 \\ -2 \end{pmatrix}$

2 a  $= \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

b  $= \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

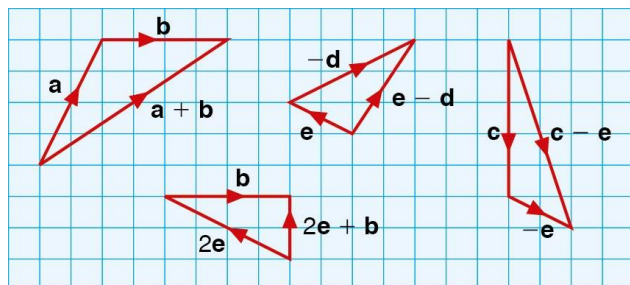
c  $= \begin{pmatrix} 0 \\ -5 \end{pmatrix}$

d  $= \begin{pmatrix} -4 \\ -2 \end{pmatrix}$

e  $= \begin{pmatrix} -2 \\ 1 \end{pmatrix}$



3



- 4 a  $\begin{pmatrix} -1 \\ 5 \end{pmatrix}$   
b  $\begin{pmatrix} 1 \\ 5 \end{pmatrix}$   
c  $\begin{pmatrix} 1 \\ 11 \end{pmatrix}$   
d  $\begin{pmatrix} 0 \\ 14 \end{pmatrix}$

## Student assessment 2

- 1 a  $|\overrightarrow{AB}| = 7.21$  units (3 s.f.)  
b  $|a| = 9.22$  units (3 s.f.)  
 $|b| = 8.06$  units (3 s.f.)  
 $|c| = 13$  units
- 2 a 17.5 units (3 s.f.)  
b 2.69 units (3 s.f.)
- 3  $A = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$   
 $B = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$   
 $C = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$   
 $D = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$   
 $E = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$
- 4 a  $\overrightarrow{AF}$  or  $\overrightarrow{FC}$   
b  $\overrightarrow{DF} = \frac{1}{2}\overrightarrow{BC}$   
c  $\overrightarrow{CF} = -\overrightarrow{DE}$

## Student assessment 3

- 1 a i  $\frac{1}{4}b$   
ii  $\frac{1}{4}(4a - b)$



iii  $\frac{2}{5}(b - a)$

**b** Proof

**2 a** Proof

**b** Proof

**3 a i**  $b - a$

**ii**  $a$

**iii**  $a + b$

**b i**  $4 : 25$

**ii**  $4 : 5$

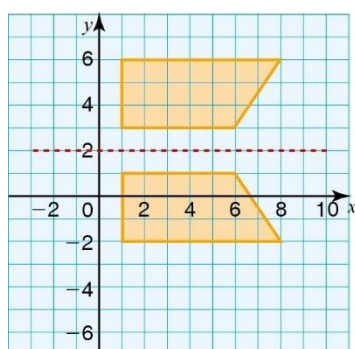


All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

## 32 Transformations

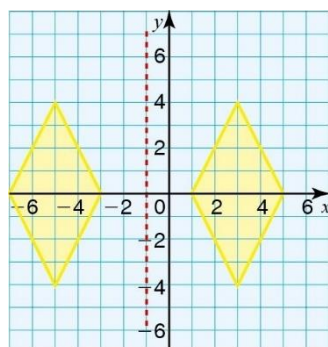
### Exercise 32.1

1 a



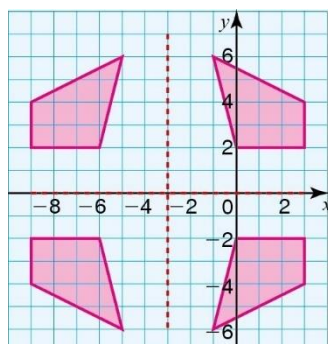
b  $y = 2$

2 a



b  $x = -1$

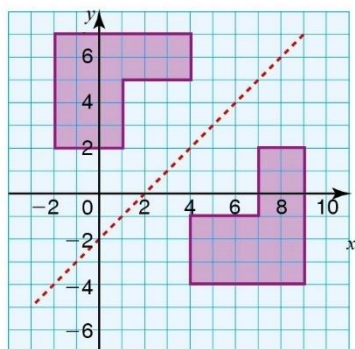
3 a



b  $y = 0, x = -3$

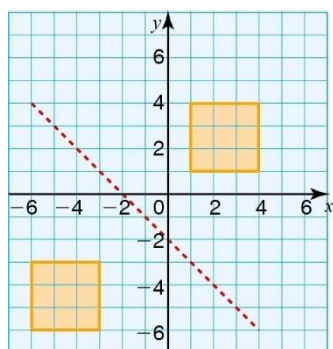


4 a



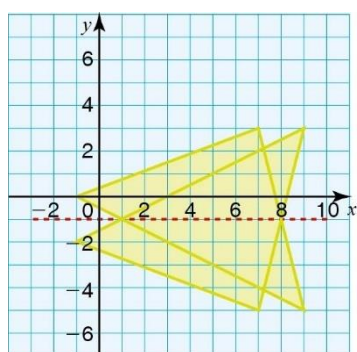
b  $y = x - 2$

5 a



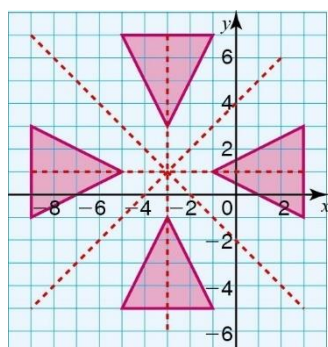
b  $y = -x - 2$

6 a



b  $y = -1$

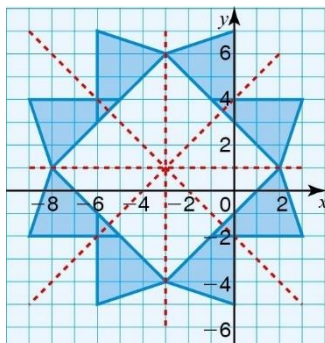
7 a





**b**  $y = 1, x = -3, y = x + 4, y = -x - 2$

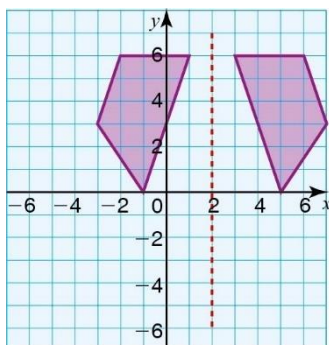
**8 a**



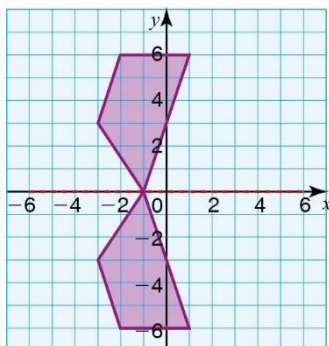
**b**  $y = 1, x = -3, y = x + 4, y = -x - 2$

## Exercise 32.2

**1 a**

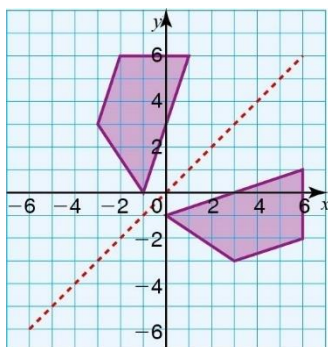


**b**

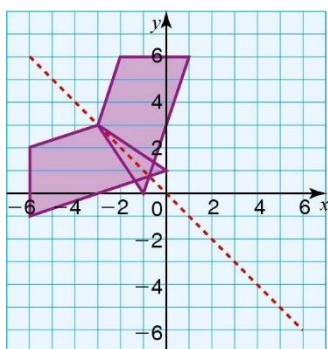




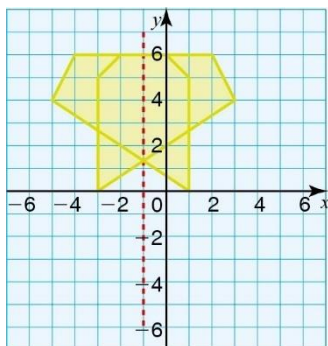
c



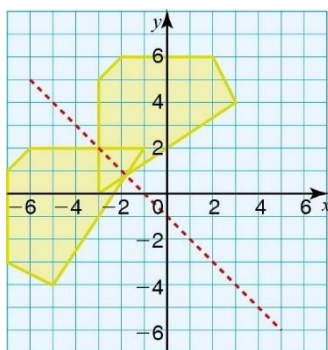
d



2 a

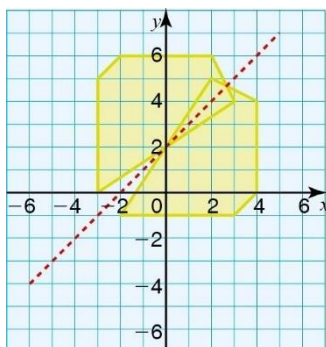


b

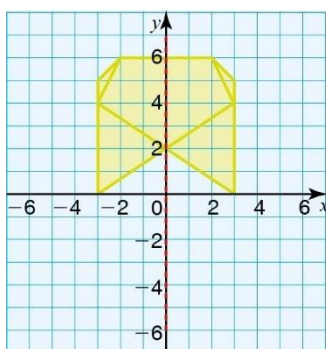




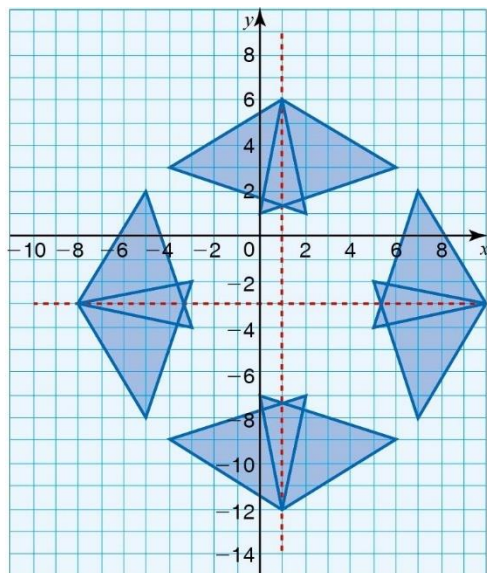
c



d



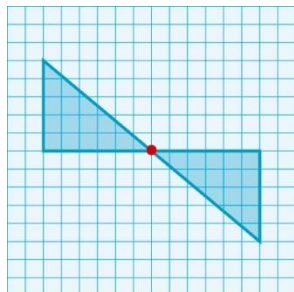
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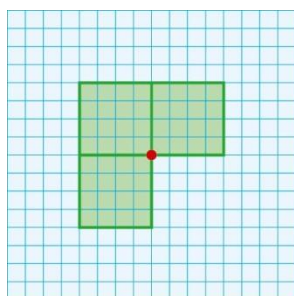


### Exercise 32.3

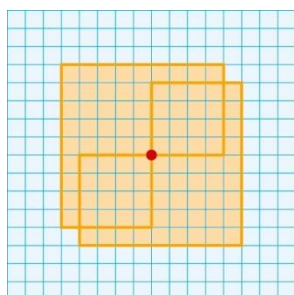
1



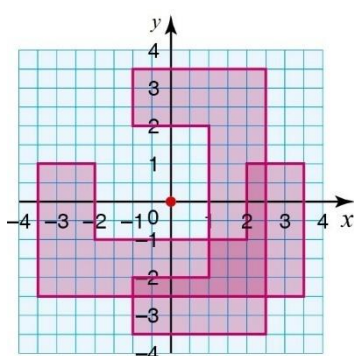
2



3

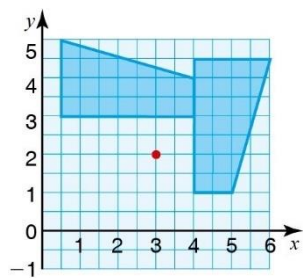


4

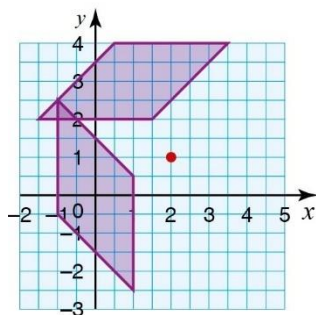




5

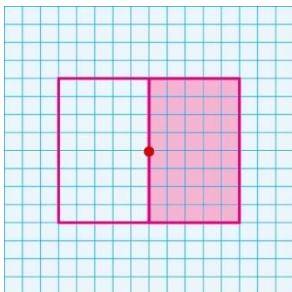


6



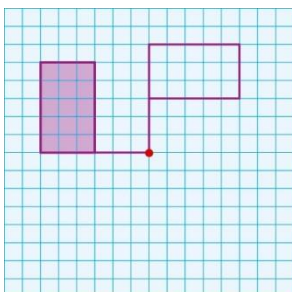
## Exercise 32.4

1 a



b  $180^\circ$  clockwise/anti-clockwise

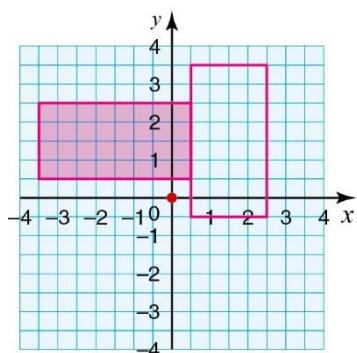
2 a



b  $90^\circ$  anti-clockwise

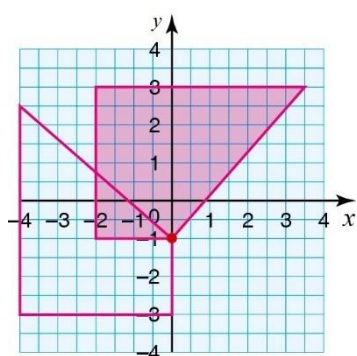


3 a



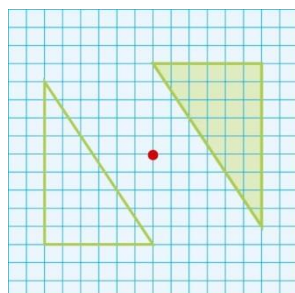
b  $90^\circ$  anti-clockwise about  $(0, 0)$

4 a



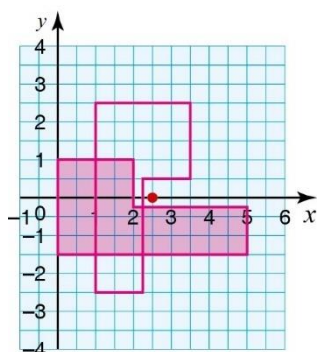
b  $90^\circ$  clockwise about  $(0, -1)$

5 a



b  $180^\circ$  clockwise/anti-clockwise

6 a



b  $90^\circ$  anti-clockwise about  $(2.5, 0)$

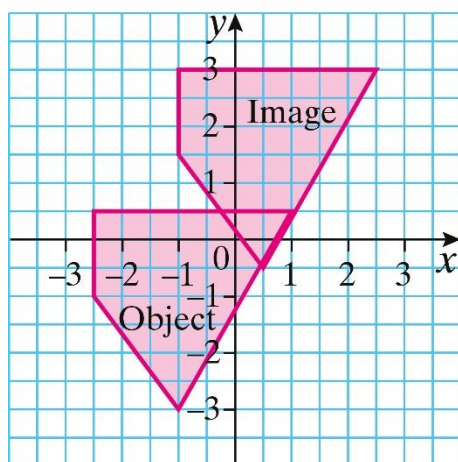


### Exercise 32.5

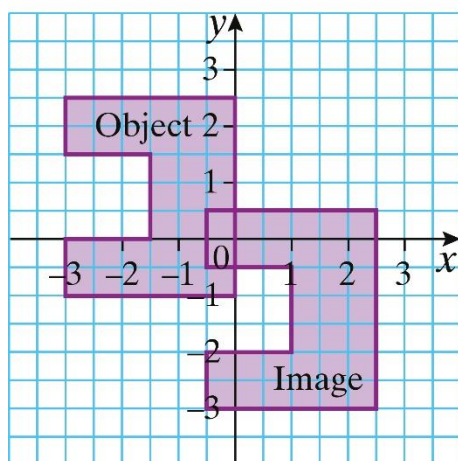
- 1  $A \rightarrow B = \begin{pmatrix} -6 \\ 0 \end{pmatrix}$      $A \rightarrow C = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$
- 2  $A \rightarrow B = \begin{pmatrix} 0 \\ -7 \end{pmatrix}$      $A \rightarrow C = \begin{pmatrix} -6 \\ 1 \end{pmatrix}$
- 3  $A \rightarrow B = \begin{pmatrix} 0 \\ 6 \end{pmatrix}$      $A \rightarrow C = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$
- 4  $A \rightarrow B = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$      $A \rightarrow C = \begin{pmatrix} -3 \\ -6 \end{pmatrix}$

### Exercise 32.6

1

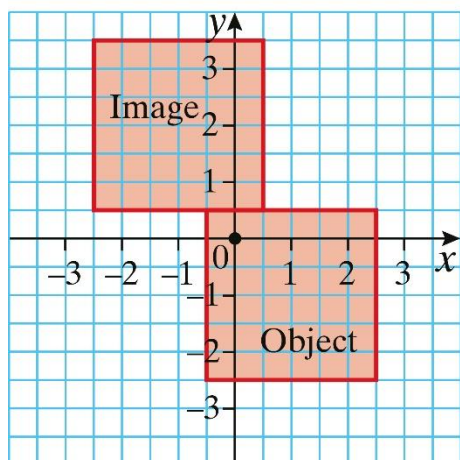


2

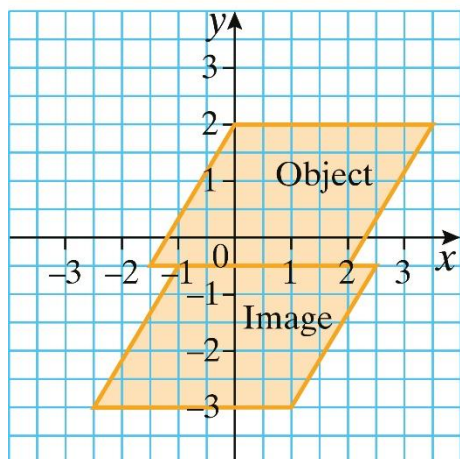




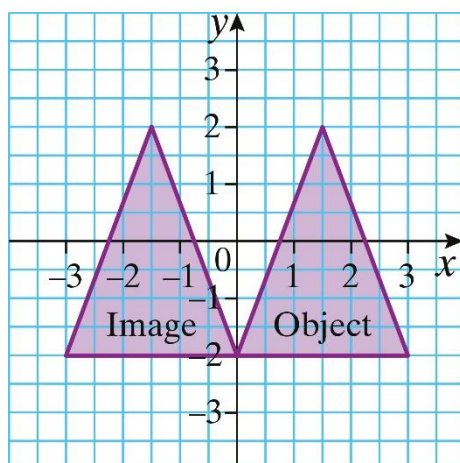
3



4

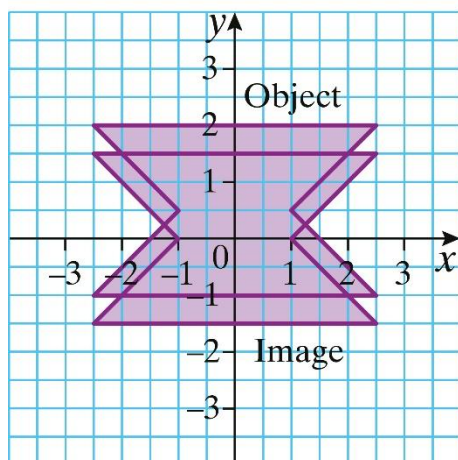


5



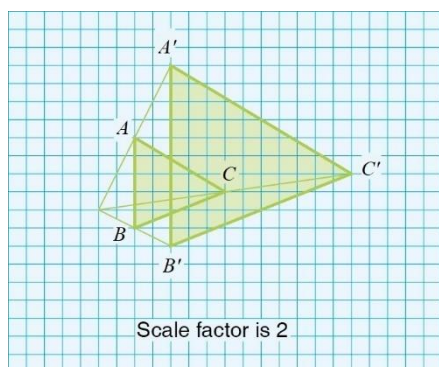


6

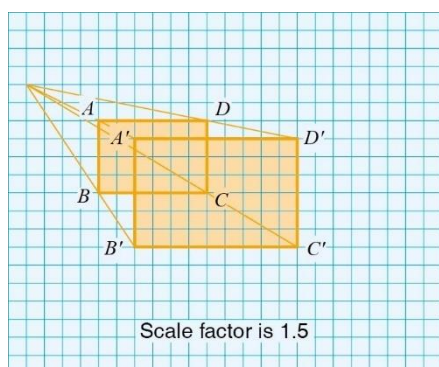


### Exercise 32.7

1

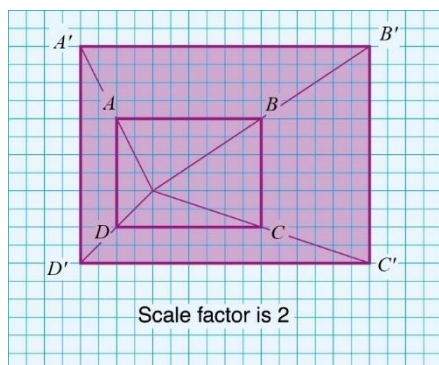


2

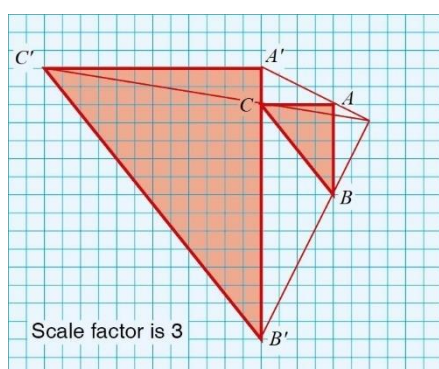




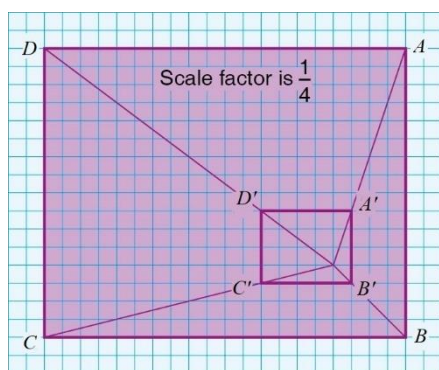
3



4



5

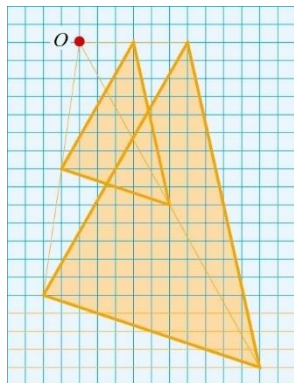




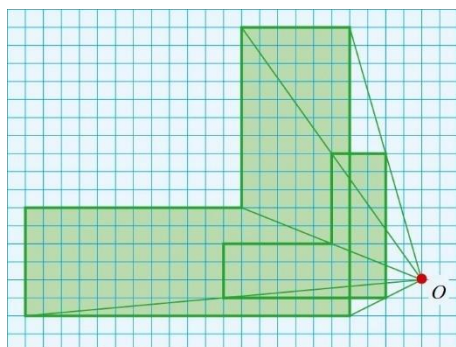
### Exercise 32.8

---

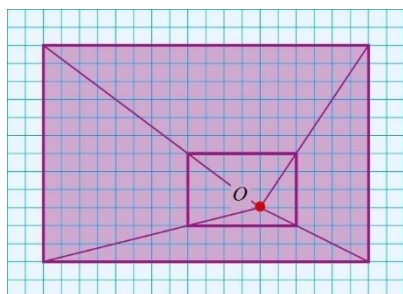
1



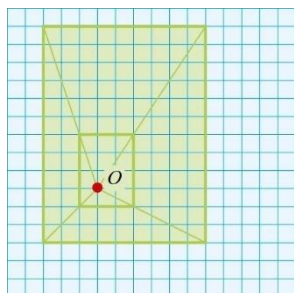
2



3



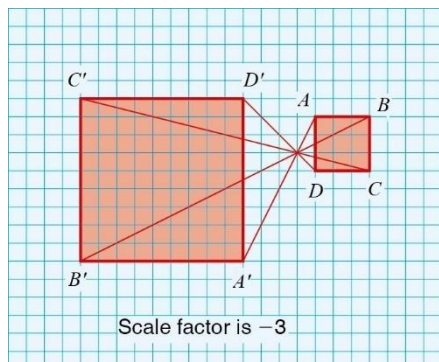
4



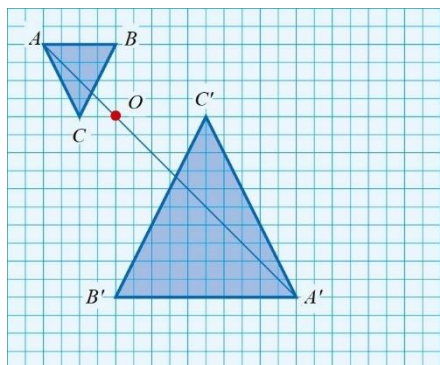


## Exercise 32.9

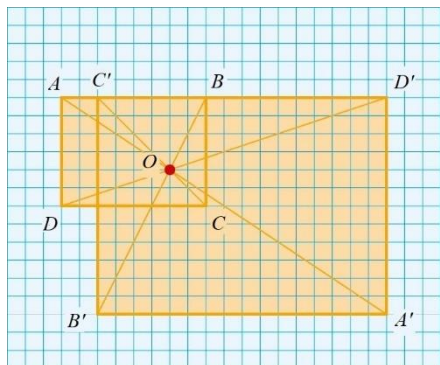
1



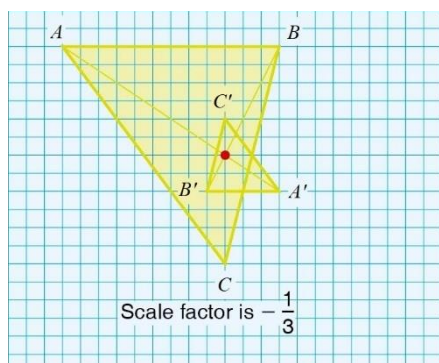
2



3

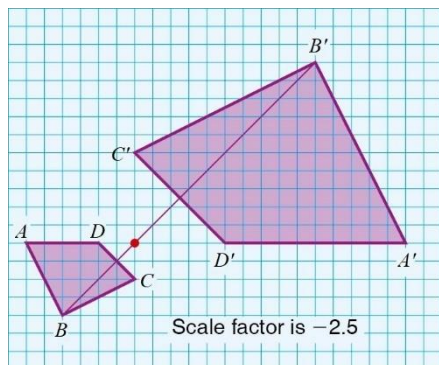


4

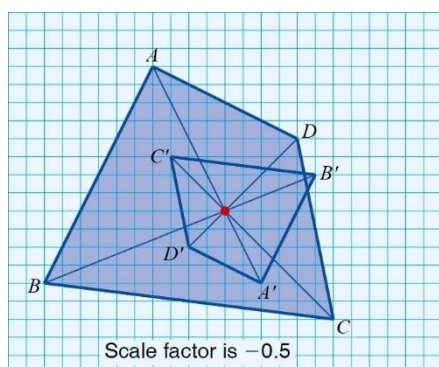




5

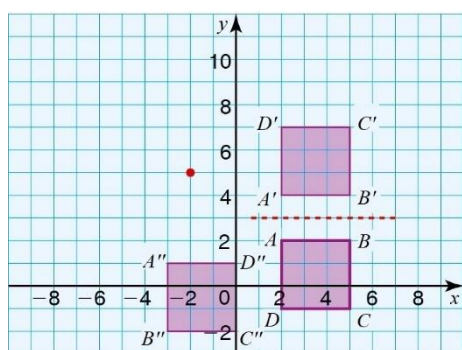


6

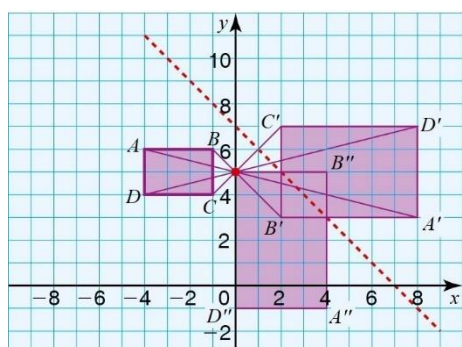


### Exercise 32.10

1



2



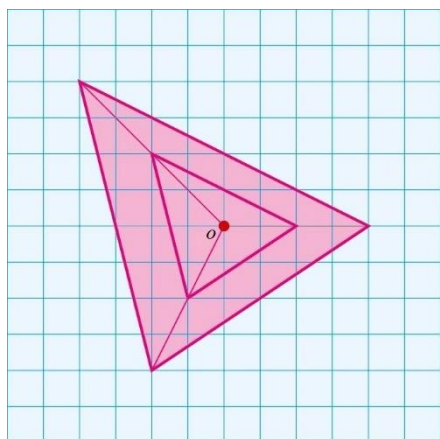


## Student assessment 1

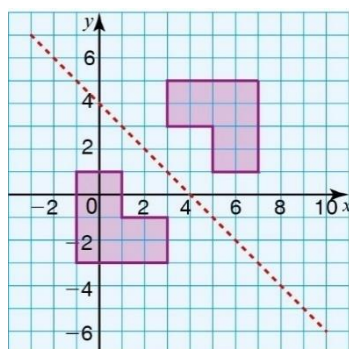
1 a  $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$

b  $\begin{pmatrix} -3 \\ -5 \end{pmatrix}$

2



3 a



$y = x - 2$  is also a mirror line

b  $y = -x + 4$

4 a  $(3, 2)$

b  $90^\circ$  clockwise

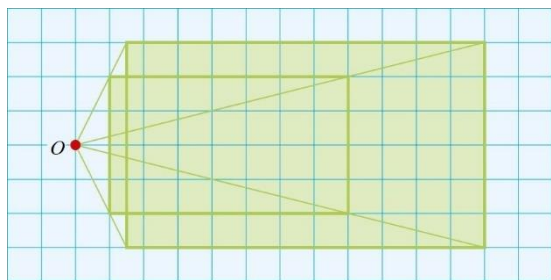
## Student assessment 2

1 a  $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$

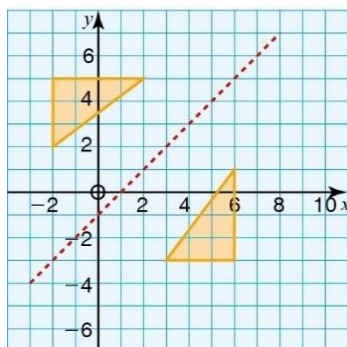
b  $\begin{pmatrix} -8 \\ 2 \end{pmatrix}$



2



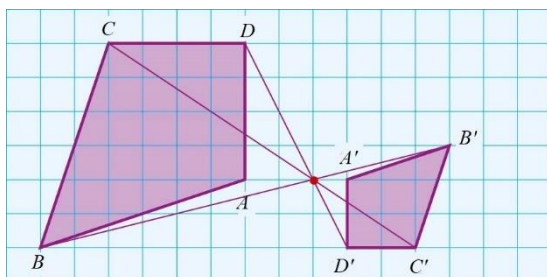
3



- 4 a An enlargement of scale factor 2. Centre of enlargement (3, 3)  
b A reflection about the line  $y = -x - 1$

### Student assessment 3

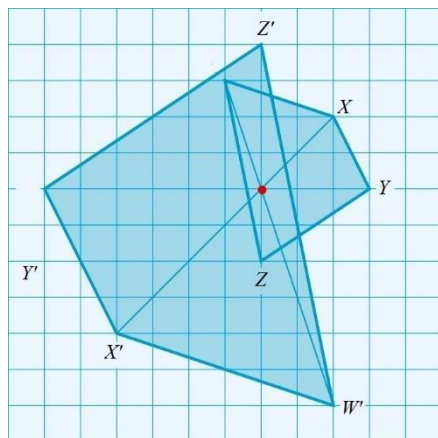
1 a



- b The scale factor of enlargement is  $-0.5$   
2 a A reflection in the line  $x = 0$   
b An enlargement by scale factor  $-0.5$ . Centre of enlargement (0,  $-1$ )



3 a



b The scale factor of enlargement is  $-2$

4 a A translation of vector  $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$

b An enlargement of scale factor 2. Centre of enlargement (6, 8)



## Mathematical investigations and ICT 7

### A painted cube

---

- 1 a 8  
b 12  
c 6  
d 1
- 2 A: 8, B: 24, C: 24, D: 8
- 3 A: 8, B: 96, C: 384, D: 512
- 4 When  $(n - 2) \geq 0$ , A: 8, B:  $12(n - 2)$ , C:  $6(n - 2)^2$ , D:  $(n - 2)^3$
- 5 When  $(l - 2) \geq 0$ ,  $(w - 2) \geq 0$  and  $(h - 2) \geq 0$ ,  
A: 8,  
B:  $4(l - 2) + 4(w - 2) + 4(h - 2)$ ,  
C:  $2(l - 2)(w - 2) + 2(l - 2)(h - 2) + 2(w - 2)(h - 2)$ ,  
D:  $(l - 2)(w - 2)(h - 2)$

### Triangle count

---

- 1 9
- 2 Student's investigation and ordered table

| Number of horizontal lines, ( $h$ ) | Number of triangles, ( $t$ ) |
|-------------------------------------|------------------------------|
| 0                                   | 3                            |
| 1                                   | 6                            |
| 2                                   | 9                            |
| 3                                   | 12                           |
| ...                                 | ...                          |

- 3  $t = 3(h + 1)$
- 4 12



## 5 Student's investigation and ordered table

| Number of horizontal lines, ( $h$ ) | Number of triangles, ( $t$ ) |
|-------------------------------------|------------------------------|
| 0                                   | 6                            |
| 1                                   | 12                           |
| 2                                   | 18                           |
| 3                                   | 24                           |
| ...                                 | ...                          |

6  $t = 6(h + 1)$

## ICT activity

---

Student's help sheet



*All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.*

## 33 Probability

### Exercise 33.1

---

- 1 Student's own drawing
- 2 Student's own answers

### Exercise 33.2

---

- 1
  - a  $\frac{1}{6}$
  - b  $\frac{5}{6}$
  - c  $\frac{1}{2}$
  - d  $\frac{5}{6}$
  - e 0
  - f 1
- 2
  - a
    - i  $\frac{1}{7}$
    - ii  $\frac{6}{7}$
  - b Total = 1; it is certain that someone is either born on a Wednesday or is not born on a Wednesday.
- 3
  - a  $\frac{1}{250}$
  - b  $\frac{1}{2}$
  - c  $\frac{151}{250}$
  - d 1
- 4
  - a  $\frac{5}{8}$
  - b  $\frac{3}{8}$
- 5
  - a  $\frac{1}{13}$
  - b  $\frac{5}{26}$
  - c  $\frac{21}{26}$
  - d  $\frac{3}{26}$
  - e Student's own answer
- 6  $\frac{1}{6}$
- 7
  - a
    - i  $\frac{1}{10}$



ii  $\frac{1}{4}$

b i  $\frac{1}{19}$

ii  $\frac{3}{19}$

8 a  $\frac{1}{37}$

b  $\frac{36}{37}$

c  $\frac{18}{37}$

d  $\frac{1}{37}$

e  $\frac{21}{37}$

f  $\frac{12}{37}$

g  $\frac{17}{37}$

h  $\frac{11}{37}$

9 a RCA RAC CRA CAR ARC ACR

b  $\frac{1}{6}$

c  $\frac{1}{3}$

d  $\frac{1}{2}$

e  $\frac{1}{24}$

10 a  $\frac{1}{4}$

b  $\frac{3}{4}$

c  $\frac{1}{13}$

d  $\frac{1}{26}$

e  $\frac{3}{13}$

f  $\frac{1}{52}$

g  $\frac{5}{13}$

h  $\frac{4}{13}$

### Exercise 33.3

---

1 a 140

b i  $\frac{1}{5}$

ii  $\frac{3}{70}$



iii  $\frac{6}{7}$

2 a 32

b i  $\frac{9}{16}$

ii  $\frac{5}{32}$

iii  $\frac{3}{8}$

iv  $\frac{5}{8}$

3 a 70

b i  $\frac{11}{35}$

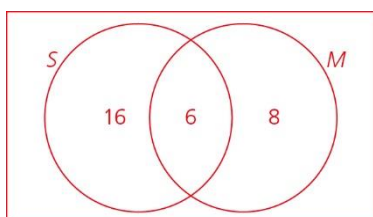
ii  $\frac{2}{7}$

iii  $\frac{5}{7}$

iv  $\frac{33}{35}$

### Exercise 33.4

1 a



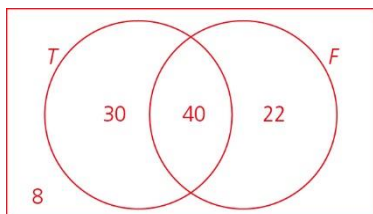
b i  $\frac{22}{30} = \frac{11}{15}$

ii  $\frac{14}{30} = \frac{7}{15}$

iii  $\frac{6}{30} = \frac{1}{5}$

iv  $\frac{30}{30} = 1$

2 a



b i  $\frac{70}{100} = \frac{7}{10}$

ii  $\frac{40}{100} = \frac{4}{10}$

iii  $\frac{38}{100} = \frac{19}{50}$



iv  $\frac{8}{100} = \frac{2}{25}$

3 a  $\frac{28}{60} = \frac{7}{15}$

b  $\frac{6}{60} = \frac{1}{10}$

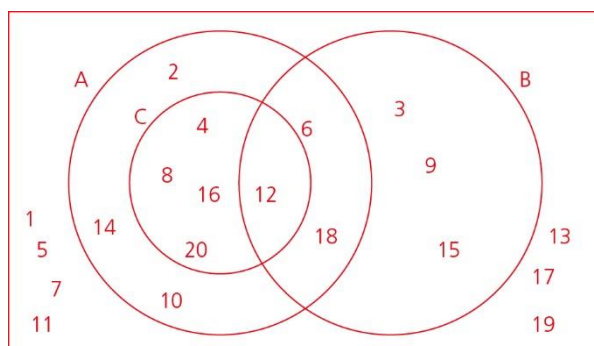
c  $\frac{46}{60} = \frac{23}{30}$

d  $\frac{2}{60} = \frac{1}{30}$

e  $\frac{37}{60}$

f  $\frac{5}{60} = \frac{1}{12}$

4 a



b i  $\frac{10}{20} = \frac{1}{2}$

ii  $\frac{3}{20}$

iii  $\frac{1}{20}$

iv  $\frac{10}{20} = \frac{1}{2}$

v  $\frac{7}{20}$

vi 1

### Exercise 33.5

1 2

2 25

3 a  $\frac{1}{16}$

b  $\frac{7}{16}$

c  $\frac{1}{2}$

d  $\frac{15}{16}$

e  $\frac{11}{16}$

4 a  $\frac{14}{45}$



**b**  $\frac{7}{45}$

**c**  $\frac{13}{30}$

**d**  $\frac{1}{15}$

**e**  $\frac{14}{15}$

**5** 35 blue, 28 red, 21 yellow, 49 green, 7 white

**6** 300

**7** 14

**8** 200

**9** 2

### Student assessment 1

---

**1 a**  $\frac{1}{6}$

**b**  $\frac{5}{6}$

**c**  $\frac{2}{3}$

**d** 0

**2 a**  $\frac{1}{4}$

**b**  $\frac{1}{13}$

**c**  $\frac{1}{2}$

**d**  $\frac{3}{13}$

**e**  $\frac{4}{13}$

**3 a**  $\frac{1}{250}$

**b**  $\frac{1}{50}$

**c**  $\frac{2}{25}$

**d**  $\frac{3}{10}$

**e** 1

**4 a**  $\frac{11}{40}$

**b**  $\frac{1}{8}$

**c**  $\frac{1}{4}$

**d**  $\frac{4}{5}$

**5** 160 red, 96 blue, 64 green



- 6 a  $\frac{41}{150}$   
b  $\frac{29}{150}$   
c  $\frac{1}{25}$   
d  $\frac{4}{15}$   
e  $\frac{11}{15}$   
f  $\frac{137}{150}$   
g  $\frac{1}{30}$
- 7 a Jas's results are likely to be more reliable as she repeated the experiment more times.  
b It is likely that the spinner is biased towards the number 3.
- 8 a  $\frac{49}{100}$   
b  $\frac{23}{100}$   
c  $\frac{8}{100} = \frac{2}{25}$   
d  $\frac{9}{100}$   
e  $\frac{76}{100} = \frac{19}{25}$

## Student assessment 2

---

- 1 a  $\frac{1}{8}$   
b  $\frac{7}{8}$   
c  $\frac{5}{8}$   
d 0
- 2 a 32 cards in pack  
b i  $\frac{1}{8}$   
ii  $\frac{3}{8}$   
iii  $\frac{1}{4}$   
iv  $\frac{1}{8}$   
v  $\frac{11}{32}$
- 3 a  $\frac{1}{180}$   
b  $\frac{1}{20}$   
c  $\frac{1}{12}$



d  $\frac{2}{9}$

e 1

4 a  $\frac{9}{32}$

b  $\frac{3}{8}$

c  $\frac{21}{32}$

5 128 red, 80 blue, 112 green

6 a  $\frac{3}{20}$

b  $\frac{19}{20}$

c  $\frac{7}{10}$

7 a 50 000

b You have to assume that all entrants have an equal chance of winning.

8 a 180

b 300



All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

## 34 Further probability

### Exercise 34.1

1 a

|        |   | Dice 1 |      |      |      |
|--------|---|--------|------|------|------|
|        |   | 1      | 2    | 3    | 4    |
| Dice 2 | 1 | 1, 1   | 2, 1 | 3, 1 | 4, 1 |
|        | 2 | 1, 2   | 2, 2 | 3, 2 | 4, 2 |
|        | 3 | 1, 3   | 2, 3 | 3, 3 | 4, 3 |
|        | 4 | 1, 4   | 2, 4 | 3, 4 | 4, 4 |

b  $\frac{1}{4}$

c  $\frac{1}{4}$

d  $\frac{9}{16}$

2

| Die 2 | 6 | 1, 6  | 2, 6 | 3, 6 | 4, 6 | 5, 6 | 6, 6 |
|-------|---|-------|------|------|------|------|------|
|       | 5 | 1, 5  | 2, 5 | 3, 5 | 4, 5 | 5, 5 | 6, 5 |
|       | 4 | 1, 4  | 2, 4 | 3, 4 | 4, 4 | 5, 4 | 6, 4 |
|       | 3 | 1, 3  | 2, 3 | 3, 3 | 4, 3 | 5, 3 | 6, 3 |
|       | 2 | 1, 2  | 2, 2 | 3, 2 | 4, 2 | 5, 2 | 6, 2 |
|       | 1 | 1, 1  | 2, 1 | 3, 1 | 4, 1 | 5, 1 | 6, 1 |
|       |   | 1     | 2    | 3    | 4    | 5    | 6    |
|       |   | Die 1 |      |      |      |      |      |

a  $\frac{1}{36}$

b  $\frac{1}{6}$

c  $\frac{1}{18}$

d  $\frac{1}{6}$

e  $\frac{1}{4}$

f  $\frac{3}{4}$

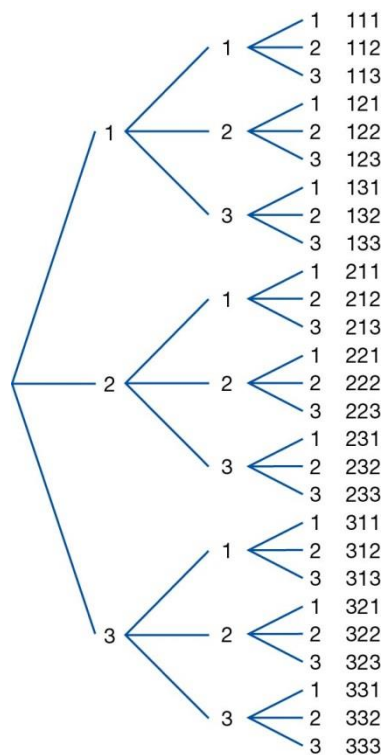
g  $\frac{5}{18}$



- h  $\frac{1}{6}$   
i  $\frac{11}{18}$

## Exercise 34.2

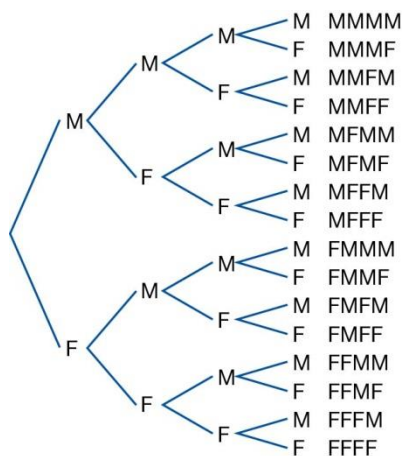
1 a



- b i  $\frac{1}{27}$   
ii  $\frac{1}{3}$   
iii  $\frac{1}{9}$   
iv  $\frac{1}{3}$   
v  $\frac{5}{9}$



2 a



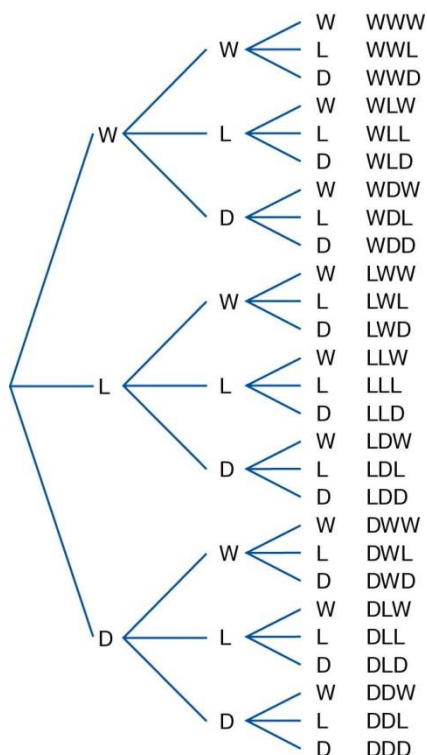
b i  $\frac{1}{16}$

ii  $\frac{3}{8}$

iii  $\frac{15}{16}$

iv  $\frac{5}{16}$

3 a



b i  $\frac{1}{27}$

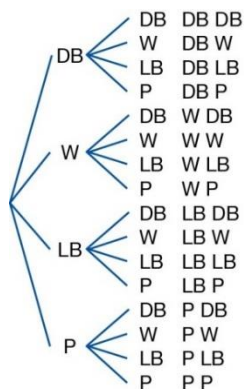
ii  $\frac{10}{27}$

iii  $\frac{19}{27}$



iv  $\frac{8}{27}$

4 a



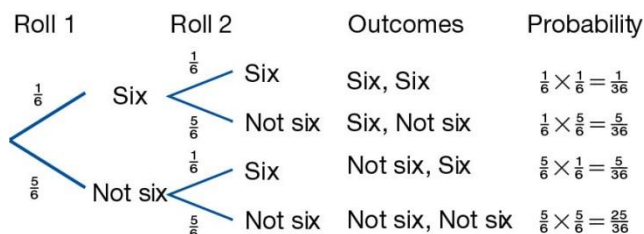
b i  $\frac{1}{16}$

ii  $\frac{1}{4}$

iii  $\frac{1}{8}$

### Exercise 34.3

1 a



b i  $\frac{1}{6}$

ii  $\frac{11}{36}$

iii  $\frac{5}{36}$

iv  $\frac{125}{216}$

v  $\frac{91}{216}$

c Add up to 1; you are certain to either start the game within 3 rolls or to not start the game within 3 rolls

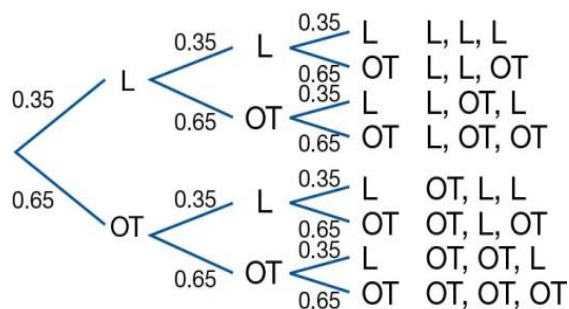
2 a  $\frac{4}{25}$

b  $\frac{54}{125}$

c  $\frac{98}{125}$



3 a



b i 0.275 (3 s.f.)

ii 0.123 (3 s.f.)

iii 0.444 (3 s.f.)

iv 0.718 (3 s.f.)

4 a 0.0588 (3 s.f.)

b 0.0129 (3 s.f.)

c 0.414 (3 s.f.)

d 0.586 (3 s.f.)

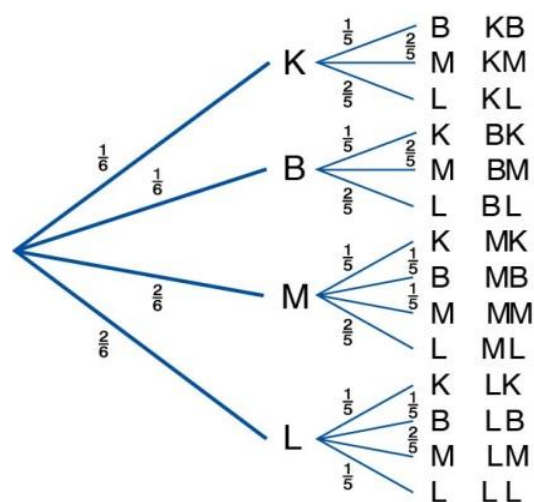
5 0.027

6 a 0.563 (3 s.f.)

b 0.422 (3 s.f.)

c 0.0563 (3 s.f.)

7 a



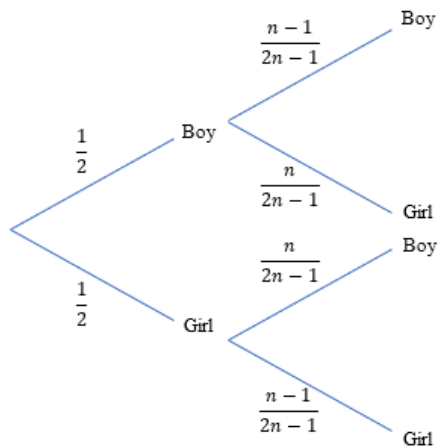
b i  $\frac{1}{15}$

ii  $\frac{1}{15}$

iii  $\frac{3}{5}$

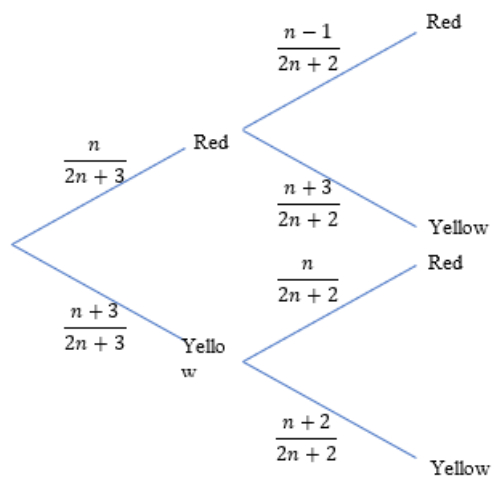


8 a



b  $P(G, G) = \frac{1}{2} \times \frac{n-1}{2n-1} = \frac{n-1}{2(2n-1)}$

9 a



b  $\frac{(n+3)(n+2)}{(2n+3)(2n+2)}$

### Exercise 34.4

1 a  $\frac{14}{24} = \frac{7}{12}$

b  $\frac{5}{24}$

c  $\frac{3}{14}$

d 0

2 a  $\frac{29}{60}$

b  $\frac{48}{60} = \frac{4}{5}$



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c  $\frac{15}{60} = \frac{1}{4}$

d  $\frac{15}{29}$

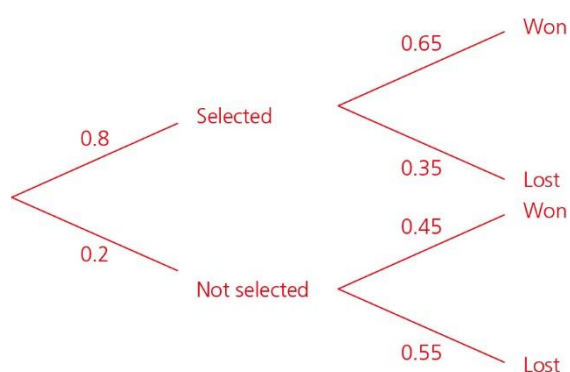
e  $\frac{15}{42} = \frac{5}{14}$

f  $\frac{3}{42} = \frac{1}{14}$

g  $\frac{5}{31}$

h  $\frac{38}{42} = \frac{19}{21}$

3 a

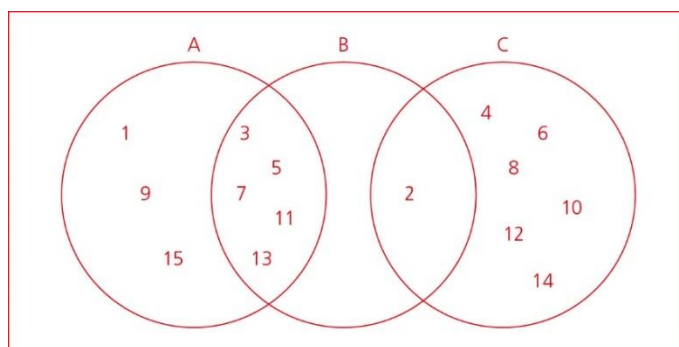


b 61

c 0.65

d 0.85 (2 d.p.)

4 a



b i  $\frac{6}{15} = \frac{2}{5}$

ii  $\frac{1}{15}$

iii 0

iv  $\frac{5}{6}$

v  $\frac{1}{7}$

vi  $\frac{1}{6}$

vii 1

c Set A and set C do not overlap as an odd number cannot be a multiple of two.



## Student assessment 1

1 a

|        |   | Dice 1 |   |   |    |    |    |
|--------|---|--------|---|---|----|----|----|
|        |   | 1      | 2 | 3 | 4  | 5  | 6  |
| Dice 2 | 1 | 2      | 3 | 4 | 5  | 6  | 7  |
|        | 2 | 3      | 4 | 5 | 6  | 7  | 8  |
|        | 3 | 4      | 5 | 6 | 7  | 8  | 9  |
|        | 4 | 5      | 6 | 7 | 8  | 9  | 10 |
|        | 5 | 6      | 7 | 8 | 9  | 10 | 11 |
|        | 6 | 7      | 8 | 9 | 10 | 11 | 12 |

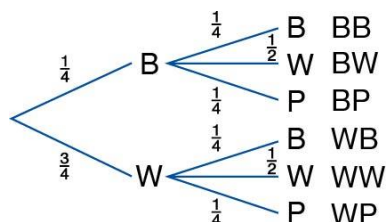
- b i  $\frac{1}{36}$   
 ii  $\frac{1}{6}$   
 iii  $\frac{1}{12}$   
 iv  $\frac{7}{12}$

c 25

- 2 a i  $\frac{1}{4}$   
 ii  $\frac{3}{8}$

- b i  $\frac{3}{64}$   
 ii  $\frac{3}{32}$   
 iii  $\frac{7}{16}$   
 iv  $\frac{39}{64}$

3 a



- b i  $\frac{1}{16}$   
 ii  $\frac{3}{8}$   
 iii  $\frac{3}{16}$   
 iv  $\frac{7}{8}$

4 a  $\frac{1}{12}$



# Cambridge IGCSE™ Mathematics Core and Extended Answers to Student's Book

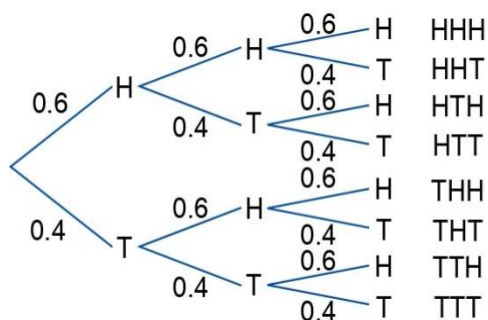
**b**  $\frac{1}{6}$

**c**  $\frac{1}{6}$

**5 a** 0.72

**b** 0.729

**6 a**



**b i** 0.216

**ii** 0.064

**iii** 0.648

**7 a i** 0.8

**ii** 0.7

**iii** 0.3

**b i** 0.06

**ii** 0.56

**iii** 0.38

**c** 0.3

**d** 0.631 (3 s.f.)

**8 a i** 0.06

**ii** 0.39

**iii** 0.2

**b i** 0.590 (3 s.f.)

**ii** 0.393 (3 s.f.)

**iii** 0.0164 ((3 s.f.))

**9 a** 0.68

**b** 0.26

**c** 0.265 (3 s.f.)

**d** 0.647 (3 s.f.)



## Mathematical investigations and ICT 8

### Probability drop

---

- 1 Tray 1: LLL  
Tray 2: LLRLRL RLL  
Tray 3: LRR RLR RRL  
Tray 4: RRR
- 2 Tray 1: LLLL  
Tray 2: LLLR LLRL LRLL RLLL  
Tray 3: LLRR LRLR LRRL RLLR RLRL RRLL  
Tray 4: RRRL RRLR RLRR LRRR  
Tray 5: RRRR
- 3  $\frac{1}{16}$  because there are 16 possible routes and only one results in the marble landing in Tray 1.
- 4 Tray 2:  $\frac{4}{16} = \frac{1}{4}$   
Tray 3:  $\frac{6}{16} = \frac{3}{8}$   
Tray 4:  $\frac{4}{16} = \frac{1}{4}$   
Tray 5:  $\frac{1}{16}$
- 5 Student's investigation
- 6  $\frac{210}{1024} = \frac{105}{512}$
- 7 Each number in each row of Pascal's triangle corresponds to the number of routes to landing in each tray of the game.
- 8 The binomial expansion generates the numbers in Pascal's triangle and therefore the number of routes to landing in each tray of the game.



## Dice sum

1

|        |   | Dice 1 |   |   |    |    |    |
|--------|---|--------|---|---|----|----|----|
|        |   | 1      | 2 | 3 | 4  | 5  | 6  |
| Dice 2 | 1 | 2      | 3 | 4 | 5  | 6  | 7  |
|        | 2 | 3      | 4 | 5 | 6  | 7  | 8  |
|        | 3 | 4      | 5 | 6 | 7  | 8  | 9  |
|        | 4 | 5      | 6 | 7 | 8  | 9  | 10 |
|        | 5 | 6      | 7 | 8 | 9  | 10 | 11 |
|        | 6 | 7      | 8 | 9 | 10 | 11 | 12 |

2 36

3 7

4  $\frac{3}{36} = \frac{1}{12}$

5  $\frac{6}{36} = \frac{1}{6}$

6 You are four times more likely to get a 5 than a 2.

7

|        |   | Dice 1 |   |   |   |
|--------|---|--------|---|---|---|
|        |   | 1      | 2 | 3 | 4 |
| Dice 2 | 1 | 2      | 3 | 4 | 5 |
|        | 2 | 3      | 4 | 5 | 6 |
|        | 3 | 4      | 5 | 6 | 7 |
|        | 4 | 5      | 6 | 7 | 8 |

8 16

9 5

10  $\frac{4}{16} = \frac{1}{4}$

11 Student's investigation

12 a  $m^2$

b  $m + 1$

c  $\frac{m}{m^2} = \frac{1}{m}$

13 a  $m \times n$

b The total can take any integer value in the range  $(n + 1)$  to  $(m + 1)$ .

c  $\frac{n}{nm} = \frac{1}{m}$



### ICT activity: Buffon's needle experiment

---

**1–9** Student's experiment and results entered in a spreadsheet

**10** The value of  $\frac{2}{p}$  should tend to  $\pi$ .



*All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.*

## 35 Mean, median, mode and range

### Exercise 35.1

---

- 1 Mean = 1.67 (3 s.f.)  
Median = 1  
Mode = 1  
Range = 5
- 2 Mean = 6.2  
Median = 6.5  
Mode = 7  
Range = 9
- 3 Mean = 14 yrs 3 mths  
Median = 14 yrs 3 mths  
Mode = 14 yrs 3 mths  
Range = 8 mths
- 4 Mean = 26.4  
Median = 27  
Mode = 28  
Range = 5
- 5 Mean = 13.9 s (3 s.f.)  
Median = 13.9 s  
Mode = 13.8 s  
Range = 0.6 s
- 6 91.1 kg
- 7 103 points

### Exercise 35.2

---

- 1 Mean = 3.35  
Median = 3  
Mode = 1 and 4  
Range = 5
- 2 Mean = 7.03
- 3 a Mean = 6.33 (3 s.f.)  
Median = 7



Mode = 8

Range = 5

- b** The mode, as it gives the highest number of flowers per bush

## Exercise 35.3

|            |                    |                  |                           |                                       |
|------------|--------------------|------------------|---------------------------|---------------------------------------|
| <b>1 a</b> | <b>Height (m)</b>  | <b>Frequency</b> | <b>Mid-interval value</b> | <b>Frequency × mid-interval value</b> |
|            | $1.8 < H \leq 1.9$ | 2                | 1.85                      | 3.7                                   |
|            | $1.9 < H \leq 2.0$ | 5                | 1.95                      | 9.75                                  |
|            | $2.0 < H \leq 2.1$ | 10               | 2.05                      | 20.5                                  |
|            | $2.1 < H \leq 2.2$ | 22               | 2.15                      | 47.3                                  |
|            | $2.2 < H \leq 2.3$ | 7                | 2.25                      | 15.75                                 |
|            | $2.3 < H \leq 2.4$ | 4                | 2.35                      | 9.4                                   |

**b** Mean = 2.13 m (3 s.f.)

**c** Modal class =  $2.1 < H \leq 2.2$

**2 a** Mean = 33 h (2 s.f.)

**b** Modal class = 30–39 hours

**3 a** Mean = 6.2 cm (1 d.p.)

**b** Modal class =  $6.0 < L \leq 6.5$

## Student assessment 1

**1** Mean = 86.8 m

Median = 90.5 m

Mode = 93 m

Range = 18 m

**2 a** 26

**b i** Mean = 7.73 (3 s.f.)

**ii** Median = 7.5

**iii** Mode = 10

**iv** Range = 6

**c** Both teachers are correct. Each is using a different type of average. The class teacher stating an 'average' of 10/10 is using the modal value, whilst the other teacher is using the median value as the average.

**3** 6, 12, 12, 12, 18

**4 a** Mean = 10.0 kg (3 s.f.)

**b** Modal class is  $10.0 \leq M < 10.1$



All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

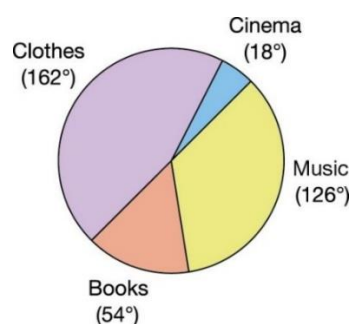
## 36 Collecting, displaying and interpreting data

### Exercise 36.1

1

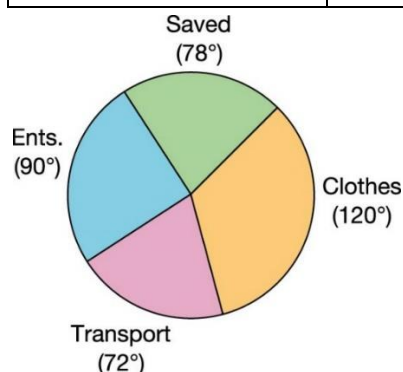
|              | Sleep  | Meals | Sport  | TV  | School |
|--------------|--------|-------|--------|-----|--------|
| <b>Ayse</b>  | 8 h 20 | 2 h   | 5 h    | 2 h | 6 h 40 |
| <b>Ahmet</b> | 8 h 40 | 2 h   | 5 h 20 | 2 h | 6 h    |

2



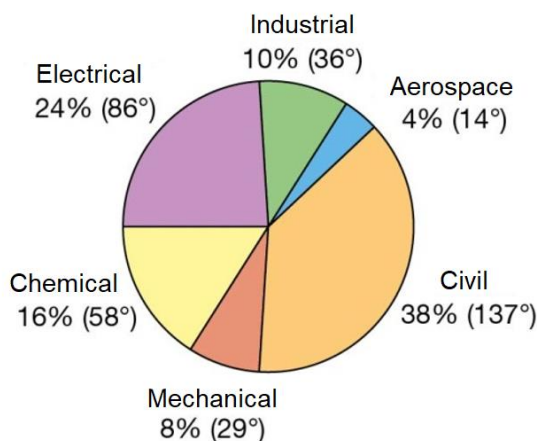
3

|                               | Fraction        | \$  | Degrees |
|-------------------------------|-----------------|-----|---------|
| <b>Clothes</b>                | $\frac{1}{3}$   | 800 | 120     |
| <b>Transport</b>              | $\frac{1}{5}$   | 480 | 72      |
| <b>Entertainments (Ents.)</b> | $\frac{1}{4}$   | 600 | 90      |
| <b>Saved</b>                  | $\frac{13}{60}$ | 520 | 78      |





University A



- 5 a

### Club 1

[illegible]

**Key:** 2 | 1 | 4 represents 12 years (club 1) and 14 years (club 2)

- b**
- i** Club 1 range = 63      Club 2 range = 67
  - ii** Club 1 median = 22      Club 2 median = 57
- c** Club 1 is likely to be the athletics club as although it has a similar range of ages to Club 2, the median is considerably lower.

- 6 a 14**

- b 3**

- c** Student's own answer

- d** Student's own bar chart

- 7 a** Fishing boat 2 as it caught 651 kg of fish type A, compared to Boat 1's 304 kg.

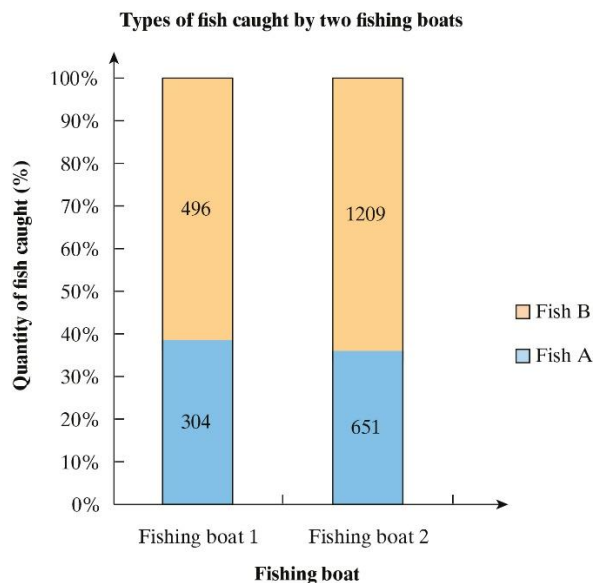
- b** Fishing boat 1, percentage of fish type A =  $\frac{304}{800} = 38\%$

Fishing boat 2, percentage of fish type A =  $\frac{651}{1860} = 35\%$

Therefore boat 1 had a higher percentage of fish type A in its catch.



c



## Exercise 36.2

- 1 Student's own survey results and pie charts
- 2 Student's report

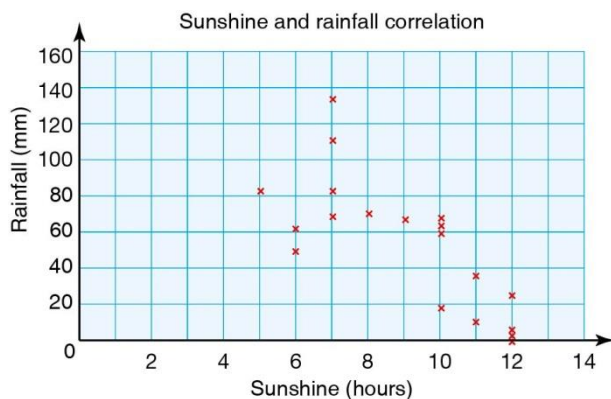
## Exercise 36.3

- 1 Student's answers may differ from those given below.
  - a Possible positive correlation (strength depending on topics tested)
  - b No correlation
  - c Positive correlation (likely to be quite strong)
  - d Negative correlation (likely to be strong). Assume that motorcycles are not rare/vintage.
  - e Factors such as social class, religion and income are likely to affect results. Therefore little correlation is likely.
  - f Negative correlation (likely to be strong)
  - g 0–16 years likely to be a positive correlation
  - h Strong positive correlation



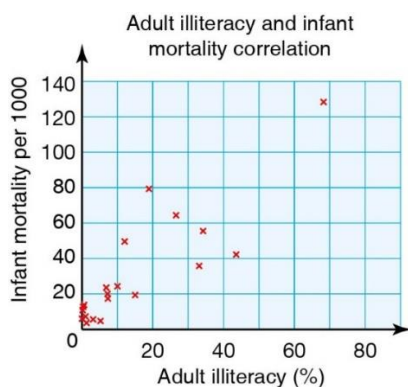
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2 a



b Graph shows a very weak negative correlation. The more sunshine there is, the less rainfall.

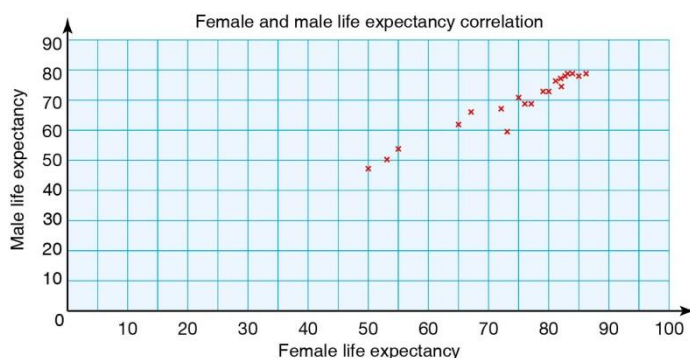
3 a



b Positive correlation

c Student's answer. However, although there is a correlation, it doesn't imply that one variable affects the other

d



4 a Moderate/strong positive correlation

b Approx. 31 tomatoes

c Approx. 60 cm



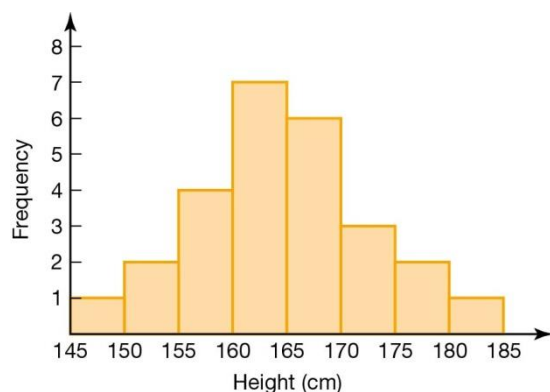
- 5 a Moderate positive correlation
- b There is no evidence that the Gold medal winners are part of the over-weight population. A wealthy country is more likely to have an over-weight population but also spend more on sport development. Winning gold medals (the effect) is not likely to be a result of being over-weight (the cause). Just because there is a positive correlation between two variables, does not mean that one variable causes the result of the other.
- c Student's analysis

## Exercise 36.4

1



2



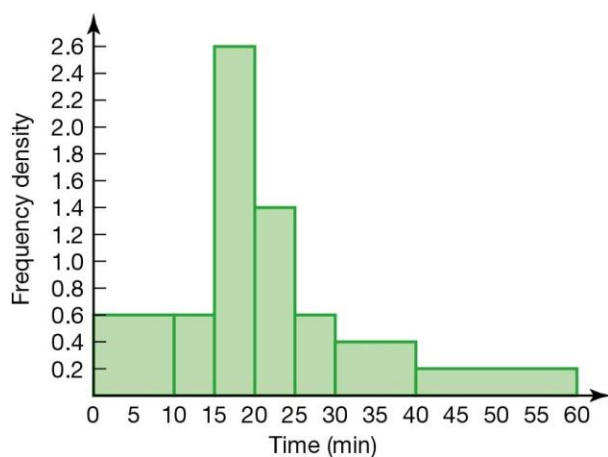


### Exercise 36.5

1 a

| Time (min)       | Frequency | Frequency density |
|------------------|-----------|-------------------|
| $0 \leq t < 10$  | 6         | 0.6               |
| $10 \leq t < 15$ | 3         | 0.6               |
| $15 \leq t < 20$ | 13        | 2.6               |
| $20 \leq t < 25$ | 7         | 1.4               |
| $25 \leq t < 30$ | 3         | 0.6               |
| $30 \leq t < 40$ | 4         | 0.4               |
| $40 \leq t < 60$ | 4         | 0.2               |

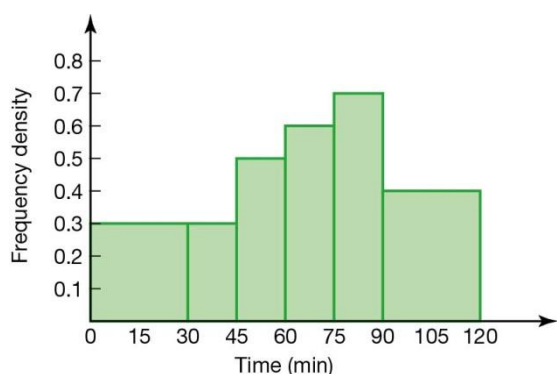
b



2 a

| Time (min)        | Frequency | Frequency density |
|-------------------|-----------|-------------------|
| $0 \leq t < 30$   | 8         | 0.3               |
| $30 \leq t < 45$  | 5         | 0.3               |
| $45 \leq t < 60$  | 8         | 0.5               |
| $60 \leq t < 75$  | 9         | 0.6               |
| $75 \leq t < 90$  | 10        | 0.7               |
| $90 \leq t < 120$ | 12        | 0.4               |

b



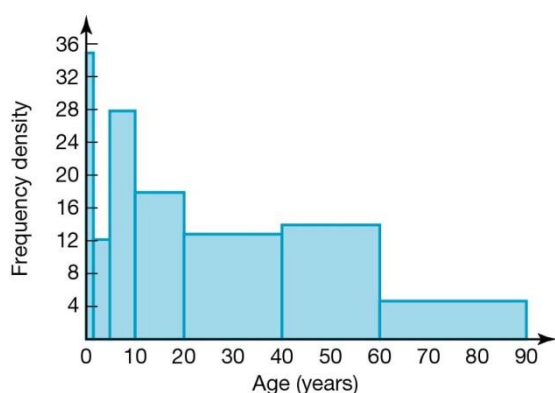


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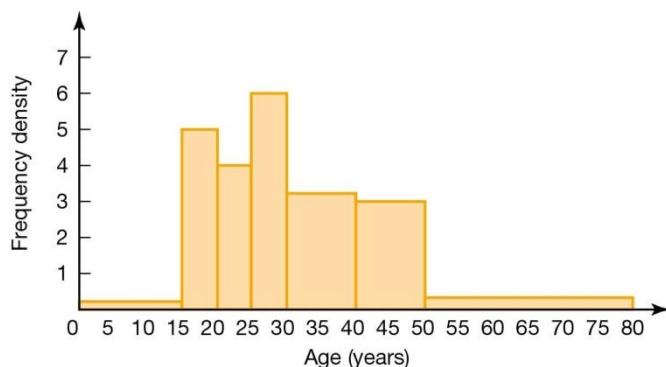
3 a

| Age (years)      | Frequency | Frequency density |
|------------------|-----------|-------------------|
| $0 \leq y < 1$   | 35        | 35                |
| $1 \leq y < 5$   | 48        | 12                |
| $5 \leq y < 10$  | 140       | 28                |
| $10 \leq y < 20$ | 180       | 18                |
| $20 \leq y < 40$ | 260       | 13                |
| $40 \leq y < 60$ | 280       | 14                |
| $60 \leq y < 90$ | 150       | 5                 |

b



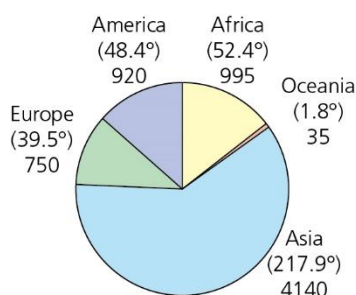
4 a



b Student's own answers

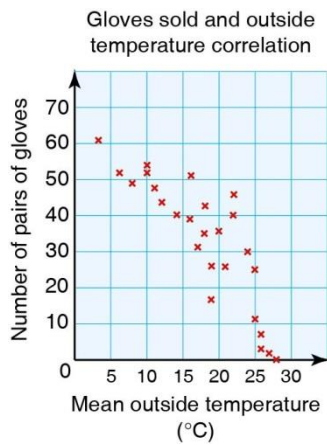
## Student assessment 1

1





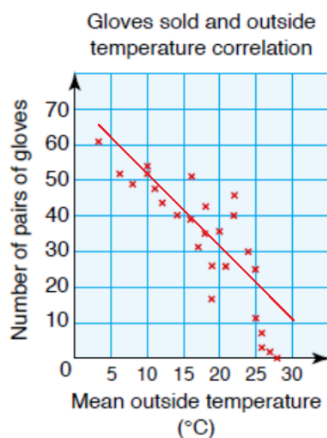
2 a



b Negative correlation

c Student's own answer

d



Approx. 30 pairs

3 a

|   |   |   |   |   |   |   |   |   |   |   |   |  |
|---|---|---|---|---|---|---|---|---|---|---|---|--|
| 0 | 9 |   |   |   |   |   |   |   |   |   |   |  |
| 1 | 1 | 1 | 1 | 1 | 4 | 6 | 6 | 7 | 8 |   |   |  |
| 2 | 1 | 2 | 2 | 3 | 4 | 6 | 7 | 7 | 8 | 8 | 8 |  |
| 3 | 0 | 3 | 3 | 3 | 4 | 6 | 6 | 7 | 8 | 8 | 9 |  |

Key: 2 | 1 represents 21 marks

b Average can refer to either the mean, median or mode.

Mean result = 24.9 therefore 15 students would have a re-test

Median result = 26.5 therefore 16 students would have a re-test

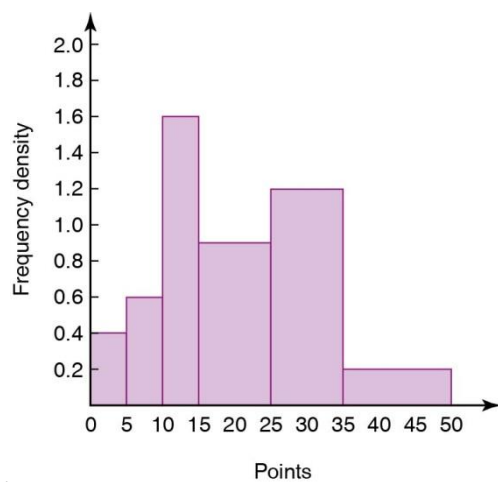
Modal result = 11, therefore only 1 student would have a re-test



4 a

| Points           | Number of games | Frequency density |
|------------------|-----------------|-------------------|
| $0 \leq p < 5$   | 2               | 0.4               |
| $5 \leq p < 10$  | 3               | 0.6               |
| $10 \leq p < 15$ | 8               | 1.6               |
| $15 \leq p < 25$ | 9               | 0.9               |
| $25 \leq p < 35$ | 12              | 1.2               |
| $35 \leq p < 50$ | 3               | 0.2               |

b





All non-exact answers are rounded to 3 significant figures and non-exact angles are rounded to 1 decimal place unless otherwise stated.

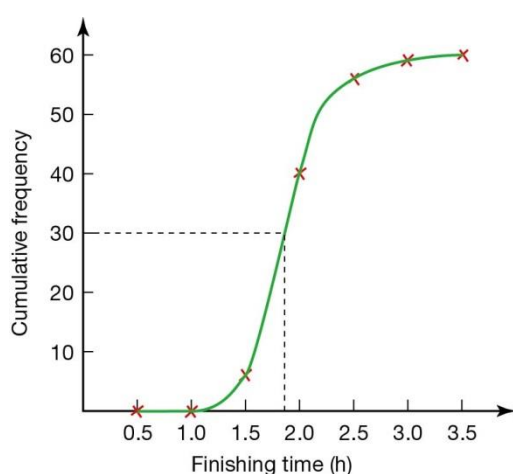
## 37 Cumulative frequency

### Exercise 37.1

1 a

| Finishing time (h) | Frequency | Cumulative freq. |
|--------------------|-----------|------------------|
| $0 \leq h < 0.5$   | 0         | 0                |
| $0.5 \leq h < 1.0$ | 0         | 0                |
| $1.0 \leq h < 1.5$ | 6         | 6                |
| $1.5 \leq h < 2.0$ | 34        | 40               |
| $2.0 \leq h < 2.5$ | 16        | 56               |
| $2.5 \leq h < 3.0$ | 3         | 59               |
| $3.0 \leq h < 3.5$ | 1         | 60               |

b



c Median  $\approx 1.8$  h

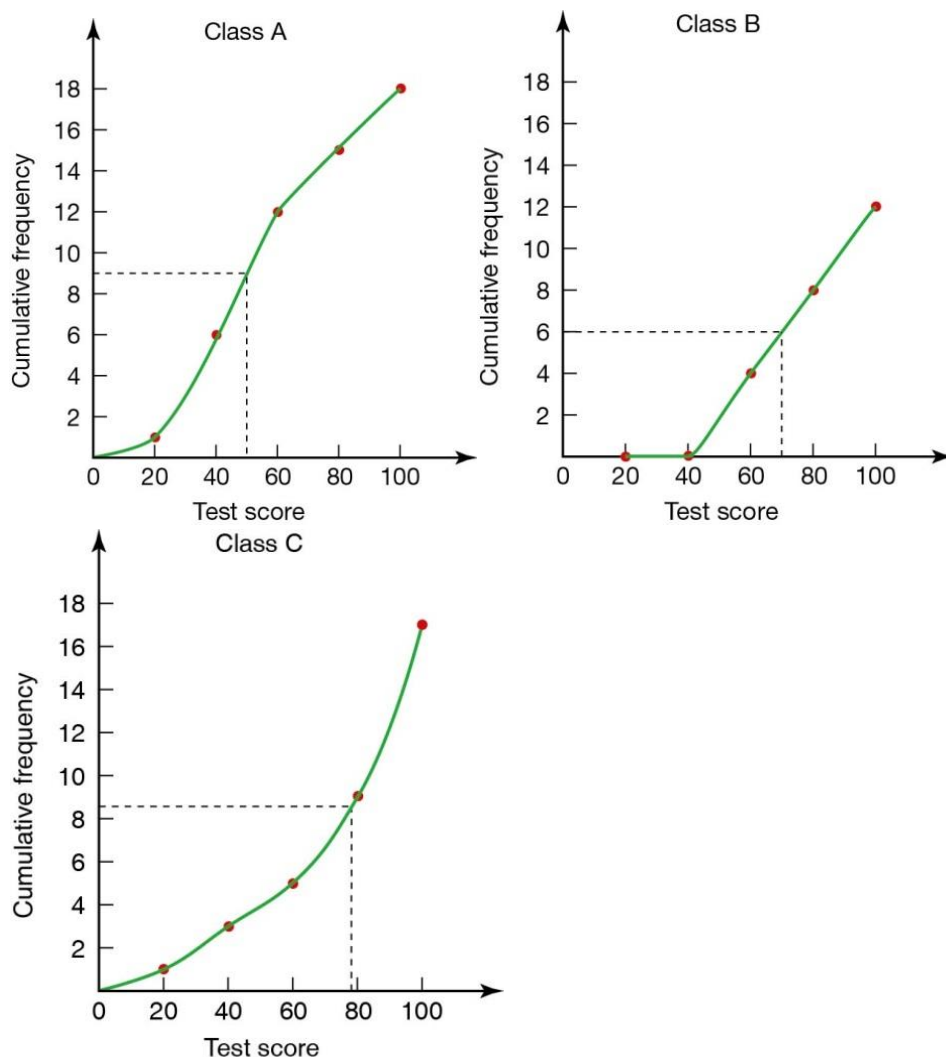
d 50% of athletes took longer than 1.8 hours and 50% took less than 1.8 hours.

2 a

| Score             | Class A |            | Class B |            | Class C |            |
|-------------------|---------|------------|---------|------------|---------|------------|
|                   | Freq.   | Cum. freq. | Freq.   | Cum. freq. | Freq.   | Cum. freq. |
| $0 \leq x < 20$   | 1       | 1          | 0       | 0          | 1       | 1          |
| $20 \leq x < 40$  | 5       | 6          | 0       | 0          | 2       | 3          |
| $40 \leq x < 60$  | 6       | 12         | 4       | 4          | 2       | 5          |
| $60 \leq x < 80$  | 3       | 15         | 4       | 8          | 4       | 9          |
| $80 \leq x < 100$ | 3       | 18         | 4       | 12         | 8       | 17         |



b



c Class A median  $\approx 50$

Class B median  $\approx 70$

Class C median  $\approx 78$

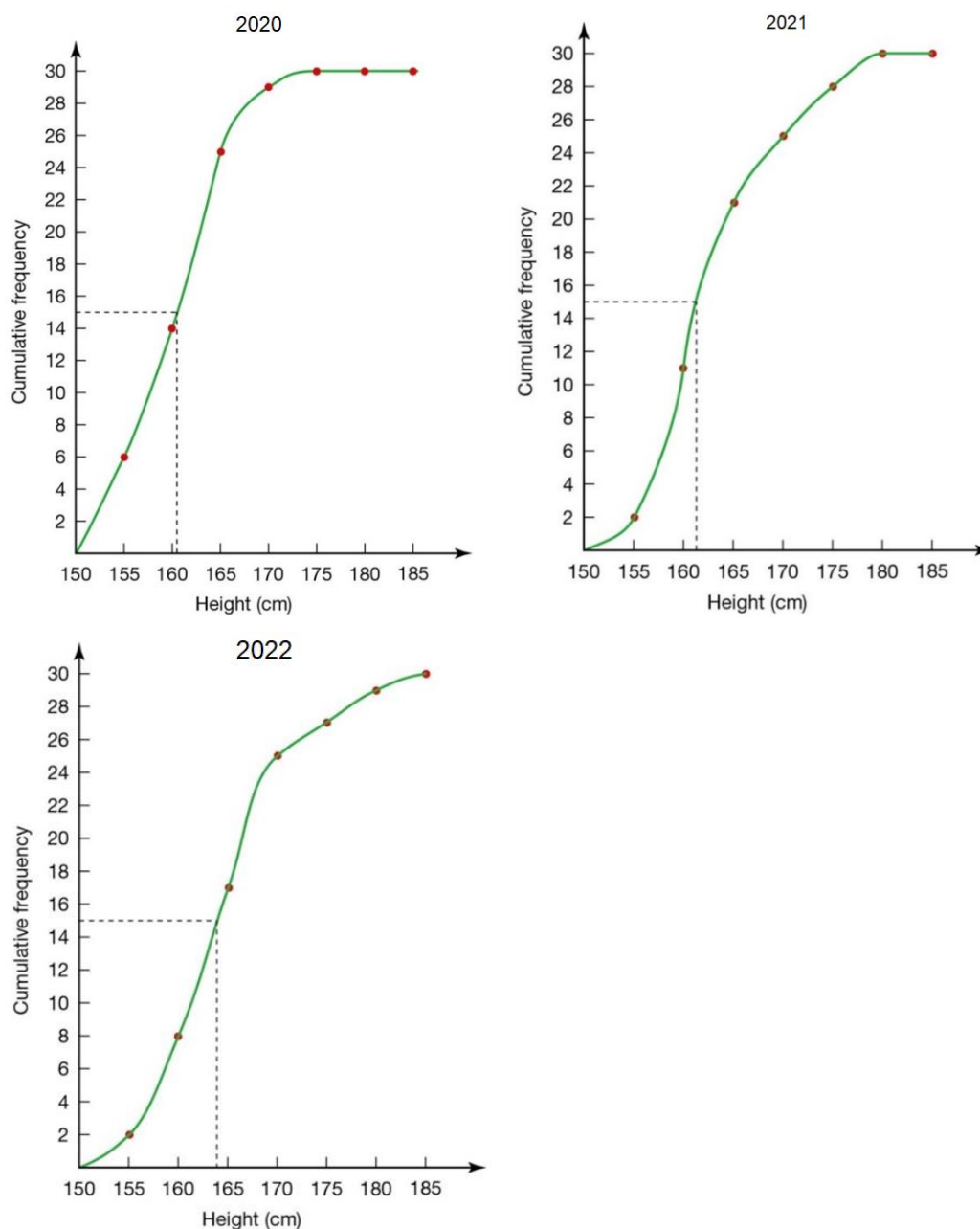
d Class C had a higher average test score.

3 a

| Height (cm)        | 2020  |            | 2021  |            | 2022  |            |
|--------------------|-------|------------|-------|------------|-------|------------|
|                    | Freq. | Cum. freq. | Freq. | Cum. freq. | Freq. | Cum. freq. |
| $150 \leq h < 155$ | 6     | 6          | 2     | 2          | 2     | 2          |
| $155 \leq h < 160$ | 8     | 14         | 9     | 11         | 6     | 8          |
| $160 \leq h < 165$ | 11    | 25         | 10    | 21         | 9     | 17         |
| $165 \leq h < 170$ | 4     | 29         | 4     | 25         | 8     | 25         |
| $170 \leq h < 175$ | 1     | 30         | 3     | 28         | 2     | 27         |
| $175 \leq h < 180$ | 0     | 30         | 2     | 30         | 2     | 29         |
| $180 \leq h < 185$ | 0     | 30         | 0     | 30         | 1     | 30         |



**b**



- c** Median (2007)  $\approx$  161 cm  
Median (2008)  $\approx$  162 cm  
Median (2009)  $\approx$  164 cm
- d** The average heights of the students increased by 3 cm over the three years.

## Exercise 37.2

- 1 a** Class A  $\approx$  30      Class B  $\approx$  30      Class C  $\approx$  40  
**b** Student's own responses
- 2 a** 2007  $\approx$  7 cm      2008  $\approx$  8 cm      2009  $\approx$  8 cm



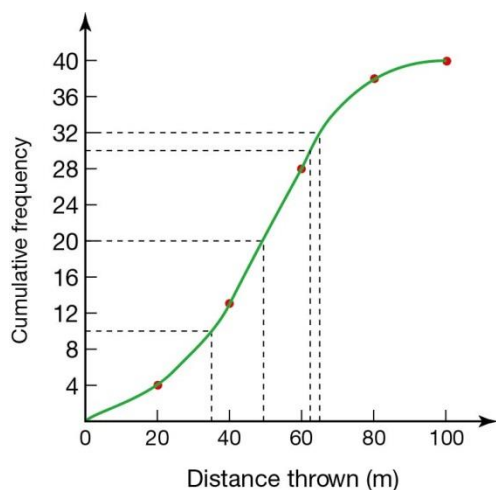
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b Student's own responses

3 a

| Distance thrown ( $d$ ) | $0 \leq d < 20$ | $20 \leq d < 40$ | $40 \leq d < 60$ | $60 \leq d < 80$ | $80 \leq d < 100$ |
|-------------------------|-----------------|------------------|------------------|------------------|-------------------|
| Frequency               | 4               | 9                | 15               | 10               | 2                 |
| Cumulative freq.        | 4               | 13               | 28               | 38               | 40                |

b



c Qualifying distance  $\approx 66$  m

d Interquartile range  $\approx 28$  m

e Median  $\approx 50$  m

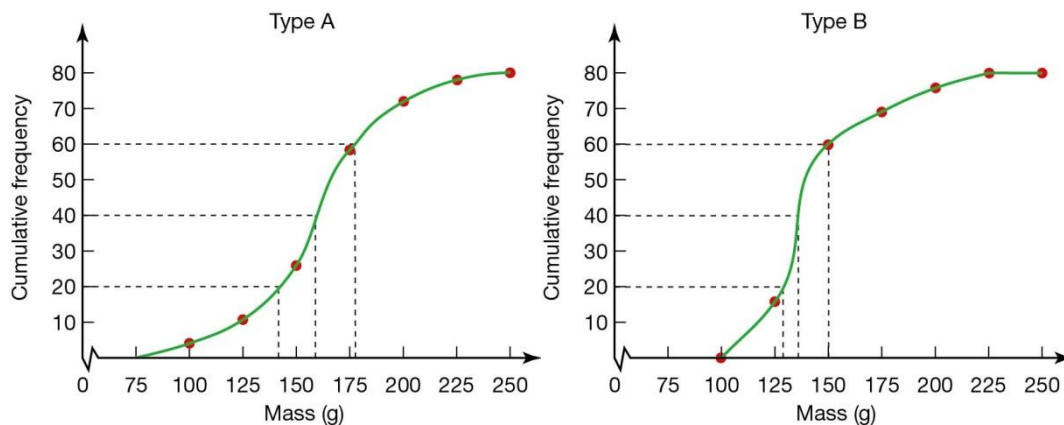
4 a

| Type A             |           |            |
|--------------------|-----------|------------|
| Mass (g)           | Frequency | Cum. freq. |
| $75 \leq m < 100$  | 4         | 4          |
| $100 \leq m < 125$ | 7         | 11         |
| $125 \leq m < 150$ | 15        | 26         |
| $150 \leq m < 175$ | 32        | 58         |
| $175 \leq m < 200$ | 14        | 72         |
| $200 \leq m < 225$ | 6         | 78         |
| $225 \leq m < 250$ | 2         | 80         |

| Type B             |           |            |
|--------------------|-----------|------------|
| Mass (g)           | Frequency | Cum. freq. |
| $75 \leq m < 100$  | 0         | 0          |
| $100 \leq m < 125$ | 16        | 16         |
| $125 \leq m < 150$ | 43        | 59         |
| $150 \leq m < 175$ | 10        | 69         |
| $175 \leq m < 200$ | 7         | 76         |
| $200 \leq m < 225$ | 4         | 80         |
| $225 \leq m < 250$ | 0         | 80         |



**b**



**c** Median type A  $\approx 157$  g

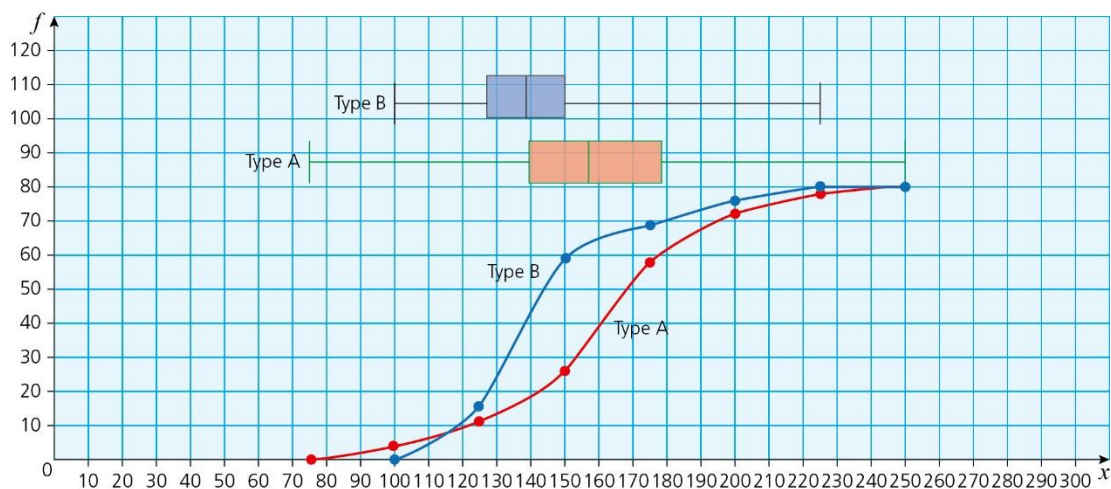
Median type B  $\approx 137$  g

**d i** Lower quartile type A  $\approx 140$  g      Lower quartile type B  $\approx 127$  g

ii Upper quartile type A  $\approx 178$  g      Upper quartile type B  $\approx 150$  g

iii Interquartile type range type A  $\approx 38$  g      Interquartile type range type B  $\approx 23$  g

**e**



The range in mass of type B oranges is smaller than that of type A. This can be seen in the overall length of the box-plots. The interquartile range of type B oranges is also smaller as shown in the size of the central box of the box plot. This implies that type B oranges are more consistent in mass. However, the median of type A oranges is greater as the central line is further to the right than that of type B oranges.

**5 a** Student's own explanation

**b** Student's own explanation

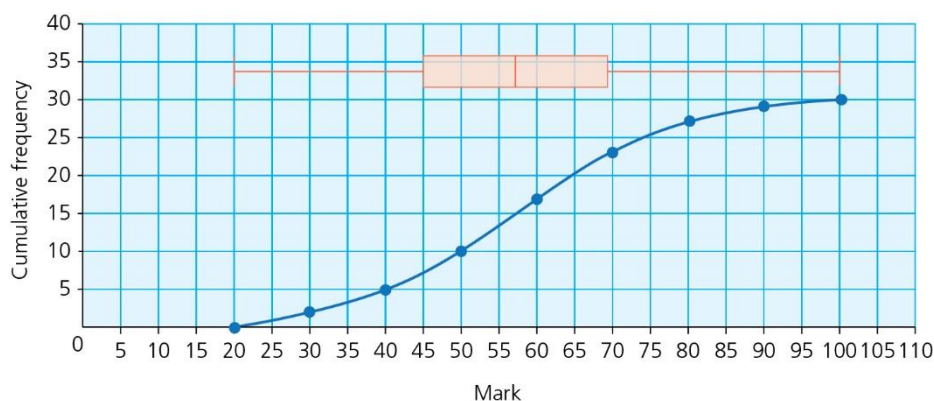


## Student assessment 1

1 a

| Mark (%)             | $20 \leq m < 30$ | $30 \leq m < 40$ | $40 \leq m < 50$ | $50 \leq m < 60$ | $60 \leq m < 70$ | $70 \leq m < 80$ | $80 \leq m < 90$ | $90 \leq m < 100$ |
|----------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|-------------------|
| Frequency            | 2                | 3                | 5                | 7                | 6                | 4                | 2                | 1                 |
| Cumulative frequency | 2                | 5                | 10               | 17               | 23               | 27               | 29               | 30                |

b, d



- c i Median  $\approx 57\%$   
 ii Lower quartile  $\approx 45\%$   
 Upper quartile  $\approx 69\%$   
 iii Interquartile range  $\approx 24\%$

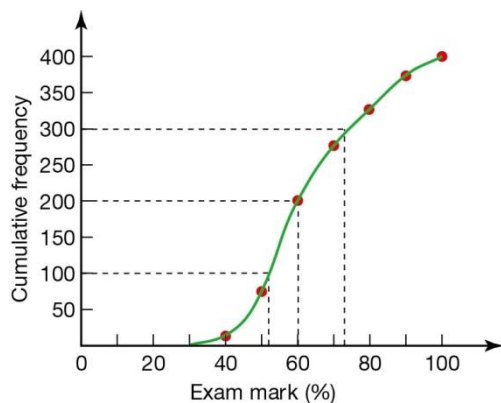
2 a

| Mark (%)          | Frequency | Cumulative frequency |
|-------------------|-----------|----------------------|
| $31 \leq m < 40$  | 21        | 21                   |
| $41 \leq m < 50$  | 55        | 76                   |
| $51 \leq m < 60$  | 125       | 201                  |
| $61 \leq m < 70$  | 74        | 275                  |
| $71 \leq m < 80$  | 52        | 327                  |
| $81 \leq m < 90$  | 45        | 372                  |
| $91 \leq m < 100$ | 28        | 400                  |



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**b**

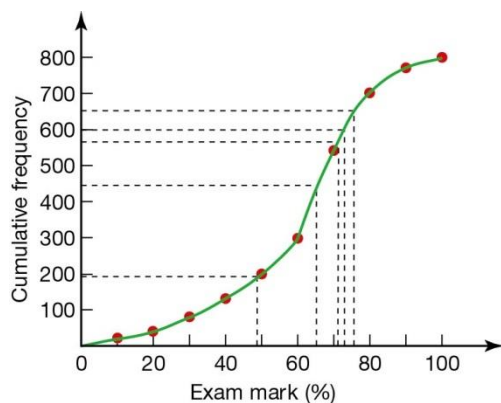


- c**
- i** Median  $\approx 60\%$
  - ii** Lower quartile  $\approx 52\%$   
Upper quartile  $\approx 73\%$
  - iii** Interquartile range  $\approx 21\%$

**3 a**

| Mark (%)          | Frequency | Cumulative frequency |
|-------------------|-----------|----------------------|
| $1 \leq m < 10$   | 10        | 10                   |
| $11 \leq m < 20$  | 30        | 40                   |
| $21 \leq m < 30$  | 40        | 80                   |
| $31 \leq m < 40$  | 50        | 130                  |
| $41 \leq m < 50$  | 70        | 200                  |
| $51 \leq m < 60$  | 100       | 300                  |
| $61 \leq m < 70$  | 240       | 540                  |
| $71 \leq m < 80$  | 160       | 700                  |
| $81 \leq m < 90$  | 70        | 770                  |
| $91 \leq m < 100$ | 30        | 800                  |

**b**



- c** 'A' grade  $\approx 75\%$



- d** Lower boundary  $\approx 66\%$   
Upper boundary  $\approx 72\%$
- e** Interquartile range  $\approx 25\%$

- 4**
- a** Graph D
  - b** Graph A
  - c** Graphs B and C
  - d** Graphs A and B



## Mathematical investigations and ICT 9

### Heights and percentiles

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- 1 Approx. 167 cm
- 2 Approx. 168 cm (Note: This corresponds to the 25th percentile not the 75th.)
- 3 Approx. 151 cm
- 4 Student's calculations could include mean, median, inter-quartile range and comparisons with printed charts.
- 5 It is likely that different cultures have different charts as some races are taller on average than others.

### Reading ages

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- 1 Possible answers include length of sentences, number of words with 3 or more syllables, size of type, etc.
- 2 Student's choices
- 3 Student's calculations
- 4 Students should choose articles on a similar topic. Ignore proper nouns. Choose more than one article from each paper.

### ICT activity

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- 1–6 Student's data, graph and comparisons